Landau Damping

part 2

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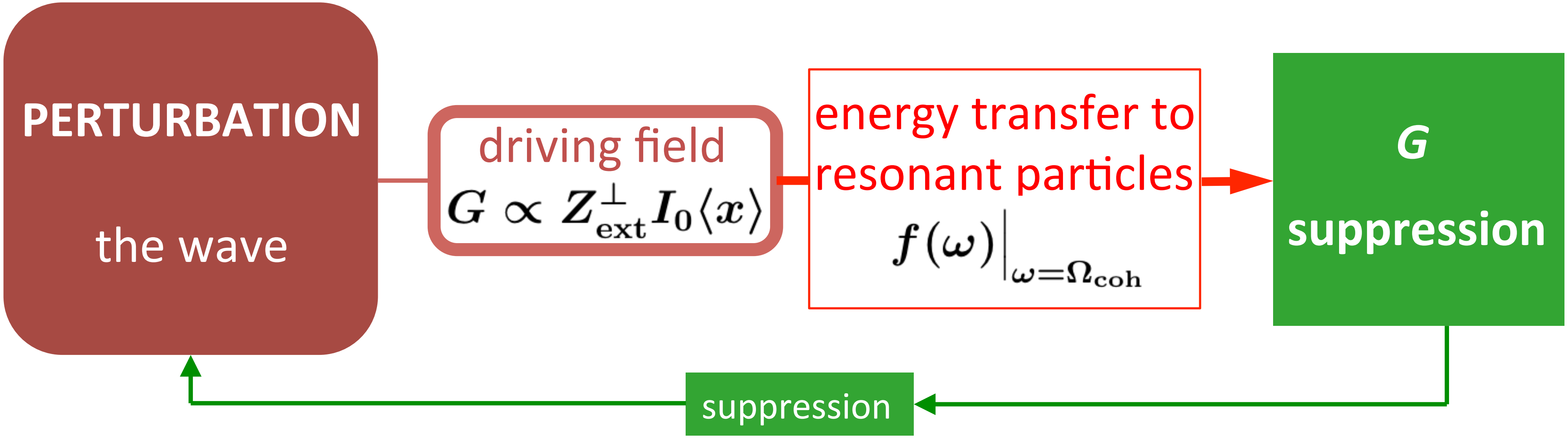
Landau Damping

Incomplete (!) mechanism of Landau Damping in beams
for the end of the first part



Main ingredients of Landau damping:

✓ wave−particle collisionless interaction: Impedance driving field

✓ energy transfer: the wave ⟷ the (few) resonant particles

Existence of Landau damping

In any accelerator, there are many Re(Z) sources

In any beam, there are many unsuppressed eigenmodes

$$\mathrm{Im}(\Delta Q_{\mathrm{coh}}) = \frac{\lambda_0 r_p}{\gamma Q_0} \frac{\mathrm{Re}(Z^{\perp})}{Z_0/R}$$

driving dipole impedance here

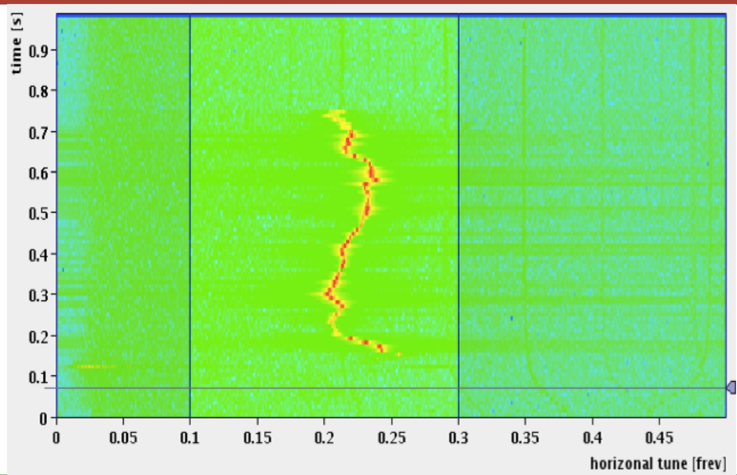
Still, the beams are often stable without an active mitigation

There must be a fundamental damping mechanism in beams

Existence of Landau damping

Additionally to Re(Z), deliberate excitation is often applied (tune measurements, optics control, …)

Energy is directly transferred to the beam, mostly at the beam resonant frequencies



Tune measurements at PS, kick every 10 ms, M.Gasior, et al, CERN

The beams are stable and absorb some energy

There must be a fundamental damping mechanism in beams

Damping

Basic consideration of a collective mode stability

$$\Delta\Omega = \Delta\Omega_{\mathrm{Re}} + i\gamma_{\mathrm{drive}} + i\gamma_{\mathrm{damping}}$$

change the parameters and the source of the driving mechanism

use and enhance the intrinsic damping mechanism

$\gamma_{\mathrm{drive}} + \gamma_{\mathrm{damping}}$	$>$	0	Instability
$\gamma_{\mathrm{drive}} + \gamma_{\mathrm{damping}}$	$<$	0	Stabilized mode
γ_{drive}	$>$	0	Driven (unsuppressed) mode
γ_{drive}	$<$	0	Mode suppressed by its drive

Another Leading Actor: Decoherence

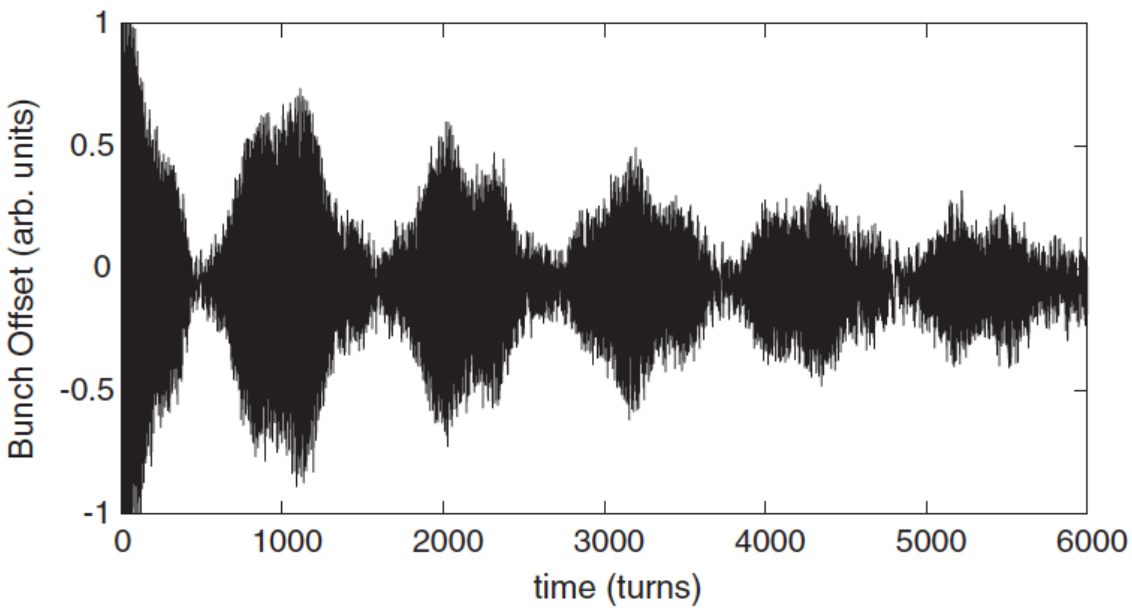
K.Y.Ng, Physics of Intensity Dependent Beam Instabilities, 2006
A.Hofmann, Proc. CAS 2003, CERN-2006-002
A.Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993
A.W. Chao, et al, SSC-N-360 (1987)

Decoherence

Collective beam oscillations after a short (one turn or shorter) kick

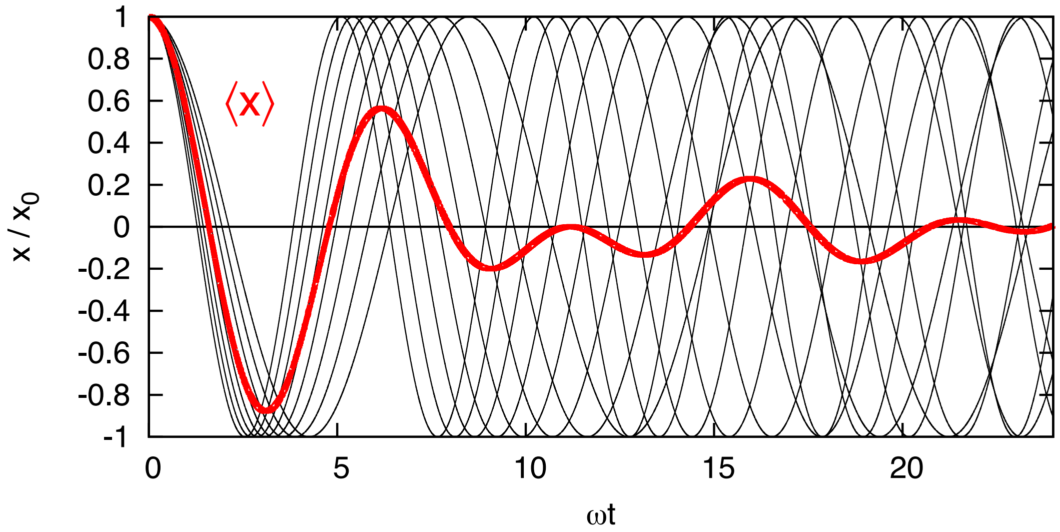
- Usually the beam displacement is comparable to the beam size
- In reality, signals can be very complicated
- A common diagnostics
- **Decoherence** is a **process** affected by many **effects** and **mechanisms**

A bunched beam Ar^{18+} in SIS18.
Transverse signal after a kick



V.Kornilov, O.Boine-F., PRSTAB 15, 114201 (2012)

Pulse Response



8 particles with different frequencies

Betatron oscillations: frequency spread

$$\delta\omega = Q_0 \xi \omega_0 \delta_p$$

$$g(t) = \frac{\langle x(t) \rangle}{x_0}$$

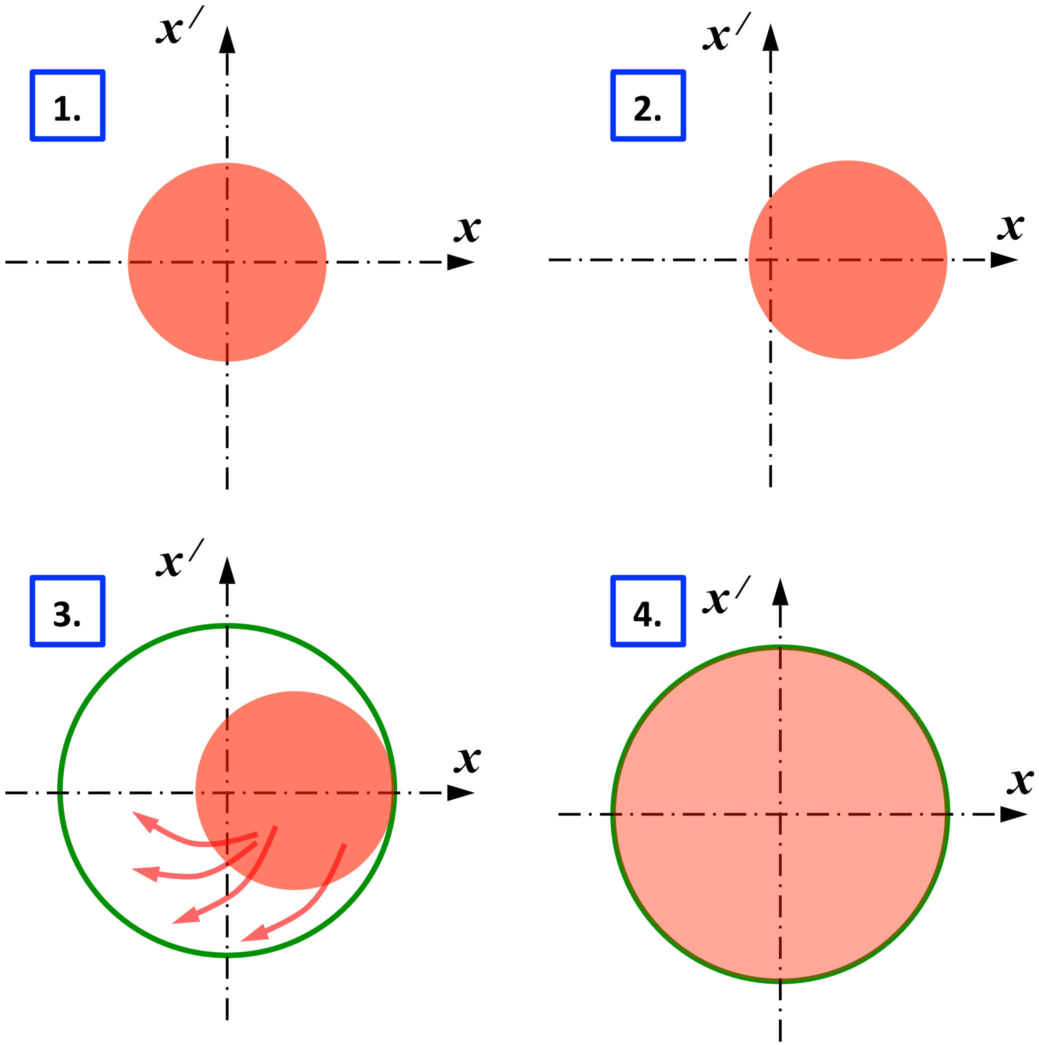
$$g(t) = \int f(\omega) \cos(\omega t) d\omega$$

$$g(t) = \text{Fourier}^{-1}\left\{R(\omega)\right\} = \frac{1}{2\pi} \int R(\omega) e^{-i\omega t} d\omega$$

The Pulse Response is the Fourier image of BTF

Phase-Mixing (Filamentation)

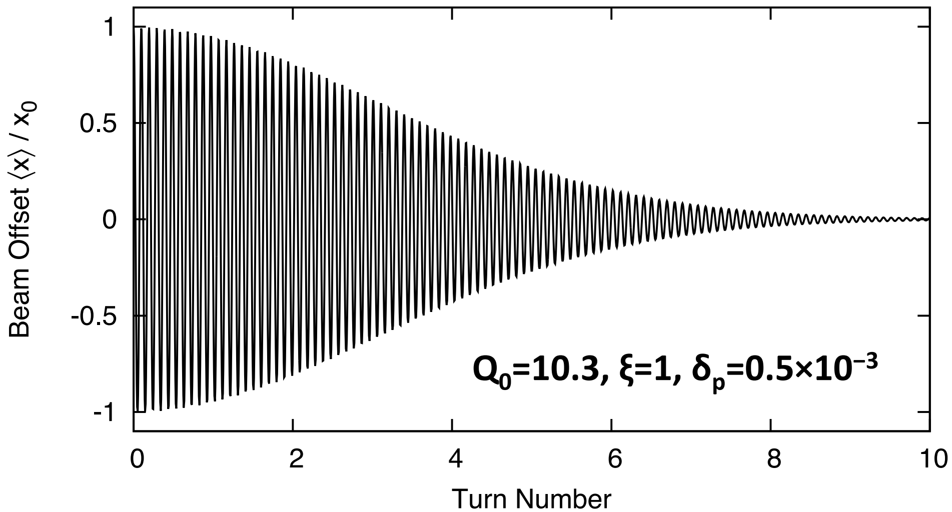
Phase-mixing of non-correlated particles with a tune spread



- Beam offset oscillations decay
- Beam size increases (blow-up)

Phase-Mixing, Coasting Beam

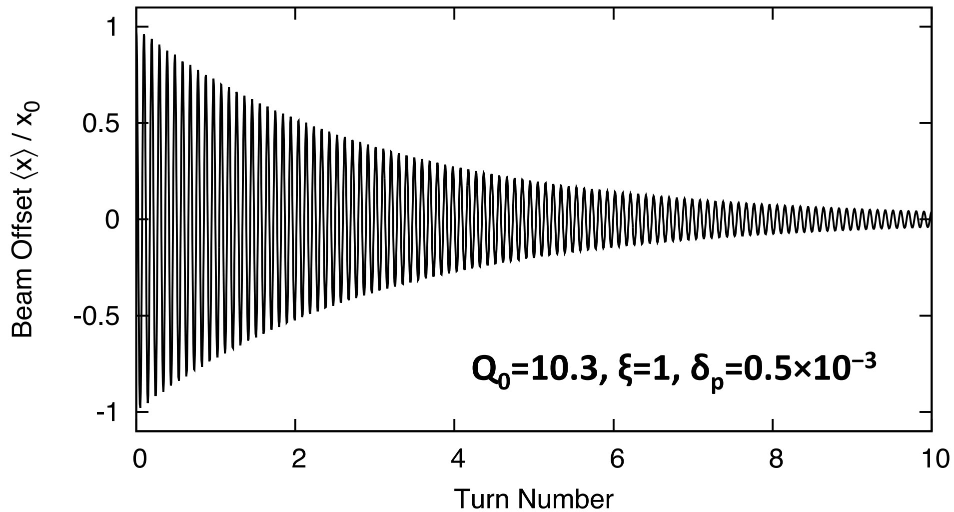
Gaussian Distribution



$$f(\omega_\beta) = \frac{1}{\sqrt{2\pi}\delta\omega^2} e^{-\omega_\beta^2/2\delta\omega^2}$$

$$g(t) = e^{-\delta\omega^2 t^2/2} \cos(\omega_\beta t)$$

Lorentz Distribution



$$f(\omega_\beta) = \frac{1}{\pi\ \delta\omega} \frac{1}{1 + \omega_\beta^2/\delta\omega^2}$$

$$g(t) = e^{-\delta\omega\ t} \cos(\omega_\beta t)$$

This is the case without any collective interactions:
phase-mixing of non-correlated particles

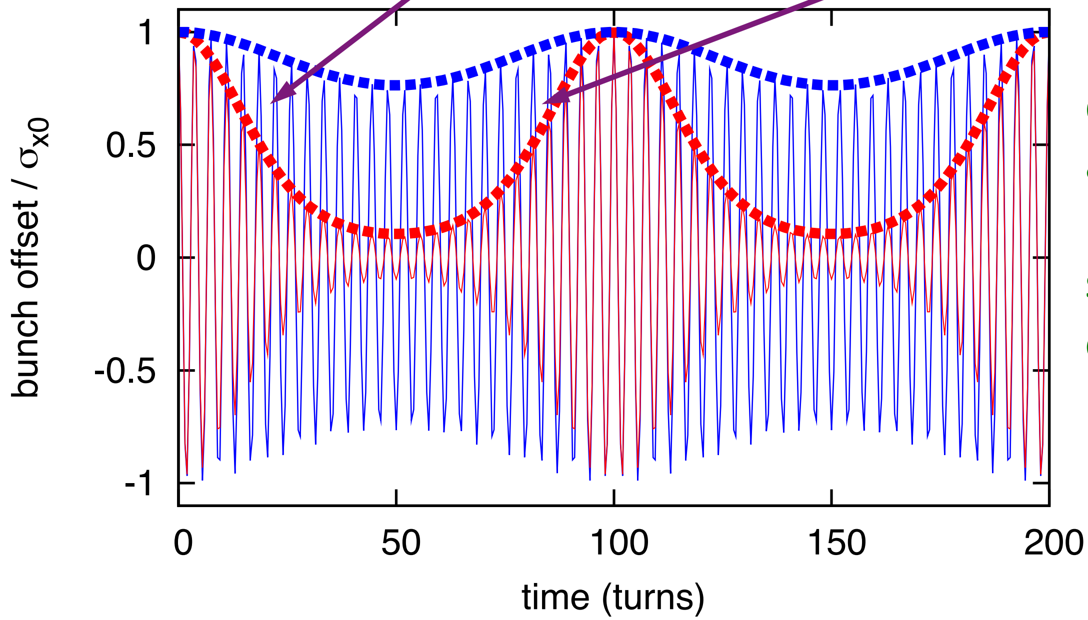
Phase-Mixing, Bunched Beam

Due to the particle synchrotron motion,
the initial positions return after a synchrotron period

$$A(N) = A_0 \exp\left\{-2\left(\frac{\xi Q_0 \delta_p}{Q_s}\sin(\pi Q_s N)\right)^2\right\}$$

In literature, named as "**decoherence**" and "**recoherence**"

ξ=0.5
ξ=1.4
N_s=100
Q_s=0.01



dashed lines: amplitude

solid lines: offset

In reality, the decoherence is very different (other effects)

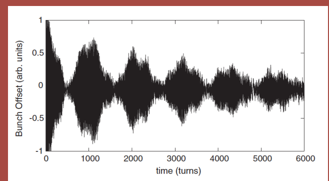
Decoherence

Mechanisms | **Process** | **Effects**

Mechanisms:
- Landau Damping
- Phase-mixing

Process:

Decoherence



Effects:
- Chromaticity
- Octupoles
- Space-charge
- Image charges
- Impedances
- Other nonlinearities
- Impedances

K.Y.Ng, Physics of Intensity Dependent Beam Instabilities, 2006
A.Hofmann, Proc. CAS 2003, CERN-2006-002
A.W. Chao, et al, SSC-N-360 (1987)
V.Kornilov, O.Boine-F., PRSTAB 15, 114201 (2012)
I.Karpov, V.Kornilov, O.Boine-F., PRAB 19, 124201 (2016)

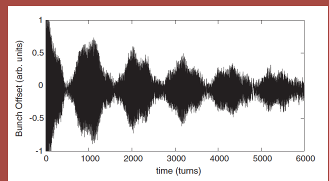
Decoherence

Mechanisms | **Process** | **Effects**

Mechanisms:
- Landau Damping
- Phase-mixing

Process:

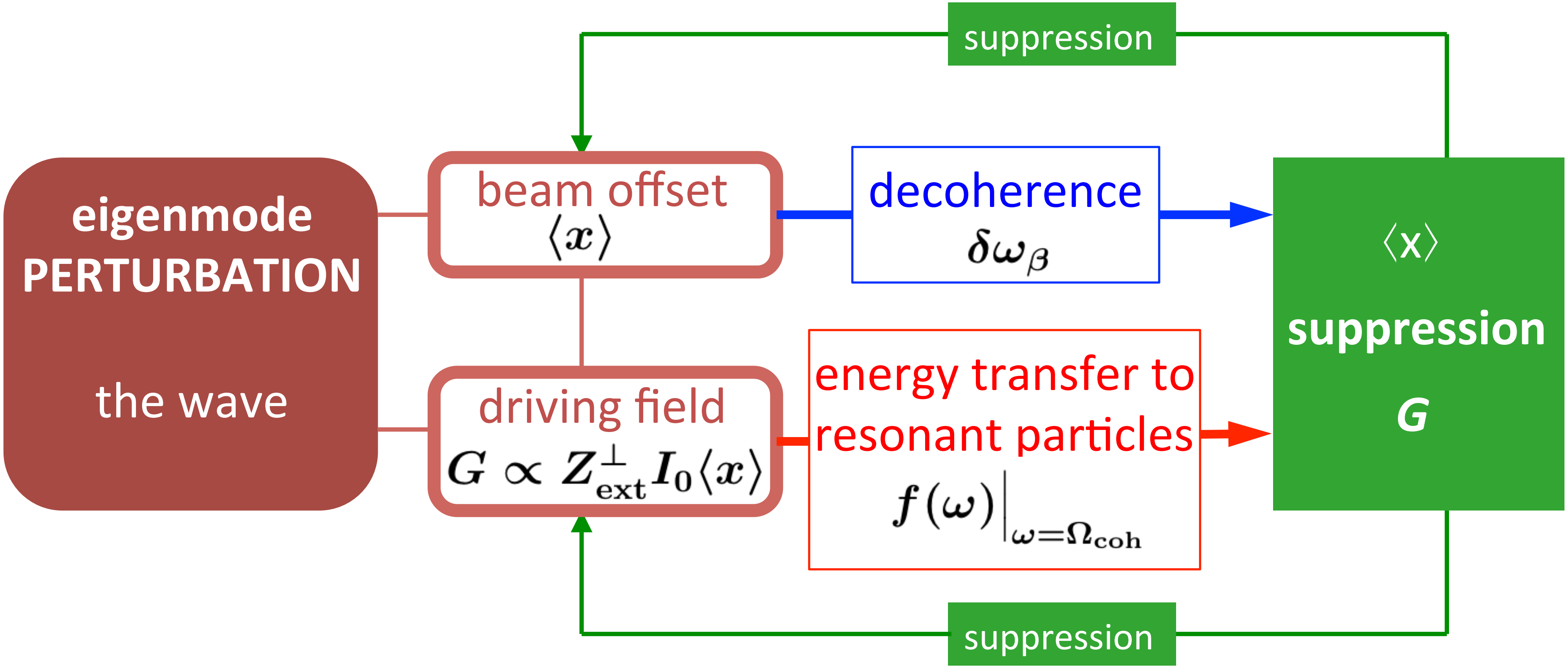
Decoherence



Effects:
- Chromaticity
- Octupoles
- Space-charge
- Image charges
- Impedances
- Other nonlinearities
- Impedances

After a kick, we observe the decoherence, which is a mixture of effects.
We do not observe pure Landau Damping.
(infinitesimal perturbations)

Landau Damping of 1st Type



Main ingredients of Landau damping:

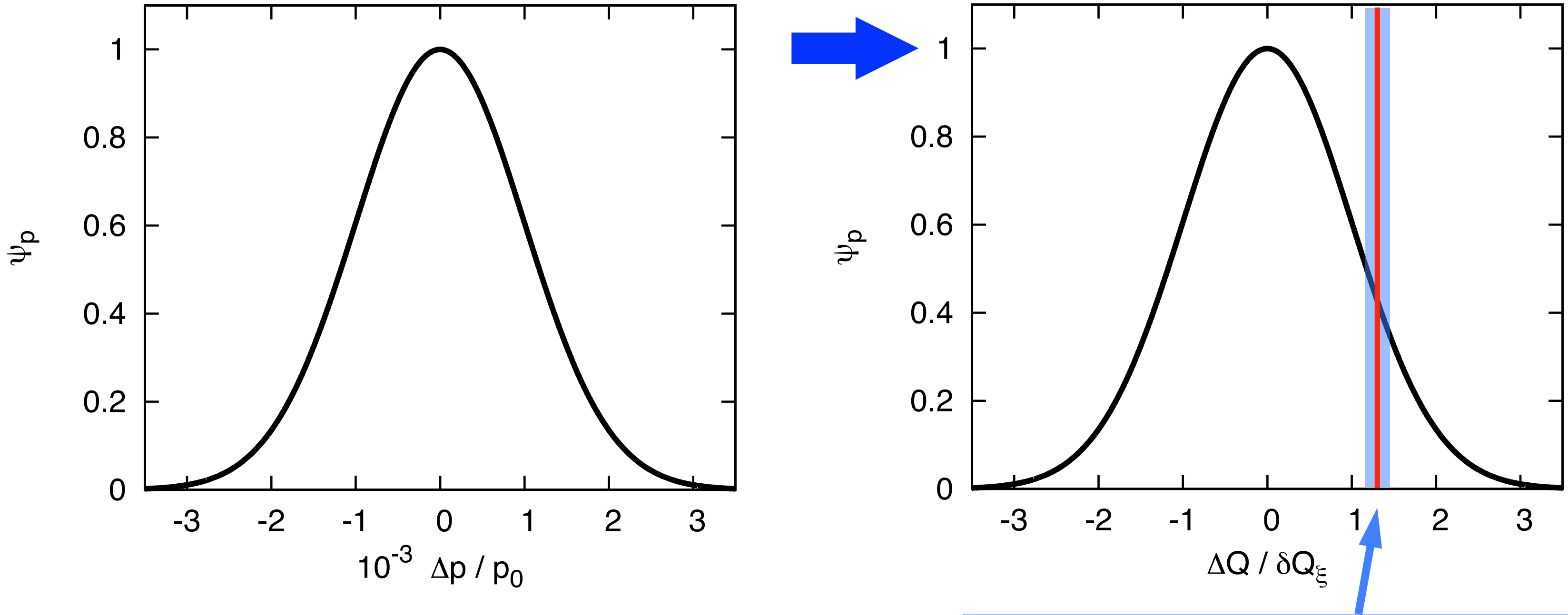
- ✓ wave−particle collisionless interaction: Impedance driving field
- ✓ energy transfer: the wave ↔ the (few) resonant particles

Landau Damping:
the wave & the tune spread

Landau Damping

Momentum spread translates into tune spread

$$\delta Q_\xi = \left|\eta(n - Q_0) + Q_0\xi\right|\delta_p$$

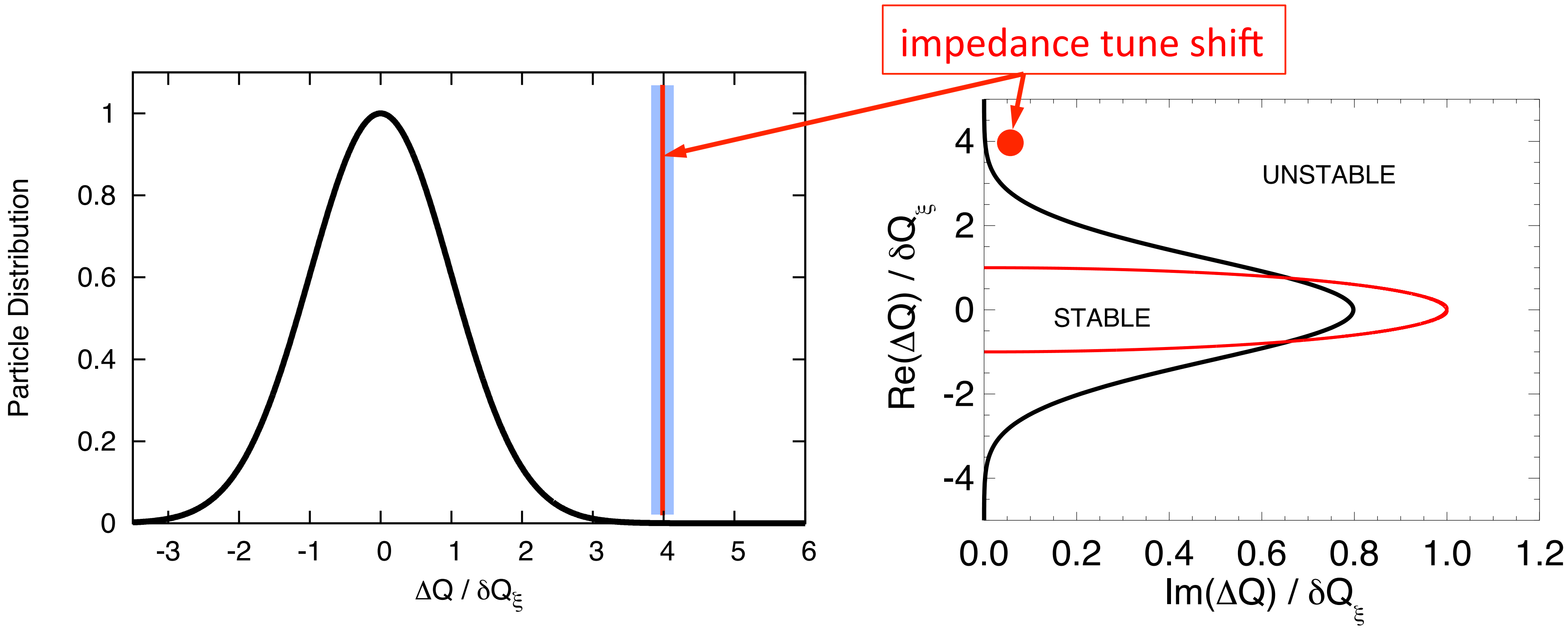


$$\Delta Q_{\mathrm{coh}}\omega_0 \int \frac{f(\omega)d\omega}{\omega - \Omega} = 1$$

resonant particles from both sides contribute to the energy transfer, thus ***f(ω)***

Landau Damping

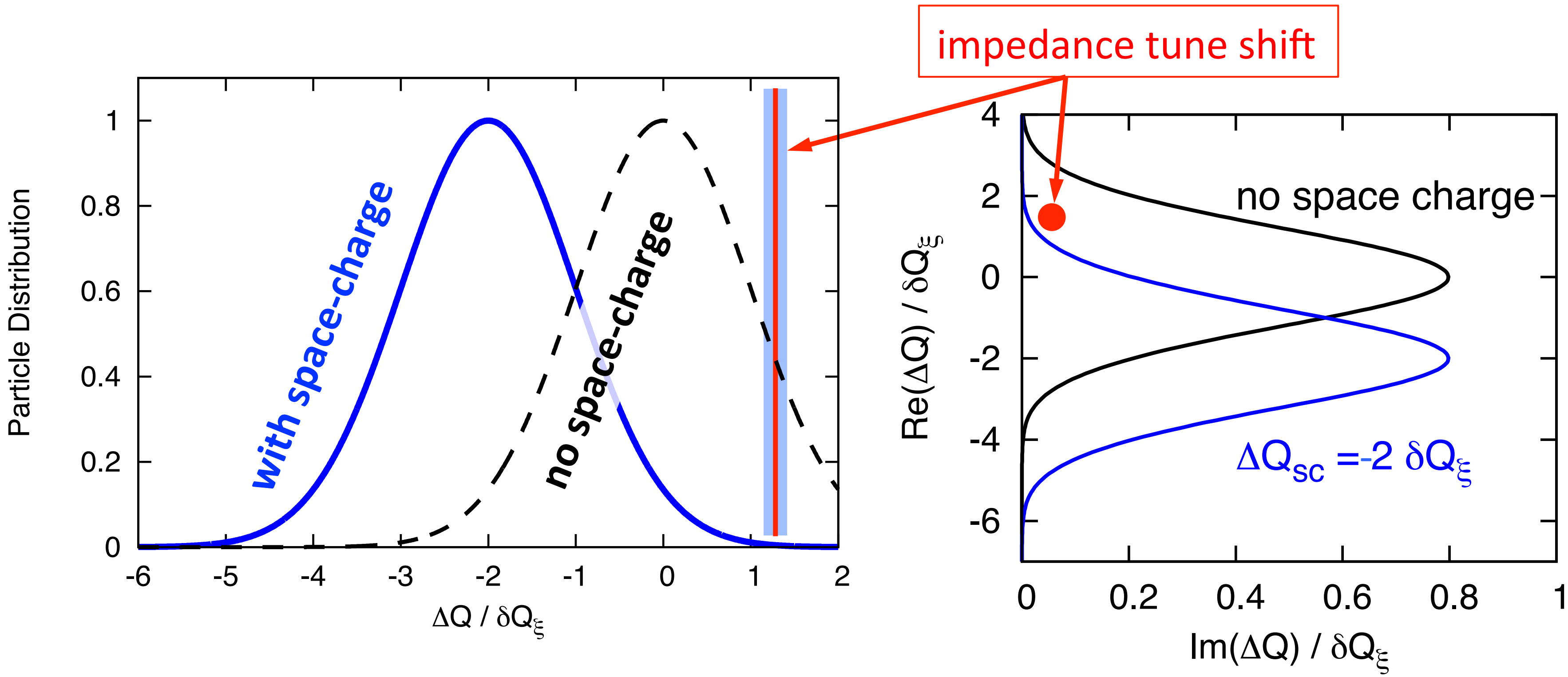
Loss of Landau damping due to coherent tune shift



there is still tune spread,
but no resonant particles for this wave → no Landau damping

Landau Damping

Loss of Landau damping due to space-charge



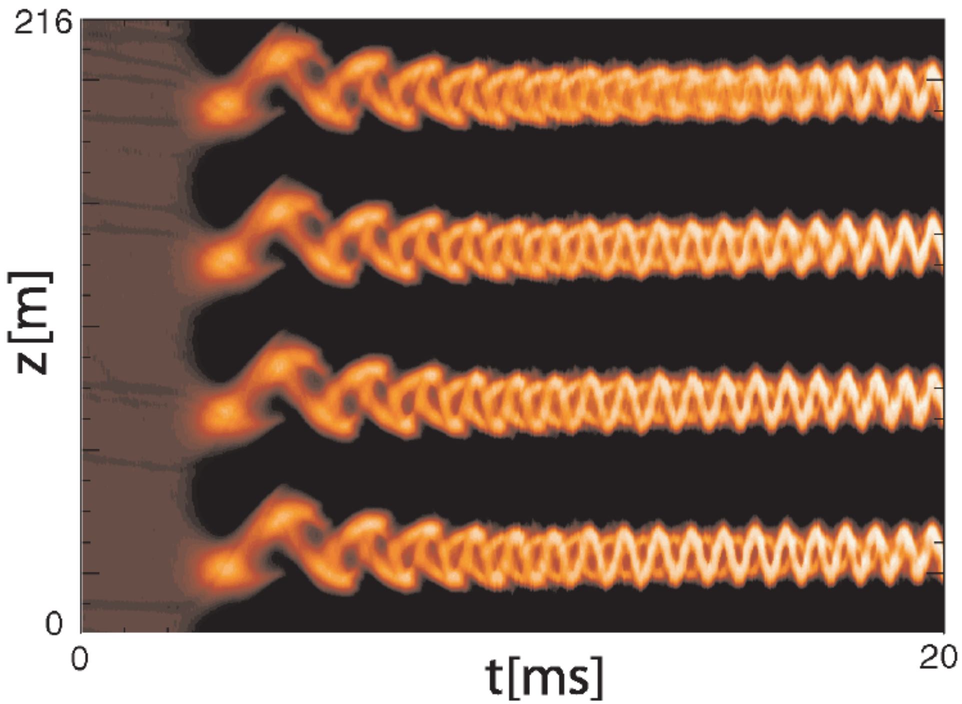
there is still tune spread,
but no resonant particles for this wave → no Landau damping

Landau Damping

Loss of Landau damping due to space-charge

Coherent dipole oscillations of four O^{8+} bunches in the SIS18 of GSI Darmstadt at the injection energy 11.4 MeV/u

O.Boine-F., T.Shukla, PRSTAB **8**, 034201 (2005)



there is still tune spread,
but no resonant particles for this wave → no Landau damping

Landau Damping

Remark 1

Some analytical models or simulations predict Landau **antidamping**:

unstable oscillations for zero or negative impedance

No antidamping for Gaussian-like distributions!

D.Pestrikov, NIM A 562, 65 (2006)
V.Kornilov, et al, PRSTAB 11, 014201 (2008)
A.Burov, V.Lebedev, PRSTAB 12, 034201 (2009)

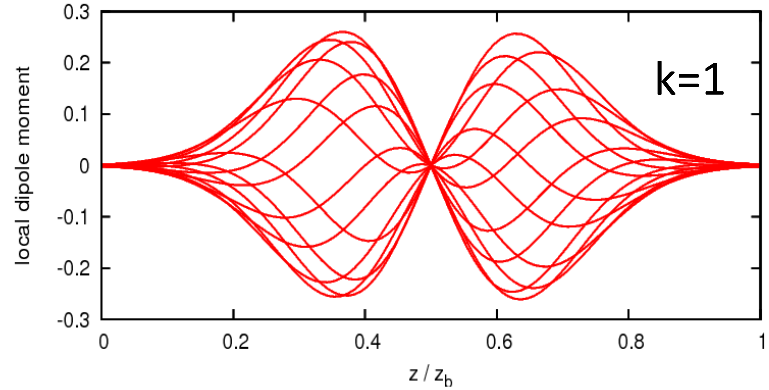
Landau Damping

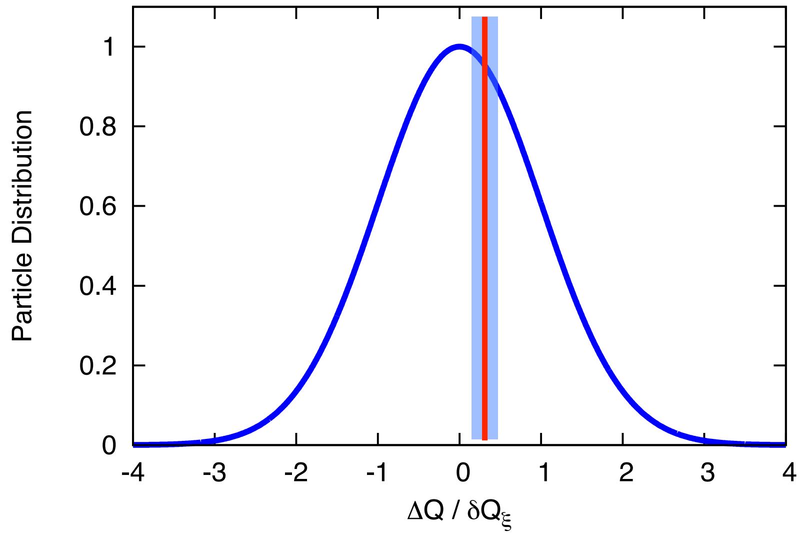
Remark 2

Head-tail modes in bunches:

- there is a **tune spread** (due to chromaticity ξ)
- the coherent frequency overlaps with the incoherent spectrum

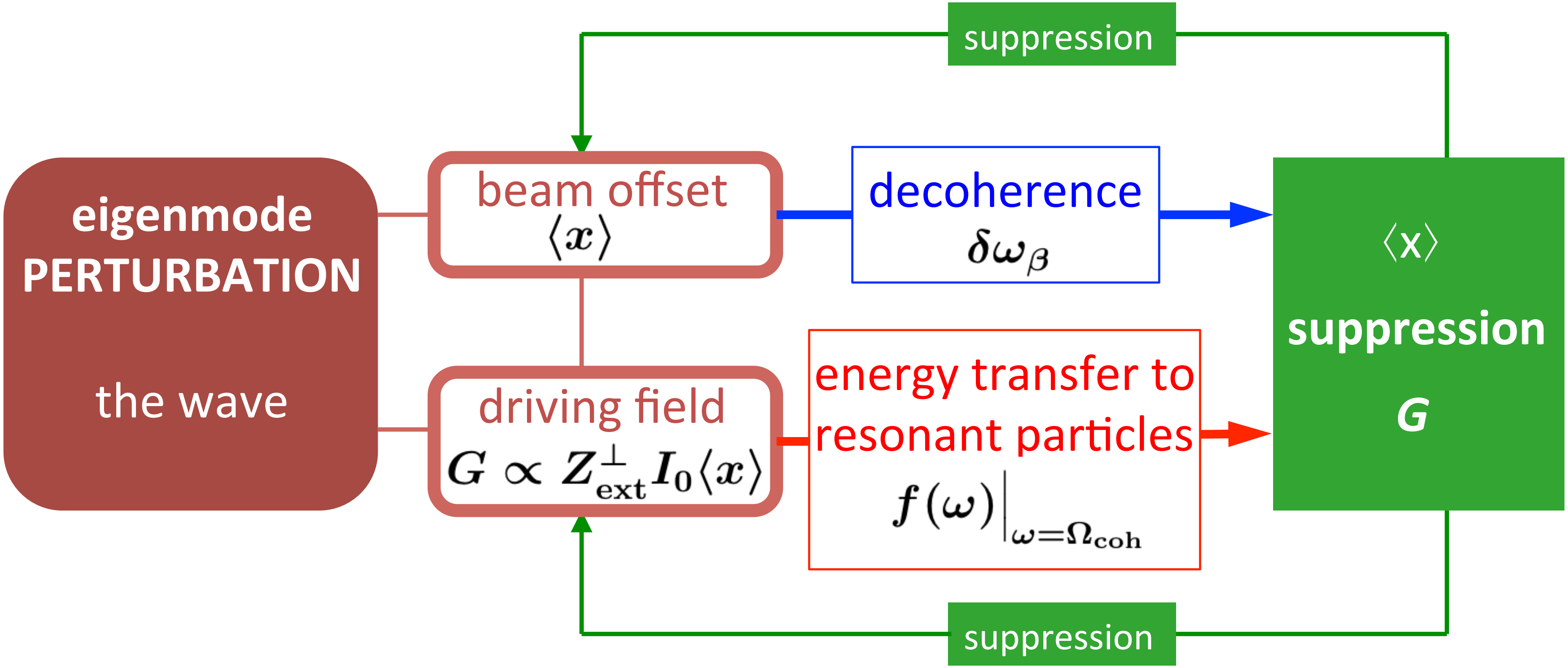
still, there is
NO Landau Damping!





a tune spread does not automatically means Landau Damping

Landau Damping of 1st Type



Main ingredients of Landau damping:

- ✓ wave−particle collisionless interaction: Impedance driving field
- ✓ energy transfer: the wave ↔ the (few) resonant particles

Landau Damping in Beams of 2nd type

Landau Damping of 2nd type

Different situation from ξ+δp:
Tune spread due to amplitude-dependent tune shifts

For example, an octupole magnet:

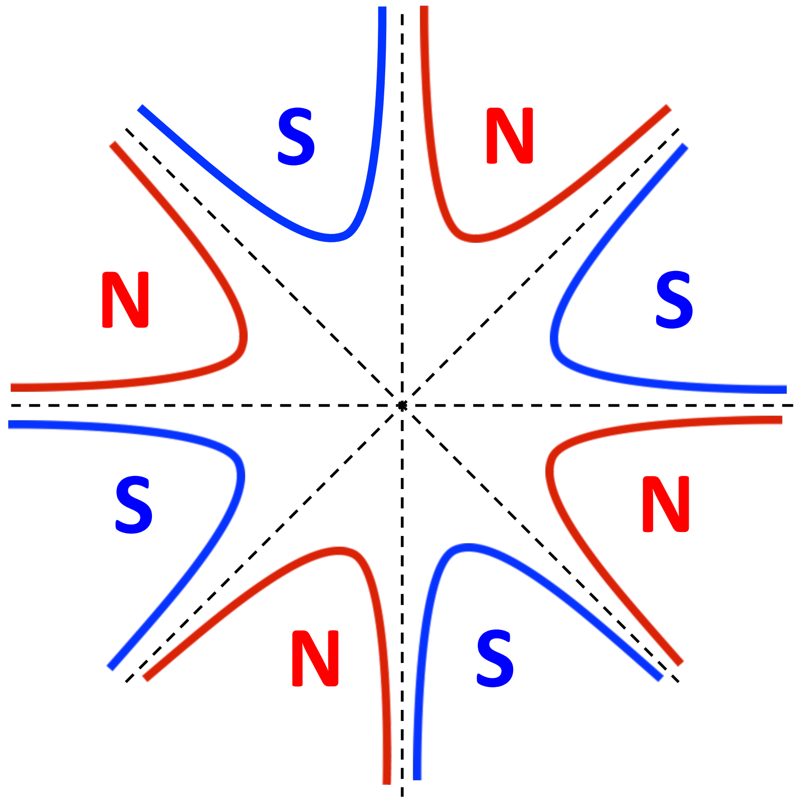
$$\begin{aligned} B_x &= O_3(3x^2y - y^3) \\ B_y &= O_3(x^3 - 3xy^2) \end{aligned}$$

First-order frequency shift of the anharmonic oscillations

$$\ddot{x} + \omega_0^2 x = \varepsilon x^3$$

$$\omega \approx \omega_0 - \frac{3}{8}\frac{\varepsilon A^2}{\omega_0}$$

Amplitude-dependent betatron tune shifts



Schematic yoke profile of an octupole magnet

Decoherence

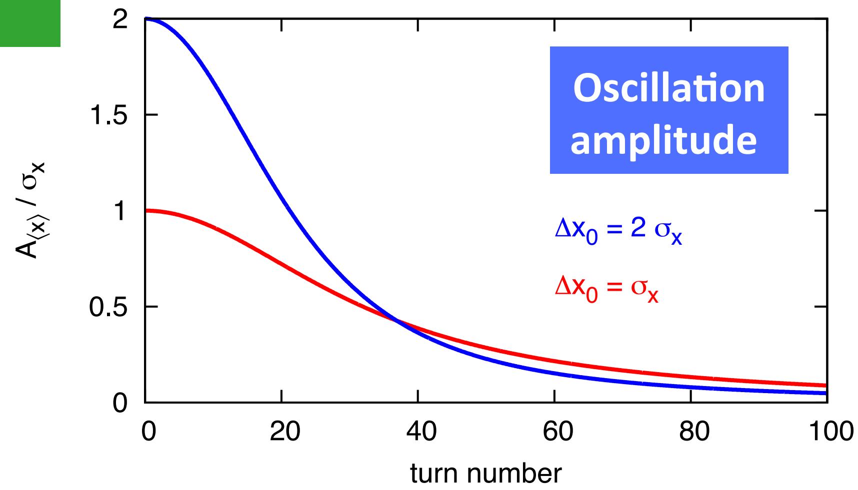
Phase-Mixing is very different from the chromaticity case

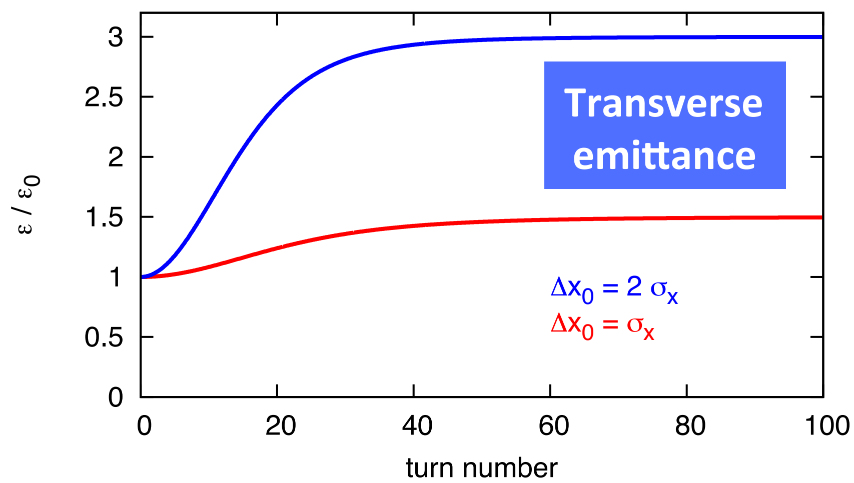
- No recoherence
- Kick amplitude dependent
- Analytical model for:

$$Q(a) = Q_0 - \mu \left(\frac{a}{\sigma_x} \right)^2$$

$$A(N) = \frac{A_0}{1+\theta^2} \exp\left\{ -\frac{A_0^2}{2\sigma_x^2} \frac{\theta^2}{1+\theta^2} \right\}$$

$$\theta = 2\pi\mu\, N$$





A.W. Chao, et al, SSC-N-360 (1987)
V.Kornilov, O.Boine-F., PRSTAB 15, 114201 (2012)
I.Karpov, V.Kornilov, O.Boine-F., PRAB 19, 124201 (2016)

Landau Damping of 2nd type

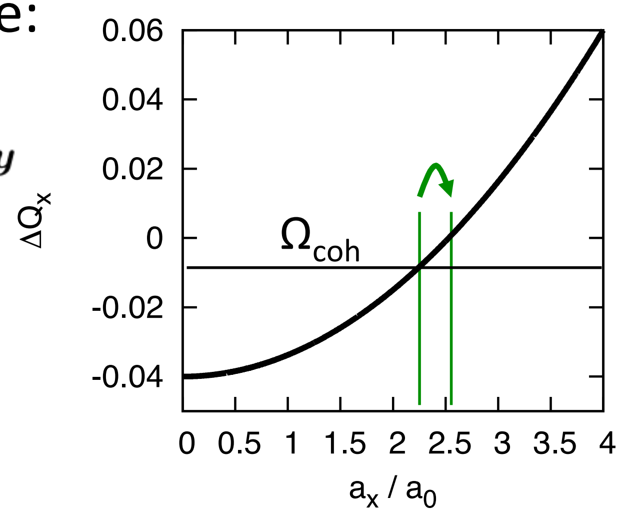
Tune spread due to amplitude-dependent tune shifts

Amplitude-dependent betatron tune shifts in a lattice:

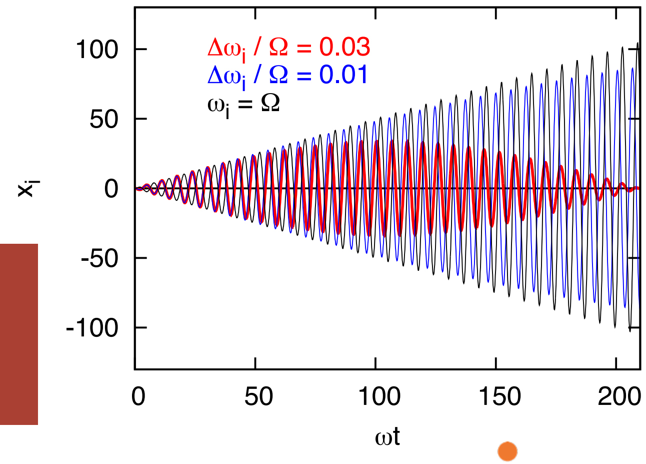
$$\Delta Q_x^{\text{oct}} = \left(\int \frac{K_3 \beta_x^2}{16\pi} ds\right) J_x - \left(\int \frac{K_3 \beta_x \beta_y}{8\pi} ds\right) J_y$$

$$K_3 = \frac{6 O_3}{B\rho}$$

$$x(s) = \sqrt{2 J_x \beta_x(s)} \cos(\phi_x)$$



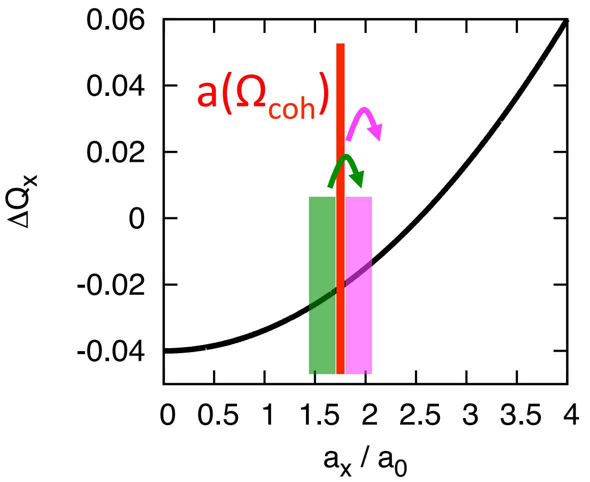
Every particle has a different amplitude (J_x, J_y)
→ tune spread → Landau damping?

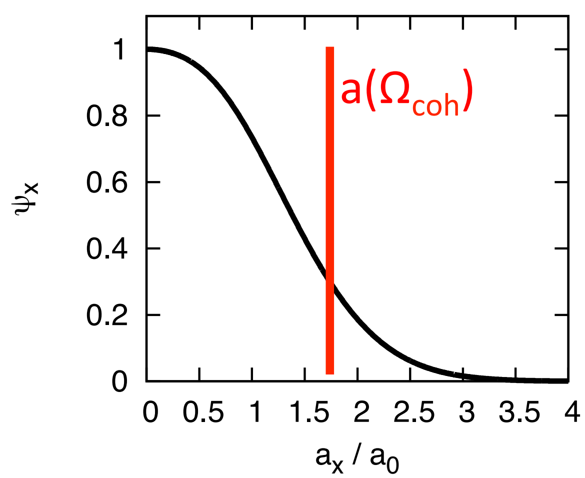


The resonant particles drift away in tune from the resonance as they get excited

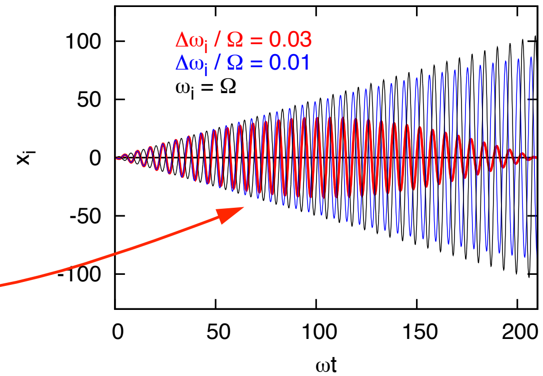
Landau Damping of 2nd type

Particle excitation for amplitude-dependent tune shifts





Once the particle is driven away from the resonance, the energy is transferred back to the wave



We already guess: the distribution slope (*df/da*) might be involved

Landau Damping of 2nd type

The dispersion relation

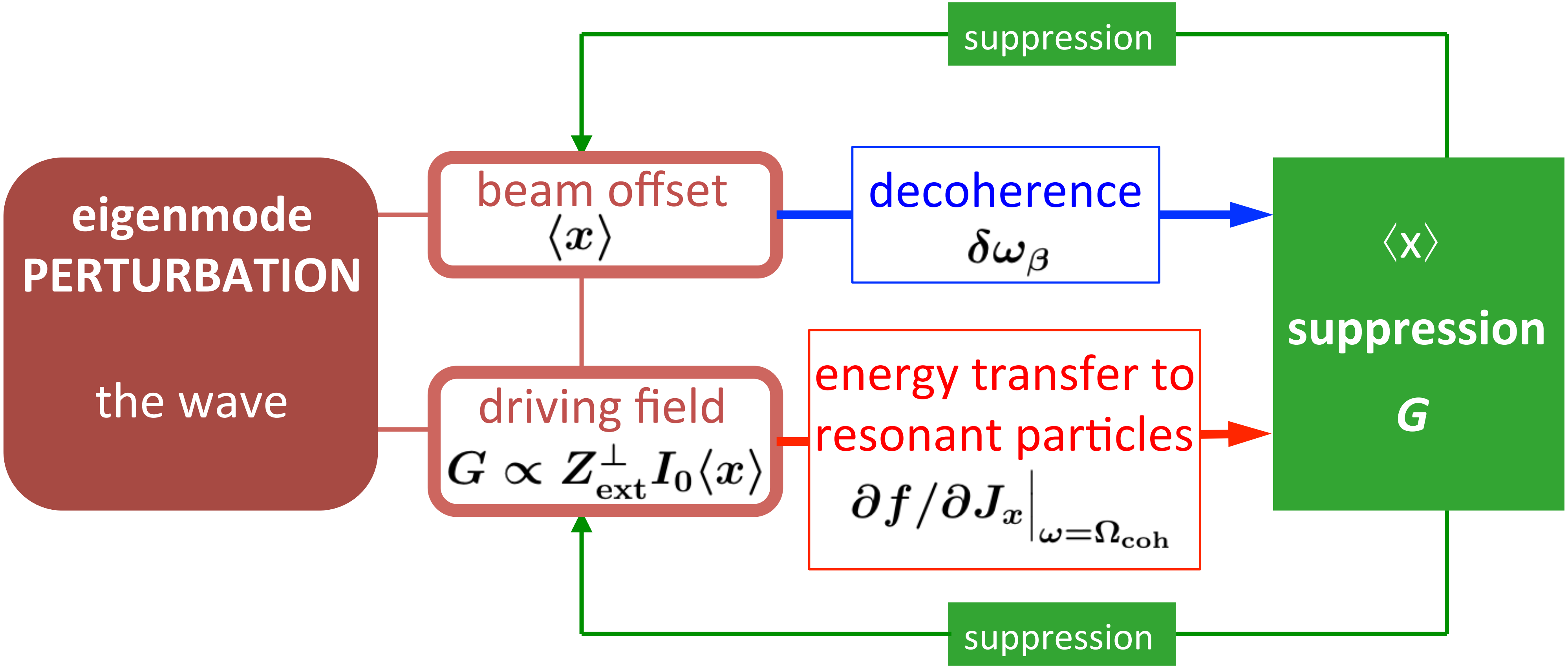
$$\Delta Q_{\mathrm{coh}} \int \frac{1}{\Delta Q_{\mathrm{ex}} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} dJ_x dJ_y = 1$$

ΔQ_{coh} : coherent no-damping tune shift imposed by an impedance
$\Delta Q_{ex}(J_x,J_y)$: external (lattice) incoherent tune shifts

L.Laslett, V.Neil, A.Sessler, 1965
D.Möhl, H.Schönauer, 1974
J.Berg, F.Ruggiero, CERN SL-96-71 AP 1996

The resulting damping is a complicated 2D convolution of the distribution $\{df(J_x,J_y)/dJ_x\}$ and tune shifts $\Delta Q_{ext}(J_x,J_y)$

Landau Damping of 2nd type



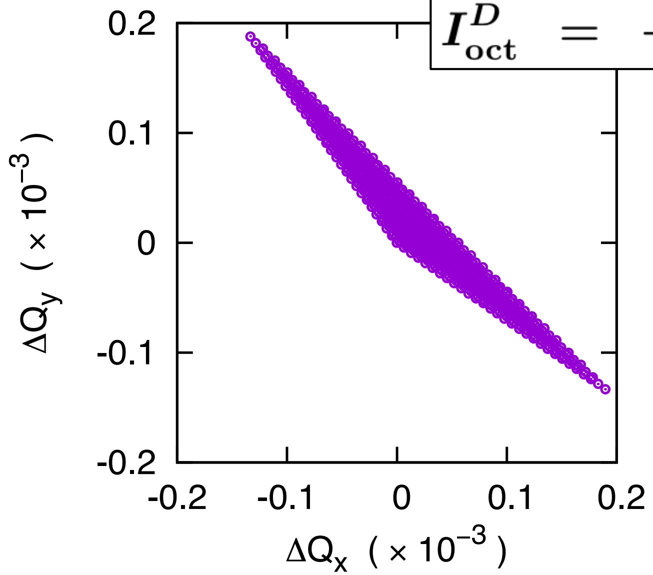
Main ingredients of Landau damping:

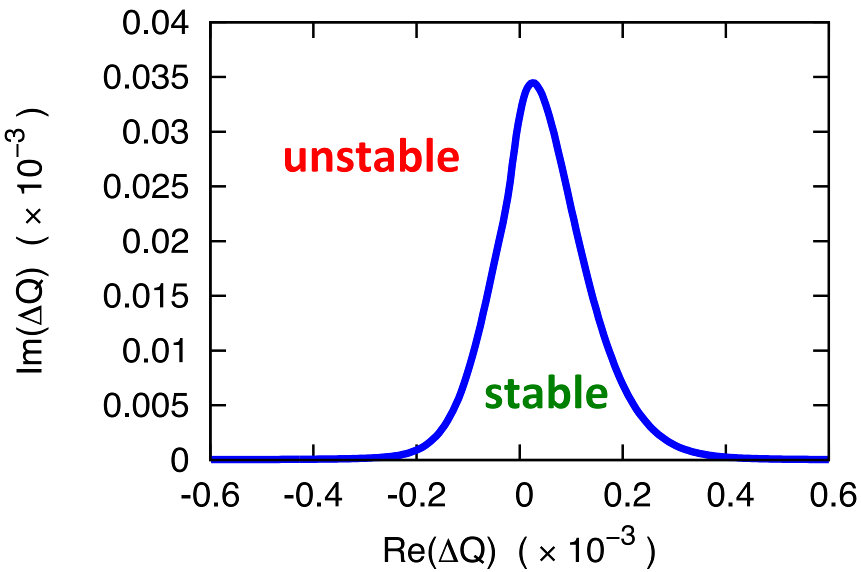
- ✓ wave–particle collisionless interaction: Impedance driving field
- ✓ energy transfer: the wave ↔ the (few) resonant particles

Landau Damping of 2nd type

An example of the application:
Beam stability studies for FCC

$$I_{oct}^{F} = +500\ \text{A}$$
$$I_{oct}^{D} = +500\ \text{A}$$





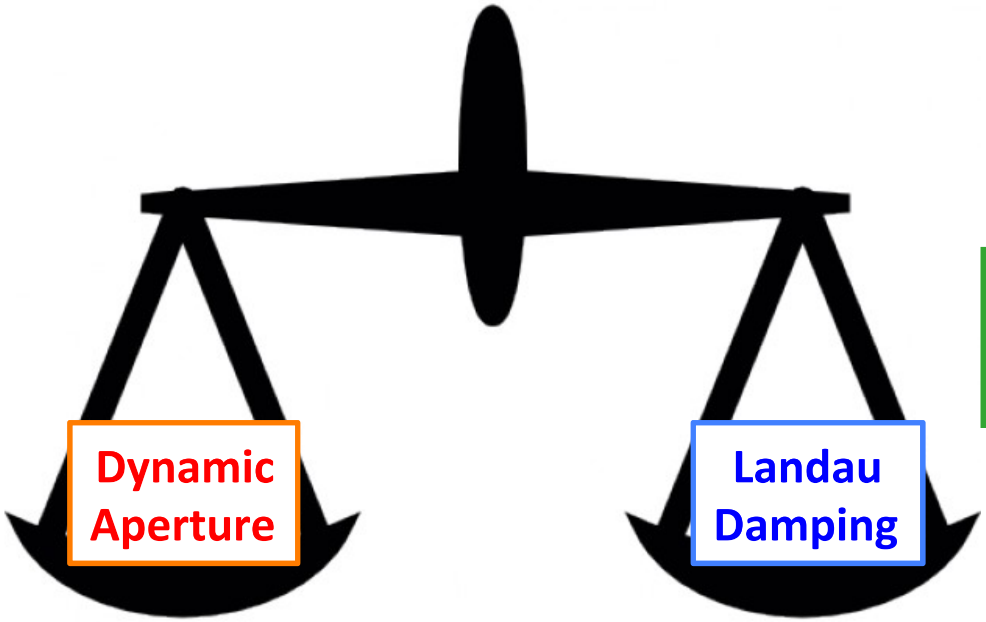
Octupole Tune Footprint

Resulting Stability Diagram

V.Kornilov, FCC Week 2017, Berlin

Landau Damping of 2nd type

Drawback: octupoles, like other nonlinearities, can reduce Dynamic Aperture and Lifetime.

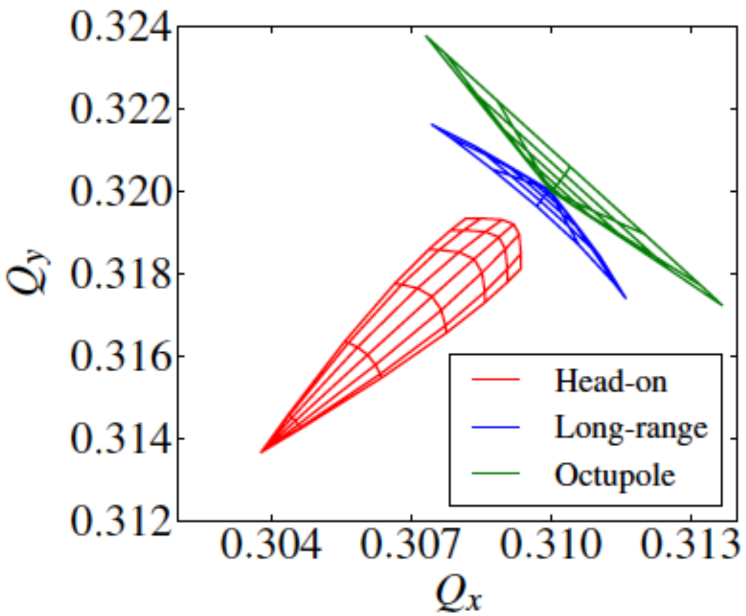


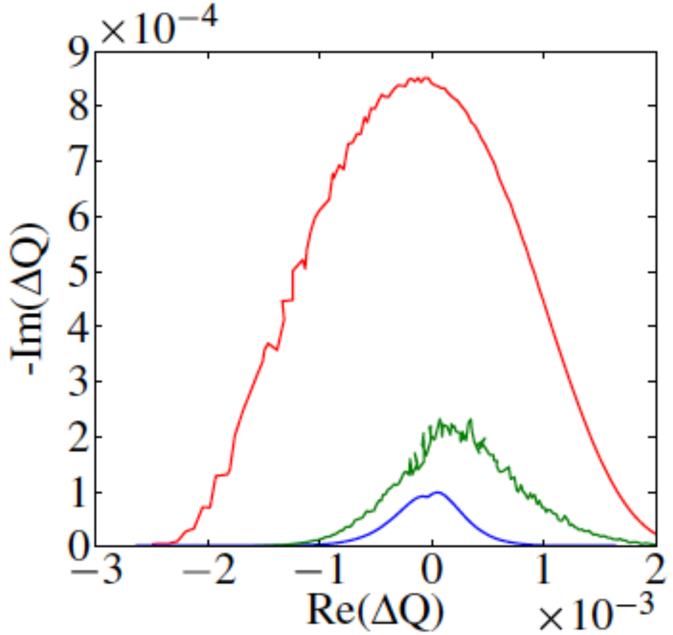
A balance is necessary

Octupoles are the essential part of the beam stability in many machines

Amplitude-Dependent Detuning

Another source of amplitude-dependent tune-shifts:
Tune-spread due to Beam-Beam effects produces Landau Damping





Tune Footprints

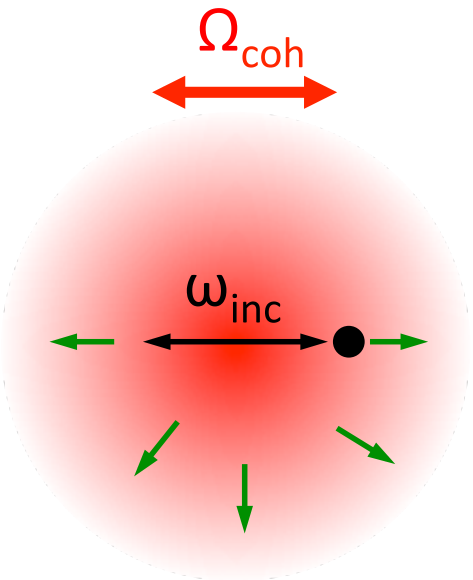
Resulting Stability Diagrams

X.Buffat, et al, PRSTAB 17, 111002 (2014)

Landau Damping in Beams of 3rd type

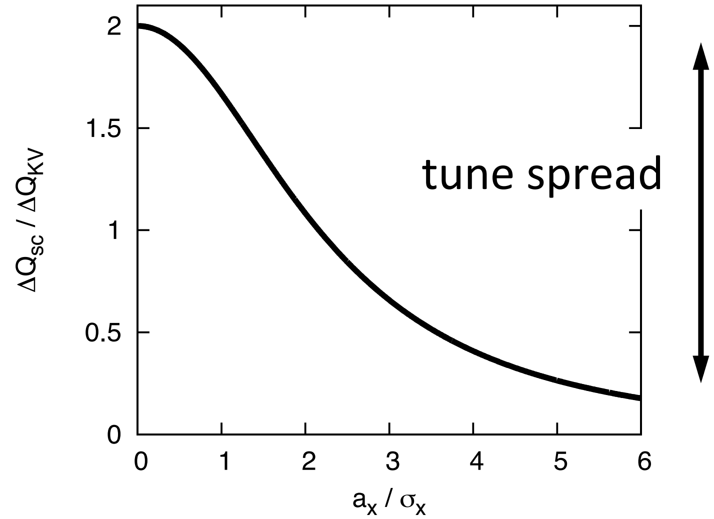
Landau Damping of 3rd type

Different situation:
Wave–Particel interaction is due to space-charge



Electric field
of the self-field
space charge

Space-charge tune shift



For the resonant particles $Q_{inc} \approx Q_{coh}$,
wave↔particles energy transfer should be possible

Landau Damping of 3rd type

The dispersion relation

$$\int \frac{\Delta Q_{\mathrm{coh}} - \Delta Q_{\mathrm{sc}}}{\Delta Q_{\mathrm{ex}} + \Delta Q_{\mathrm{sc}} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} \mathrm{d}J_x \mathrm{d}J_y \mathrm{d}p = 1$$

$f(J_x, J_y, p)$

ΔQ_{coh} :	no-damping coherent tune shift imposed
$\Delta Q_{ex}(J_x, J_y, p)$:	external (lattice) incoherent tune shift
$\Delta Q_{sc}(J_x, J_y)$:	space-charge tune shift

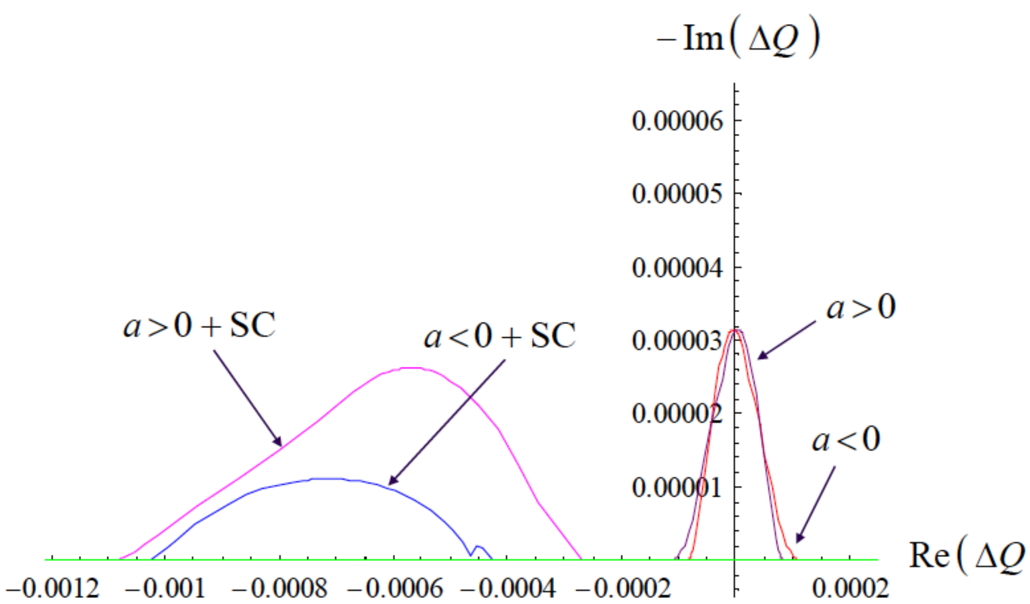
L.Laslett, V.Neil, A.Sessler, 1965
D.Möhl, H.Schönauer, 1974

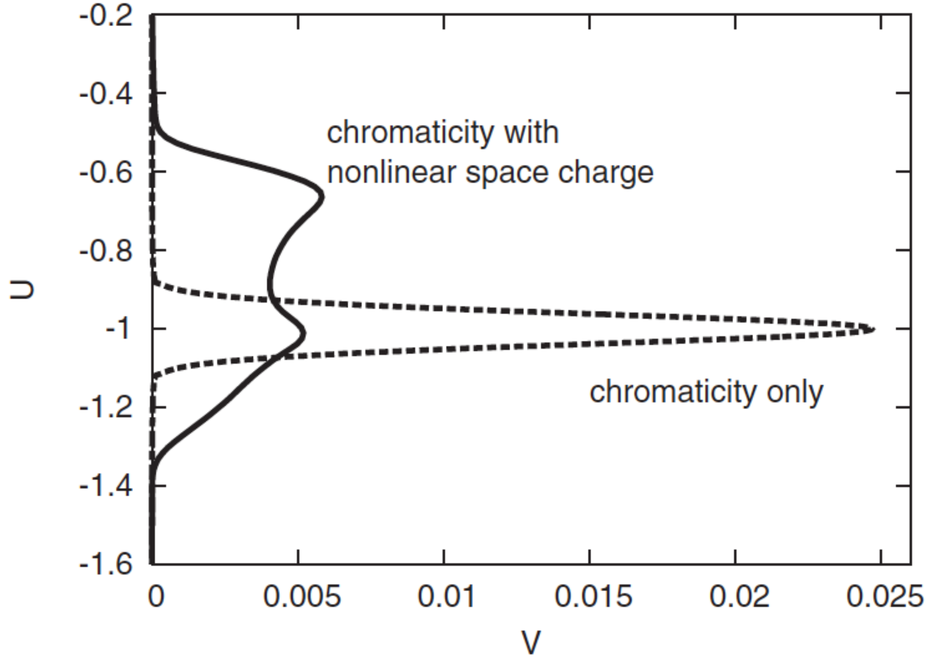
The resulting damping is a complicated 2D convolution of the distribution $\{df(J_x, J_y)/dJ_x\}$ and tune shifts $\Delta Q_{sc}(J_x, J_y)$, $\Delta Q_{ext}(J_x, J_y)$

Landau Damping of 3rd type

$$\int \frac{\Delta Q_{\rm coh} - \Delta Q_{\rm sc}}{\Delta Q_{\rm ex} + \Delta Q_{\rm sc} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} {\rm d}J_x {\rm d}J_y {\rm d}p = 1$$

<u>Solution examples</u>





E.Metral, F.Ruggiero, CERN-AB-2004-025 (2004) **V.Kornilov, O.Boine-F, I.Hofmann, PRSTAB 11, 014201 (2008)**

Landau Damping of 3rd type

The dispersion relation

$$\int \frac{\Delta Q_{\rm coh} - \Delta Q_{\rm sc}}{\Delta Q_{\rm ex} + \Delta Q_{\rm sc} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} {\rm d}J_x {\rm d}J_y {\rm d}p = 1$$

$\Delta Q_{ex}=0$: no pole, no damping!

Momentum conservation in a closed system

Even if Ω_{coh} is inside the spectrum,
and there are resonant particles $Q_{inc} \approx Q_{coh}$,
there is no Landau damping in coasting beams only due to space-charge

L.Laslett, V.Neil, A.Sessler, 1965
D.Möhl, H.Schönauer, 1974
V.Kornilov, O.Boine-F, I.Hofmann, PRSTAB 11, 014201 (2008)
A.Burov, V.Lebedev, PRSTAB 12, 034201 (2009)

Landau Damping

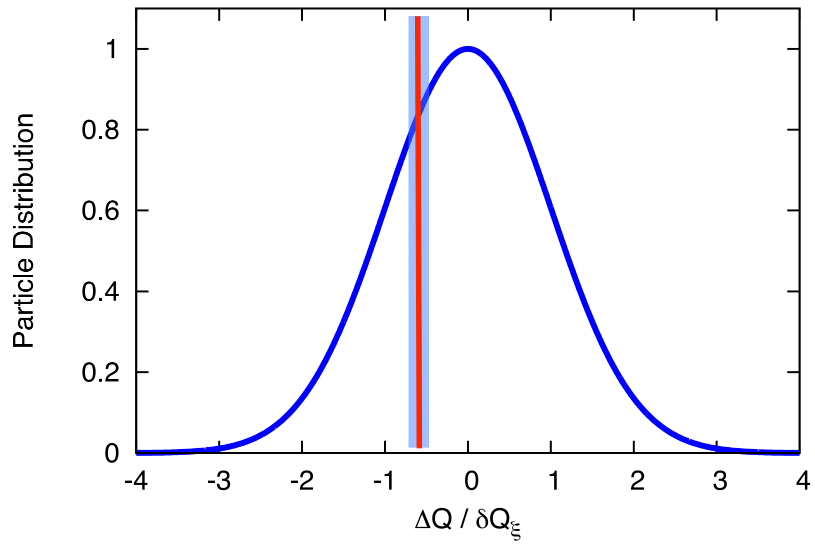
Remark

Thus, we discussed two examples:

1. Head-tail modes and the chromaticity tune-spread
2. Nonlinear space-charge in coasting beams

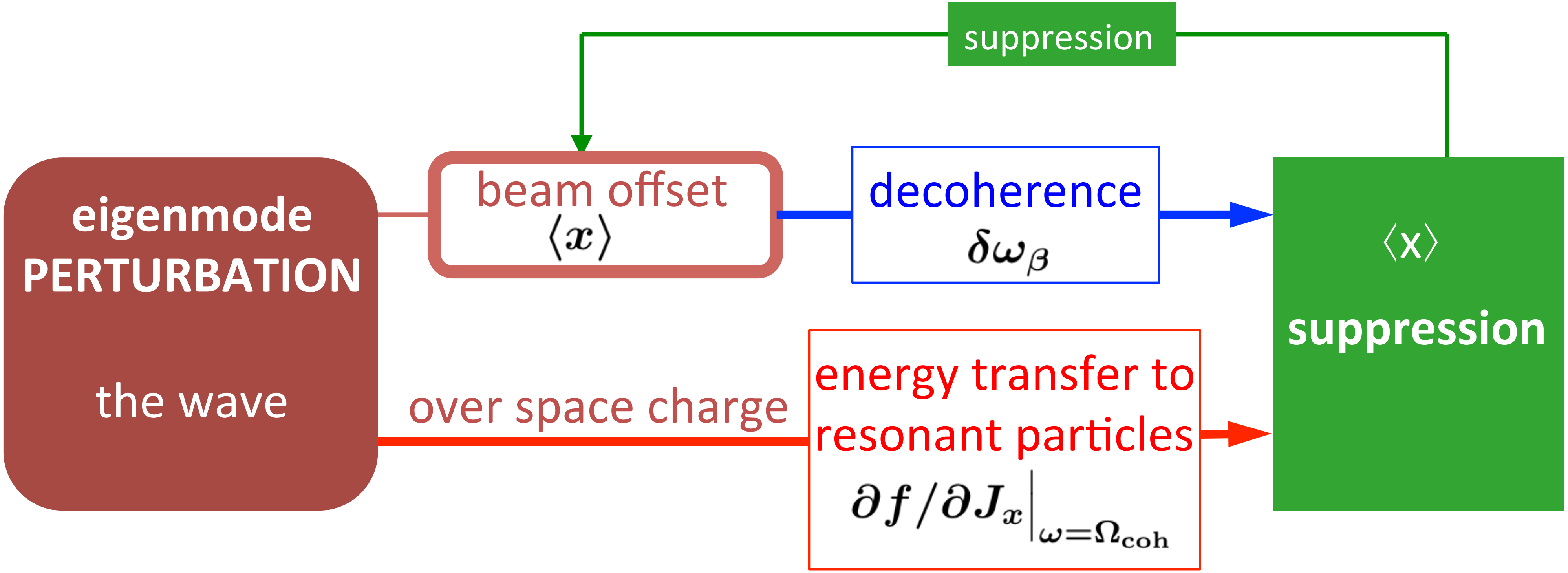
- there is a **tune spread**
- the coherent frequency overlaps with the incoherent spectrum

still, there is **NO Landau Damping!**



a tune spread does not automatically means Landau Damping

Landau Damping of 3rd type

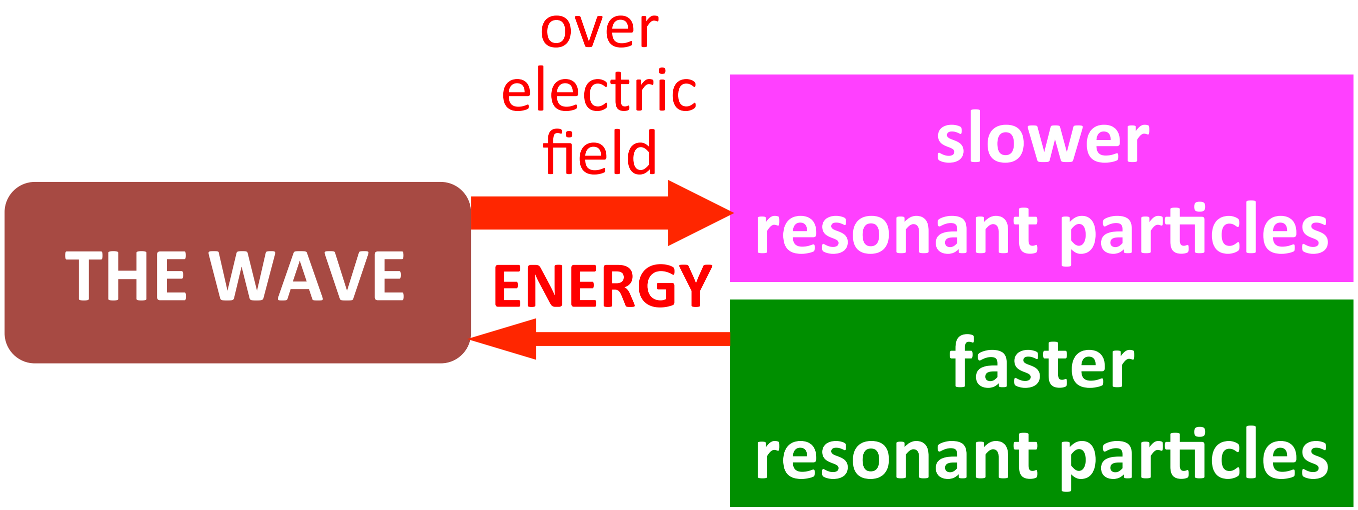


Main ingredients of Landau damping:

- ✓ wave−particle collisionless interaction: ***E***-field of Space-charge
- ✓ energy transfer: the wave ↔ the (few) resonant particles

Landau Damping of 3rd type

If simplified, very similar to Landau damping in plasma:



$$\partial f / \partial J_x \big|_{\omega=\Omega_{\mathrm{coh}}}$$

Main ingredients of Landau damping:

✓ wave−particle collisionless interaction:
  the electric field of space-charge

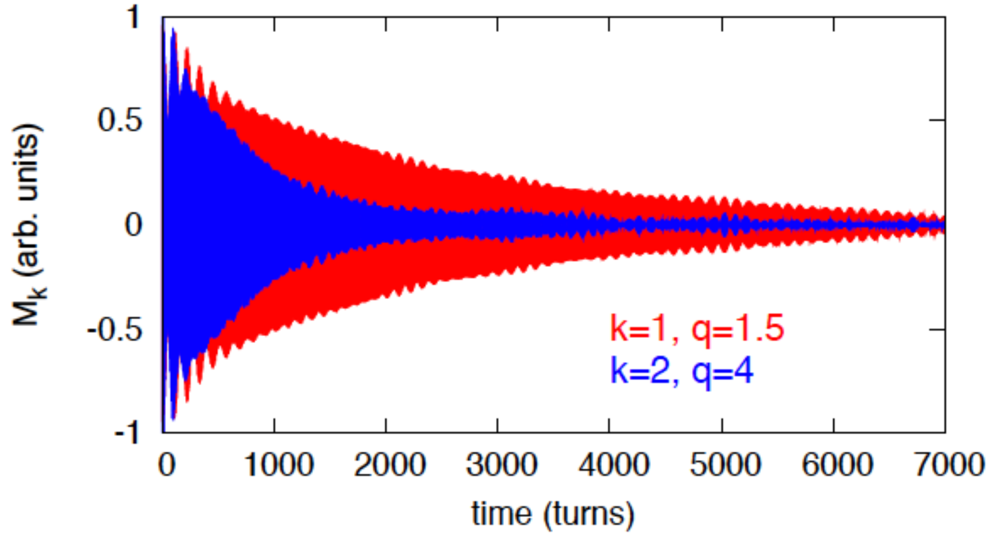
✓ energy transfer: the wave ↔ the (few) resonant particles

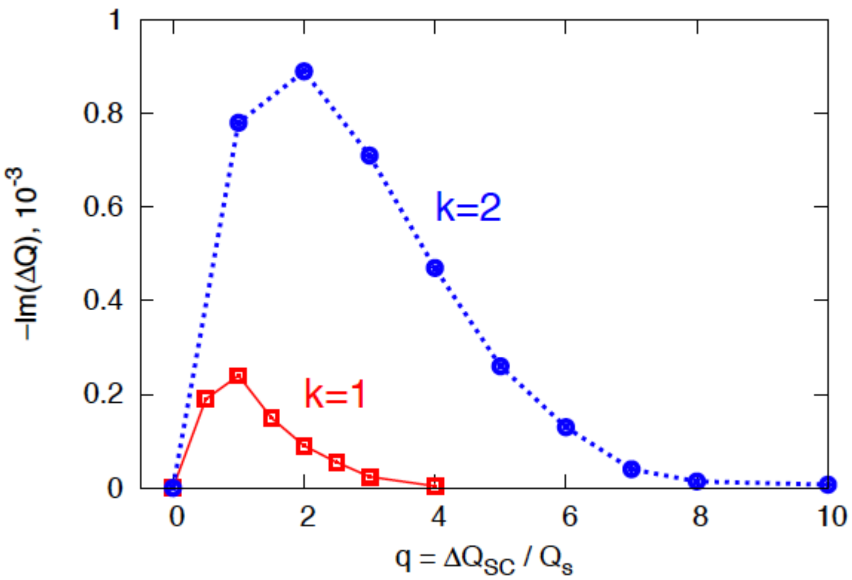
in addition, the decoherence, other $\Delta Q_{ex}(J_x, J_y, p)$, the mix with $\boldsymbol{G}$

Landau Damping of 3rd type

Landau damping in bunches

- There is damping due to only space charge
- Space-charge tune spread due to longitudinal bunch profile





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Landau Damping in beams

- Waves: collective oscillations in a medium of particles
- Wave ↔ particles collisionless interaction
- Dispersion relation and Stability Diagram
- Decoherence is a result of Phase-Mixing, Landau damping, etc
- Collective Frequency ↔ Tune Spread
- Tune spreads of different nature produce different types of Landau damping
- A tune spread does not automatically means Landau Damping
- A special role of Space-Charge as a provider of a tune spread and the wave↔particles interaction