ELECTRON SOURCES AND INJECTORS

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Lecture 1:

- The role of electron sources and injectors in light source
- Requirements for electron sources and injectors
- Injector beam dynamics

Lecture 2:

- Injector components
  - Cathodes systems
  - Electron guns
  - Compression systems
  - Focusing systems
  - Accelerating systems
  - Diagnostics systems
- Challenges and required R&D

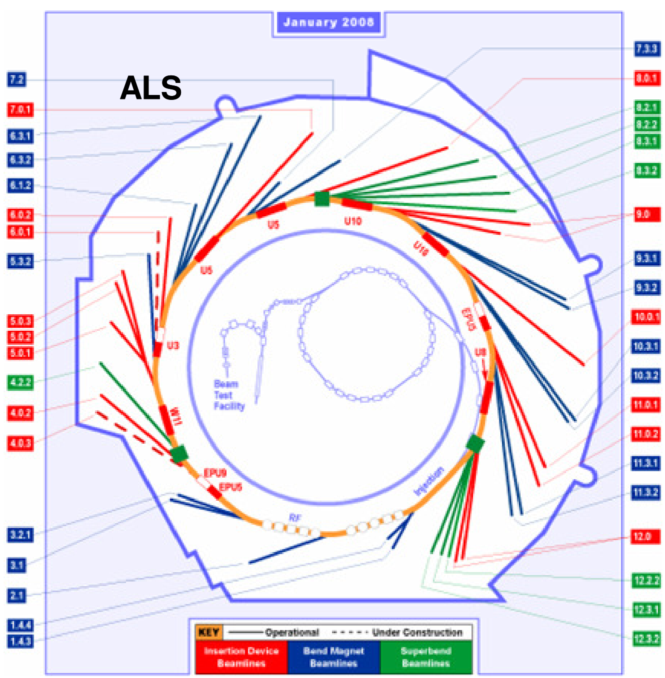
A lot of material for just two lectures!

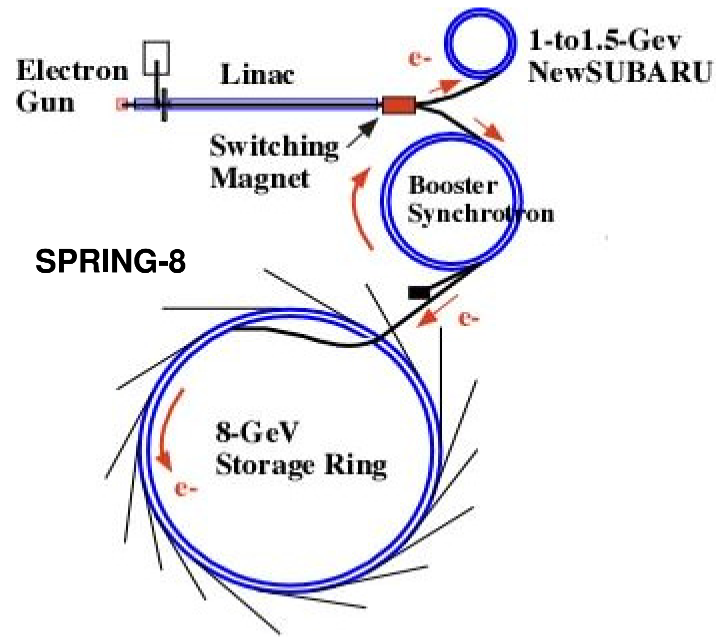
It forces to minimize the amount of explicit derivations and to concentrate in the explanation of main concepts and issues.

References will be given for the complete derivation and a more detailed examination

The Role of the Electron Injector in Light Sources

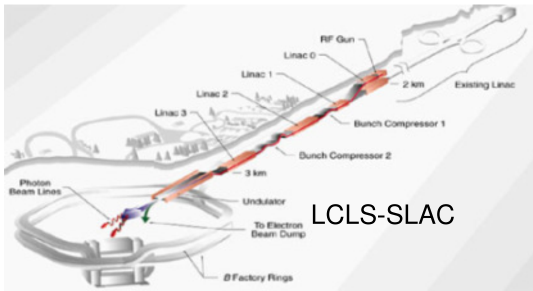
In 1st, 2nd and 3rd generation light sources, electron sources are part of the injector chain that typically includes a small linac and a "booster" ring.
The beam generated by the electron gun goes through the linac and is then accelerated and stored in the booster for a time long enough that the 6D beam phase-space distribution is fully defined by the characteristics of the booster and not of the electron source.

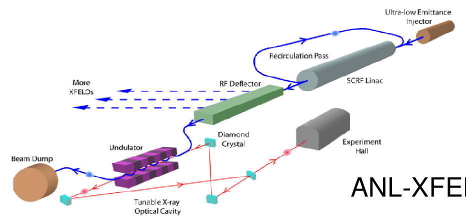


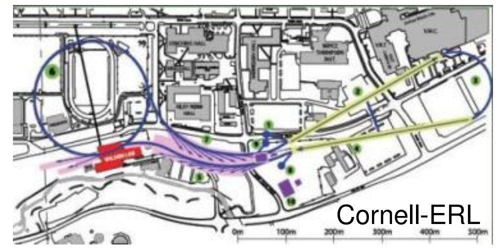


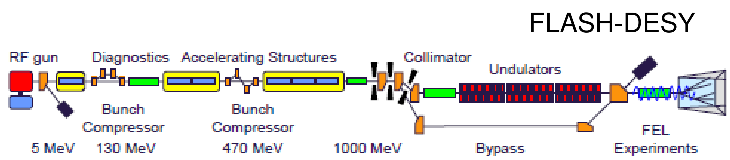
The Role of the Electron Injector in Light Sources

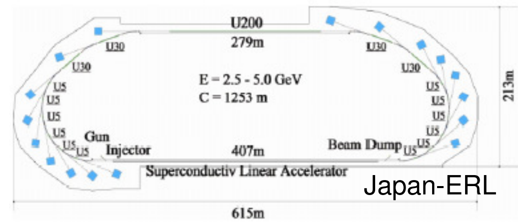
In linac based 4th generation light sources, such as free electron lasers (FELs) and energy recovery linacs (ERLs), the situation can be quite different. Indeed, in such a case, the final beam quality is set by the linac and ultimately by its injector and electron source.











In such facilities, the requirements for a large number of quasi-"monochromatic" electrons, concentrated in very short bunches, with small transverse size and divergence, translate into high particle density 6D phase-space, or in other words, in high brightness B:

$$B = \frac{N_e}{\varepsilon_{nx}\varepsilon_{ny}\varepsilon_{nz}}$$

with N_e the number of electrons per bunch and $\varepsilon_{nx,\,ny,\,nz}$ the normalized emittances for the planes x, y, and z

The brightness generated at the electron source represents the ultimate value for such a quantity, and cannot be improved but only spoiled along the downstream accelerator

X-Ray 4th Generation Light Sources, the Most Challenging Electron Injector Case

- In FELs, the matching condition for transverse emittance drives towards small normalized emittances.

$$\varepsilon \approx \frac{\lambda}{4\pi} \Rightarrow \frac{\varepsilon_n}{\beta\gamma} \approx \frac{\lambda}{4\pi}$$

- The minimum obtainable value for ε_n defines the energy of the beam ($\gamma = E/mc^2$).

(with β the electron velocity in speed of light units, and assuming that an undulator with the proper period λ_u and undulator parameter K exist: $\lambda = \lambda_u / 2\gamma^2(1 + K^2/2)$)

- We will see later, that for the present electron gun technologies: $\varepsilon_n <$ ~1 μm for the typical < ~1 nC charge/bunch.

For X-Ray machines ($\lambda <$ ~1 nm) that implies GeV-class electron beam energy, presently obtainable by long and expensive linacs.

- Similar transverse emittance requirements apply also to ERLs.

- In X-Ray FELs the matching condition for the energy spread requires a fairly low energy spread as well

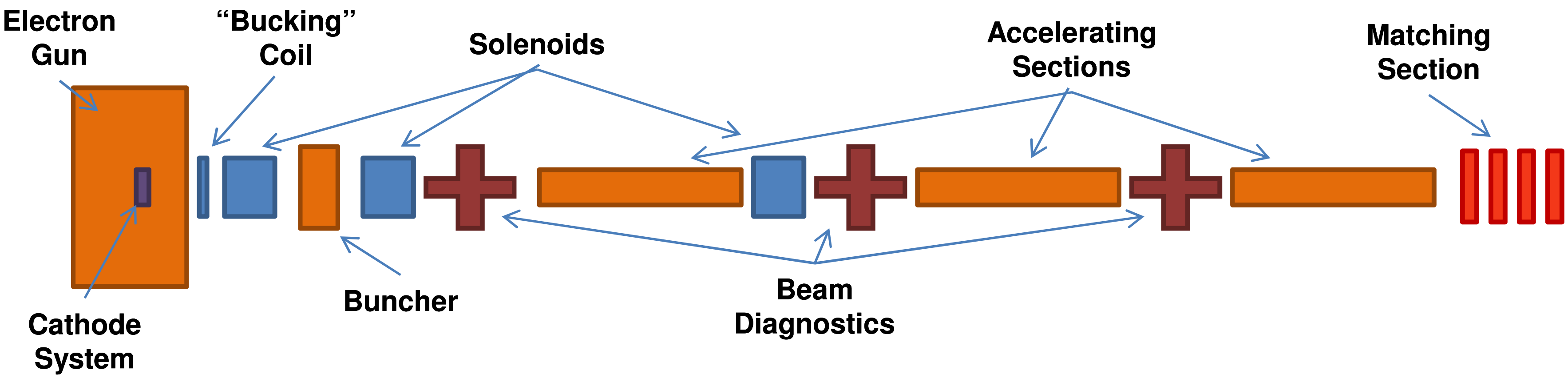
$$\frac{\sigma_E}{E} < \sim \rho_{Pierce} < \sim 10^{-3}$$

- Achieving the necessary FEL gain requires high peak current (~ 1 kA), and hence high charge/bunch and short bunches.

- In both ERLs and FELs, high-time resolution user-experiments require extremely short X-Ray pulses (down to sub-fs) imposing the need for small and linear longitudinal emittances to allow for the proper compression along the linac.

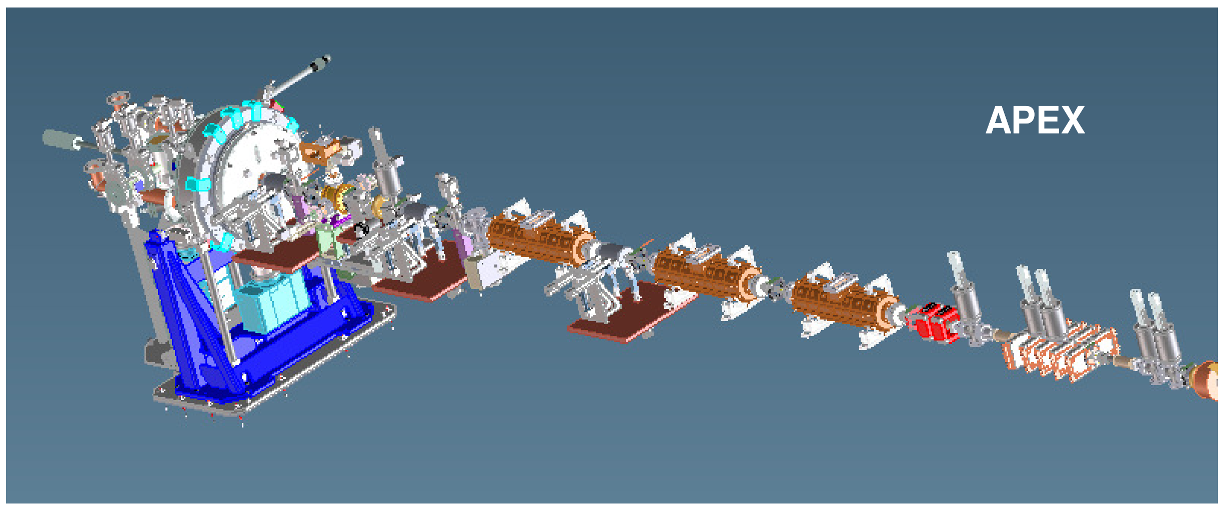
In summary, 4th generation X-Ray facilities challenge the performance of electron injectors. These lectures from now on will focus on such a type of injectors

The Typical High-Brightness Injector Layout



Injector Sub-Systems:

- Cathode system
- Electron gun
- Focusing system
- Compression system
- Accelerating system
- Diagnostics system



Much more details in Lecture 2!

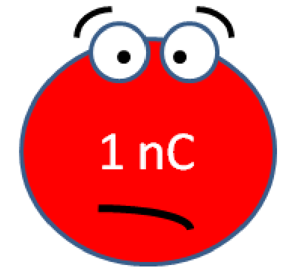
Requirements for the electron injector

Injector Requirement Summary Table

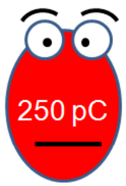
Repetition rate	from ~ 10 Hz to ~ 1 MHz (FELs) up to ~ 1GHz and beyond (ERLs)
Charge per bunch (depending on the operation mode)	from ∼ 1 pC to ∼ 1 nC
	(repetition rate and charge per bunch together:) up to several 100s mA average current (ERLs)
Normalized transverse emittance (slice)	sub 10^{-7} m to 10^{-6} m (from low to high charge/bunch)
Normalized longitudinal emittance	~ several µm at low charge outside the MBI regime
Beam energy at the gun exit (to control space charge effects)	$\gtrsim$ 500 keV
Beam energy at the injector exit	$\gtrsim$ 100 MeV
Accelerating electric field at the cathode (to overcome the space charge limit)	$\gtrsim$ 10-15 MV/m
Dark current	minimization is critical for high duty cycle injectors
Bunch length at the cathode (to control space charge effects and for different modes of operation)	from ∼ 100 fs to tens of ps
Peak current at the injector exit	tens of A in FEL's injectors
Compatibility with magnetic fields in the cathode and gun regions (for emittance compensation and/or exchange techniques)	
Operational vacuum pressure at the electron gun (compatible with damage-sensitive cathodes)	10^{-7} - 10^{-9} Pa (∼ 10^{-9} - ∼ 10^{-11} Torr)
'Easy and fast' replacement of cathodes at the electron gun	
High reliability required to operate in an user facility	

Charge per Bunch

In order to exploit all the different modes of operation of ERLs and FELs, the injector must operate within a very broad range of charge/bunch.

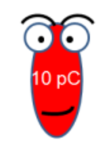


- **For example, experiments in FELs and ERLs requiring large number of photons per pulse or very narrow transform limited photon bandwidth in seeded FEL schemes require longer bunches and hence higher charges per bunch that can approach the nC.**



- **The main operational mode for X-Ray FELs relies on a charge/bunch of a few 100s pC (~ 100 pC for ERLs), where a satisfactory tradeoff between the number of photons/pulse and a moderate transverse emittance increase at the injector due to space charge forces can be found.**

- **Smaller charges per bunch (from few tens of pC down to the pC) have being proposed as an alternative/complementary mode of operation. Because of the lower charge/bunch, space charge effects can be more efficiently controlled making electron guns capable of generating beams with smaller transverse and longitudinal normalized emittances. The resulting higher 6D brightness allows for shorter FEL gain lengths at a relatively moderate electron beam energy.**



In ERLs the low emittance potentially obtainable with few tens of pC charge/bunch allows for modes of operation with X-Ray pulse with full transverse coherence.

Emittance

We previously saw that the major objective for electron injectors is to maximize the brightness B

$$B = \frac{N_e}{\varepsilon_{nx}\varepsilon_{ny}\varepsilon_{nz}}$$

For a fixed charge/ bunch that translate in minimizing the emittance in each of the planes.

The **normalized emittance** in each plane is proportional to the area in the phase space occupied by the beam.

$$\varepsilon_{nw} = \sigma_w \frac{\sigma_{pw}}{mc} \qquad w = x, y, z$$

In **Hamiltonian systems the normalized emittance is an invariant of motion.**

For a constant energy beam, the **geometric emittance** is an invariant of motion and is defined (in the transverse plane) as:

$$\varepsilon_w = \frac{\varepsilon_{nw}}{\beta\gamma} = \sigma_w \frac{\sigma_{pw}}{\beta\gamma\, mc} = \sigma_w \frac{\sigma_{pw}}{p} = \sigma_w \sigma_{w'} \quad with \quad w = x, y \;\; and \quad w' = \frac{p_w}{p}$$

For a given set of particles (beam) the **r.m.s. geometric emittance** is defined as

$$\varepsilon_{w\,rms} = \sqrt{\langle w^2 \rangle \langle w'^2 \rangle - \langle w w' \rangle^2} \qquad w = x, y$$

The r.m.s. emittance is not conserved in the presence of nonlinear forces

Transverse Emittance

• **Small normalized transverse emittances are extremely important in X-Ray FELs because the required matching between the small X-Ray photon emittance and the electron beam geometric emittance**

$$\varepsilon \approx \frac{\lambda}{4\pi} \;\Rightarrow\; \frac{\varepsilon_n}{\beta\gamma} \approx \frac{\lambda}{4\pi}$$

can be achieved at lower beam energies
(assuming that undulators with the required period are feasible).

• **Also, small emittances in SASE FELs allow for shorter gain lengths and thus for shorter undulators.**

$$\rho^{1D}_{Pierce} = \frac{1}{\gamma}\left[\frac{1}{64\pi^2}\frac{I_e}{I_A}\frac{1}{\varepsilon_x \beta_x}\lambda_u^2 K^2 JJ^2\right]^{1/3}; \qquad \frac{1}{\rho^{1D}_{Pierce}} \approx \begin{matrix} Number\ of\ undulator\ periods \\ required\ for\ saturation \end{matrix}$$

(3D effects add further dependence of the gain length on the geometric emittance)

• **In ERLs high electron brightness translates directly into high photon brightness.**

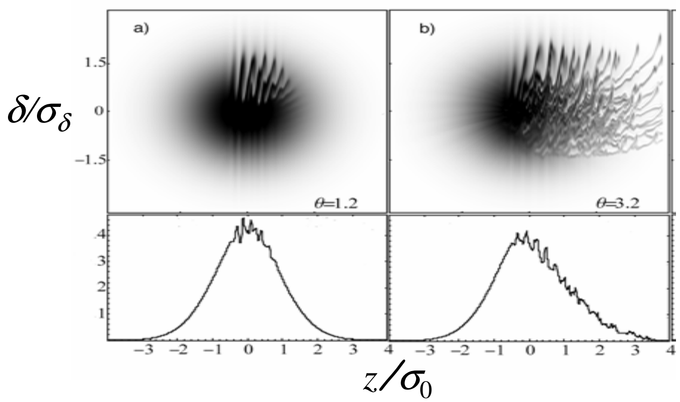
• **These benefits are particularly important because they allow to effectively reduce the size and the cost of expensive X-Ray FEL and ERL facilities.**

• **The minimum achievable ε_n depends on the charge/bunch.**
Indeed, at lower charges, it is possible to reduce the beam size at the cathode while still keeping under control space charge.
We will see later that smaller sizes at the cathode imply smaller emittances.

Longitudinal Emittance

- When discussing **longitudinal emittance** requirements for injectors, **two different cases need to be considered.**

- In the relatively high charge/bunch regime (**few hundreds of pC and above**), the rising of the **microbunching instability** (MBI) in linac magnetic compressors, forces to use 'heating' techniques (e.g. laser heating) to increase the uncorrelated energy spread and damp the instability. In this situation, the **longitudinal emittance at the injector exit is not relevant**.



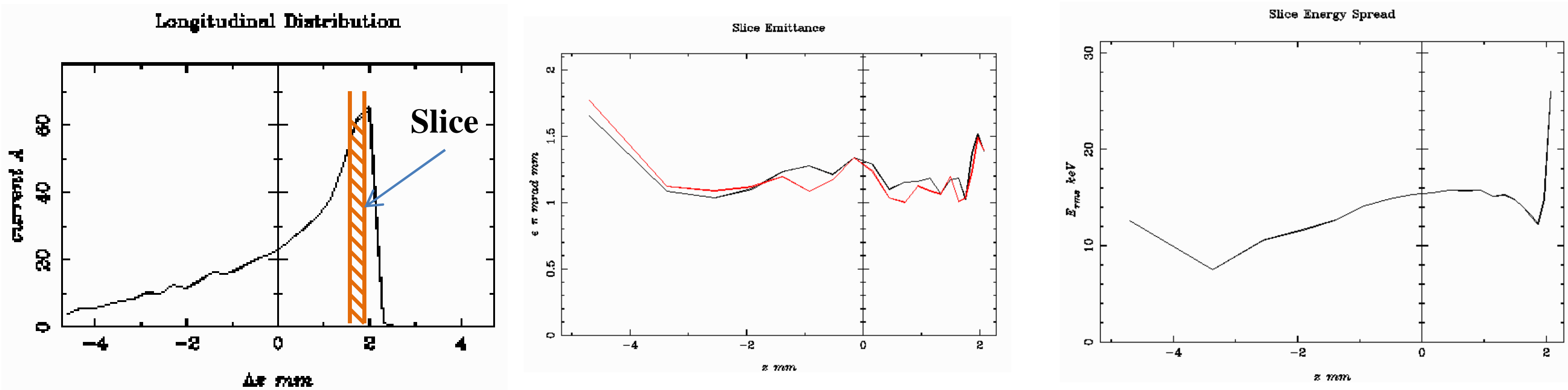
Simulation by Marco Venturini

- **At lower charges per bunch**, MBI can be generally controlled and no beam heating is required anymore. Lower longitudinal emittances become now possible and available resulting in an increased 6D brightness and thus in a reduction of the FEL gain length.

In this low charge regime, normalized longitudinal emittances in the µm range are desirable.

"Slice" and "Projected" Quantities

- An important remark: in an FEL, the **requirements** on beam parameters such as emittance, peak current and energy spread **need to be satisfied only in the longitudinal portion of the electron beam where lasing is desired.**



- The length of this region must be greater than the electron to photon slippage along the undulator, but it is ultimately defined by the FEL mode of operation, the experimental tolerances and the fluctuations of the relevant parameters.
- For example, in seeded FEL schemes, such a length must be longer than the seeding laser pulse convoluted with the total jitter between the electron and laser pulses. The term '**slice**' is usually associated with a beam quantity measured within this 'lasing' part of the beam (or to a fraction of it), while the term '**projected**' is referred to a property of the whole beam.
- On the contrary, **in ERLs are the projected characteristics** to be **important**

Brightness

Brightness: density of particles in the phase space.
I.e. number of particles per unit of phase space volume.

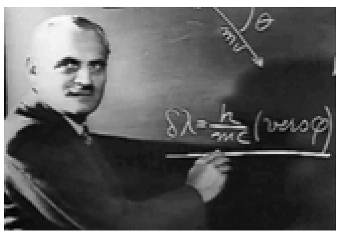
$$B = \frac{N}{\varepsilon_{nx}\varepsilon_{ny}\varepsilon_{nz}}$$

Heisenberg uncertainty principle: *"it is impossible to determine with precision and simultaneously, the position and the momentum of a particle"*. Applied to emittances:

$$\varepsilon_{nw} \geq \lambda_c / 4\pi \qquad w = x, y, z$$

$$\lambda_c \equiv Compton\ wavelength = h/m_0 c = 2.426\ pm\ \ for\ electrons$$

This can be interpreted as the fact that the phase space volume occupied by a particle is given by: $(\lambda_c/4\pi)^3$ = elementary phase space volume



Degeneracy Factor, δ: if the phase space is expressed in elementary phase space volume units, the brightness becomes a dimensionless quantity δ representing the **number of particles per elementary volume.**

$$\delta = B\left(\frac{\lambda_C}{4\pi}\right)^3$$

Because of the Pauli exclusion principle the **limit value of δ** is: infinity for bosons and **1 for non polarized fermions** (electrons). Quantum limited brightness for fermions.

High Brightness Electron Sources?

Most of the edge electron beam applications (accelerators, free electron lasers, microscopes, ...) are limited by the performance of the electron source in:

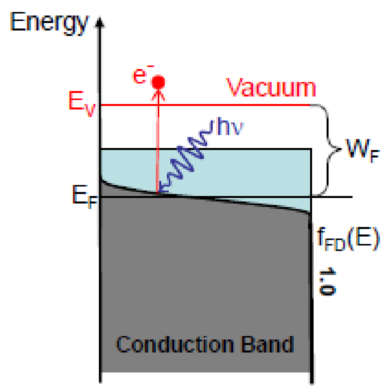
- Lowest emittance
- Smallest energy spread
- Highest brightness

} **Highest Degeneracy Factor δ** ⇒

- Conv. thermionic: $\delta \sim 10^{-14}$
- SEM: $\delta < 10^{-14}$
- Photo-RF guns: $\delta \sim 10^{-11}$
- Field emission: $\delta \sim 10^{-5}$

The degeneracy factor inside a metal cathode is ~ 1.

!!!



How do we lose all of that?

Extraction Mechanism

Coulomb interaction

ERLs and FELs require high charge/bunch sources for high photon flux. For those charges a $\delta \sim 10^{-11}$ is the best that can be presently obtained.

Energy

• Most of the emittance increase due to space charge happens in the low energy part of the injector, the electron gun.

• Space charge effect intensity scales as γ^{-2} so higher energies at the gun are beneficial in minimizing such effects.

• Extensive simulation work and experimental evidence (Shintake gun) showed that an energy of at least 500 keV is necessary to achieve the required beam quality within the charge/bunch range of interest.

• In a high brightness injector the final electron beam energy must be high enough to make residual space charge effects negligible. The actual value for such an energy depends on the bunch characteristics but it is typically found to be around 100 MeV or more.

• Bazarov, I .V., and Sinclair, C .K., Phys. Rev. ST Accel. and Beams 8 034202 (2005).
• T. Shintake, et al., Phys. Rev. ST Accel. and Beams 12, 070701 (2009).

Accelerating Gradient at the Cathode

- During emission at the cathode, the electric field E_{SC} due to the already emitted electrons presents opposite direction with respect to E_a , the accelerating field in the gun.

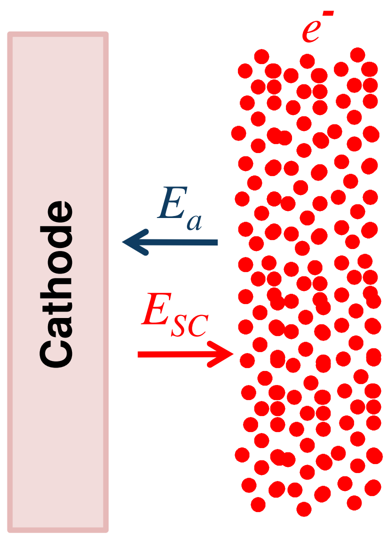
  The emission can continue until E_{SC} cancels E_a.

  The max charge density that can be emitted by a given E_a is known as the 'space-charge limit' and scales linearly with E_a.

  Higher gradients are required to extract higher charge/bunch and preserve beam quality.

  (1 nC bunch with $\varepsilon_{xn,\,yn}$ = 1 mm requires E_a > ~ 10-15 MV/m)

  More on the space-charge limit later in the lecture.



- Also, larger gradients allow for a 'faster' acceleration of the beam towards higher energies minimizing the deleterious effects of space charge forces.

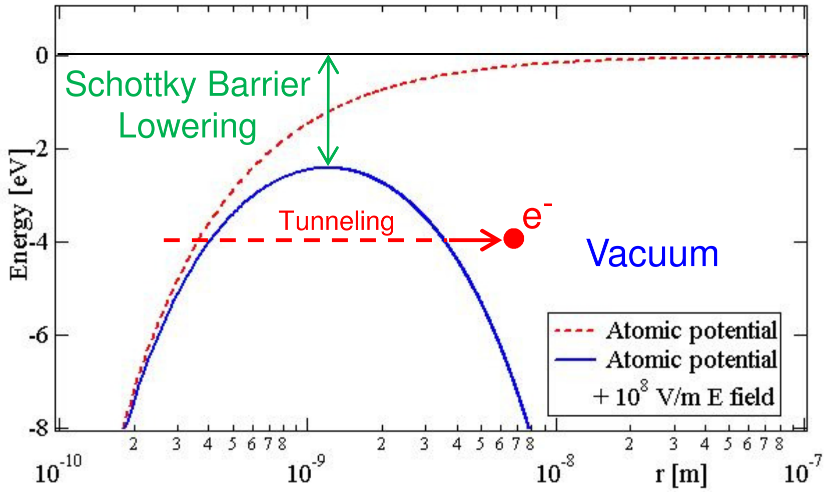
- There is a special mode of operation, the so-called 'beam blowout' where a pancake like beam is emitted and evolves under the action of its own space charge forces.

  Such a mode of operation requires relatively high gradients at the cathode.

  More later in the lecture.

Dark Current

Dark current is mainly generated by field emission from the accelerator parts.

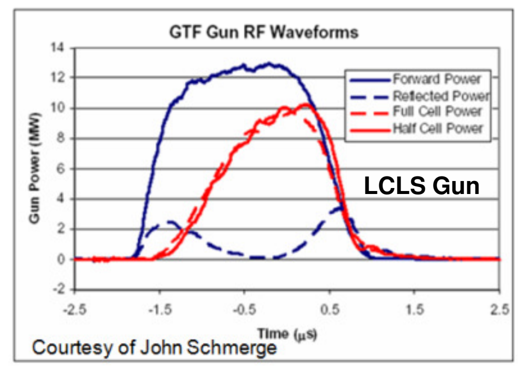


$$U_p = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} + e\left|\bar{E}\right|r$$

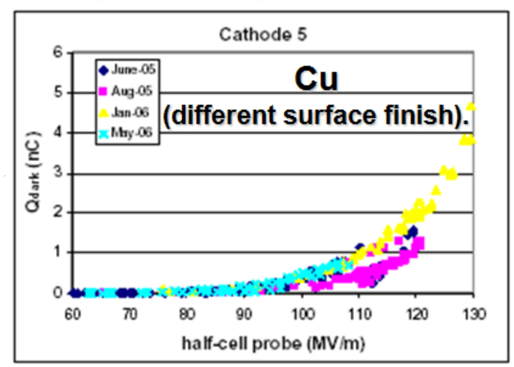
$$\left|\bar{E}\right| = \text{constant}$$

$$\left|\bar{E}\right| >\sim 10^8 \div 10^9\ V/m$$

- **Dark current can be relatively tolerated in pulsed injectors but can represent a serious issue in injectors running in continuous wave (CW) mode that can generate damage, quenching, and high radiation levels in the downstream accelerator.**



- **While no definitive 'cure' for dark current exists, the best techniques known for minimizing it should be used (surface finish, geometry, materials, …). In particular, high accelerating fields in the cathode/gun area, which can potentially generate field emission, should be carefully evaluated in terms of dark current.**

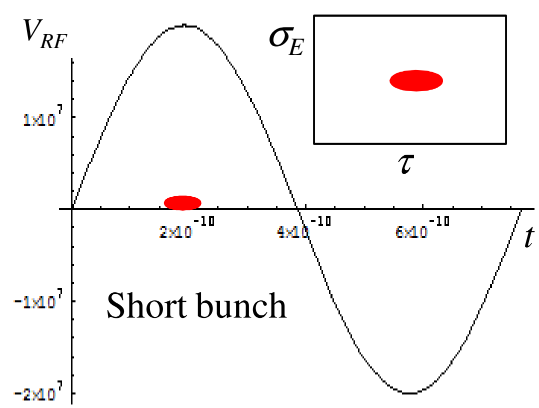


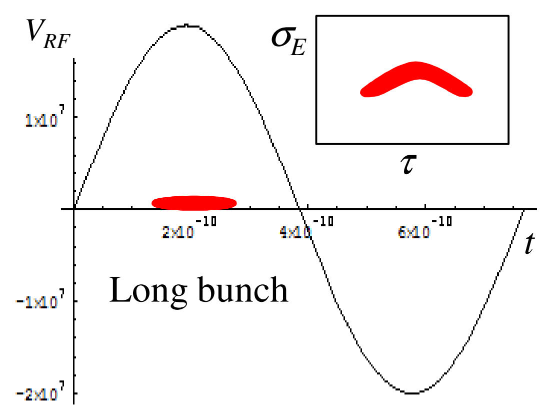
Bunch Length

• The **bunch length is an important knob for controlling the charge density** of the electron beam and **hence space charge effects** along the injector.

• In particular, **longer bunches at the cathode** can be used for mitigating space-charge induced emittance increase, especially when **relatively low accelerating gradients** at the electron gun are available.

• **Other factors** can also limit the **maximum bunch length**. For example, longer bunches sample more **RF nonlinearities** in the accelerating RF sections. Also as we will see later, **transverse emittance dilution due to RF** scales with the **square of the bunch length.**





• The capability of **controlling the longitudinal and transverse beam distributions** is **also important for the beam dynamics performance.**

• For this requirement, photo-cathode systems represents an appealing choice because they allow controlling the electron beam distribution by shaping the pulse of the laser used for the photoemission (more on cathode systems later).

Peak Current

- **FEL gain depends on the bunch peak current.**
For high-gain FEL schemes, values of up to 1 kA are typically required.

$$P(z) = P_0 e^{z/L_G} \qquad L_G = \frac{\lambda_u}{4\sqrt{3}\pi\rho^{1D}_{Pierce}} \quad gain\ \ length$$

$$\rho^{1D}_{Pierce} = \frac{1}{\gamma}\left[\frac{1}{64\pi^2}\frac{I_e}{I_A}\frac{1}{\varepsilon_x\beta_x}\lambda_u^2 K^2 JJ^2\right]^{1/3}$$

- **Oscillator FEL schemes include storage cavities for the X-Ray pulses, and are usually operated in low-gain regime hence requiring smaller peak currents.**

- **Such high-peak currents are obtained by compressing the bunch length along the accelerator in several stages.**
Typical schemes include one or more magnetic chicanes in the linac, plus buncher systems and/or velocity bunching in the injector.

- **Typical peak currents required at the injector exit, compatible with reasonable compression factors in the downstream linac, are many tens of A.**

- **More on compression later in the lecture.**

Transverse Beam Size

Transverse emittance dilution due to RF scales with the square of the transverse rms size of the bunch.

(More later in the lecture).

- **Larger beam sizes are more prone to sample regions with stronger nonlinearities in the transport channel potentially generating an increase of the transverse rms emittance.**

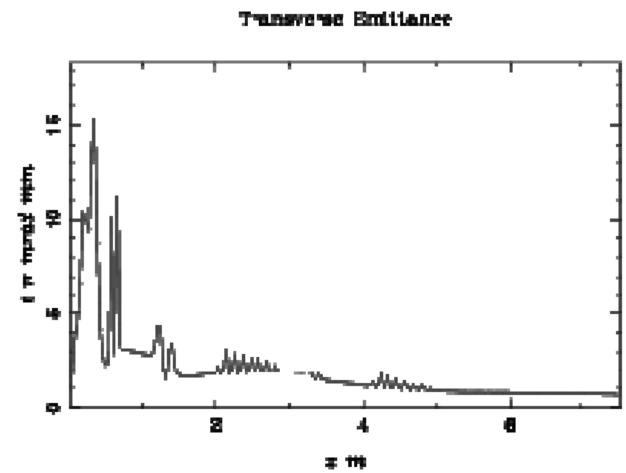
- **Larger beam sizes requires larger vacuum chamber cross-sections, larger bore magnetic components, making such parts bigger and more expensive.**

- **The rms beam size inside the typical high brightness gun ranges from few hundreds microns up to few mm.**
- **The average beam size along the injector decreases linearly with the beam energy due to the geometric emittance scaling with energy.**

Compatibility with Magnetic Field

- **High-brightness injector schemes should be compatible with the application of magnetic fields in the cathode/gun area.**

- **Indeed, preserving the gun brightness along the injector requires techniques such as emittance compensation that requires magnetic fields in the gun region.**
  (More later in the lecture)

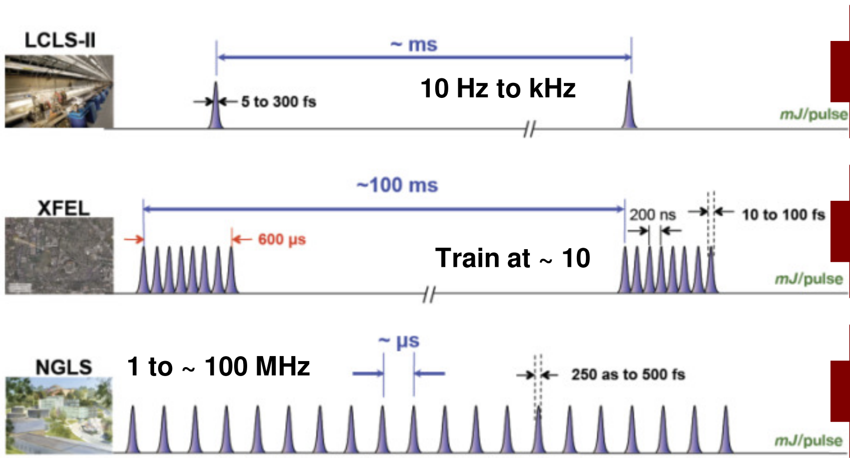


- **Additionally, some of the proposed emittance exchange techniques requires the presence magnetic fields in the cathode region.**
  (More later in the lecture).

Repetition Rate

The repetition rate is a parameter that deeply impacts the technological choices for a 4th generation light source.

Indeed, it determines the injector and linac technologies and has a relevant impact on the facility cost, and also, as it will be discussed later, on the electron beam beam dynamics.

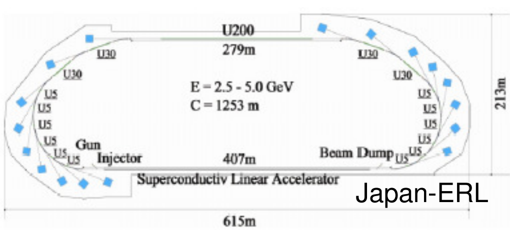


Normal-conducting linacs (S, C or X-Band); low repetition-rate gun.

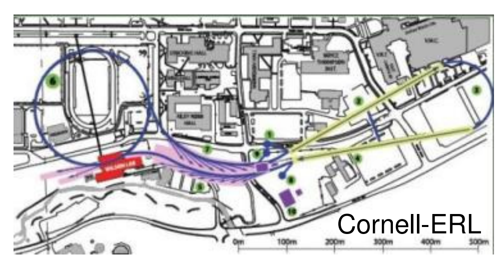
Normal or super-conducting linac; low repetition-rate gun.

Super-conducting linac; high repetition-rate gun.
(presently proposed XFEL ocillators requires ~ 1 MHz)

ERLs target 100s of mA average currents requiring GHz-class repetition rates



Japan-ERL



Cornell-ERL

and hence super-conducting linac and high repetition-rate gun.

Vacuum Performance

As it will be discussed later in Lecture 2, the large majority of cathode systems used in high-brightness injectors are based on photo-cathodes.

In the low repetition-rate case, metal cathodes (mainly copper) have been extensively used because of their relatively simple preparation and robustness.

In the high repetition-rate case, metals cannot be used because of their low quantum efficiency QE in the 10^{-5} range (number of photo-emitted electrons per impinging photon), which would require unrealistically powerful lasers.

Higher QE materials, in the 10^{-2} range, are required in the high repetition-rate case.
Several materials have been already successfully developed and tested and many other promising candidates are under investigation.

Most of such materials are very reactive and/or their emitting surface is sensitive to damage by ion back-bombardment. Indeed, in order to achieve lifetimes compatible with the operation of a user facility, vacuum pressures ranging from 10^{-7} to 10^{-9} Pa (~10^{-9} to 10^{-11} Torr) are necessary (with even lower partial pressures for reactive residual gas molecules such as H_2O, O_2, CO_2).

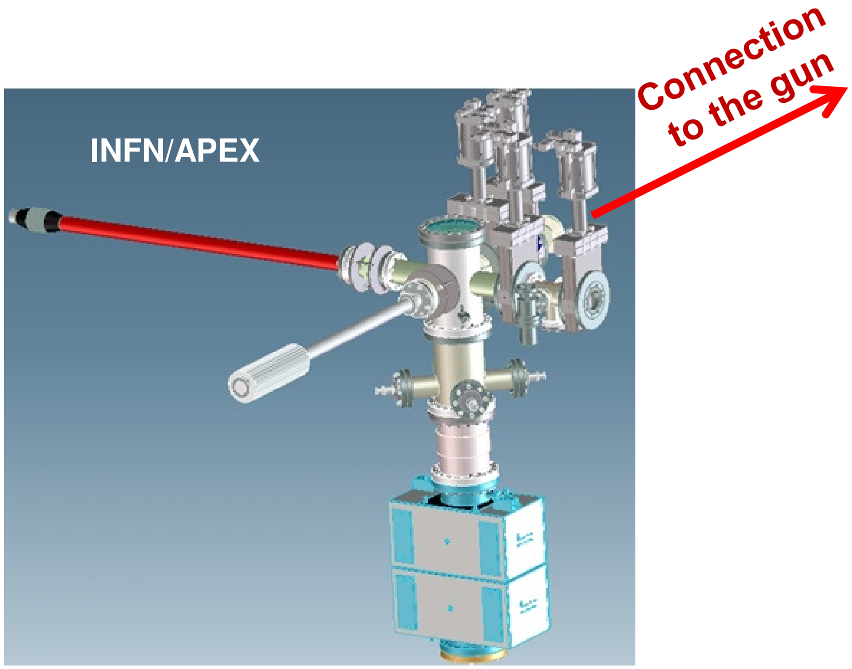
Reliability, Easiness of Operation

Independently from the gun/injector technology choice, the selected scheme should offer the required reliability and robustness to operate in an user facility and guarantee the proper continuity to the experimenters.

Also, replacement of parts with reduced lifetime should be performed as fast and efficiently as possible.

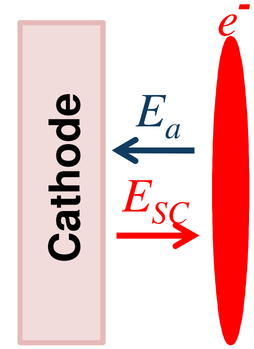
For example, in the case of delicate high-QE cathodes it is necessary to periodically replace and/or regenerate/activate the cathodes without breaking the vacuum pressure inside the gun.

To make that possible in a relatively straightforward and timely way, a vacuum load-lock system is usually required.



Beam Dynamics

Space Charge Limit



• **During emission at the cathode**, the electric field E_{SC} due to the already emitted electrons presents opposite direction with respect to E_a, the accelerating field in the gun.

The **emission can continue until** E_{SC} **cancels** E_a.

The **max charge density that can be emitted by a given** E_a is known as the '**space-charge limit**' $\sigma_{SC\,MAX}$.

• Assuming a '**pancake' beam** longitudinally thin and transversely wide (Gaussian) we can estimate the field due to space charge by:

$$E_{SC} \approx \frac{\sigma_{SC}}{2\varepsilon_0} \approx \frac{Q}{4\pi\varepsilon_0\sigma_r^2} \quad \Rightarrow \quad \sigma_{SC}^{MAX} = \left(\frac{Q}{2\pi\sigma_r^2}\right)_{MAX} \approx 2\varepsilon_0 E_a \quad \Rightarrow \quad \sigma_r^{\min} \approx \sqrt{\frac{Q}{4\pi\,\varepsilon_0 E_a}}$$

Q is the charge per bunch, σ_r the rms transverse beam size and ε_0 the vacuum permittivity.

• We will see later that the emittance at the cathode is proportional to σ_r and to a quantity $f(T_i)$ that depends on the cathode used and on the emission mechanism.

$$\varepsilon_n = \sigma_r f(T_i) \quad \Rightarrow \quad \varepsilon_n^{\min} = \sigma_r^{\min} f(T_i) \approx f(T_i)\sqrt{\frac{Q}{2\pi\,\varepsilon_0\,E_a}}$$

$$B_{4D}^{\max} \propto \frac{Q/e}{\left(\varepsilon_n^{\min}\right)^2} \approx \frac{2\pi\,\varepsilon_0\,E_a}{e\,f^2(T_i)}$$

Space charge limits the min emittance and the max brightness obtainable at the cathode for a given E_a. The brightness is also independent from charge.

Bazarov, PRL 102, 104801 (2009)

Space Charge Forces Downstream the Cathode

- **Interaction between the electromagnetic field of the particles** in a beam can be divided into two main categories:
  - **Space charge forces** or **self-field forces**: the force on a particular particle resulting from the combination of the fields from all other particles in the beam. Such a force is Hamiltonian and the low order terms of it can be compensated.
  - **Scattering** (**Boersch effect**): a particle in the beam scatters (interacts) with another particle in the beam. This is a stochastic and hence non-Hamiltonian process, that generates an increase of the 'Liouville' emittance ('heating') that cannot be compensated.
- In a plasma (the beam is a nonneutral plasma), the **Debye length** λ_D represents the length beyond that the screening from the other particles in the plasma cancels the field from an individual particle.

$$\lambda_D = \left(\frac{\varepsilon_0 \gamma k_B T}{e^2 n} \right)^{\frac{1}{2}}$$

with n the electron density, k_B the Boltzmann constant, T the electron beam 'temperature' in the rest frame with $m\sigma_v^2 = k_B T$

If $\lambda_D < \sim n^{-1/3}$ = average electron distance ➡ scattering is prevalent

If $\lambda_D >> n^{-1/3}$ ➡ scattering can be neglected

For more info, see for example: Rieser, Theory and Design of Charged Particle Beams, Wiley, chapter 4.1.

Transverse Space Charge Forces

- It can be shown that for a beam with Gaussian linear charge density λ_C and for $|x| << \sigma_x$ and $|y| << \sigma_y$, (beam core) the transverse space charge fields are:

$$E_x = \frac{1}{2\pi\varepsilon_0}\frac{\lambda_C}{\sigma_x(\sigma_x+\sigma_y)}x, \quad E_y = \frac{1}{2\pi\varepsilon_0}\frac{\lambda_C}{\sigma_y(\sigma_x+\sigma_y)}y, \qquad B_x = -\frac{\mu_0}{2\pi}\frac{\lambda\beta c}{\sigma_y(\sigma_x+\sigma_y)}y, \quad B_y = \frac{\mu_0}{2\pi}\frac{\lambda\beta c}{\sigma_x(\sigma_x+\sigma_y)}x$$

Such space charge fields exert forces on the beam particles, and the intensity of such a forces are given by the Lorentz equation:

$$\vec{F} = q\left(\vec{E}+\vec{v}\times\vec{B}\right)$$

By comparing the previous relation one finds:

$$B_x = -\frac{\beta}{c}E_y \qquad B_y = \frac{\beta}{c}E_x$$

$$F_x = q\left(E_x - \beta c B_y\right) = qE_x\left(1-\beta^2\right) \propto \lambda_C\left(1-\beta^2\right)x$$

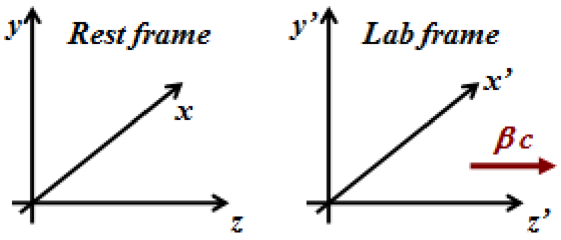
$$F_y = q\left(E_y + \beta c B_x\right) = qE_y\left(1-\beta^2\right) \propto \lambda_C\left(1-\beta^2\right)y$$

- The **force dependence on the $(1 - \beta^2) = 1/\gamma^2$** term is actually quite general and shows that the **transverse space charge forces become negligible for relativistic beams**.

- The above equations also show that in the **'core' of the beam the forces are linear**. This **implies that they can be compensated** by linear focusing elements (solenoids, quadrupoles)

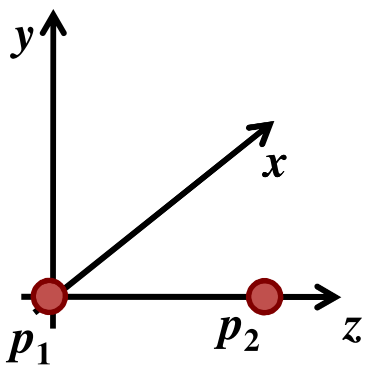
For more information, see for example: Wiedemann, Particle Accelerator Physics, Springer, 3rd edit., chapter 18.2.

Longitudinal Space Charge Forces



The longitudinal component of the Lorentz force in the lab frame is given by:

$$F_z' = q\left[E_z + \frac{1}{1 - v_z\,\beta/c}\left[v_x\left(B_y + \frac{\beta}{c}E_x\right) - v_y\left(B_x - \frac{\beta}{c}E_y\right)\right]\right]$$



In the rest frame, two particles are resting as in the figure.

The field acting on p_2 due to p_1 is:

$$E_z = \frac{q}{4\pi\varepsilon_0}\frac{1}{z^2};\; E_x = 0;\; E_y = 0$$

$$\overline{B} = 0$$

Using this result in the previous expression:

$$F_z' = qE_z = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{z^2} = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{(\gamma z')^2} = \frac{1}{\gamma^2}\frac{q^2}{4\pi\varepsilon_0}\frac{1}{z'^2}$$

• **Similarly to the transverse case, the $1/\gamma^2$ term shows that also the longitudinal space charge force becomes negligible for relativistic beams.**

Linear Space Charge Regime

- At the **injector energies**, the beam is not fully relativistic and the **space charge forces play a relevant role**.
- In the case of linear space charge forces the effect is that of a linear defocusing in both planes, and an analytical expression for the **rms beam envelope σ** can be derived. In the case **of cylindrical symmetric continuous beam:**

$$\sigma'' + \sigma' \frac{\gamma'}{\beta^2 \gamma} + K_r \sigma - \frac{\kappa}{\sigma \beta^3 \gamma^3} - \frac{\varepsilon_n^2}{\beta^2 \gamma^2 \sigma^3} = 0 \qquad \kappa = \frac{I}{2I_0} \equiv perveance \qquad \frac{\partial f}{\partial z} = f'$$

where the second term on the LHS is the accelerating adiabatic damping, K_r is a linear focusing term (given for example by a solenoid), ε_n the normalized emittance, I the beam current and $I_0 \sim 17$ kA the Alfven current.

- In the case of a **bunched beam**, we previously saw that in its 'core' space charge forces are with good approximation linear, and the **envelope equation above can be used for the core replacing I with the beam peak current I_p**.
- **The envelope equation shows how in the linear space charge case a proper focusing can be used to control space charge forces**
- A similar equation can be derived for the longitudinal beam envelope.

For more info, see for example: Rieser, Theory and Design of Charged Particle Beams, Wiley, chapters 4 and 5.
J. D. Lawson, The Physics of Charged Particle Beams, 2nd ed., Oxford University Press, New York, 1988.

Is the Space Charge Linear Regime Realistic?

• **Linear transverse space charge forces** are generated by the **Kapchinski-Vladimirski or K-V distribution**, where the charge density is uniform on the surface of a hyper-ellipsoid in the 4D transverse phase space and zero elsewhere.

• In the longitudinal plane the **Neuffer distribution** plays a similar role **generating linear space-charge forces and a parabolic linear longitudinal charge density.**

• **The best approximation of the above distributions, projected in the 3D spatial reference frame, is represented by a 3D ellipsoidal beam with uniform particle density inside and zero elsewhere.**

•**This distribution generates linear space charge forces in both transverse and longitudinal planes, and no emittance increase.**

• Uniform ellipsoidal charge densities are **experimentally pursued** by **shaping the laser in photocathode system**, or by the so-called **beam-blowout** regime.

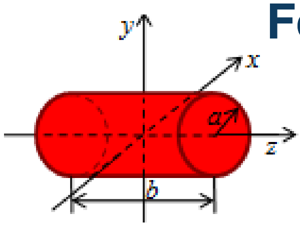
• In such a mode, that can be used with photo-cathode systems, a very short laser pulse (~100 fs) is sent on the cathode. The resulting 'pancake' of photo-emitted electrons is accelerated in the gun and simultaneously under the action of its own space-charge field evolves in a 3D uniform ellipsoidal charge distribution.

At the present time, this mode of operation has been experimentally demonstrated for charges per bunch smaller than ~100 pC

For more info, see for example: Rieser, Theory and Design of Charged Particle Beams, Wiley, chapters 4 and 5.

Beam-blowup, see: P. Musumeci, *et al*., Phys. Rev. Letters 100, 244801 (2008), and references in there.

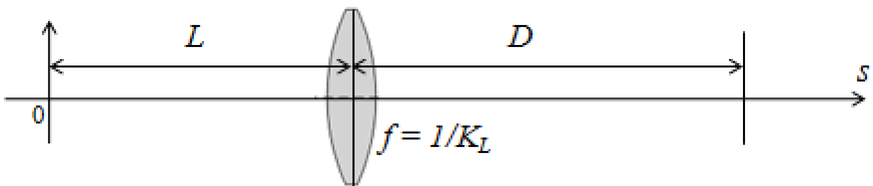
Emittance Compensation



For a cylindrical beam with radius a, length b, linear charge density $\lambda_C(z)$ slow changing function of z, and uniform transverse charge density,

Teng showed that: $$for\ |z| << b \Rightarrow F_r(z) = \frac{e\lambda_C(z)}{2\varepsilon_0}\frac{(1-\beta^2)}{a^2}r$$

with $I = \lambda_C v_z = \lambda_C \beta c$ ➡ $$F_r(z) = \frac{e}{2c\varepsilon_0}\frac{1}{\beta\gamma^2}\frac{I(z)}{a^2}r = \frac{e}{2c\varepsilon_0}\frac{1}{\beta\gamma^2}J(z)\,r = K_{SC}(z)r$$

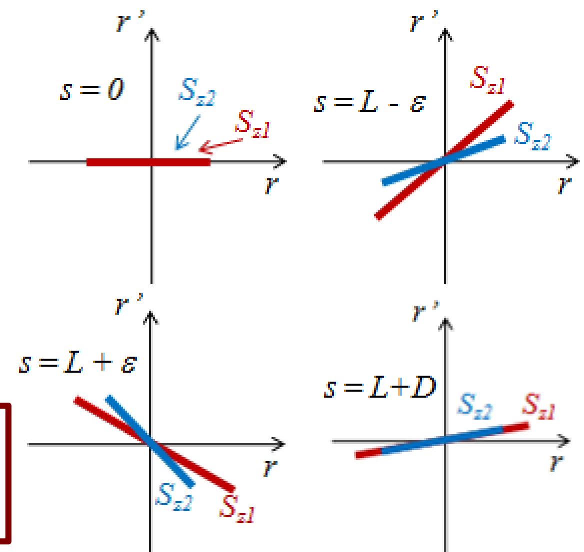


The transverse motion equations for the 'slice' at position z through the system are (assuming zero transv. emittance $r'(z,0) = 0$):

$$r(z, s = L+D) = (1-K_L D)r(z,0) + \frac{1}{2\beta^2}\frac{F_r(z)}{mc^2}\left[(L+D)^2 - K_L D L^2\right]$$

$$r'(z, L+D) = -K_L r(z,0) + \frac{1}{2\beta^2}\frac{F_r(z)}{mc^2}\left[2(L+D) - K_L L^2\right]$$

If $K_L = \frac{2(L+D)}{D^2}$ ➡ $$\tan(\alpha_{rr'}) = \frac{r'(z,L+D)}{r(z,L+D)} = \frac{2(L+D)}{D(2L+D)}$$



The slope in the phase space is the same for all slices.
Minimum of projected emittance

By placing an accelerating section in proximity of the minimum the emittance can be 'frozen' at its minimum value.

Emittance Compensation References

B. E. Carlsten, Nucl. Instr. and Meth. Phys. Res., Sect. A 285, 313 (1989).

L. Serafini, and J. B. Rosenzweig, Physical Review E 55, 7565 (1997).

The Final Injector Emittance

- The final transverse emittance at the injector output is given by:

$$\varepsilon_{nw} = \sqrt{\varepsilon^2_{nw\ Cathode} + \varepsilon^2_{nr\ Bz\ at\ Cathode} + \varepsilon^2_{nw\ Space\ Charge} + \varepsilon^2_{nw\ Beam\ Optics} + \varepsilon^2_{nw\ RF}} \qquad w = x, y$$

- The first term is often referred as the **cathode thermal or intrinsic emittance.**

Such emittance term is proportional to the beam size at cathode and to a momentum term defined by the emission process used

$$\varepsilon_{nw\ Cathode} = \sigma_w \frac{\sqrt{\langle p_w^2 \rangle}}{mc} \qquad w = x, y$$

The game in present injectors is to make all the emittance terms negligible respect to the cathode term.

- The 2nd term is due to the presence of solenoidal B_z field at the cathode (Palmer, *et al.*, PAC97, p. 2843)

$$\varepsilon_{nr\ Bz\ at\ Cathode} = \frac{e}{2mc} \sigma_r^2 B_z$$

- We already learned how to minimize the space charge term by removing the linear space charge contribution by the emittance compensation. The residual part is due to non linear space charge forces.

- The Beam Optics term is due to nonlinear components in the focusing and deflecting components along the injector. A proper design of such components can make this term negligible

- The last term is due to the RF fields along the injector

Transverse Emittance Dilution Due to RF

- RF cavities generate the longitudinal electric field E_z to accelerate the particles. Due to Maxwell equations also a radial and an azimuthal fields exist:

$$E_r = -\frac{r}{2}\frac{\partial E_z}{\partial z}; \qquad B_\theta = \frac{r}{2c^2}\frac{\partial E_z}{\partial t}$$

- Such fields component generates a radial Lorentz force, which is stronger in the RF fringes, and that affects the transverse momentum of the particles

$$F_r = e\left(E_r - \beta c B_\theta\right)$$

- That generates an increase in the transverse normalized emittance. For a Gaussian beam:

$$\varepsilon_{nr\,RF} = \frac{e}{2\sqrt{2}mc^4} E_0 \omega_{RF}^2 \sigma_r^2 \sigma_z^2$$

with e and m the electron charge and rest mass respectively, c the speed of light, $\omega_{RF}/2\pi$ the RF frequency, E_0 the accelerating field, and σ_r and σ_z the rms transverse and longitudinal beam sizes.

- For example, in a 1.3 GHz accelerating section with $E_0 = 20$ Mv/m, a beam with $\sigma_r = 1$ mm and rms bunch length of 10 ps will experience a normalized emittance increase of $\varepsilon_{nr\,RF} \sim 10^{-7}$ m.

K. J. Kim, *NIM*, A275, 201 (1989)

Longitudinal Emittance Dilution Terms

- As in the transverse case, also the **longitudinal emittance** is affected by **RF** and **space charge dilution.**

- The **increase** of the **normalized longitudinal emittance** due to RF is given by:

$$\varepsilon_{nz\,RF} = \frac{\sqrt{3}}{c^2}\left(\gamma_{exit} - 1\right)\omega_{RF}^2 \sigma_z^3$$

with e and m the electron charge and rest mass respectively, c the speed of light, $\omega_{RF}/2\pi$ the RF frequency, E_0 the accelerating field, and σ_r and σ_z the rms transverse and longitudinal beam sizes.

- Such a longitudinal emittance increase is mainly due to a quadratic energy/position correlation that can be removed by using a harmonic cavity downstream in the linac.

- The **increase** of the **normalized longitudinal emittance** due to space charge is instead given by:

$$\varepsilon_{nz}^{SC} = \frac{\pi}{4}\frac{1}{\sin\varphi_0}\frac{2mc^2}{e\,\hat{E}_z^{RF}}\frac{I}{I_A} f\left(\frac{\sigma_x}{\sigma_z}\right) \quad with \ \ \varphi_0 \equiv emission \ \ phase \qquad f\left(A\right) = \frac{1}{1 + 4.5A + 2.9A^2}$$

- For example for a 1 nC, 10 ps bunch with a 1/3 aspect ratio, 120 MV/m field, emitted at 90 deg phase, the normalized emittance increase is ~ 15 μm.
- This is significantly larger than the cathode thermal emittance contribution of ~ 3 μm that a cathode with $\sigma_{pz}/mc \sim 10^{-3}$ would have for that beam transverse size.

K. J. Kim, NIM A275, 201 (1989

Boersch Effect

- After the emission from the cathode, the electron beam presents an isotropic distribution of temperatures

$$kT_{\perp i} = kT_{//i} = kT_i = m\sigma_v^2$$

- The subsequent acceleration does not affect the transverse temperature but dramatically decreases the longitudinal one:

$$kT_{//f} \approx \frac{\gamma_i^3}{\beta_f^2 \gamma_f^3} \frac{\left(kT_{//i}\right)^2}{mc^2}$$

- As a consequence, the longitudinal temperature becomes soon negligible, and Coulomb collisions start to reestablish the thermal equilibrium in the beam transferring momentum from the transverse to the longitudinal plane.
- This phenomenon is known as the **Boersch effect**.

$$T_{\perp f} \cong 2/3\, T_{\perp i}\left(1 + 0.5 e^{-3t/\tau}\right)$$
$$T_{//f} \cong 2/3\, T_{\perp i}\left(1 - e^{-3t/\tau}\right)$$

$$\tau = \frac{4.44\times 10^{20}\left(0.307\, kT_{\perp i}/mc^2\right)^{3/2}}{n \ln\left[5.66\times 10^{21}\left(kT_{\perp i}/mc^2\right)^{3/2} n^{-1/2}\right]}$$

$$\sigma_E = \left(\frac{\beta_f^2 \gamma_f^3}{\gamma_i^3} mc^2\, kT_{//f}\right)^{\frac{1}{2}}$$

where k is the Boltzmann constant n the electron density, and i and f stay for 'final' and 'initial' respectively.

- For a 1 nC, 10 ps bunch with a 1/3 aspect ratio, kT_i 1 eV, the temperature relaxation time τ is ~ 300 ns (~ 100 m of accelerator!), but for a beam accelerated up to 1 MeV, 1 m downstream of the cathode, σ_E ~ 600 eV!

Rieser, Theory and Design of Charged Particle Beams, Chapter 6.4.1, Wiley

Low & High Gun Gradient Regimes

• To preserve brightness, it is desirable to accelerate the beam as quickly as possible, thus 'freezing-in' the space charge forces, before they can significantly dilute the phase space.

• In the case of high repetition rate injectors, as it will be discussed in Lecture 2, technology limitations and/or or dark current mitigation, significantly reduces the peak accelerating gradients at the cathode with respect to those in pulsed low repetition rate systems.

• This situation can have a significant impact on beam dynamics.

• Space charge can be controlled by reducing the beam charge density, especially in the cathode region where the beam energy is small.

The use of larger transverse beam sizes at the cathode to reduce the density is carefully minimized because it increases the cathode thermal emittance.

• Instead, the bunch length is used, and longer bunches are required for lower gradients.

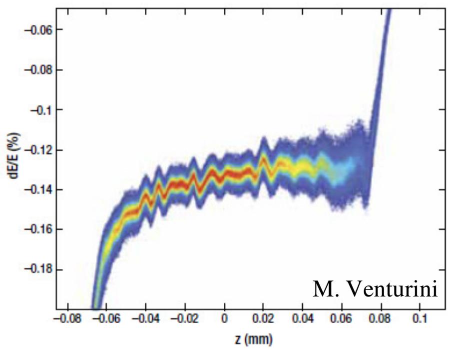
That increases the longitudinal emittance, but for most cases this is tolerable.

• As a consequence, in high repetition rate injectors, the bunch length at the cathode can be significantly longer than required at the FEL undulator entrance. This in turn necessitates relatively larger compression factors both at the injector and in the main linac.

Bunch Length Compression

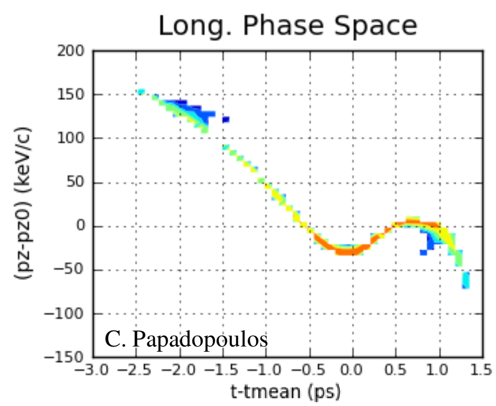
- In all FEL or ERL schemes the bunch length at the gun is longer than that required at the undulator position.

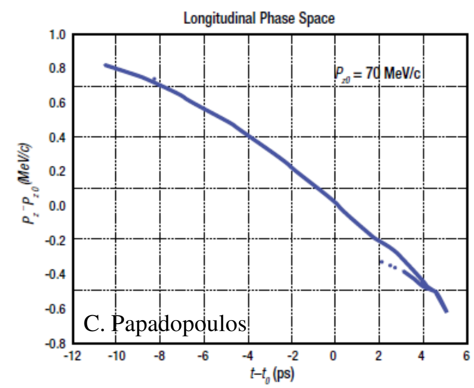
Longitudinal compression is then necessary and can be performed in the linac and /or in the injector and/or in the arcs in the case of ERLs.



- **Magnetic compressors** such as chicanes or arcs can be subjected to **microbunching instability** and to **emittance growth due to coherent synchrotron radiation** in the case of high compression factors.

- Excessive **compression in the injector** can **generate space charge induced transverse emittance increase and longitudinal phase space distortions** making the final compression in the linac challenging.



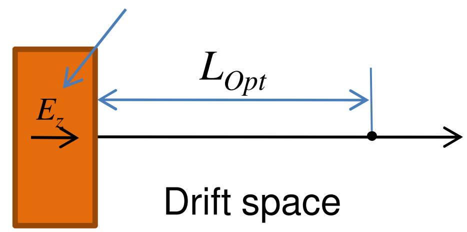


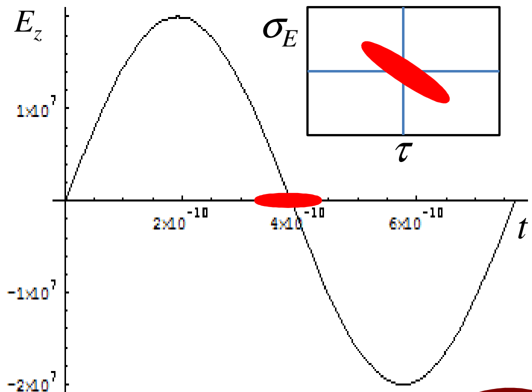
- The proper balance between these compression strategies must be found.

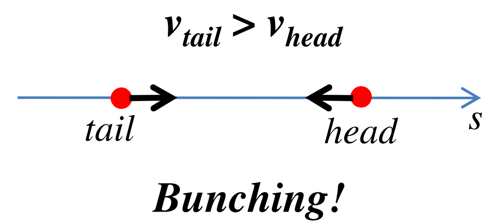
- Methods for compressing the beam in the injector include a dedicated **buncher** section and/or the use of a technique referred as **velocity bunching**.

"Zero-Crossing" Buncher

One effective method to compress bunches when the beam is not fully relativistic consists in using a 'buncher' (or prebuncher) cavity.
In a buncher the most linear part of the RF field ('zero crossing') is used for creating an energy 'chirp' in the beam with no net acceleration of the bunch.







- **It can be shown that the optimal bunching drift length L_{Opt} is given by:**

$$L_{Opt} = mc^2 \gamma\left(\gamma^2 - 1\right) \frac{1}{\left.\frac{dW}{ds}\right|_{\varphi=\frac{\pi}{2}}}$$

where dW/ds is the particle energy variation per unit longitudinal displacement in the cavity.

- **For an optimized pill box cavity (gap = $\beta\,\lambda_{RF}/2$) with electric field**

$$E_z = \hat{E}_z \cos\left(\omega_{RF} t + \varphi\right) \quad \Rightarrow \quad \left.\frac{dW}{ds}\right|_{\varphi=\frac{\pi}{2}} = 2\beta \hat{E}_z$$

- **For a non-relativistic beam and for λ_{RF} sufficiently long, to the linear energy chirp corresponds a linear velocity chirp, and the compression is symmetric.**
For more relativistic beams the velocity chirp becomes non-linear and the compression asymmetric.

Compressing more relativistic beams in the injector is still possible:

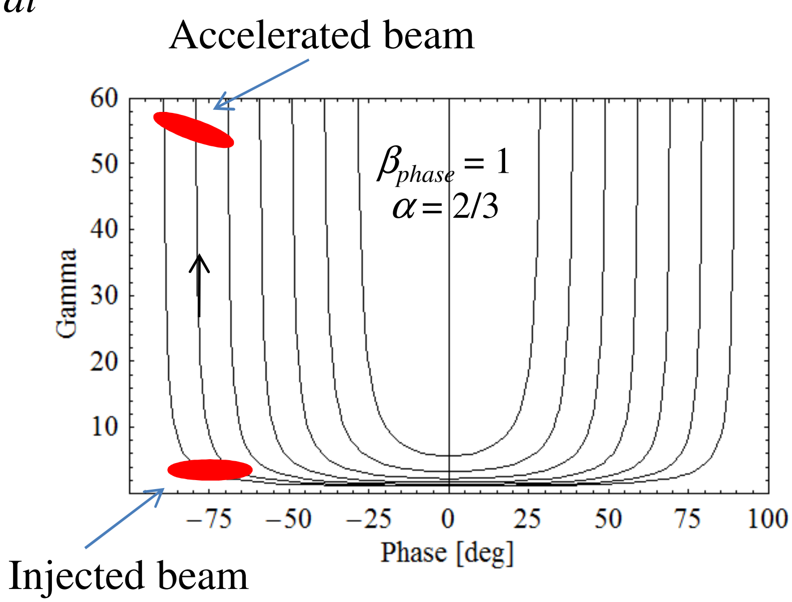
$$E_z = \hat{E}_z \sin(\omega t - kz - \psi_0) \quad with \quad \frac{\omega}{k} = c\beta_{phase}$$

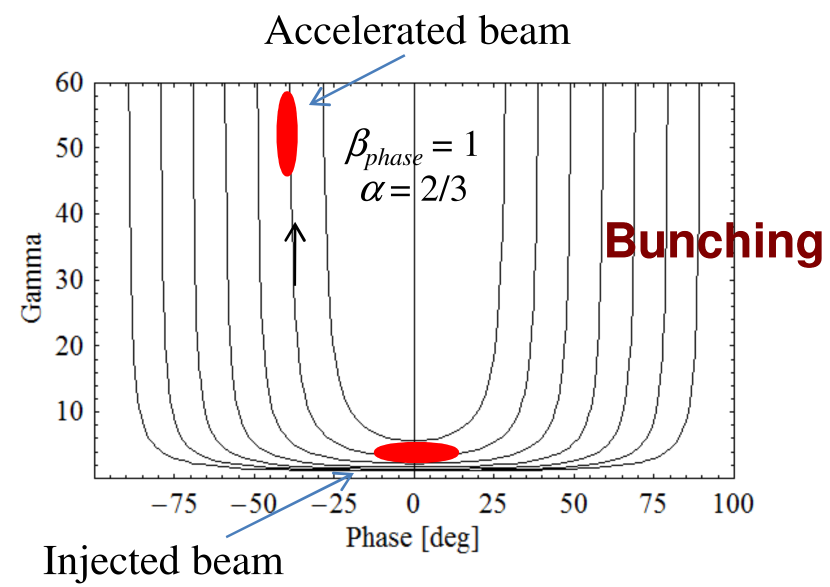
$$\begin{cases} \dfrac{d\varphi}{dt} = kc(\beta - \beta_{phase}) \\ \dfrac{dp}{dt} = -e\hat{E}_z \sin\varphi \quad with \ \varphi = \omega t - kz - \psi_0 \end{cases}$$

$$H = \gamma - \beta_{phase}\sqrt{\gamma^2 - 1} - \alpha\cos\varphi \qquad with \ \ \alpha = \frac{e\hat{E}_z}{kmc^2}$$

$$and \ \ H \equiv constant$$

Particles trajectories in the γ, φ phase space





- **The method can generate compression factors of more than 10, and can be used also with standing-wave accelerating sections.**

- B. Aune and R. H. Miller, Report No. SLAC-PUB 2393, 1979.
- L. Serafini and M. Ferrario, AIP Conf. Proc. 581, 87 (2001).

Emittance Manipulation

- It should be clear now that **emittance** (in all its dimensions) is one of the **fundamental parameters** in present 4th generation light sources and that the major part of **the game for this parameter is played in the injector**.

- Schemes have been conceived and tested to generate **flat beams** in injectors, to match the requirements of linear colliders and diffraction limited x-ray sources based on spontaneous undulator radiation,

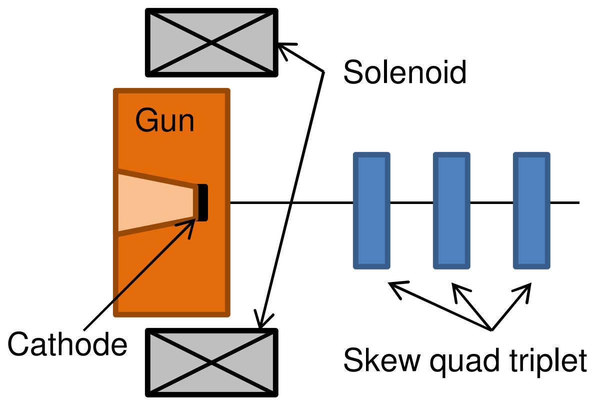
- or, to be used in combination with techniques that **exchange emittances** from one plane to another (typically from the longitudinal to the transverse) to best match the application requirements.

- The above are just particular applications of the more general concept of manipulating **eigen-emittances**, that are motion invariant quantities in the 6D phase space for linear Hamiltonian systems.

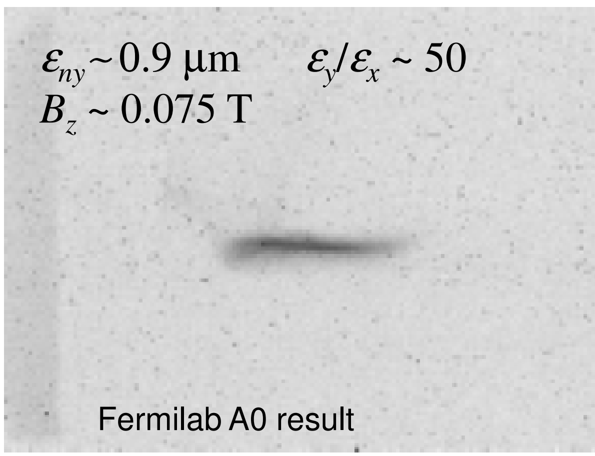
- Recently, compressor and "echo"-like schemes based on multiple emittance-exchange/flat-beam steps have been proposed.

Flat Beams

Flat beams from round beams at the cathode can be obtained by the following scheme:



- **The presence of a solenoidal field at the cathode couples the transverse planes, and the skew triplet 'exploits' this correlation to generate the flat beam. It must be remarked that in the process the horizontal and vertical emittances are respectively increased and decreased with respect to their values at the cathode.**



It can be seen that the emittance ratio is given by:

$$\frac{\varepsilon_x}{\varepsilon_y} = \frac{\varepsilon_{nx}}{\varepsilon_{ny}} = 1 + \frac{2\sigma^2_{r\ Gun}}{\beta_F^2 \sigma^2_{r'\ Gun}}$$

with the optical function:

$$\beta_F = \frac{2 p_Q}{e B_z} \quad with \quad p_Q \equiv \begin{matrix} momentum \\ at\ quad\ entrance \end{matrix}$$

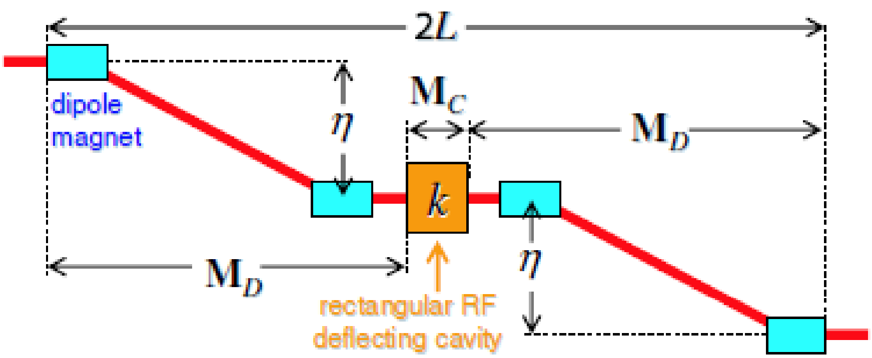
If everything is linear:

$$\varepsilon_{nr} = \sqrt{\varepsilon_{nx}\varepsilon_{ny}}$$

Ya. Derbenev, "Adapting Optics for High Energy Electron Collider", UM-HE-98-04, Univ. Of Michigan, 1998.
R. Brinkmann, et al., "A Flat Beam Electron Source for Linear Colliders", TESLA-99-09.
D. Edwards et al., "The flat beam experiment at the FNAL photoinjector", Linac 2000, Monterey.
Ph. Piot, Y.-E Sun, and K.-J. Kim, Phys. Rev. ST Accel. Beams 9, 031001 (2006).

Emittance Exchange

In some applications (FELs in the microbunching instability regime or ERL modes where the longitudinal emittance is not very important) it can be convenient to exchange a smaller longitudinal emittance with a larger transverse emittance.



$$1 + k\eta = 0$$

$$k \equiv transverse\ kick\ strength$$

$$\eta \equiv transverse\ dispersion$$

Several schemes have been proposed for exchanging the longitudinal with one of the transverse emittances (we will assume the horizontal plane in what follows). A dispersive section ('dogleg') is first used to create a correlation z-x, followed by a deflecting cavity that gives a transverse kick proportional to the particle z position, a second dogleg (as in the figure) removes undesired energy-position correlations generating a complete emittance exchange between x and z.

M. Cornacchia and P. Emma, Phys. Rev. ST Accel. Beams 5, 084001 (2002).
K.-J. Kim and A. Sessler, AIP Conf. Proc. No. 821 (AIP, New York, 2006), pp. 115–138.
P. Emma, Z. Huang, K.-J. Kim, P. Piot, Phys. Rev. ST Accel. Beams 9, 100702 (2006)

Eigen-Emittances

- Calculating the **beam evolution through a generic beamline**, where only linear fields are present, is in general a **6D problem.**

- For a **large number of cases**, the motion in the **different planes** can be considered **decoupled** and one can deal with a **2D problem.**

- In this **2D case the rms emittance is an invariant in each of the planes.**

$$\varepsilon_{w\,rms} = \sqrt{\langle w^2 \rangle \langle w'^2 \rangle - \langle w w' \rangle^2} \qquad w = x, y, z$$

- In the **general 6D case** (including coupling between all planes) the concept of rms emittance can be generalized to produce **three invariants (eigen-emittances).** Such **eigen-emittances are made out of second order moments of the beam distributions, and form a complete set.**
In the uncoupled case, the eigen-emittances reduce to the 2D rms emittances.

- Schemes proposing to generate the **proper correlations** already at the **cathode/injector** and to manipulate the emittances between the planes to obtain at the linac exit the desired emittances have been proposed.

- The cases of flat beam and emittance exchange techniques presented before can be derived as particular applications of the 6D eigen-emittance theory.

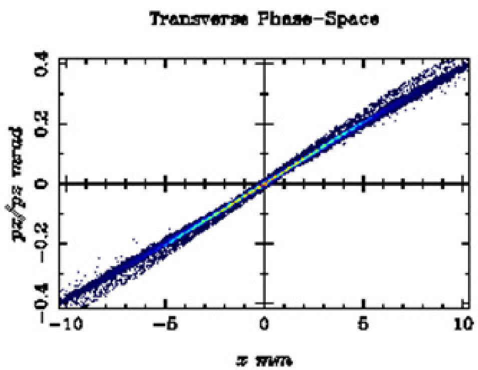
G. Rangarajan, F. Neri, and A. Dragt, "Generalized emittance invariants" PAC1989.

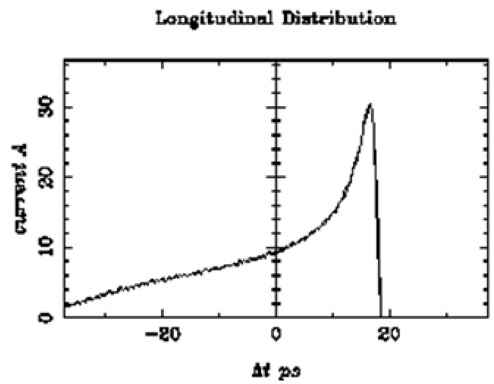
F. Neri, and G. Rangarajan, Phys. Rev. Letters 64, 1073, 1990.

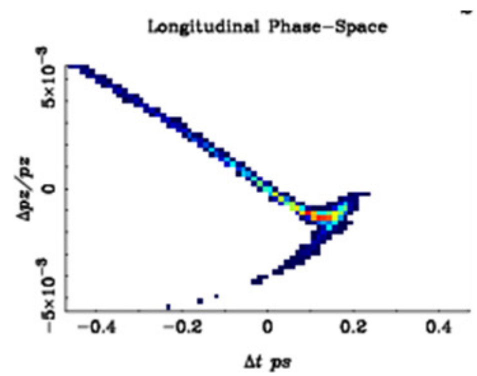
Yampolsky, Carlsten, Ryne, Bishofberger, Russell, Dragt, arXiv:1010.1558 [physics.acc-ph] 7 Oct 2010

• So far we have prevalently dealt with linear beam dynamics cases.
But real injector components and space charge fields are generally nonlinear.

• Nonlinearities generates phase space **filamentation, halos** and **non-Gaussian longitudinal tails** in the beam.







• **Filamentation generates rms emittance increase** and can make **compression difficult.**

• **Particles in the halo and tail** of the beam can go out of the accelerator acceptance and **can be lost generating radiation** or causing undesired effects such as **quenching** in superconducting RF structures.
(particularly important in high duty cycle accelerator)

• **In other words nonlinearities effects need to controlled and minimized.**

Simulation Tools

At this point, we had the chance to realize how complex is beam dynamics in the presence of space charge, even in the linear regime.
As we just mentioned, if more realistic nonlinear problems are considered, the complexity increases even further and accurate evaluation can be pursued only by simulation tools.

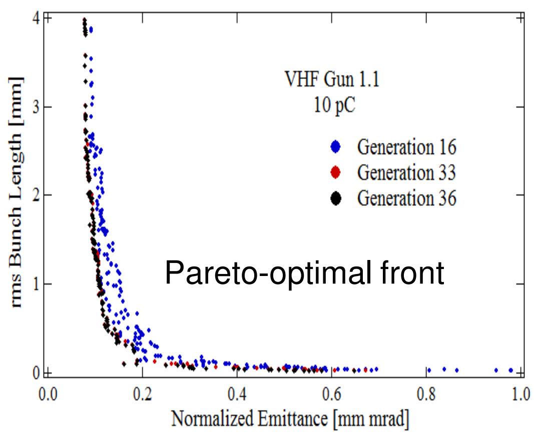
Indeed, a number of simulation codes with space charge have been developed over the years to address the problem, and are heavily used in the design and optimization of high-brightness electron injectors.

An incomplete list of 'popular' codes include (alphabetical order):

- **ASTRA**: free downloadable at http://www.desy.de/~mpyflo/
- **GPT**: commercial (http://www.pulsar.nl/gpt/)
- **IMPACT-T**: free, contact Ji Qiang (jqiang@lbl.gov),
- **PARMELA**: free, export limitations apply. http://laacg1.lanl.gov/laacg/services/
- …

Multi-Objective Optimization

- The **design and optimization of an electron injector** requires the tuning of a **large number of knobs** targeting simultaneously **multiple objectives**. In other words is a **multi-objective optimization problem**.
  - For example, for a given charge/bunch, transverse emittance and bunch length at the injector exit need to be minimized by tuning laser spot size and pulse length at the cathode, phase and electric field intensity for the buncher, field intensity of the emittance compensation solenoid, position, phase and field of the first accelerating section, …
  - **Systematic scanning** of the multi-dimensional parameter space is **unrealistic** with existing computer power and more efficient method should be used.
- A notable example, is represented by **multi-objective genetic algorithms** (MOGA) that mimics the natural selection process.
- A population of possible solutions, is ranked towards the objective functions and a new generation of solutions is produced by 'mating' the highest ranking solutions. Random solution mutation is also applied. After a number of generations, the population of solutions converges to the so-called **Pareto optimal** front that represents the set of possible solutions trading-off between objectives. MOGA finds all the **globally optimal solutions** and allows for a ***a posteriori*** tradeoff selection.



Bazarov, I .V., and Sinclair, C .K., Phys. Rev. ST Accel. and Beams 8 034202 (2005).