Plasma injection schemes for laser-plasma accelerators

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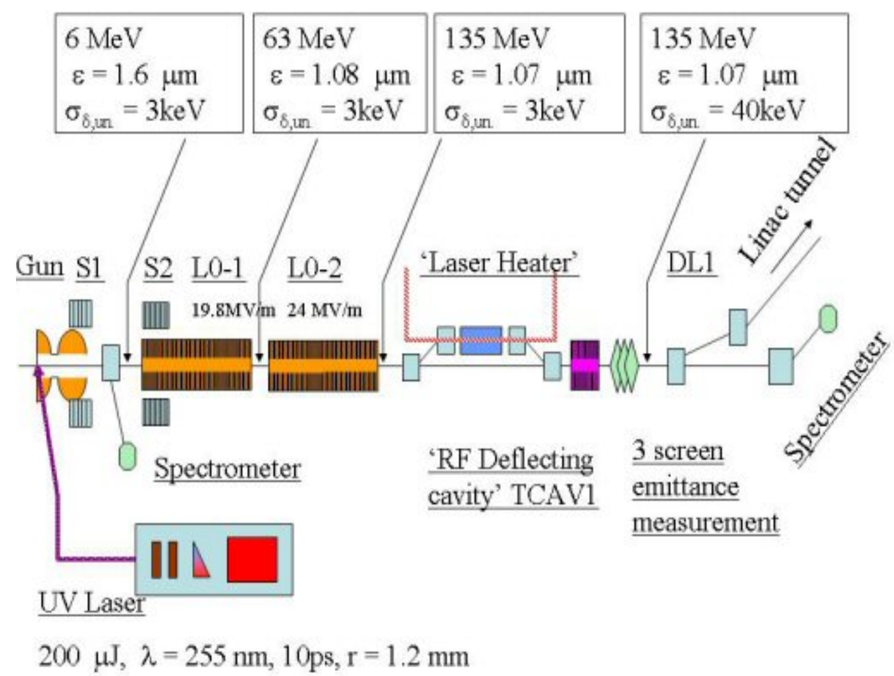
Motivations: why is injection so important ?

- Any high energy accelerator starts with an injector (MeV level)
- Injector strongly influences the performances of the accelerators: rep. rate, charge, beam quality (emittance, energy spread)

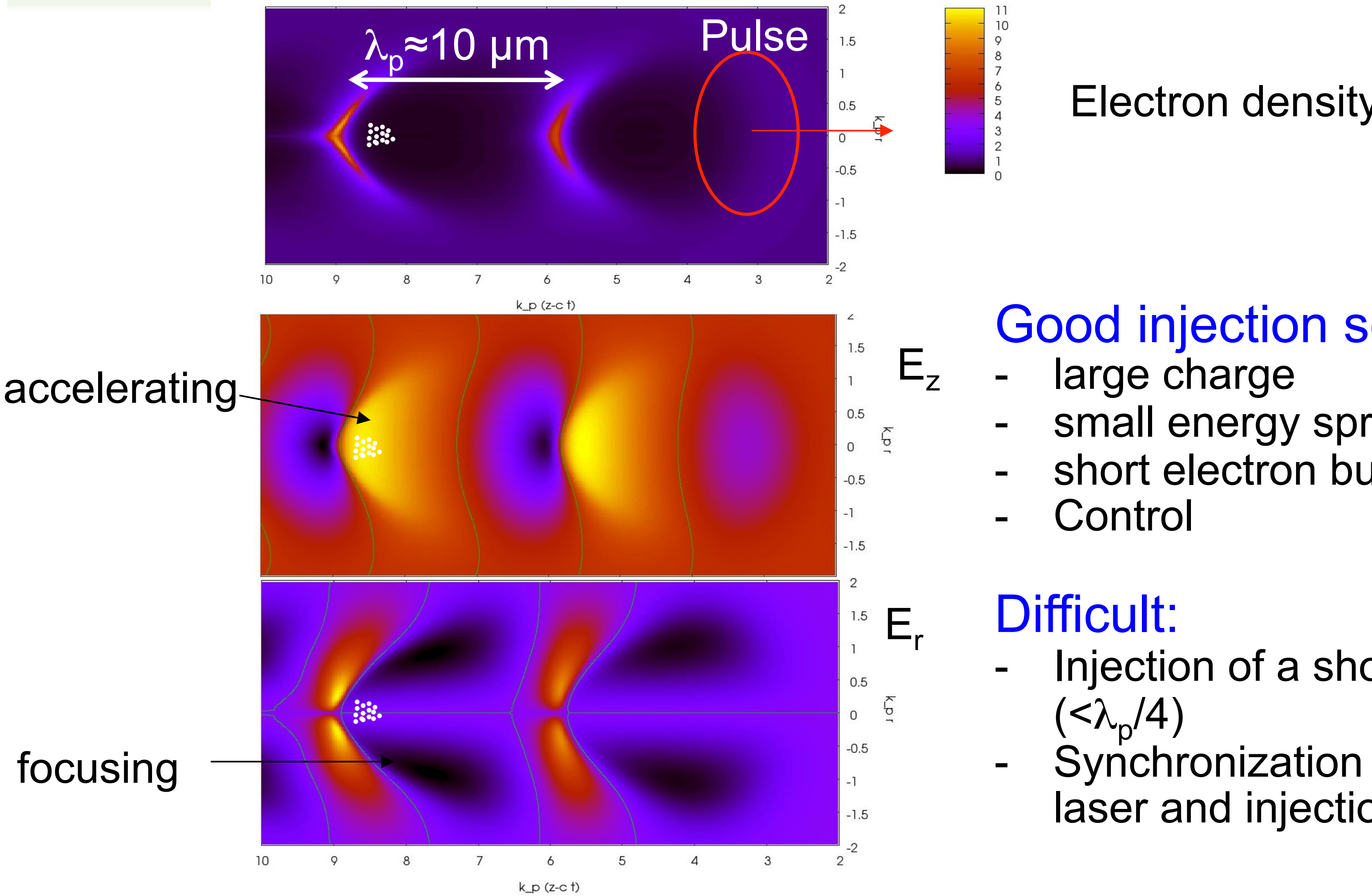
In a laser plasma accelerator, it is important to decouple the injection from the acceleration mechanism

- Better stability
- More control
- Possibility to tune the beam parameters independently
- Getting away from self-injection



LCLS photoinjector

How do we inject plasma electrons into wakefields ?



Good injection scheme:

- large charge
- small energy spread
- short electron bunch
- Control

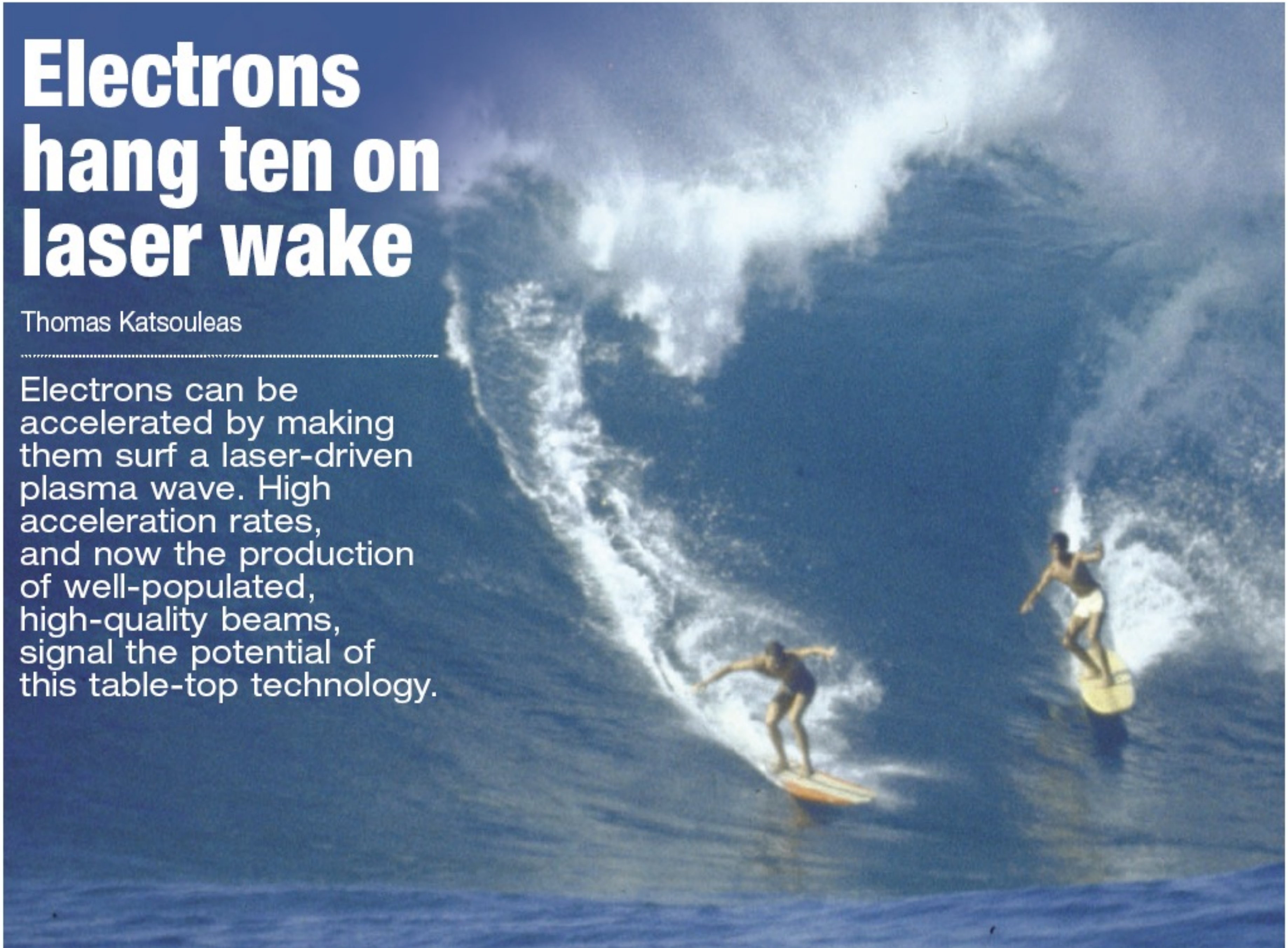
Difficult:

- Injection of a short bunch ($<\lambda_p/4$)
- Synchronization between laser and injection beam

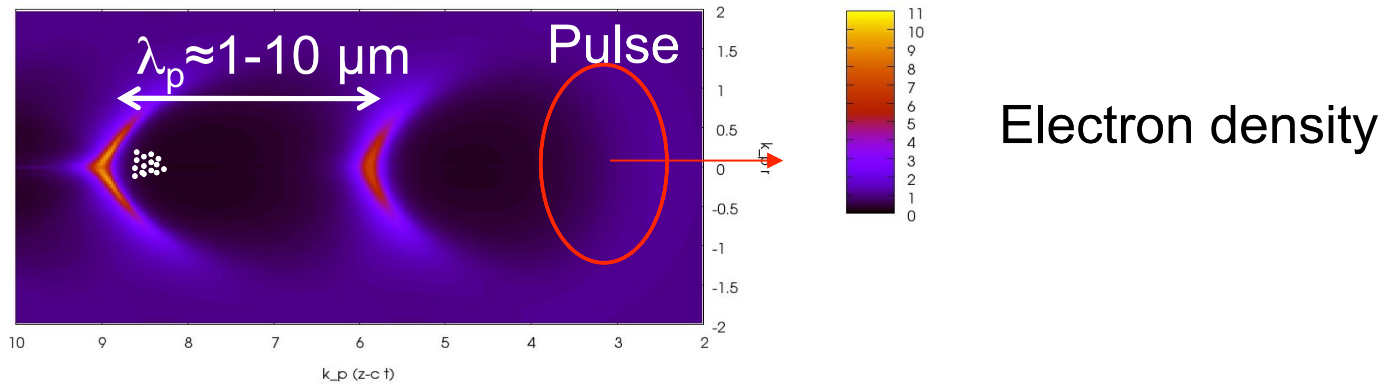
Electrons hang ten on laser wake

Thomas Katsouleas

Electrons can be accelerated by making them surf a laser-driven plasma wave. High acceleration rates, and now the production of well-populated, high-quality beams, signal the potential of this table-top technology.



How do we inject electrons into wakefields ?



Three fundamental methods for injecting electrons in the wake

- Give a initial kick to electron (paddling surfer) → colliding pulse injection
- Drop (dephase) electrons in the wake at the right phase → ionization injection
- Slow down the wakefield → injection in density gradient

Outline

- 1 D Hamiltonian model for electrons interacting with laser field and plasma wave
- Ionization injection
- Colliding pulse injection
- Injection by slowing down the wakefield:
  - Density gradient injection
  - Injection caused by laser pulse evolution

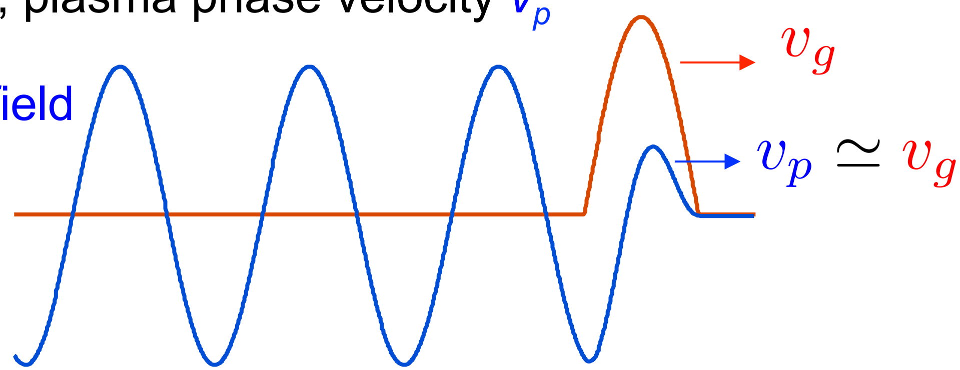
Summary / reminder of notation

- Intensity and normalized vector potential a (linear pol.) $$a = \frac{q_e A}{m_e c} = 8.5 \times 10^{-10} \lambda[\mu m] I^{1/2} [W/cm^2]$$

- Wakefield amplitude: comes from charge separation → define scalar potential for the plasma wave and its normalized counterpart

$$\Phi \to \phi = \frac{q_e \Phi}{m_e c^2}$$

- Laser group velocity v_g, plasma phase velocity v_p



- Define Lorentz factor

$$\gamma_p = \frac{1}{(1 - v_p^2/c^2)^{1/2}}$$

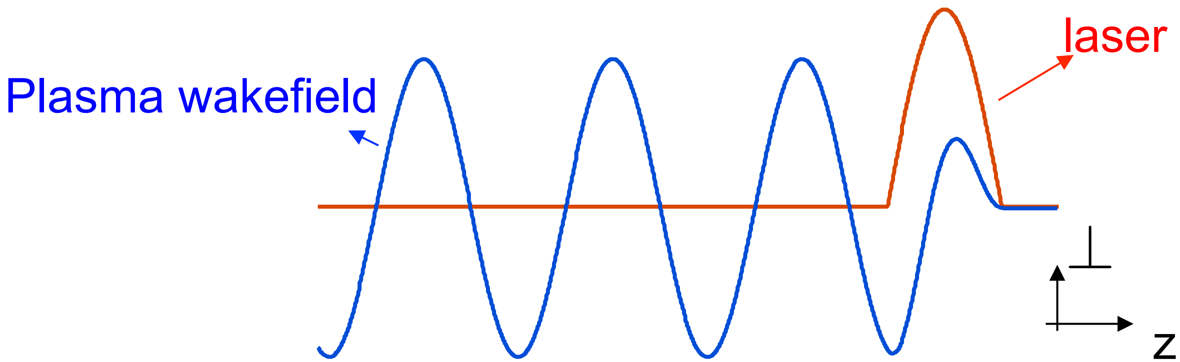
$$\beta_p = v_p/c$$

We use a 1D fluid model for the plasma wave

- We will start with a 1D model: we will only consider the motion of electrons along the longitudinal coordinates. We will neglect the role of the radial electric fields. In this case, the wakefield potential ϕ is only dependent on z and t.

- For simplicity, we will assume that the driver does not change during its propagation. A consequence of this is that the plasma wakefield is also stationary along the propagation. This is important because it will allow us to use a conservation of energy law.

Electron motion:
Transverse $p_\perp$
longitudinal p_z



1D Model: Plasma wave

- Low intensity limit ($a^2 \ll 1$), the potential is solution of $$\left(\frac{\partial^2}{\partial\zeta^2} + k_p^2\right)\phi = k_p^2\frac{\hat{a}^2}{4}$$

$\zeta = z - v_g t$ Co-moving coordinate

$k_p = \omega_p / v_g$ Plasma wave vector (ω_p plasma frequency)

- One can use a 1D nonlinear fluid theory which works for a > 1

$$\frac{\partial^2\phi}{\partial\zeta^2} = k_p^2\gamma_p^2\left[\beta_p\left(1 - \frac{1+a^2}{\gamma_p^2(1+\phi)^2}\right)^{-1/2} - 1\right]$$

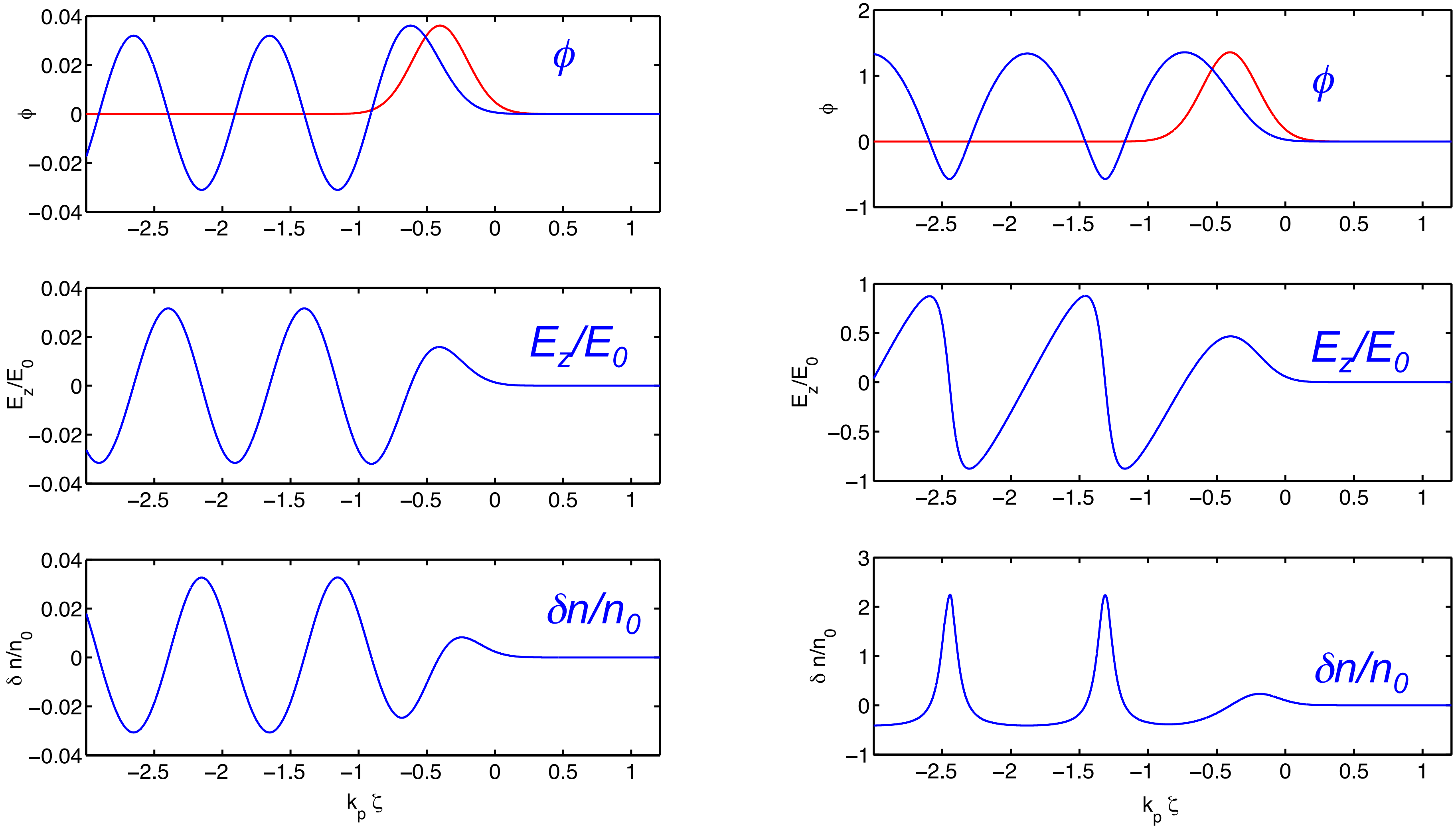
Integrate numerically this equation for a gaussian pulse

$$a(\zeta) = a_0 \exp(-\zeta^2/2L_0^2)\cos(k_0 z - \omega_0 t)$$

Example of 1D nonlinear plasma wave

$$\zeta = z - v_g t$$

a=0.3 a=2



Outline

- 1 D Hamiltonian model for electrons interacting with laser field and plasma wave

References:

E. Esarey and M. Pilloff, Phys. Plasmas **2**, 1432 (1995)

E. Esarey et al., IEEE Trans. Plasm. Sci. **24**, 252 (1996)

Hamiltonian of electron in laser and plasma wave with potential Φ

$$H = m_e c^2(\gamma - 1) - q_e \Phi(z - v_g t)$$

Let's normalize the Hamiltonian

kinetic energy potential energy

$$H = \gamma - \phi(z - v_g t) = \sqrt{1 + u_\perp^2 + u_z^2} - \phi(z - v_g t)$$

$$u_{\perp,z} = p_{\perp,z}/mc$$

- H depends on time but in a a particular manner (z-$v_g t$)
  - Eliminate time using a canonical transformation $(z,\, u_z) \rightarrow (\zeta, u_z)$
  - With generating function $F_2(z, u_z) = u_z \times (z - v_g t)$

- New Hamiltonian: $$H' = H + \frac{1}{c}\frac{\partial F_2}{\partial t}$$

Hamiltonian's basic properties

- New Hamiltonian is then: $H = \sqrt{1 + u_\perp^2 + u_z^2} - \phi(\zeta) - \beta_p u_z$
- Define the momentum conjugate to the position (or canonical momentum) $\mathbf{P} = \mathbf{p} + \mathbf{q}\mathbf{A}$
- In our case, *q*=-q_e and *A* is a transverse laser field. This translates in normalized units into

$$\mathcal{U}_\perp = u_\perp - a(\zeta)$$

- Hamiltonian expressed in terms of canonical momentum

$$H = \sqrt{1 + (\mathcal{U}_\perp + a)^2 + u_z^2} - \phi(\zeta) - \beta_p u_z$$

Hamiltonian's basic properties

$$H = \sqrt{1 + (\mathcal{U}_\perp + a)^2 + u_z^2} - \phi(\zeta) - \beta_p u_z$$

- From Hamilton's equations, one finds that in 1D the transverse canonical momentum is conserved (constant of motion)

$$\dot{\mathcal{U}}_\perp = -\frac{\partial H}{\partial r_\perp} = 0 \rightarrow u_\perp - a(\zeta) = Cste$$

- For electrons initially at rest in front of the laser pulse, *Cste*=0 and

$$\mathcal{U}_\perp = 0 \rightarrow u_\perp(\zeta) = a(\zeta)$$

Trajectories in the wakefield

$$H = \sqrt{1 + u_\perp^2 + u_z^2} - \phi(\zeta) - \beta_p u_z$$

- The Hamiltonian does not depend on time → constant of motion H_0

We want to find the electron trajectories in phase space: $u_z(\zeta)$

- Solving the Hamiltonian for u_z, one finds

$$(H_0 + \phi + \beta_p u_z)^2 = 1 + u_\perp^2 + u_z^2 = \gamma_\perp^2 + u_z^2$$

- 2nd degree polynomial equation with solution

$$u_z = \beta_p \gamma_p^2 (H_0 + \phi) \pm \gamma_p \sqrt{\gamma_p^2 (H_0 + \phi)^2 - \gamma_\perp^2}$$

(Exercise)

- If $a(\zeta)$ and $\phi(\zeta)$ and H_0 are known then the initial conditions are known the trajectory in phase space $u_z(\zeta)$ is known

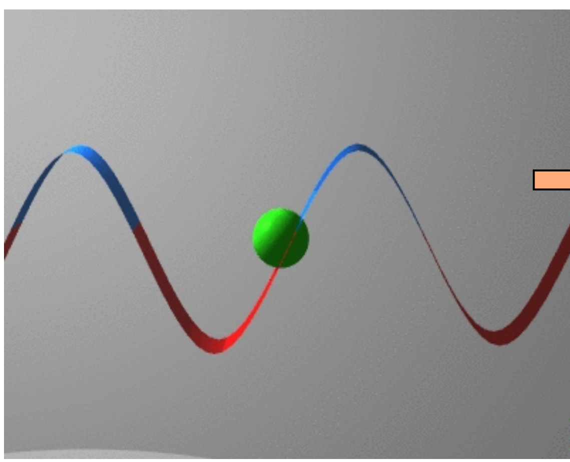
Fluid trajectories: electrons initially at rest in front of the laser

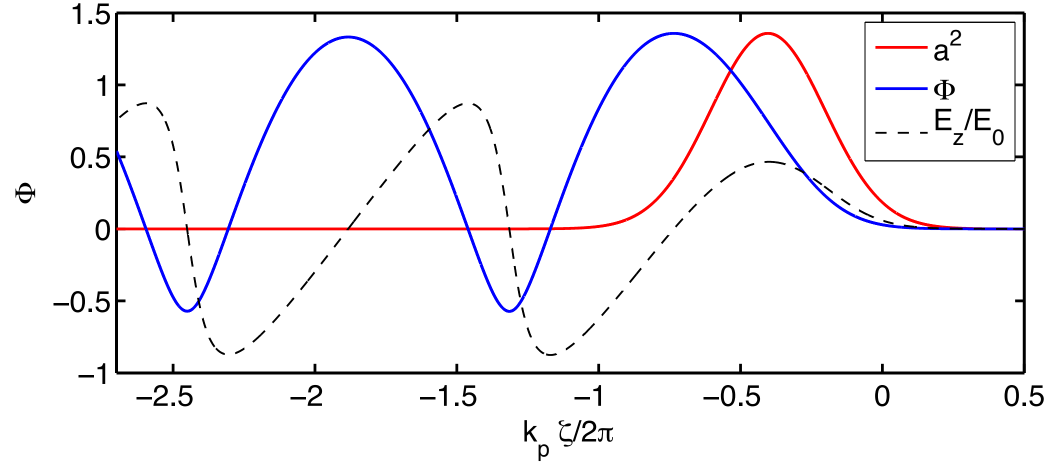
$$\zeta_i = +\infty$$

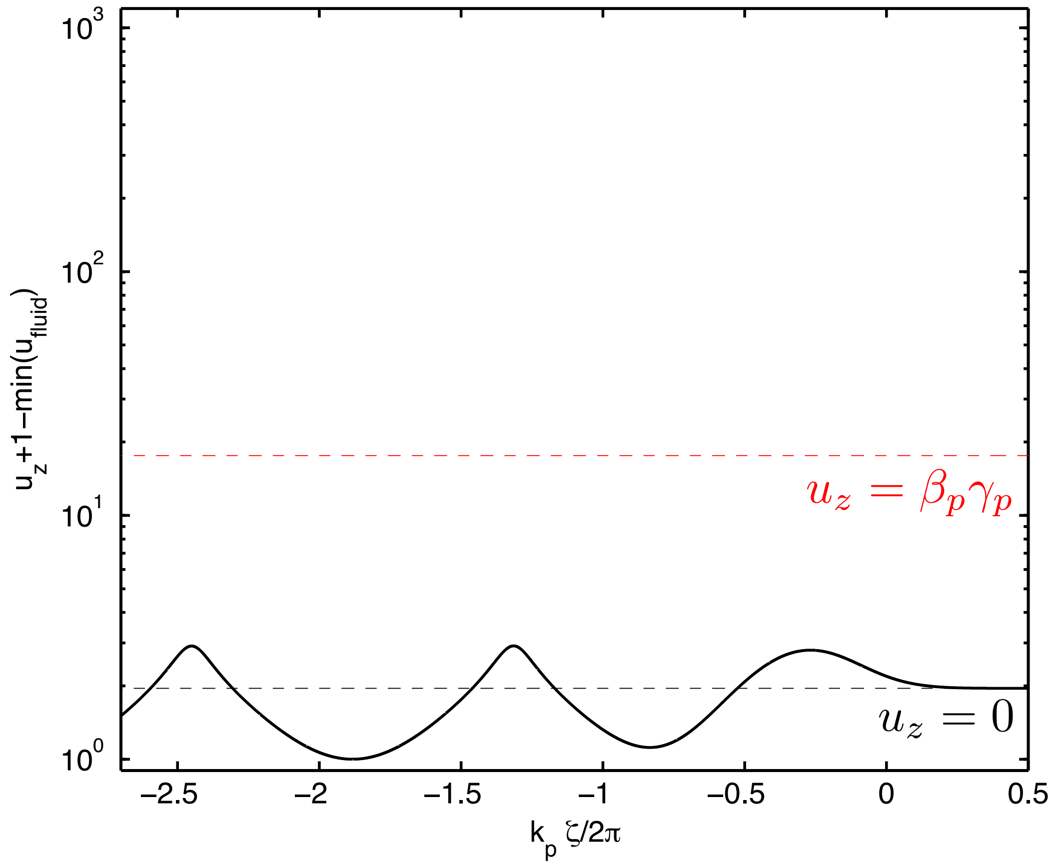
$$u_\perp(\zeta_i) = u_z(\zeta_i) = 0$$

$$H_0 = 1$$

Fluid electrons





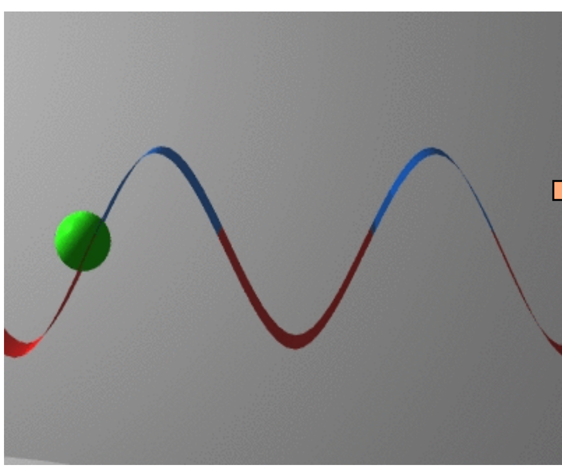


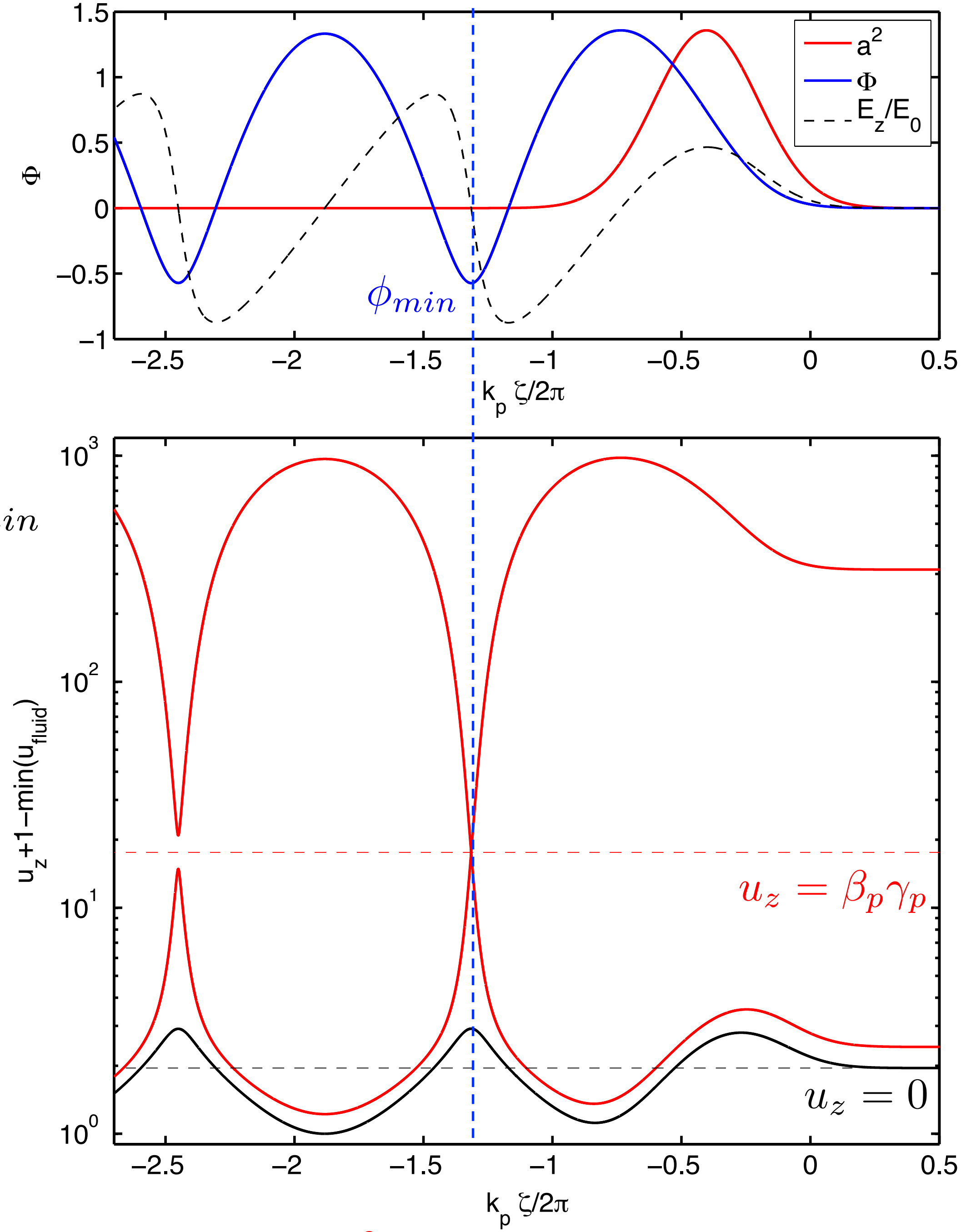
$a = 2,\ n_e/n_c = 0.44\%,\ \lambda = 0.8\mu\text{m},\ \tau = 20\text{fs}$

The separatrix: limit of trapped trajectories

$$\phi(\zeta_{min}) = \phi_{min}$$
$$E_z(\zeta_{min}) = 0$$
$$u_\perp(\zeta_{min}) = a(\zeta_{min})$$
$$u_z(\zeta_{min}) = \beta_p \gamma_p$$
$$H_0 = H_{sep} = \gamma_\perp(\zeta_{min})/\gamma_p - \phi_{min}$$

Trapped electrons=
Paddling surfer



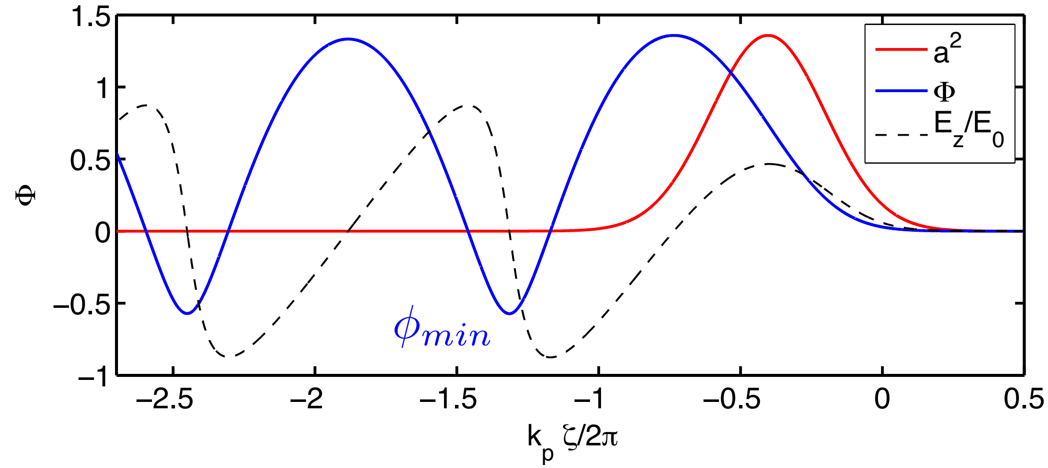


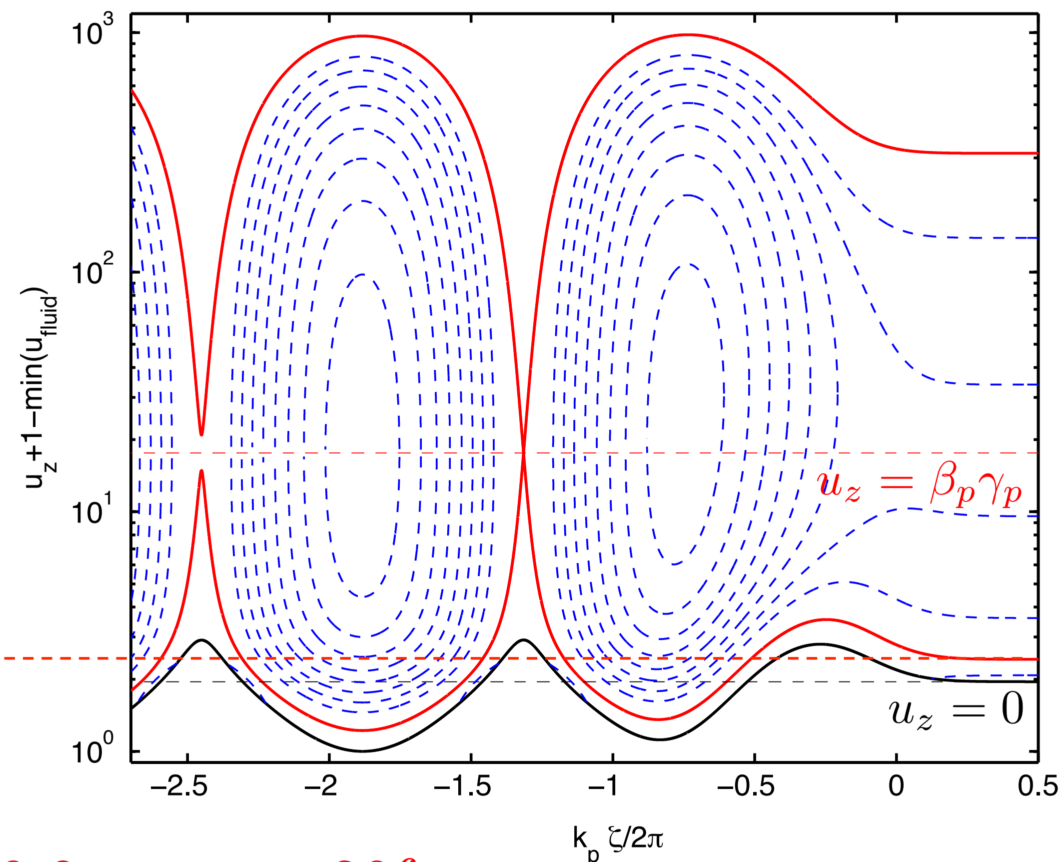
$a = 2,\ n_e/n_c = 0.44\%,\ \lambda = 0.8\mu\text{m},\ \tau = 20\text{fs}$

Trapped orbits

$$H_0 < H_{sep}$$

(Exercise)





Trapped electrons have initial kinetic energy

$$u_{z,sep}(\zeta = +\infty) > 0$$

$$a = 2,\ n_e/n_c = 0.44\%,\ \lambda = 0.8\mu\text{m},\ \tau = 20\text{fs}$$

Injection threshold

Calculate $u_{z,sep}(+\infty)$ and obtain minimum energy for trapping

$$E_{trap} = m_e c^2 (\sqrt{1 + u_{z,sep}^2(+\infty)} - 1)$$

We start from

$$u_z = \beta_p \gamma_p^2 (H_0 + \phi) \pm \gamma_p \sqrt{\gamma_p^2 (H_0 + \phi)^2 - \gamma_\perp^2}$$

Electrons in front of the laser pulse and on the separatrix

$$H_0 = H_{sep} = \gamma_\perp(\zeta_{min})/\gamma_p - \phi_{min}$$

$$a = 0\,, \phi = 0$$

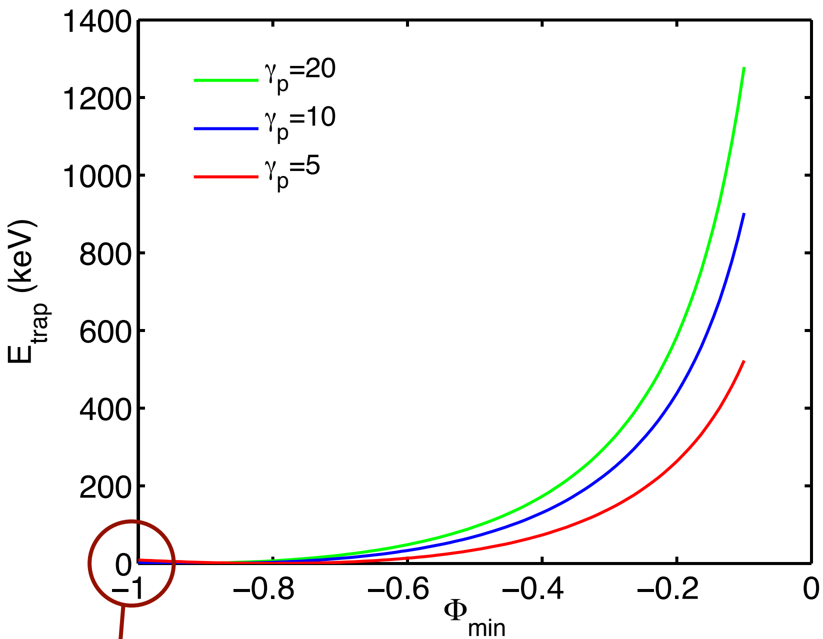
$$u_\perp = 0 \rightarrow \gamma_\perp^2 = 1 + u_\perp^2 = 1$$

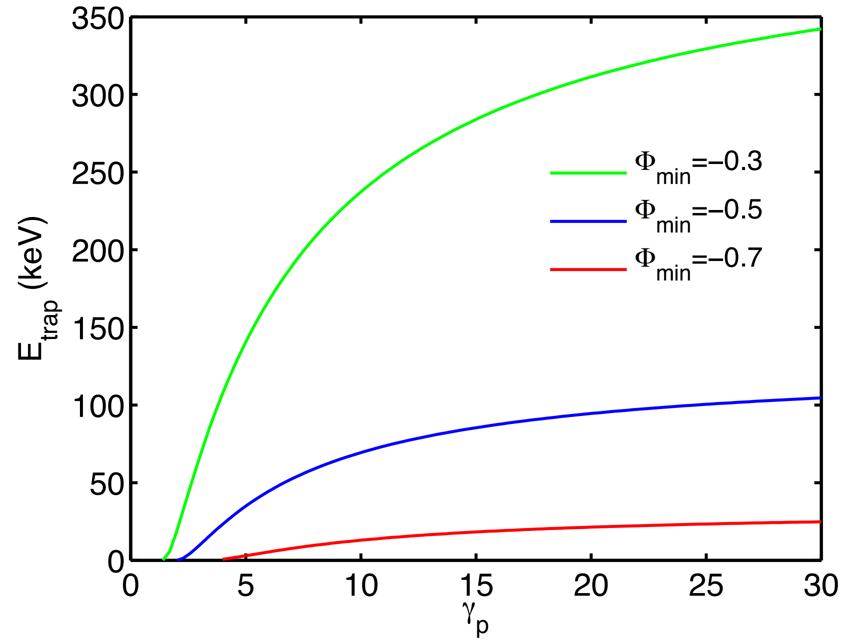
We can easily calculate

(Exercise)

$$u_{z,sep}(+\infty) = \beta_p \gamma_p^2 H_{sep} - \gamma_p \sqrt{\gamma_p^2 H_{sep}^2 - 1}$$

Injection thresholds





Larger wake amplitudes

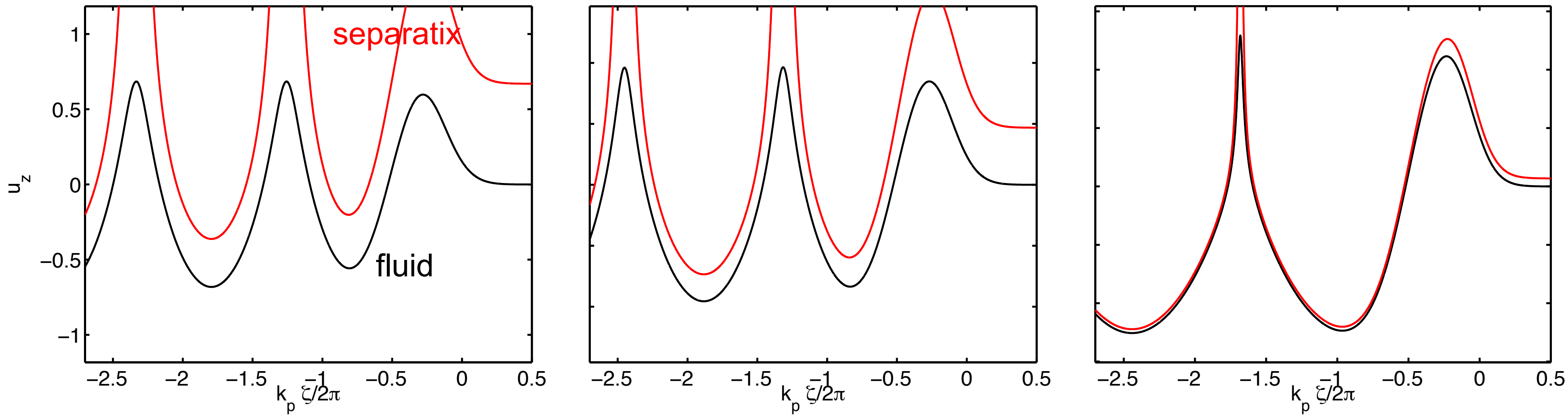
Slower wakes

Injection threshold is lower for large amplitude and slow wakefields

Wavebreaking ?

Wavebreaking as an injection mechanism

Zoom on the fluid trajectory



$a = 1.6\,,\phi_{min} = -0.45$ $\qquad$ $a = 2\,,\phi_{min} = -0.57$ $\qquad$ $a = 4\,,\phi_{min} = -0.8$

- As the wake amplitude increases ($|\phi_{min}|$ increases), the fluid trajectory gets closer to the separatrix
- 1D wavebreaking occurs when fluid and separatrix overlap
- All plasma electrons are then injected and accelerated

Wavebreaking

Outline

- Ionization injection

References:

Experiments

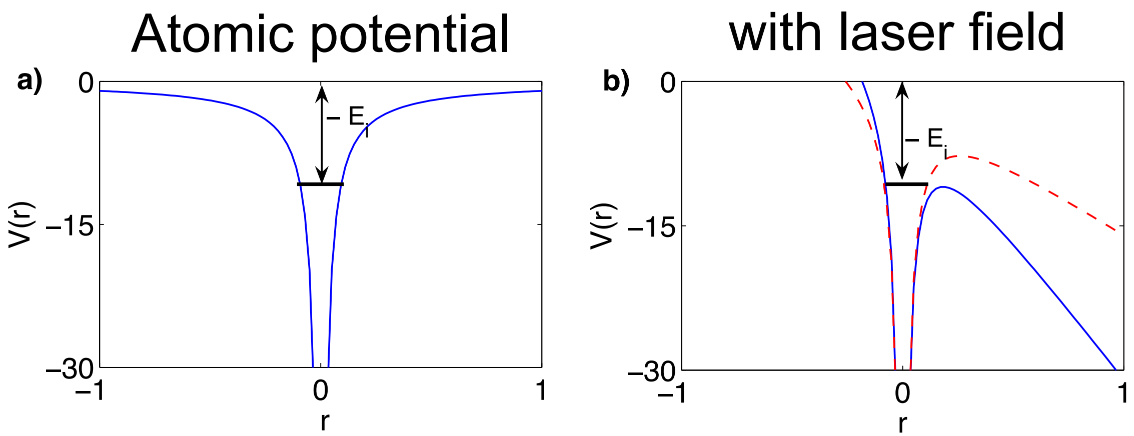
C. McGuffey et al., Phys. Rev. Lett. **104**, 025004 (2010)

A. Pak et al., Phys. Rev. Lett. **104**, 025003 (2010)

Theory

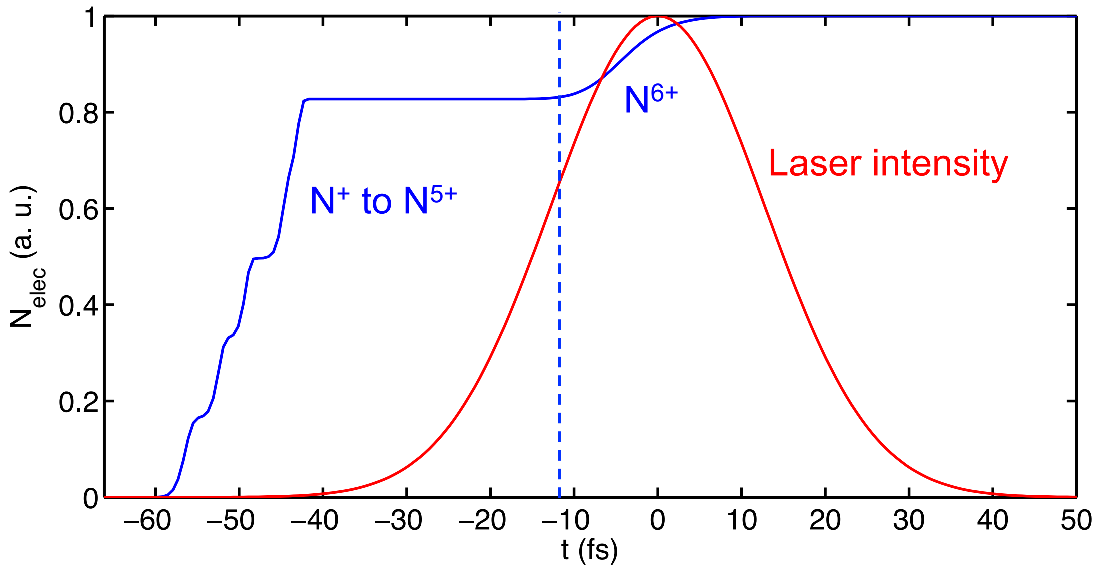
M. Chen et al., Phys. Plasmas **19**, 033101 (2012)

Ionization by barrier suppression



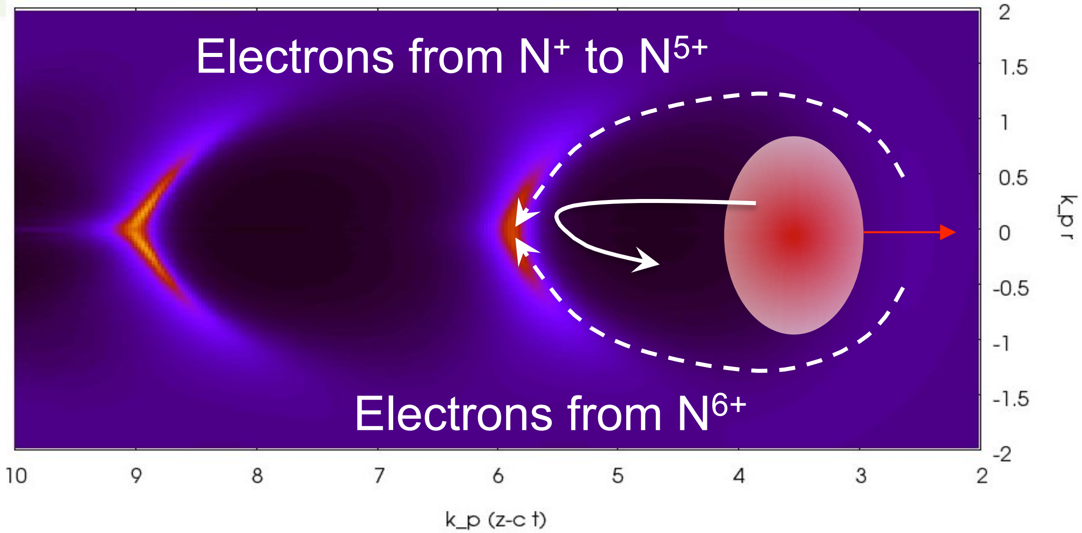
$$I[W/cm^2] = 4 \times 10^9 E_i^4[eV]/Z^{*2}$$

Example: Nitrogen
$I = 10^{19}$ W/cm^2



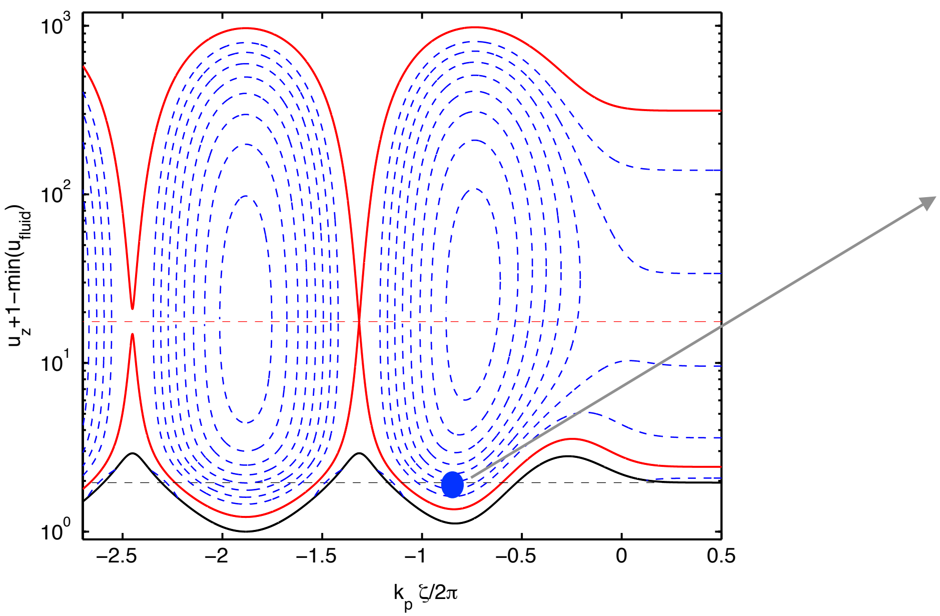
N^{6+} electrons are created
In the middle of the laser pulse

Principle of ionization injection



Ionized electrons are born in the laser and in the wake itself

- They have different initial conditions compared to fluid electrons
- "Dropping" them at the right phase so they can be trapped



Main idea:
If we drop an electron at rest at this phase, it will be on a trapped orbit

Calculation of phase space trajectories for ionization injection

$$u_z = \beta_p \gamma_p^2 (H_0 + \phi) \pm \gamma_p \sqrt{\gamma_p^2 (H_0 + \phi)^2 - \gamma_\perp^2}$$

We just have to plug in the right initial conditions:

- Assume electrons are born at phase ζ_{ion}
- Born at rest $\rightarrow u_\perp(\zeta_{ion}) = u_z(\zeta_{ion}) = 0$
- Born close to the peak of the laser field a $\rightarrow a(\zeta_{ion}) \simeq 0$

$$\rightarrow \gamma_\perp(\zeta)^2 = 1 + u_\perp(\zeta)^2 = 1 + a(\zeta)^2$$

- Initial Hamiltonian $H = \sqrt{1 + u_\perp^2 + u_z^2} - \phi(\zeta) - \beta_p u_z$

$$H_{ion} = 1 - \phi(\zeta_{ion})$$

Trajectories of trapped electrons

$a = 2,\ n_e/n_c = 0.44\%$

Condition for trapping:

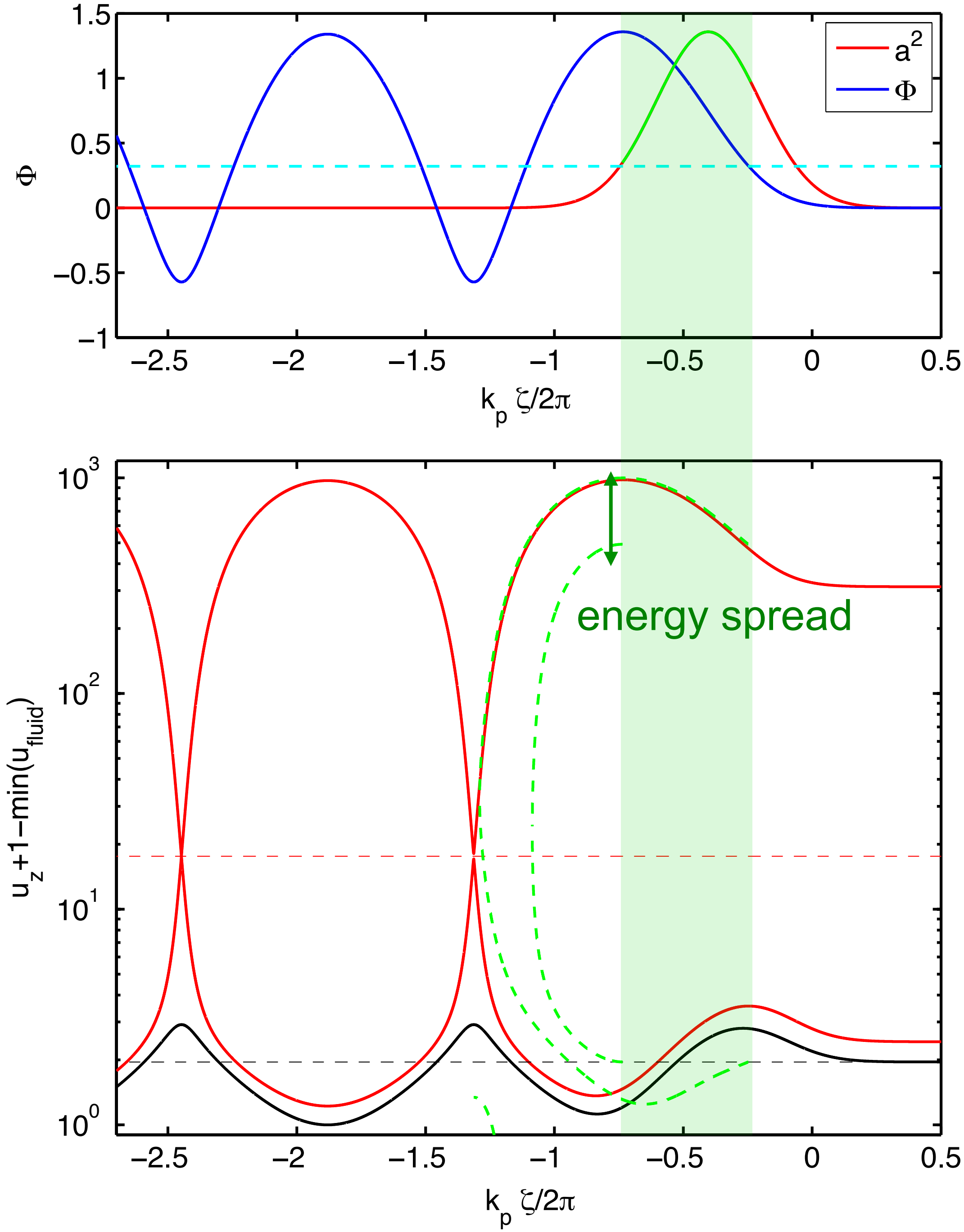
- Electrons should be ionized

  $a(\zeta_{ion}) > a_{threshold}$

- Initial condition for trapping:

  $H_{ion} < H_{sep}$

- Defines a trapping region in phase space



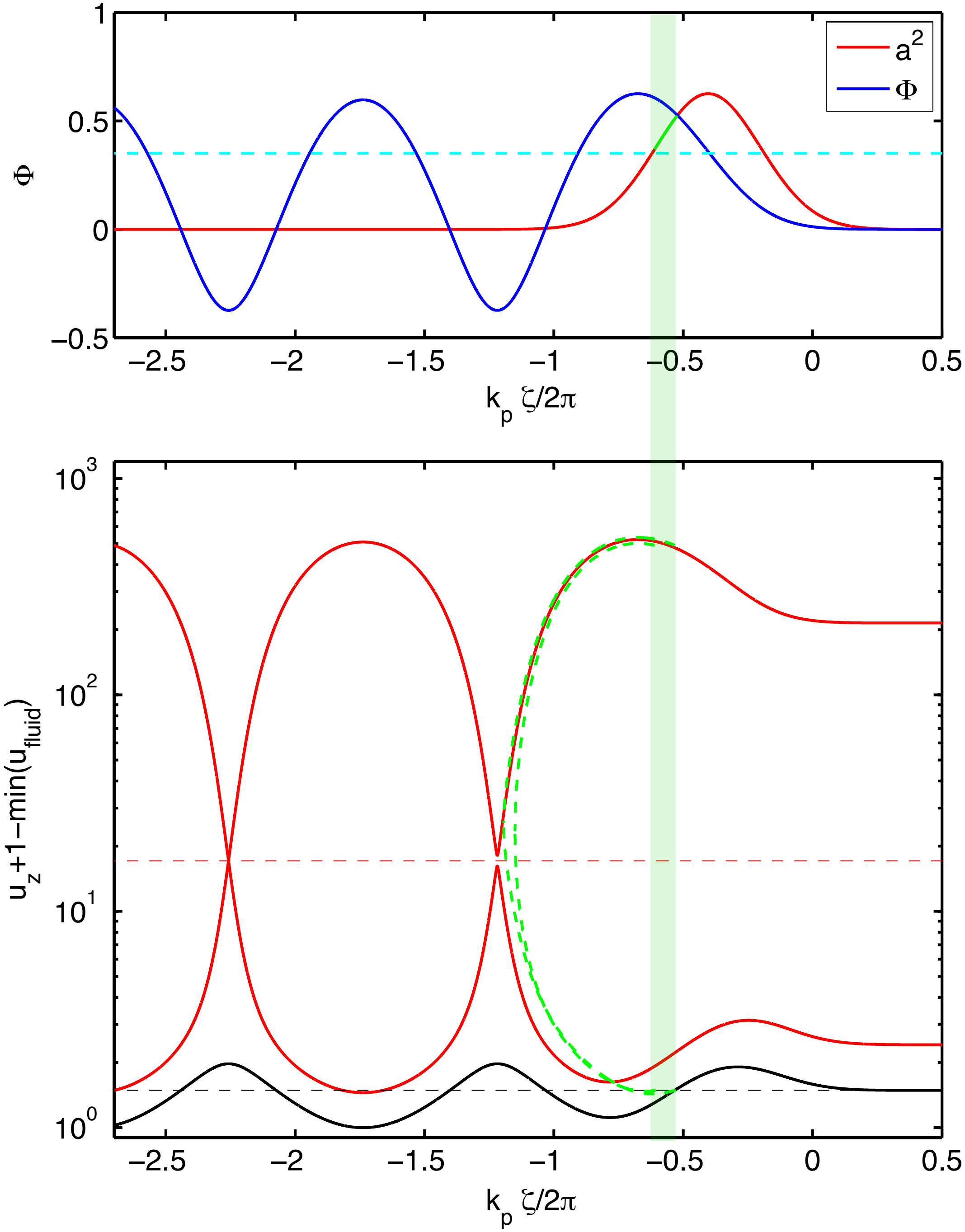
Same thing at lower laser intensity

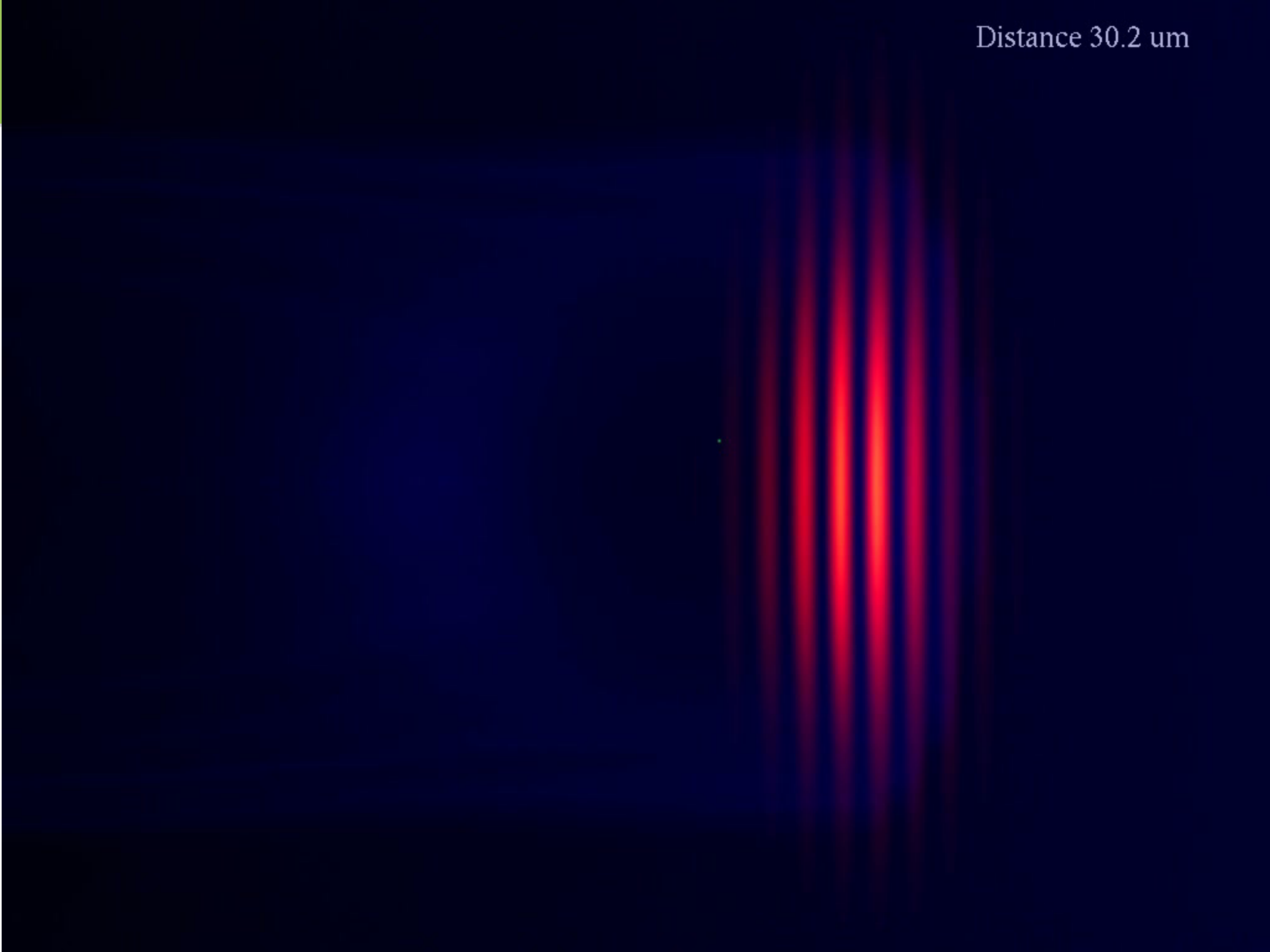
$a = 1.3,\ n_e/n_c = 0.44\%$

- Smaller trapping region
- Better energy spread

- Ionization injection requires typically

$a > 1$




Distance 30.2 um

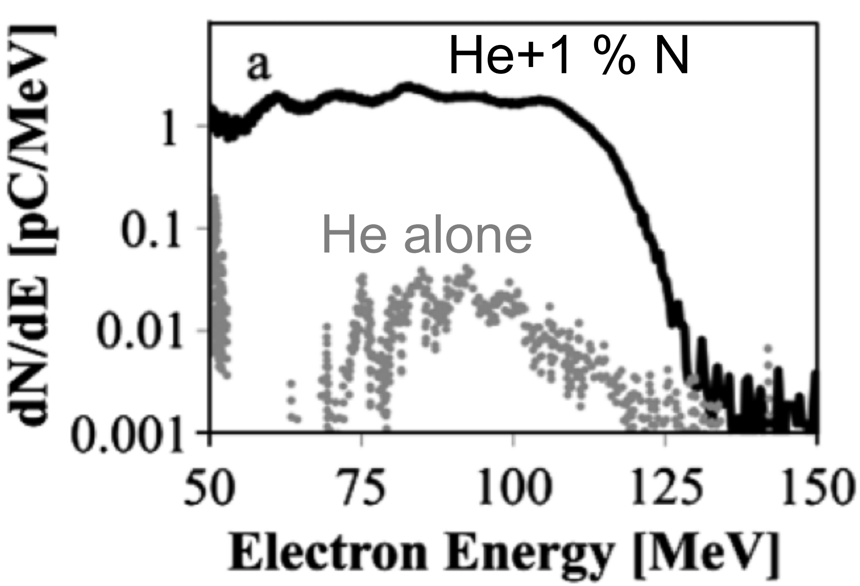
Example of experimental results

From C. McGuffey et al., Phys. Rev. Lett. **104**, 025004 (2010)

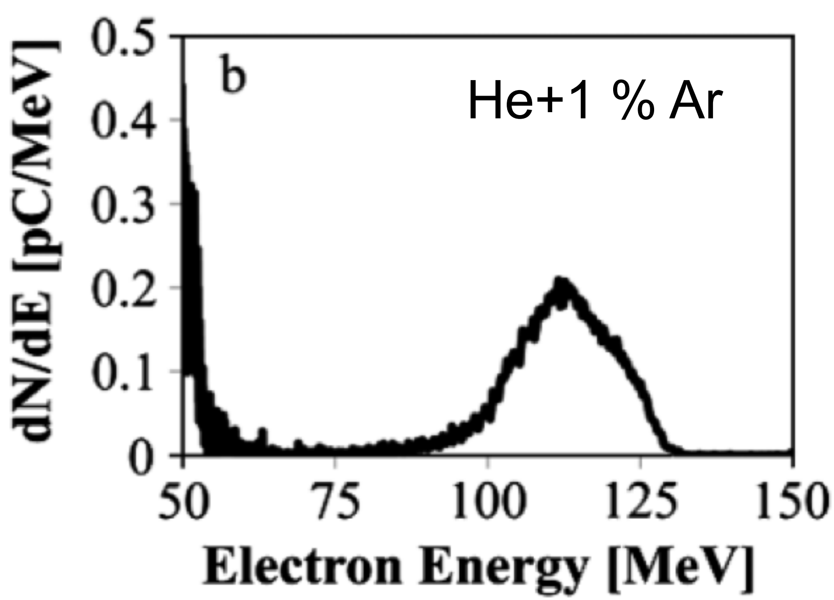
$n_e \approx 10^{19}$ cm^{-3}

Higher charge with ionization injection: trapping is easier

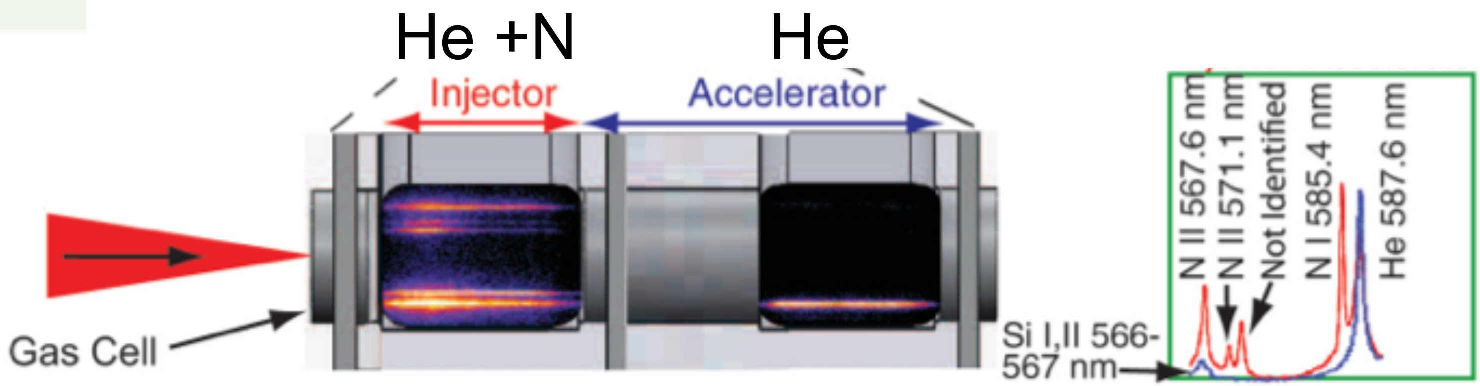
$I \approx 3.5 \times 10^{19}$ W/cm^2



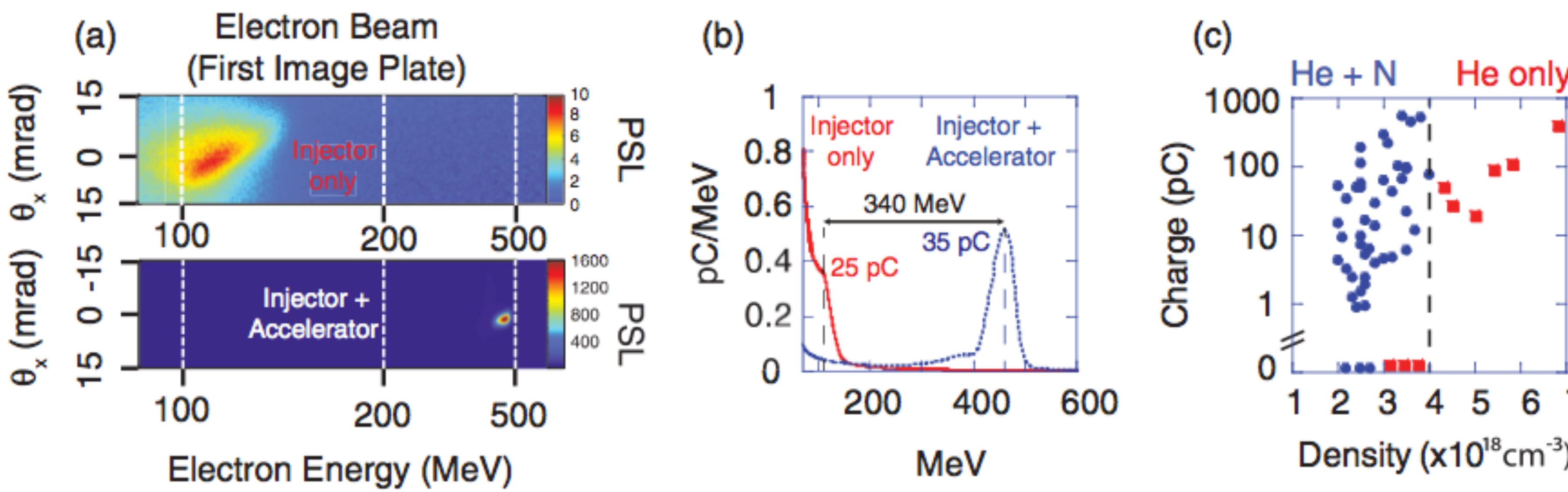
$I \approx 2.8 \times 10^{19}$ W/cm^2



Ionization injection as a injector for a 2 stage laser-plasma accelerator



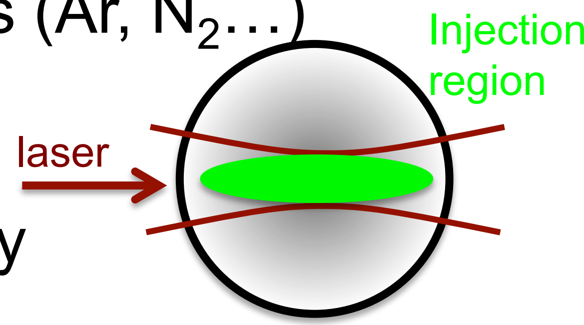
60 fs, 50-100 TW



From Pollock et al., Phys. Rev. Lett. **107**, 045001 (2011)

Ionization injection: conclusion

- Easy to implement: just add some high Z gas (Ar, N_2…)
- Good for increasing the charge



- Injection region is controlled by laser intensity
  - Difficult to control
  - Fluctuations in laser intensity directly impacts charge, energy spread, emittance …

  - Difficult to obtain high quality beams (high energy spread):
    - Requires to perform ionization in a very localized region (a small slice of plasma with a high Z gas)

→ Good research project: invent a device for ionization injection giving small energy spread (percent level)

Outline

- Colliding pulse injection

References:

Theory

Esarey et al. Phys. Rev. Lett. **79**, 2682 (1997)

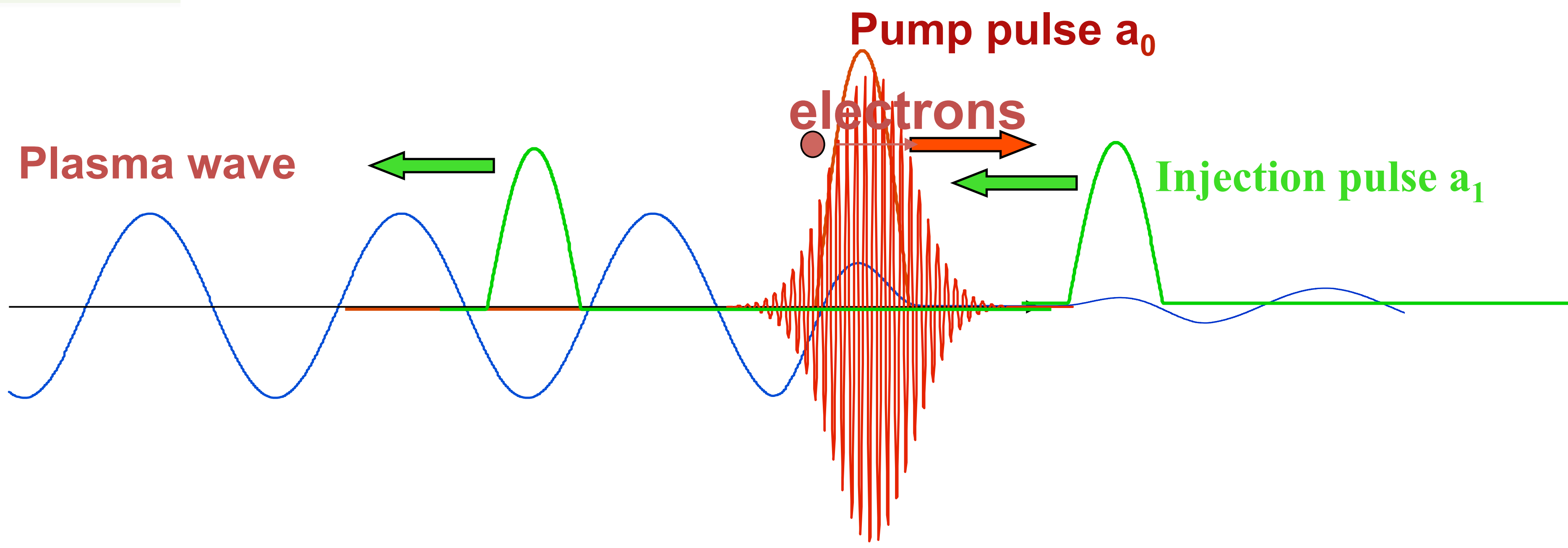
Fubiani et al., Phys. Rev. E **70**, 016402 (2004)

Experiments

Faure et al., Nature **444**, 737 (2006)

Rechatin et al., Phys. Rev. Lett. **102**, 164801 (2009)

Principle of colliding pulse injection



Ponderomotive force in the beatwave: $F_p \sim 2a_0a_1/\lambda_0$

The beatwave pre-accelerates electrons locally and injects them

INJECTION is local and short (30 fs) → monoenergetic beams

Hamiltonian in the laser beat wave

- Assume two counter propagating laser pulses a_0 and a_1

$$H_{beat} = \sqrt{1 + u_\perp^2 + u_z^2}$$
$$= \sqrt{1 + (\mathbf{a_0} + \mathbf{a_1})^2 + u_z^2}$$

(reminder conservation of canonical momentum)
$$\mathbf{u}_\perp = \mathbf{a}$$

- Assume same wavelength and circular polarization

$$\mathbf{a_0} = \frac{a_0}{\sqrt{2}}\left(\cos(k_0 z - \omega_0 t)\mathbf{e_x} + \sin(k_0 z - \omega_0 t)\mathbf{e_y}\right)$$
$$\mathbf{a_1} = \frac{a_1}{\sqrt{2}}\left(\cos(k_0 z + \omega_0 t)\mathbf{e_x} - \sin(k_0 z + \omega_0 t)\mathbf{e_y}\right)$$

$$(\mathbf{a_0} + \mathbf{a_1})^2 = \frac{a_0^2 + a_1^2}{2} + a_0 a_1 \cos(2k_0 z)$$

- Hamiltonian is conserved (does not depend on time)

Separatrix in the beatwave

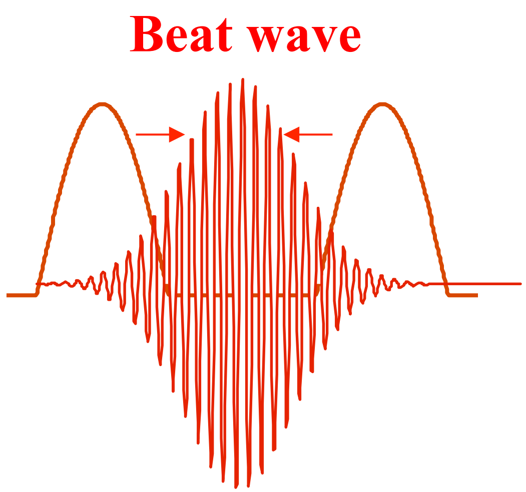
(Exercise)

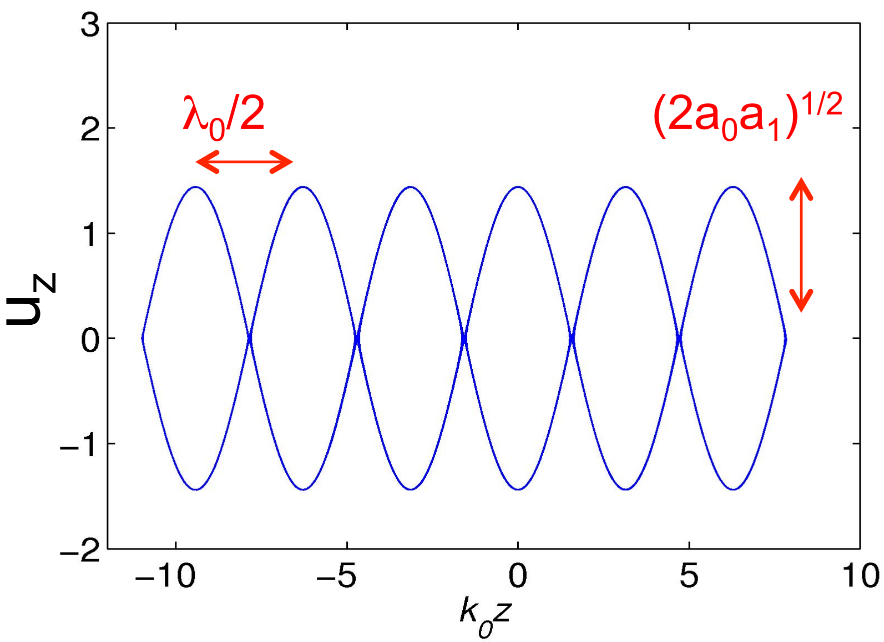
Solve for u_z with initial conditions

$$u_z = 0 \text{ for } z = 0$$

$$u_{beat,sep} = \pm\sqrt{a_0 a_1 (1 - \cos(2k_0 z))}$$

$$u_{beat,max} = \sqrt{2a_0 a_1}$$



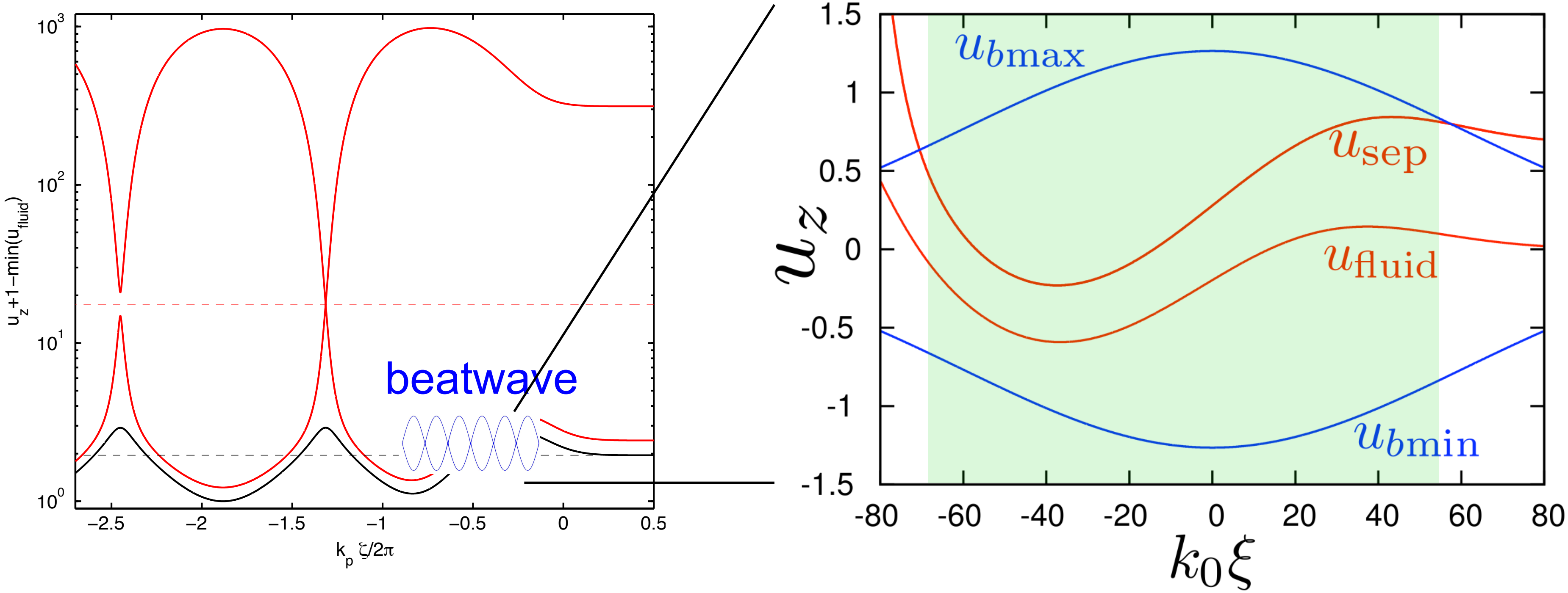


Typical energy gain in beatwave a_0=2, a_1=0.3

$E_{beat} \approx$ 250 keV

Injection condition:

Kick electrons from fluid to trapped orbits

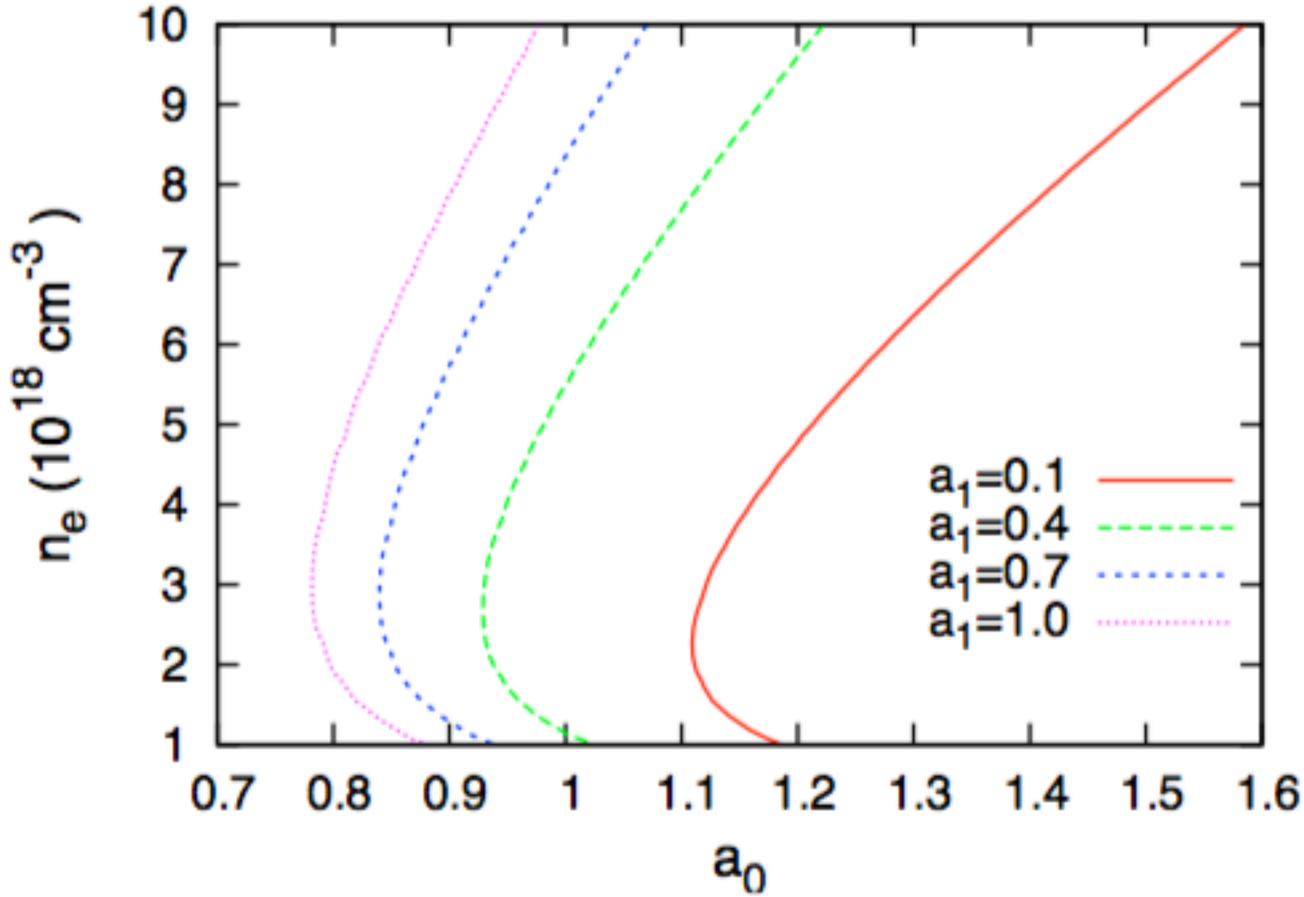


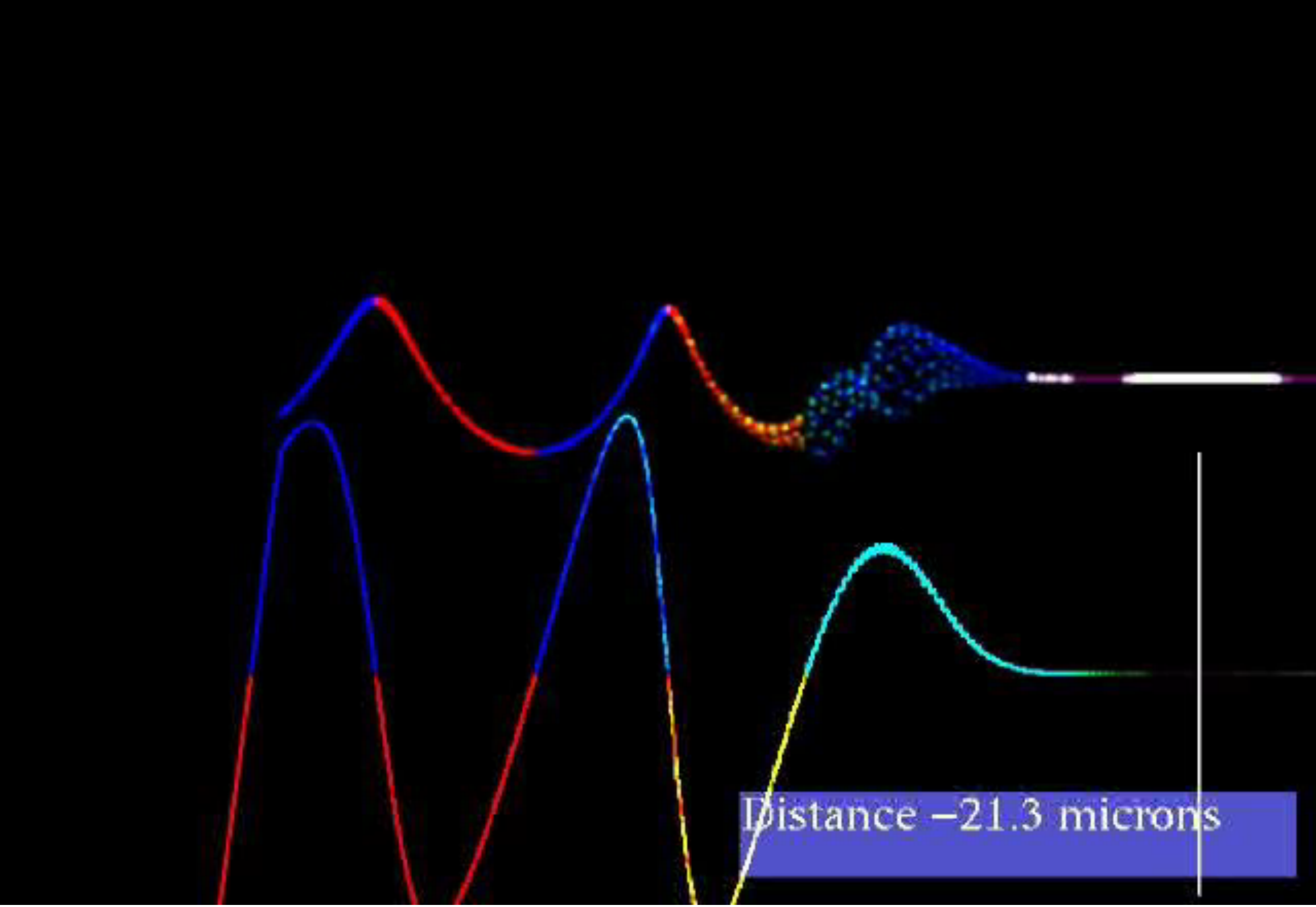
- Injection region defined for ζ such as

$$u_{beat,max}(\zeta) > u_{sep}(\zeta)$$

$$u_{beat,min}(\zeta) < u_{fluid}(\zeta)$$

Thresholds for colliding pulse injection for a 30 fs laser pulse

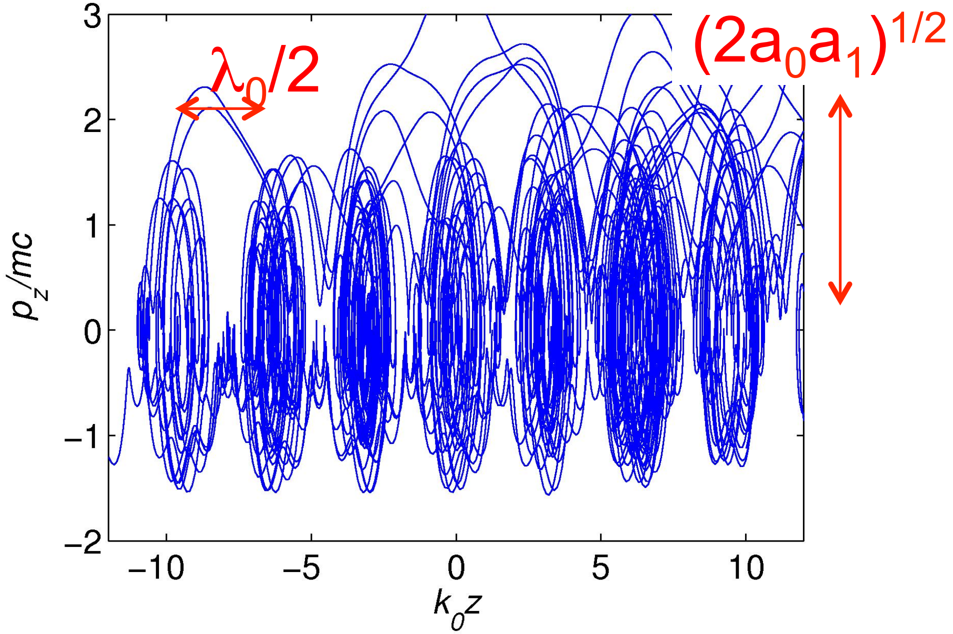



Distance –21.3 microns

Things we did not mention

- With linear polarization, electron motion in the beat wave is chaotic

- Heating is more efficient with linear pol. (see experiments)

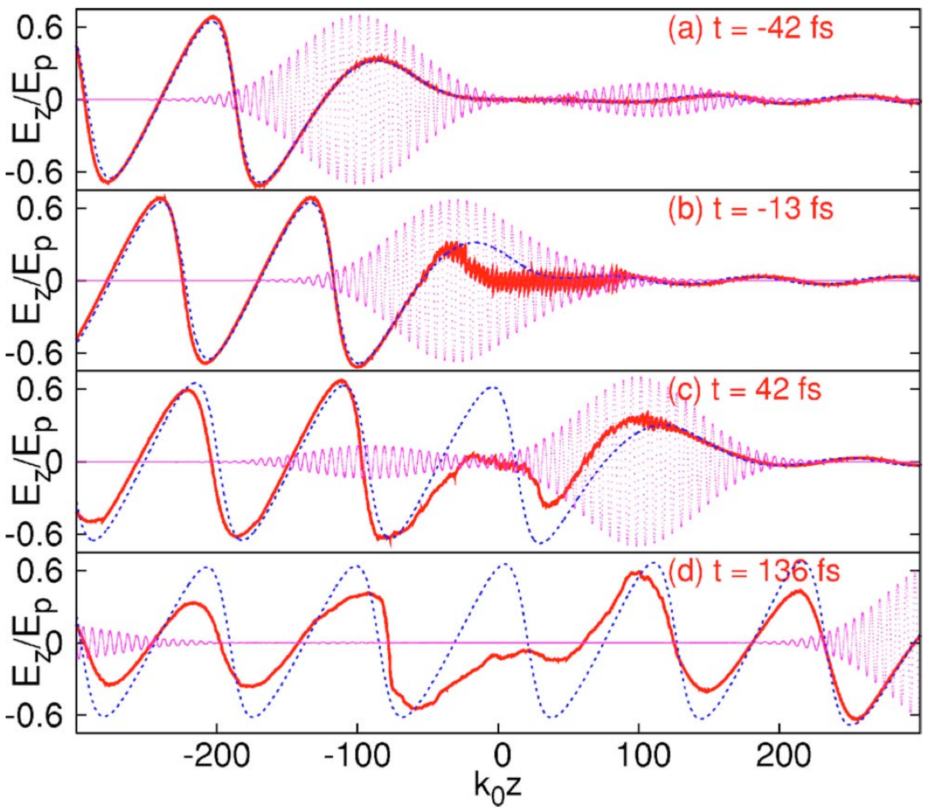


- The wakefield is inhibited during the collision

Things we did not mention (2)

- The wakefield is inhibited during the collision

Rechatin et al., PoP 14, 060702 (2007)

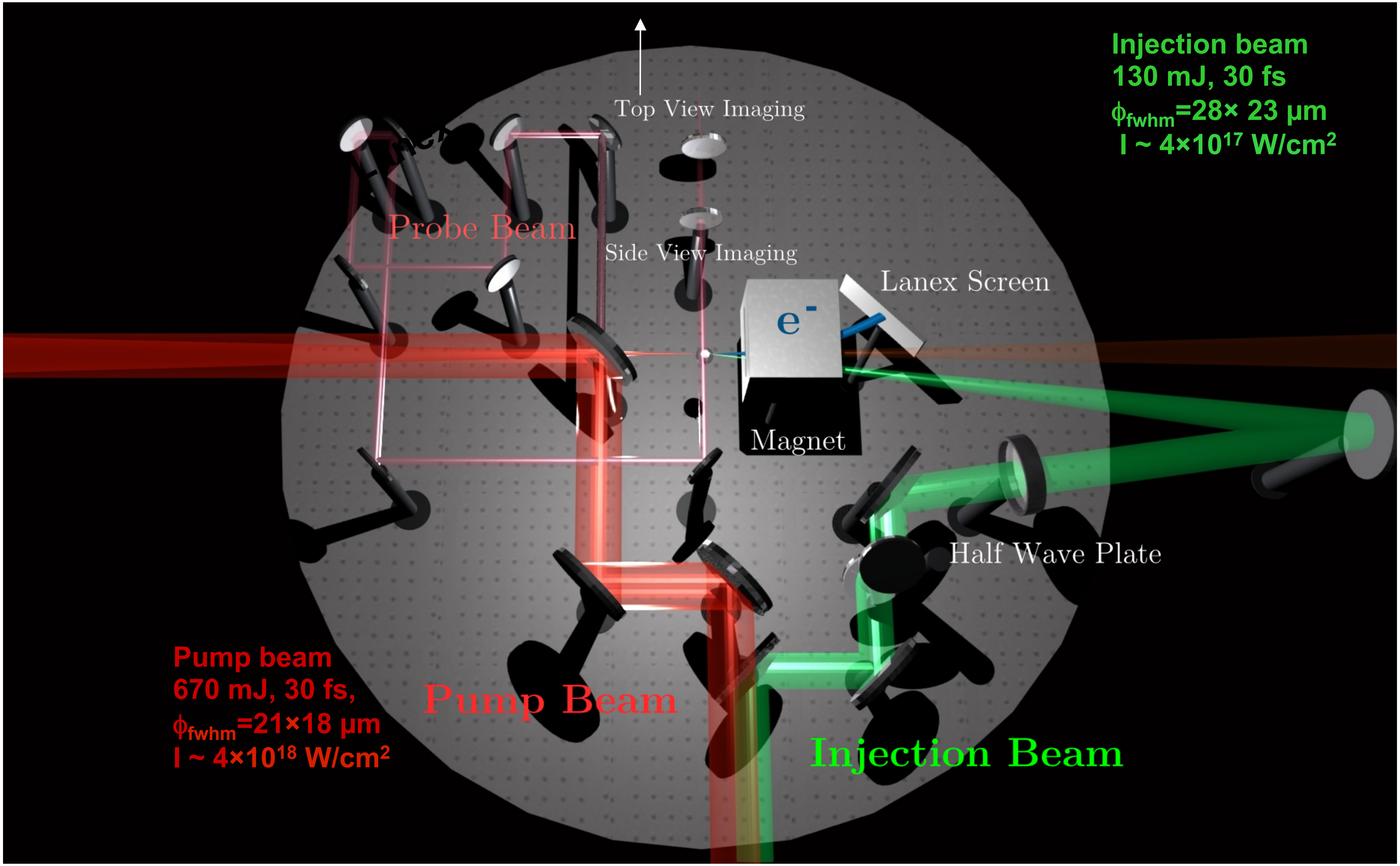


- Physics is 3D

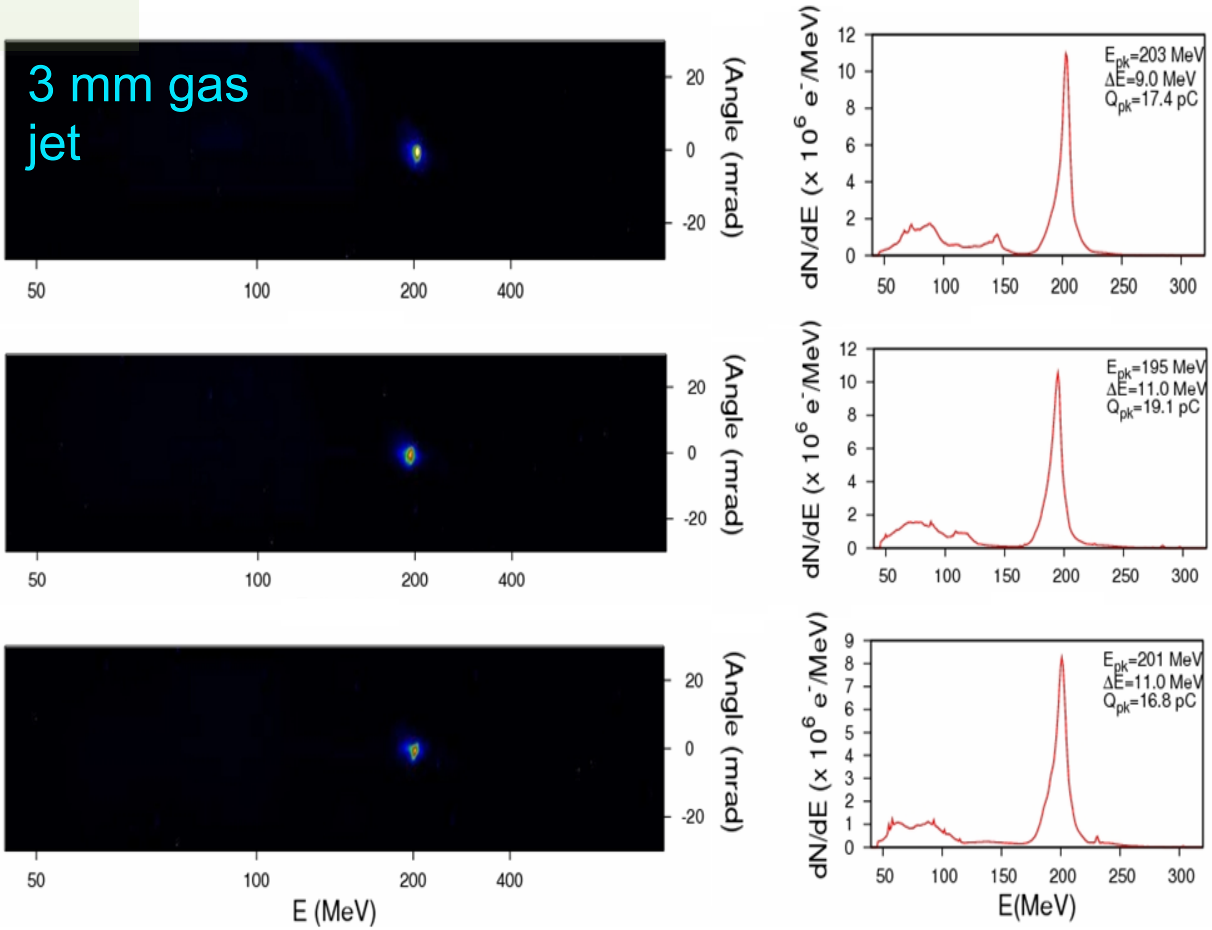
Davoine et al., PRL 102, 065001 (2009)

Full modeling requires self-consistent 3D PIC simulations

Experimental set up



Stable monoenergetic beams



Statistics (30 shots):

E = 206 +/- 11 MeV

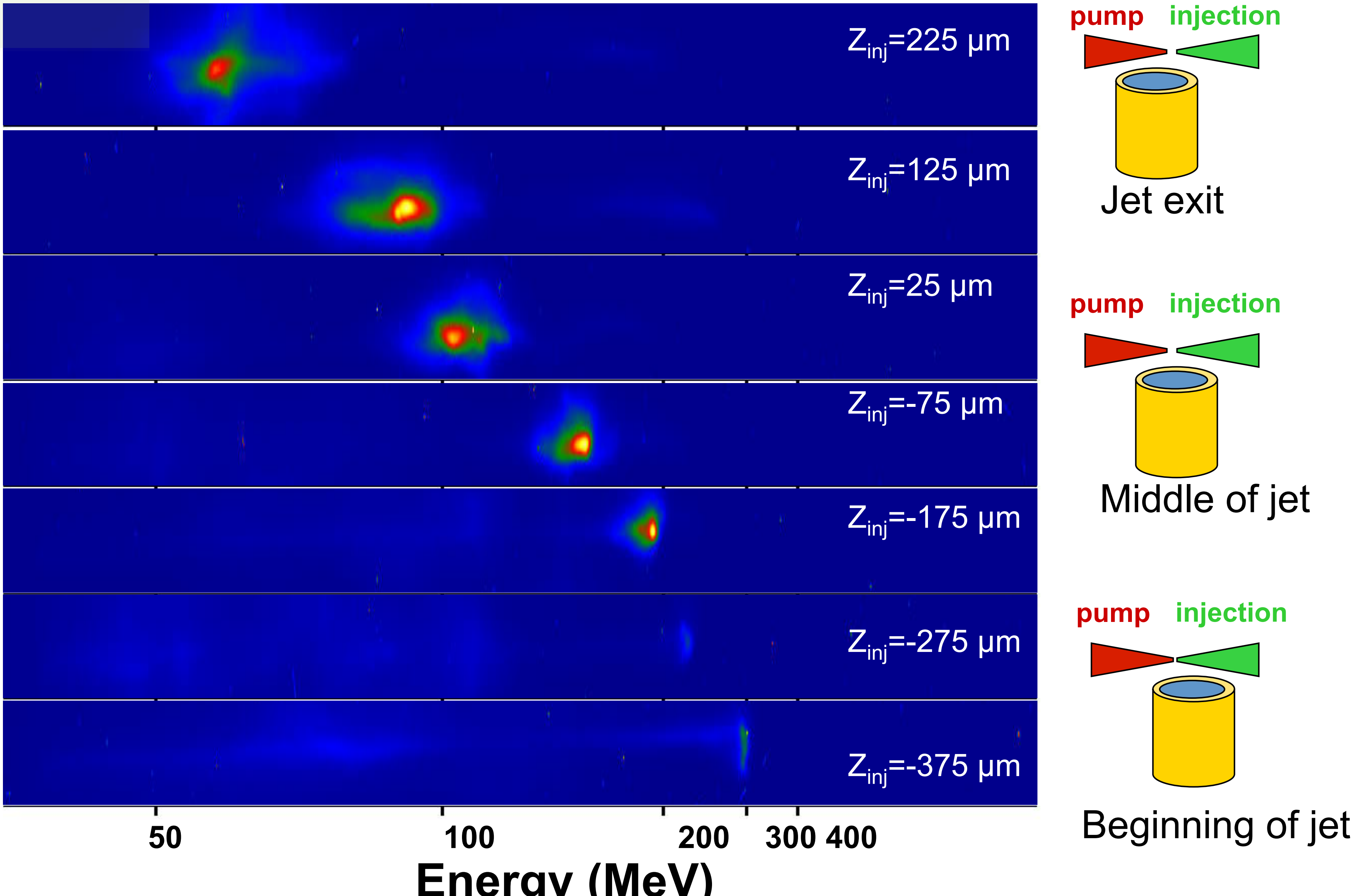
charge = 13+/- 4 pC

δE = 14 +/- 3 MeV

δE/E = 6%

Very little electrons at low energy, δE/E=5% limited by spectrometer

Tuning the beam energy

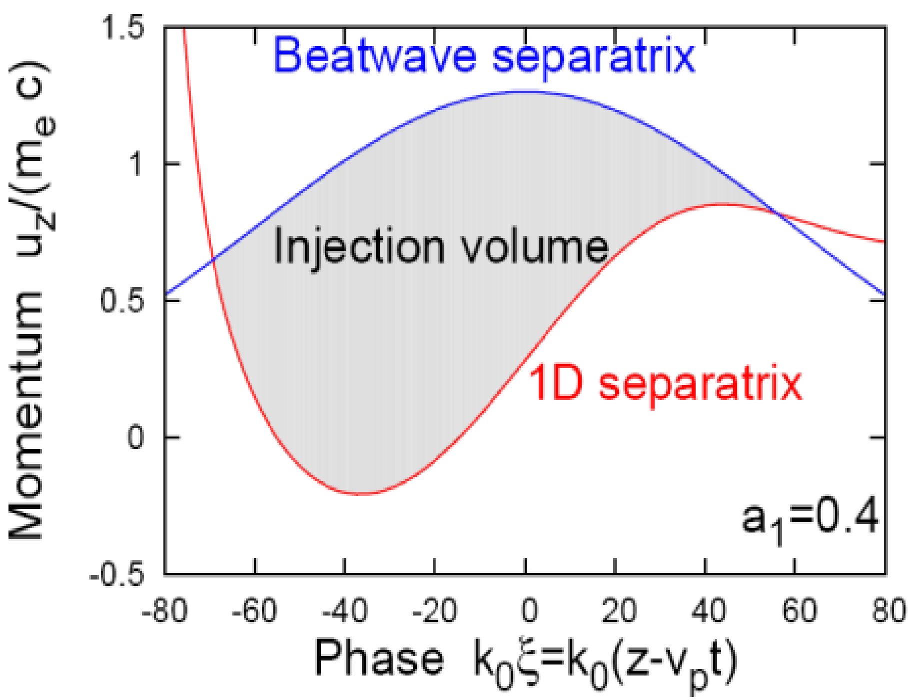


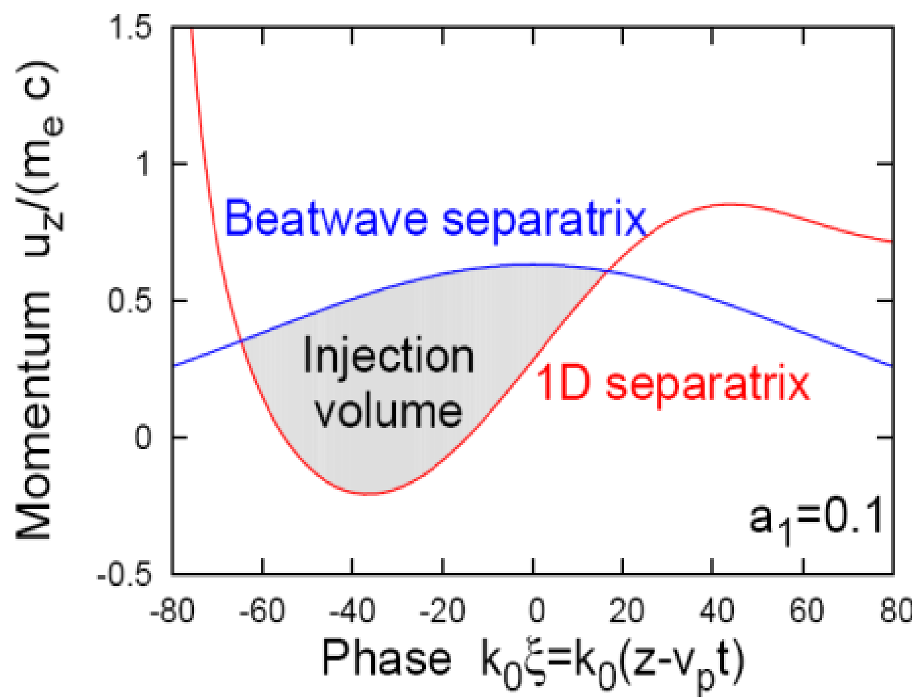
Controlling charge and energy spread

- Charge can be controlled by
  - Modifying how much electrons are heated at the collision

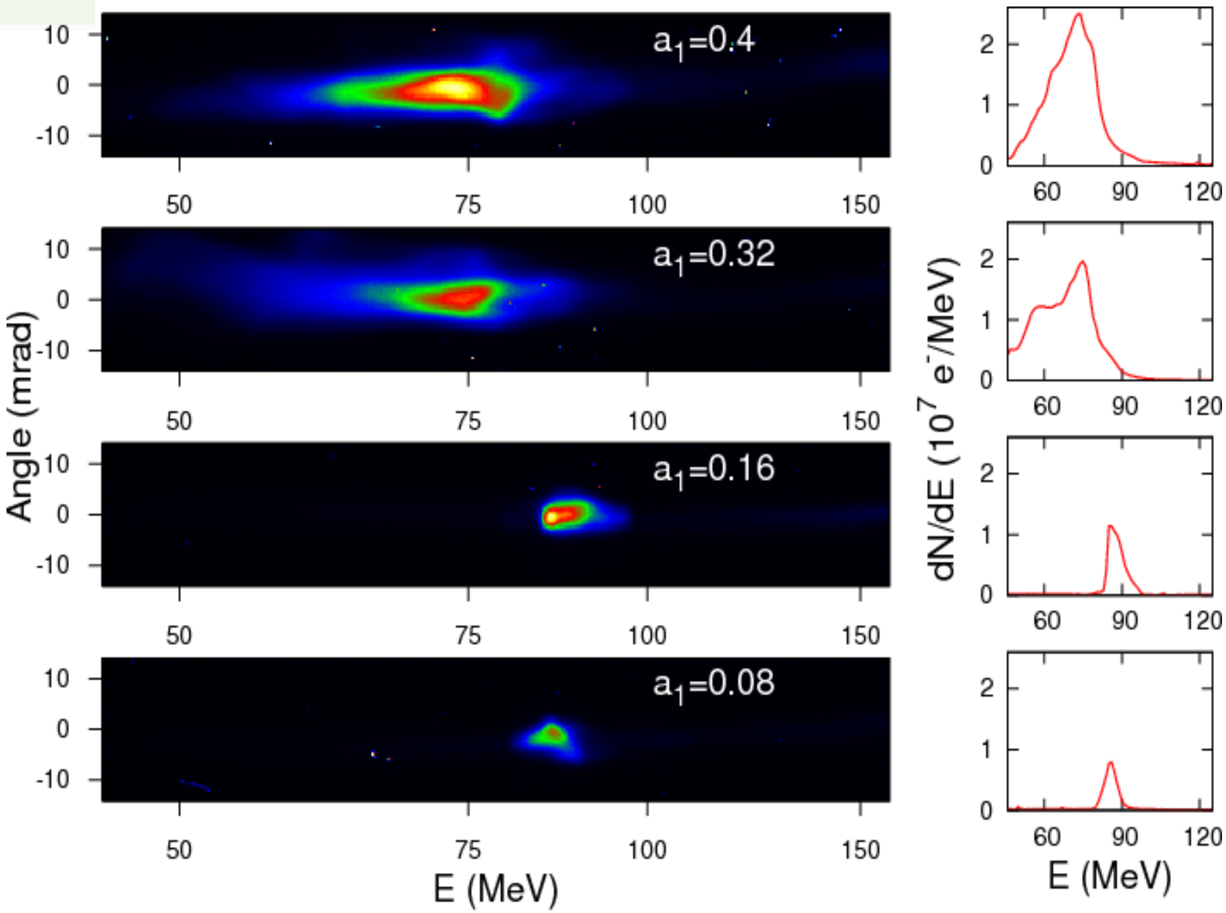
  →by modifying the intensity of the injection pulse, one can control the amount of heating: $E_{beat} \sim (2a_0a_1)^{1/2}$

- dE/E also follows the variation of the charge





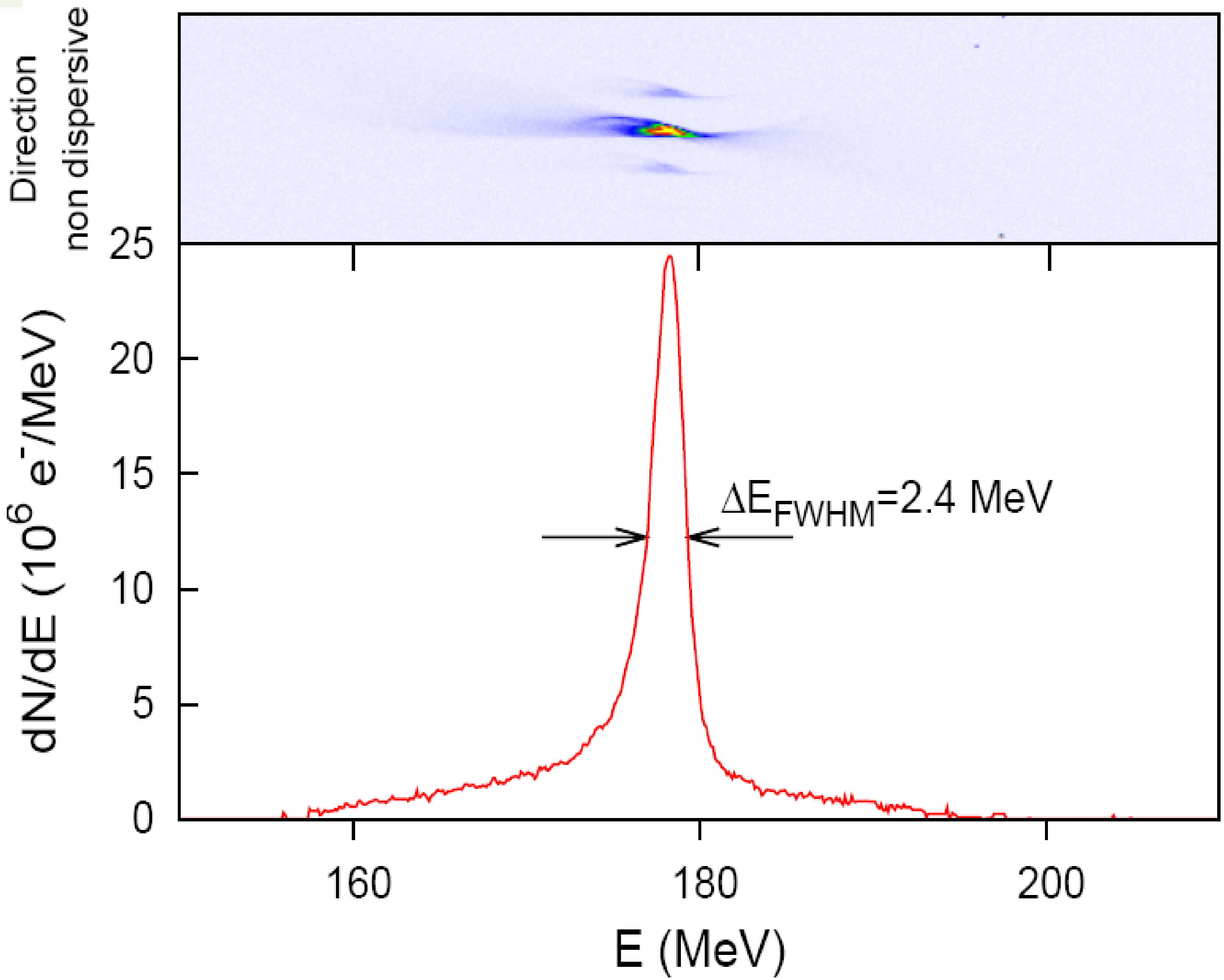
Tuning the charge with the injection pulse



In practice, charge and energy spread are correlated

C. Rechatin et al, PRL 2009

Reduction of energy spread down to 1 %



Conclusion on colliding pulse

- Harder to implement: required 2 intense laser pulses + temporal and spatial overlap

- Injection is local in time and space: can lead to monoenergetic beam, high beam quality

- Possible to control the injection region optically
    - by tuning the injection pulse (energy, polarization)…

→ Energy, charge, energy spread can be controlled optically

Outline

- Injection by slowing down the wakefield:
  - Density gradient injection
  - Injection caused by laser pulse evolution

References:

Theory

Bulanov et al. Phys. Rev. E. **58**, R5257 (1997)

Fubiani et al., Phys. Rev. E **73**, 026402 (2006)

Brantov et al., Phys. Plasmas **15**, 073111 (2008)

Experiments

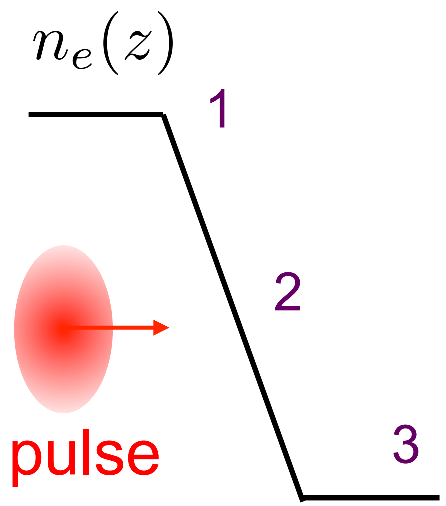
Chien et al., Phys. Rev. Lett. **94**, 115003 (2005)

Geddes et al., Phys. Rev. Lett. **100**, 215004 (2008)

Faure et al., Phys. Plasmas **17**, 083107 (2010)

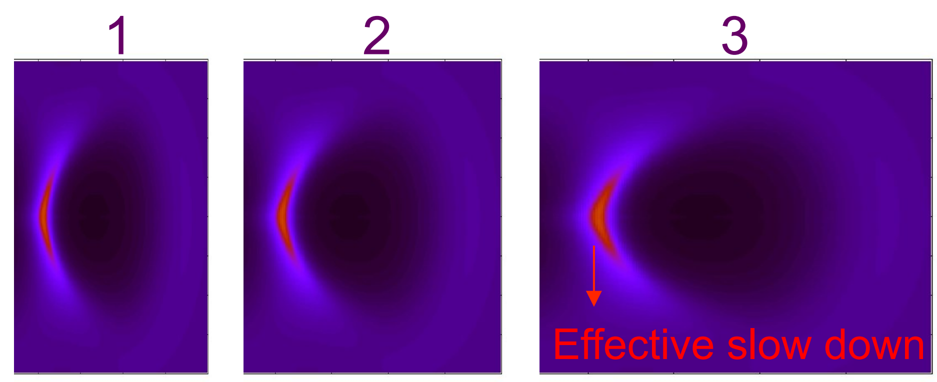
Schmidt et al., PRSTAB **13**, 091301 (2010)

Principle of density gradient injection



Gradient scale length L_{grad}

$$\lambda_p(z) \propto 1/\sqrt{n_e(z)}$$



In the density gradient, λ_p increases

- causes the plasma wave to elongate
- effective slow down of the back of the plasma wave
- effective decrease of the phase velocity

→ Facilitates trapping

→ Decreases the threshold for self-injection

Fluid model with quasi-static approximation

In the quasi-static approximation, i.e for a gentle gradient
The plasma wave equation becomes

$$\left(\frac{\partial^2}{\partial\zeta^2} + k_p^2(z)\right)\phi = k_p^2(z)\frac{\hat{a}^2}{4}$$

$$a^2 \ll 1$$

$$k_p L_{grad} \ll 1$$

The solution of this equation behind the laser pulse is

$$\phi(\zeta, z) = \phi_0(z)\sin\left[k_p(z)(z - v_g t)\right]$$

Consider the phase of the sinusoid

$$\Phi = k_p(z)(z - v_g t)$$

- Local oscillation frequency $\omega = -\partial\Phi/\partial t = k_p(z)/v_g = \omega_p(z)$
- Wave vector $k = \partial\Phi/\partial z = k_p(z) + \partial k_p/\partial z(z - v_g t)$

→ *k(z,t)* !!!

Local plasma wave phase velocity in the density gradient

$\omega = \omega_p(z)$ does not depend on time

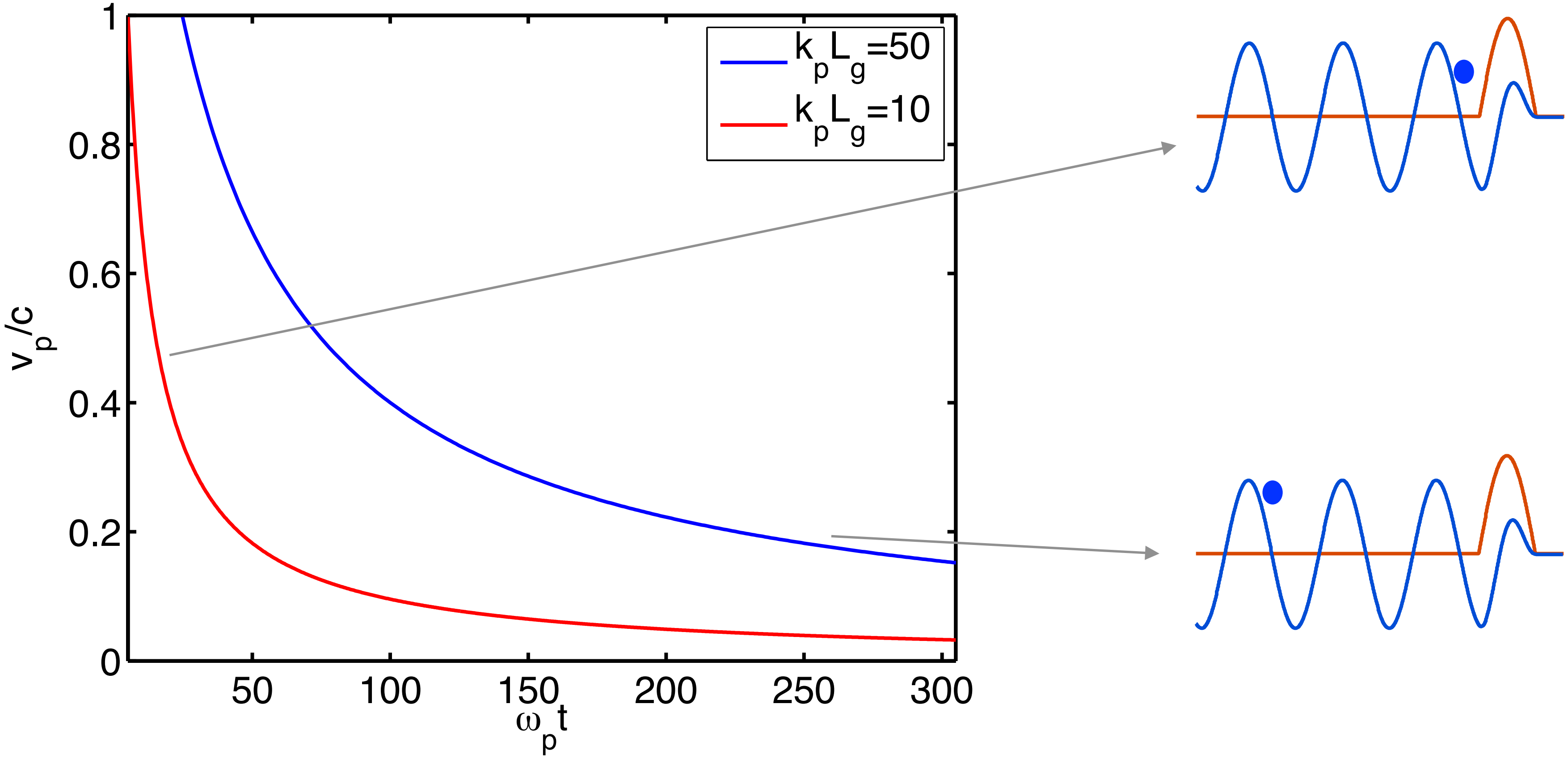
$k = k_p(z) + \partial k_p/\partial z(z - v_g t)$ increases with time

$$v_p(z,t) = \frac{\omega}{k} = v_g \times \frac{1}{1 + \frac{1}{k_p}\frac{\partial k_p}{\partial z}(z - v_g t)}$$

$$v_p(z,t) \propto 1/(A + Bt)$$

Plasma wave slows down with time:
Injection always occurs in a gradient (one has to wait long enough)

Short versus long gradient



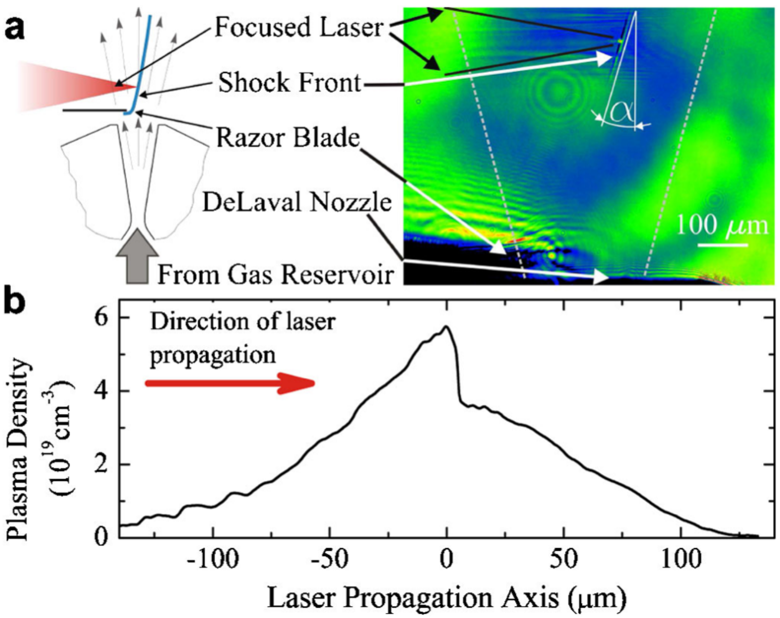
Plasma wave slows down faster for short gradient

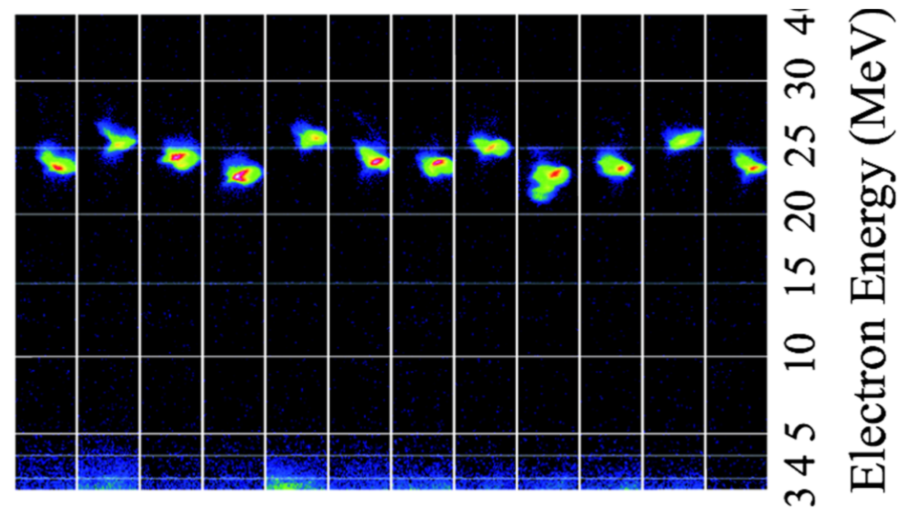
→ Trapping occurs earlier, possibly in first plasma bucket

→ For slow gradients, trapping can occur far behind the pulse

Example of experimental results

65 mJ, 8 fs, I=2.5×10^{18} W/cm^2



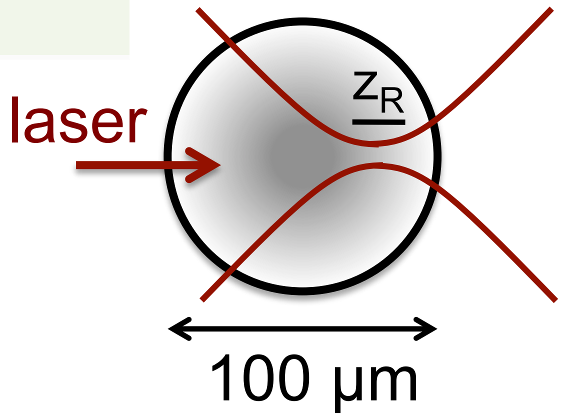


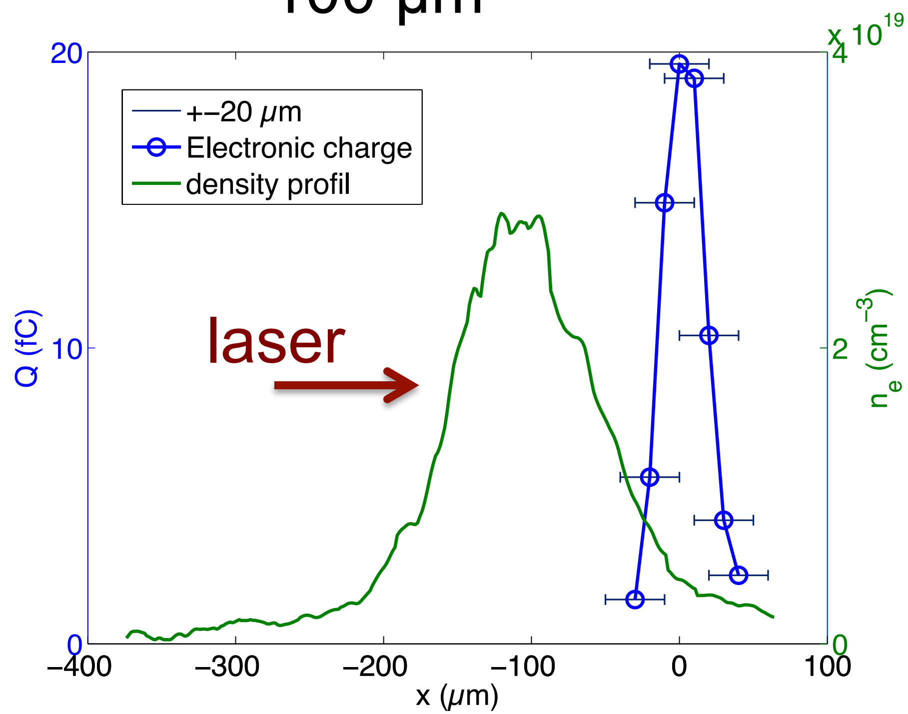
Shock in the gas flow

Stable
Relatively narrow energy spread

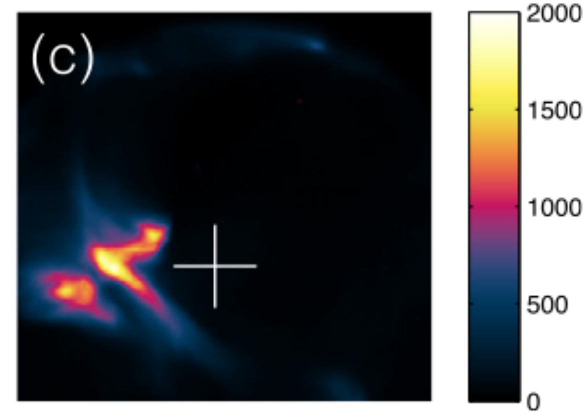
From Schmidt et al., PRSTAB **13**, 091301 (2010)

Injection in gradient with < 10 mJ laser pulses

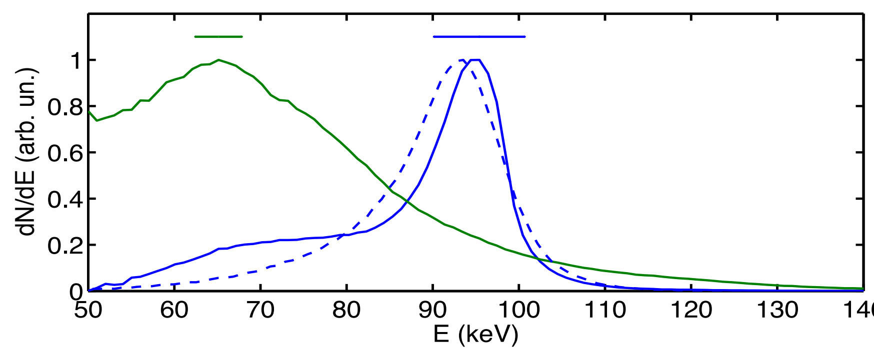




E-beam distribution



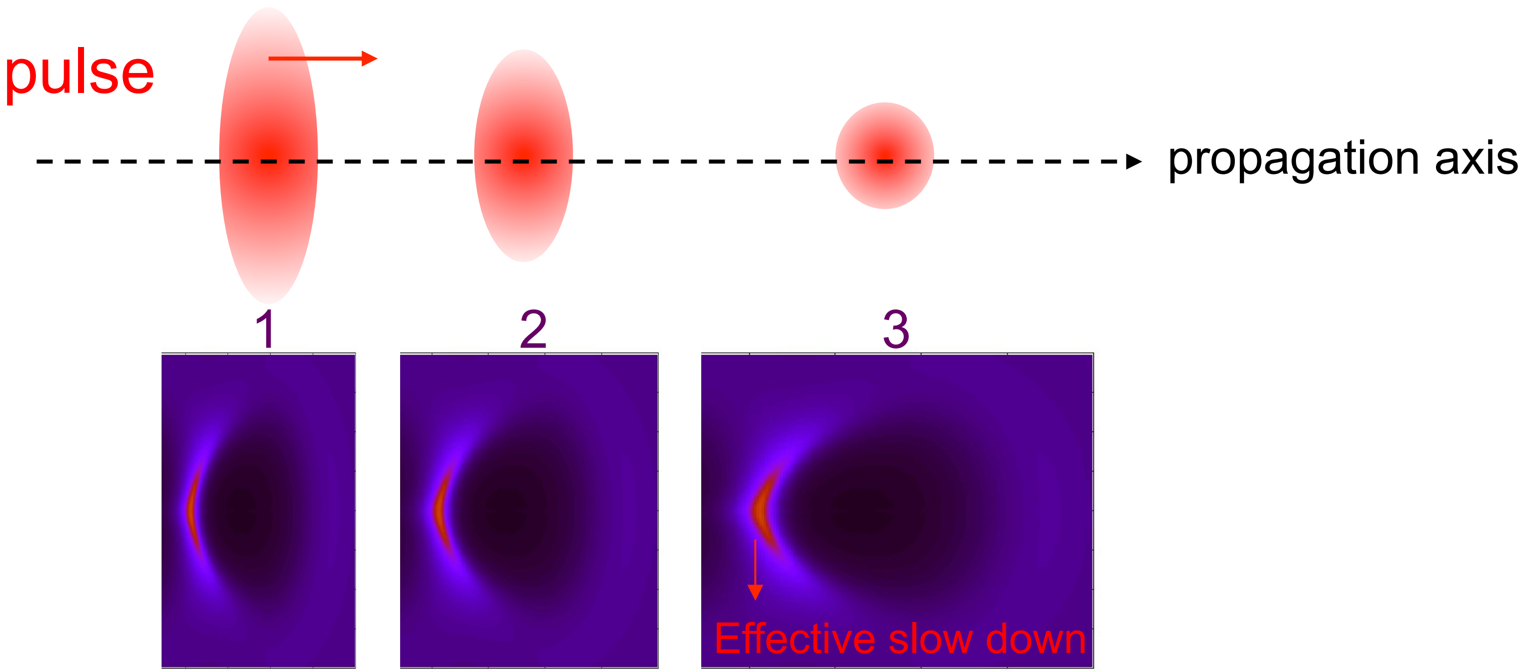
@ 100 keV



From Z. He et al., NJP **15** 05316 (2013)

Injection due to a self-focusing laser pulse

Kalmylov et al., PoP **18**, 056704 (2011)



- Intensity increases → more nonlinear plasma wave
- causes the plasma wave to elongate
- effective decrease of the phase velocity

→ Difficult to observe but probably the cause of self-injection in most experiments

Injection due to a self-focusing laser pulse PIC simulation example

Courtesy R. Lehe

Conclusion on injection in density gradients

- Relatively easy to implement (work on gas target design)
- Works well: increases stability.
- Some level of control: injection location corresponds to the location of the gradient
- More difficult: control of the injection in time (in which buckets, how many bunches are injected)

→ A good way to control injection without using other laser beams. Requires smart design of gas targets

General conclusion

Injection schemes:

- Still an active area of research
- Important for increasing beam stability
- Important for controlling beam parameters

- First demonstration have been performed but
- Room for improvement
- New schemes always needed
- Example: combining several methods for more knobs
  - Colliding pulse + gradient injection ?
  - Other ideas …