Particle interactions with matter

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based on slides by A. Ferrari and F. Cerutti

Calculations based on the FLUKA Monte Carlo code

CAS, Erice, Italy

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Contents

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Why do particle-matter interactions matter?

Eventually all beam particles and/or their secondary products will interact with surrounding media ...

- **Beam disposal on a dump/stopper**
- **Particle impact on protection or beam manipulation devices**
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    - ⇒ halo cleaning
    - ⇒ radioprotection
    - ⇒ background reduction
    - ⇒ machine protection (in case of equipment malfunctions)
  - **Stripping foils**, **crystals** to extract the beam
- **Beam directed on targets**
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  - **Collisions in interaction points (luminosity)**
  - **Synchrotron radiation**
  - **Interactions with residual gas molecules**
  - **Interactions with macroparticles**
  - ...

SPS dump

(105-450 GeV/c protons)

LHC injection protection absorber

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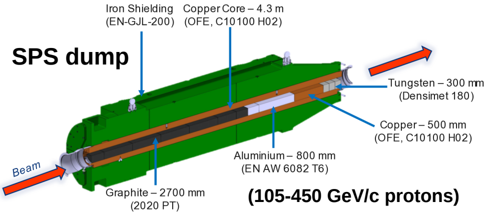

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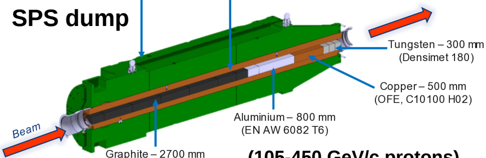

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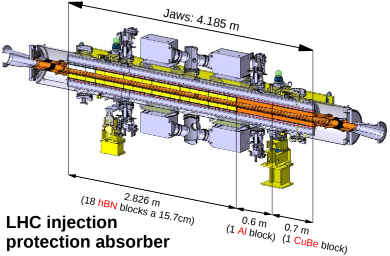

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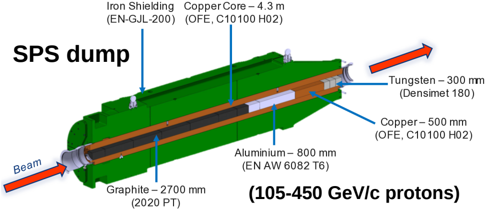

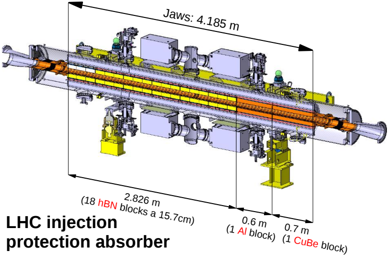

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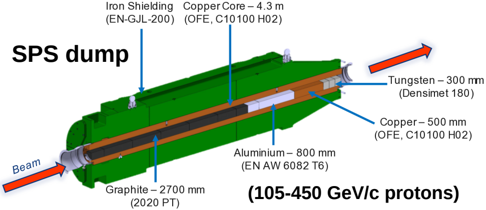

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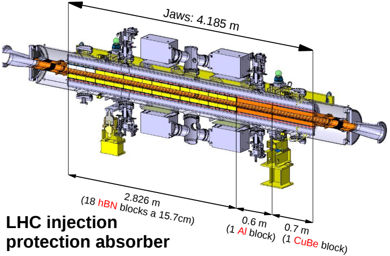
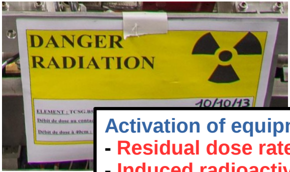


LHC injection protection absorber

Consequences & relevant macroscopic quantities

A non-exhaustive list



Activation of equipment:
- **Residual dose rate**
- **Induced radioactivity**

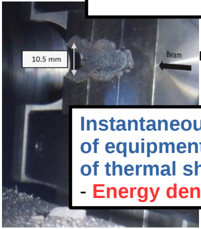
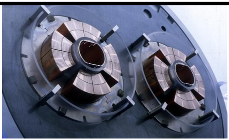


Figure courtesy of A. Bertarelli

Instantaneous damage of equipment because of thermal shock:
- **Energy density**

Quench of superconducting magnets:
- **Energy density** (transient losses)
- **Power density** (steady state losses)



Gas production:
- **Residual nuclei production**

Oxidation, radiolysis:
- **Energy deposition**

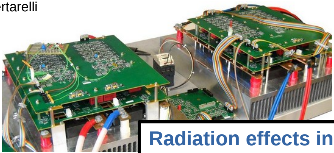


Fig. courtesy of TE/EPC

Radiation effects in electronics:
- **High-energy hadron fluence** (single event effects)
- **Total ionizing dose** (cumulative effects)
- **Si 1 MeV neutron equivalent fluence** (cumulative effects)

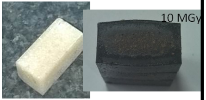


Fig. courtesy of P. Fessia

Long-term radiation damage of equipment:
- **Displacement per Atom** (non-organic materials)
- **Dose** (insulators)

The particle zoo

Some properties:

- **Hadrons:**
  - **Proton (p)** 938 MeV/c^2 stable
  - **Neutron (n)** 940 MeV/c^2 τ=886 s
  - **Charged pions (π^+, π^-)** 140 MeV/c^2 τ=2.6$\times 10^{-8}$ s ($c\tau$=780 cm) [mainly $\pi \rightarrow \mu\nu_\mu$]
  - **Neutral pions (π^0)** 135 MeV/c^2 τ=8.4$\times 10^{-17}$ s ($c\tau$=25 nm) [mainly $\pi \rightarrow \gamma\gamma$]
  - **Charged and neutral kaons, (anti)hyperons, antiprotons, antineutrons ...**

- **Photons (γ)**, stable, m=0

- **Leptons:**
  - **Electron, positron (e^-, e^+)** 511 keV/c^2 stable
  - **Muons (μ^-, μ^+)** 106 MeV/c^2 τ=2.2$\times 10^{-6}$ s ($c\tau$=687 m) [mainly $\mu \rightarrow e\nu_e\nu_\mu$]

Cross section and mean free path

- **Cross section per atom/nucleus σ (microscopic cross section)** [area]

For a given particle with energy E,
on an atom with atomic/mass number Z/A

$$\sigma = \sigma(E, Z, A)$$

(common unit: 1 barn (b) = 10^{-24} cm^2)

- **Mean free path λ** [length]

$$\lambda = \frac{1}{N\sigma} = \frac{M}{\rho N_A \sigma}$$

Molar mass [mass/mol]

Atom density [1/volume]

Microscopic cross section [area]

Avogadro's constant [6.022 x 10^{23} mol^{-1}]

Material density [mass/volume]

= average distance travelled by a particle between two successive collisions

- **Macroscopic cross section Σ** [inverse length]

$$\Sigma = \frac{1}{\lambda} = N\sigma$$

- **Remark:**
  - The amount of material traversed by a particle is often expressed as **surface density** $\Rightarrow$ **length** $\times$ **density** ρ [$cm \times g/cm^3 = g/cm^2$]

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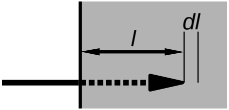
Interaction probability

Assume that particles are normally incident on a homogeneous material and that they are subject to a process with a mean free path λ between collisions:

- **Path length distribution**

Note : $\int_0^\infty p(l')dl' = 1$

$$p(l)dl = \frac{1}{\lambda} exp\left(-\frac{l}{\lambda}\right) dl$$



$p(l)dl$ = probability that a particle has an interaction between l and $l + dl$

- **Cumulative interaction probability**

P(λ) = 63.2%
P(2λ) = 86.5%
P(3λ) = 95.0%
P(4λ) = 98.2%

$$P(l) = \int_0^l p(l')\mathrm{d}l' = 1 - exp\left(-\frac{l}{\lambda}\right)$$

Survival probability:

$$P_s(l) = 1 - P(l) = exp(-l/\lambda)$$

$P(l)$ = probability that a particle interacts before reaching a path length l

- **In case of a thin target (thickness $d \ll \lambda$)**

$$P_{target} = P(d) \approx \frac{d}{\lambda}$$

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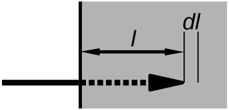

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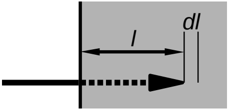

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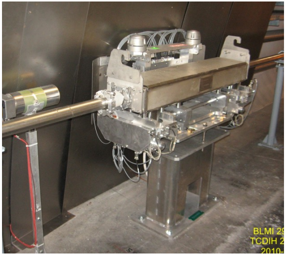
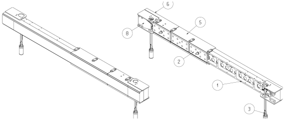
Interaction probability: example

- **Example: beam loss during SPS-to-LHC transfer (protons@450 GeV)**
  - Assume that **288 bunches** with **1.3×10^{11} protons/bunch** are intercepted by a collimator during SPS-to-LHC transfer ($288 \cdot 1.3\times10^{11} =$ **3.74×10^{13} protons**).
  - How many protons will have an **inelastic nuclear collision** in the collimator?





Material	Density	Active length
Graphite	1.84 g/cm^3	1.20 m

Answer:

- *Inelastic p-C cross section (@450GeV):*

$$\sigma = \sim 245\,\mathrm{mb}$$

- *Mean free path (=inelastic scattering length):*

$$\lambda = \frac{\overbrace{12.0107\,\mathrm{g/mol}}^{\textit{Molar mass carbon}}}{\underbrace{1.84\,\mathrm{g/cm^3}}_{\textit{Density}} \cdot \underbrace{6.022\times10^{23}\,\mathrm{mol^{-1}}}_{\textit{Avogadro constant}} \cdot \underbrace{245\times10^{-27}\,\mathrm{cm^2}}_{\sigma}} = 44\,\mathrm{cm}$$

- *Number of interacting protons (out of 3.74×10^{13}):*

$$N_i = \left[1 - exp\left(-\frac{\overbrace{120\,\mathrm{cm}}^{\textit{Length}^*}}{\underbrace{44\,\mathrm{cm}}_{\lambda}}\right)\right] \cdot 3.74\times10^{13} = 3.5\times10^{13}\,\mathrm{p}$$

* *For simplicity, we assume that the path of protons in the collimator is straight, i.e. no elastic scattering.*

Contents

Photon interactions: basics

- **Photons can be produced in a variety of processes, e.g.:**
  - **Bremsstrahlung**
  - **Gamma-deexcitation after nuclear reactions**
  - **Radiative neutron capture**
  - **Electron-positron annihilation**
  - **Particle decay (e.g. π^0's from nuclear reactions)**
  - **...**

- **Relevant processes for photon scattering and absorption:**

Coherent (Rayleigh) scattering:

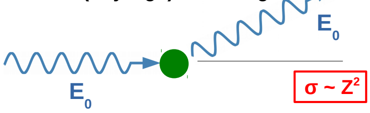
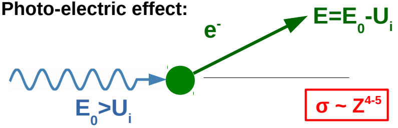
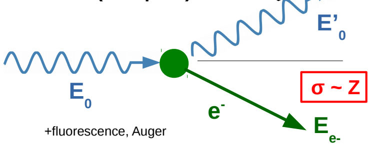


Photo-electric effect:



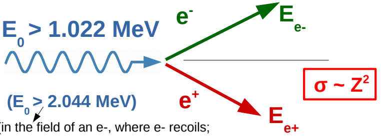
+fluorescence, Auger

Incoherent (Compton) scattering:



+fluorescence, Auger

Electron-positron pair production:



(in the field of an e-, where e- recoils;
also called triplet production)

+ photo-nuclear reactions

Photon interactions: cross sections

$\sigma_{p.e.}$= **Photo-electric effect**

$\sigma_{Rayleigh}$= **Coherent scattering**

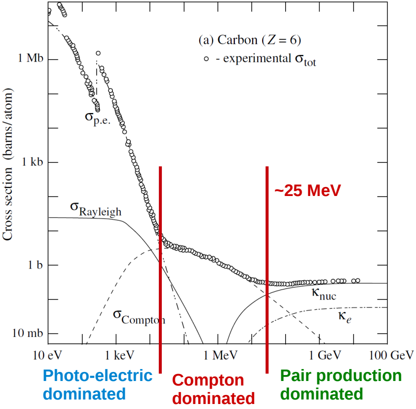
$\sigma_{Compton}$= **Incoherent scattering**

κ_{nuc}= **Pair production in field of nucleus**

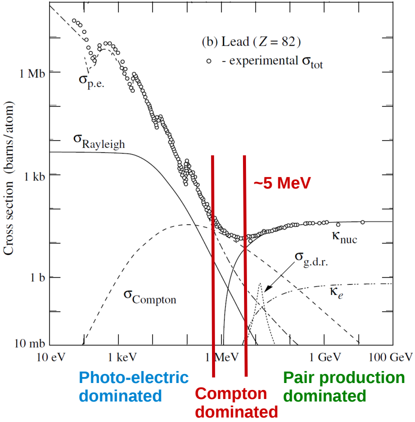
κ_e= **Pair production in field of electron**

$\sigma_{g.d.r}$= **Giant Dipole Resonance**

Carbon:



Lead:



Figures from: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016).

Photon interactions: absorption length

Mass attenuation length $[\frac{mass}{area}]$ **(absorption length)**

$$\lambda\rho = \frac{M}{N_A \sigma_{tot}} \quad where \quad \sigma_{tot} = \sum_i \sigma_i$$

(sum over all processes discussed on previous pages)

Mass attenuation coefficient $[\frac{area}{mass}]$

$$\mu_m = \frac{1}{\lambda\rho}$$

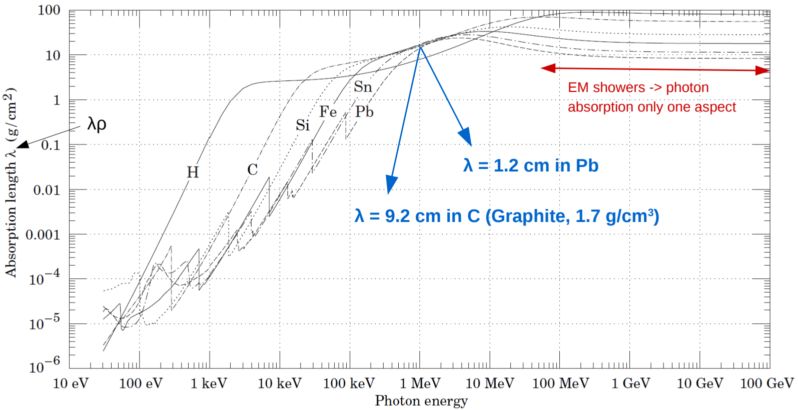


Figure from: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016).

Charged particle interactions: basics

- **Coulomb interactions with electrons and nuclei**

  - **Excitation** or **ionisation** of atoms (energetic electrons: **δ-rays**)
  - Dominate† **energy loss** up to energies where radiative losses become important

    $\Rightarrow$ up to a few **10 MeV** for **$e^{+/-}$**
    $\Rightarrow$ up to a few **100 GeV** for $\mu^{+/-}$
    (up to even higher E for ch. hadrons††)

    = electronic energy loss $\Rightarrow$ *heating*

  - Dominate the **angular deflections** of charged particles
  - **Energy loss** $\ll$ electronic one, except for low-energy heavy projectiles (ions keV/u)

    = non-ionizing energy loss (NIEL)

    $\Rightarrow$ *displacement damage*

† *Except for low-energy heavy projectiles where NIEL can be higher.*
†† *But: high-energy hadrons are subject to nuclear interactions.*

- **Radiative processes**

  - For **$e^{+/-}$** above a few **10 MeV**: **energy loss** dominated by **Bremsstrahlung** processes
  - For $\mu^{+/-}$ above a few **100 GeV**: **Bremsstrahlung**, **e^-/e^+pair production**, **photo-nuclear**

Heavy charged particles ($M \gg m_e$): example muon stopping

Mass stopping power: $\frac{dE}{dx}\frac{1}{\rho}$ in units : $\left[\frac{MeV}{cm}\frac{1}{g/cm^3} = \frac{MeV\ cm^2}{g}\right]$

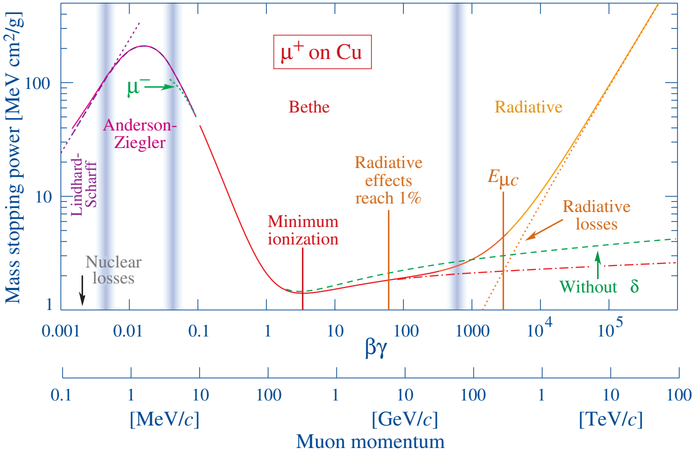


Figure from: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016).

Bethe equation:

$\Rightarrow$ to a first order:

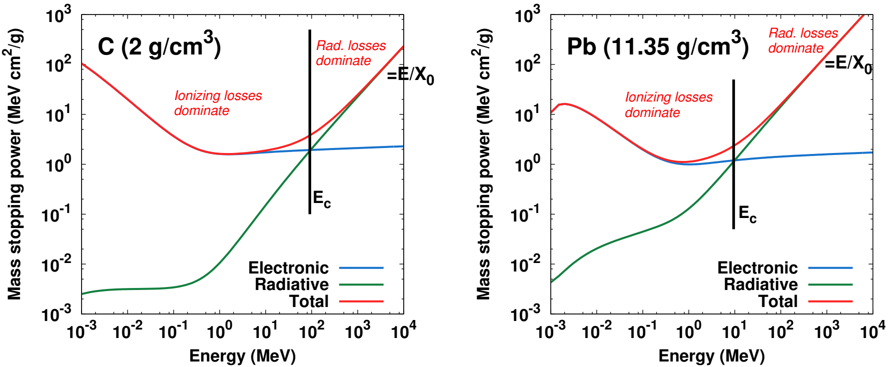
- it has a weak material dependency ($\propto$Z/A)
- depends only on projectile charge not mass

$\Rightarrow$ Min. at $\beta\gamma \approx$3-3.5 (MIPs = min. ionizing particles) **1-2 $MeV\ cm^2/g$**

Energy loss fluctuations in Coulomb interactions not discussed here, but are equally important for the energy loss.

Electrons: electronic and radiative stopping

For electrons $\rightarrow$ radiative losses already important at *much lower energies*



Source: http://physics.nist.gov/PhysRefData/Star/Text/intro.html

Critical energy E_c:

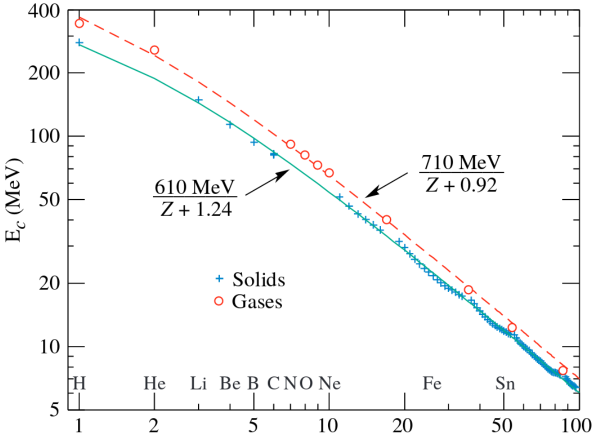
$$\frac{dE}{dx}(E_c)\bigg|_{ioni} = \frac{dE}{dx}(E_c)\bigg|_{brems}$$

X_0 is the radiation length, which will be introduced later.

Electrons: critical energy E_c

$$E_c = \frac{610(710)\ MeV}{Z + 1.24(0.92)}$$

$\longleftarrow$ *commonly used fit for solids (gases) for Rossi's definition*[†] *of* E_c *(similar fits exist for* μ*)*



Atomic number Z

[†] Rossi uses a slightly different definition for E_c, however the difference is small:

$$\frac{dE}{dx}(E_c)\Big|_{ioni} = \frac{E}{X_0}$$

Figure from: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016).

Radiation length X_0

- **Radiation length X_0**
  - **Is a characteristic length for both bremsstrahlung and pair production:**

High energy electrons: $-\frac{dE}{dz}\Big|_{rad} = \frac{E}{X_0}$

$$\langle E(z)\rangle = E_0 \cdot \exp\left(-\frac{z}{X_0}\right)$$

X_0 = average distance needed to reduce the energy of a high-energy electron by a factor of 1/e

$$i.e.\ \ \langle E(X_0)\rangle = 36.8\% E_0$$

High energy photons: $\sigma_{pp} \approx \frac{7}{9}\frac{M}{\rho N_A X_0}$

$$\langle I(z)\rangle = I_0 \cdot \exp\left(-\frac{7}{9}\frac{z}{X_0}\right)$$

X_0 = 7/9 of the mean free path for pair production by a high-energy photon

$$i.e.\ \ \langle I(X_0)\rangle = 45.9\% I_0$$

- **Material dependency**
  - **Common approximation (Dahl):**

$$X_0\rho\,[g/cm^2] = \frac{716.4 g/cm^2 A}{Z(Z+1)\ln(287/\sqrt{Z})}$$

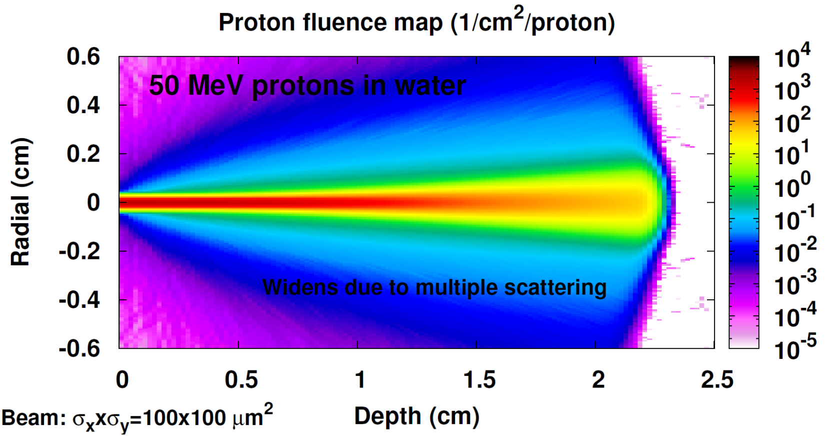
ρ = density, Z = atomic number, A = mass number

Material	Z	Density	X_0
Graphite	6	2.21 g/cm^3	19.32 cm
Al	13	2.699 g/cm^3	8.90 cm
Fe	26	7.874 g/cm^3	1.76 cm
Cu	29	8.96 g/cm^3	1.44 cm
W	74	19.30 g/cm^3	0.35 cm
Pb	82	11.35 g/cm^3	0.56 cm

Source: pdg.lbl.gov/AtomicNuclearProperties/

Multiple scattering

- **Coulomb interactions with nuclei**
  - ⇒ Particles scatter more when their energy decreases
  - ⇒ Lighter particles scatter more if they have the same βc as heavier particles
  - ⇒ Well described by multiple scattering theory of **Moliere**



Contents

Hadron-nucleus interactions: cross-section & mean free path

Elastic + non-elastic interactions: in the latter new particles are produced and/or the internal structure of the target/projectile are changed

⇒ Microscopic cross section for non-elastic had-nucleus collisions scales as

$$\sigma \propto A^{2/3}$$

(geometrical cross section)

⇒ Mean free path of non-elastic nuclear interactions scales as:

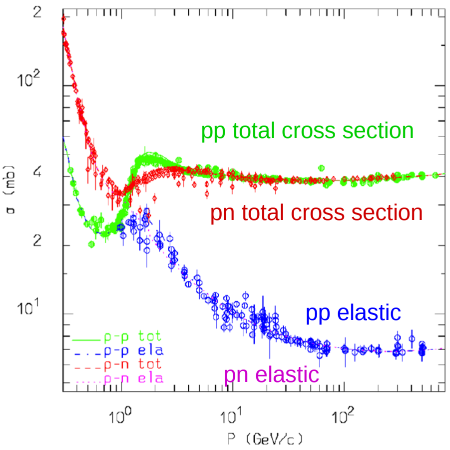
$$\lambda_I \rho \propto A^{1/3}$$

Also called **inel. scattering length**

Mat	Z	Density	X_0	λ_I
C	6	2.2 g/cm^3	21.4 cm	37.3 cm
Al	13	2.7 g/cm^3	8.90 cm	35.4 cm
Fe	26	7.8 g/cm^3	1.76 cm	15.1 cm
Cu	29	8.96 g/cm^3	1.44 cm	13.9 cm
W	74	19.3 g/cm^3	0.35 cm	8.9 cm
Pb	82	11.4 g/cm^3	0.56 cm	15.7 cm

Source: FLUKA, λ_I for 7 TeV protons

Nucleon-nucleon cross sections:



Hadron-nucleus interactions: basics (simplified picture!)

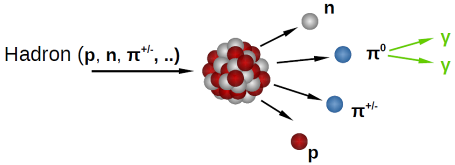
Fast stage (10^{-22} s)

Hadron interacts with nucleons: particle production possible (mainly **π**)

e.g. nucleon-nucleon: π production opens at 290 MeV for a free nucleon (somewhat lower for nucleons in nucleus)

Intra-nuclear cascade of p, n, π:

→ energetic particles can leave nucleus (→ forward directed)
→ others can deposit energy in nucleus (→ excited state)



fast particle multiplicity ~ log(E)

Hadronic + EM cascades

Pre-compound

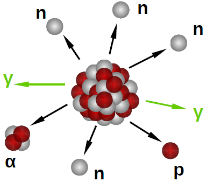
Equilibrium

Slow stage (10^{-16} s)

Evaporation
(n, light fragments)
γ-deexcitation

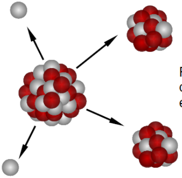
isotropic emission

few MeV



compete

Fission
(heavy elements)

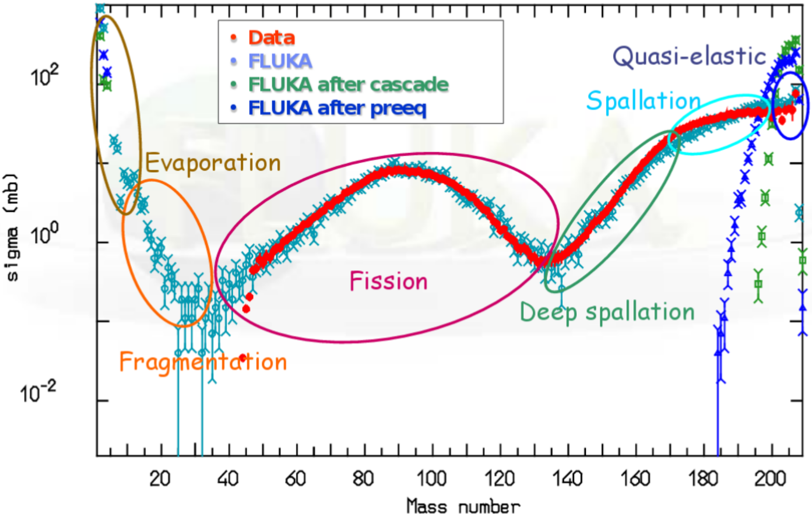


Fission products can also undergo evaporation

→ residuals can be radioactive

Residual nuclei production: example of fission/evaporation

1 A GeV ^{208}Pb + p reactions Nucl. Phys. A 686 (2001) 481-524



Neutrons: low-energy cross sections ($< 20\,\text{MeV}$)

- Only "stable" neutral hadron $\rightarrow$ **very penetrating**
- Mainly slow down (mainly in elastic coll. $\rightarrow$ recoil) until they thermalize and are captured

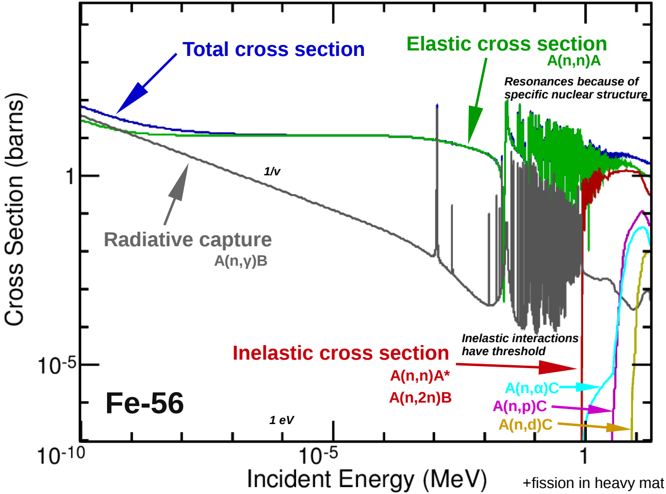


Figure from: ENDF/B-VII.1, `http://www.nndc.bnl.gov/exfor/endf00.jsp`

Contents

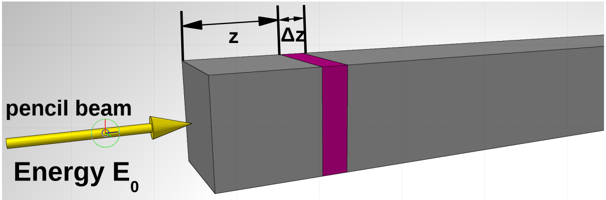
Energy deposition: general remarks

- **Energy deposition in a material**
  - Mediated by Coulomb interactions of **charged particles** put in motion by atomic and nuclear processes
  - **Energy loss $\neq$ energy deposition** $\rightarrow$ energy can be transported away by secondaries
- **Longitudinal energy deposition profiles shown in the following:**
  - It is assumed that a pencil beam impacts on a **laterally infinite material block**
  - Longitudinal profiles are expressed as:

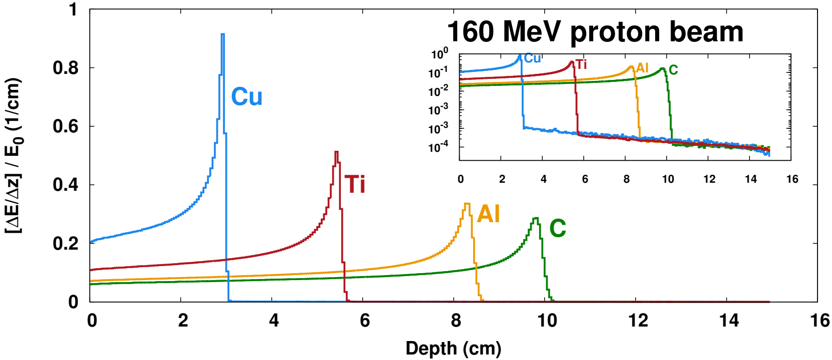
$$\varepsilon(z) = \frac{\Delta E}{\Delta z} \frac{1}{E_0}(z)$$

Unit = [1/length]

ΔE = energy deposited in layer
Δz = layer thickness
z = depth inside target
E_0 = beam energy



Example: Protons at 160 MeV (LINAC4 at CERN)

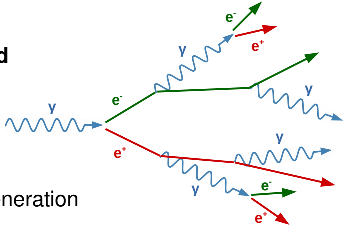


$\Rightarrow$ **at higher energies (GeV) showers dominate the energy deposition profile**

EM showers: basics

- **Relevant processes:**
  - **High-energy e^-/e^+ lose energy mainly through bremsstrahlung**
    - $\Rightarrow$ above **10 MeV** in heavy materials
    - $\Rightarrow$ above **100 MeV** in light materials
  - **For photons at such energies, dominant interaction is pair production**
- **Cascade development:**
  - **At high energy ($\gtrsim$ GeV), these processes lead to particle multiplication**

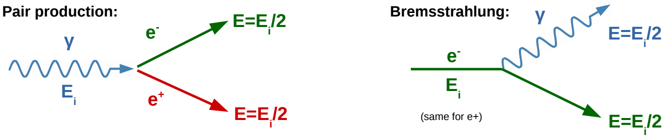
    = electromagnetic (EM) shower



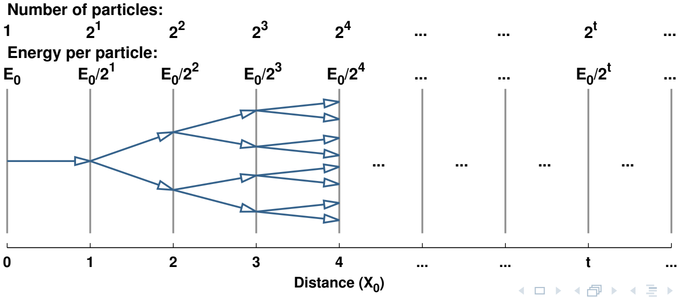
  - Energy/particle decreases from generation to generation
  - Multiplication stops when the **energy of e^-/e^+ falls below $\sim E_c$**
    - $\Rightarrow$ below E_c they dissipate energy mainly through ionization/excitation
    - $\Rightarrow$ **shower maximum = location where number of particles is maximum**
    - $\Rightarrow$ characteristic length $\rightarrow$ X_0

EM showers: longitudinal profile (Heitler model)

- **Qualitative features can be derived from Heitler's model, which assumes:**
  - interactions (bremsstrahlung, pair production) take always place after a distance X_0
  - at each interaction the energy is equally split between the two outgoing particles



- **Particle multiplication vs depth (expressed as $t = z/X_0$):**



EM showers: longitudinal profile (Heitler model cont'd)

- **Location of shower maximum predicted by model:**
  - **Assume shower (i.e. multiplication) stops when energy/particle = E_c:**

$$E_{av}(t)\Big|_{t=t_{max}} = \frac{E_0}{2^{t_{max}}} = E_c$$

t_{max} = # of X_0 *required to reach shower maximum*

  - **Depth of shower maximum increases logarithmically with energy**

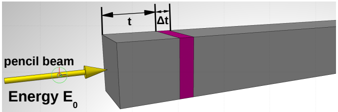
$$t_{max} \propto ln\left(\frac{E_0}{E_c}\right)$$

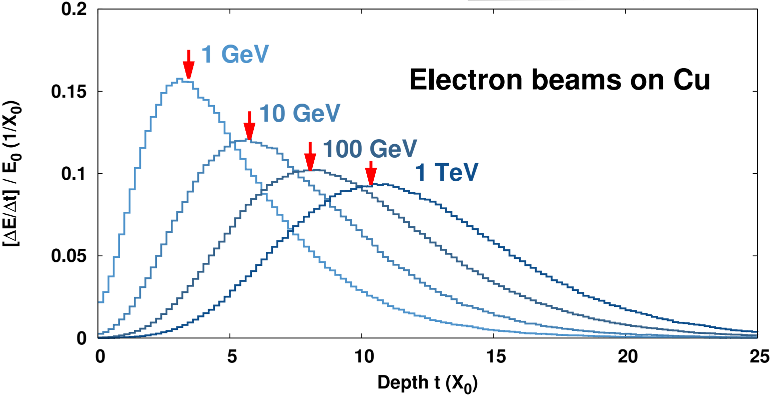
$\Rightarrow$ **since EM showers scale with X_0, they are shorter the higher the atomic number and the material density** ($X_0 \propto A/(Z^2\rho)$)

- **Note:**
  - Although it correctly predicts the logarithmic dependence, the model has many limitations (e.g. electron/photon ratios not correctly predicted)

EM showers: longitudinal profile (Monte Carlo simulation)

Longitudinal energy deposition profile
(all curves have same area)





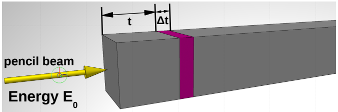
Red arrows:

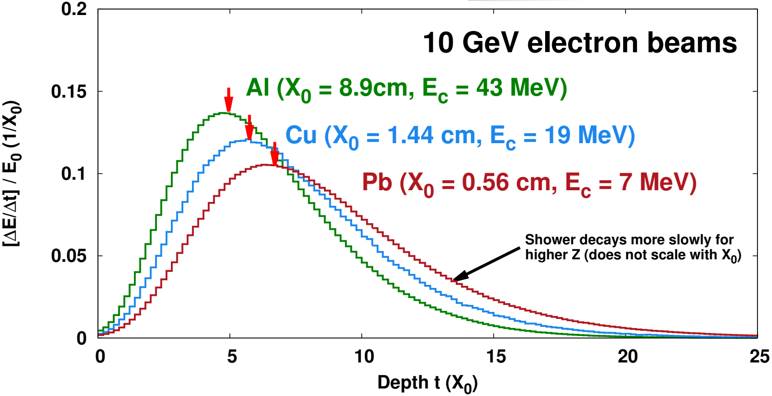
$$t_{max} = \frac{z_{max}}{X_0} = log\left(\frac{E_0}{E_c}\right) - 0.5 \quad \longleftarrow$$

commonly used expression to estimate the depth of shower max

(note: photon-induced: -0.5 $\rightarrow$ 0.5)

Longitudinal energy deposition profile
(all curves have same area)





Red arrows:

$$t_{max} = \frac{z_{max}}{X_0} = \log\left(\frac{E_0}{E_c}\right) - 0.5$$

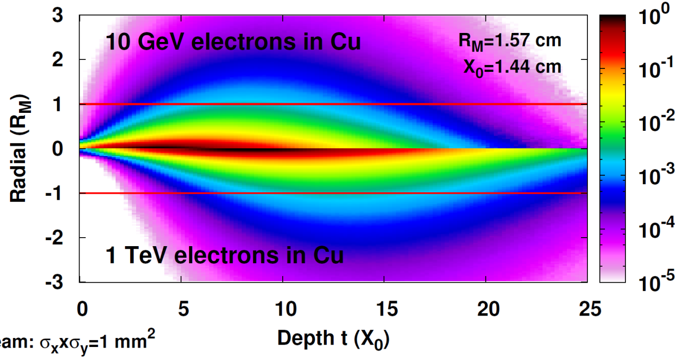
Higher-Z materials: multiplication down to lower energies (lower E_c)

EM showers: transverse profile (Monte Carlo simulation)

r-z energy deposition map (normalized to peak value)

Transverse profile of shower core around longitudinal peak:

- Roughly energy-independent
- Governed by multiple scattering of (lower-energy) electrons



Well described by Moliere radius:

$$R_M = \frac{E_s^\dagger}{E_c} X_0 = \frac{21\ MeV}{E_c} X_0$$

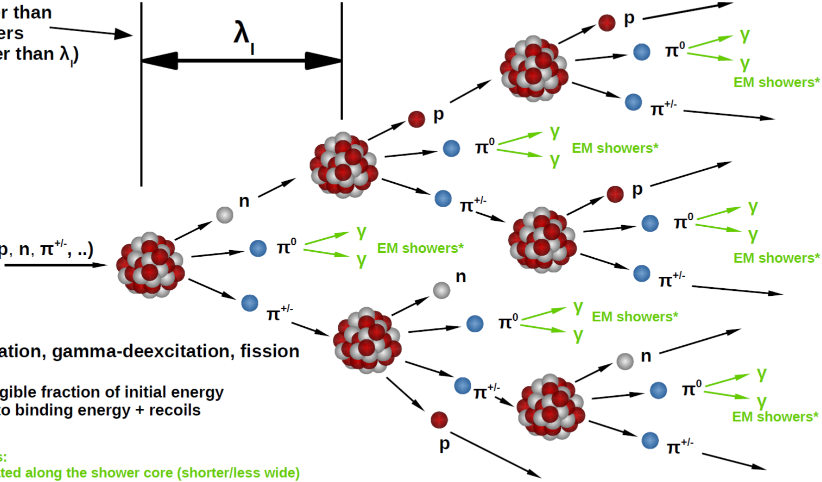
= average lateral deflection of electrons with $E=E_c$ after traversing one X_0 (90% of energy deposition within $\sim$1 R_M)

† $E_s = \sqrt{4\pi/\alpha}\, m_e c^2$, where α is the fine structure constant

Hadronic showers: basics

Are longer than EM showers (X_0 smaller than λ_I)

Hadron (p, n, $\pi^{+/-}$, ..)



+Evaporation, gamma-deexcitation, fission

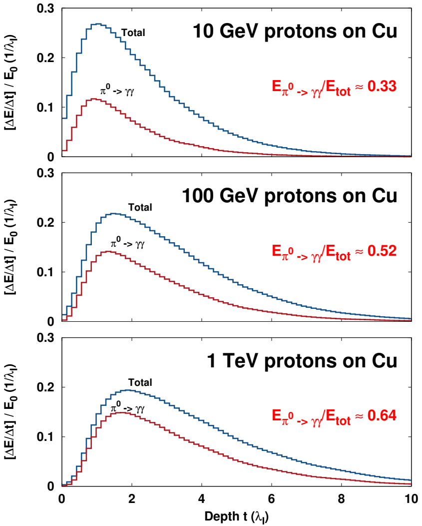
Non-negligible fraction of initial energy can go into binding energy + recoils

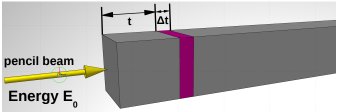
*EM showers:
→ concentrated along the shower core (shorter/less wide)
→ ~ do not give rise to hadronic showers (photo-nuclear cross section small)
→ not only π^0 but also other particles like η

roughly continues until particle energy falls below pion production threshold

Hadronic showers: longitudinal profile (Monte Carlo sim.)

Longitudinal energy deposition profiles

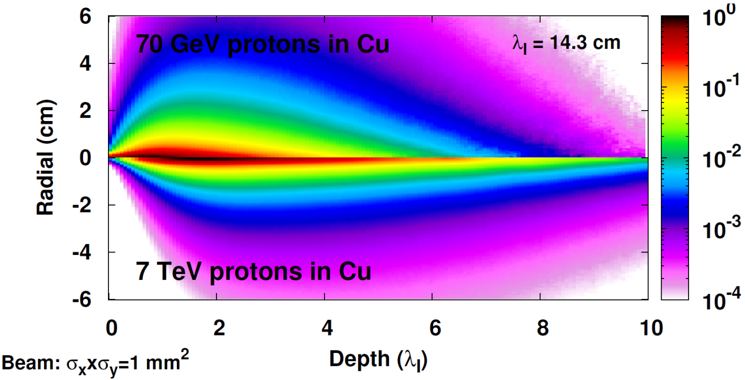




- **Depth of shower maximum:**
  - Like for EM showers, scales roughly with ***log(E_0)***
- **Relative EM shower contribution to energy deposition:**
  - The higher E_0, the more interactions needed to go below a few GeV
  - Since at each interaction ∼1/3 of energy goes into π^0's, relative EM shower contribution **increases with increasing E_0**

Hadronic showers: transverse profile (Monte Carlo sim.)

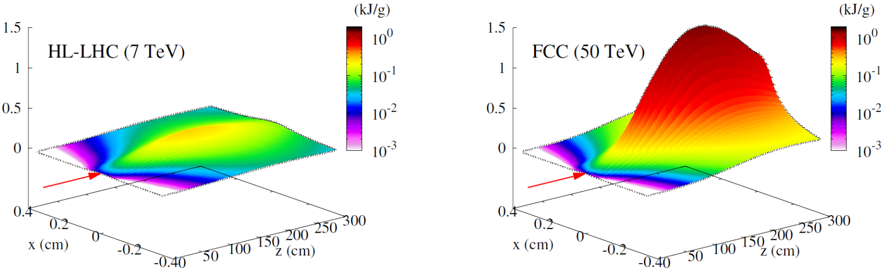
r-z energy deposition map (normalized to peak value)



Transverse shower profile:

- The **transverse momentum** of hadrons produced in nuclear collisions is more or less **invariant with energy** (average 300-400 MeV/c)
- Shower opening angle becomes **narrower with increasing energy**

Challenges ahead: LHC bunch vs FCC bunch



Figures: Energy density in 3 m-long Graphite (1.83 g/cm^3) for one nominal proton bunch (σ=400μm), comparing HL-LHC (top) and FCC (bottom).

Thank you very much for your attention!