Timing, Synchronization & Longitudinal Aspects I

H. Damerau

CERN

CAS Course on
Beam Injection, Extraction and Transfer

13 March 2017

Outline

- Introduction
- General concepts
  - Signals with noise, transmission of RF signals
  - Phase detectors and dividers
- Beam transfer
  - Fundamental periodicity
  - Transfer between circular lepton accelerators
- Transfer between hadron accelerators
  - Beam phase loop, bucket numbering
  - Transfer process: Synchronization, transfer triggers
  - Longitudinal matching
- Summary

Introduction

Introduction

- **Two or more people must be synchronized to meet**
  - → **Calendar item: date, time and location**
  - → **Typical uncertainty: some minutes**

- **Slightly more precision required to have a meeting with a particle beam**
  - → **Typical uncertainty: some nanoseconds down to femtoseconds**

→ To be at the right time in the right place

→ Set conditions and generate timings and RF signals with a given time relation with respect to the beam

→ Make beam feel comfortable in its new accelerator

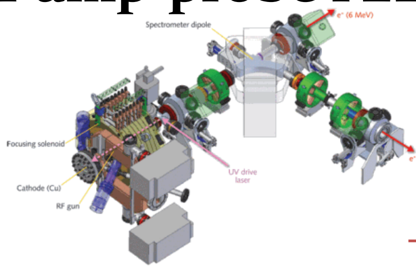
Timescales

Proton synchrotrons at medium energy

Electronics
Low-level RF systems

SASE FELs

Pump-probe FELs



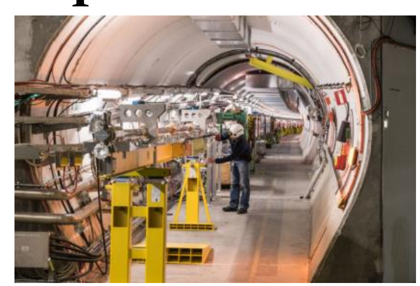
10 ns
1 ns
100 ps
10 ps
1 ps
100 fs
10 fs

Proton bunches in low energy synchrontrons

Hadron colliders

Electron storage rings

Plasma wakefield experiments



→ Geometrical size: few meters to some km

Synchronization for beam transfer

- **How to get the beam trough the accelerator?**

Source → Exit

- **How to transfer beam from accelerator A to B?**

Accelerator A → Accelerator B

- **Beam passes many elements on its way:**

→ RF structures → Must be in phase

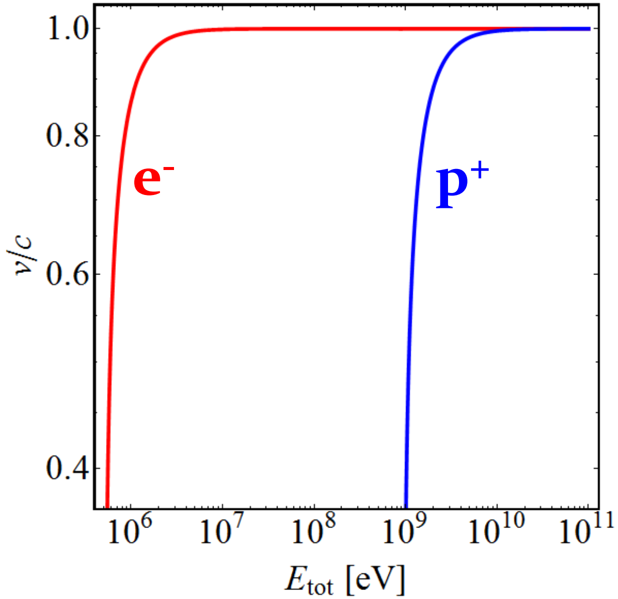
→ Septa, bumper and kicker magnet → Trigger

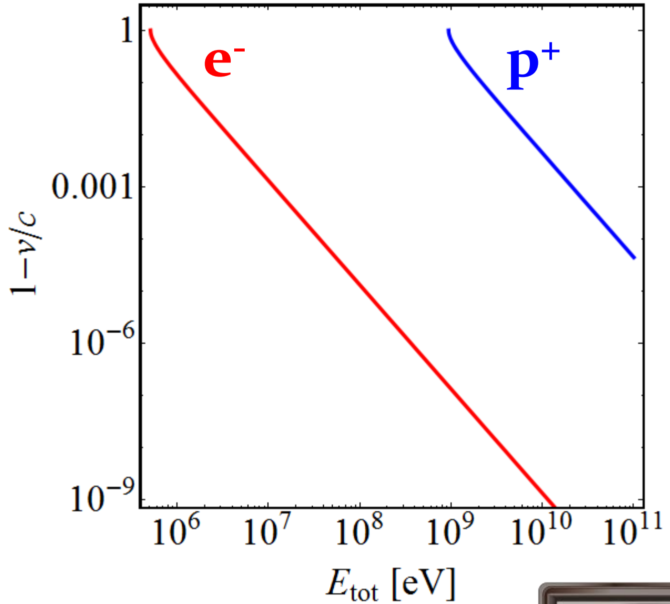
→ Fast beam instrumentation → Trigger

→ RF systems in source and target accelerator → Correct phase with respect to beam

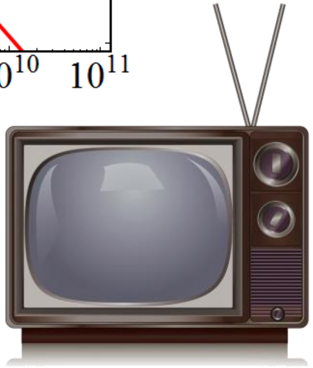
Particle velocity

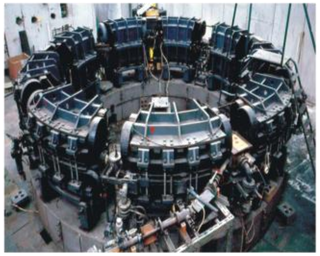
- **Particle velocity depends on its type:** $\beta = v/c = \sqrt{1-(E_0/E)^2}$





- **Old television set (30 kV):** **Electrons at 30% of c_0**
  Protons just at 0.7%
- **Small synchrotron (500 MeV):** **Electrons at 99.99995%**
  Protons at 75.8%





→ **Many electron accelerators at 'fixed' frequency**

Synchronization needs for particle types

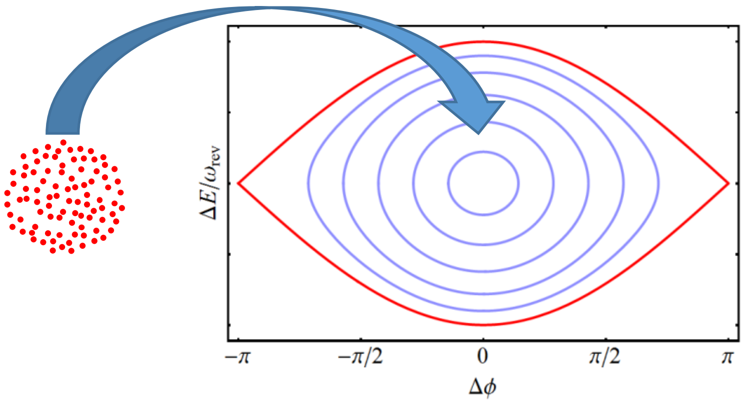
Lepton accelerators	Hadron accelerators

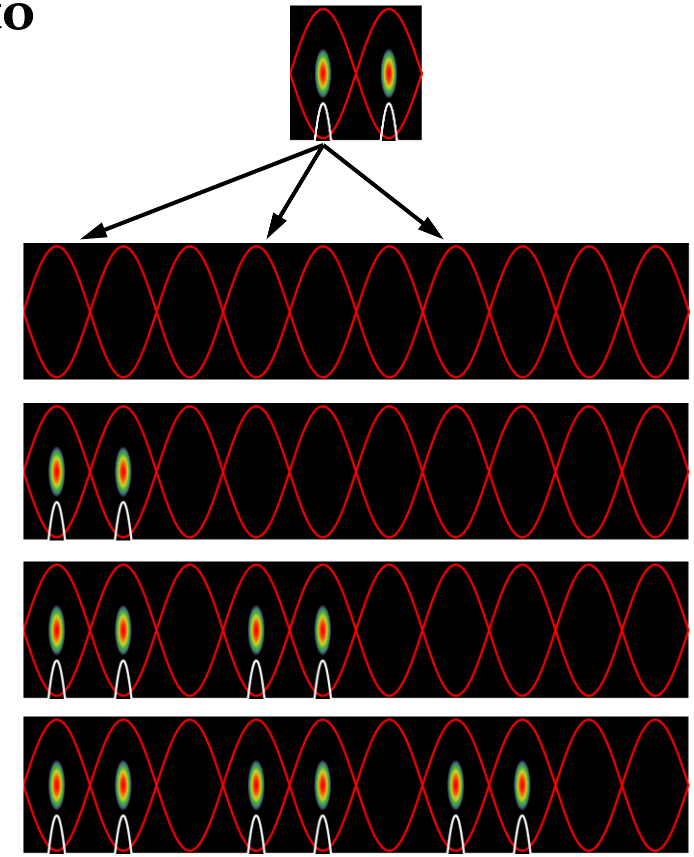
• Velocity $v \approx c$ in high energy accelerators
• Synchrotron radiation damping (mainly circular accelerators)
• Short bunches
  • Storage rings: ~10...100 ps
  • Linear free electron lasers: 50...200 fs
	• Slow, even velocity change relevant to the multi-GeV range
	• Negligible or small damping from synchrotron radiation
	• Long bunches
	  • Synchrotrons: 1...1000 ns (depends on RF frequency)
	  • Linear accelerators: typically few ns

→ Fixed frequencies → High precision	→ Variable (sweeping) frequencies → Moderate precision

Bunch-to-bucket transfer

- **Bunch from sending accelerator into the bucket of receiving**





Advantages:

→ **Particles always subject to longitudinal focusing**

→ **No need for RF capture of de-bunched beam in receiving accelerator**

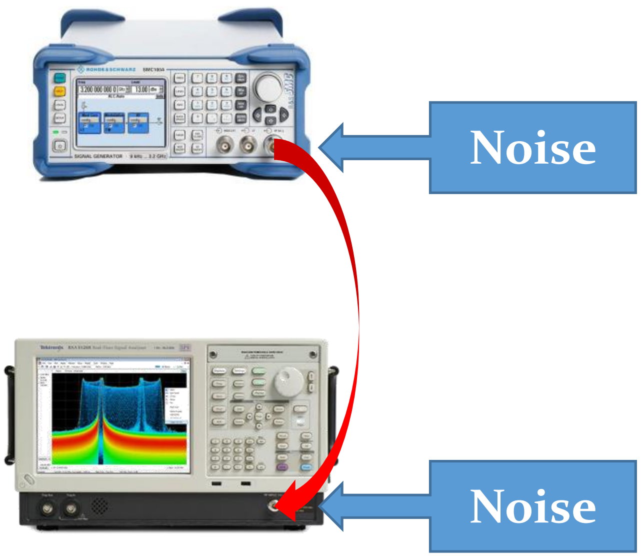
→ **No particles at unstable fixed point**

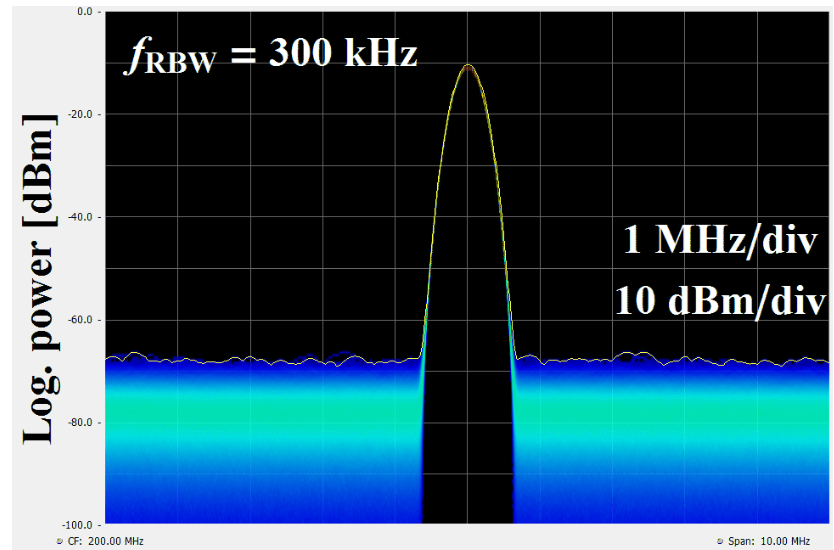
→ **Time structure of beam preserved during transfer to the next**

Noise on signals

Noisy signals

- Degradation of signal quality due to noise
  - Amplitude and/or phase jitter
- What is the difference between a coherent signal and noise?





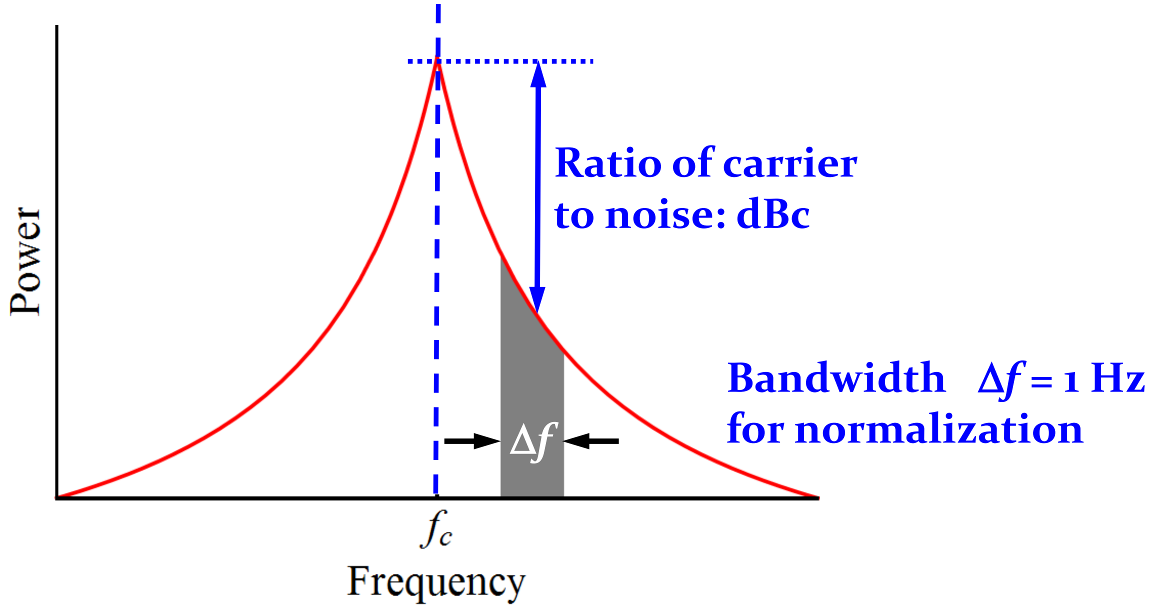
→ Amplitude of coherent, quasi monochromatic signal (at 200 MHz) is independent of observation bandwidth

→ Incoherent noise power (dominated by spectrum analyzer front-end amplifier/mixer) is proportional to bandwidth

→ Thermal noise power $\frac{P}{\Delta f} = k_{\mathrm{B}}T = 1.38 \cdot 10^{-23}\ \mathrm{J/K} \cdot 296\ \mathrm{K} \simeq -174\ \mathrm{dBm/Hz}$

Analysis of phase noise

- **Compare noise power with carrier power as reference**



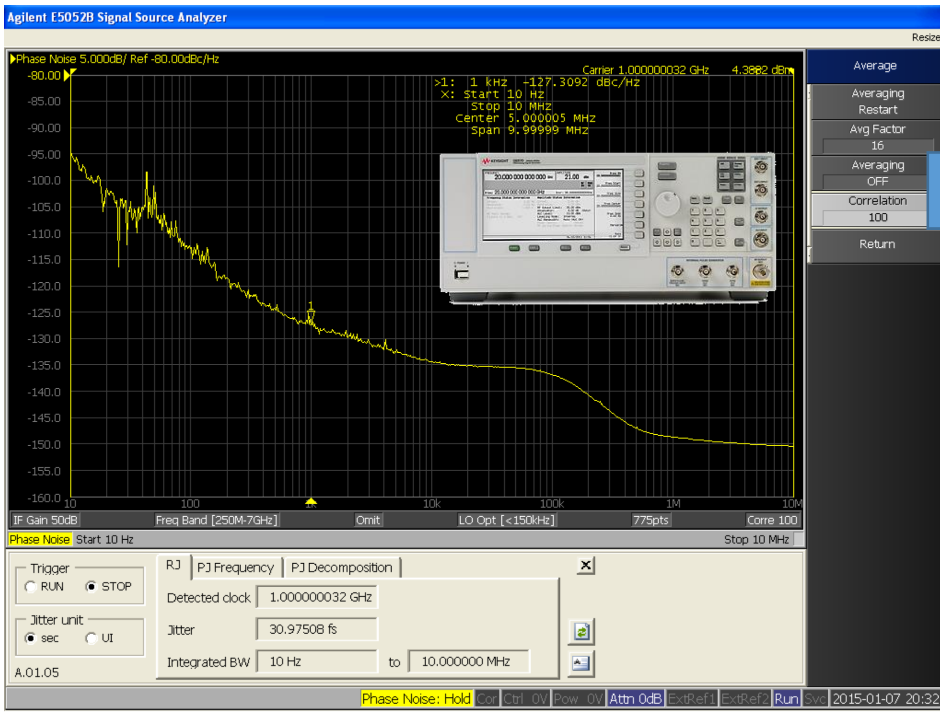
- **Noise power density** $$\mathcal{L}(f) = \frac{\text{Power density}}{\text{Carrier power}} \left[\frac{\text{dBc}}{\text{Hz}}\right] = \frac{1}{2} S_\phi(f)$$

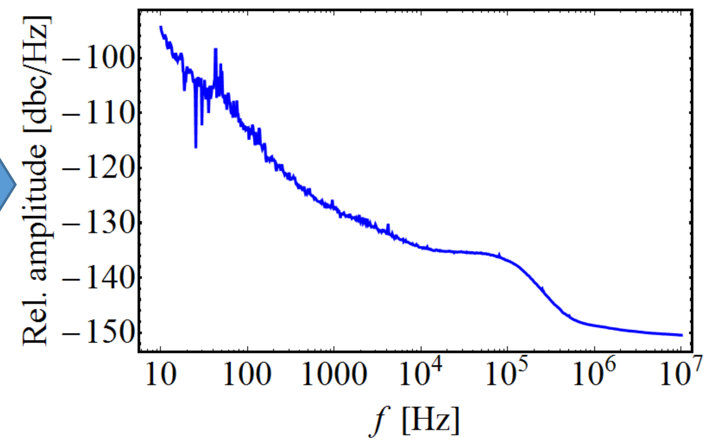
$\rightarrow$ **Its integral is the phase jitter and using** $\Delta t = \dfrac{\Delta\phi}{2\pi f_c}$

the jitter in time becomes $$\Delta t_{\text{rms}} = \frac{1}{2\pi f_c} \sqrt{\int_{f_1}^{f_2} S_\phi(f)\, df}$$

Typical phase noise plots

- **Measure phase noise of a synthesized lab generator**





Frequency range	Δt_{rms} **[fs]**
10…100 Hz	12.4
100 Hz …1 kHz	5.4
1…10 kHz	5.4
10…100 kHz	11.1
100 kHz…1 MHz	13.0
Total	31.0

→ **Note: jitter values can be added as square root of quadratic sum**

$$\Delta t_{\mathrm{rms}} = \sqrt{\Delta t^2_{\mathrm{rms},1} + \Delta t^2_{\mathrm{rms},2} + \ldots}$$

→ **Convenient split to relevant ranges**

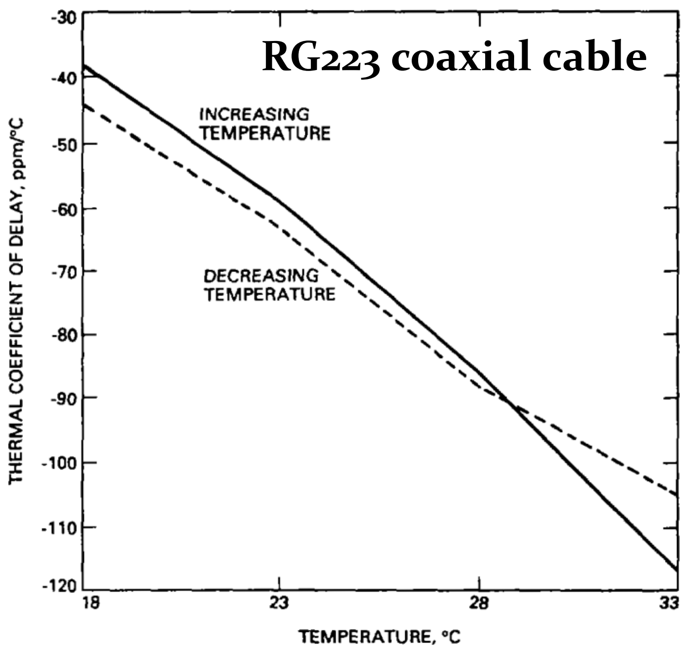
Signal transmission

Transmission of reference signals

- Thermal drift of long coaxial cables or optical fibres

- Thermal coefficient of delay:

$$\mathrm{TCD} = \frac{\Delta\tau}{\tau} \cdot \frac{1}{\Delta T} = \frac{\Delta\phi}{\phi} \cdot \frac{1}{\Delta T}$$



Lutes/Diener, NASA TDA Progress Report 42-99

- Example: 2 km long RG223 cable with ~10 μs delay

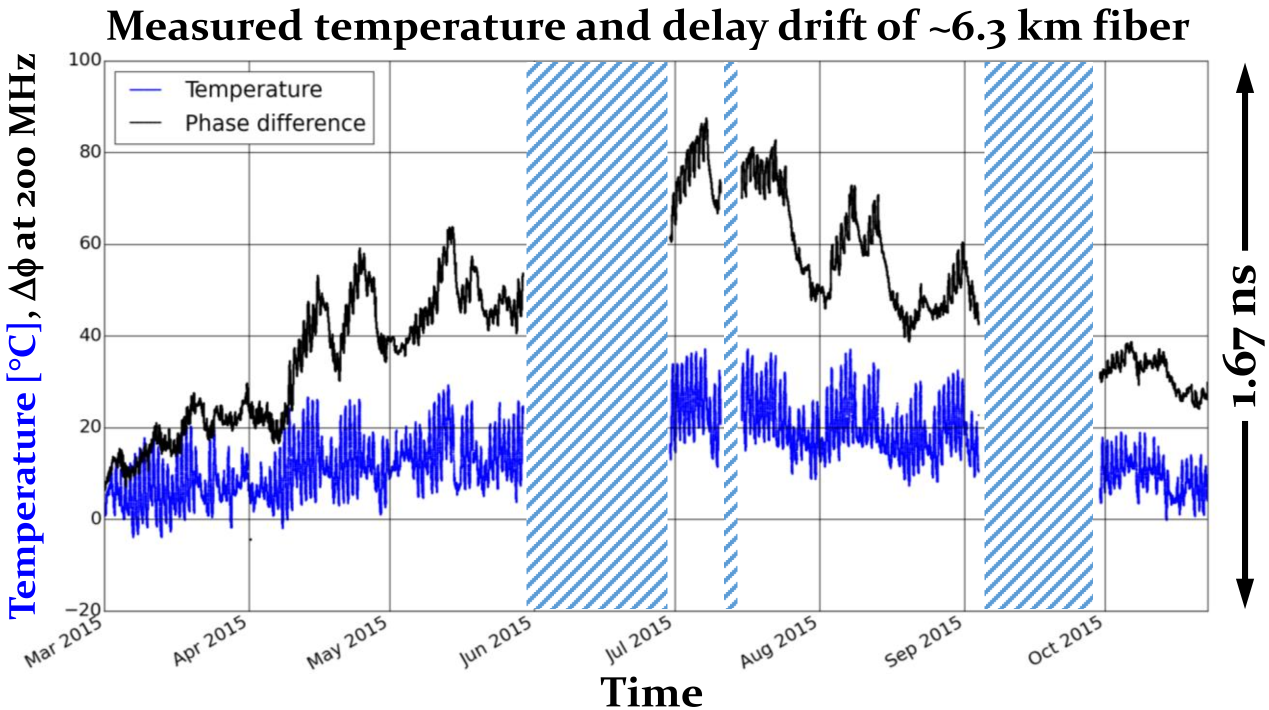
→ ΔT of only 1° C (room temperature) changes delay by ~0.5 ns

→ 1.8° at 10 MHz (CERN PS), but 73° at 400 MHz (LHC)

- Optical fibres are typically 10...100 times more stable
- What to do if this is still not sufficient?

Transmission of reference signals

- Measured drift of optical fibres over long distance standard optical fibre

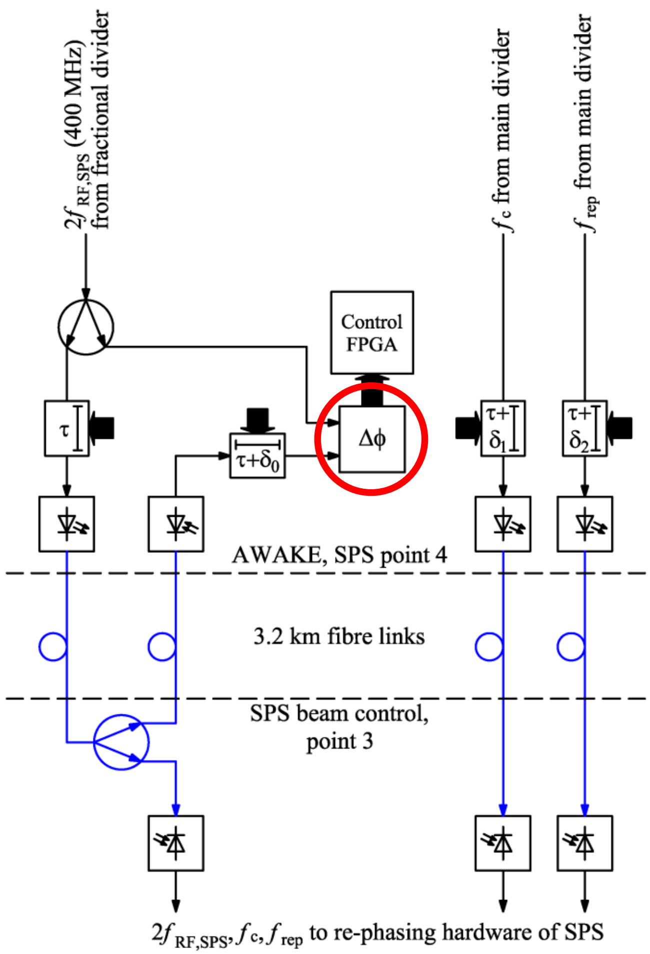


- Drift by about 1 ns insufficient for requirements of setup

→ Active compensation of delay

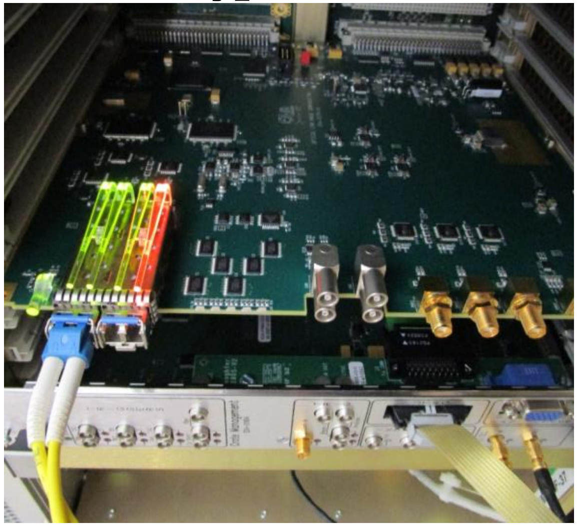
Example: Active drift compensation

Simplified diagram



- **Precise synchronization of proton beam from CERN SPS with plasma wake-field experiment AWAKE**

Prototype hardware

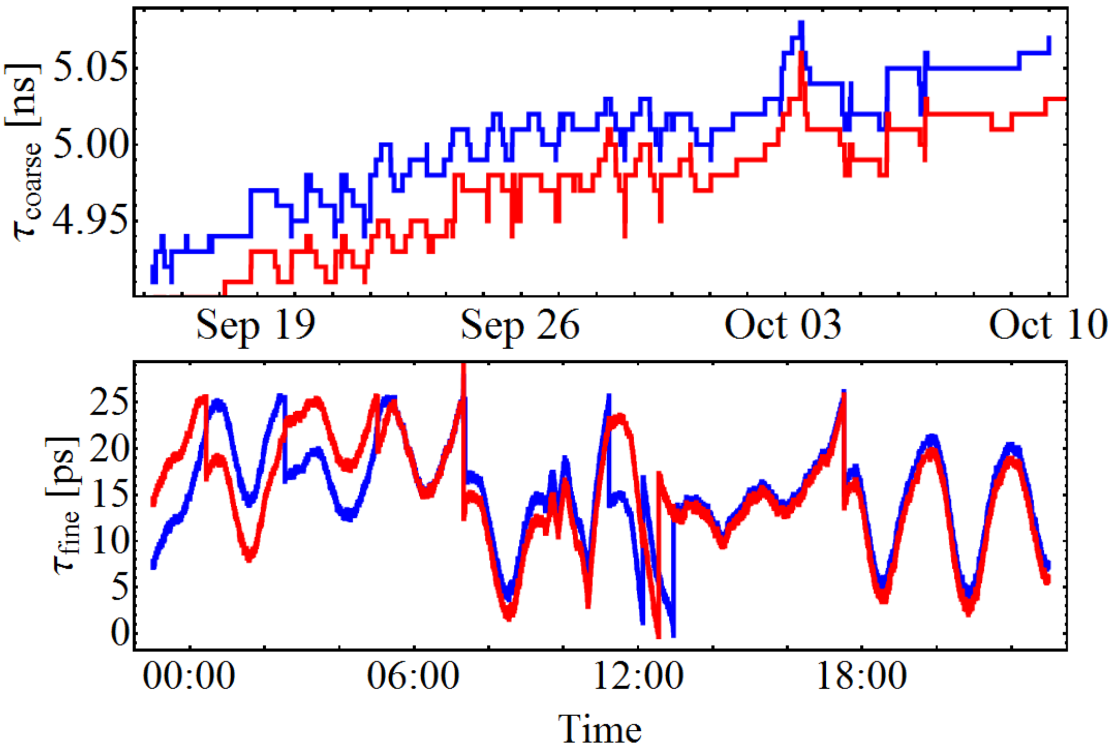


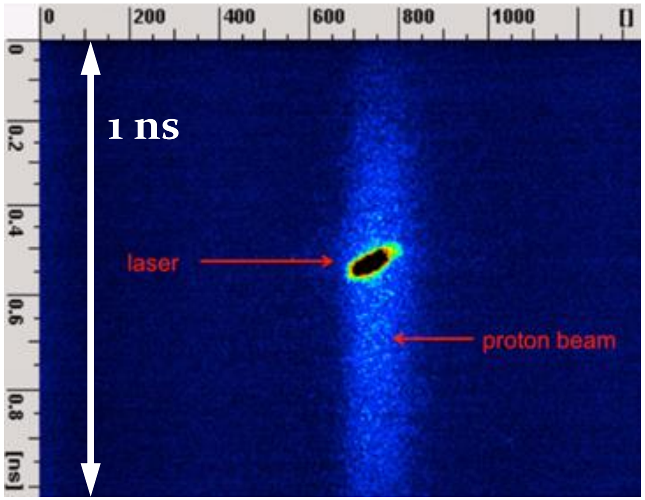
→ **Expect picosecond precision over several kilometres**

D. Barrientos, J. Molendijk

Transmission of reference signals

- **Total delay composed of coarse (steps of 10 ps) and fine ~30 ps range: $\tau = \tau_{coarse} + \tau_{fine}$**





→ **Precision difficult to evaluate without 2nd 'reference' link**

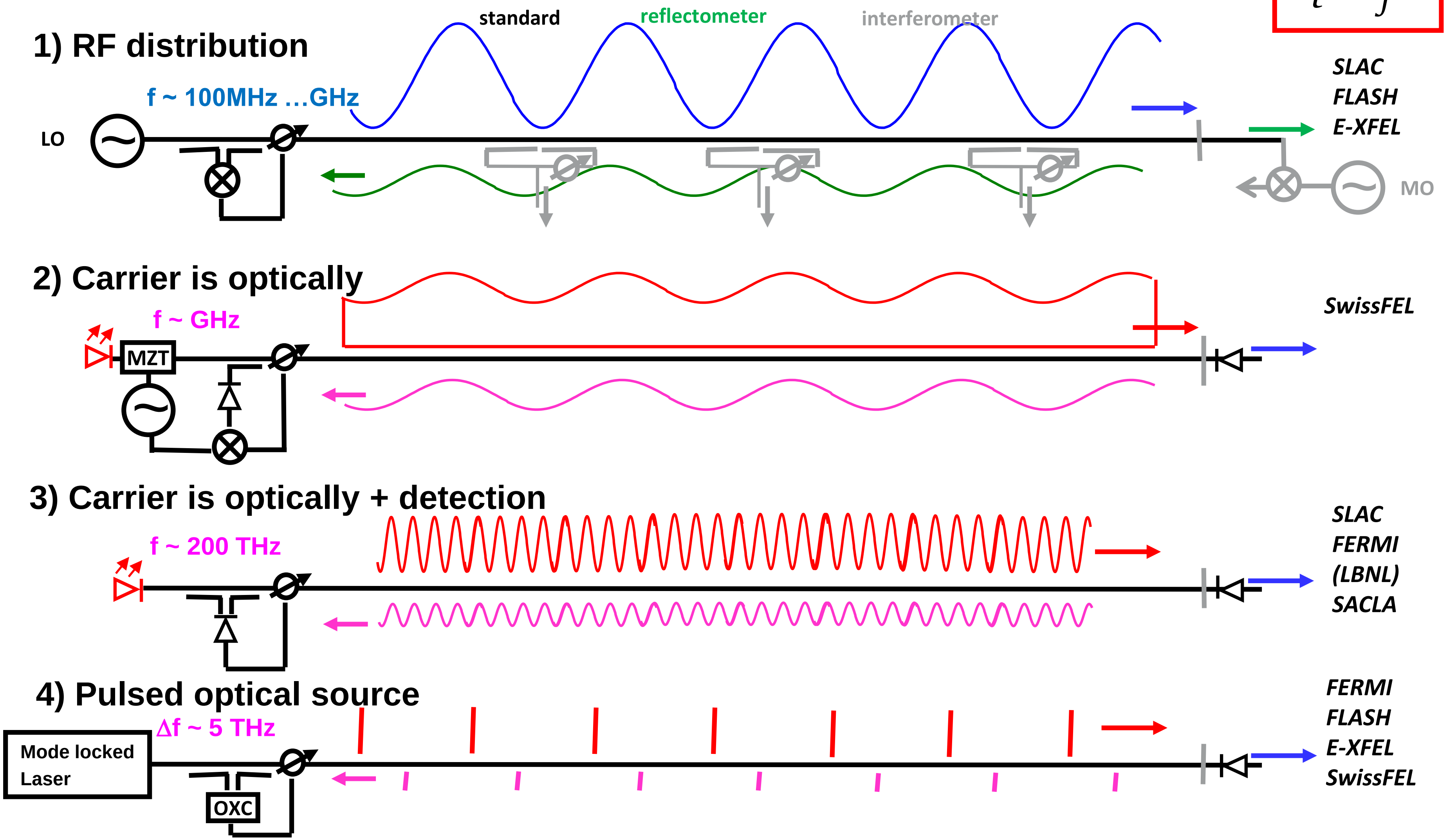
→ **Arrival of two beams in AWAKE experiment stable to better ~100 ps over months**

D. Barrientos, J. Molendijk

Overview of transmission methods

Various approaches:

$$\frac{\Delta t}{t} = \frac{\Delta f}{f}$$

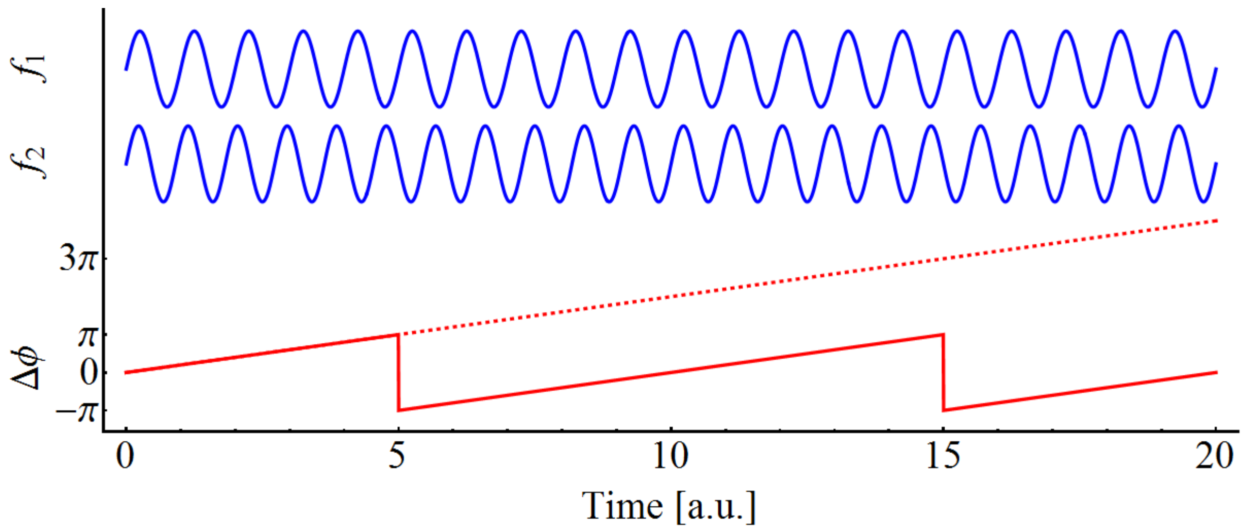


H. Schlarb

Phase detection

Frequency and phase

- **Two signals at different frequencies ω_1 and ω_2**



→ **Phase difference, $\Delta\phi$, between both signals changes linearly**

→ **Ambiguity to distinguish between $\Delta\phi$ = $-\pi$, π, -3π, 3π,…**

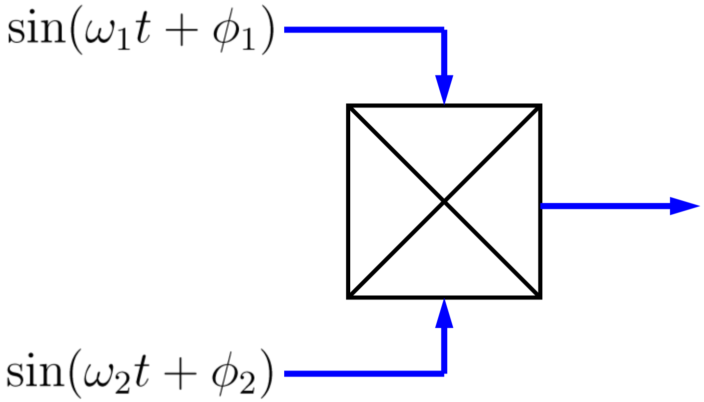
→ **Saw-tooth in phase means constant frequency difference**

→ **Equivalence of frequency and phase**

$$\omega = \frac{d\phi}{dt} \quad \leftrightarrow \quad \phi = \int \omega \, dt$$

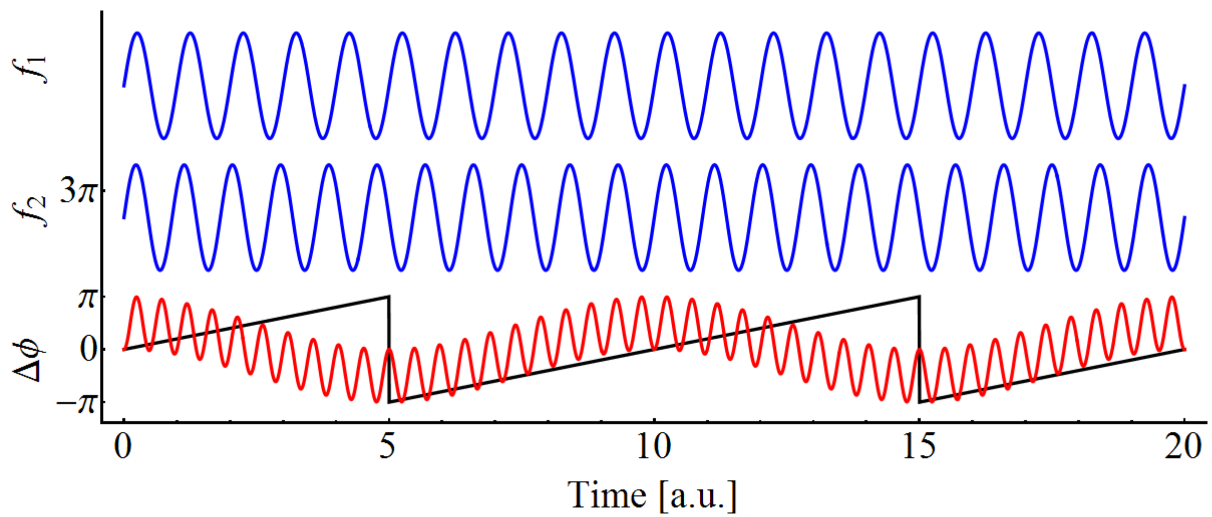
How to detect phase differences?

- **Example: analogue 4 quadrant multiplier and low pass filter**



$$\frac{1}{2}\{\cos\left[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)\right] - \cos\left[(\omega_1 + \omega_2)t + (\phi_1 + \phi_2)\right]\}$$

- **Signals:**



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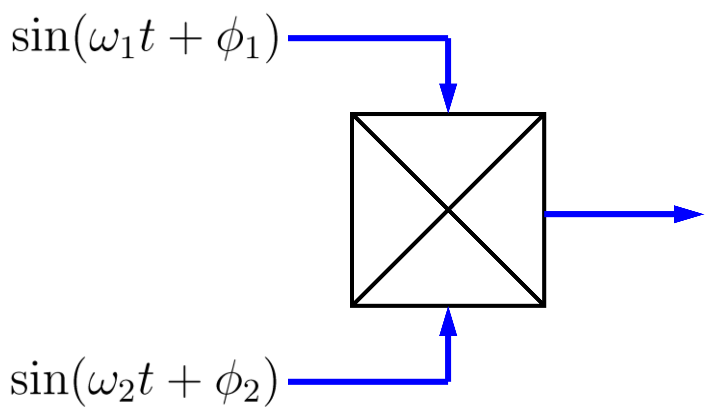
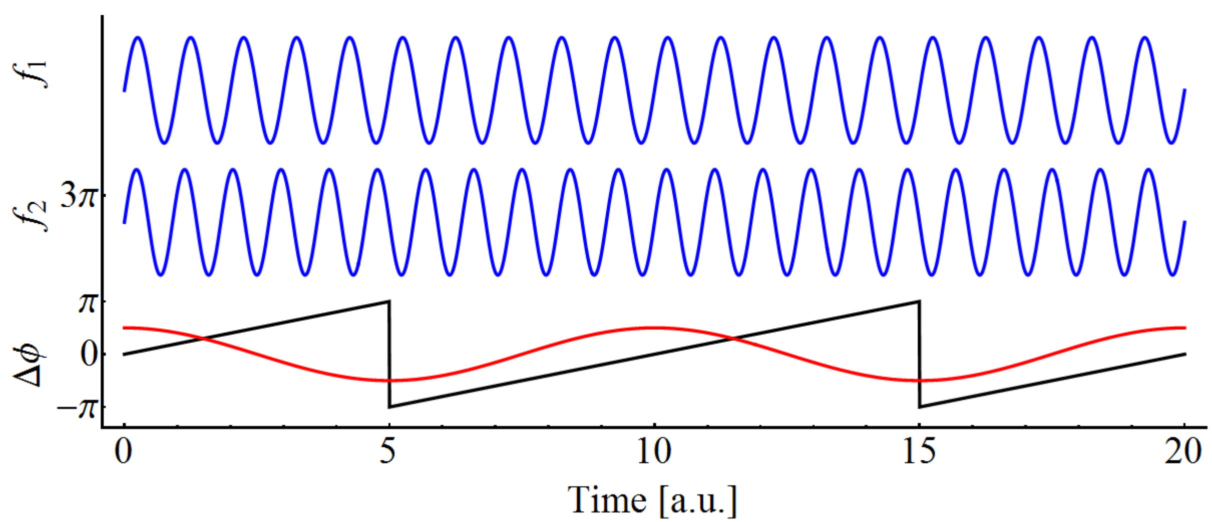


$$\frac{1}{2}\{\cos\left[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)\right]$$

$$\cancel{\cos\left[(\omega_1 + \omega_2)t + (\phi_1 + \phi_2)\right]\}}$$

Remove ripple → Low-pass filter

- **Signals:**



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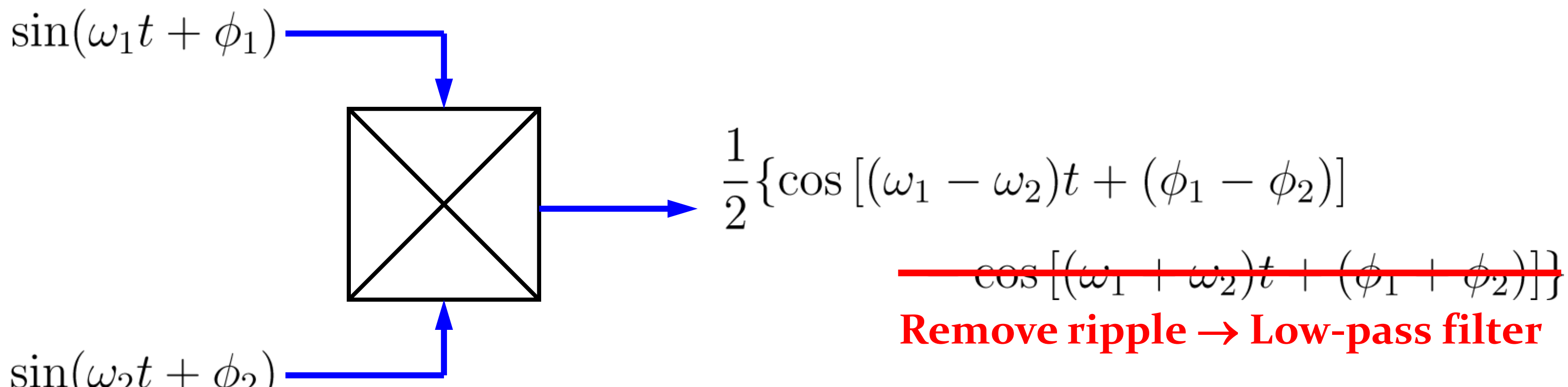
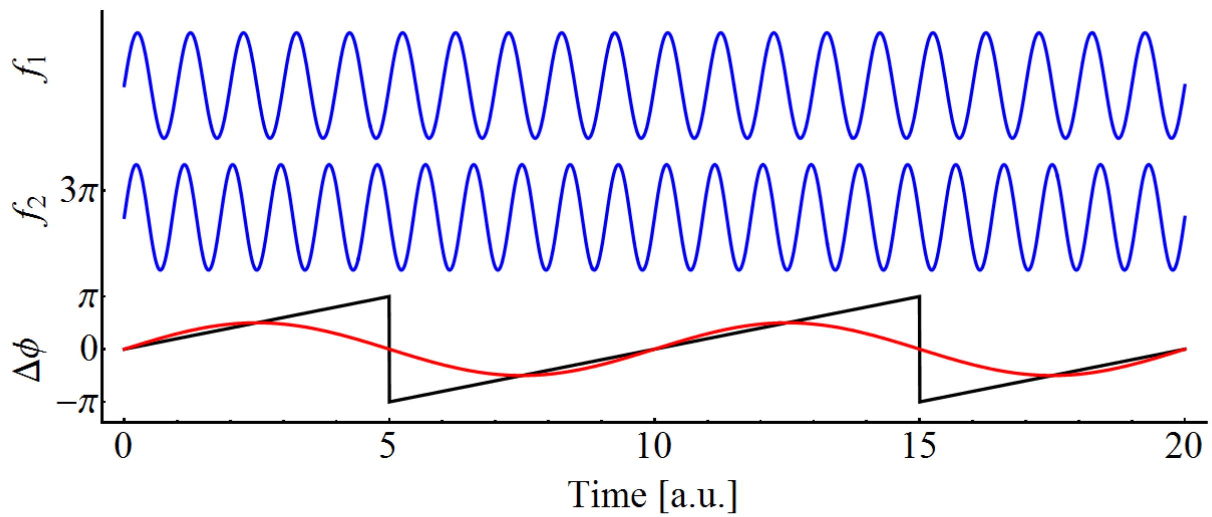


Remove ripple → Low-pass filter

Relative: arbitrary shift by 90°

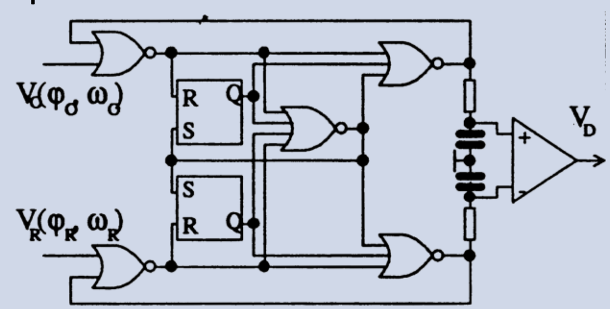
- **Signals:**



- **Phase discriminator in approximately +/-90° range**

Further phase detection techniques

Multitude of different phase discriminators

Type	Range	Behavior
Analogue 4 quadrant multiplier	π	Sinusoidal: $s_{out} \sim \cos \phi$
Exclusive OR gate	π	Linear: $s_{out} \sim \phi - 3\pi/2$, or $s_{out} \sim -\phi + \pi/2$
Sample and hold	π	Sinusoidal: $s_{out} \sim \sin \phi$
Flip-flop phase detector	π	Linear: $s_{out} \sim \phi - \pi$
Tri-state double flip-flop	2π	Linear: $s_{out} \sim \phi$ 
Balanced optical microwave phase detector (Sagnac loop)	$<\pi$	Sinusoidal: $s_{out} \sim \sin \phi$ (clipped)

- Full phase coverage of 2π range excludes ambiguity of $\pm\pi$

$\rightarrow$ Avoids locking of phase loop with unwanted offset

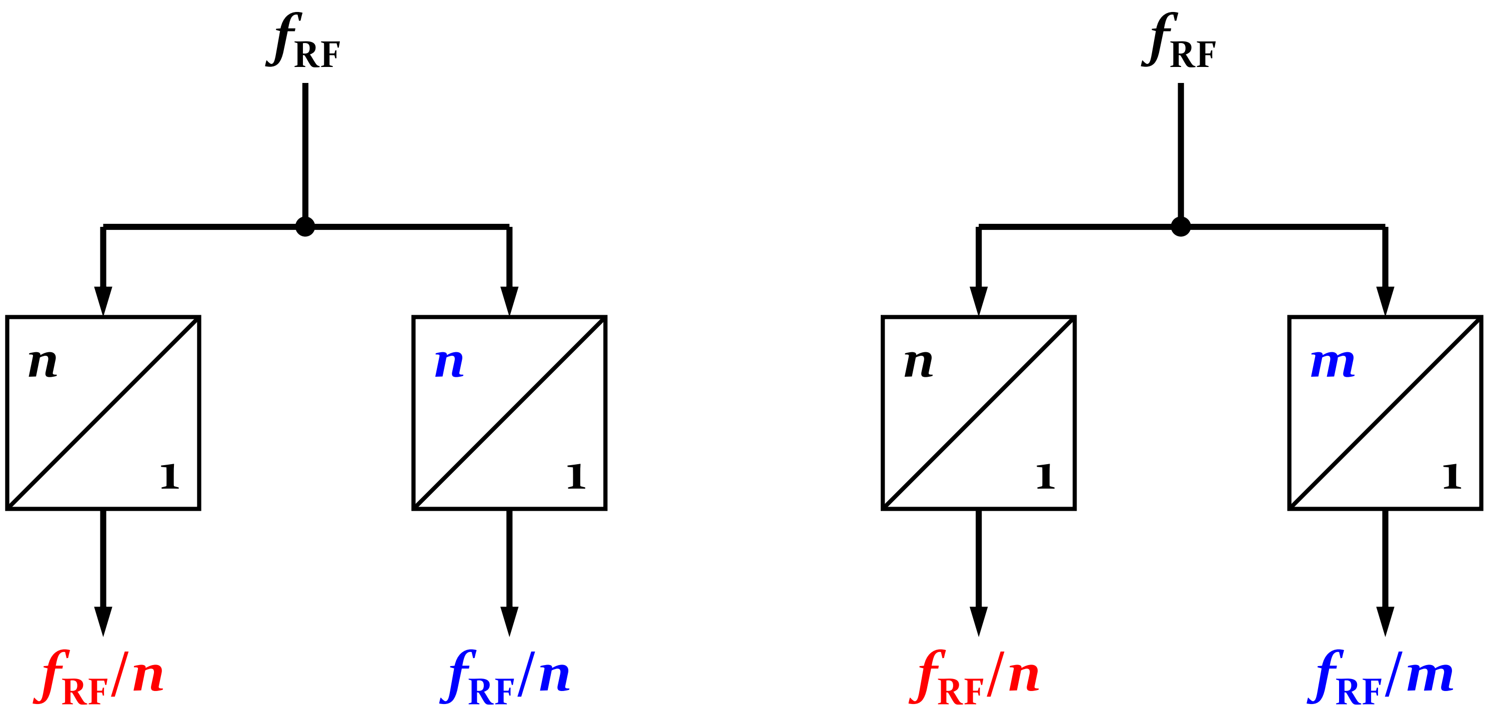
- Measure phase at high frequencies for precision

R. Garoby

Dividers

Frequency dividers

- Generate signals using frequency division from f_{RF}

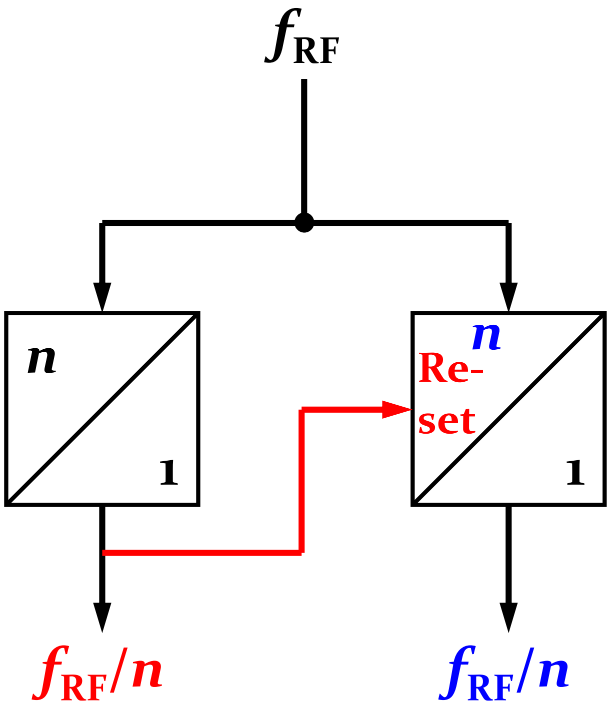


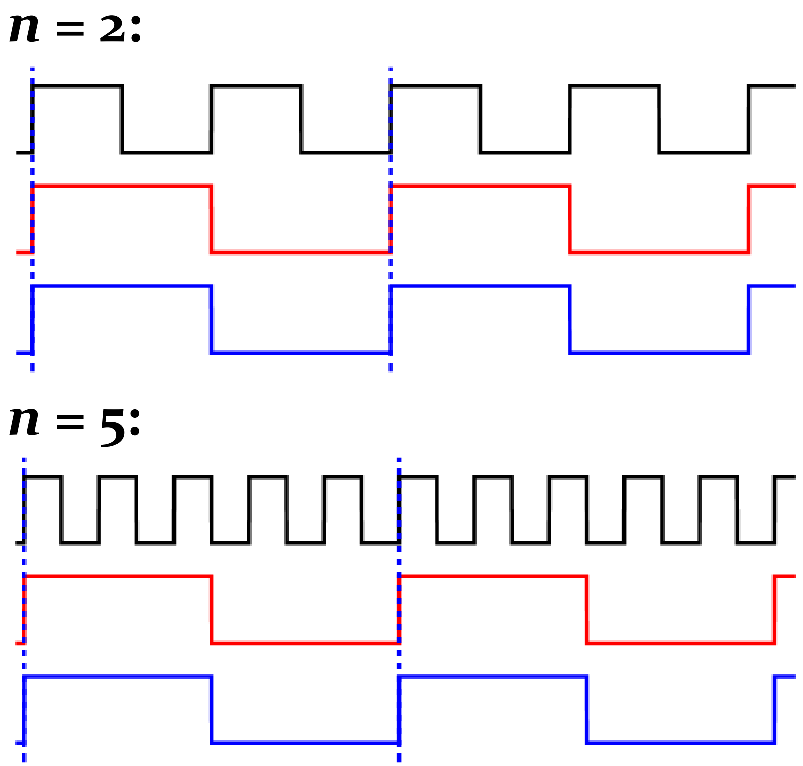
- Works (well, on paper), so what is the problem?

$\rightarrow$ Dividers are nothing but counters! Initial value?

Synchronizing multiple dividers

- Generate signals using frequency division from f_{RF}



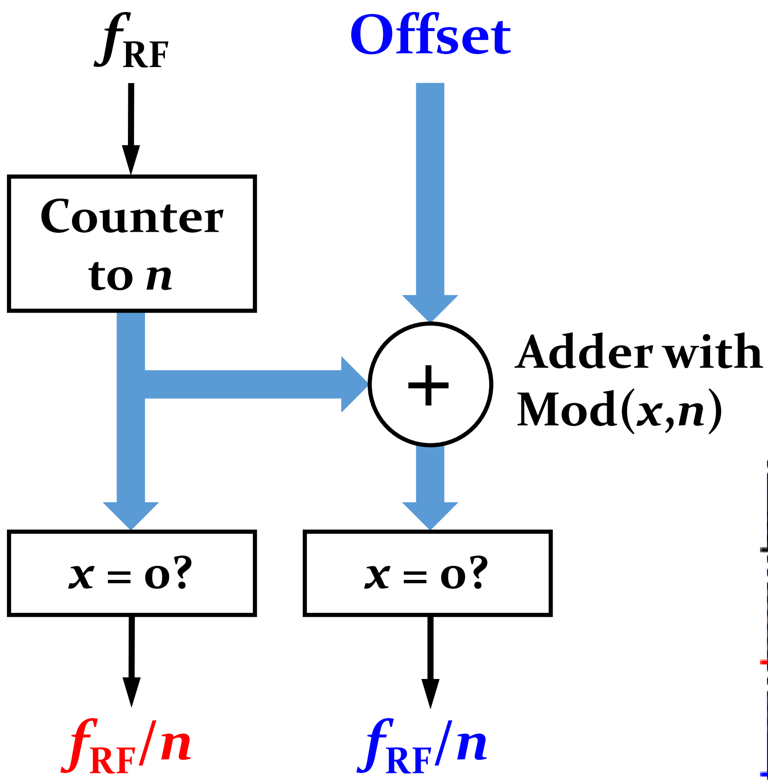


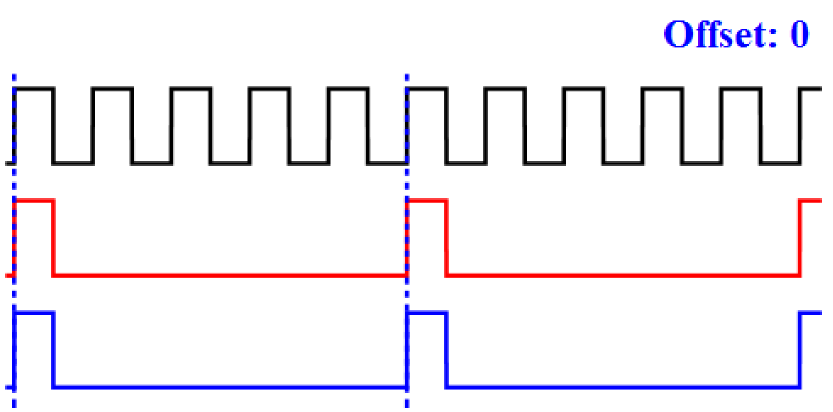
- How to fix?
- Reset from master to slave divider(s) to force initial condition

→ Never more than one divider without reset!

Multiple divider with counting offset

- **Counter with programmable offset value**





→ **Single counter/divider split in two output branches**

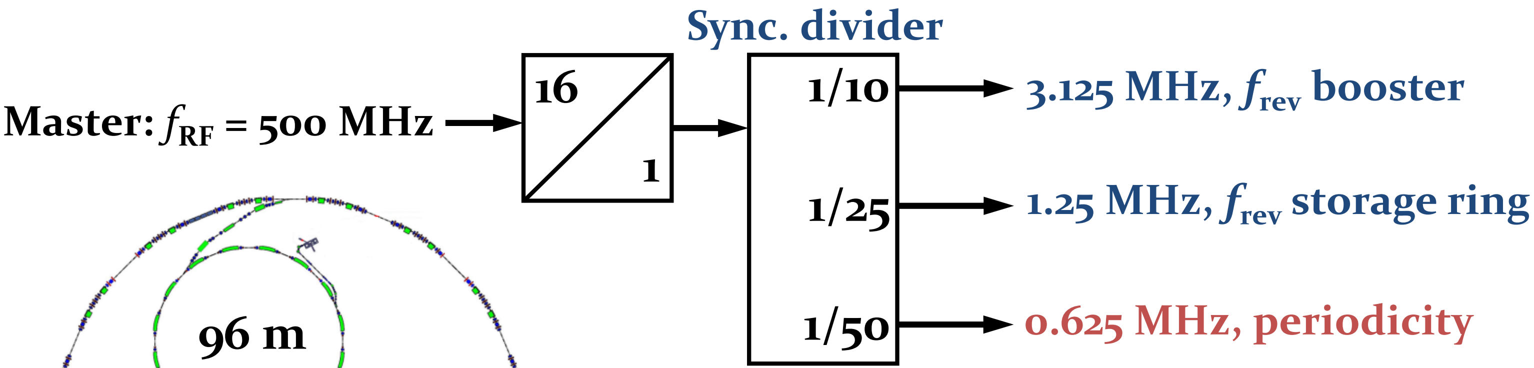
→ **Impossible to lose relative phase of outputs**

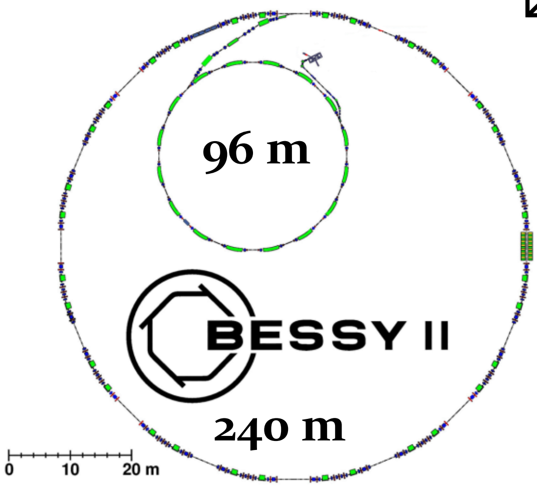
→ **More complicated set-up allows also f_{RF}/m and f_{RF}/n, etc.**

Fundamental periodicity

Example: BESSY II booster and storage ring

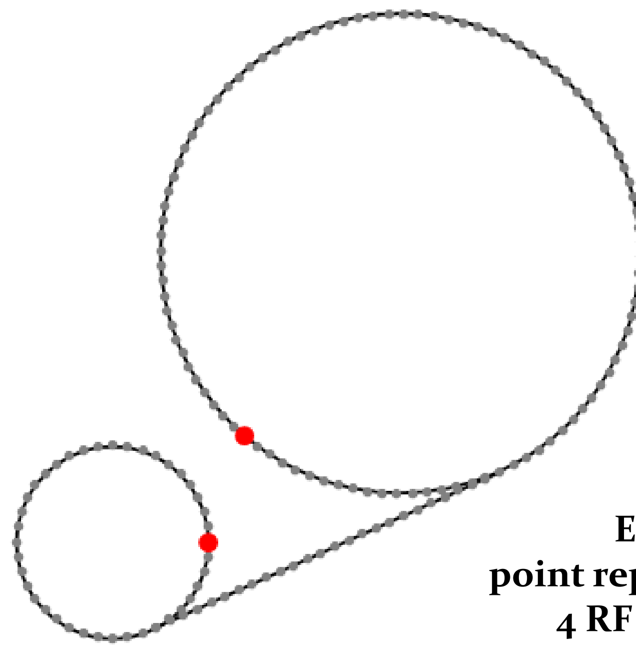
- Storage ring circumference 240 m, f_{RF} = 499.6 MHz
- Circumference ratio of Booster and storage ring: 2/5





→ **Everything repeats with periodicity of**

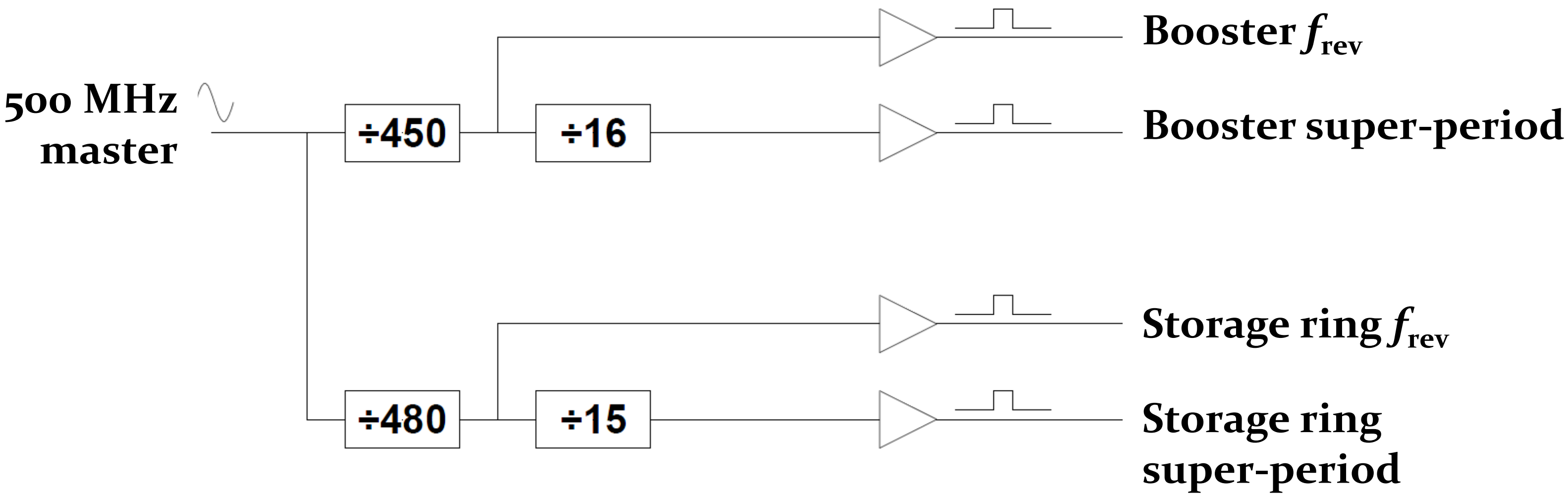
- 5 turns in booster
- 2 turns in storage ring



Each gray point represents 4 RF buckets

Example: SLS booster and storage ring

- Storage ring circumference 288 m, f_{RF} = 499.6 MHz
- Circumference ratio of Booster and storage ring: 15/16



K. Babak, C. Gough

→ Fundamental periodicity (super-period)

16 turns of booster corresponding to 15 turns in storage ring

Fundamental periodicity for transfer

- **Two accelerators with revolution periods $T_{\mathrm{rev},1}$ and $T_{\mathrm{rev},2}$**

$$T_{\mathrm{rev},2} = \frac{m}{n} T_{\mathrm{rev},1} \quad \rightarrow \quad T_{\mathrm{super}} = T_{\mathrm{common}} = T_{\mathrm{fiducial}} = m T_{\mathrm{rev},1} = n T_{\mathrm{rev},2}$$

→ **Beam transfer may take place at every period $mT_{\mathrm{rev},1}$ or $nT_{\mathrm{rev},2}$**

→ **This periodicity is, depending on the accelerator and laboratory, called super-period, common or fiducial period**

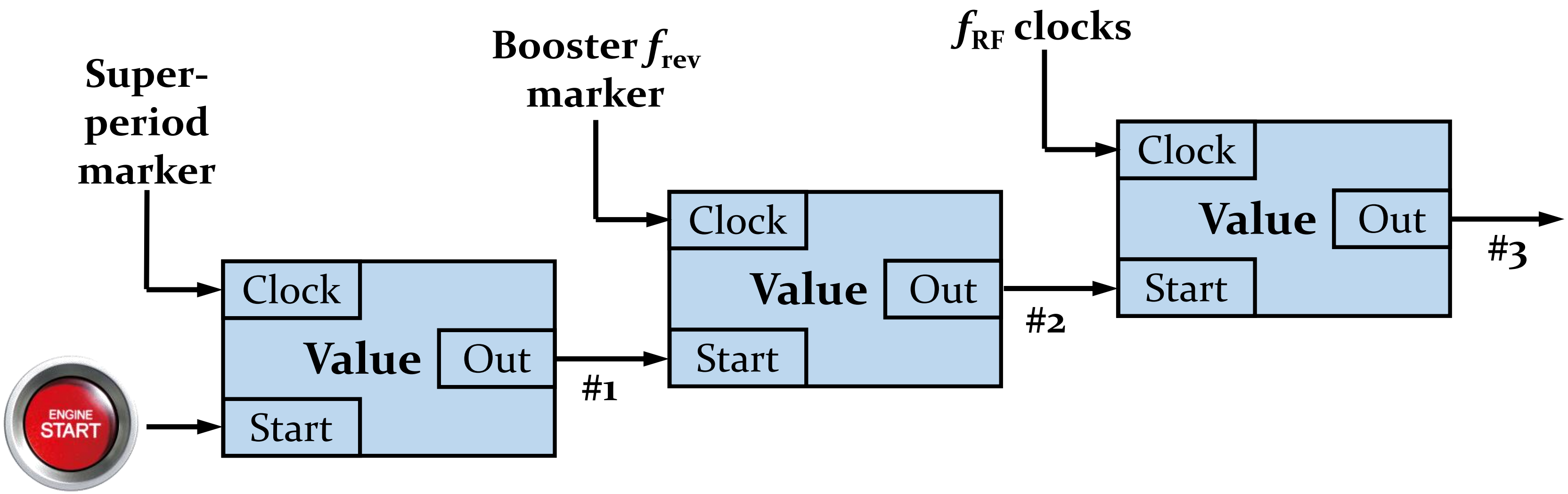
→ **In case of integer ratio of revolution frequencies, beam can be transferred once every turn of the larger accelerator**

Sending	Receiving	Ratio	Remark
BESSY booster	BESSY SR	2/5	Fixed frequency
SLS booster	SLS SR	15/16	Fixed frequency
J-PARC RCS	J-PARC MR	2/9	Profit from ratio for bucket selection
PS booster	PS	1/4	
PS	SPS	1/11	
PS	AD	3/1	Particle type and energy change at transfer
SPS	LHC	7/27	f_c as low 1.6 kHz

Synchronous triggers

How to generate beam synchronous triggers?

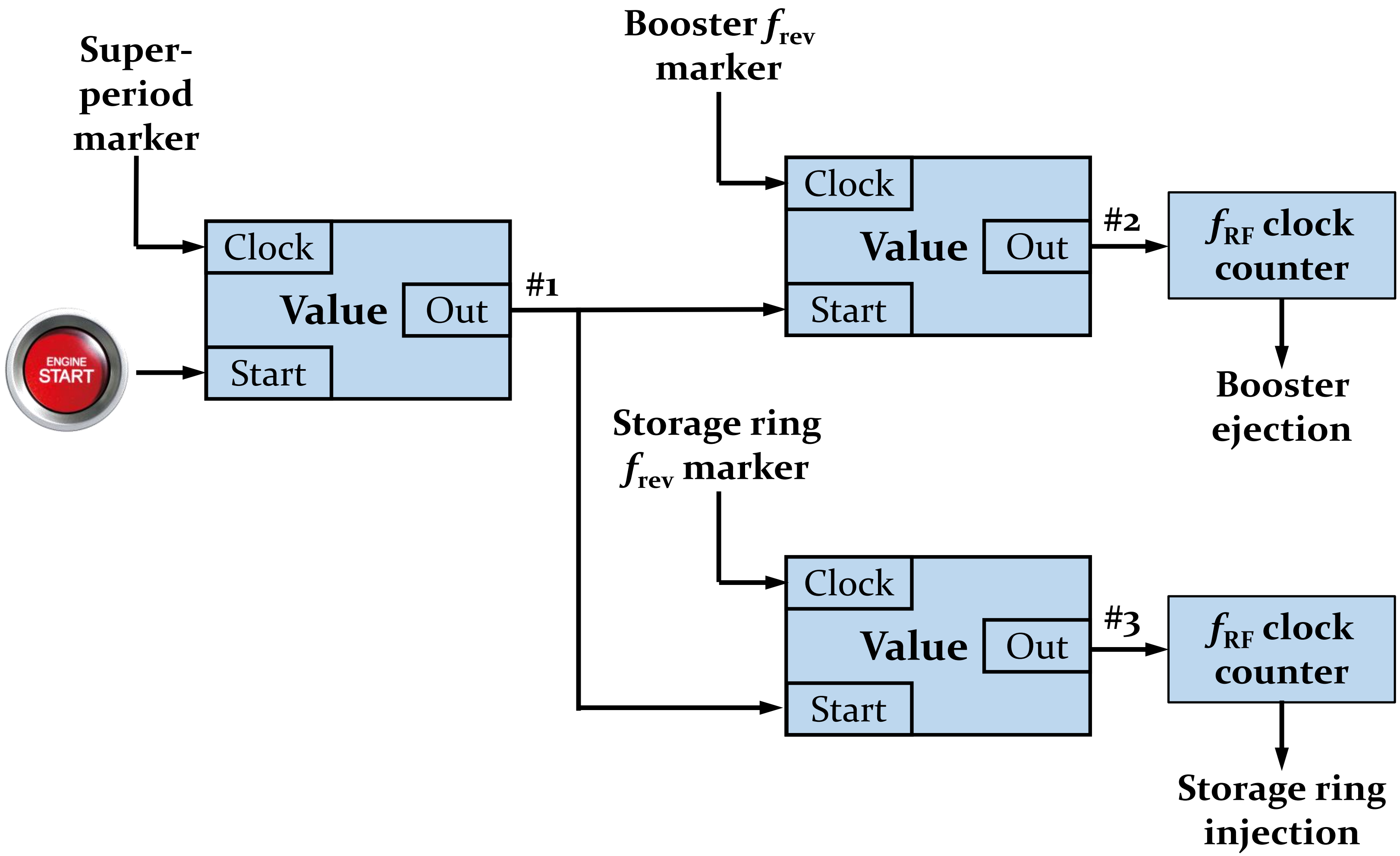
→ Chains of counters to re-synchronize timings



Each step re-synchronizes with respect counter clock

- ‘Start engine button’ synchronous to nothing
- Complete system of two accelerators periodic with timing #1
- Timing #2 marks, e.g., a delay in number of turns
- Timing #3 counts f_{RF} clocks to fine adjust, e.g., bucket number

Synchronous trigger trees



- **Timing counters may use different clocks, as long as the clocks are derived from the same source**

→ **Reproducible delay between clock #2 and #3**

→ **Tree structures of timings**

Circular electron/lepton accelerators

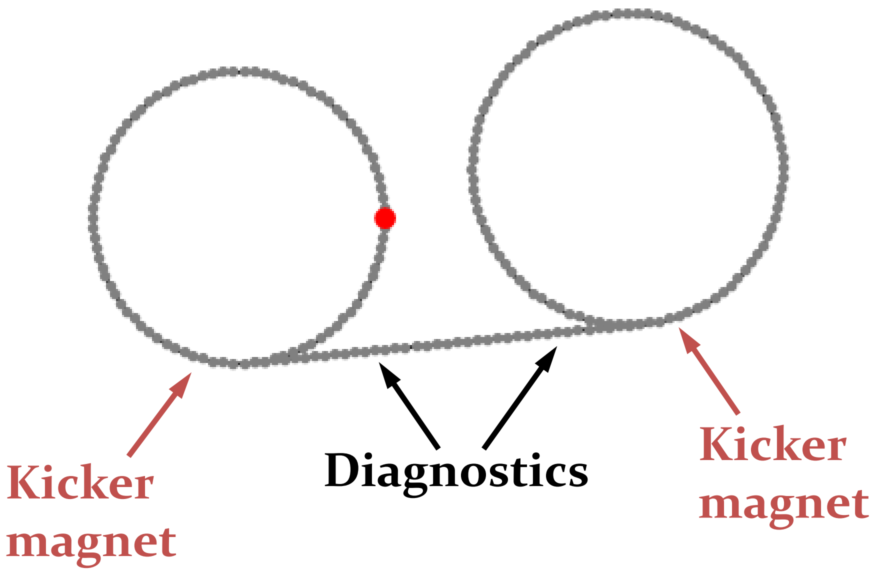
- **Simplification for most electron accelerators:**

→ **Leptons are practically at speed of light**

→ **Synchrotron radiation damping forces bunches into buckets**

→ **Beam synchronous timing triggers can be derived by counting RF master clock (or its sub-multiples)**

→ **Everything is predictable from the beginning**



Each gray point represents 5 RF buckets

→Let's get frequencies moving

Transfer between hadron accelerators

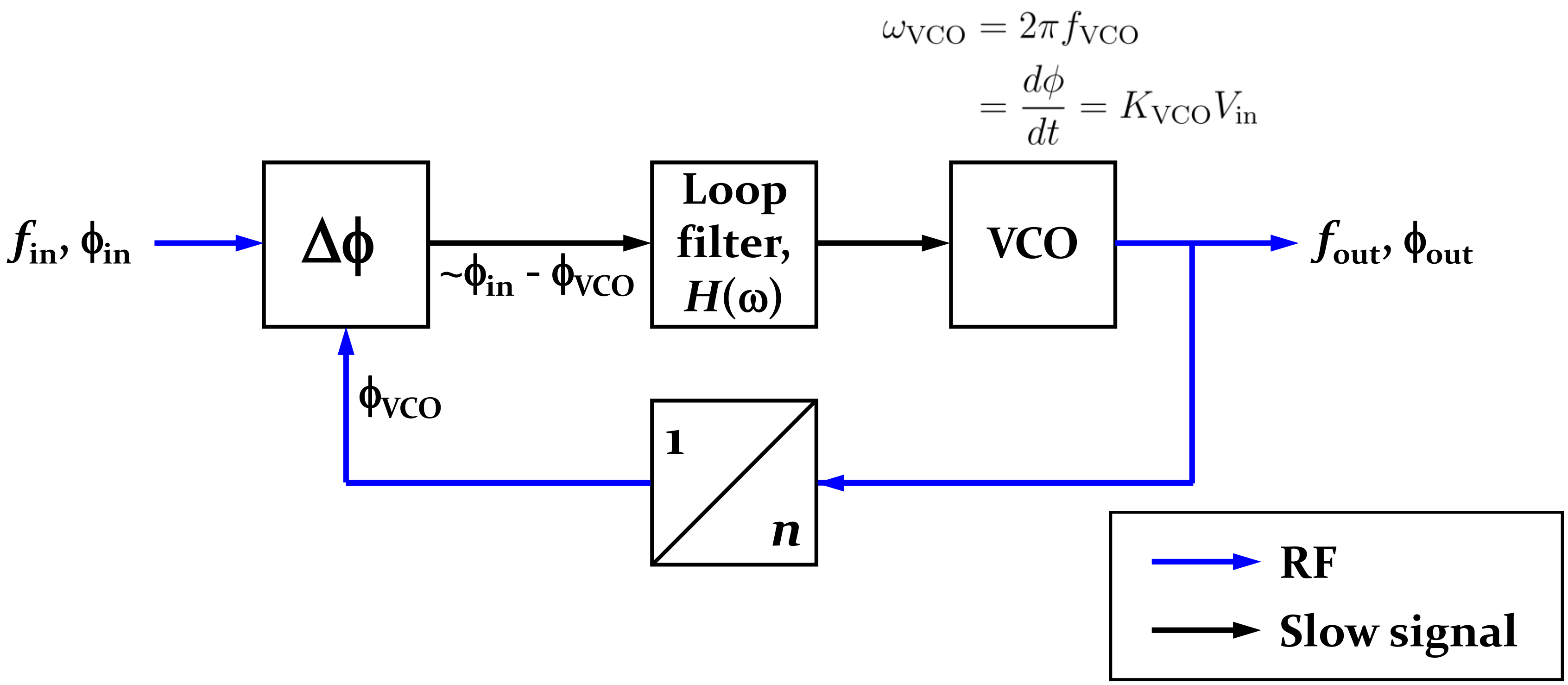
Synchronous triggers and bucket counting

- Circular hadron accelerators: master clock sweeps
- Need again synchronous timings with respect to beam
  - → Kicker magnets
  - → Beam instrumentation
- RF manipulations require bunches in certain buckets
  - → Beating pattern due to multiple RF harmonics
    - → Splits behaviour for different buckets
  - → Bucket numbering
- Need to know longitudinal beam position for transfer

→ Where (in phase/in time) is the beam?

Phase-locked loop

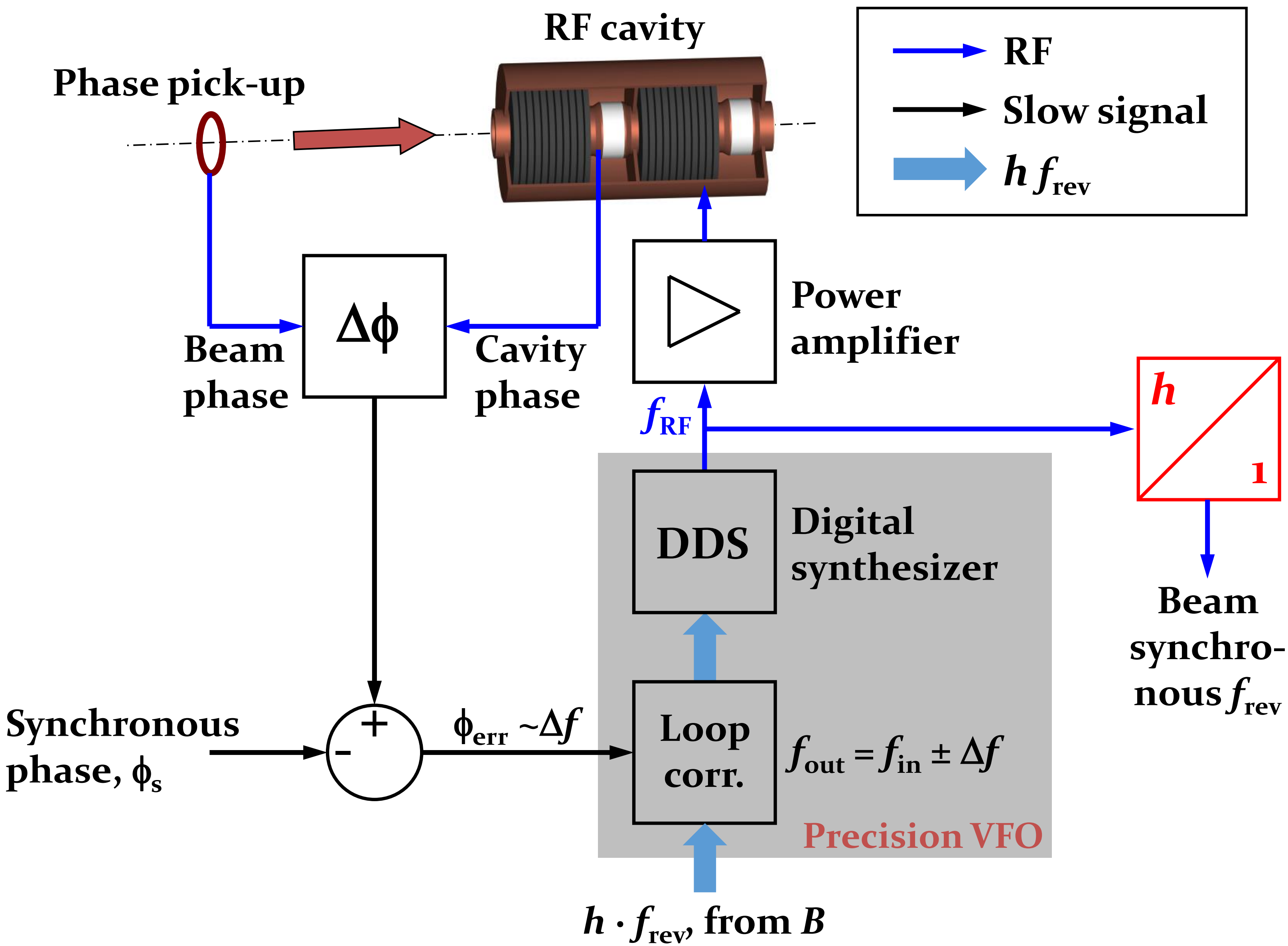
- **Frequency re-generation and multiplication**
- **Voltage controlled oscillator (VCO) locked in phase to input**



→ **Fixed phase relationship:** $\phi_{out}/n - \phi_{in} = \text{const.}$

→ **Optional divider:** $f_{out} = n \cdot f_{in}$

Beam phase loop



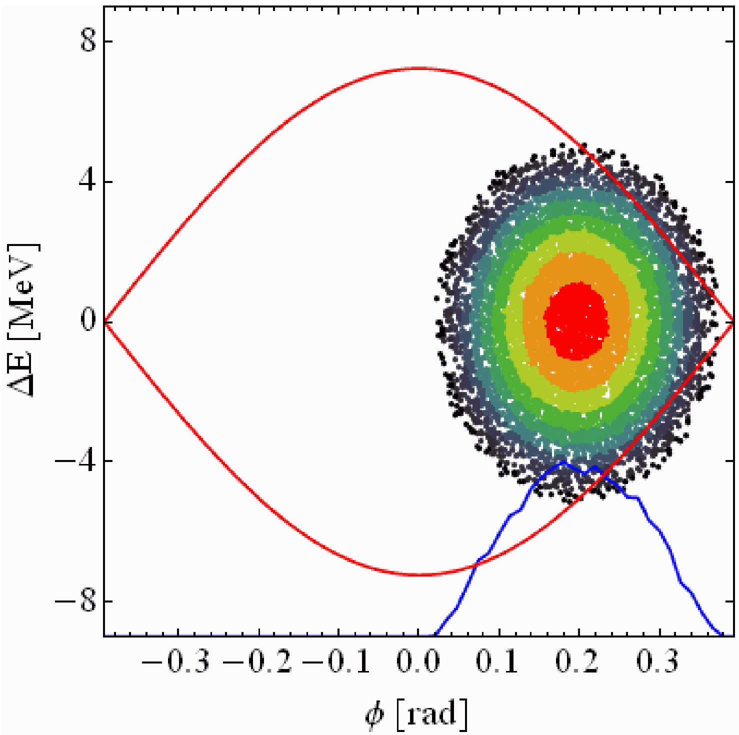
→ Phase-locked loop with beam phase as reference for RF system

Benefits of beam phase loop at transfer

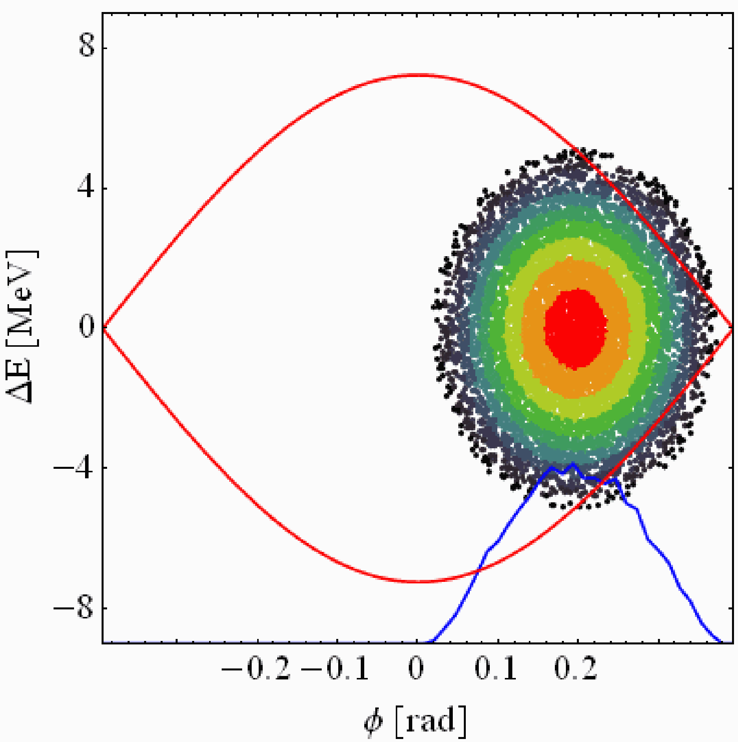
- Adapt RF phase to bunch phase before beam blows-up

→ Fast compared to timescale of synchrotron frequency, f_s

Rigid RF, no phase loop



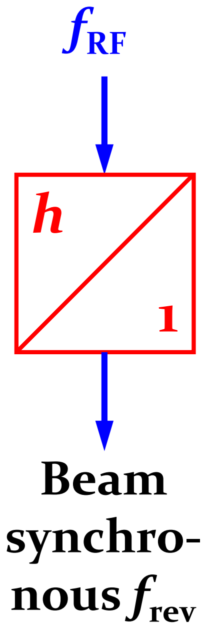
With phase loop



→ Even large transients (injection, transition) can be controlled

→ Small longitudinal emittance blow-up

Start counting with injection

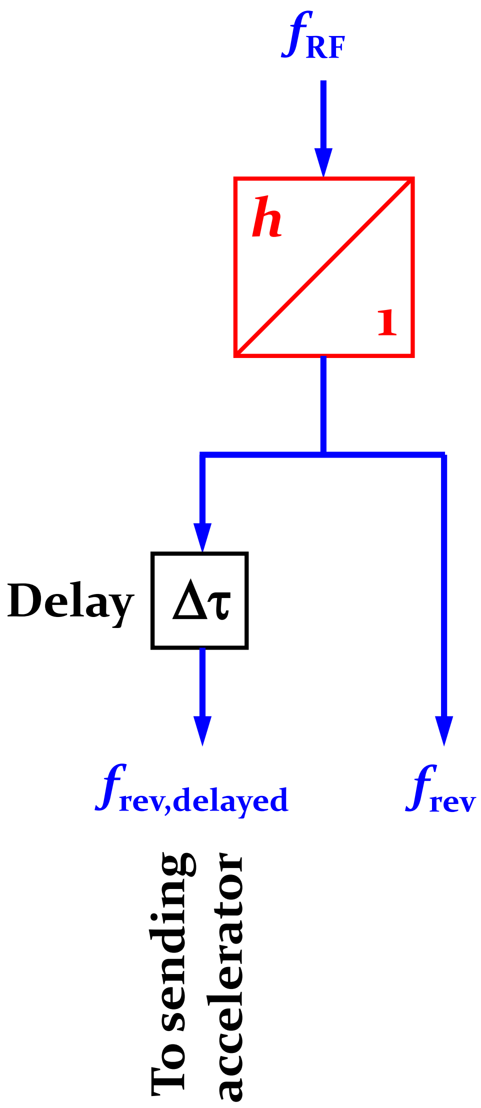


- **Start of divider/counter?**

→ Get it right from injection

→ Use output from divider as reference for incoming beam

Start counting with injection



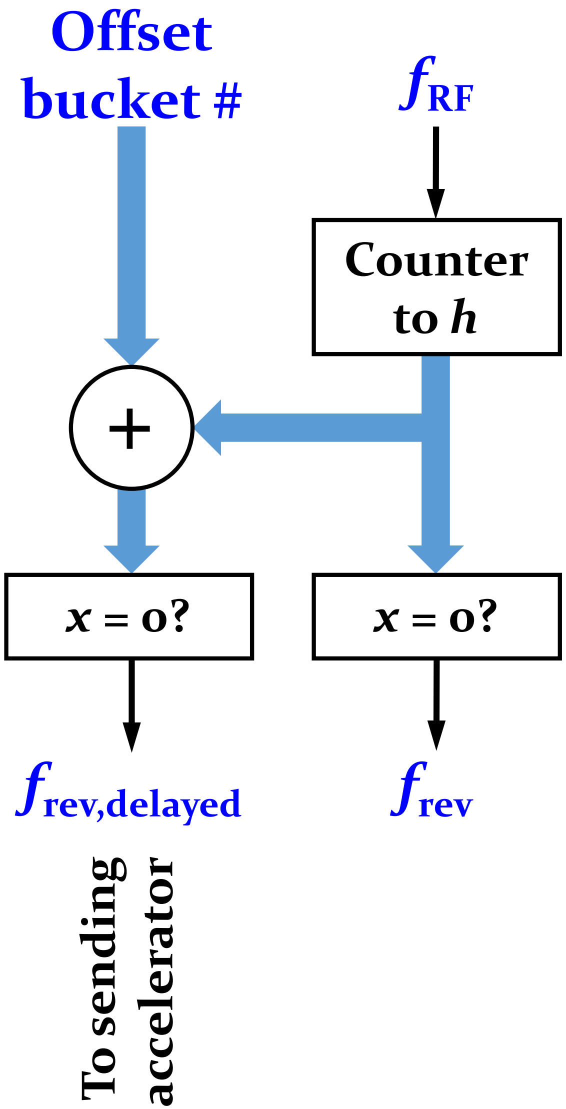
- Start of divider/counter?

→ Get it right from injection

→ Use output from divider as reference for incoming beam

- Before injection:
  - → Distribute delayed revolution frequency to sending accelerator
  - → Bunches are injected synchronously with $f_{rev,delayed}$
  - → Shifted with respect to f_{RF} and f_{rev}

Start counting with injection



- **Start of divider/counter?**

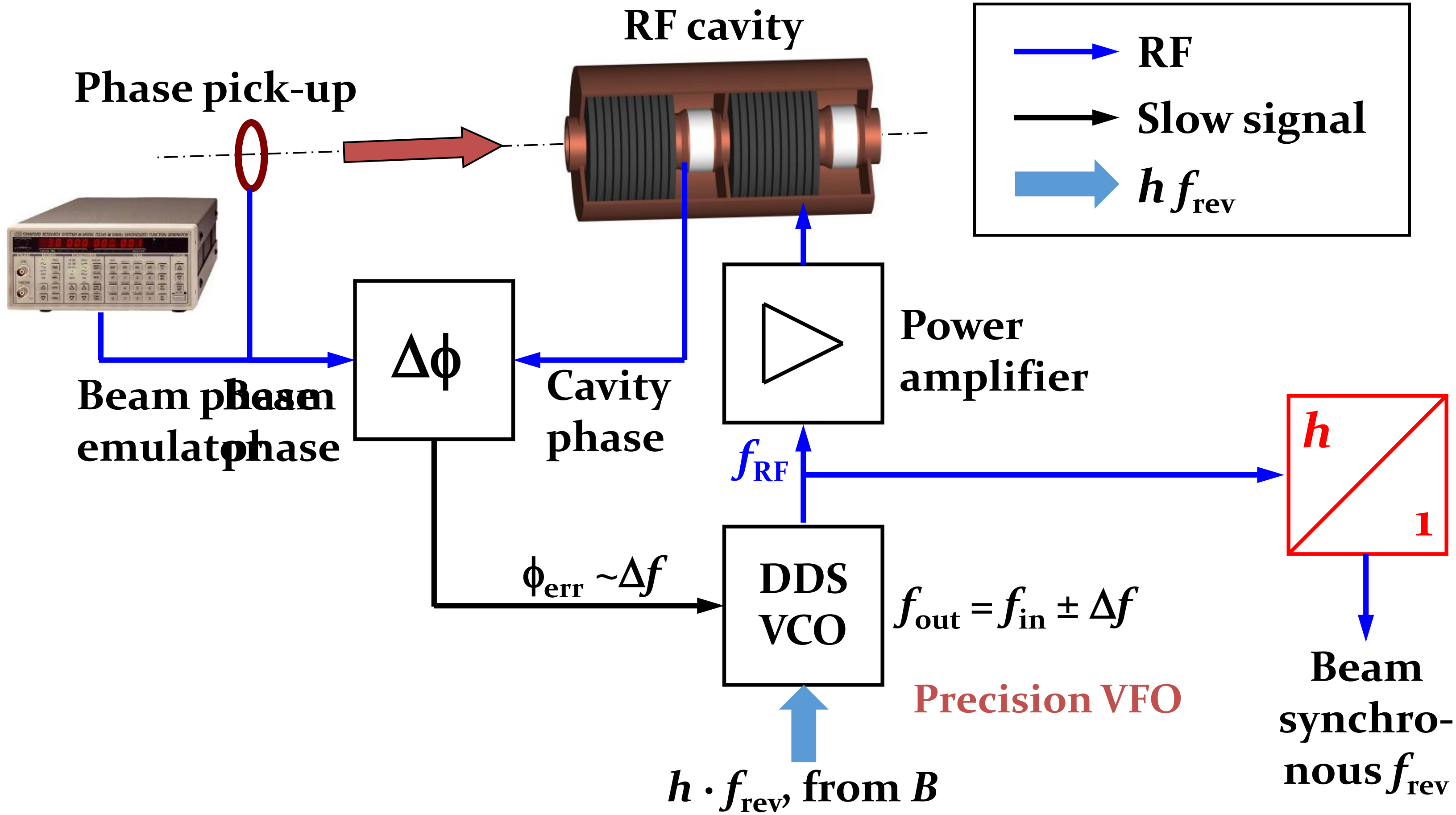
→ **Get it right from injection**

→ **Use output from divider as reference for incoming beam**

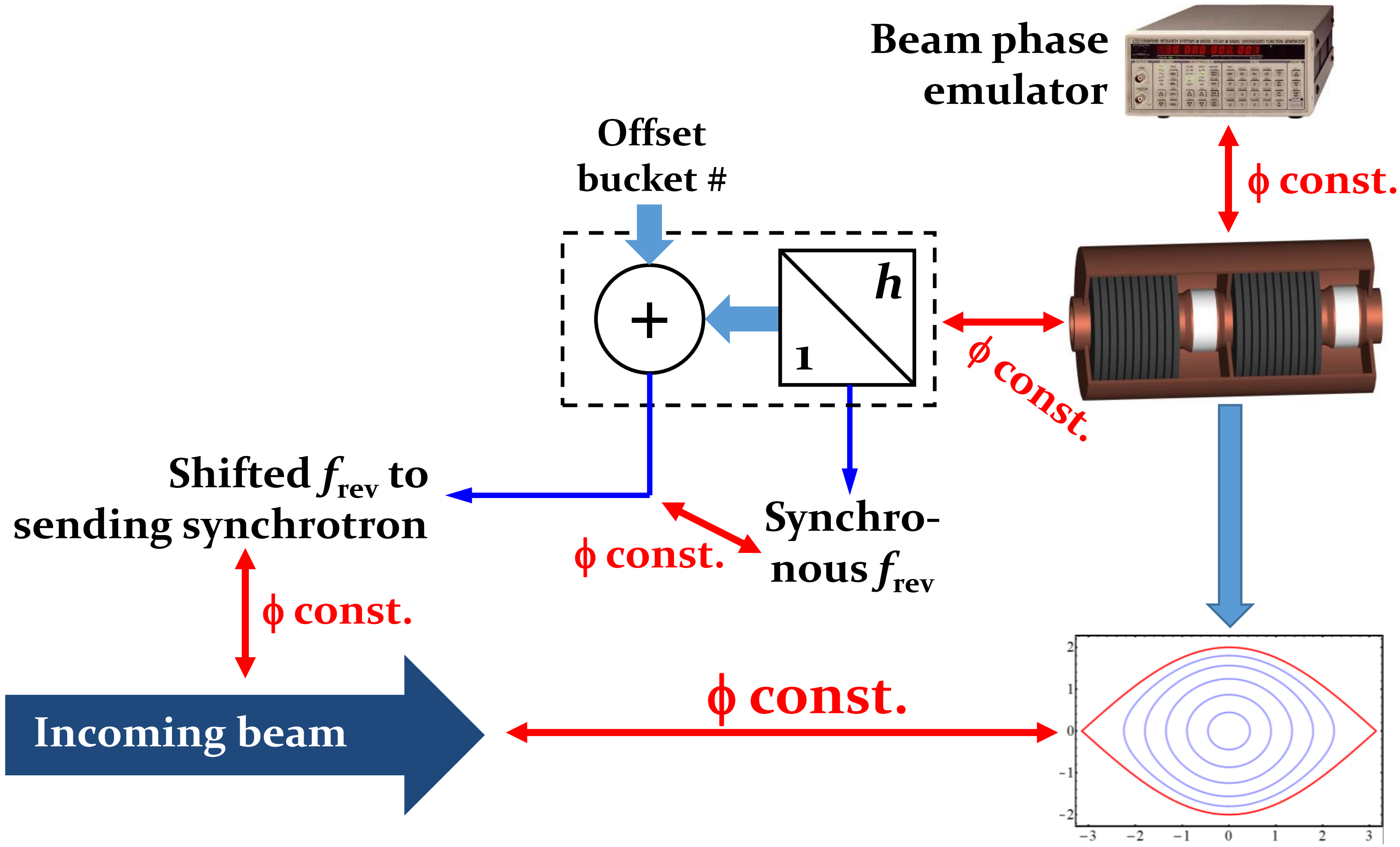
- **Before injection:**
  - → **Distribute delayed revolution frequency to sending accelerator**
  - → **Bunches are injected synchronously with $f_{\mathrm{rev,delayed}}$**
  - → **Shifted with respect to f_{RF} and f_{rev}**

Beam phase loop without beam?

→ Just replace beam by a simple RF generator!



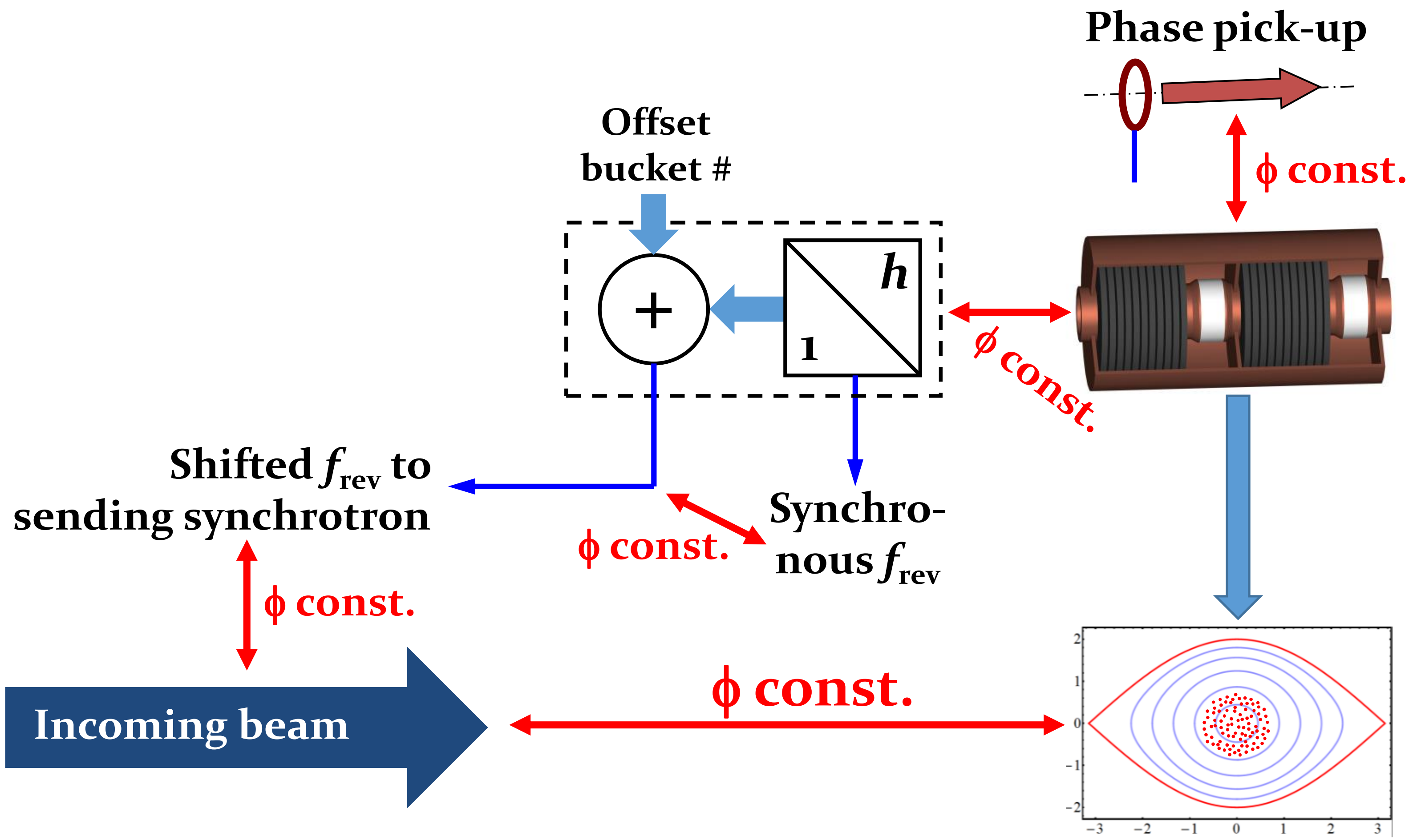
Synchronization chain for bucket counting



- Incoming beam has reproducible phase with respect to RF bucket, synchronous f_{rev} and beam phase emulating generator

→ Straightforward switch to beam signals, already locked in phase

Synchronization chain for bucket counting



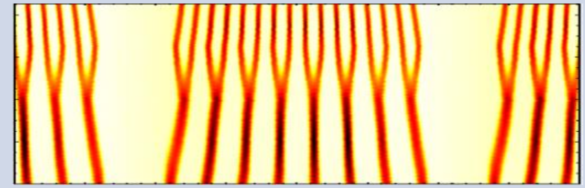
- **Incoming beam has reproducible phase with respect to RF bucket, synchronous f_{rev} and beam phase emulating generator**

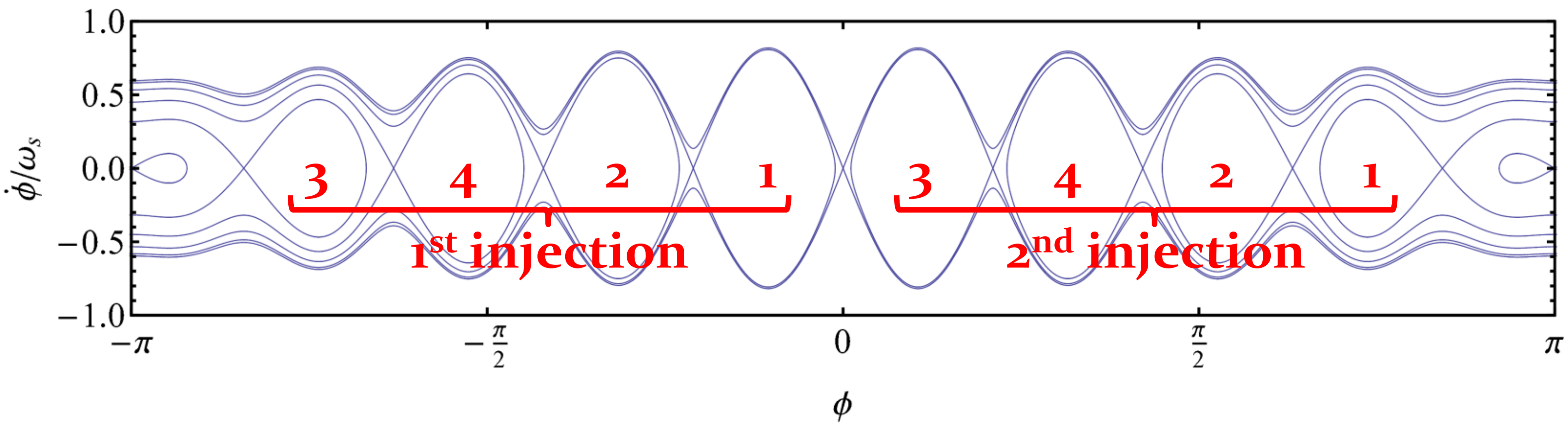
→ **Straightforward switch to beam signals, already locked in phase**

→ Beam phase with respect to f_{rev} always known

Bucket numbering

Bucket numbering for RF manipulations

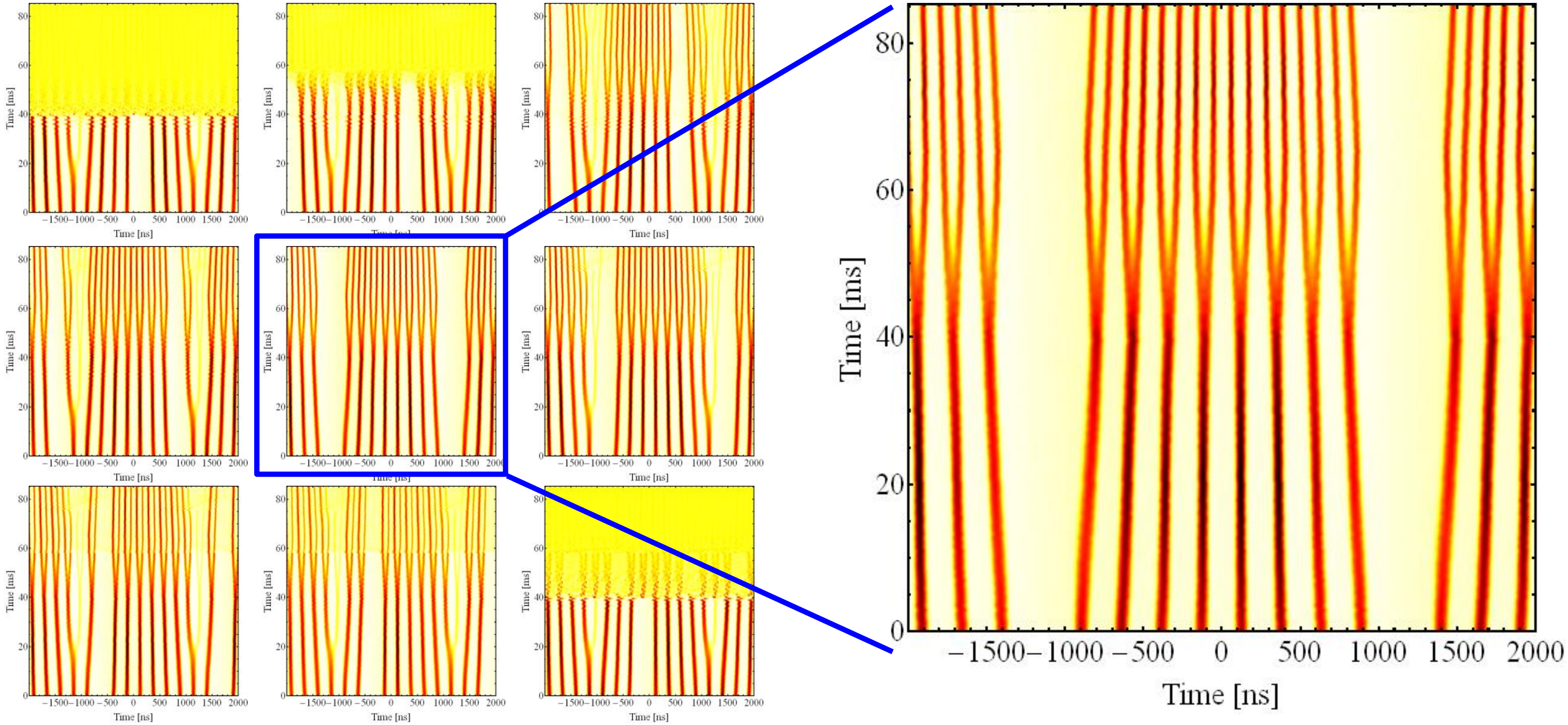
	Triple splitting	Batch compression
Injection harmonic		
Periodicity of RF manipulation	**Every bucket**	**Only one beating along circumference**
Injection bucket selection	**4 buckets difference between both injections**	**Both injections into independently defined buckets**



→Must inject into the correct bucket numbers

Example: PS injection bucket selection

- **Bunches must be placed into the correct buckets numbers**
- **Harmonic number change only for even number of bunches**



→ Bucket number control during both transfers PSB → PS

→ How to handle changing number of bunches?

Intermediate summary

- **Basic techniques of signal synchronizations**

  → **Beware of dividers**

- **Beam transfer between circular lepton accelerators**

  → **Constant frequency**

  → **Predictable, independently from beam**

  → **Fundamental periodicity**

- **Beam transfer between circular hadron accelerators**

  → **Beam is reference, keep track**

Timing, Synchronization & Longitudinal Aspects II

H. Damerau

CERN

The CERN Accelerator School

CAS Course on
Beam Injection, Extraction and Transfer

13 March 2017

Outline

- Introduction
- General concepts
  - Signals with noise, transmission of RF signals
  - Phase detectors and dividers
- Beam transfer
  - Fundamental periodicity
  - Transfer between circular lepton accelerators
- **Transfer between hadron accelerators**
  - Beam phase loop, bucket numbering
  - **Transfer process: Synchronization, transfer triggers**
  - **Longitudinal matching**
- **Summary**

Synchronization and transfer

Steps of beam transfer synchronization

1.
- Set bending fields in both accelerators the to same magnetic rigidity

2.
- Synchronize sending or receiving accelerator

→ Ready for transfer

3.
- Start counting clock of fundamental periodicity
- Trigger bump and septum elements

4.
- Start counting f_{rev} clock (sending/receiving accelerator)
- Start counting bucket clock

5.
- Fine delay
- Ejection and injection kickers triggers

→ Transfer

Match bending field of both accelerator

- **Same magnetic rigidity ρ*B* of sending (1) and receiving (2) accelerators**

$$F_Z = F_L \quad \rightarrow \quad \frac{p}{q} = \rho B \qquad\qquad \boxed{\rho_1 B_1 = \rho_2 B_2}$$

→ **No rule without exception: Particle type change at transfer**

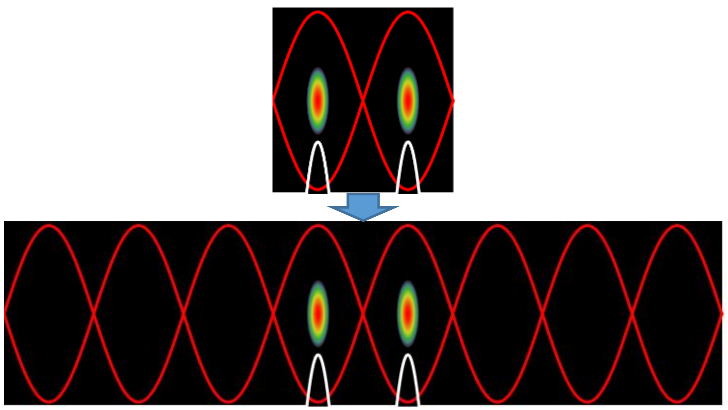
- **Proton to anti-proton conversion, e.g.,**
  120 GeV/c ≠ 8 GeV/c (Fermilab), 26 GeV/c ≠ 3.6 GeV/c (CERN),
- **Charge state change at transfer, e.g. LHC ion injector chain**
  Pb^{54+} in LEIR/PS → Pb^{82+} (in SPS)

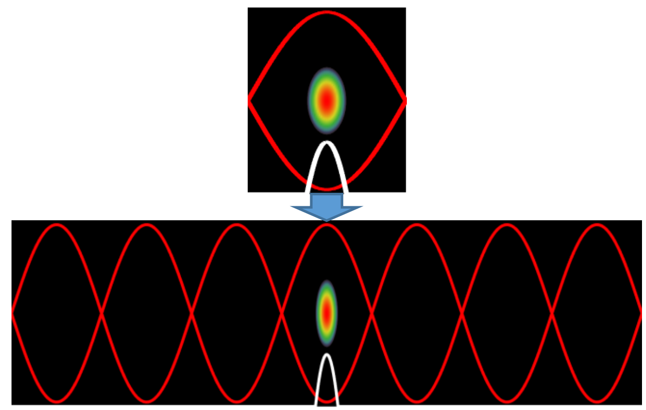
Match RF frequencies

- **RF frequencies of both accelerators must have appropriate ratio assuming that the beam velocity is unchanged**

$$f_{\text{rev}} = \frac{f_{\text{RF}}}{h} = \frac{\beta c}{2\pi R}$$

$$\beta c = 2\pi R_1 f_{\text{rev},1} = 2\pi R_2 f_{\text{rev},2} \quad \rightarrow \quad \boxed{R_1 \frac{f_{\text{RF},1}}{h_1} = R_2 \frac{f_{\text{RF},2}}{h_2}}$$





→ **Common choice of most circular electron accelerators $f_{\mathrm{RF},1} = f_{\mathrm{RF},2}$**

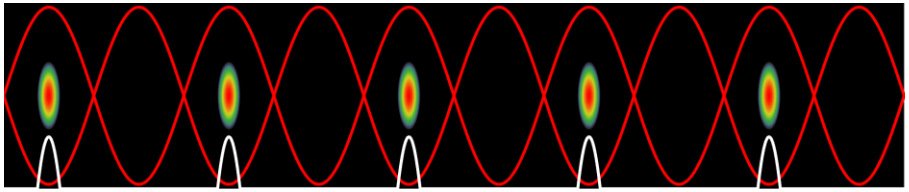
→ **Harmonic number, h, proportional to circumference, $2\pi R$**

→ **Again no rule without exception: Production of antiprotons in target in transfer line**

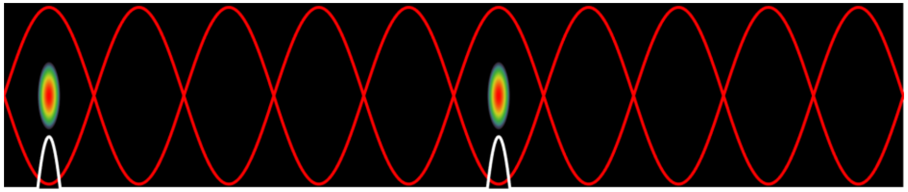
Distance between bunches

- Distance of bunches (bunch spacing, τ_{bunch}) from source accelerator must match distance of buckets

- Example: $\tau_{\text{bunch}} = 2/f_{\text{RF}}$



- Example: $\tau_{\text{bunch}} = 5/f_{\text{RF}}$



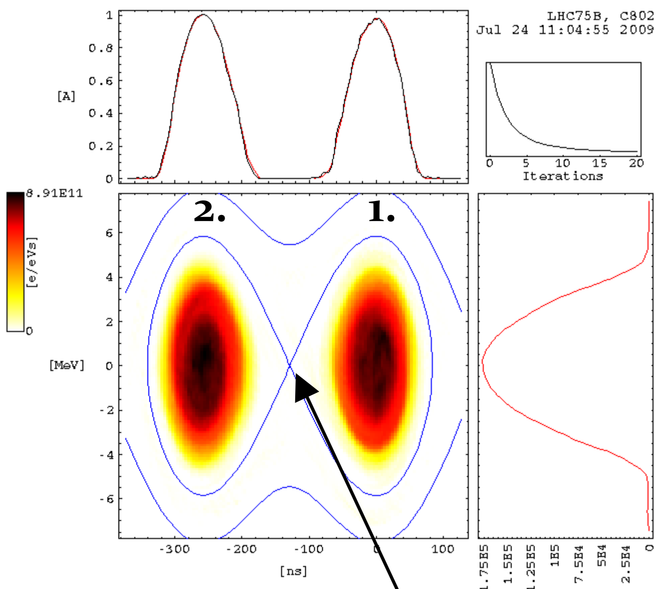
- Common case: $f_{\text{RF},2} = n \cdot f_{\text{RF},1}$
  → $f_{\text{RF,LHC}} = 2 \cdot f_{\text{RF,SPS}}$ and $f_{\text{RF,SPS}} = 5 \cdot f_{\text{RF,PS}}$

→ Several exceptional cases:
  → No bunch distance with single bunch → more flexibility
  → Adjust bunch spacing using multiple RF systems

Exception: double-harmonic RF at transfer

- Was used at CERN PSB-to-PS to transfer 2 bunches at once
- Circumference ratio $C_{PS}/C_{PSB} = 4$

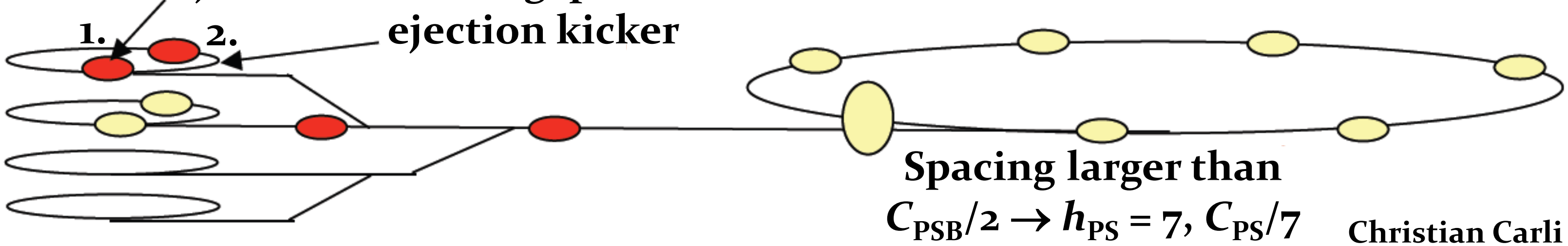
→ Ratio virtually moved to 2/7: use $h_{RF} = 2 + 1$



1. Add $h1$ component such that bunches approach to 245 ns (small spacing) → big spacing becomes 327 ns
2. Synchronize on $h1$ to the PS
3. Trigger extraction kicker in-between the small spacing
4. Eject two bunches per ring at a distance of 327 ns

Next bunch ejected

Short gap for ejection kicker



Spacing larger than $C_{PSB}/2 \rightarrow h_{PS} = 7$, $C_{PS}/7$

Christian Carli

Steps of beam transfer synchronization

1. • Set bending fields in both accelerators to the same magnetic rigidity

2. • Synchronize sending or receiving accelerator

→ Ready for transfer

3. • Start counting clock of fundamental periodicity
   • Trigger bump and septum elements

4. • Start counting f_{rev} clock (sending/receiving accelerator)
   • Start counting bucket clock

5. • Fine delay
   • Ejection and injection kickers triggers

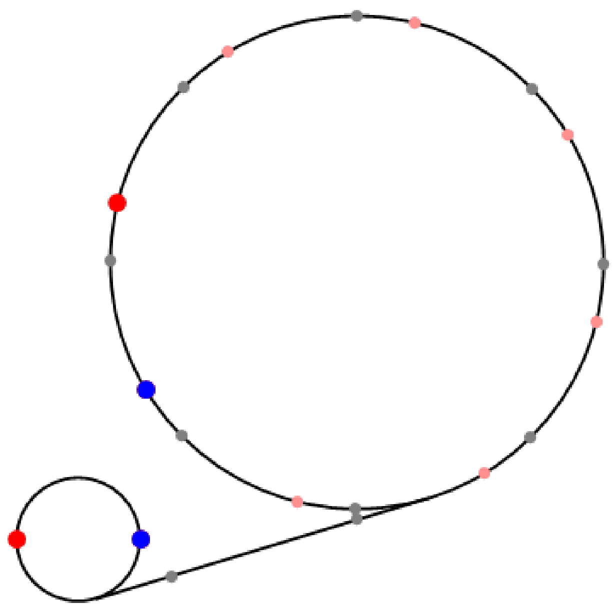
→ Transfer

Before synchronization

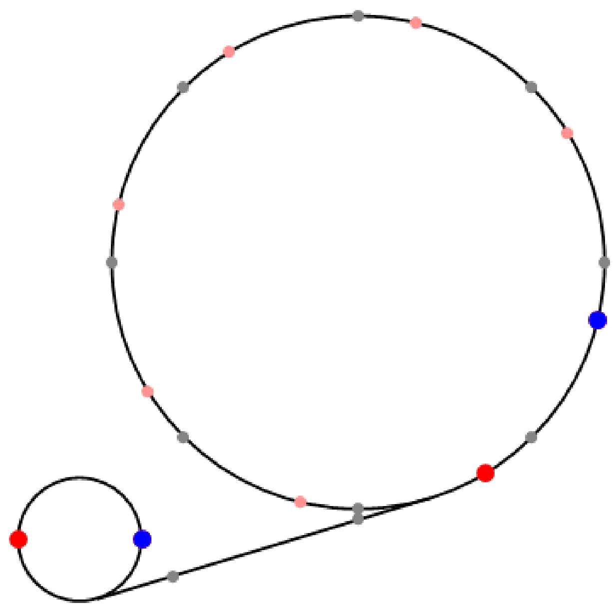
- **Even with magnetic rigidity matched: revolution frequencies not at theoretical ratio due to imperfections**

→ **Bunches and buckets slip in phase**

Slippage of bunches and bucket



Rotation in source accelerator subtracted

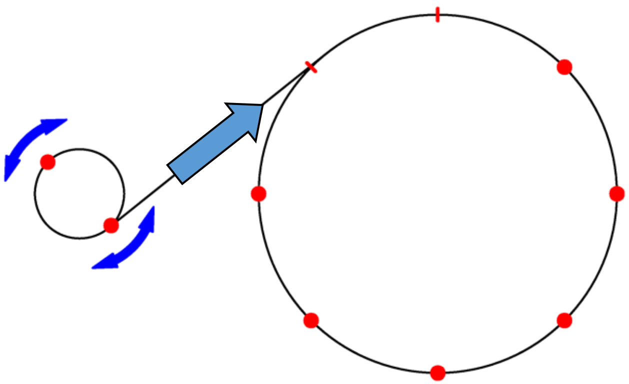


But: important question left unanswered!

Who is the boss?

- Transfer beam to a downstream machine: Bunch-to-bucket

1. Protons between synchrotrons → Synchronize accelerators



2. Move relative phase of RF together with beam between both machines to hit the empty buckets

Sending accelerator is, the boss?

Receiving accelerator is the boss?

Choice of master for transfer synchronization

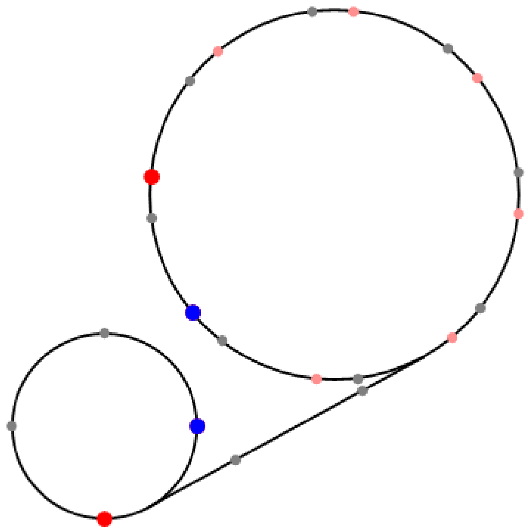
- **Sending accelerator is master of transfer**
  - → Receiving accelerator adapts to incoming beam
  - → Common choice when receiving accelerator has no beam before transfer
  - → Interesting for only single beam transfer, e.g., protons from PS → AD for antiproton production

- **Receiving accelerator is master of transfer**
  - → Sending accelerator adapts to incoming beam
  - → Common choice when receiving accelerator has already beam before transfer (multiple injections)
  - → Most common at CERN, e.g., proton injector chain PSB → PS → SPS → LHC

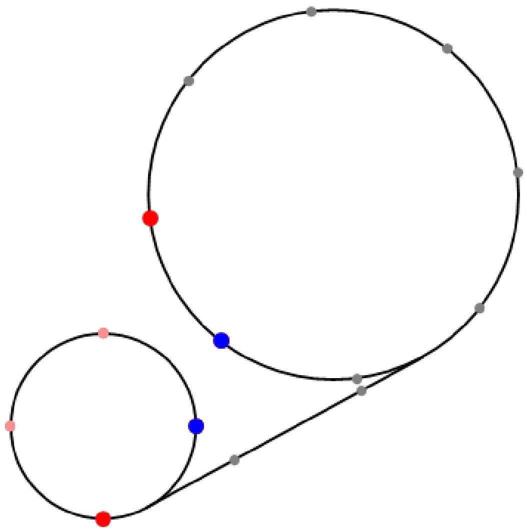
Before synchronization

- **Simple test case of circumference ratio 2: $C_2 = 2C_1$**

Source accelerator is master at transfer



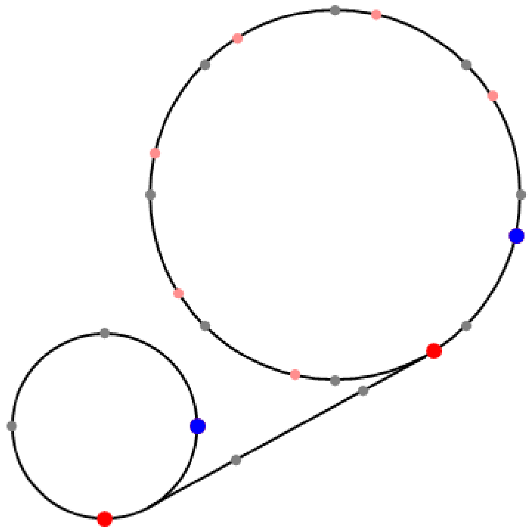
Target accelerator is master at transfer



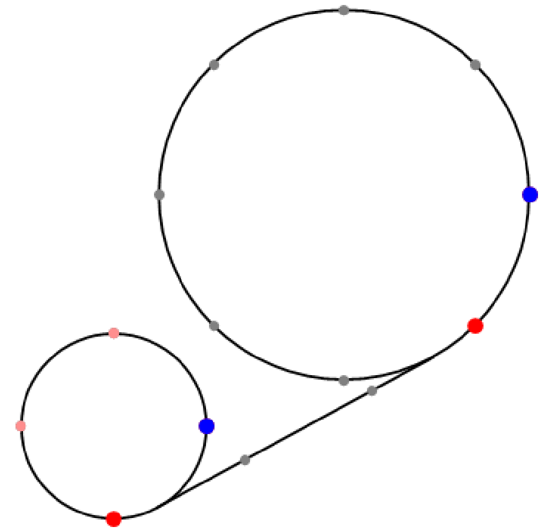
Before synchronization

- **Simple test case of circumference ratio 2: $C_2 = 2C_1$**

Source accelerator is master at transfer



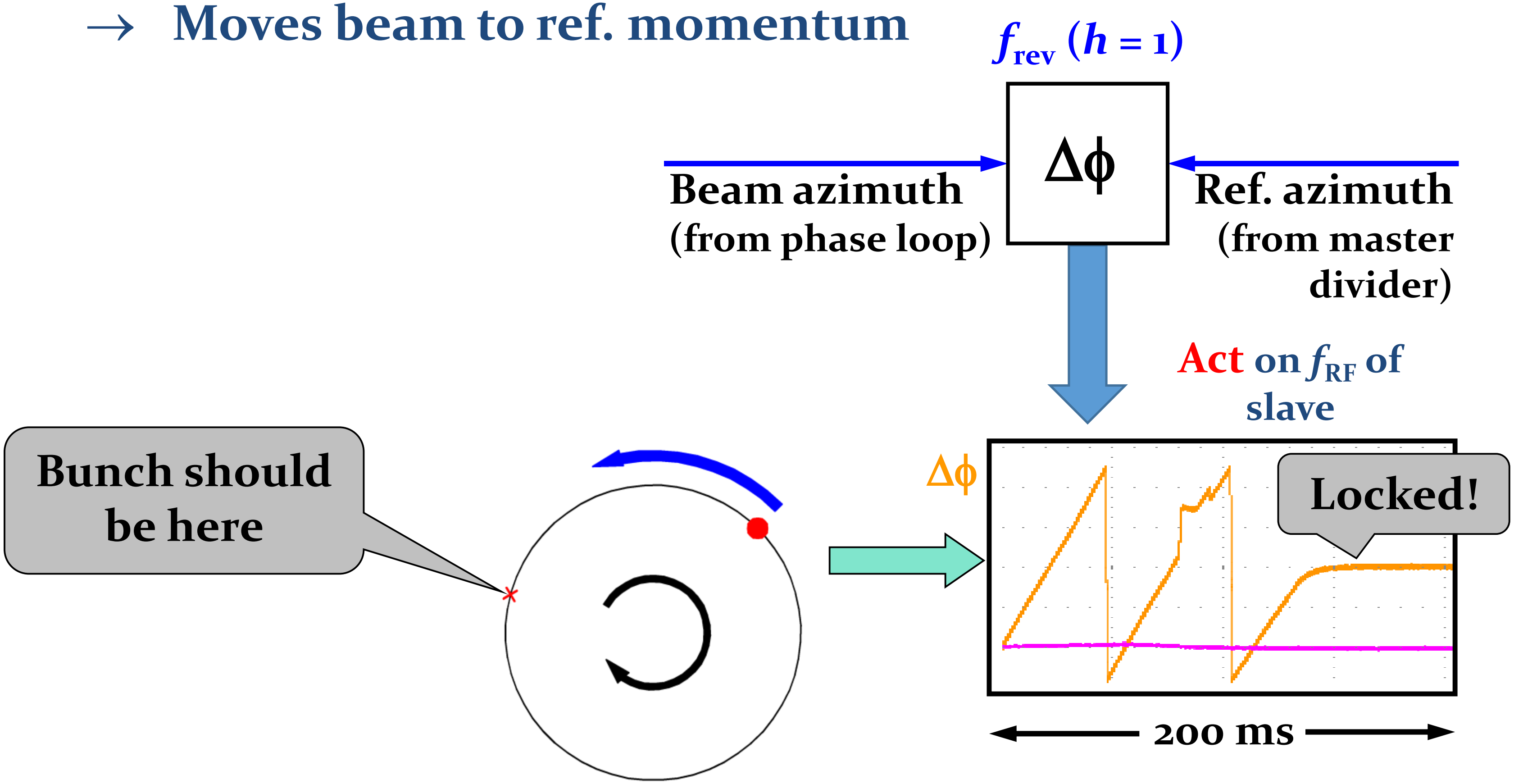
Target accelerator is master at transfer



→ Synchronize both accelerator to force: $f_{rev,1} = 2f_{rev,2}$

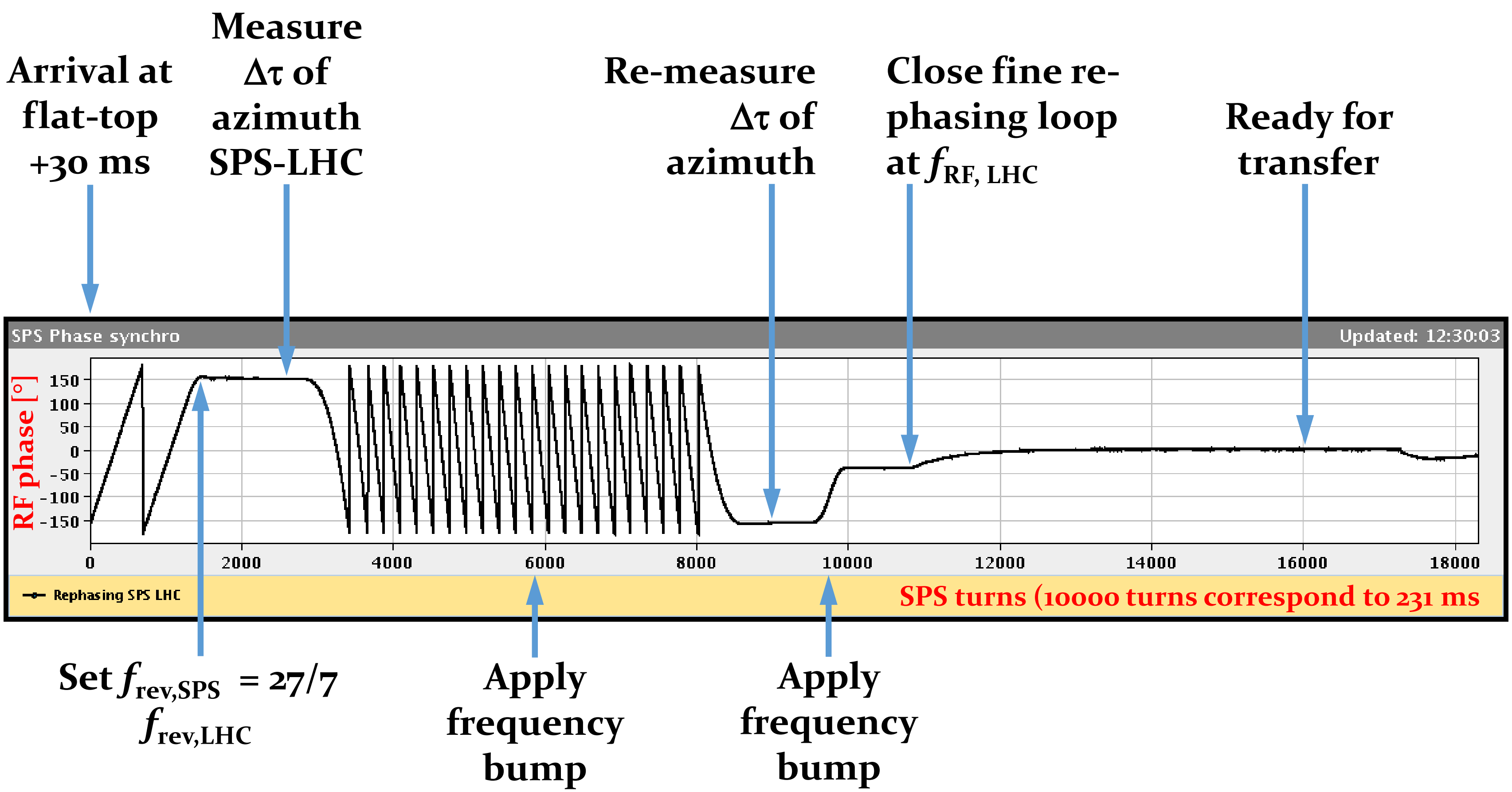
Simple synchronization process

1. **Move beam to off-momentum (B const.):** $$\frac{df}{f} = \frac{\gamma_{\mathrm{tr}}^2 - \gamma^2}{\gamma^2 \gamma_{\mathrm{tr}}^2} \frac{dp}{p}$$
   - → Well defined frequency difference between accelerators
2. **Measure azimuth error, when beam at correct azimuth**
   - → Close synchronization loop
   - → Moves beam to ref. momentum



Example: Synchronization of SPS to LHC

→ LHC is master for beam transfer from SPS

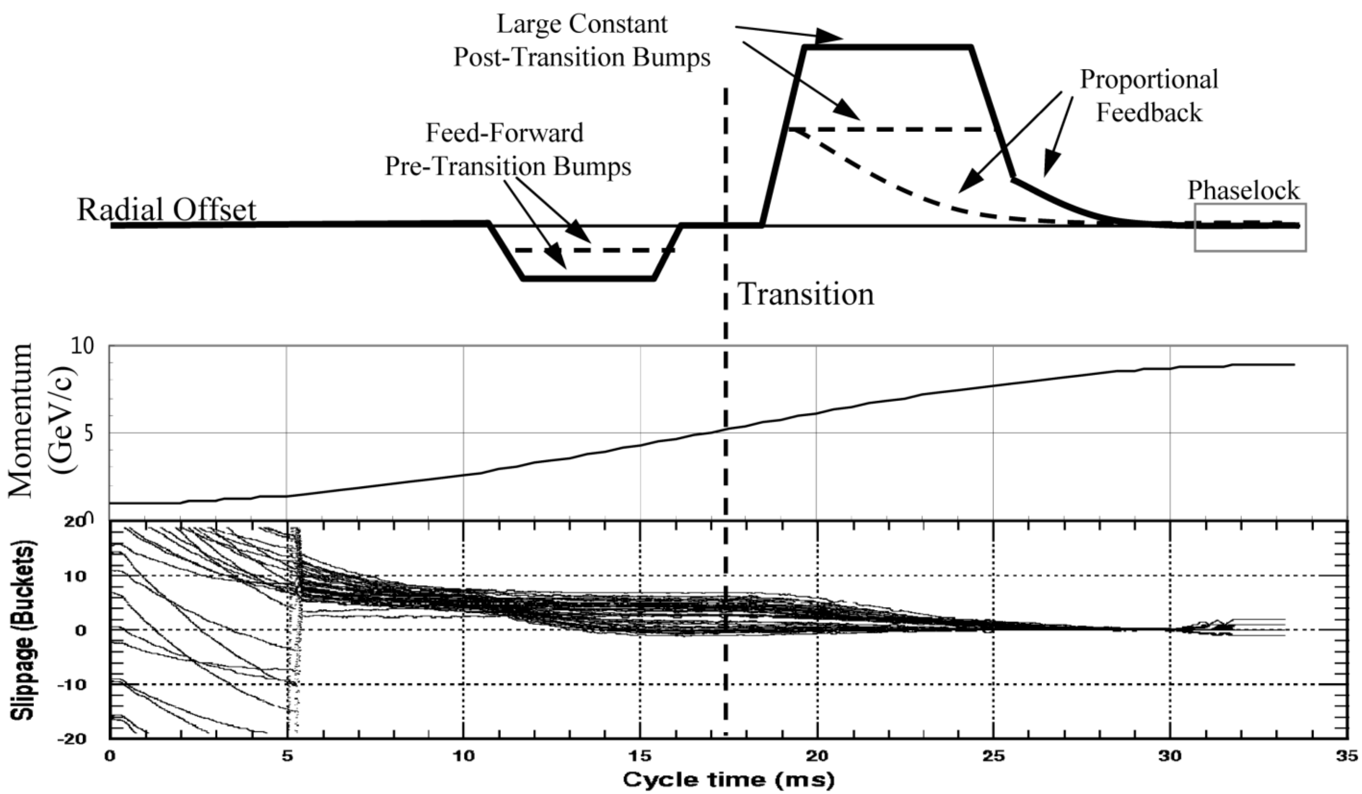


→ Coarse and fine re-phasing to perfectly align bunches with respect to target buckets (400 MHz, 2.5 ns) in LHC

→ Complete synchronization process takes about 500 ms

Example: Fast cogging of booster at FNAL

- **Rapid cycling synchrotron from 400 MeV to 8 GeV**
- **Total cycle length is only 25 ms → How to synchronize fast?**



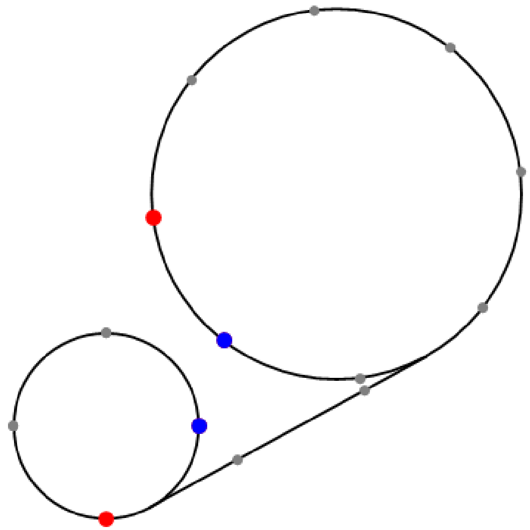
C. Drennan, R. Zwaska

1. **Measure beam phase early in the cycle and predict azimuth at flat-top**
2. **Apply radial/frequency bumps already during acceleration**

After synchronization

- **Simple test case of circumference ratio 2: $C_2 = 2C_1$**

Source or target accelerator is master at transfer



→ **Revolution frequencies coupled: $f_{\mathrm{rev},1} = 2f_{\mathrm{rev},2}$**

→ **Transfer can be triggered every turn of the target accelerator**

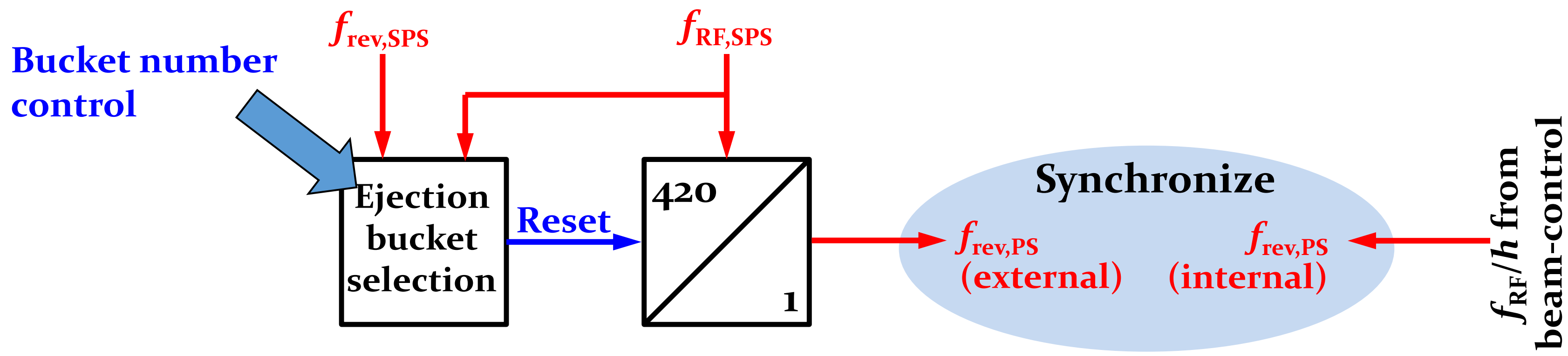
Example: Ejection bucket numbering in PS

- Azimuthal position of 1st bunch ambiguous after RF manipulations

→ Number of buckets and bunches changes during acceleration

- But: Synchronous $f_{\mathrm{rev,PS}}$ signal with reproducible phase to beam

→ 'Re-numbering' of buckets by shifting reference from SPS



→ Shift of external reference $f_{\mathrm{rev,PS}}$ adjustable in SPS bucket units

→ Synchronize external and beam synchronous $f_{\mathrm{rev,PS}}$

Example: Ejection synchronization chain

→ **Multiple 'batches' are transferred from PS to 11 times larger SPS**

Outgoing beam ← ϕ **const.** → $f_{rev,PS}$ **internal** ← ϕ **const.** → $f_{rev,PS}$ **reference** ← ϕ **const.** → $f_{rev,SPS}, f_{RF,SPS}$ **for synchro.**

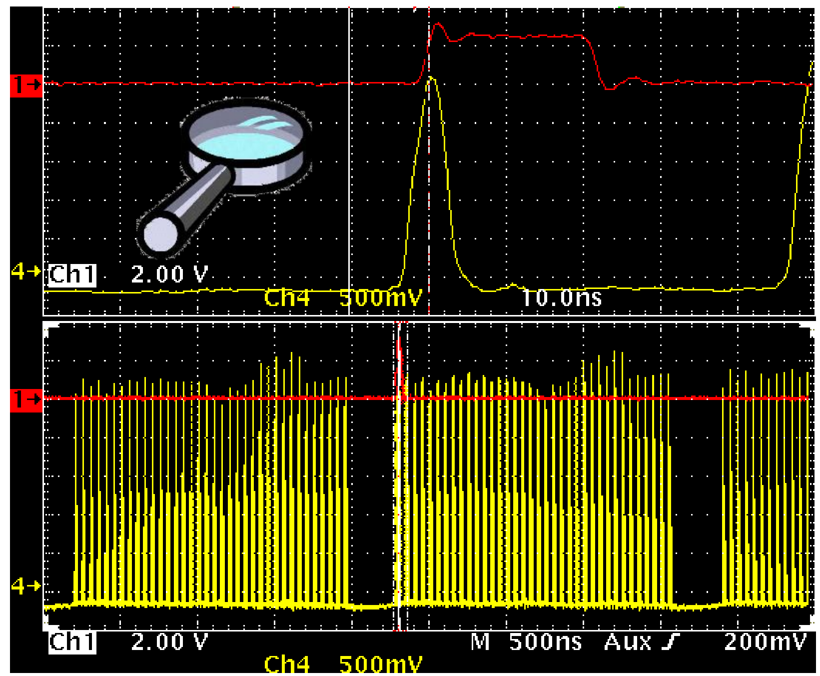
Beam phase loop | **Synchronization loop** | **Ejection divider**

Last turn before PS → SPS transfer

ϕ **const.**

$f_{rev,SPS}$ **marker** →

Outgoing beam →



→ **Beam phase with respect to f_{rev} always known**

Steps of beam transfer synchronization

1. • Set bending fields in both accelerators to the same magnetic rigidity

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→ Ready for transfer

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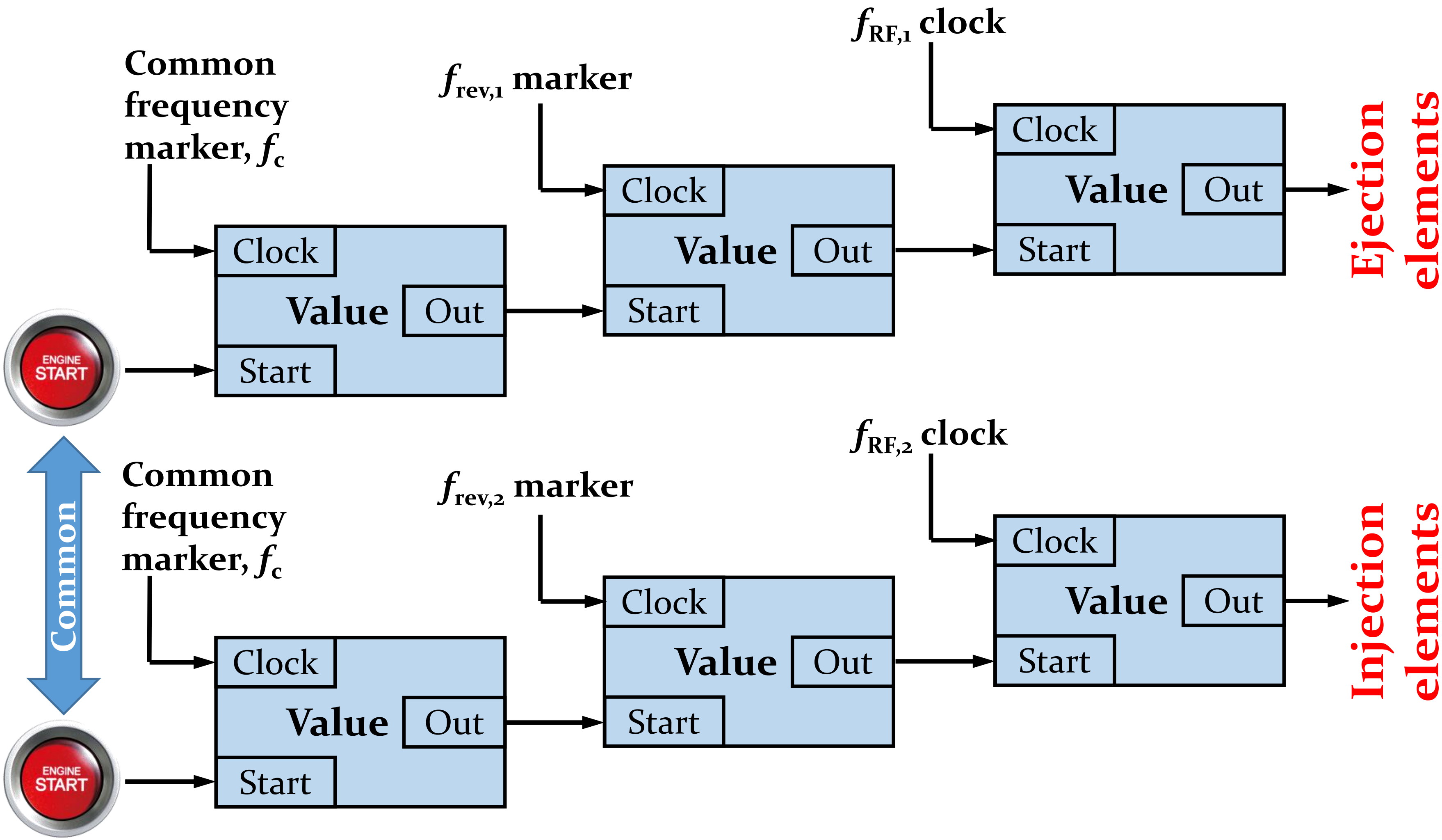
5. • Fine delay
   • Ejection and injection kickers triggers

→ Transfer

Synchronous triggers

→ **Cascade of trigger counters for fast transfer elements**

- **Very similar to transfer with lepton synchrotrons**



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   • Ejection and injection kickers triggers

→ Transfer

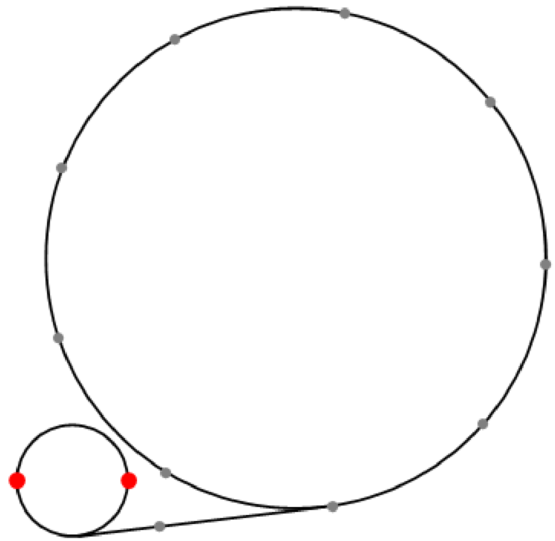
Example: Turn count control at extraction

- **J-PARC rapid cycling synchrotron and main ring ratio: 4.5**

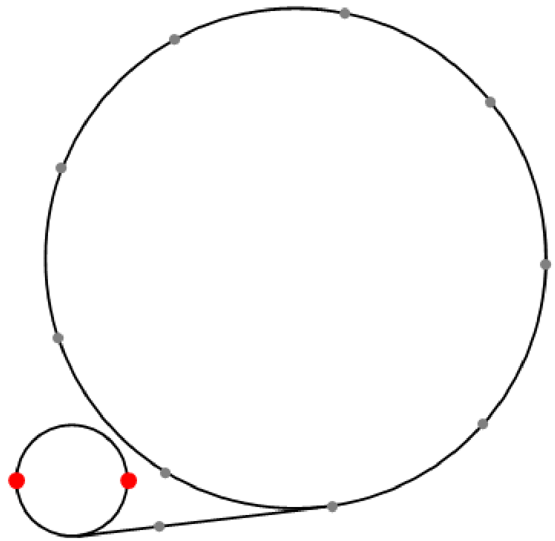
→ **Transfer possible once every two turns of main ring**

→ **Transfer of 4 times two bunches**

Counter on f_{rev} of source (RCS) set normally



Counter on f_{rev} of source (RCS) delayed by one turn



→ **Beam synchronous timing can also be used to control target azimuth (bucket number) of transferred beam**

Energy matching

Energy matching of incoming beam

- **Ideal beam circulates with the expected revolution frequency ($\Delta f = 0$) on the central orbit ($\Delta R = 0$) → $\Delta p = 0$**
- **Real beam behaviour is calculated using**

Variables	Equations
$B,\ p,\ R$	$\dfrac{dp}{p} = \gamma_{\mathrm{tr}}^2 \dfrac{dR}{R} + \dfrac{dB}{B}$
$f,\ p,\ R$	$\dfrac{dp}{p} = \gamma^2 \dfrac{df}{f} + \gamma^2 \dfrac{dR}{R}$
$B,\ f,\ p$	$\dfrac{dB}{B} = \gamma_{\mathrm{tr}}^2 \dfrac{df}{f} + \dfrac{\gamma^2 - \gamma_{\mathrm{tr}}^2}{\gamma^2} \dfrac{dp}{p}$
$B,\ f,\ R$	$\dfrac{dB}{B} = \gamma^2 \dfrac{df}{f} + (\gamma^2 - \gamma_{\mathrm{tr}}^2) \dfrac{dR}{R}$

Energy matching of incoming beam

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- **Real beam behaviour is calculated using**

Variables	Equations
$B,\ p,\ R$	$\frac{dp}{p} = \gamma_{\text{tr}}^2 \frac{dR}{R} + \cancelto{0}{\frac{dB}{B}}$
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$B,\ f,\ p$	$\cancelto{0}{\frac{dB}{B}} = \gamma_{\text{tr}}^2 \frac{df}{f} + \frac{\gamma^2 - \gamma_{\text{tr}}^2}{\gamma^2} \frac{dp}{p}$
$B,\ f,\ R$	$\cancelto{0}{\frac{dB}{B}} = \gamma^2 \frac{df}{f} + (\gamma^2 - \gamma_{\text{tr}}^2) \frac{dR}{R}$

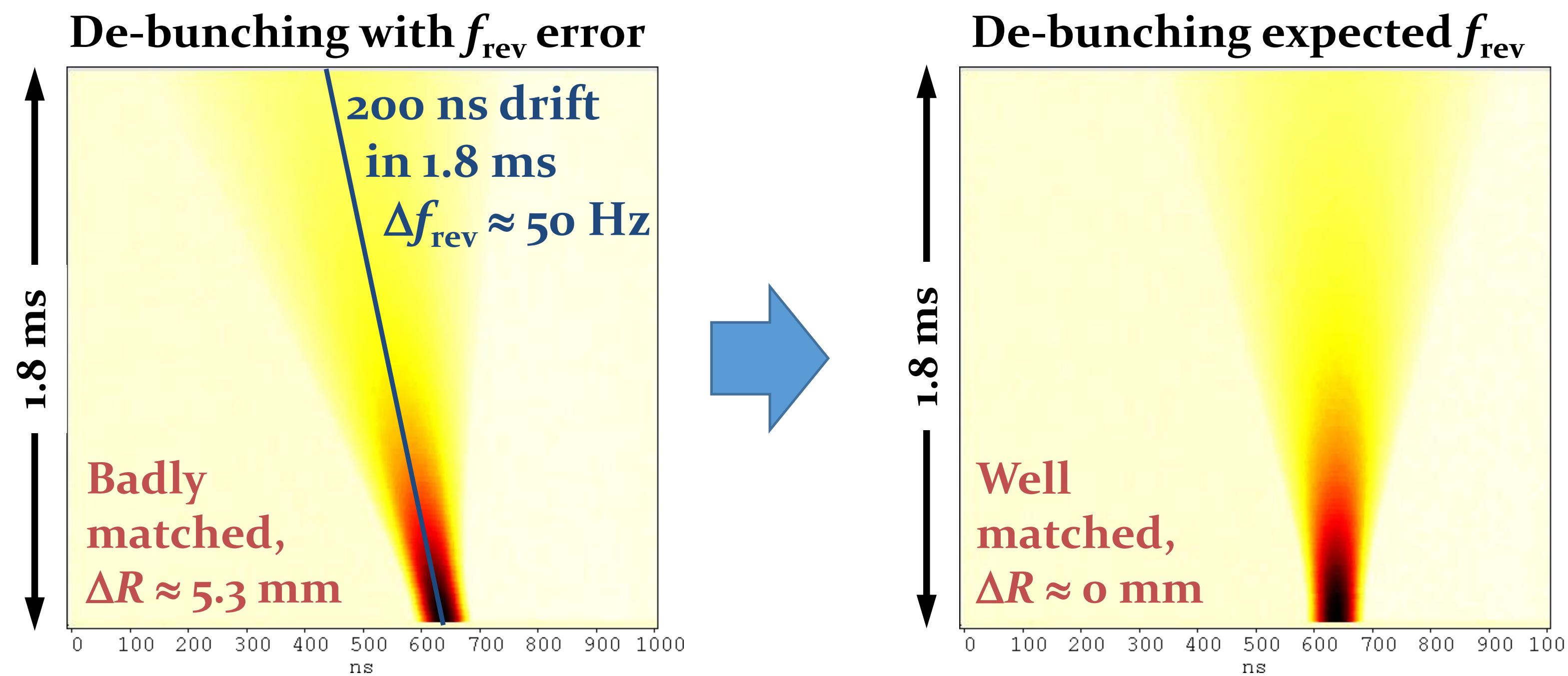
Choice of two parameters from B, p, R, f directly constrains all others

→ **Example: at fixed magnetic field ($\Delta B = 0$), revolution frequency and radial position are directly linked**

Energy matching without RF

- **Observe de-bunching (no RF) with periodic trigger at $n \cdot f_{\mathrm{rev}}$**

→ Does the beam circulate with the expected f_{rev}? at the central orbit?



- **Changing B alone insufficient, since f_{rev} and R linked (const. p)**

→ Change two parameters to fix the others, e.g., B and p or B and f

→ All parameters are constrained

Longitudinal matching equations

Recap of longitudinal beam dynamics (1)

$$\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_{\mathrm{tr}}^2}$$

For a single harmonic RF system

$$H\left(\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = -\frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2 + \frac{qV}{2\pi}\left[\cos\phi - \cos\phi_0 + (\phi - \phi_0)\sin\phi_0\right]$$

with $\phi = \phi_0 + \Delta\phi$ **it becomes**

$$H\left(\Delta\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = -\frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2 + \frac{qV}{2\pi}\left[\cos(\phi_0 + \Delta\phi) - \cos\phi_0 + \Delta\phi\sin\phi_0\right]$$

using
$$\cos(\phi_0 + \Delta\phi) = \cos\phi_0\cos\Delta\phi - \sin\phi_0\sin\Delta\phi$$
$$\simeq \cos\phi_0\left(1 - \frac{1}{2}\Delta\phi^2\right) - \sin\phi_0\Delta\phi$$

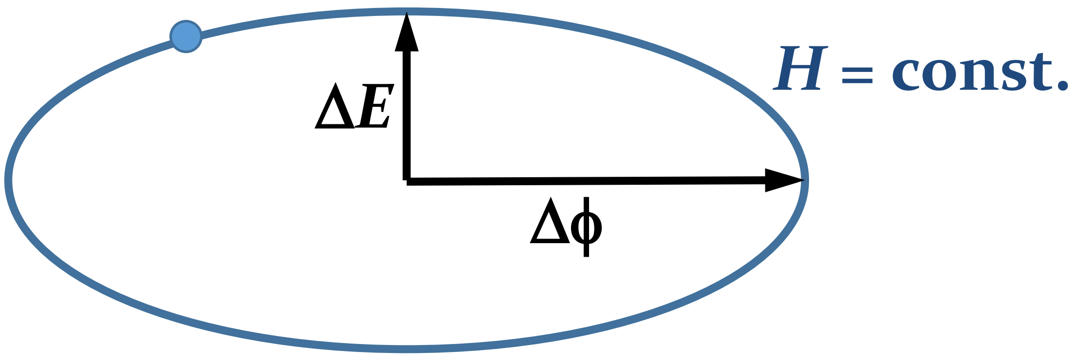
The Hamiltonian simplifies to

$$H\left(\Delta\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) \simeq -\frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2 - \frac{1}{2}\frac{qV}{2\pi}\Delta\phi^2\cos\phi_0$$

Recap of longitudinal beam dynamics (2)

$$H\left(\Delta\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) \simeq -\frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\boxed{\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2} - \frac{1}{2}\frac{qV}{2\pi}\boxed{\Delta\phi^2}\cos\phi_0$$

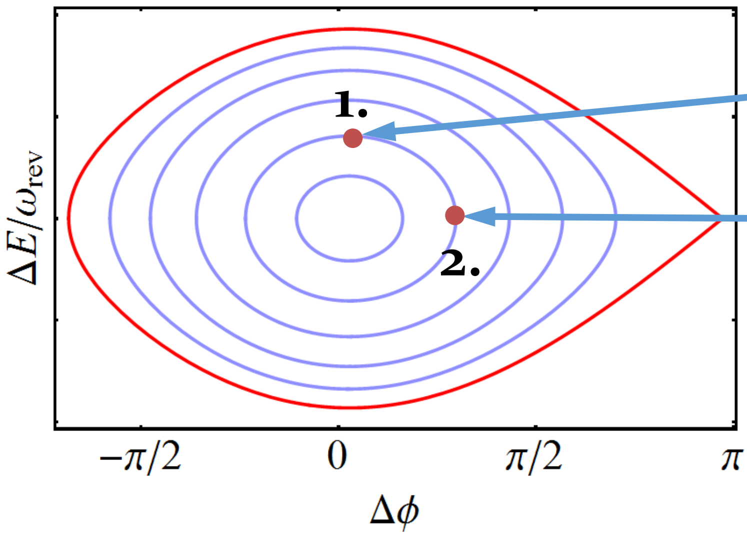
- **In the centre of the bucket, particles move on elliptical trajectories in Δϕ-Δ*E* phase space**
- **Hamiltonian is constant on these trajectories**



→ Aspect ratio of the elliptical trajectories must be identical in sending and receiving accelerator

Physical aspect ratio of bucket trajectories (1)

- **Compare two particles on the same trajectory**
  1. **No phase deviation**
  2. **No energy deviation**



$$H\left(\Delta\phi = 0, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = -\frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2$$

$$H\left(\Delta\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}} = 0\right) = -\frac{1}{2}\frac{qV}{2\pi}\Delta\phi^2\cos\phi_0$$

→ **$\Delta\phi$ depends on frequency → use physical duration $\Delta\tau$ instead**

$$\Delta\phi = 2\pi f_{\mathrm{RF}}\Delta\tau = h\omega_{\mathrm{rev}}\Delta\tau$$

→ **Also replacing** $pR = \dfrac{E\beta^2}{\omega_{\mathrm{rev}}}$

Physical aspect ratio of bucket trajectories (2)

→ **Hamiltonian equal for both extreme particles, hence**

$$-\frac{1}{2}\frac{h\eta\omega_{\text{rev}}^2}{E\beta^2}\left(\frac{\Delta E}{\omega_{\text{rev}}}\right)^2 = -\frac{1}{2}\frac{qV}{2\pi}h^2\omega_{\text{rev}}^2\Delta\tau^2\cos\phi_0$$

which can be simplified to

$$\boxed{\left(\frac{\Delta E}{\Delta\tau}\right)^2 = \frac{qV}{2\pi}E\beta^2 h\omega_{\text{rev}}^2\frac{\cos\phi_0}{\eta}}$$

→ **This aspect ratio $\Delta E/\Delta\tau$ must remain unchanged at transfer**

Matched bunch-to-bucket transfer

→ **Equating** $\left(\frac{\Delta E}{\Delta \tau}\right)^2 = \frac{qV}{2\pi} E \beta^2 h \omega_{\mathrm{rev}}^2 \frac{\cos\phi_0}{\eta}$ **for sending (1) and receiving (2) accelerator gives a general matching condition**

$$q_1 V_1 E_1 \beta_1^2 h_1 \omega_{\mathrm{rev},1}^2 \frac{\cos\phi_{0,1}}{\eta_1} = q_2 V_2 E_2 \beta_2^2 h_2 \omega_{\mathrm{rev},2}^2 \frac{\cos\phi_{0,2}}{\eta_2}$$

→ **For most cases (fixed energy and no particle type change)**

$$q_1 = q_2 \quad \beta_1 = \beta_2 \quad E_1 = E_2 \quad \cos\phi_{0,1} = \cos\phi_{0,2} = 1$$

It simplifies to the voltage ratio between RF systems:

$$\frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)^2 \left|\frac{\eta_1}{\eta_2}\right| \frac{h_2}{h_1}$$

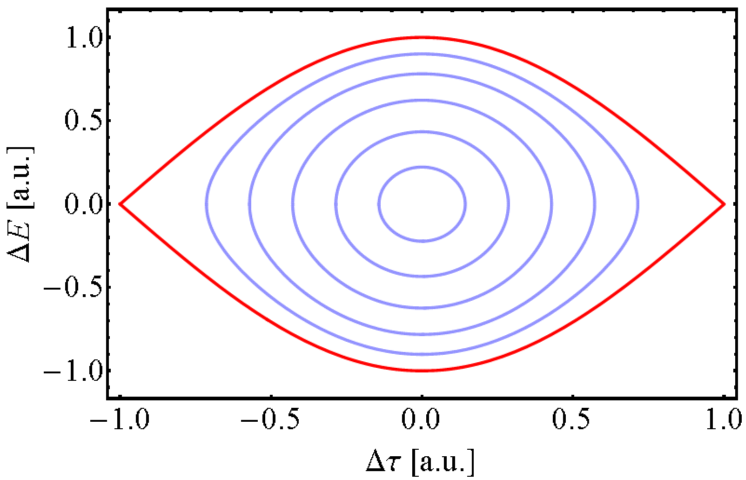
Simple matched transfer example

- **Transfer between to accelerators with $f_{RF,2} = f_{RF,1}/2$**

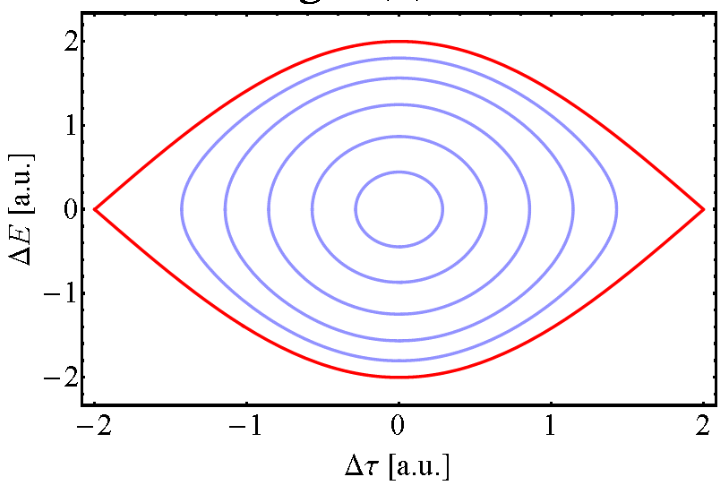
→ Phase space aspect ratio:

$$\Delta E = \beta \omega_{\text{rev}} \sqrt{\frac{qV}{2\pi} E h \left| \frac{\cos \phi_0}{\eta} \right|} \cdot \Delta\tau$$

Source (1) bucket



Target (2) bucket



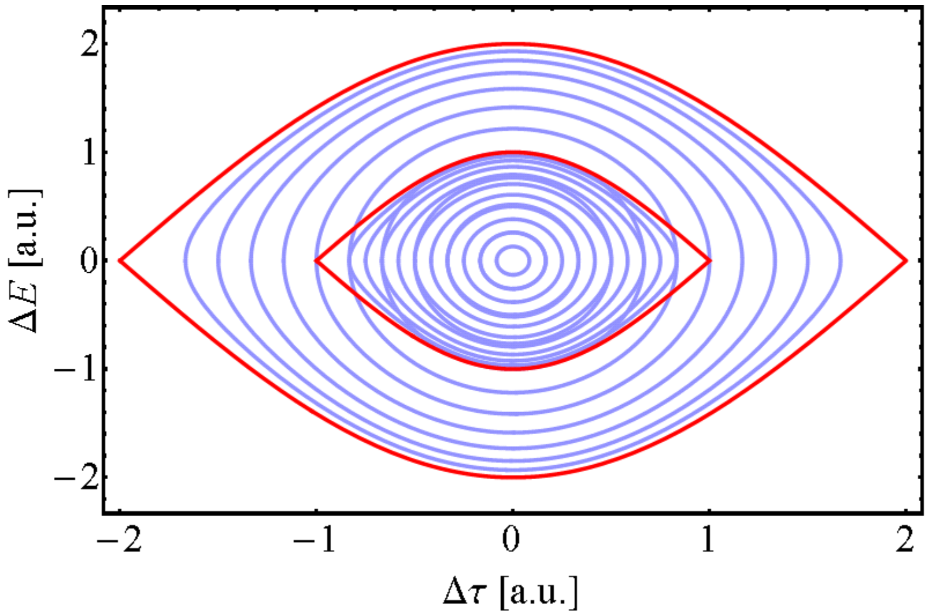
Simple matched transfer example

- **Transfer between to accelerators with $f_{RF,2} = f_{RF,1}/2$**

→ **Phase space aspect ratio:**

$$\Delta E = \beta \omega_{\mathrm{rev}} \sqrt{\frac{qV}{2\pi} E h \left| \frac{\cos \phi_0}{\eta} \right|} \cdot \Delta\tau$$

Source (1) and target (2) buckets



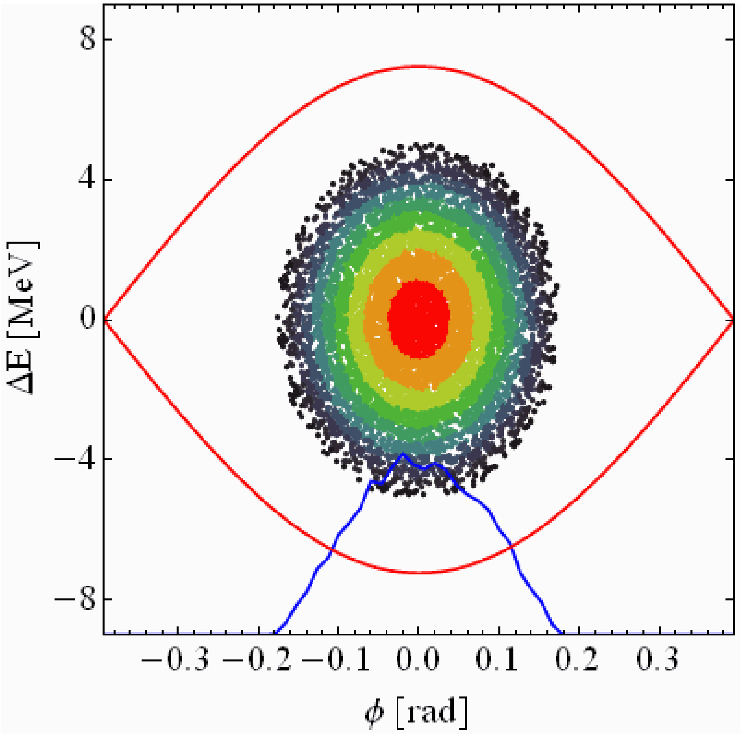
→ Obvious case of matched bunch-to-bucket transfer

Longitudinal matching

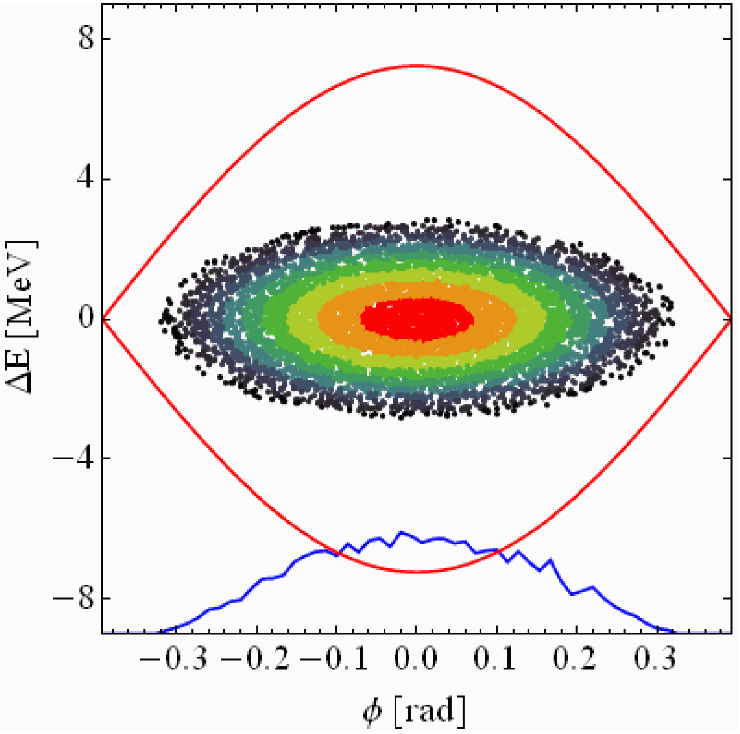
Longitudinal matching at injection

- **Long. emittance is only preserved for correct RF voltage**

Matched case



Longitudinal mismatch

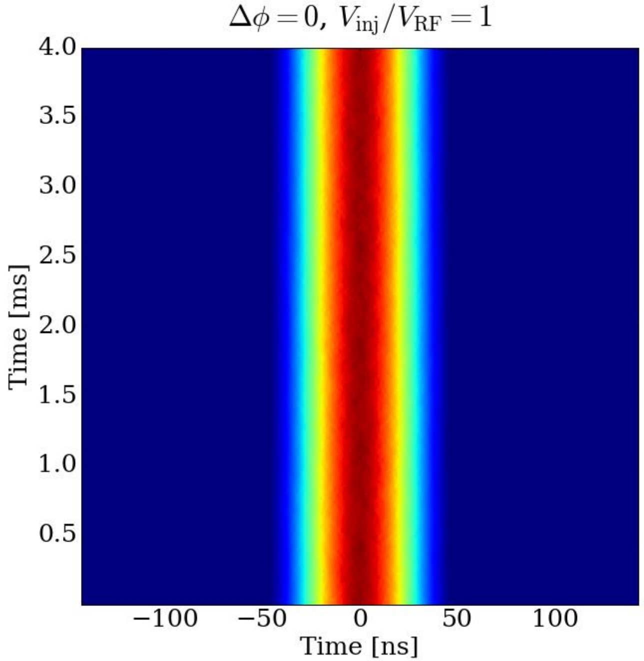


→ **Bunch is fine, longitudinal emittance remains constant**

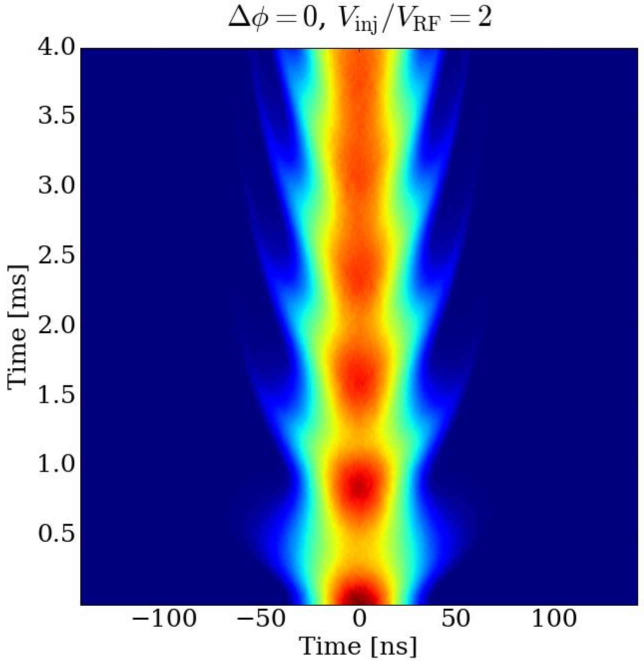
→ **Dilution of bunch results in increase of long. emittance**

Longitudinal matching

Matched case



Longitudinal mismatch

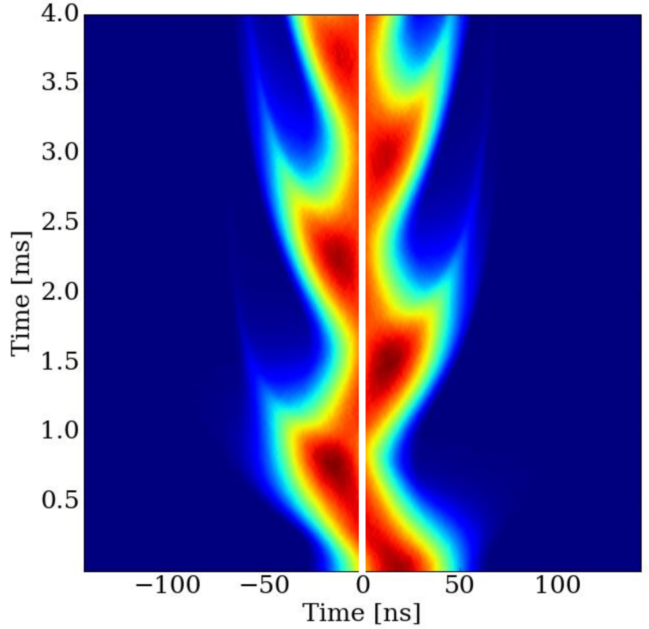


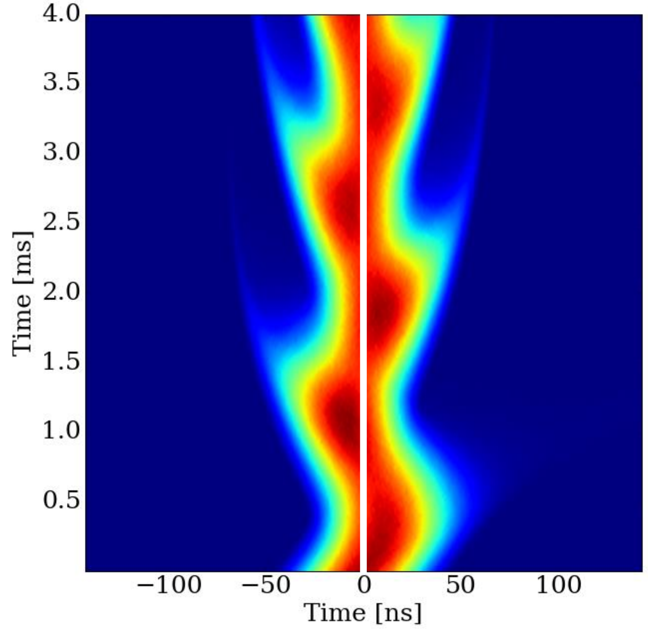
→ **Bunch is fine, longitudinal emittance remains constant**

→ **Dilution of bunch results in increase of long. emittance**

Matching of phase and energy

- **What is the difference?**





→ **-45° phase error at injection**

→ **Can be easily corrected by bucket phase**

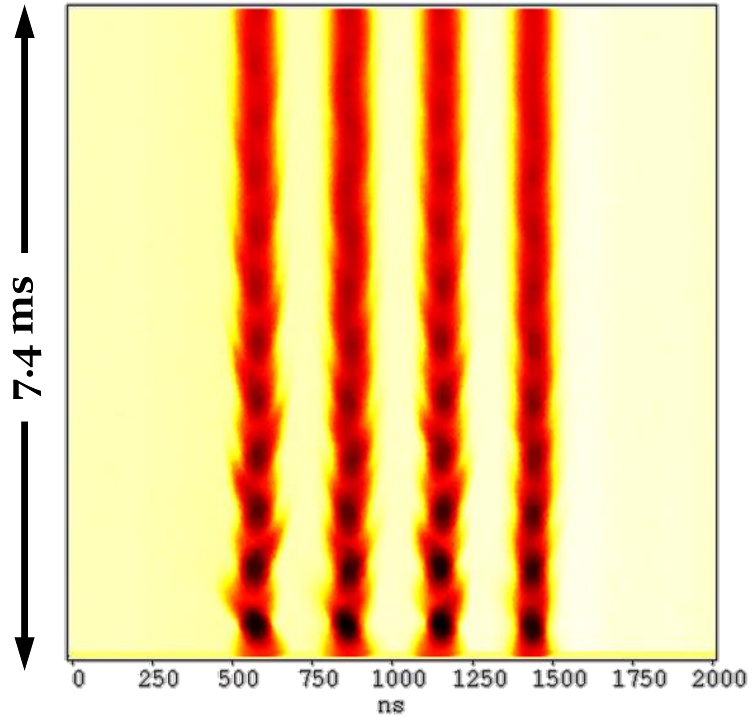
→ **Equivalent energy error**

→ **Phase does not help: requires beam energy change**

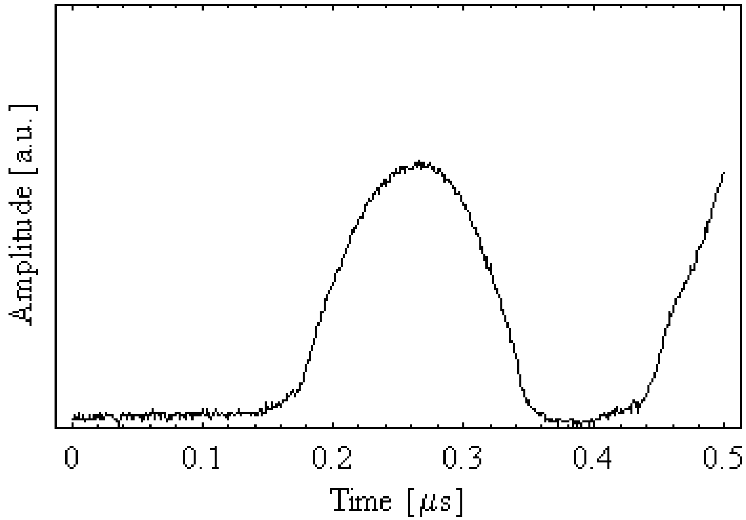
Example: mismatch at injection to PS

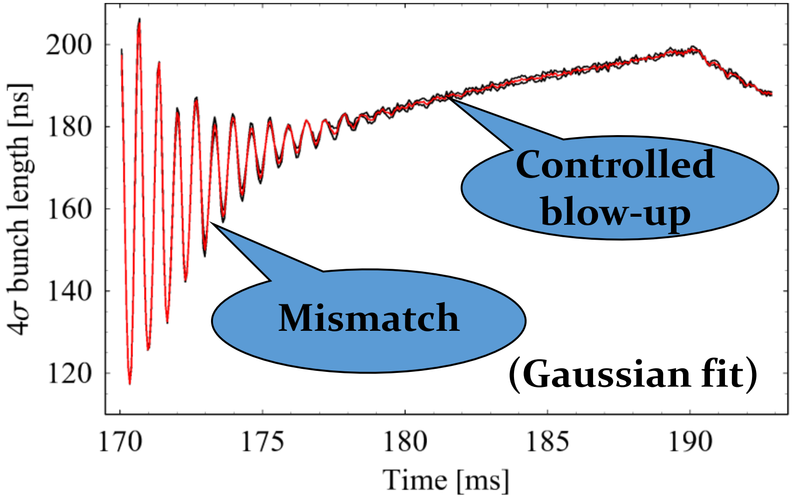
- **Deliberate longitudinal mismatch at injection for blow-up**

Mountain range



Bunch length evolution



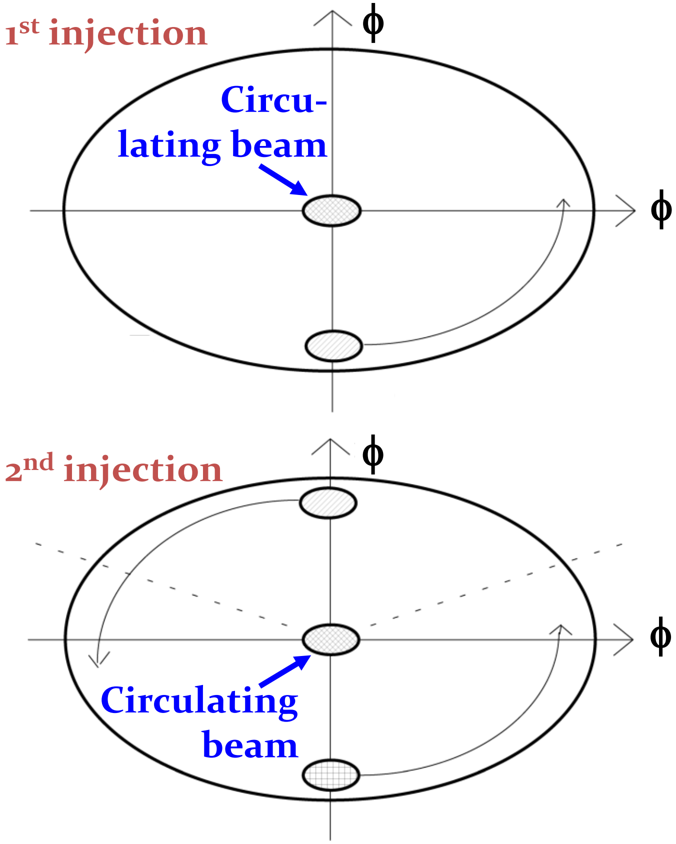


→ **Intentional mismatch contributes to controlled longitudinal blow-up**

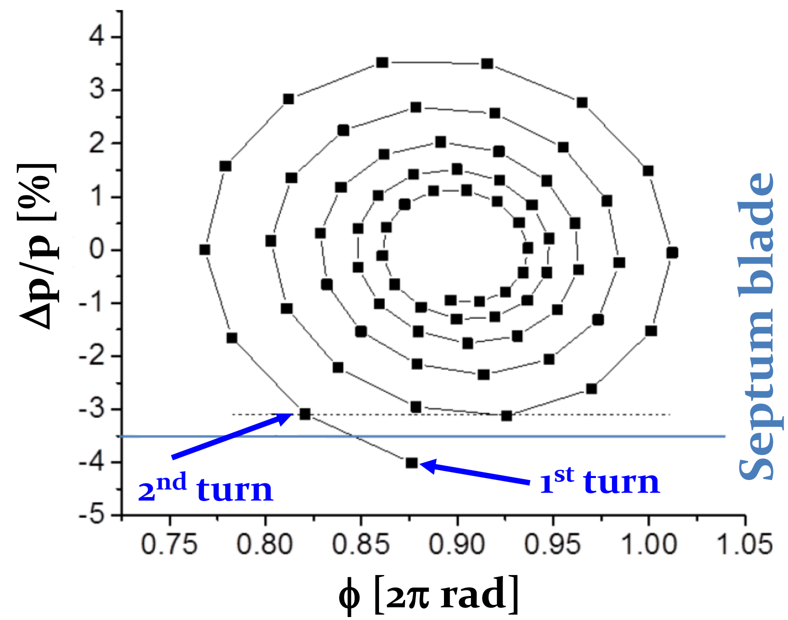
No problem with electron accelerators

- **Synchrotron radiation damping matches bunches by itself**
- **Phase and energy oscillations decay**

LEP synchrotron injection



Proposed FCC-ee injection



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→ **Mismatched injection can be a useful tool**

Summary

- **Basic techniques of signal synchronizations**
  → **Beware of dividers**

- **Beam transfer between circular lepton accelerators**
  → **Constant frequency**

- **Beam transfer between circular hadron accelerators**
  → **Variable frequency**
  → **Moving target**

- **Follow the beam**
  → **No need to measure → keep track**
  → **Matching between accelerators**

A big Thank You

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Thank you very much for your attention!

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Normalized Hamiltonian representation

- **For a single harmonic RF system**

$$H(\phi,\dot{\phi}) = \frac{1}{2}\dot{\phi}^2 + \frac{\omega_s^2}{\cos\phi_0}\left[\cos\phi_0 - \cos\phi + (\phi-\phi_0)\sin\phi_0\right]$$

with $\phi = \phi_0 + \Delta\phi$ **it becomes**

$$H(\Delta\phi,\dot{\phi}) = \frac{1}{2}\dot{\phi}^2 + \frac{\omega_s^2}{\cos\phi_0}\left[\cos\phi_0 - \cos(\phi_0+\Delta\phi) - \Delta\phi\sin\phi_0\right]$$

using $\cos(\phi_0+\Delta\phi) = \cos\phi_0\cos\Delta\phi - \sin\phi_0\sin\Delta\phi$

$$\simeq \cos\phi_0\left(1-\frac{1}{2}\Delta\phi^2\right) - \sin\phi_0\Delta\phi$$

this simplifies to $H(\Delta\phi,\dot{\phi}) \simeq \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\omega_s^2\Delta\phi^2$