Plasma Accelerators the Basics

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Introduction

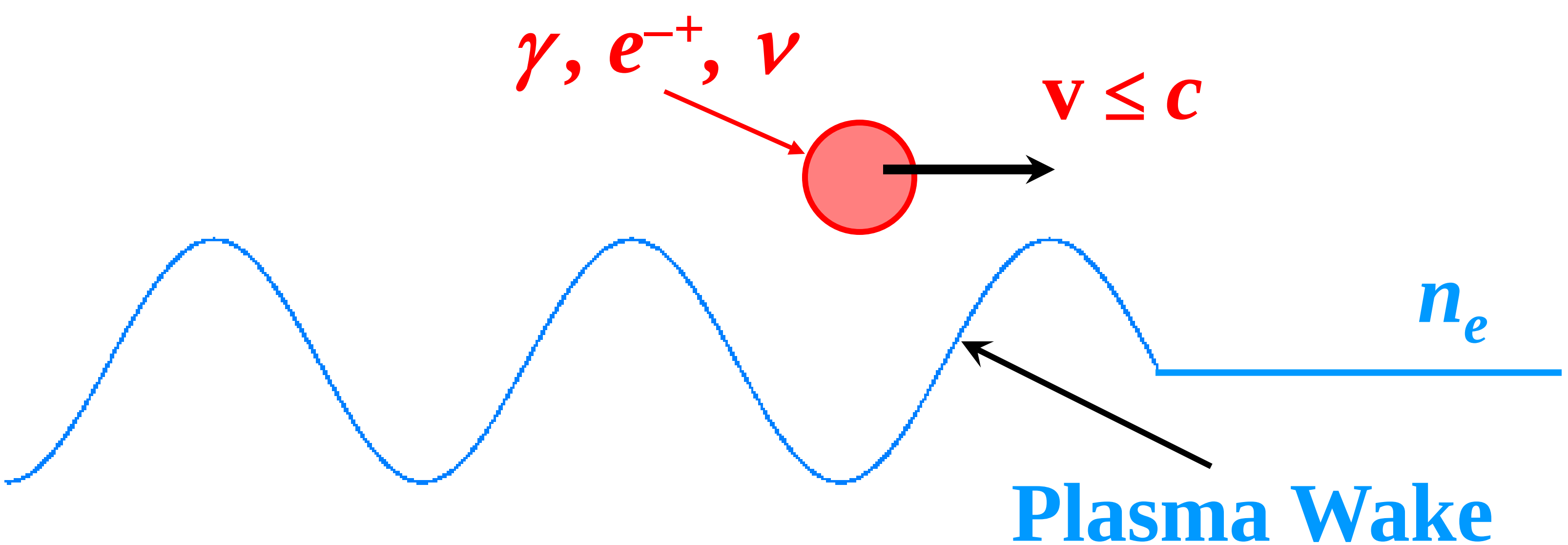
- **General Principles**
  - Laser-plasma accelerators
  - Electron-beam-plasma accelerators
  - Theory – Simulations
  - Experiments
  - Future

Plasma Wakes

photon beam
electron beam
neutrino beam
ion beams

All very similar

*** Drive Plasma Wakes**



Plasma Acceleration Technologies

Plasma Accelerators

1) Electron beam-induced Plasma Wakefield Accelerator (PWA)

2) Laser-induced Plasma Wakefield Accelerators

    a) Plasma Beat-Wave Accelerator (PBWA)

    b) Laser Plasma Wakefield Accelerator (LPWA)

    c) Self-Modulated Laser Wakefield Accelerator (SMLWA)

Particle Accelerators – compact to country size
Rich Physics and Applications

Large

- Verified Standard Model of Elem particles
- W, Z bosons
- Quarks, gluons and quark-gluon plasmas
- Asymmetry of matter and anti-matter
- In pursuit of the Higgs Boson (cause of mass)

Compact

- Medicine
  - Cancer therapy, imaging
- Industry and Gov't
  - Killing anthrax
  - lithography
- Light Sources (synchrotrons)
  - Bio imaging
  - Condensed matter science

Plasma Acceleration

- Electric fields generated in a plasma can accelerate electrons, protons to high energies
- Definition

$$\Delta T = q\, v_0\,(t).E(r_0(t),\, t)\; dt$$

ΔT is change in particle kinetic energy

E is the electric field

q is the particle charge

v_0 is the particle velocity

- Deterministic System
  - *E(r,t)* does not change significantly during the acceleration phase.
- Stochastic
  - *E(r,t)* changes in a random fashion.

Proposed Acceleration Mechanisms

Coherent $t_{acc} > t_{change}$

E-Fields

- Generated by reconnection – inductively driven field
- Double layers – conservative field cannot energise particle, can only locally accelerate them (KE at expanse of PE)

- Coherent waves
  - Acceleration mechanisms, Beat-Wave, wake field need large amplitudes or large distances of uniform plasma
- Coulombic Explosions

Wave-Particle Resonance

- Simplest is Cerenkov resonance $\omega = kv$
- corresponds to law of momentum and energy conservation particle emits wave quantum $\hbar\omega, \hbar k$

$$\Delta E = \hbar\omega = \Delta \underline{p}.\underline{v} \quad \text{where} \quad \Delta \underline{p} = \hbar \underline{k} \quad \& \quad \omega = \underline{k}.\underline{v}$$

<u>In a Magnetic Field</u>

- Resonant particle moves along a spiral on Larmor orbit $\omega_c = \frac{eB}{m}$
- Particle may be treated as an oscillator with energy moving along *B* $n\hbar\omega_c = (n = 0, 1, 2, \ldots)$
- Both energy of translational motion and oscillator energy may change

$$\Delta E = \hbar\omega = \Delta p_z v_z + n\hbar\omega_c \qquad \Delta p_z = \hbar k_z$$

$$\omega = k_z v_z + n\omega_k \qquad n = 0, \pm 1, \pm 2, \ldots$$

Plasma Accelerators – Why Plasmas?

Conventional Accelerators

- Limited by peak power and breakdown
- 20-100 MeV/m
- Large Hadron Collider (LHC) -- 27km, 2010
- Plans for “Next” Linear Collider (NLC) -- 100km ?

Plasma

- No breakdown limit
- 10-100 GeV/m

Laser

- >1 GeV in several cm

e-beam

- ~100 GeV in 1m

Drivers for Plasma Based Accelerators

- Lasers – Terawatt, Petawatt Compact Lasers 10^{12}–10^{15} Watts already exist.
  - Some with high rep. Rates *i.e*. **10 Hz**.
  - Capable of 10^{19}–10^{22} *Watts/cm*^{2} on target.
  - Future ~10^{24} *Watts/cm*^{2} using OPCPA.

- Electrons Beams – Shaped electron beams such as the Stanford/USC/UCLA experiment generate 100*GV/m* accelerating gradient using the 30 – 50*GeV* beam in a 1 meter long Lithium Plasma.

- Ion beams AWAKE experiment CERN

Laser Plasma Accelerators

- The electric field of a laser in vacuum is given by

$$E_{\perp} = 30\sqrt{I} \;\; \text{V/cm}$$

- For short pulse intense lasers,

$$P = 10 \text{ TW}, \;\; \lambda_0 = 1\,\mu\text{m}, \;\; I = 1.6\text{x}10^{18} \;\; \text{W/cm}^2$$

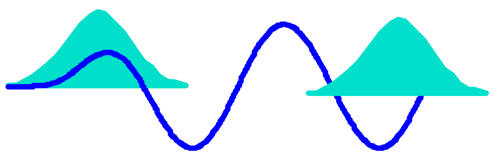
$$E_{\perp} = 40 \;\; \text{GV/cm}$$

- Unfortunately, this field is perpendicular to the direction of propagation and no significant acceleration takes place.
- The longitudinal electric field associated with electron plasma waves can be extremely large and can accelerate charged particles.

Concepts For Plasma Based Accelerators*

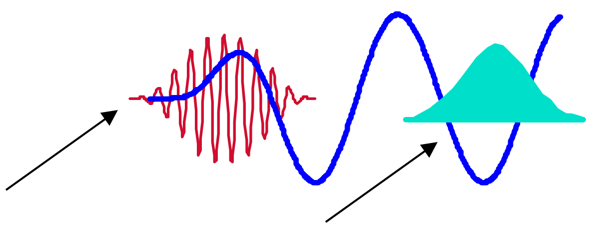
- Plasma Wake Field Accelerator(PWFA)

  A high energy electron bunch



- Laser Wake Field Accelerator(LWFA, SMLWFA, PBWA)

  A single short-pulse of photons



- Drive beam
- Trailing beam

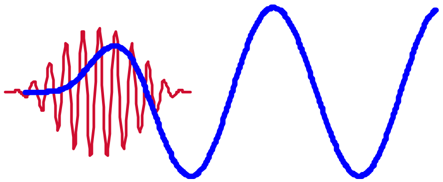
Godfather of the field: Prof. John Dawson

Physical Review Letters July 1979.

Laser Wakefield Acceleration

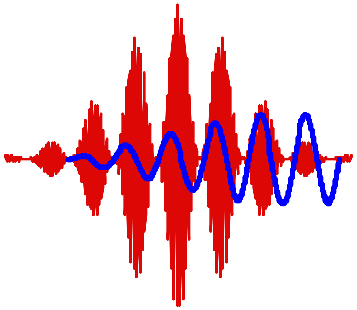
- Laser Wake Field Accelerator(LWFA)

  A single short-pulse of photons



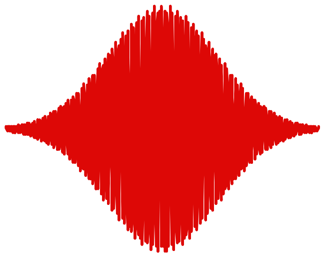
- Plasma Beat Wave Accelerator(PBWA)

  Two-frequencies, i.e., a train of pulses

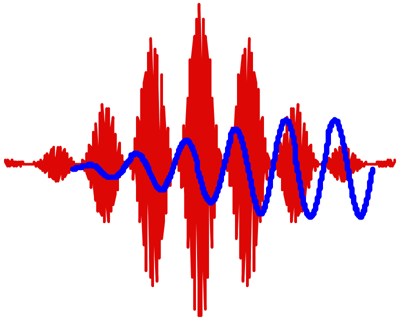


- Self Modulated Laser Wake Field Accelerator(SMLWFA)

  Raman forward scattering instability



evolves to



Plasma waves driven by electrons, photons, and neutrinos

Electron beam

$$\left(\partial_t^2 + \omega_{pe0}^2\right)\delta n_e = -\omega_{pe0}^2 n_{e-beam}$$

$\delta n_e \equiv$ Perturbed electron plasma density

Photons

$$\left(\partial_t^2 + \omega_{pe0}^2\right)\delta n_e = \frac{\omega_{pe0}^2}{2m_e}\nabla^2 \int \frac{d\mathbf{k}}{(2\pi)^3} \square \frac{N_\gamma}{\omega_{\mathbf{k}}}$$

Neutrinos

$$(\partial_t^2 + \omega_{pe0}^2)\delta n_e = \frac{\sqrt{2}\, n_{e0} G_F}{m_e}\nabla^2 n_\nu$$

Ponderomotive force

physics/9807049

physics/9807050

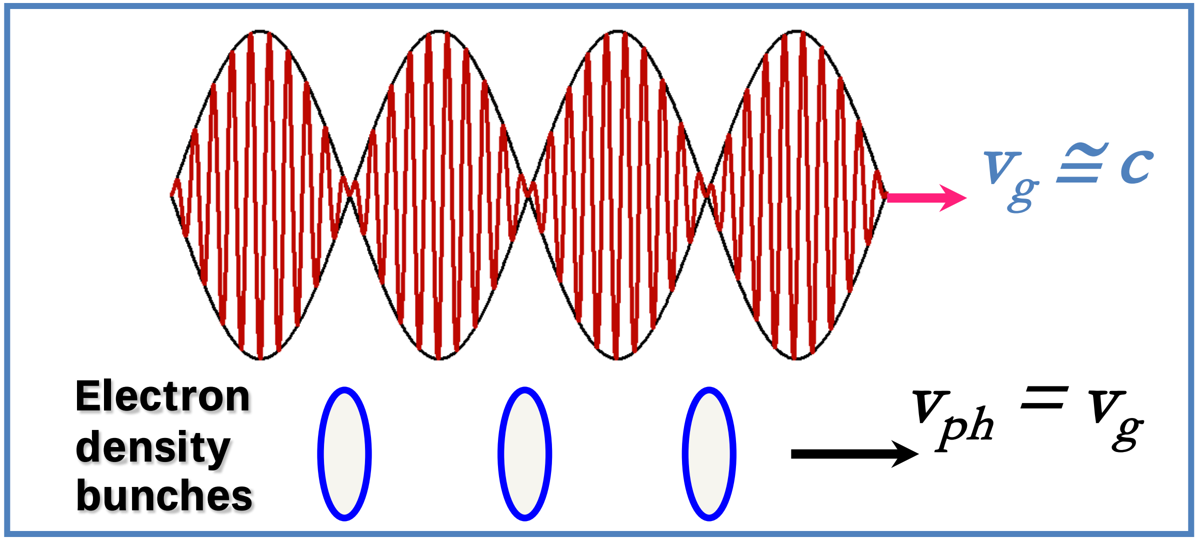
+

Kinetic/fluid equations for electron beam, photons, neutrinos coupled with electron density perturbations due to PW

Self-consistent picture of collective e,γ,ν-plasma interactions

Plasma Beat Wave Accelerator (PBWA)

- In the Plasma Beat Wave Accelerator (PBWA) a relativistic plasma wave is resonantly excited by the "ponderomotive" force of two lasers separated by the plasma frequency ω_p.
- The two laser beams beat together forming a modulated beat pattern in the plasma.



- For relativistic plasma wave the accelerating field $E_{||}$ is given by

$$E_{||} = \varepsilon\sqrt{n_0} \quad \text{V/cm}$$

ε is the fractional electron density bunching, n_0 is the plasma density. For n_0 = 10^{18} cm^{-3}, ε = 10% $\Rightarrow$ $E_{||} = 10^8$ V/cm

Plasma Beat Wave

Relativistic plasma wave driven by beating 2 lasers in a plasma

$$\omega_1 - \omega_2 \cong \omega_p \qquad \text{energy}$$

$$\underline{k}_1 - \underline{k}_2 \cong k_p \qquad \text{momentum}$$

For $\omega_1, \omega_2 >> \omega_p$ i.e. $\omega_1 = 10\omega_p \quad \omega_2 = 9\omega_p$

Then $\left.\begin{matrix} k_1 - k_2 \sim \Delta k \\ \omega_1 - \omega_2 \sim \Delta\omega \end{matrix}\right\} \quad \dfrac{\Delta\omega}{\Delta k} = \mathrm{v}_g$

$\mathbf{v_g}$ is the group velocity of the laser beat pattern.

But $\mathbf{k_1 - k_2 \sim k_p;\ \omega_1 - \omega_2 \sim \omega_p}$

$$\Rightarrow \frac{\omega_p}{k_p} = \mathrm{v}_{ph} \equiv \mathrm{v}_g \qquad \mathrm{v}_g = c\left(1 - \frac{\omega_p^2}{\omega_{1,2}}\right)^{1/2}$$

For $\omega_1, \omega_2 \rangle\rangle\, \omega_p \Rightarrow \mathbf{v_g \approx c} \Rightarrow$ "Hence relativistic"

Plasma accelerators basic equations

Relativistic fluid, Maxwell's and Poisson's equations

$$\frac{\partial n_e}{\partial t} + \nabla.\left(n_e \mathbf{v}_e\right) = 0$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_e.\nabla\right)\gamma \mathbf{v}_e + \frac{3k_B T_e}{n_o m_e}\nabla n_e = \frac{e}{m_e}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_e \times \mathbf{B}\right),$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = -\frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t},$$

$$\nabla.\mathbf{E} = 4\pi e\left(n_e - n_o\right),$$

where $\gamma = \left(1 - (v_e/c)^2\right)^{-\frac{1}{2}}$ is the relativistic Lorentz factor.

Note that the ions are not represented, we only consider high frequencies where ion dynamics are not important. Ions are important for filamentation Parametric decay etc.

Poisson's Equation

- From Poisson's equation we can estimate how large these longitudinal electron plasma waves can be

$$\underline{\nabla} \cdot \underline{E} = 4\pi e \delta n_e$$

δn_e is perturbed electron density of the plasma ions immobile on short time scale.

Largest field exists for $\delta n_e = n_0$ *i.e.* background density.

- Electron plasma waves oscillate with frequency $\omega_p = \left(4\pi n_0 e^2 / m_e\right)^{1/2}$

cgs, or $\left(n_0 e^2 / m_e \varepsilon_0\right)^{1/2}$ MKS.

Relativistic plasma waves have phase velocities close to c

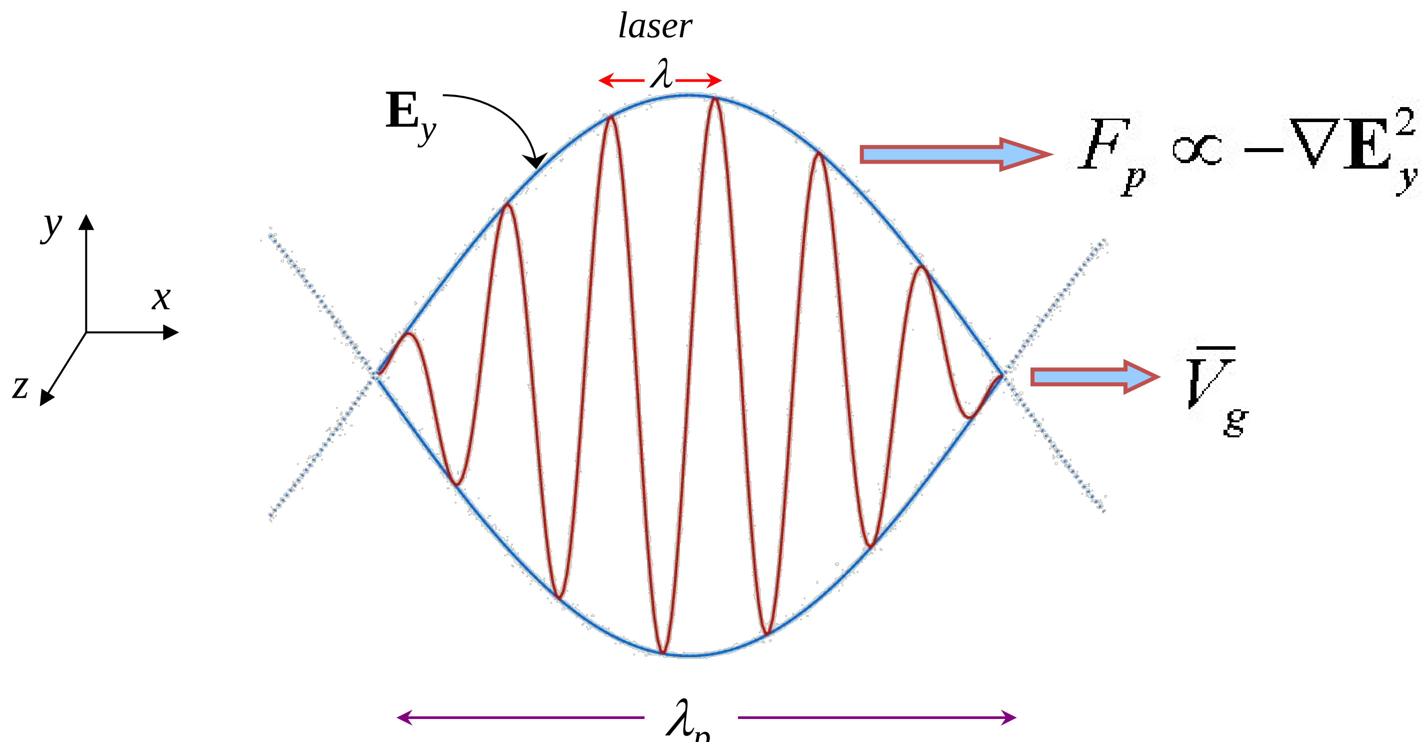
i.e. $\dfrac{\omega_p}{k_p} \cong c.$

- With Poisson's equation we get

$$eE_{MAX} \approx \frac{4\pi n_0 e^2}{\left(\omega_p / c\right)} \cong 0.97\sqrt{\mathrm{n}} \quad \mathrm{eV/cm}$$

i.e. $eE_{MAX} \approx \sqrt{\mathrm{n}} \quad \mathrm{eV/cm}$

Beat wave



- Envelope of high frequency field moving at group speed $\underline{v}_g$

$$\underline{v}_g = c\left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)^{1/2}$$

$$\omega^2 = \omega_{pe}^2 + c^2k^2$$

$$v_g = \frac{d\omega}{dk} = \frac{c^2k^2}{\omega} = c\left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)^{1/2}$$

- Ponderomotive force $F_p \propto -\nabla \mathbf{E}_y^2$ Laser field $\mathbf{E}_y$

Beat wave generation

For the plasma beat wave the equation for the plasma wave can easily be derived by introducing slowly varying amplitudes describing the laser field,

$$\mathrm{E}_{1,2} = Re\mathrm{E}'_{1,2}(x,t)\exp\left\{i\left(k_{1,2}x - \omega_{1,2}t\right)\right\}$$

and the electron density perturbation as,

$$\delta n_e = n_e - n_o = Re\delta n'_e(x,t)\exp\left\{ik_p x\right\}$$

Note we have not separated the timescales in the density perturbation since this mode can be strongly nonlinear.

The equation for the electron density perturbation is found to be,

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\delta n_e = \frac{3}{8}\omega_p^2\frac{\delta n_e^2}{n_o^2}\delta n_e - \frac{n_o}{2}\omega_{pe}^2\alpha_1\alpha_2 e^{-i\delta t}$$

where $\alpha_j = \frac{eE_j}{m_e\omega_j c}, j = 1,2$ is the normalised quiver velocity in the field of each laser $\delta = \omega_1 - \omega_2$.

The 1st term on the RHS is the relativistic mass shift in the electron plasma wave and the 2nd term is the laser beat drive term.

The Relativistic Plasma Wave is described by

$$\left(\frac{\partial^2}{\partial t^2}+\omega_p^2\right)\delta n=\frac{3}{8}\omega_p^2\frac{\delta n^2}{n_0^2}\delta n-\frac{n_0}{2}\omega_p^2\alpha_1\alpha_2 e^{-i\delta t}$$

$$\alpha_j=\frac{eE}{m_e\omega_j c}\qquad ;\ \delta=\omega_1-\omega_2$$

For $\alpha_1 = \alpha_2 =$ constant

$$\frac{\delta n}{n_0}=\frac{\delta n_0(0)}{n_0}+\frac{1}{4}\alpha_1\alpha_2\omega_p t$$

<u>Linear growth:</u> However due to 1st term on RHS i.e. cubic non-linearity wave saturates before reaching wave breaking limit $\delta n/n \sim 1$; acts as nonlinear frequency shift.

$$\frac{\delta n}{n}\max=\left(\frac{16}{3}\alpha_1\alpha_2\right)^{1/3}=\varepsilon$$

Relativistic mass increase of electrons reduces natural frequency

$$\omega_p^1=\omega_p\left(1-\frac{3}{8}\frac{v_{osc}^2}{c^2}\right)^{1/2}=\omega_p\left(1-\frac{3}{8}\frac{\delta n^2}{n^2}\right)^{1/2}$$

This results in $\omega_1 - \omega_2 \neq \omega_p \Rightarrow$ non-resonant interaction $\Rightarrow$ saturation.

Beat wave generation

The longitudinal electric field amplitude of the relativistic plasma waves can be extremely large with a theoretical limit obtained from Poisson's equation and given by

$$E = \epsilon n_o^{\frac{1}{2}} V/cm$$

where ϵ is the Rosenbluth Liu saturation value.

The growth of the plasma wave due to the beat wave mechanism is,

$$\epsilon = \int_o^t \frac{\alpha_1 \alpha_2 \omega_p}{4} dt$$

The red shift due to the relativistic mass increase of the electron in the plasma wave

$$\Delta\omega = - 3/16 \ \epsilon^2$$

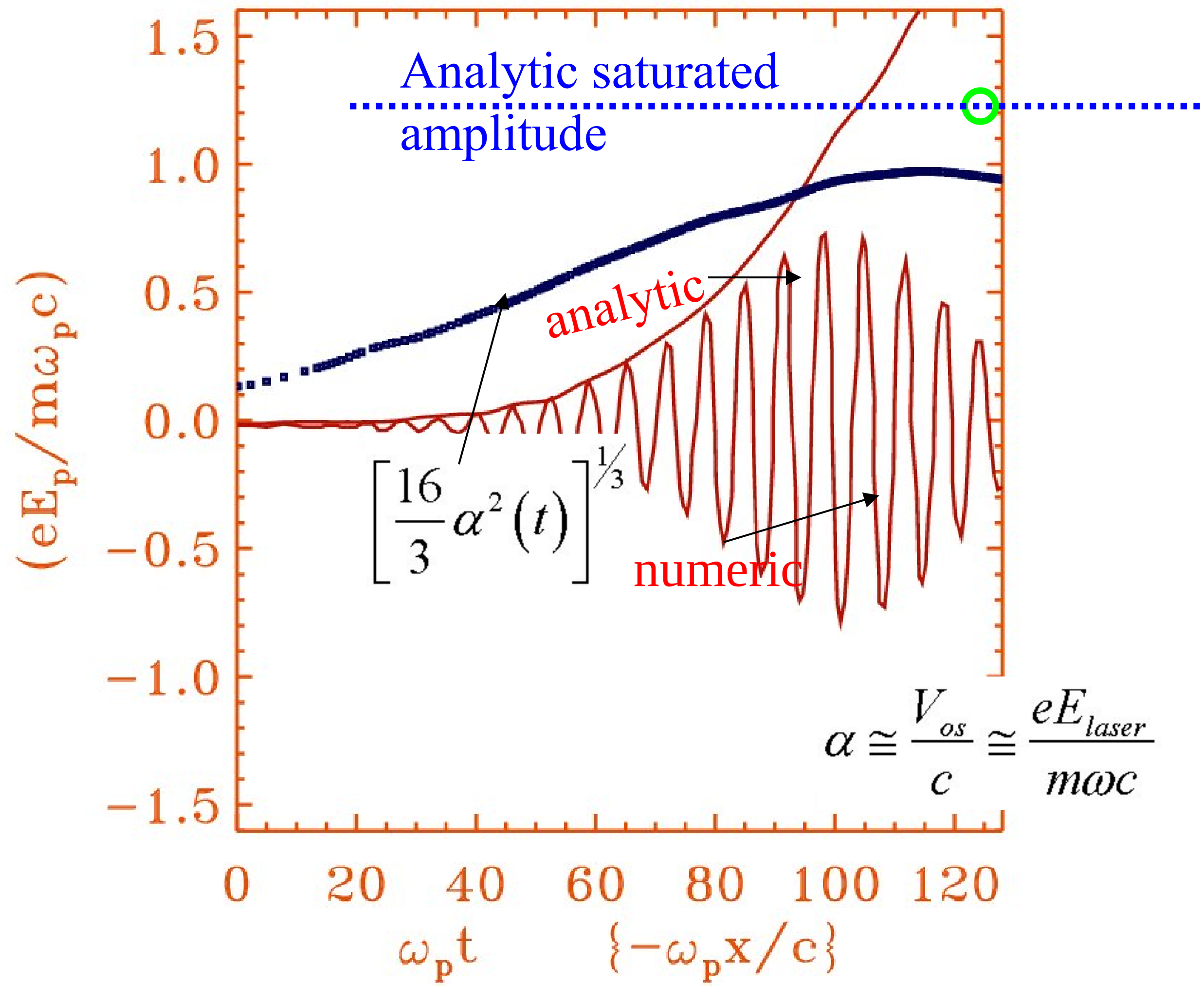
causes the wave to saturate at,

$$\epsilon_{SAT} = \left(\frac{16}{3} \alpha_1 \alpha_2 \right)^{\frac{1}{3}}$$

in a time $\tau_{SAT} = \frac{8}{\omega_p} \left(\frac{2}{3} \right)^{\frac{1}{3}} \left(\frac{1}{\alpha_1 \alpha_2} \right)^{\frac{2}{3}}$

Other factors that can limit the interaction or acceleration length is diffraction of the laser beams or pump depletion.

Beat Wave Growth



Energy Gain

- For $n_0 = 10^{18}$ cm^{-3}, $\varepsilon = n_1 / n_0 = 10\%$
- Gain in energy of electron $\Delta W \approx eE_p l$, $\Delta W = 2\,\varepsilon\,\gamma^2 m_e c^2$
- $\gamma = \omega_1/\omega_p$ is the Lorentz factor
- For a neodymium laser, ω_1/ω_p ~30 , and n_0 ~ 10^{18} cm^{-3}

$$E_p = \frac{e}{k_p}\delta n = \frac{ec}{\omega_p}\delta n$$

$$l = \frac{\lambda_p}{2}\chi \quad \Rightarrow \quad \chi = \frac{c}{c - v_{ph}} \quad \Rightarrow \quad v_{ph} \simeq c\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2}$$

$$l = \frac{\lambda_p}{2}\frac{\omega^2}{\omega_p^2} = \frac{\lambda_p}{2}\gamma^2$$

$$eEl = 2\varepsilon\gamma^2 m_e c^2$$

- Maximum energy gain $\Delta W \approx eE_p l$

$$\Delta W \approx 100\ MeV$$

UCLA Experiments

- **UCLA Plasma Beat Wave Accelerator**

CO_2 Laser, 2 wavelengths λ_1 = 10.29 μm, λ_2 = 10.59 μm, λ_p = 360 μm, n_{PLASMA} = 10^{16} cm^{-3}, α_1 = 0.17, α_2 = 0.07, τ = 150 psec.

Electrons injected at 2.1 MeV are accelerated to 30 MeV in a plasma length of 1 cm. This corresponds to an accelerating field

$$E_{||} = 30\ \text{MeV/cm}$$

Amplitude of relativistic plasma wave ε = 30% n_0 = $\delta n/n_0$

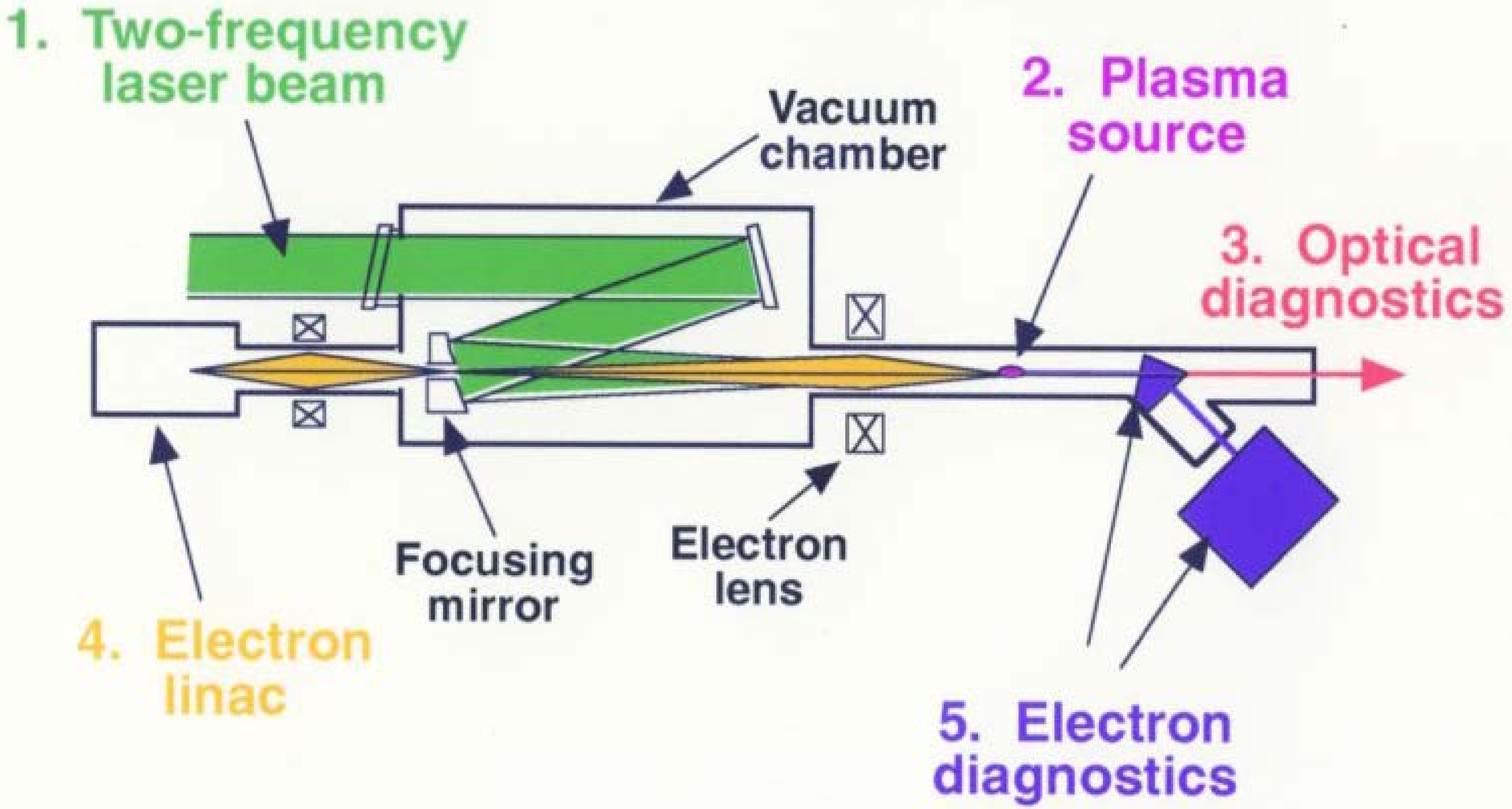
1st successful demonstration.

C. Clayton *et al.* *Phys Rev Lett.*, 70, 37 (1994)

M. Everett *et al.* *Nature*, 368, 527 (1994)

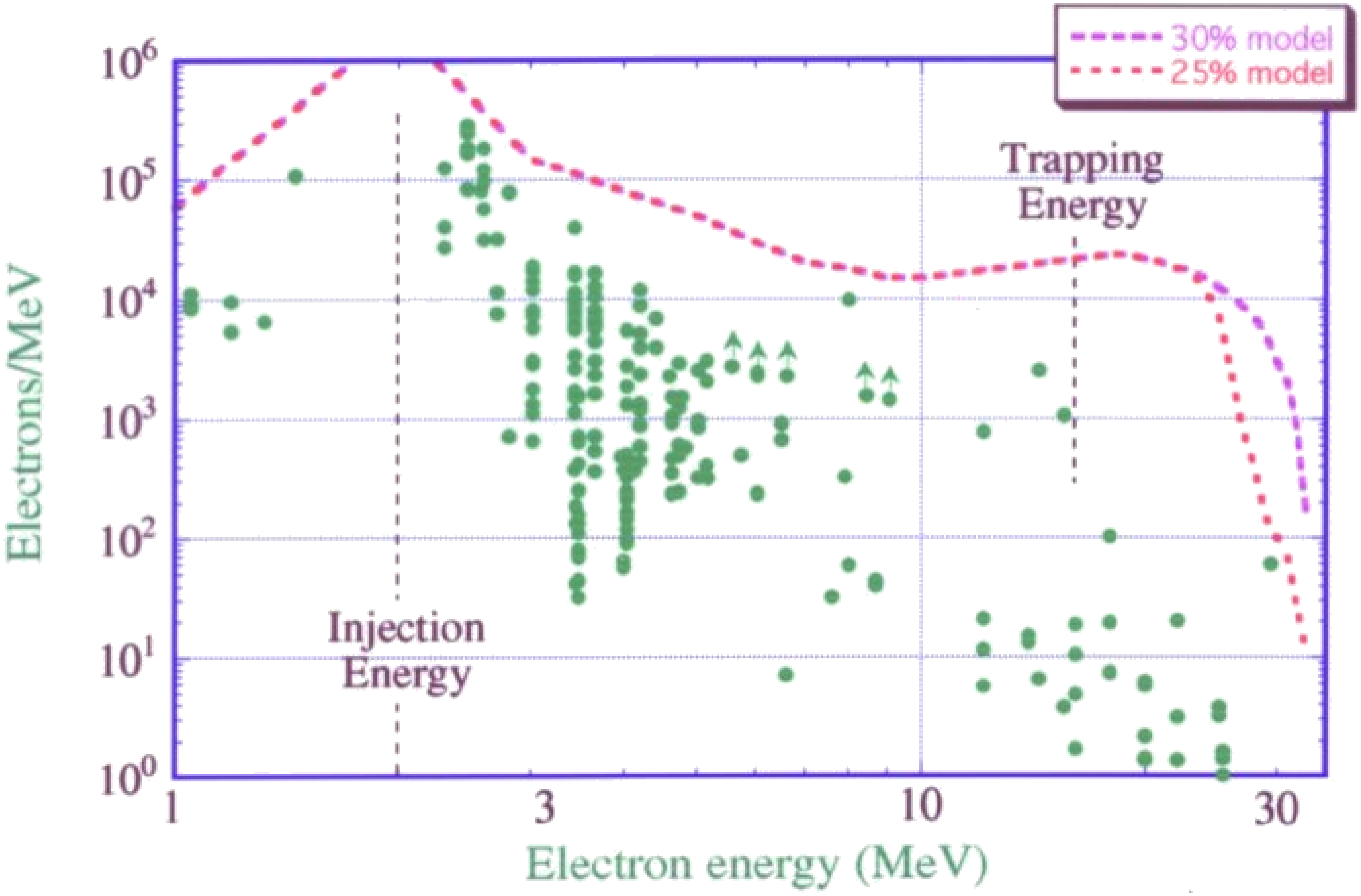
Overview of Beatwave Expt.

- Experiment consists of five major components …



UCLA Experimental Results

- Electrons injected from 0.3 GHz rf LINAC ⇒ train of pulses, <10ps duration



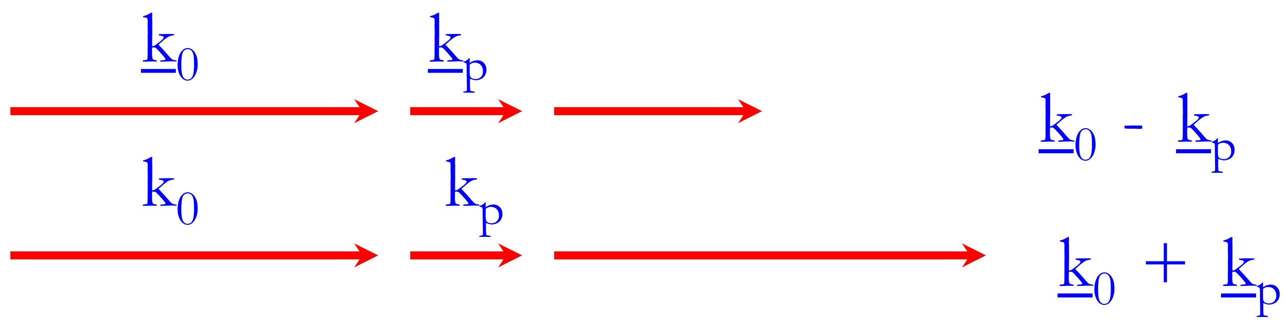
- 1% or 10^5 electrons are accelerated in the diffraction length of ~1cm.
- 2→30 MeV Gradient of 3 GeV/m

Self-Modulated Wakefield

Break-up of a long pulse $L \gg k_p$

via <u>Forward Raman Scattering</u> in 1-D limit or

via an <u>Envelope-Self-Modulation</u> instability in 2-D limit.



$\underline{k}_0 + \underline{k}_p = \underline{k}_{AS}$ Anti-Stokes

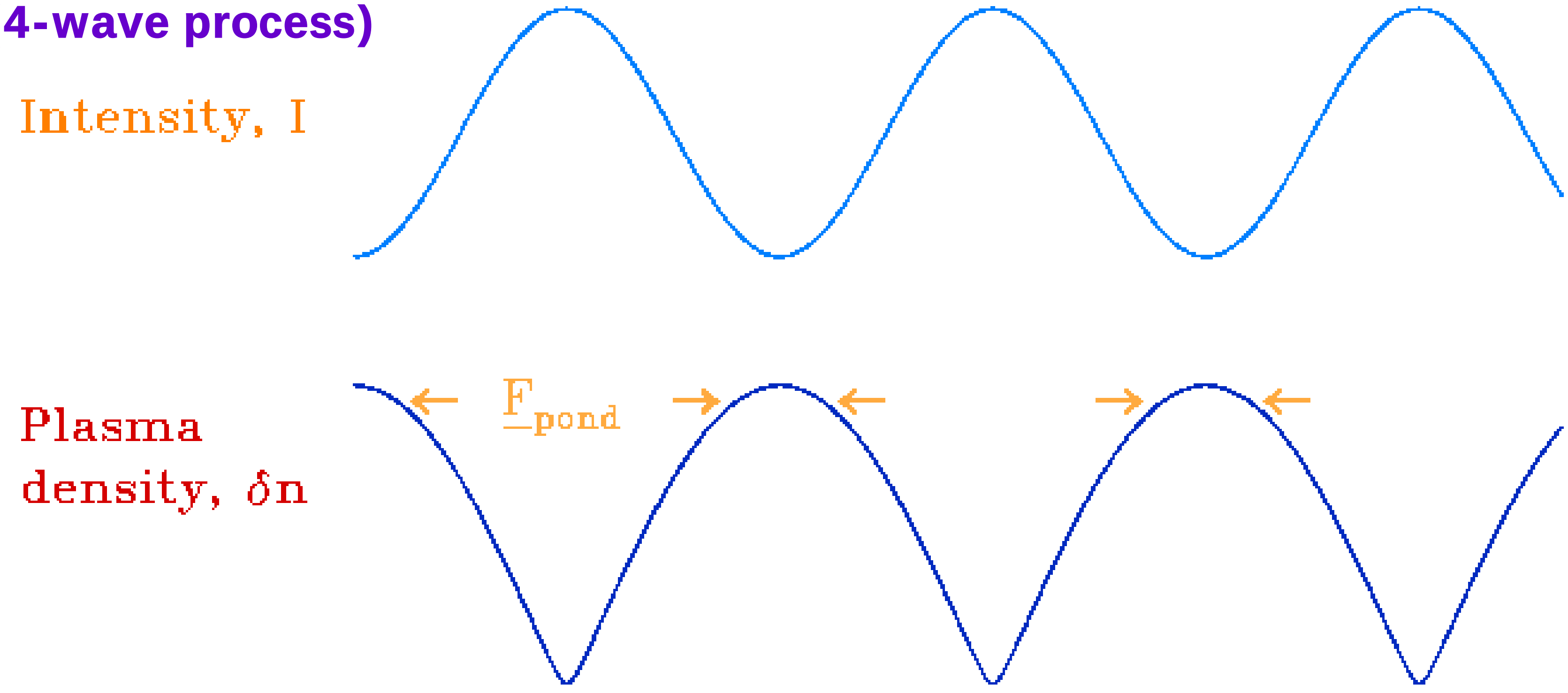
$\underline{k}_0 - \underline{k}_p = \underline{k}_S$ Stokes

<u>Pulse Power</u> > relativistic self-focusing

$$P > P_c = 17 \frac{\omega^2}{\omega_p^2} \quad \text{GW}$$

Stimulated Raman & Modulational Instability

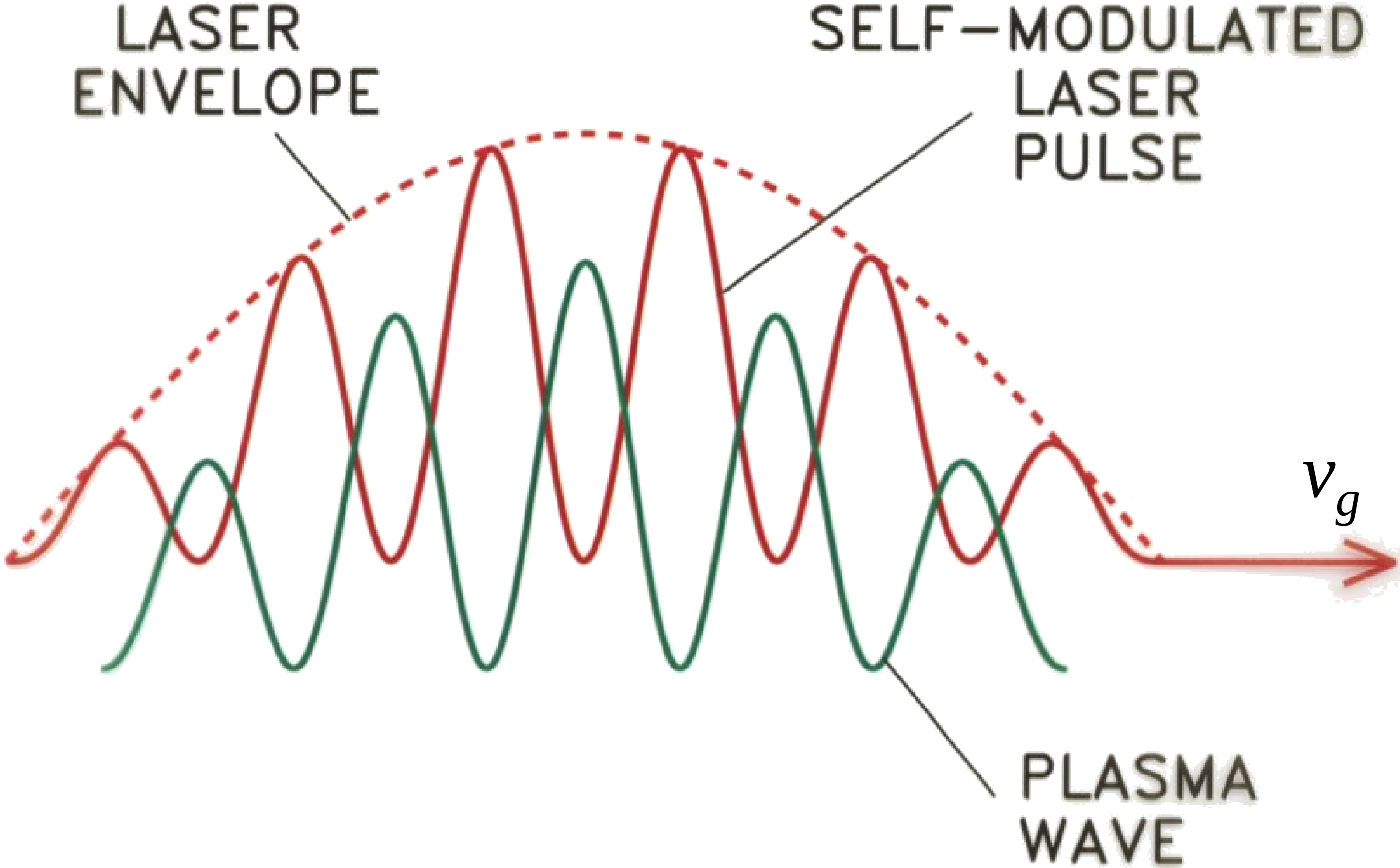
(4-wave process)



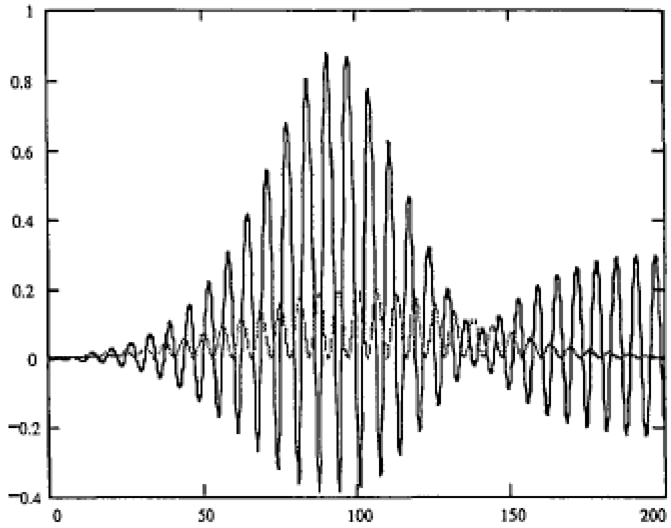
$$\underline{F}_{pond} \propto -\underline{\nabla} I$$

Physical mechanism for stimulated Raman forward scattering driven by Ponderomotive force.

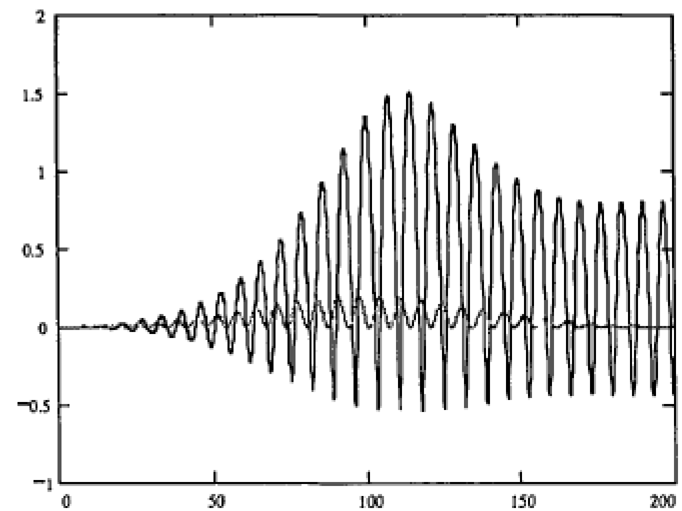
Self-modulated wakefield



Modulated laser pulse



Beat wave with long pulse laser modulated at the plasma wavelength. Laser has a constant wavelength.



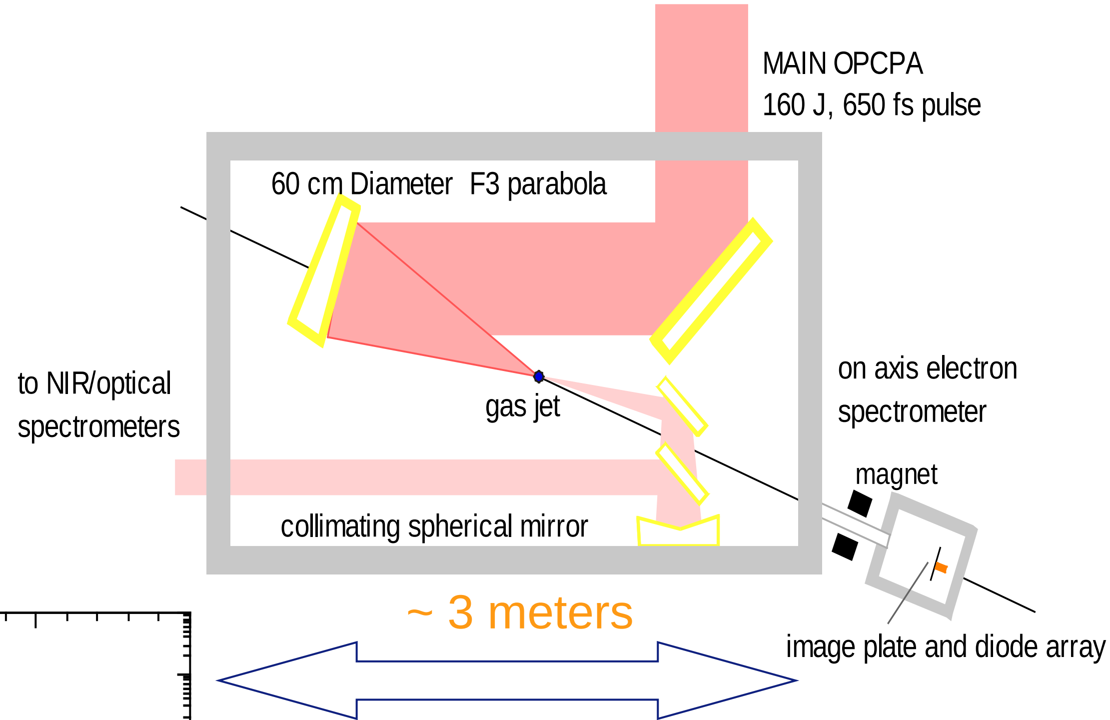
Chirped pulse laser, wavelength of the laser Increases from the front to the back of the Pulse.
Alternatively can use density ramp.

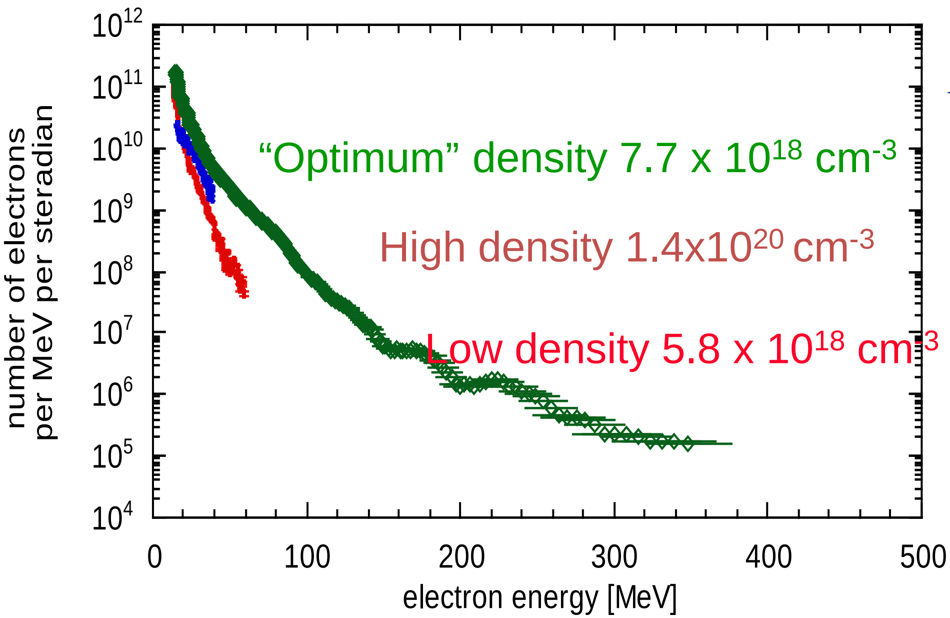
Laser acceleration experiments using the Rutherford VULCAN PetaWatt Laser

Imperial College London

Courtesy of K. Krushelnick et al.

- Vulcan@RAL: 160 J in 650 fs
- Single shot laser





- 350 MeV electrons observed
- Energy spread large

Short pulse wakefield generation

Analysis based on one fluid, relativistic hydrodynamics and Maxwell's equation

$$\frac{\partial \boldsymbol{p}}{\partial t} + v_z \frac{\partial \boldsymbol{p}}{\partial z} = -e\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}\right)$$

where

$$\boldsymbol{p} = m_0 \gamma \boldsymbol{v}, \quad \gamma = (1 + p^2/m_0^2 c^2)^{1/2},$$

$$\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{A}_\perp}{\partial t} - \hat{\boldsymbol{z}}\frac{\partial \phi}{\partial z}; \ \boldsymbol{B} = \nabla \times \boldsymbol{A}_\perp; \ \boldsymbol{A}_\perp = \hat{\boldsymbol{x}} A_x + \hat{\boldsymbol{y}} A_y,$$

Where ***A*** is the vector potential of the laser pulse, Ø is the scalar potential due to space charge separation in the plasma.

Normalise the potentials $\frac{e}{m_0 c^2} \boldsymbol{A}_\perp \equiv \boldsymbol{a}(z, t); \quad \frac{e}{m_0 c^2} \phi \equiv \Phi$

We can now derive the longitudinal component of the electron momentum equation and combine it with the continuity equation Poisson's and Maxwell's equation to form a 1 D model for laser pulse and wakefield.

Short pulse wakefield generation

In the envelope approximation assume a pulse of the form,

$$\boldsymbol{a}(z, t) = \tfrac{1}{2}\boldsymbol{a}_0(\xi, \tau)\mathrm{e}^{-i\theta} + c.c.,$$

Where the amplitude is cast in the frame of the moving laser pulse, with

$$\theta = \omega_0 t - k_0 z \qquad \xi = z - v_g t$$

The final equations for $\boldsymbol{a_o}$ and Φ are obtained on

$$\frac{\partial^2 \Phi}{\partial \xi^2} = \frac{\omega_{p0}^2}{c^2} G,$$

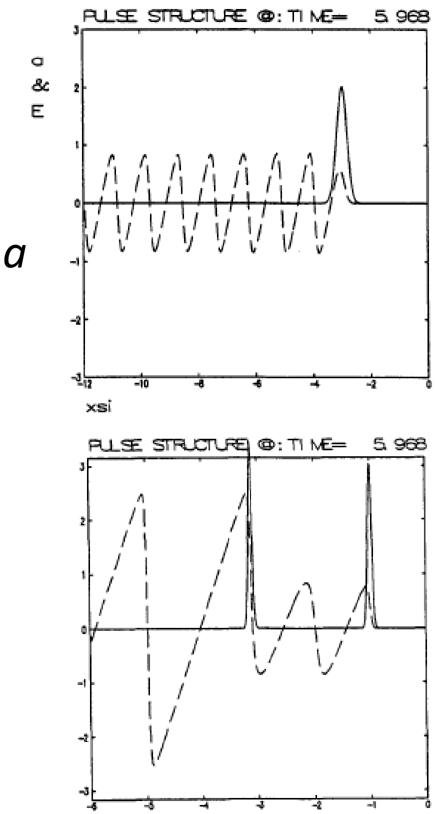
$$2i\omega_0 \frac{\partial a_0}{\partial \tau} + 2c\beta_0 \frac{\partial^2 a_0}{\partial \tau\, \partial \xi} + \frac{c^2 \omega_{p0}^2}{\omega_0^2} \frac{\partial^2 a_0}{\partial \xi^2} = -\omega_{p0}^2 H a_0$$

where

$$G = \frac{\sqrt{\gamma_\parallel^2 - 1}}{\beta_0 \gamma_\parallel - \sqrt{\gamma_\parallel^2 - 1}}, \quad H = 1 - \frac{\beta_0}{\gamma_a(\beta_0 \gamma_\parallel - \sqrt{\gamma_\parallel^2 - 1})}.$$

Valid for arbitrary laser pulses and laser intensities

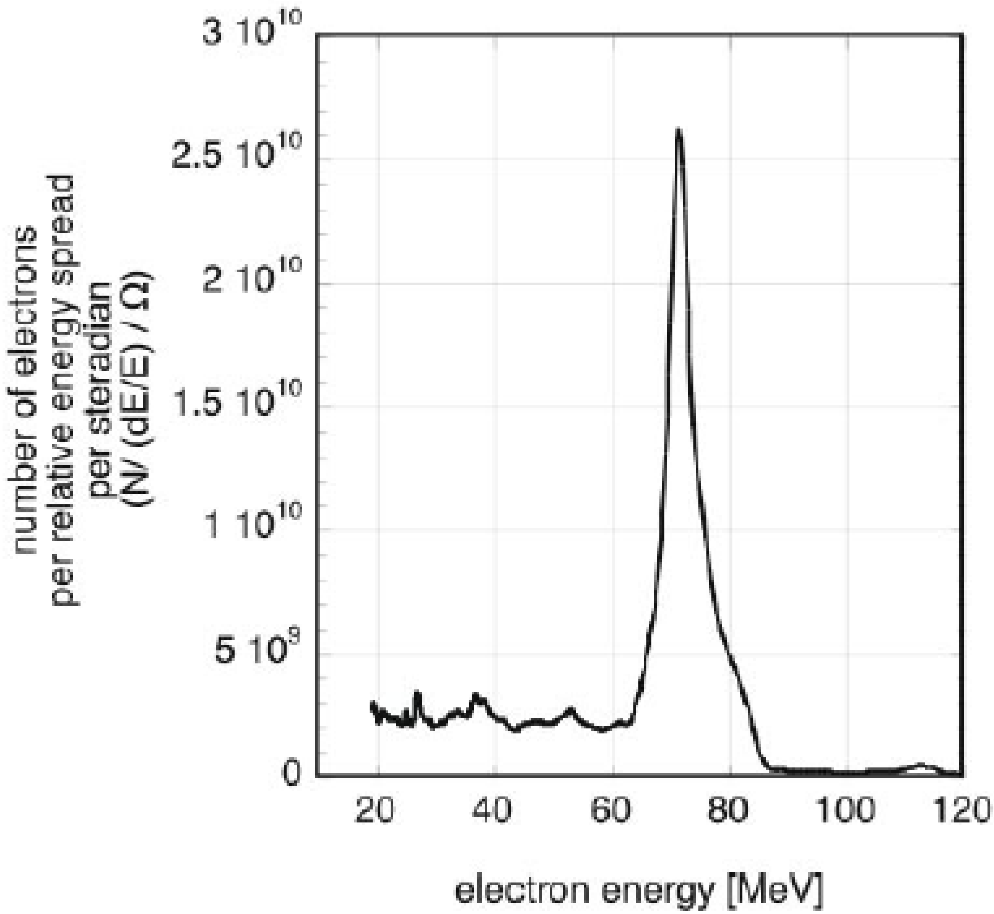
E,a



ξ

The Bubble Regime

Mono-energetic spectra can be observed at higher power ($\Delta E/E = 6\ \%$)



E ~ 500 mJ,

pulse duration ~ 40 fsec

Focal spot ~ 25 μm

Density ~ 2×10^{19} cm^{-3}

Significant shot-to-shot fluctuations in

a) energy spread

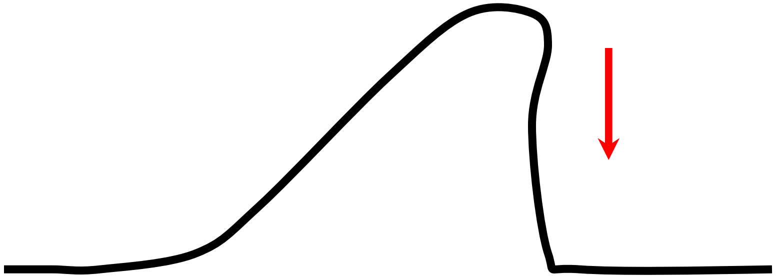
b) peak energy

Careful control of laser and plasma conditions is necessary

Courtesy: K. Krushelnick, IC

Wavebreaking Amplitude

- Wavebreaking $\Rightarrow$ crest “falls” into trough



cold plasma $$\frac{eE}{m\omega_p \mathrm{v}_{ph}} = 1$$ (Dawson 1958)

$$\frac{\partial E}{\partial x} \to \infty$$ and from Gauss’ law $$n \to \infty$$

cold relativistic oscillation

(Akhiezer & Polovin 1956)

$$\frac{eE}{m\omega_p c} = \sqrt{2}\left(\gamma_{ph} - 1\right)^{1/2}$$

$$\gamma_{ph} = \left(1 - \mathrm{v}_{ph}/c^2\right)^{-1/2}$$

warm relativistic oscillation $$\frac{eE_{\max}}{mc\omega_p} = \frac{1}{\beta^{1/4}}\left(\ln 2\gamma_{ph}^{1/2}\beta^{1/4}\right)^{1/2}$$

(Katsouleas & Mori 1988)

Linear Plasma Wakefield Theory

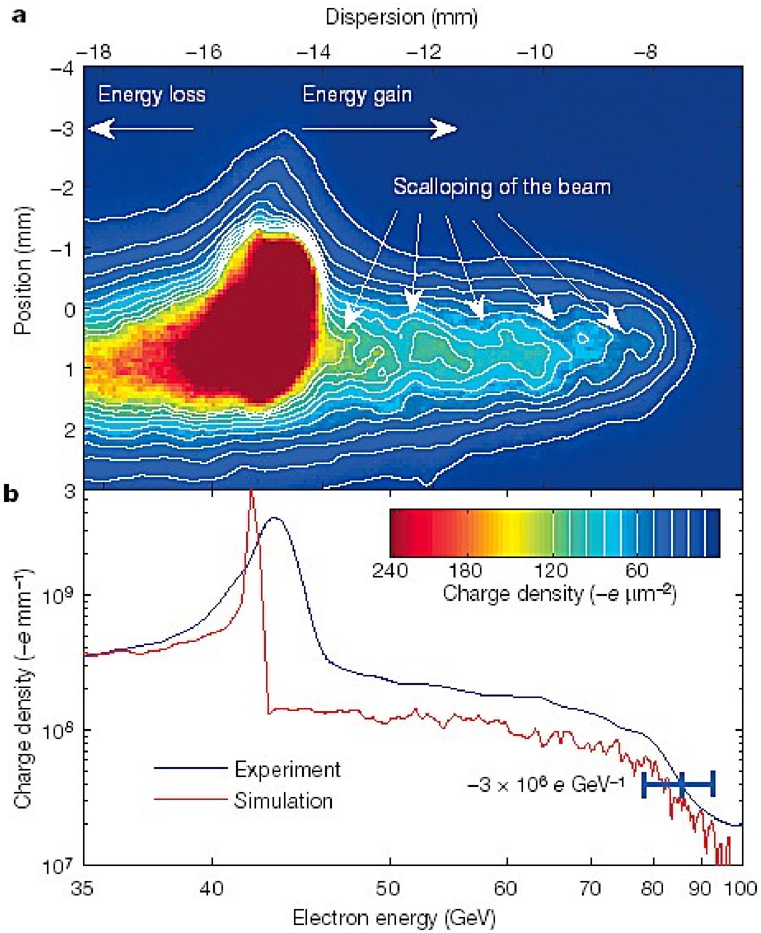
$$(\partial_t^2 + \omega_p^2)\frac{n_1}{n_o} = -\omega_p^2\left(\frac{n_b}{n_o} + k_p^2\nabla^2\sqrt{1+a_o^2}\right)$$

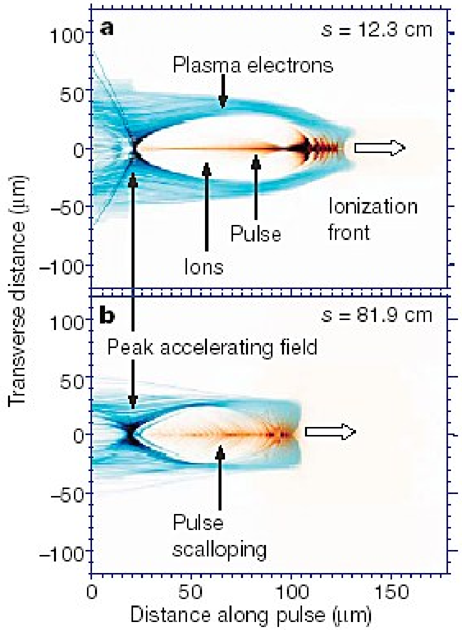
Large wake for a beam density $\mathbf{n_b \sim n_o}$ or laser amplitude $\mathbf{a_o = eE_o/m\omega_o c \sim 1}$ for τ_{pulse} of order $\omega_p^{-1} \sim 100\mathrm{fs}\,(10^{16}/n_o)^{1/2}$ and speed $\sim c = \omega_p/k_p$

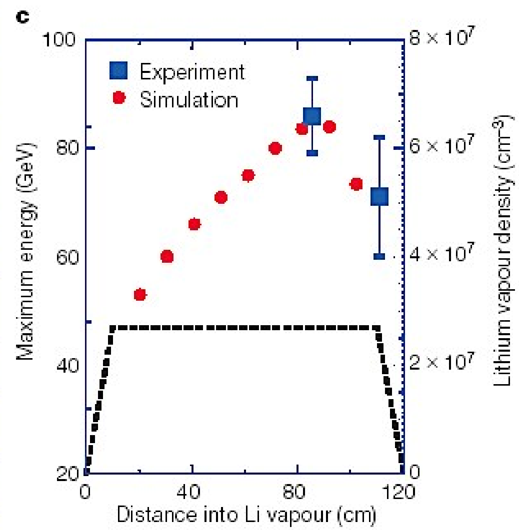
$$\nabla \bullet E = -4\pi e n_1 \Rightarrow \quad eE = \frac{n_1}{n_o}\sqrt{\frac{n_o}{10^{16}cm^{-3}}}\,10\,GeV/m\,\cos\omega_p(t - z/c)$$

But interesting wakes are very nonlinear => PIC simulations

SLAC Plasma Wakefield Expt.







a,b) Density of electron pulse (brown) and plasma electrons (blue) at two different points in the plasma (12.3 and 81.9 cm). Scalloping features are the result of increasing focusing force.

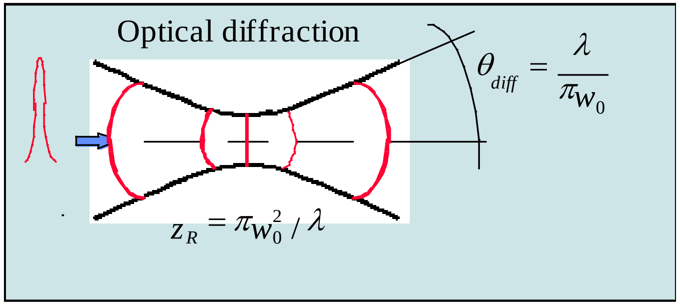
c) Maximum energy reached after 85 cm. Saturation occurs due to the beam head spreading to the point that it can no longer ionize the lithium vapour.

a) Energy spectrum of electrons in the 30-100 GeV range. Electrons reach 85 GeV ($3x10^6$ e/GeV).

b) Experimental (blue) and simulation (red)

Reference: Blumenfeld *et al. Nature* (2007)

3 Limits to Energy gain $\Delta W = eE_z L_{acc}$ (laser driver)



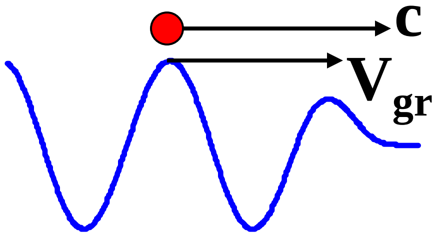
- Diffraction:

$$L_{dif} \cong \pi L_R = \pi^2 w_0^2 / \lambda$$

order mm!

(but overcome w/ channels or relativistic self-focusing)

- Dephasing:



$$L_{dph} = \frac{\lambda_p / 2}{1 - V_{gr}/c}$$

order 10 cm x $10^{16}/n_o$

- Depletion:

For small a_0 >> L_{dph}

For a_0 >~ 1 L_{dph} ~ L_{depl}

$$\Delta W_{ch}[MeV] \sim 60 (\lambda_p / w_0)^2 P[TW]$$

Requirements for High Energy Experiments

- Use Collider Parameters
- Luminosity = 10^{31} cm^{-2} sec^{-1}
- Beam Energy ~ 1 TeV
- No. of particles per pulse ~ 10^{11}
- Total Laser Energy (assuming 40% transfer efficiency) = 40 kJ/pulse
- Laser efficiency 10%
- Multiple staging required, pulse rate 1 kHz
- For a 10 stage accelerator requires 10 x 4 kJ lasers
- Power requirements :

$$P_{TOTAL} = 40 \text{ kJ} \times f \quad (\text{pulse rate second}^{-1})$$

$$P_{TOTAL} = 0.4 \text{ GW Power}$$