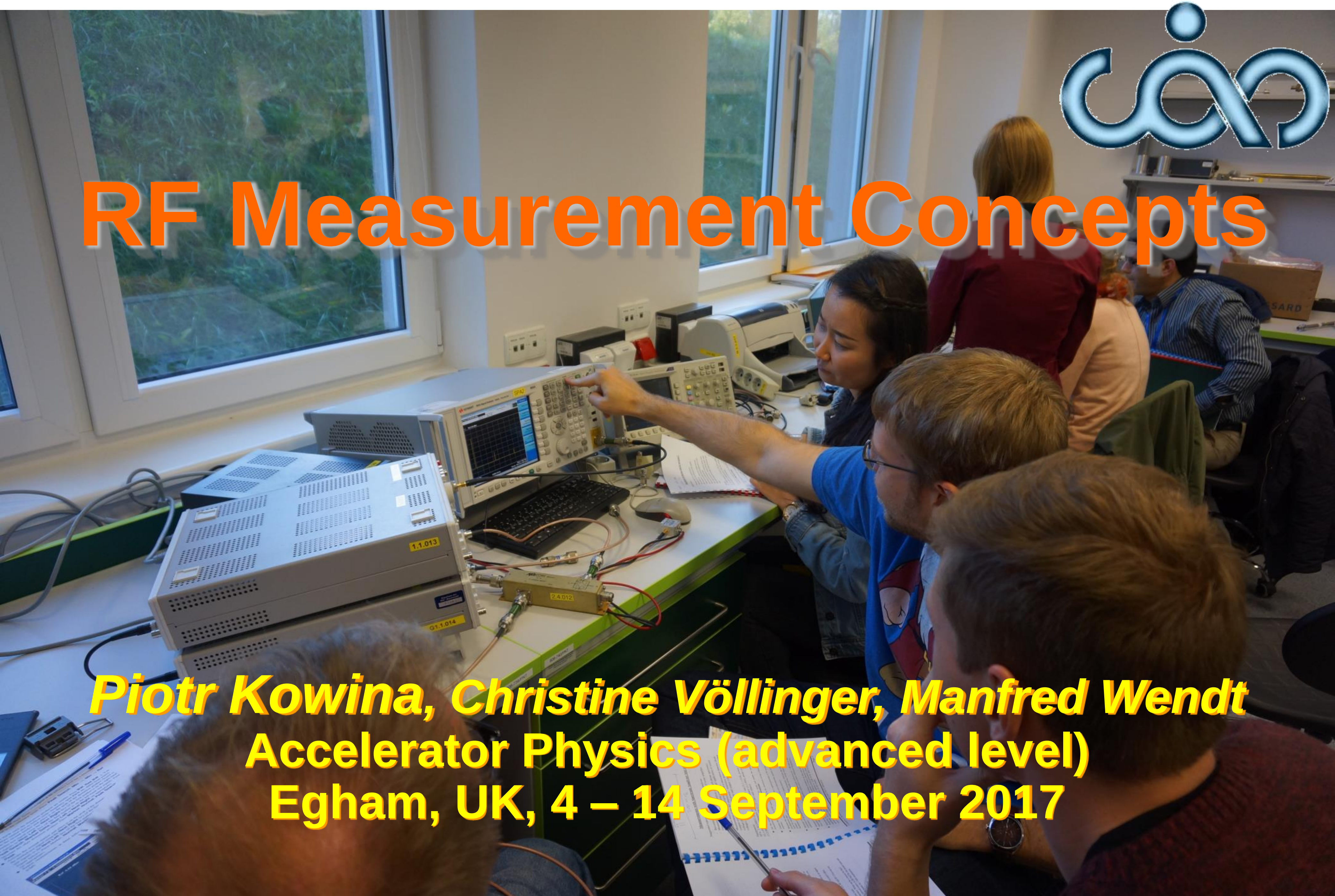

RF Measurement Concepts

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Accelerator Physics (advanced level)

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- Introduction
- RF measurement methods – some history and overview
- Super-heterodyne concept and its application(s)
- Voltage Standing Wave Ratio (VSWR)
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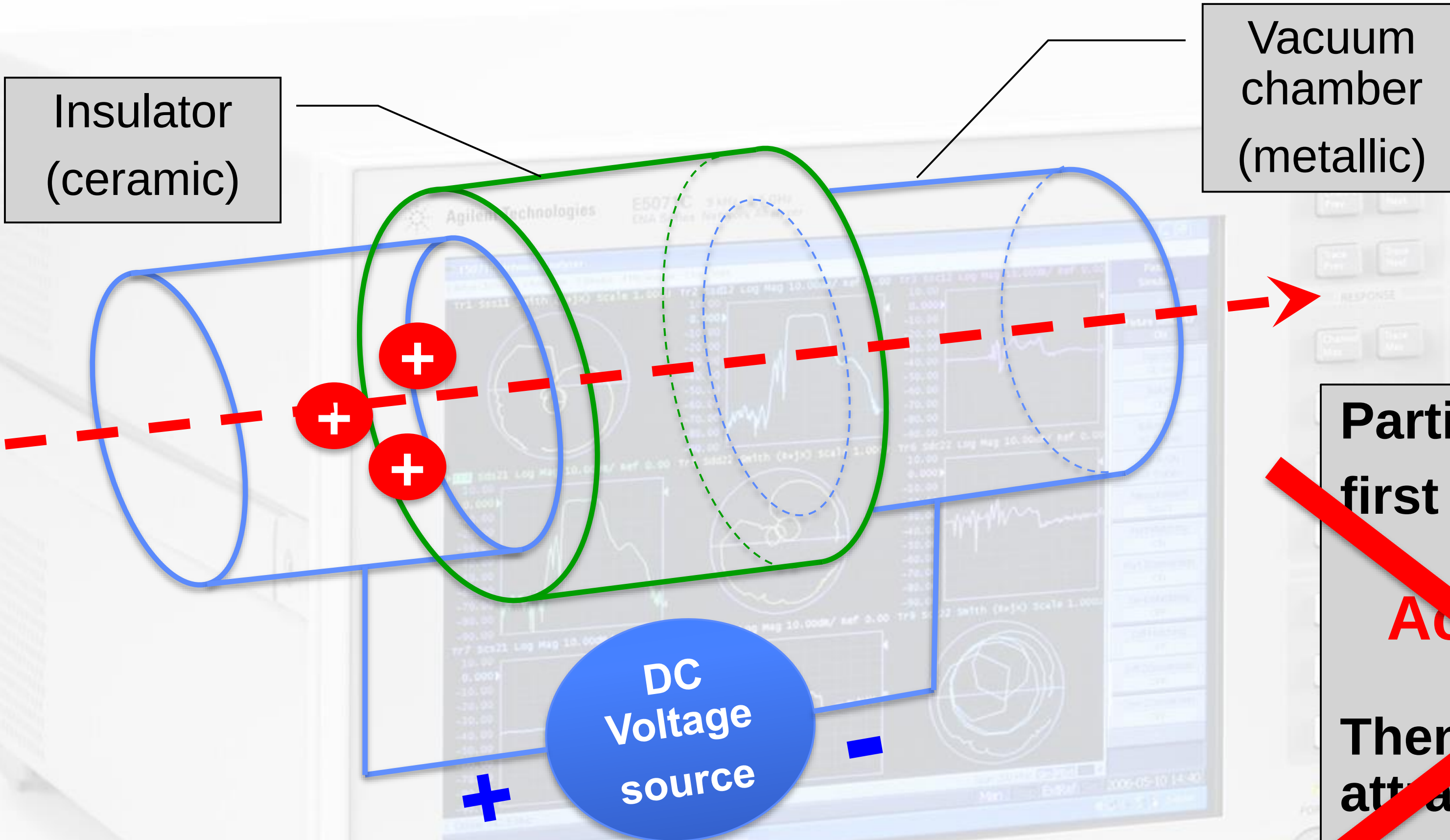
- The *Smith* chart and its applications
- Appendices

Introduction – DC Acceleration



Particles get first attracted:

Acceleration

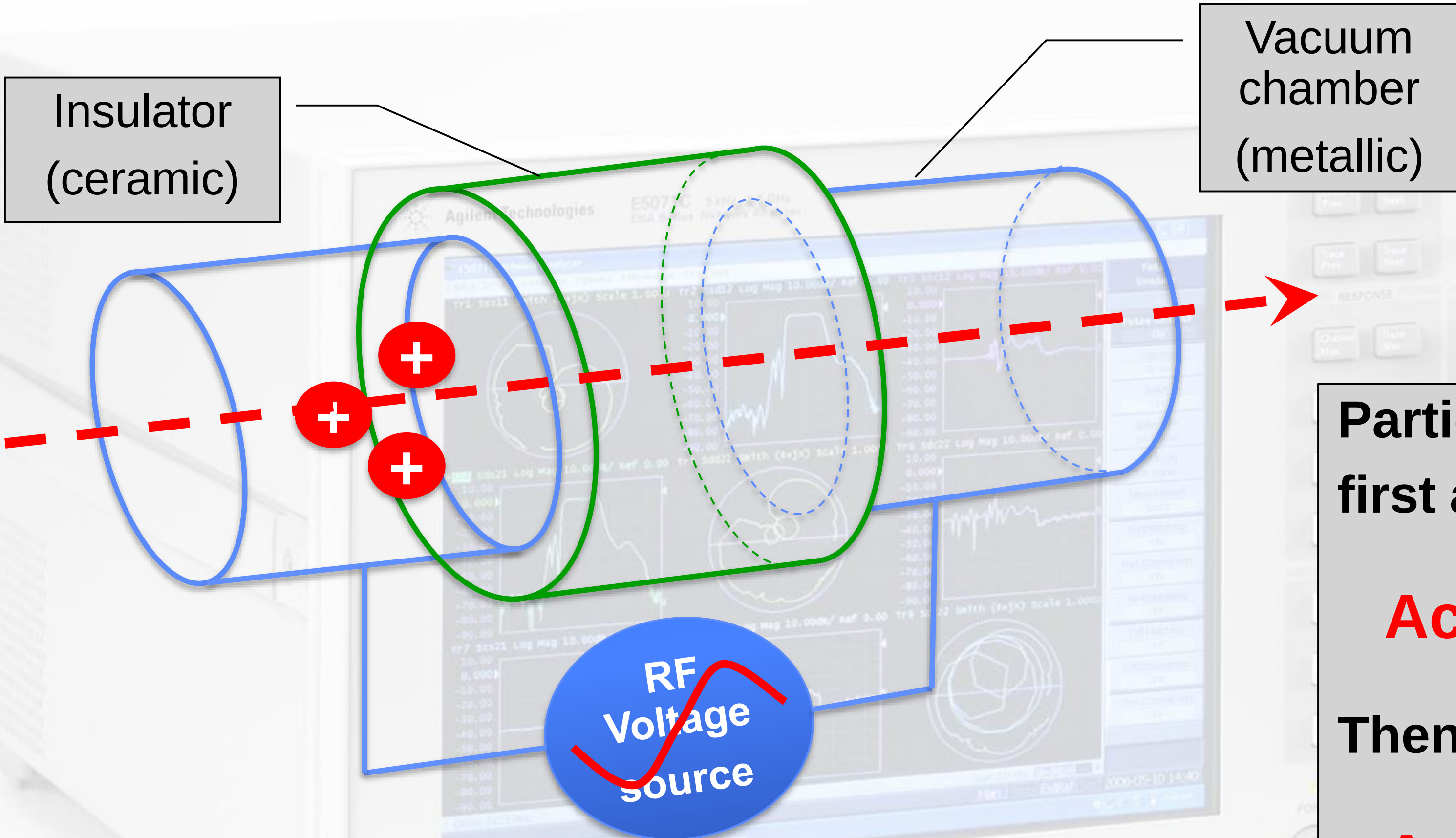
Then again attracted:

Deceleration

Net result:

No Acceleration

Introduction – RF Acceleration



Particles get first attracted:

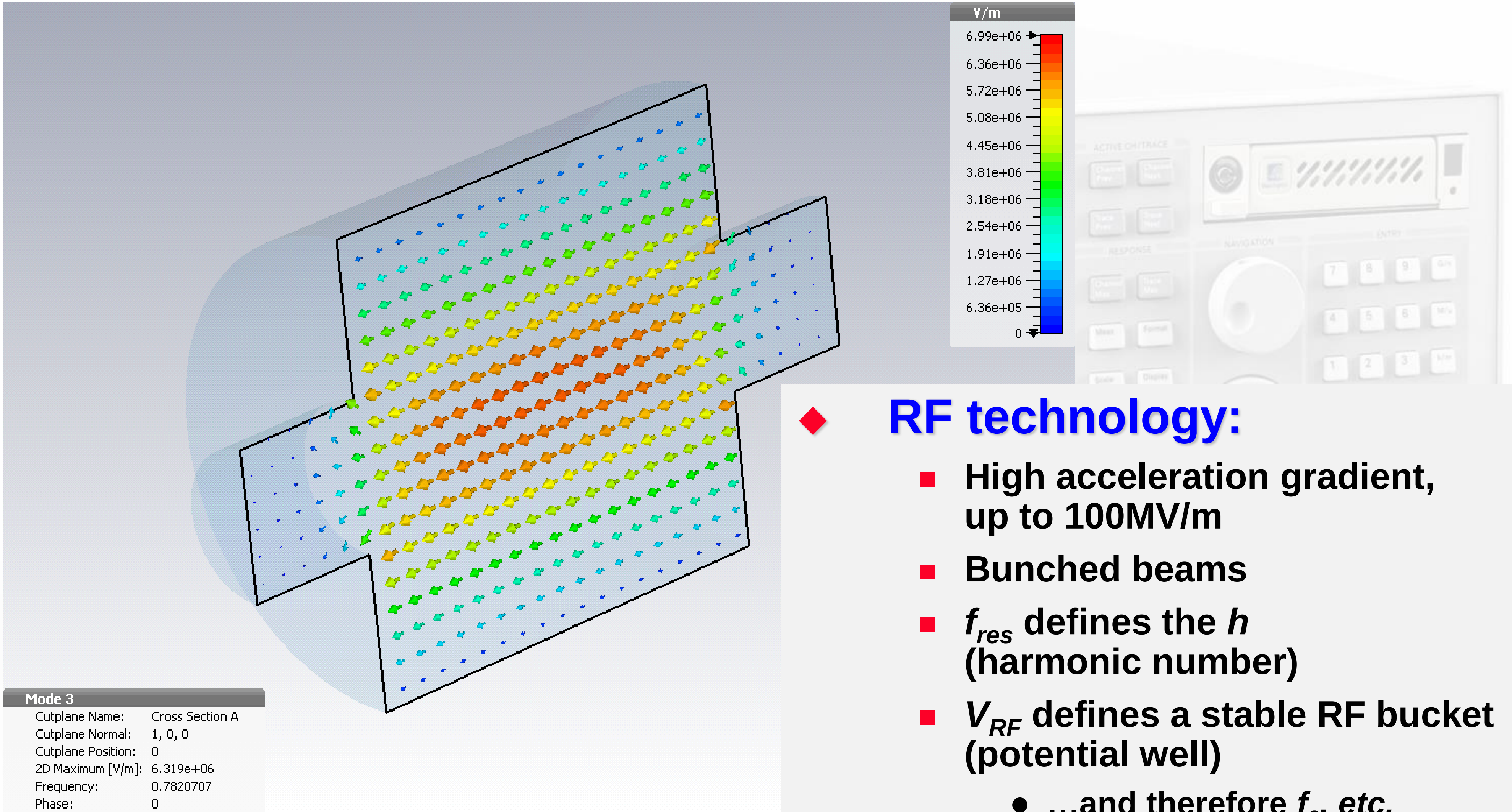
Acceleration

Then repelled:

Acceleration

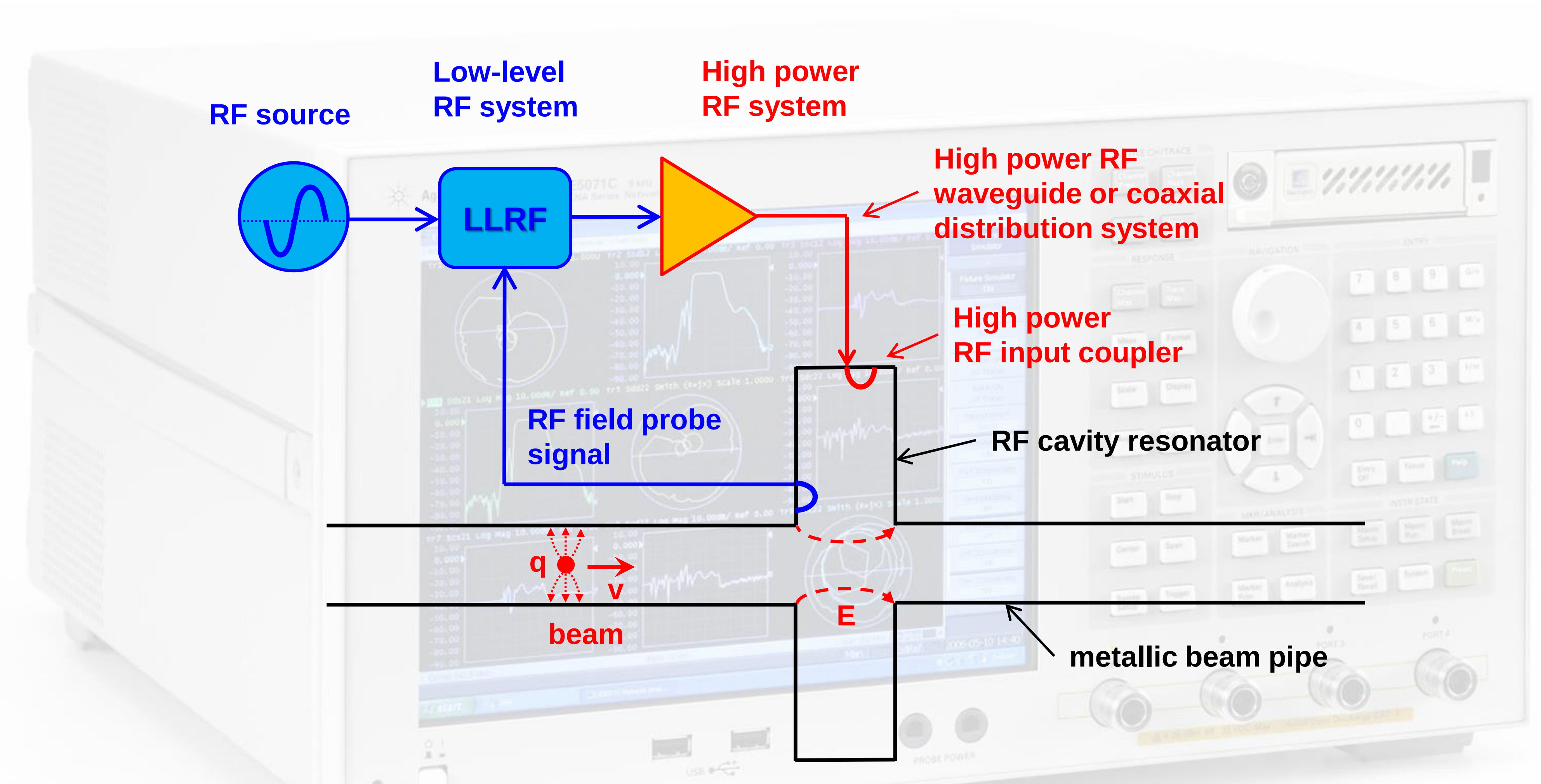
Attention:
It is almost never a good idea to locate an unshielded ceramic gap in a beam pipe!

Introduction – Resonant Cavity



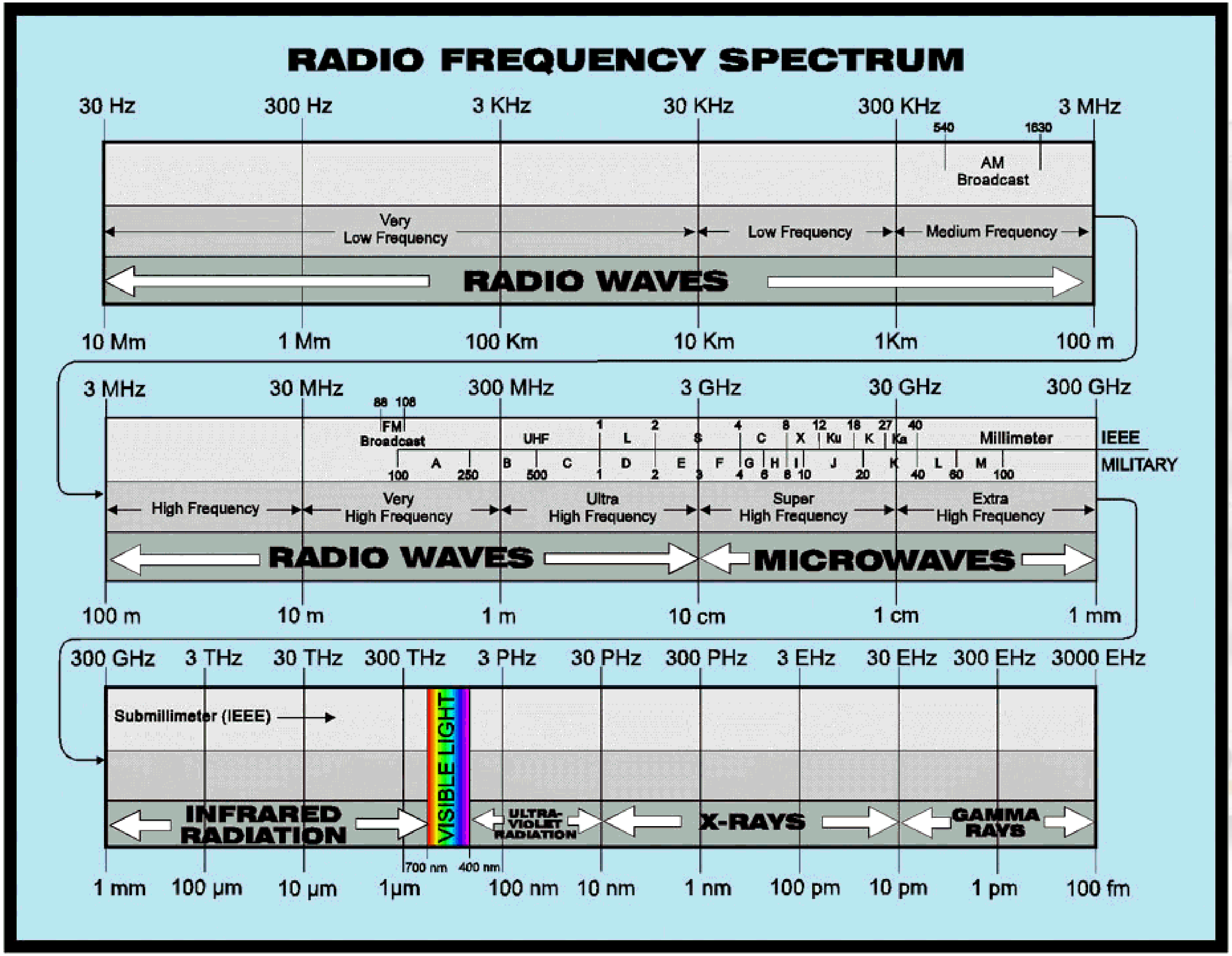
- **RF technology:**
  - **High acceleration gradient, up to 100MV/m**
  - **Bunched beams**
  - ***f_{res}* defines the *h* (harmonic number)**
  - ***V_{RF}* defines a stable RF bucket (potential well)**
    - ***…and therefore f_s, etc.***

Introduction – Simple RF System



Things get a bit more complicated in the real world: pulsed power RF, multi-cell resonators or traveling wave structures, non-relativistic beams, HOM's, etc.

Introduction – What are Radio Frequencies?



Free space wavelength:

$$\lambda = \frac{c}{f}$$

We care about RF concepts if the physical dimensions of an apparatus is $> \lambda/10$

RF Measurements Methods (1)

There are many ways to observe RF signals.
Here some typical tools:

- **Oscilloscope: to observe signals in time domain**
  - **periodic signals**
  - **burst and transient signals with arbitrary waveforms**
  - **application: direct observation of signals from a beam pick-up, test generator and other sources, shape of a waveform, evaluation of non-linear effects, etc.**
- **Spectrum analyzer: to observe signals in frequency domain**
  - **sweeps in equidistant steps through a given frequency range**
  - **application: observation of spectrum from the beam, or from a signal generator or RF source, or the spectrum emitted from an antenna to locate EMI issues in the accelerator tunnel, etc.**
  - **Requires periodic signals**

RF Measurements Methods (2)

- **Dynamic signal analyzer (FFT analyzer)**
  - **Acquires the signal, often after down-conversion, in time domain by fast sampling**
  - **Further numerical treatment in digital signal processors (DSPs)**
  - **Spectrum calculated using Fast Fourier Transform (FFT)**
  - **Combines features of an oscilloscope and a spectrum analyzer: Signals can be observed directly in time or in frequency domain**
  - **Contrary to the SA, also the spectrum of non-periodic signals and transients can be measured**
  - **Application: Observation of tune sidebands, transient behavior of a phase locked loop, single pass beam signal spectrum, etc.**
  - **Digital oscilloscopes and FFT analyzers share similar technologies, i.e. fast sampling and digital signal processing, and therefore can provide similar measurement options**

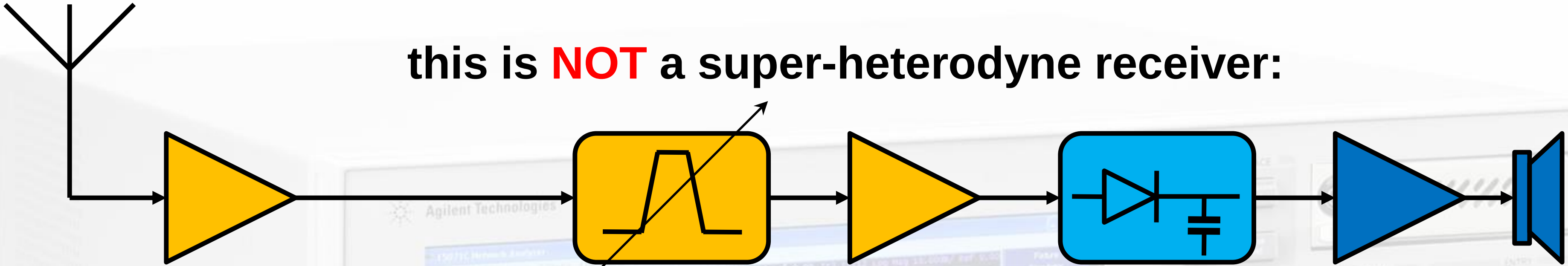
RF Measurements Methods (2)

Tools to characterize RF components and sub-systems:

- **Coaxial (or waveguide) measurement transmission-line**
  - **For study and illustration purposes only – not anymore used in today's RF laboratory environment.**
- **Vector Network Analyzer (VNA)**
  - **Excites a *Device Under Test* (DUT, e.g. circuit, antenna, amplifier, etc.) network at a given *Continuous Wave* (CW) frequency, and measures the response in magnitude and phase => determines the S-parameters**
    - **What are S-parameters?!**
  - **Covers a selectable frequency range by measuring step-by-step at subsequent frequency points (similar to the spectrum analyzer)**
  - **Applications: characterization of passive and active RF components, *Time Domain Reflectometry* (TDR) by Fourier transformation of the reflection response, etc.**
  - **The VNA is the most versatile and comprehensive tool in the RF laboratory**

The Super-Heterodyne Receiver (1)

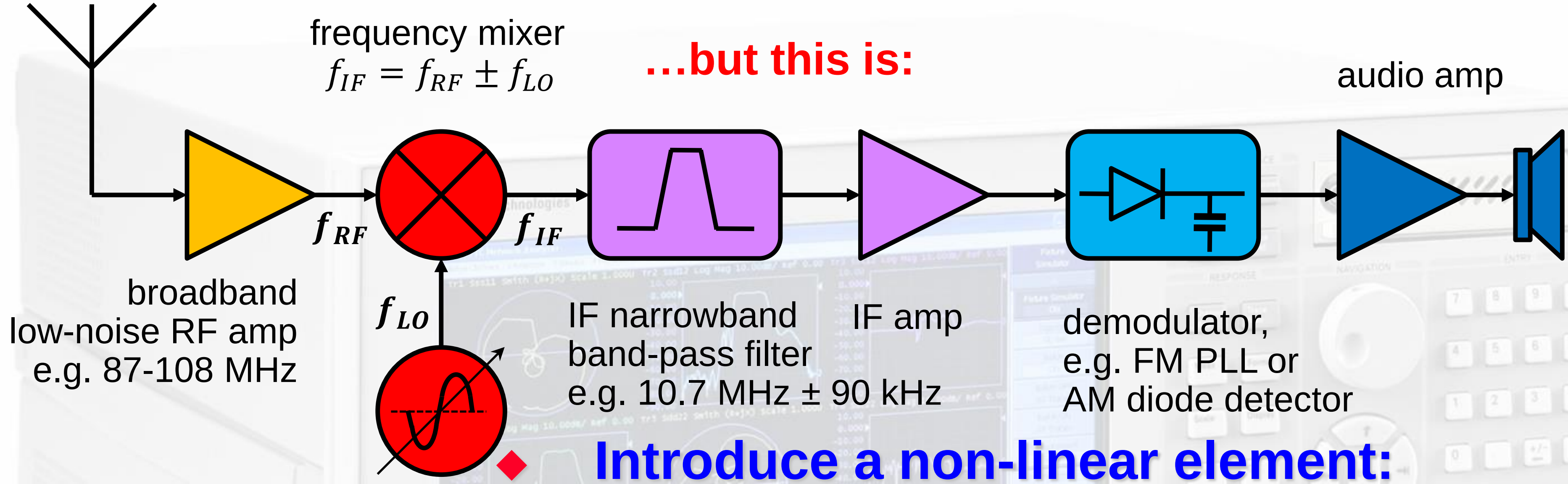
this is NOT a super-heterodyne receiver:



- **…or: 'How does a "traditional" analog radio works?**
  - **It was, and still is, difficult to make precisely tunable narrowband, band-pass filters for high frequencies (~100 MHz)!!**
  - **high frequency low-noise amplifiers are expensive!**
  - **high frequency demodulators are not trivial.**

  - **direct detection of radio and RF signals is challenging!**

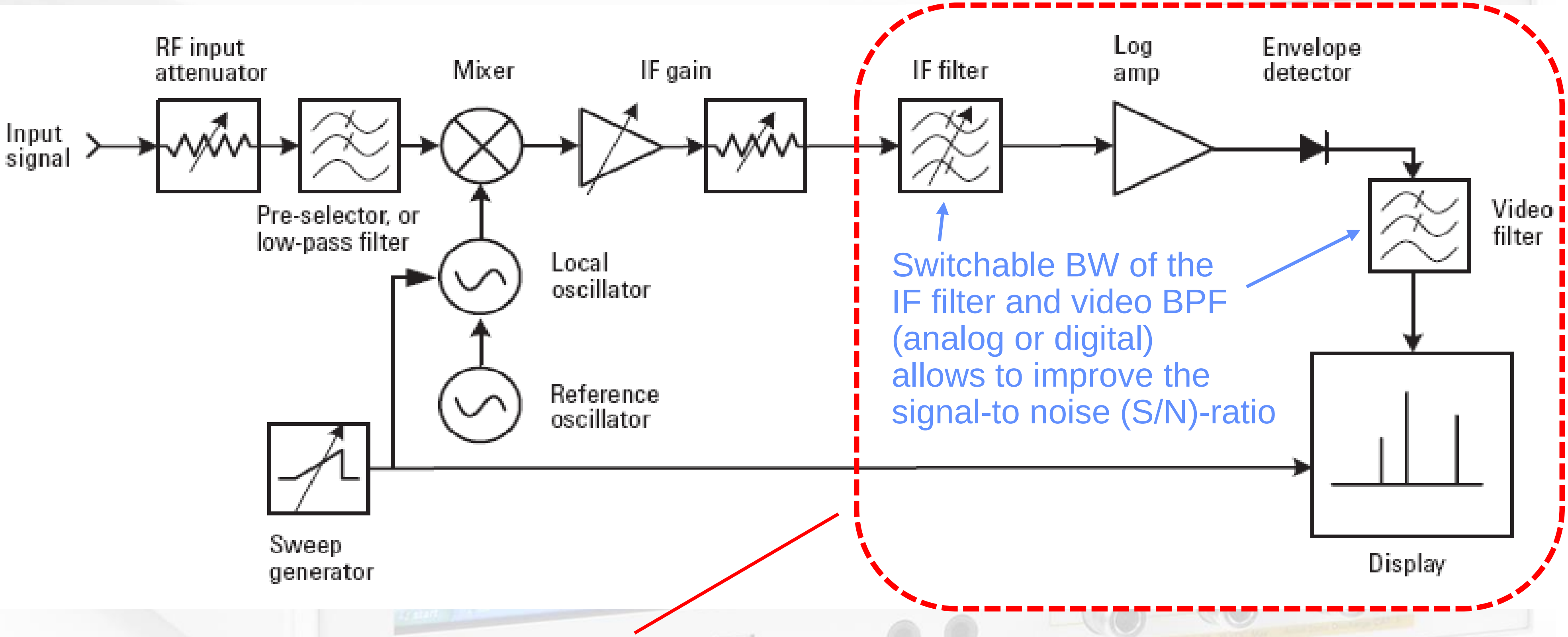
The Super-Heterodyne Receiver (2)



- **Introduce a non-linear element: the (frequency) mixer!**
  - **”down-convert” the RF band to a fixed ”intermediate” frequency (IF)** $f_{IF} = f_{RF} \pm f_{LO}$
  - **requires a tunable local oscillator (LO)**
  - **well manageable IF section:**
    - **narrowband band-pass filter(s) (BPF) and amplifier(s)**
  - **RF telecommunication standard**
  - **Often multiple mixing stages are used in modern RF instruments, line spectrum analyzers**

Simplified Spectrum Analyzer

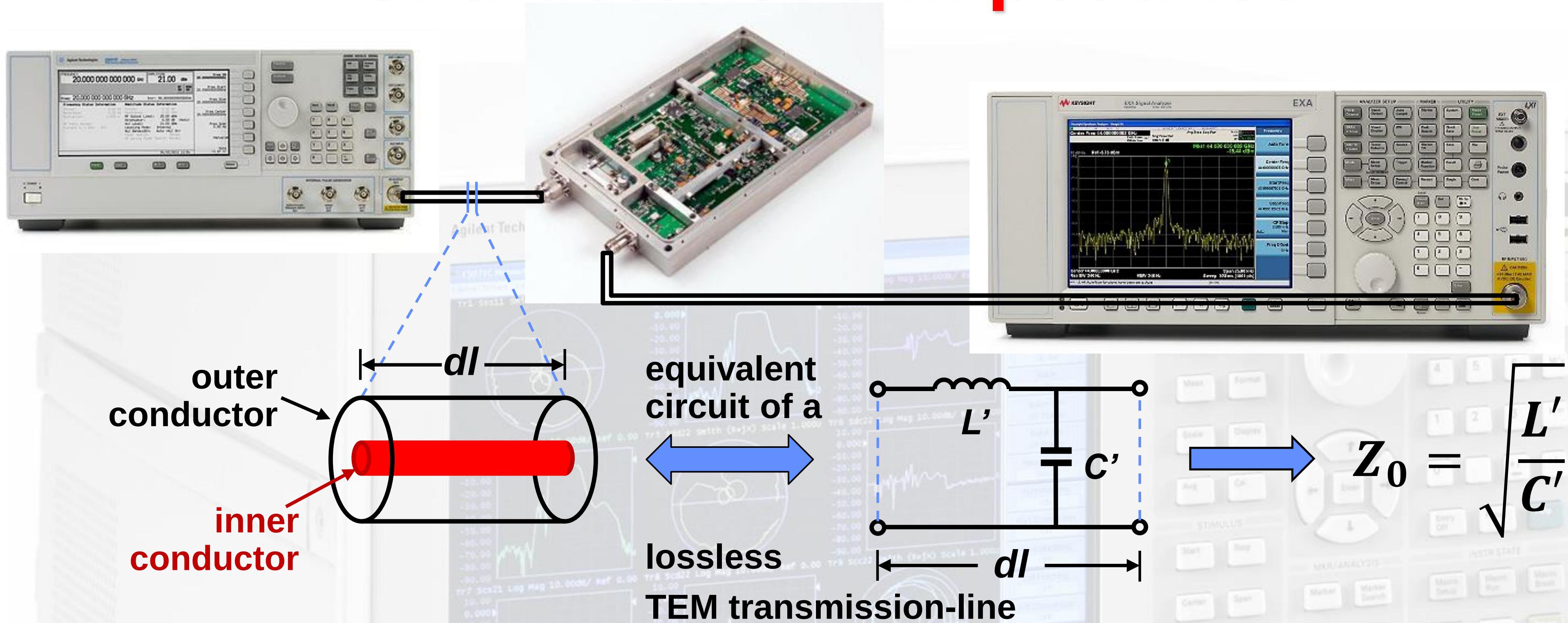
- **based on the super-heterodyne principle**



Today, the IF, demodulation, video and display sections of a spectrum analyzer are realized digitally

- **Requires an analog-digital converter (ADC) with sufficient dynamic range**

Characteristic Impedance

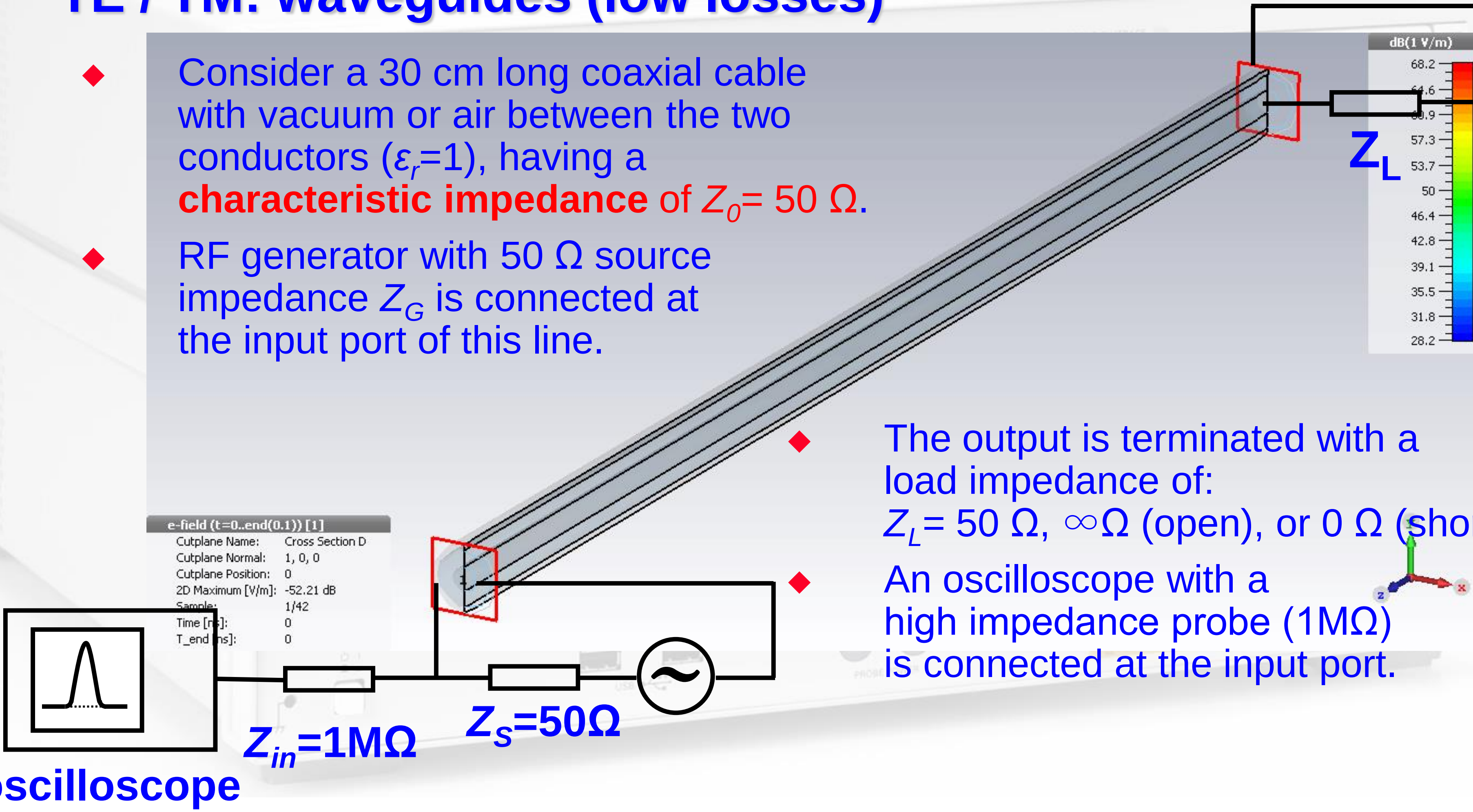


$$Z_0 = \sqrt{\frac{L'}{C'}}$$

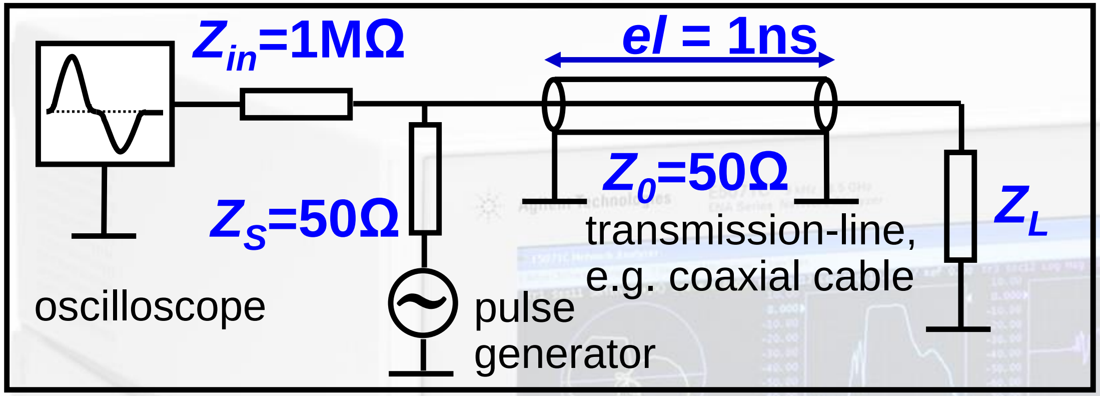
- **The reference impedance Z_0 in a RF system is defined by the characteristic impedance of the interconnect cables**
  - **often coaxial cables of Z_0=50Ω (compromise: high voltage / high power handling)**
- **The characteristic impedance of a TEM transmission-line is defined by the cross-section geometry**
  - **Ratio of H- and E-field, represented by *L’* [H/m] and *C’* [F/m] in the equivalent circuit of a line segment *dl***
  - **The characteristic impedance Z_0 has the unit Ohm [Ω]**

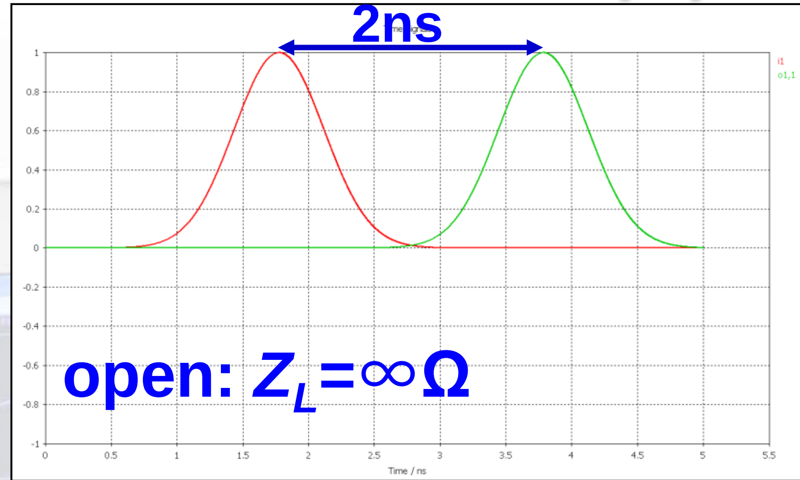
Transmission-lines in Time Domain (1)

- **TEM: coaxial cables, striplines, micro-striplines, etc.**
- **TE / TM: waveguides (low losses)**
  - Consider a 30 cm long coaxial cable with vacuum or air between the two conductors (ε_r=1), having a **characteristic impedance** of Z_0= 50 Ω.
  - RF generator with 50 Ω source impedance Z_G is connected at the input port of this line.
  - The output is terminated with a load impedance of: Z_L= 50 Ω, ∞Ω (open), or 0 Ω (short)
  - An oscilloscope with a high impedance probe (1MΩ) is connected at the input port.

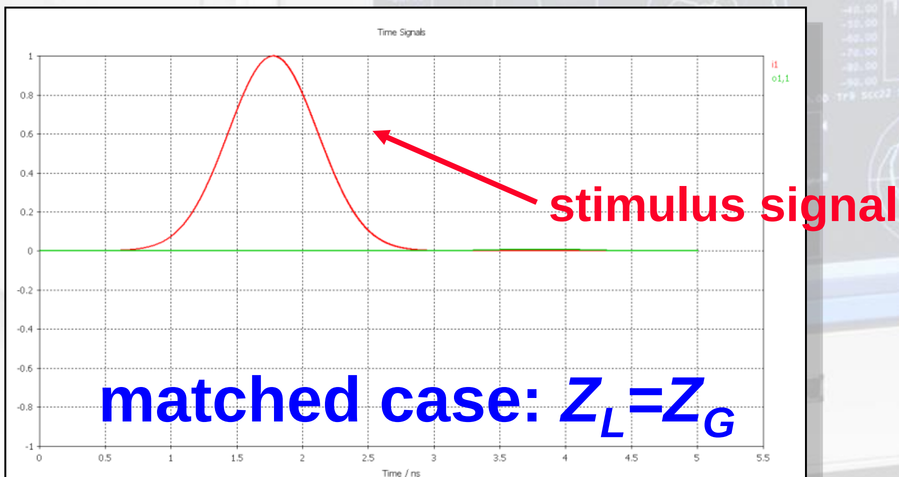


Transmission-lines in Time Domain (2)

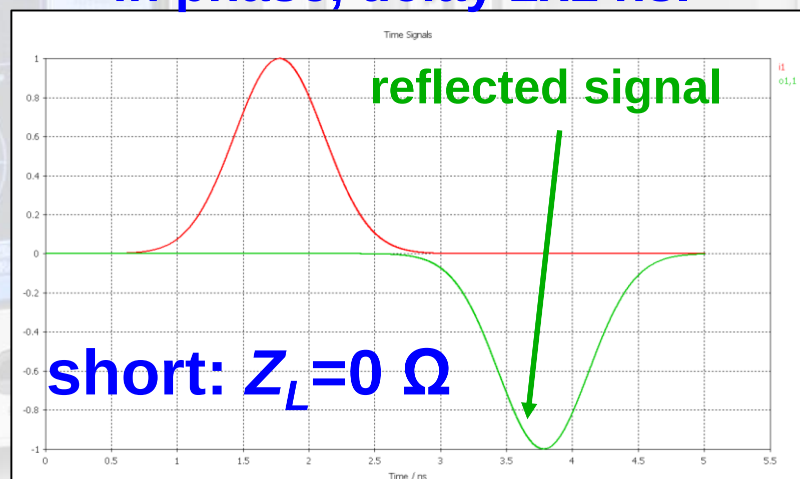




total reflection; reflected signal in phase, delay 2x1 ns.

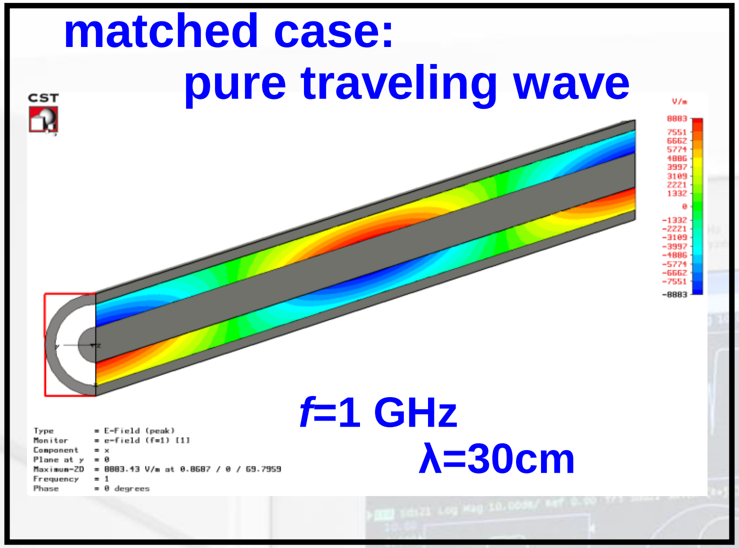


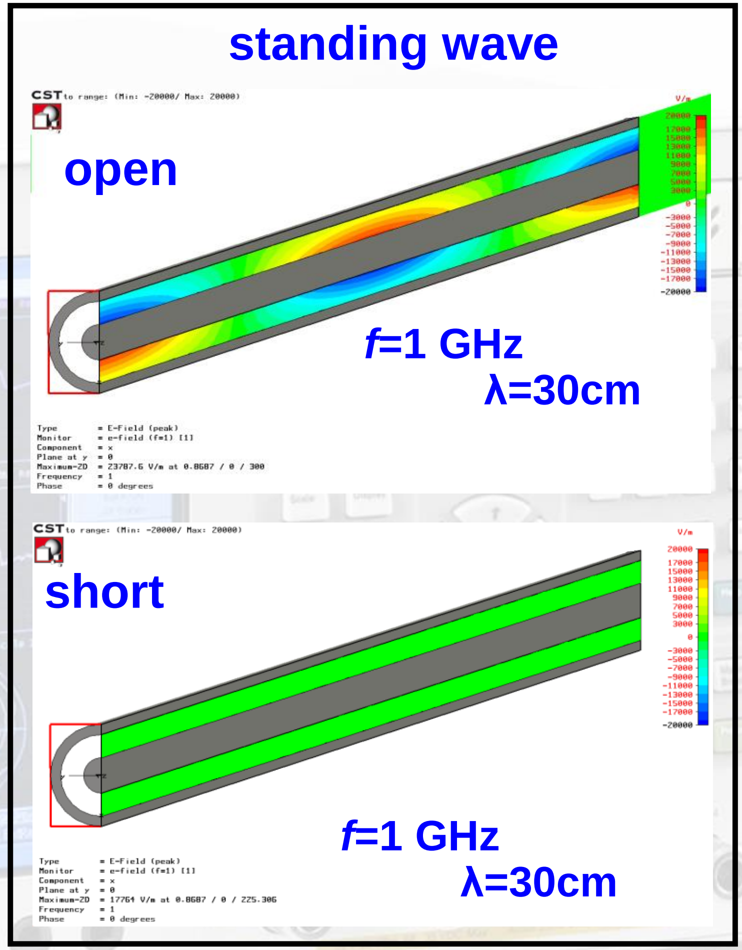
no reflection



total reflection; reflected signal in contra phase

Transmission-lines in Frequency Domain





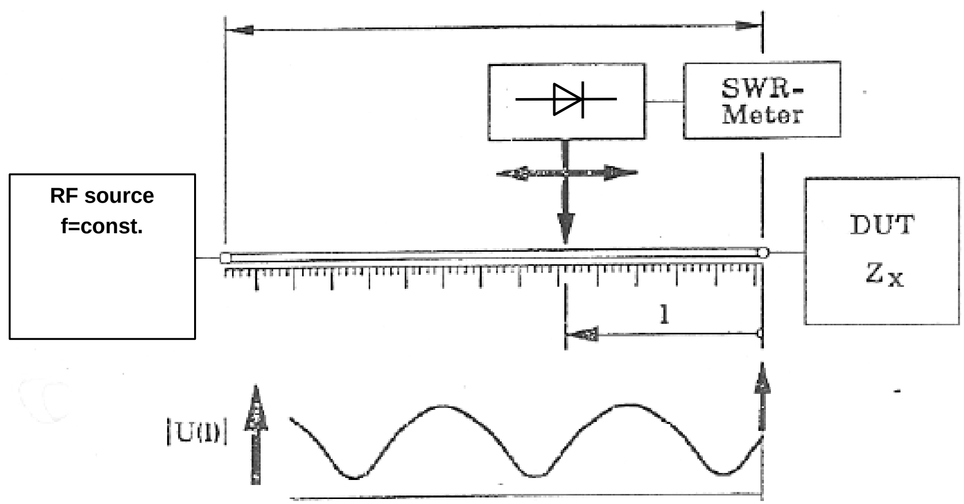
- **Standing and traveling waves:**
  - **The patterns for the short and open case are equal; only the phase is opposite, which correspond to different position of nodes.**
  - **In case of perfect matching:**
    - **traveling wave only.**
  - **Otherwise:**
    - **mixture of traveling and standing waves.**

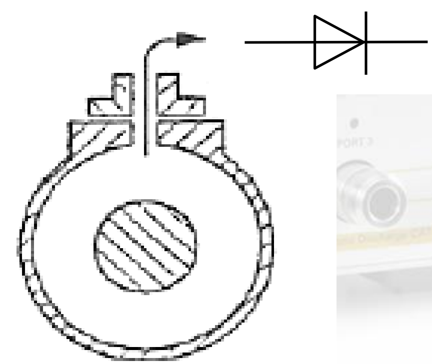
Caution: the color coding corresponds to the radial electric field strength – these are not scalar equipotential lines, which are anyway not defined for time varying fields

Voltage Standing Wave Ratio VSWR (1)

- **On a transmission-line (single frequency, CW):**
  - Superposition of forward *a* (E^{inc}) and backward *b* (E^{refl}) traveling waves $\Rightarrow$ standing waves
- **Slotted coaxial air-line is used as standing wave detector**
  - Probes the radial electric field along the slotted line.
  - Measurement of E-field minima's E_{min} and maxima's E_{max} with a diode detector, thus detect $|V_{min}|$ and $|V_{max}|$ along the line.
  - Evaluate the reflection coefficient Γ of a DUT of unknown Z_L at the end of the line

$$\Gamma = \frac{E^{\mathrm{refl}}}{E^{\mathrm{inc}}} = \frac{Z_L - Z_0}{Z_L + Z_0}$$





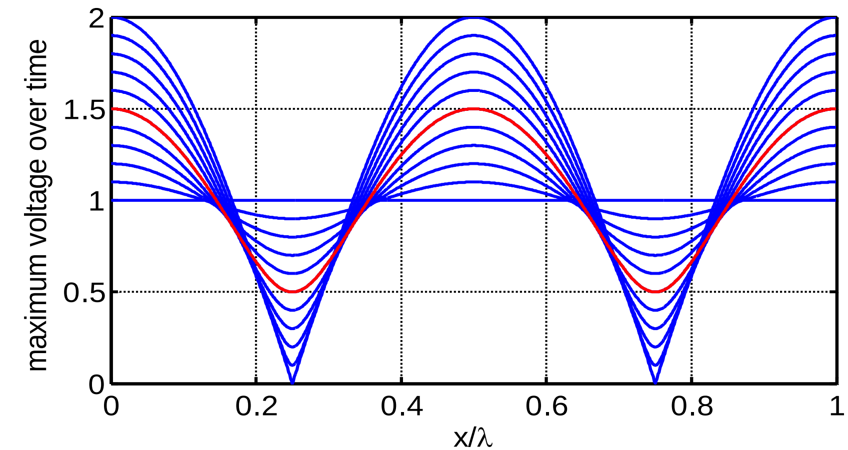
Voltage Standing Wave Ratio VSWR (2)

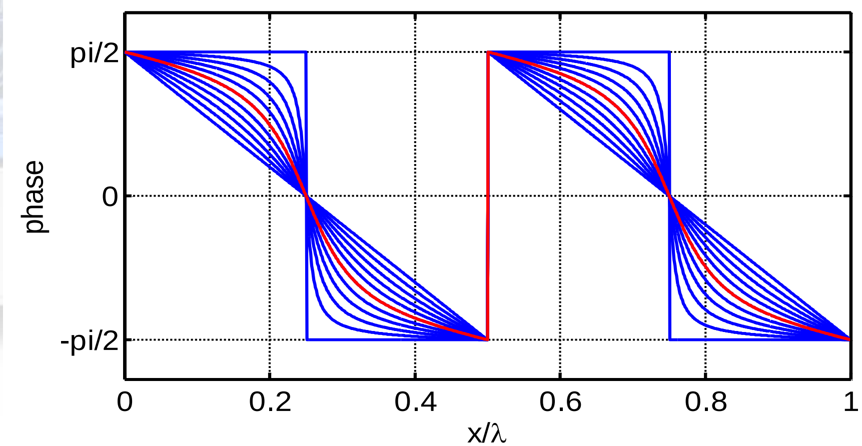
- **The VSWR is defined as:**

$$VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{|a| + |b|}{|a| - |b|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

  - **The phase of the detected E-field along the lossless coaxial line is purged by the diode detection.**
    - **Requires a mixer as detector!**

Γ	Return Loss [dB]	*VSWR* = Z_L/Z_0	Refl. Power $1-|\Gamma|^2$
0.0	∞	1.00	1.00
0.1	20	1.22	0.99
0.2	14	1.50	0.96
0.3	10	1.87	0.91
0.4	8	2.33	0.84
0.5	**6**	**3.00**	**0.75**
0.6	4	4.00	0.64
0.7	3	5.67	0.51
0.8	2	9.00	0.36
0.9	1	19	0.19
1.0	0	∞	0.00



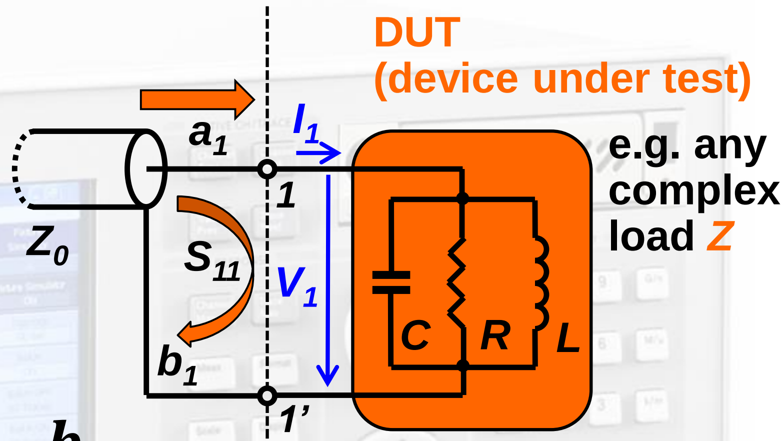


S-Parameters – Introduction (1)

- **Light falling on a car window:**
  - Some parts of the incident light is reflected (you see the mirror image)
  - Another part of the light is transmitted through the window (you can still see inside the car)
- **Optical reflection and transmission coefficients of the window glass define the ratio of reflected and transmitted light**
- **Similar:**
  Scattering (S-) parameters of an *n*-port electrical network (DUT) characterize reflected and transmitted (power) waves

S-Parameters – Introduction (2)

- **Electrical networks**
  - 1…*n*-ports circuits
  - Defined by voltages $V_n(\omega)$ or $v_n(t)$ and currents $I_n(\omega)$ or $i_n(t)$ at the ports
  - Characterized by circuit matrices, e.g. ABCD, Z, Y, H, etc.
- **RF networks**
  - 1…n-port RF DUT circuit or subsystem, e.g. filter, amplifier, transmission-line, hybrid, circulator, resonator, etc.
  - Defined by incident $a_n(\omega, s)$ and reflected waves $b_n(\omega, s)$ at a reference plane *s* (physical position) at the ports
  - Characterized by a scattering parameter (S-parameter) matrix of the reflected and transmitted power waves
  - Normalized to a reference impedance $\sqrt{Z_0}$ of typically $Z_0 = 50\ \Omega$



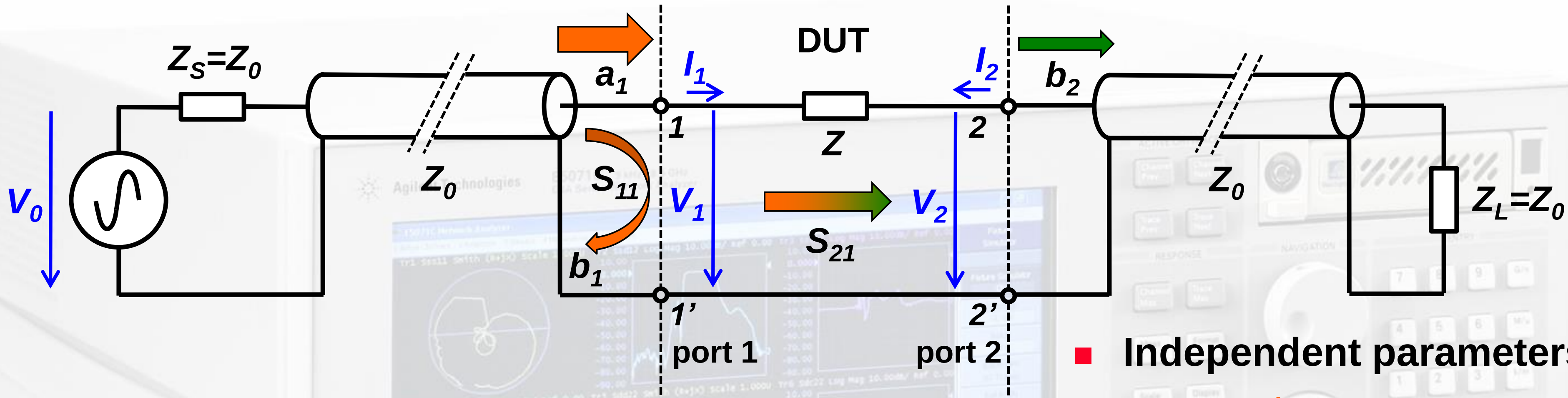
$$S_{11} = \frac{b_1}{a_1} = \Gamma$$

1-1' reference plane (port 1)

1-port DUT example

- S-Parameters allow to characterize the DUT with the measurement equipment to be located at some distance
- All high frequency effects of distributed elements are taken into account with respect to the reference plane

S-Parameters – Example: 2-port DUT



- **Independent parameters:**

$$a_1 = \frac{V_1^{inc}}{\sqrt{Z_0}} = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}}$$

$$a_2 = \frac{V_2^{inc}}{\sqrt{Z_0}} = \frac{V_2 + I_2 Z_0}{2\sqrt{Z_0}}$$

- **Dependent parameters:**

$$b_1 = \frac{V_1^{refl}}{\sqrt{Z_0}} = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}}$$

$$b_2 = \frac{V_2^{refl}}{\sqrt{Z_0}} = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}}$$

Analysis of the forward S-parameters:

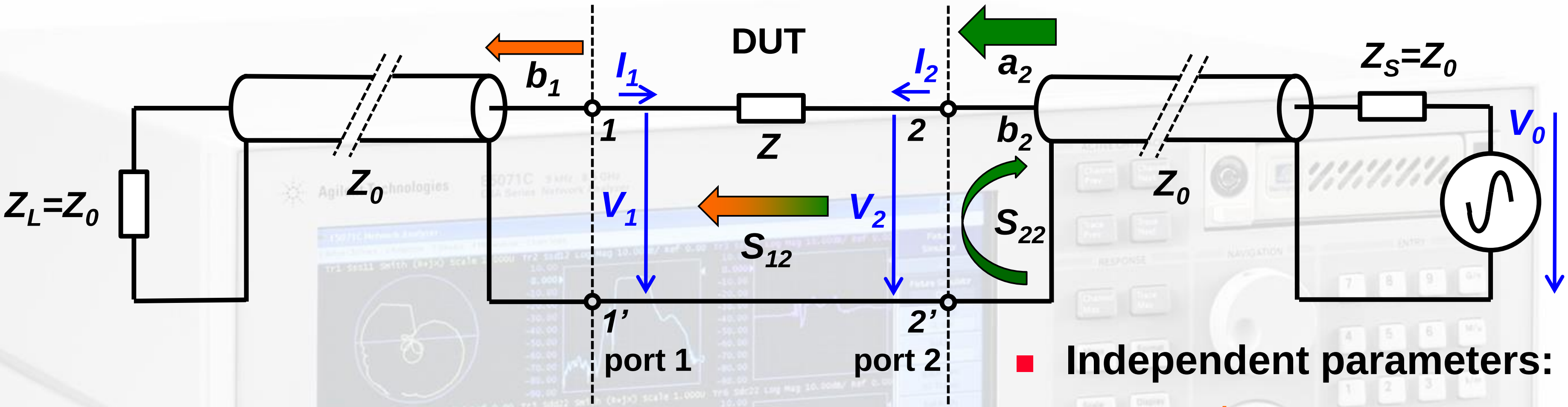
$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \equiv \text{input reflection coefficient}$$

$$(Z_L = Z_0 \Rightarrow a_2 = 0)$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \equiv \text{forward transmission gain}$$

- **Examples of 2-ports DUT: filters, amplifiers, attenuators, transmission-lines (cables), etc.**
- **ALL ports ALWAYS need to be terminated in their characteristic impedance!**

S-Parameters – Example: 2-port DUT



- **Analysis of the reverse S-parameters:**

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \equiv \text{output reflection coefficient}$$

$$(Z_L = Z_0 \Rightarrow a_1 = 0)$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \equiv \text{backward transmission gain}$$

- *n*-port DUTs still can be fully characterized with a 2-port VNA, but again: **don't forget to terminate unused ports!**

- **Independent parameters:**

$$a_1 = \frac{V_1^{inc}}{\sqrt{Z_0}} = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}}$$

$$a_2 = \frac{V_2^{inc}}{\sqrt{Z_0}} = \frac{V_2 + I_2 Z_0}{2\sqrt{Z_0}}$$

- **Dependent parameters:**

$$b_1 = \frac{V_1^{refl}}{\sqrt{Z_0}} = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}}$$

$$b_2 = \frac{V_2^{refl}}{\sqrt{Z_0}} = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}}$$

S-Parameters – Definition (1)

- **Linear equations for the 2-port DUT:**

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

  - **with:**

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \equiv \text{input reflection coefficient}$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \equiv \text{output reflection coefficient}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \equiv \text{forward transmission gain}$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \equiv \text{backward transmission gain}$$

S-Parameters – Definition (2)

- **Reflection coefficient and impedance at the *n*th-port of a DUT:**

$$S_{nn} = \frac{b_n}{a_n} = \frac{\frac{V_n}{I_n} - Z_0}{\frac{V_n}{I_n} + Z_0} = \frac{Z_n - Z_0}{Z_n + Z_0} = \Gamma_n$$

$$Z_n = Z_0 \frac{1 + S_{nn}}{1 - S_{nn}} \text{ with } Z_n = \frac{V_n}{I_n} \text{ being the input impedance at the } n^{th} \text{port}$$

- **Power reflection and transmission for a *n*-port DUT**

$$|S_{nn}|^2 = \frac{\text{power reflected from port } n}{\text{power incident on port } n}$$

$$|S_{nm}|^2 = \text{transmitted power between ports } n \text{ and } m$$

with all ports terminated in their characteristic impedance Z_0 and $Z_S=Z_0$

Here the US notion is used, where power = $|a|^2$.
European notation (often): power = $|a|^2/2$
These conventions have no impact on the S-parameters, they are only relevant for absolute power calculations

The Scattering Matrix (1)

- **Waves traveling towards the *n*-port:** $(\boldsymbol{a}) = (\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_2}, \ldots \boldsymbol{a_n})$
- **Waves traveling away from the *n*-port:** $(\boldsymbol{b}) = (\boldsymbol{b_1}, \boldsymbol{b_2}, \boldsymbol{b_2}, \ldots \boldsymbol{b_n})$

- The relation between a_i and b_i (i = 1..n) can be written as a **system of n linear equations**
(a_i = the independent variable, b_i = the dependent variable)

$$
\begin{array}{ll}
\text{one-port} & b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + S_{14}a_4 + \ldots \\
\text{two-port} & b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + S_{44}a_4 + \ldots \\
\text{three-port} & b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3 + S_{44}a_4 + \ldots \\
\text{four-port} & b_4 = S_{41}a_1 + S_{42}a_2 + S_{33}a_3 + S_{44}a_4 + \ldots
\end{array}
$$

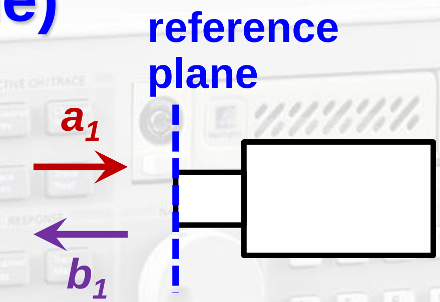
- **in compact matrix form follows**

$$(\boldsymbol{b}) = (\boldsymbol{S})(\boldsymbol{a})$$

The Scattering Matrix (2)

- The simplest form is a passive **1-port (2-pole)**

$$(S) = S_{11} \quad \Rightarrow \quad b_1 = S_{11} a_1$$

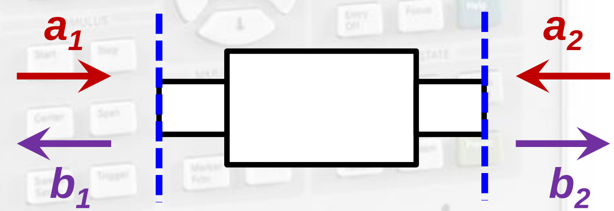


  - with the **reflection coefficient:**

$$\Gamma = S_{11} = \frac{b_1}{a_1}$$

- **2-port (4-pole) DUT:**

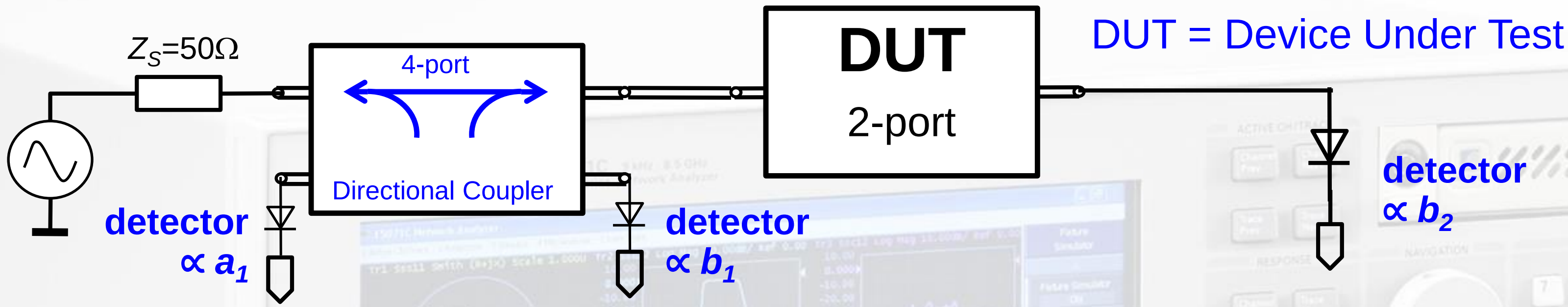
$$(S) = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \quad \Rightarrow \quad \begin{aligned} b_1 &= S_{11} a_1 + S_{12} a_2 \\ b_2 &= S_{21} a_1 + S_{22} a_2 \end{aligned}$$



  - An unmatched load, present at port 2 with a **reflection coefficient** Γ_{load} transfers to the input port as:

$$\Gamma_{in} = S_{11} + \frac{S_{21} \Gamma_{load} S_{12}}{1 - S_{22} \Gamma_{load}}$$

How to measure S-Parameters?



- **Performed in the frequency domain**
  - **Single or swept frequency generator, stand-alone or as part of a VNA or SA**
  - **Requires a directional coupler and RF detector(s) or receiver(s)**
- **Evaluate S_{11} and S_{21} of a 2-port DUT**
  - **Ensure $a_2=0$, i.e. the detector at port 2 offers a well matched impedance**
  - **Measure incident wave a1 and reflected wave b1 at the directional coupler ports and compute for each frequency**
  - **Measure transmitted wave b_2 at DUT port 2 and compute**

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

- **Evaluate S_{22} and S_{12} of the 2-port DUT**
  - **Perform the same methodology as above by exchanging the measurement equipment on the DUT ports**

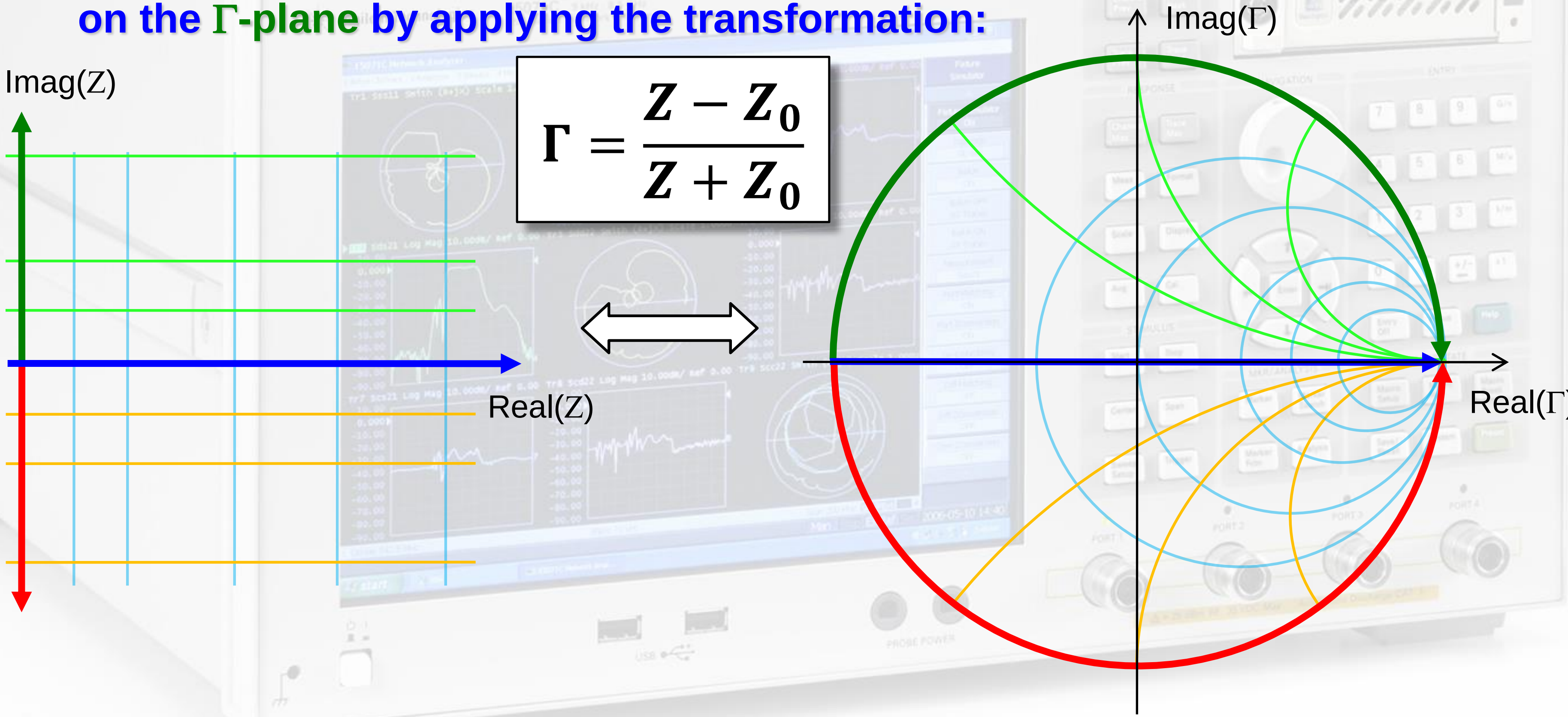
S-Parameters – Summary

- **Scattering parameters (S-parameters) characterize an RF component or system (DUT) by a matrix.**
  - ***nxn* matrix for *n*-port device**
  - **Based on incident (a_n) and reflected (b_n) power waves**
- **ALL ports need to be terminated in their characteristic (reference) impedance Z_0**
  - **For a proper S-parameter measurement or numerical computation all ports of the Device Under Test (DUT), including the generator port, must be terminated with their characteristic impedance to assure, waves traveling away from the DUT *(b_n-waves) are not reflected twice or multiple times, and convert into a_n-waves. (cannot be stated often enough…!)***
- **Typically S-parameters and DUT characteristics are ”measured” and characterized in the frequency domain**
  - **S-parameters, as wells as DUT circuit elements are described in complex notation with the frequency variable $\omega=2\pi f$**
  - **Frequency transformation (iDFT) allows time domain measurements with a ”modern” vector network analyzer (VNA).**

The *Smith* Chart (1)

- The *Smith* Chart (in impedance coordinates) represents the complex Γ-plane within the unit circle.
- It is a conformal mapping of the complex *Z*-plane on the Γ-plane by applying the transformation:

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$



- ⇒ the real positive half plane of *Z* is thus transformed (*Möbius*) into the interior of the unit circle!

The *Smith* Chart (2)

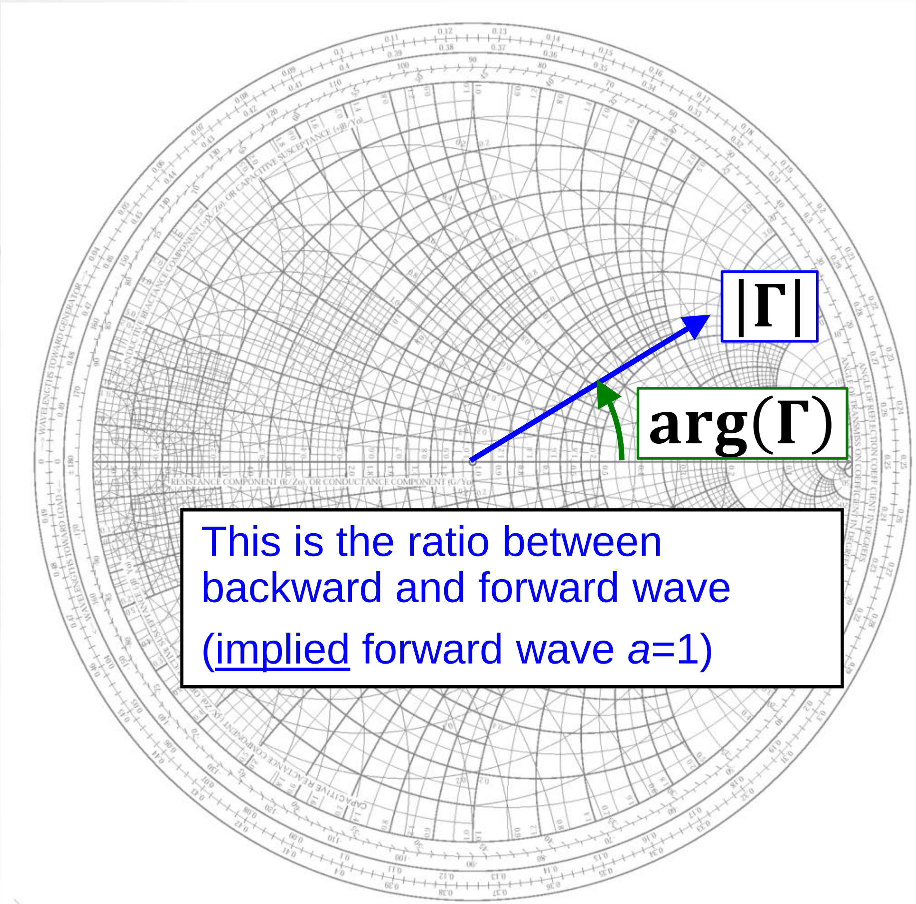
- **The Impedance Z is usually normalized** $$z = \frac{Z}{Z_0}$$ **to a reference impedance Z_0, typically the characteristic impedance of the coaxial cables of Z_0=50Ω.**
- **The normalized form of the transformation follows then as:**

$$\Gamma = \frac{z-1}{z+1} \quad \text{resp.} \quad \frac{Z}{Z_0} = z = \frac{1+\Gamma}{1-\Gamma}$$

<u>This mapping offers several practical advantages</u>:

- The diagram includes all "passive" impedances, i.e. those with positive real part, from zero to infinity in a handy format.
  - Impedances with negative real part ("active device", e.g. reflection amplifiers) would be outside the (normal) *Smith* chart.
- The mapping converts impedances or admittances into reflection factors and vice-versa. This is particularly interesting for studies in the radiofrequency and microwave domain where electrical quantities are usually expressed in terms of "incident" or "forward", and "reflected" or "backward" waves.
  - This replaces the notation in terms of currents and voltages used at lower frequencies.
- Also the reference plane can be moved very easily using the *Smith* chart.

The *Smith* Chart (3)



This is the ratio between backward and forward wave (implied forward wave a=1)

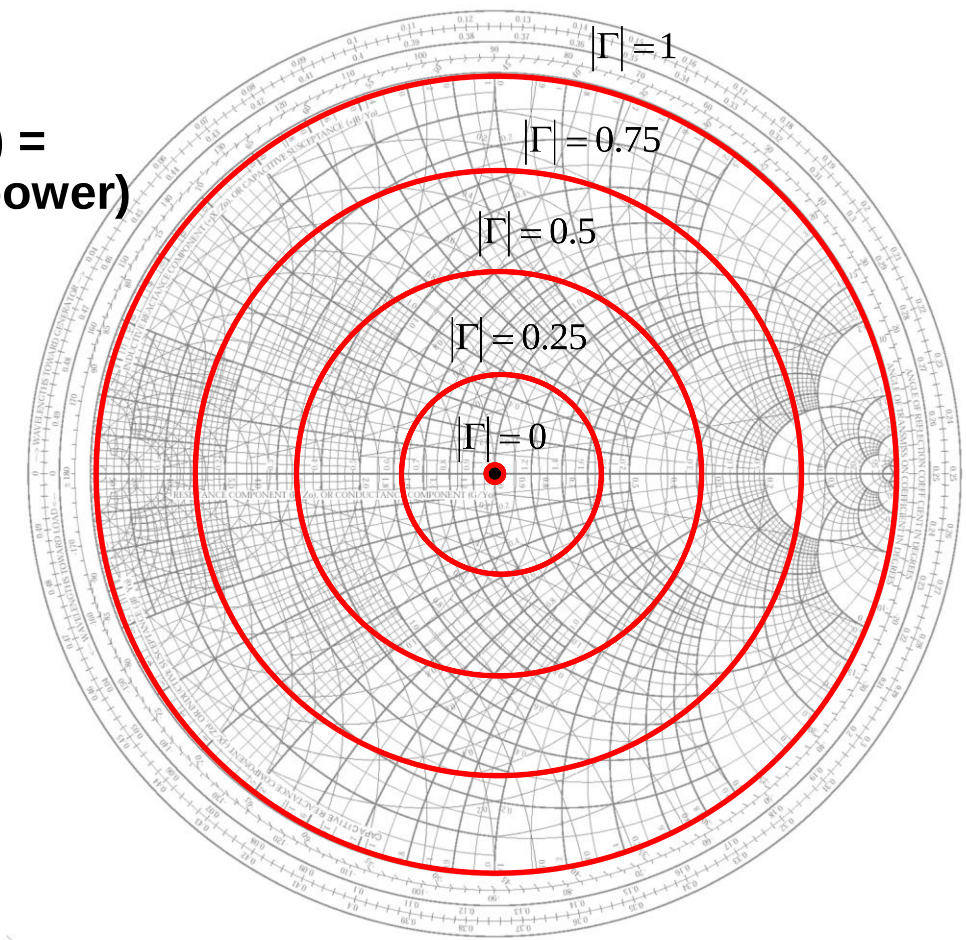
- **In the *Smith* chart, the complex reflection factor**

$$\Gamma = |\Gamma| e^{j\varphi} = \frac{b}{a}$$

is expressed in linear cylindrical coordinates, representing the ratio of backward vs. forward traveling waves.

The *Smith* Chart (4)

- The distance from the center of the directly proportional to the magnitude of the reflection factor |Γ|, and permits an easy visualization of the matching performance.
  - In particular, the perimeter of the diagram represents total reflection: |Γ|=1.
  - (power dissipated in the load) = (forward power) – (reflected power)



$$P = |a|^2 - |b|^2$$
$$= |a|^2 \left(1 - |\Gamma|^2\right)$$

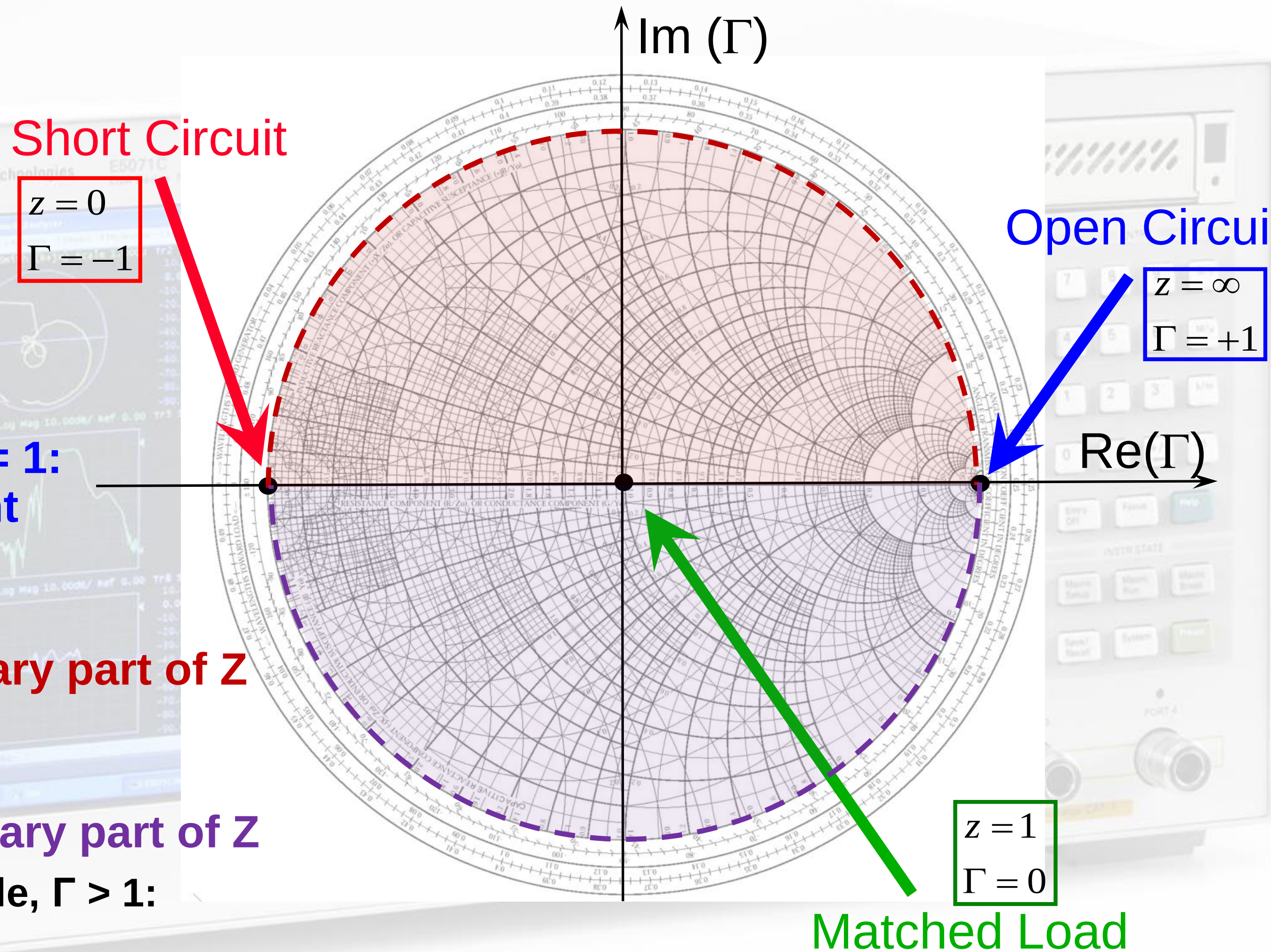
available source power

mismatch losses

The *Smith* Chart – "Important Points"

Important Points:

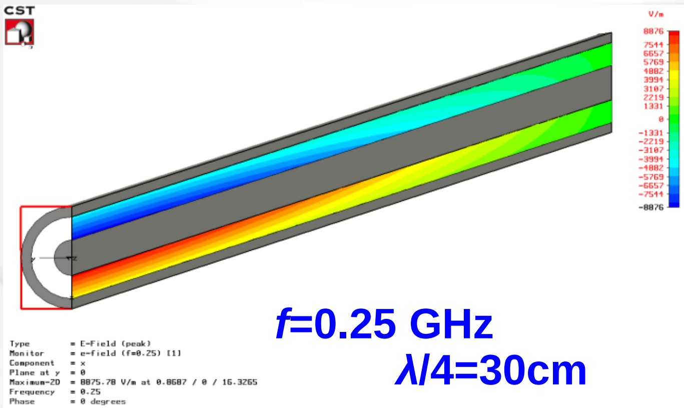
- **Short Circuit**
  $\Gamma = -1$, $z = 0$
- **Open Circuit**
  $\Gamma = +1$, $z \to \infty$
- **Matched Load**
  $\Gamma = 0$, $z = 1$
- **On the circle $\Gamma = 1$:**
  lossless element
- **Upper half:**
  "inductive" =
  positive imaginary part of Z
- **Lower half:**
  "capacitive" =
  negative imaginary part of Z
  - **Outside the circle, $\Gamma > 1$:**
    active element,
    for instance tunnel diode reflection amplifier



Coming back to our Example…

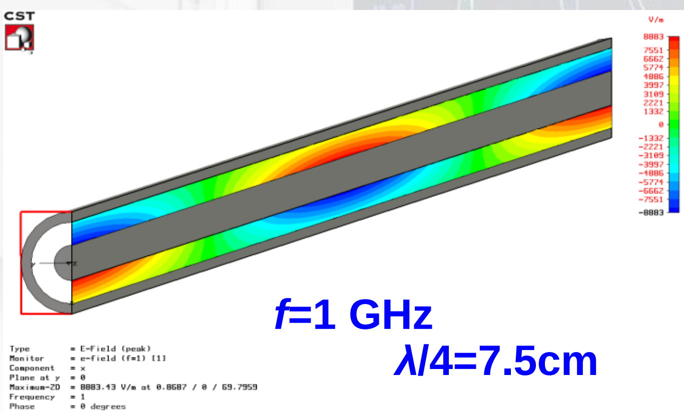
matched case:

pure traveling wave=> no reflection



***f*=0.25 GHz**

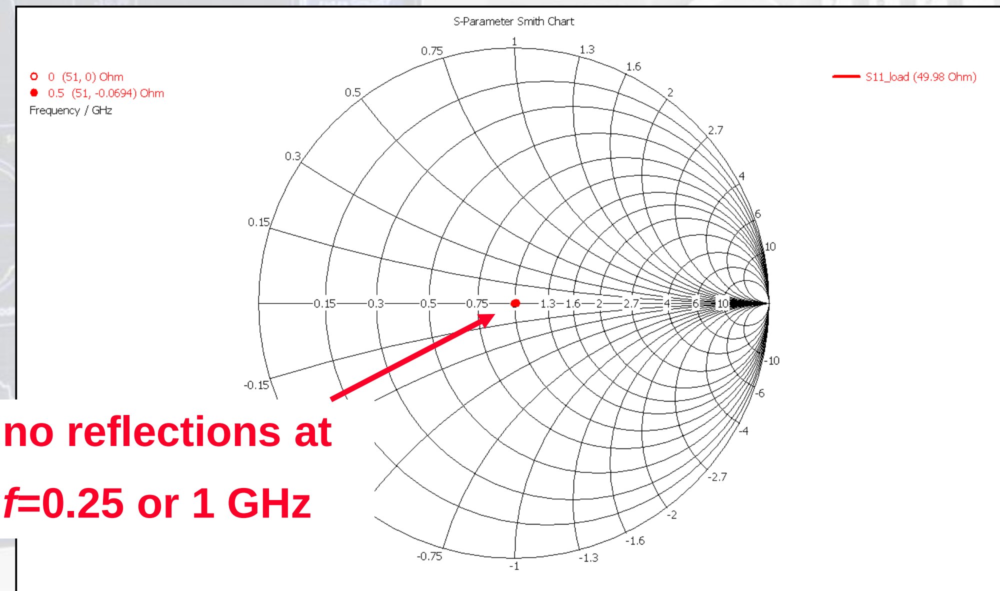
***λ*/4=30cm**



***f*=1 GHz**

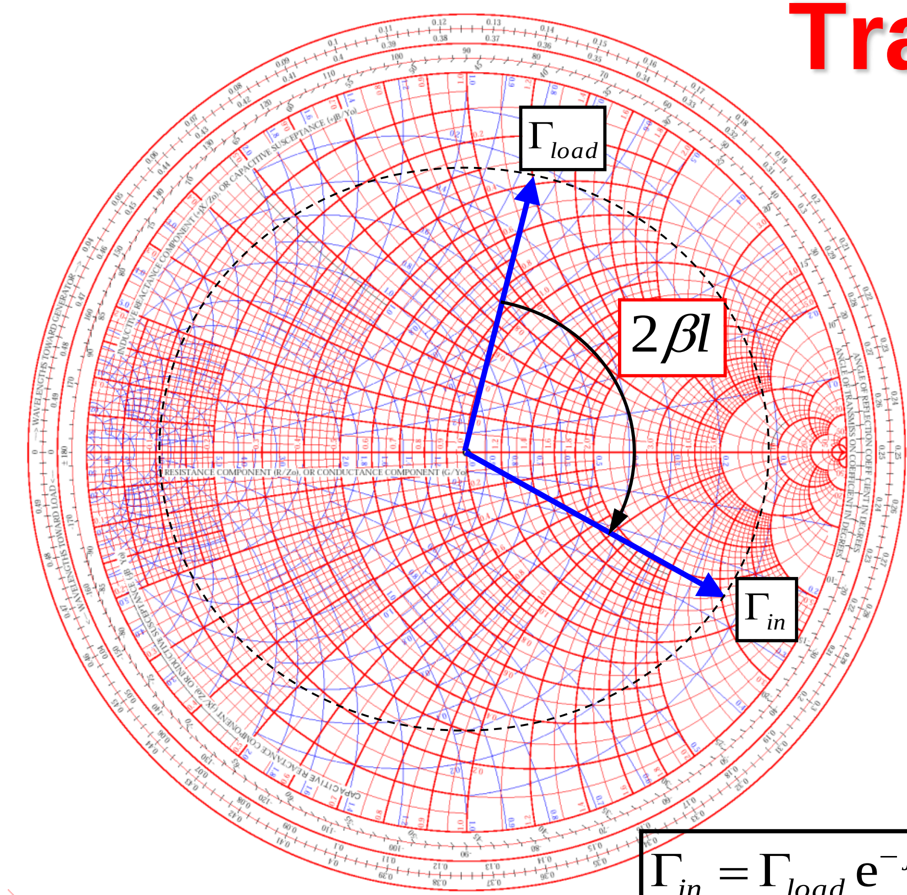
***λ*/4=7.5cm**

Coax cable with vacuum or air with a lenght of 30 cm



no reflections at *f*=0.25 or 1 GHz

Impedance Transformation using Transmission-lines



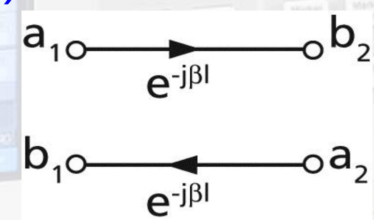
$$\Gamma_{in} = \Gamma_{load}\, e^{-j2\beta l}$$

The S-matrix for an ideal, lossless transmission line of length l is given by

$$\mathbf{S} = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix}$$

where $\beta = 2\pi / \lambda$

is the propagation coefficient with the wavelength λ (this refers to the wavelength on the line containing some dielectric).

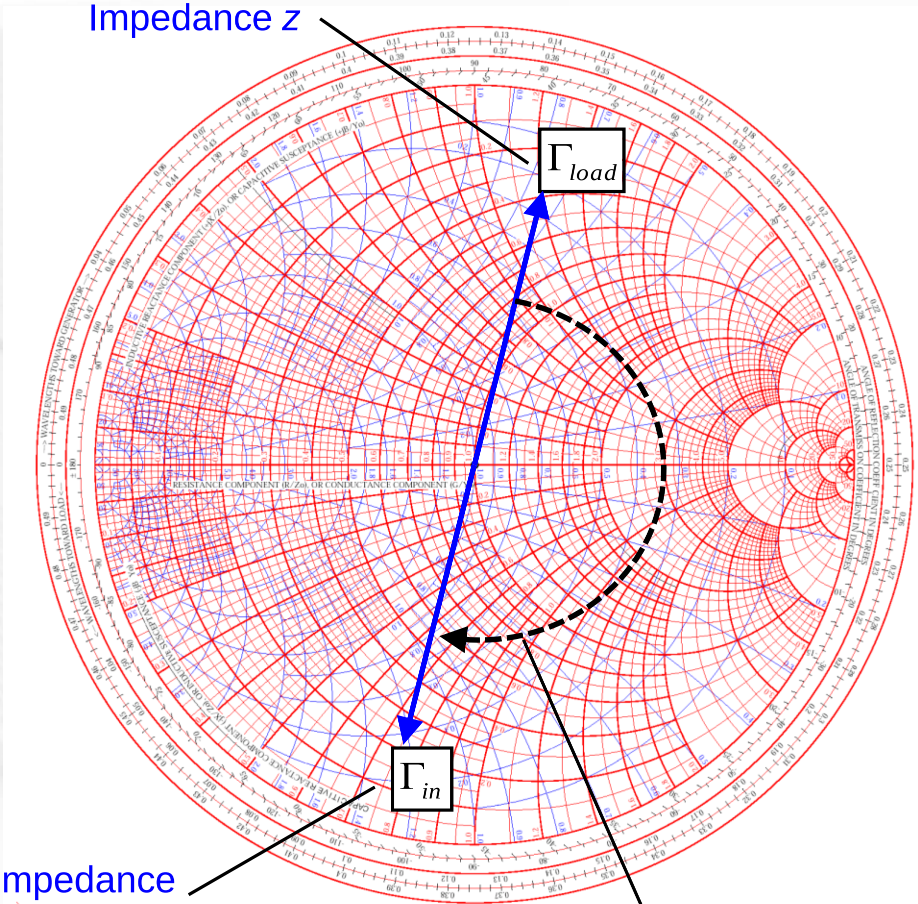


How to remember when adding a section of transmission line, we have to turn clockwise: assume we are at Γ= -1 (short circuit) and add a short piece of e.g. coaxial cable. We actually introduced an inductance, thus we are in the upper half of the *Smith*-Chart.

N.B.: The reflection factors are evaluated with respect to the characteristic impedance Z_0 of the line segment.

λ/4-line Transformations

Impedance z



Impedance $1/z$

when adding a transmission line to some terminating impedance we rotate clockwise through the *Smith*-Chart.

A transmission line of length

$$l = \lambda / 4$$

transforms a load reflection Γ_{load} to its input as

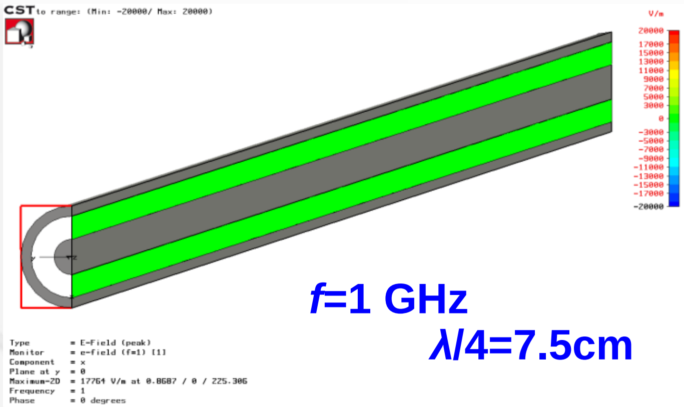
$$\Gamma_{in} = \Gamma_{load}\, e^{-j2\beta l} = \Gamma_{load}\, e^{-j\pi} = \underline{-\Gamma_{load}}$$

This results, a <u>normalized</u> load impedance z is transformed into $1/z$.

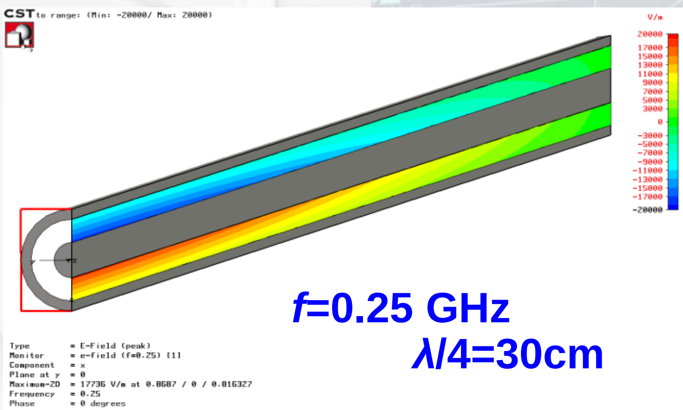
In particular, a short circuit at one end is transformed into an open circuit at the other. This is the principle of λ/4-resonators.

Again our Example: Short at the end

short : standing wave

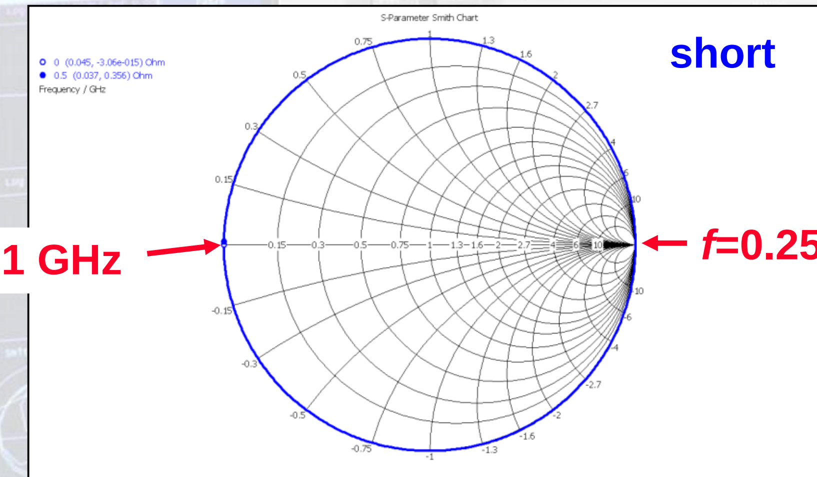


f=1 GHz
$\lambda/4$=7.5cm



f=0.25 GHz
$\lambda/4$=30cm

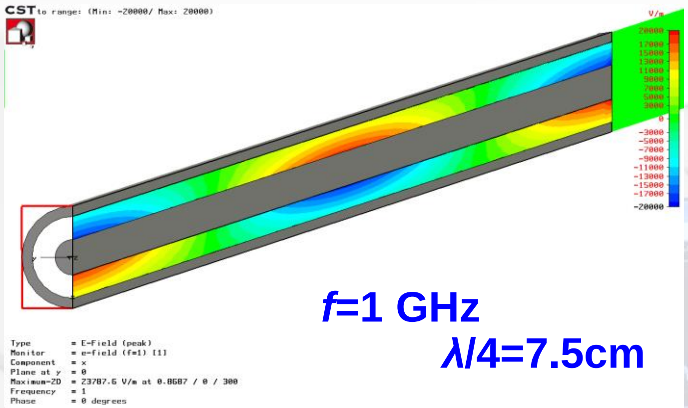
Coax cable with vacuum or air with a length of *l*=30 cm

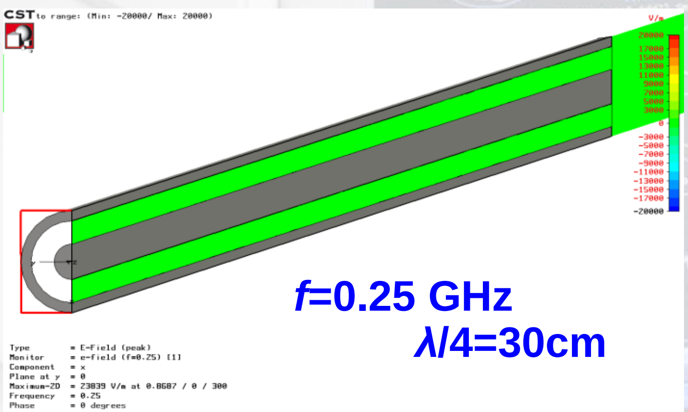


- If length of the transmission line changes by $\lambda/4$ a short circuit at one side is transformed into an open circuit at the other side.

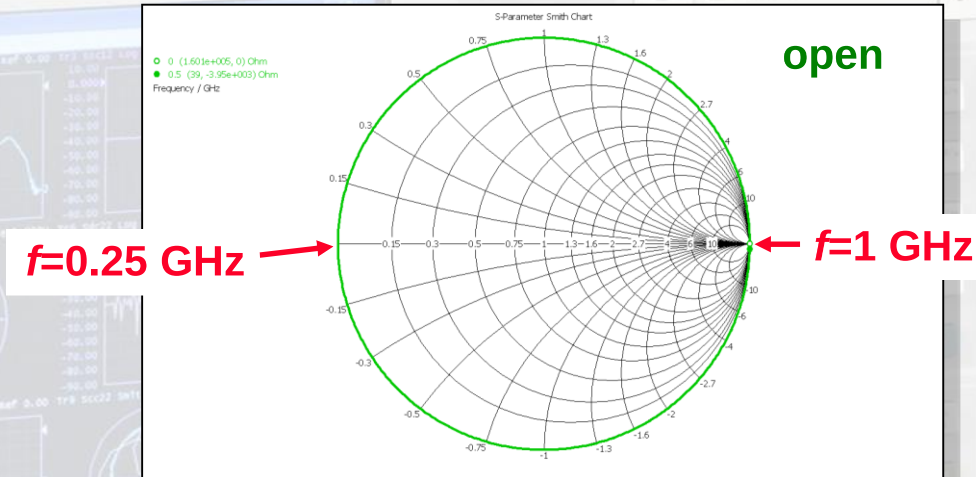
Again our Example: Open end

open : standing wave





Coax cable with vacuum with a length of 30 cm



- The patterns for the short and open terminated case appear similar; However, the phase is shifted which correspond to a different position of the nodes.
- If the length of a transmission line changes by *λ*/4, an open becomes a short, and vice versa!

More Examples: See Appendix

Transmission-line of Z=50Ω, length l=λ/4

$$(S)=\begin{bmatrix} 0 & -\mathrm{j} \\ -\mathrm{j} & 0 \end{bmatrix} \qquad \begin{aligned} b_1 &= -\mathrm{j}\,a_2 \\ b_2 &= -\mathrm{j}\,a_1 \end{aligned}$$

Attenuator 3dB, i.e. half output power

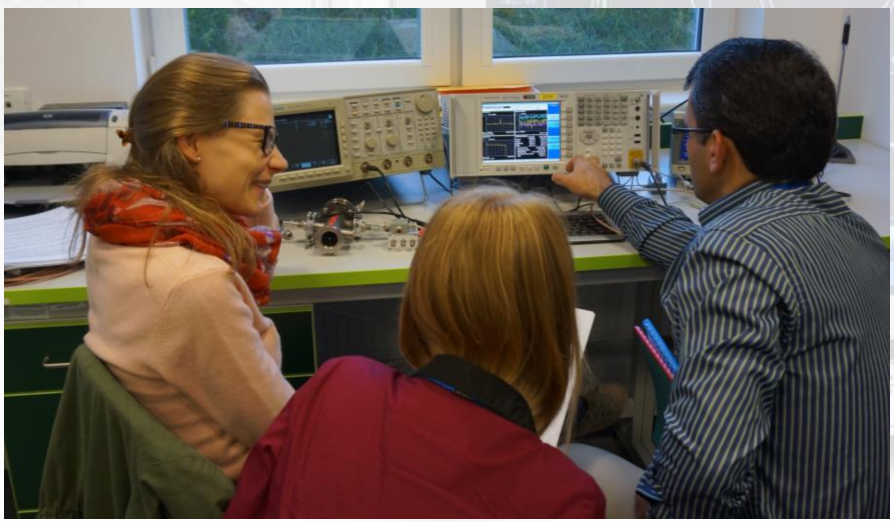
$$(S)=\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \begin{aligned} b_1 &= \frac{1}{\sqrt{2}}a_2 = 0.707a_2 \\ b_2 &= \frac{1}{\sqrt{2}}a_1 = 0.707a_1 \end{aligned}$$

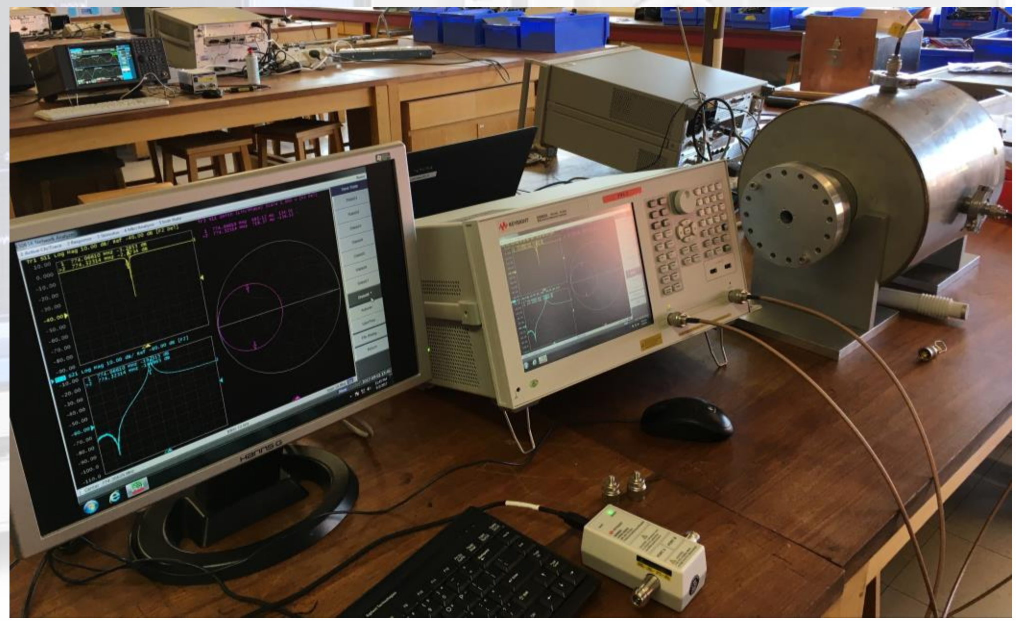
3-port circulator

$$(S)=\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad \begin{aligned} b_1 &= a_3 \\ b_2 &= a_1 \\ b_3 &= a_2 \end{aligned}$$

What awaits you?

- **Hands-on RF and microwave lab experiments!**
  - 1 ton(!) of RF hardware shipped to RHUL
  - From ”vintage” surplus
    to the latest, greatest state-of-the-art RF measurement equipment!
  - 6 test stands for 6 groups, each 3-4 students
    - 3x VNA, 3x SA & oscilloscope, plus slotted waveguide transmission-line
    - Plus: numerical simulations in the computer lab (CST Studio, QUCS)
- **Learning by doing!**





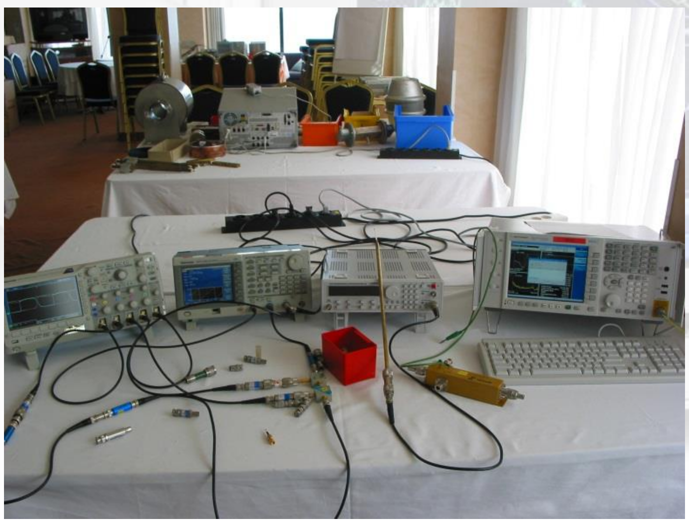
Invent your own Experiment!

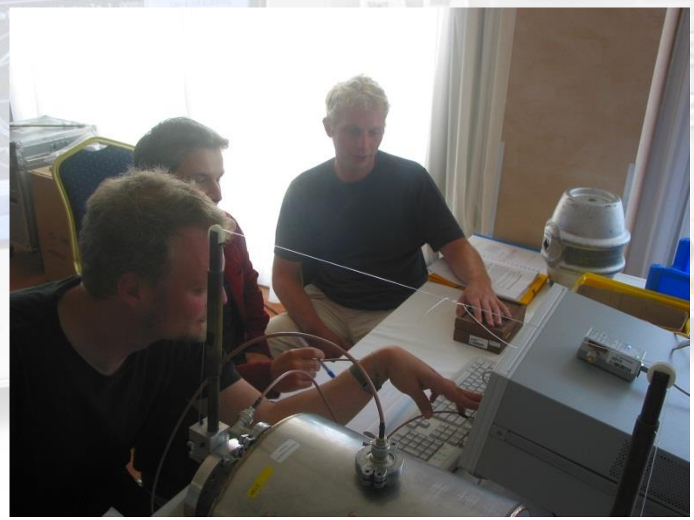
- **Build e.g. a Doppler traffic radar**
  - **Example from the CAS2011 RF-lab, CHIOS. It really worked!**

- **…or a ”tobacco”-box resonator**

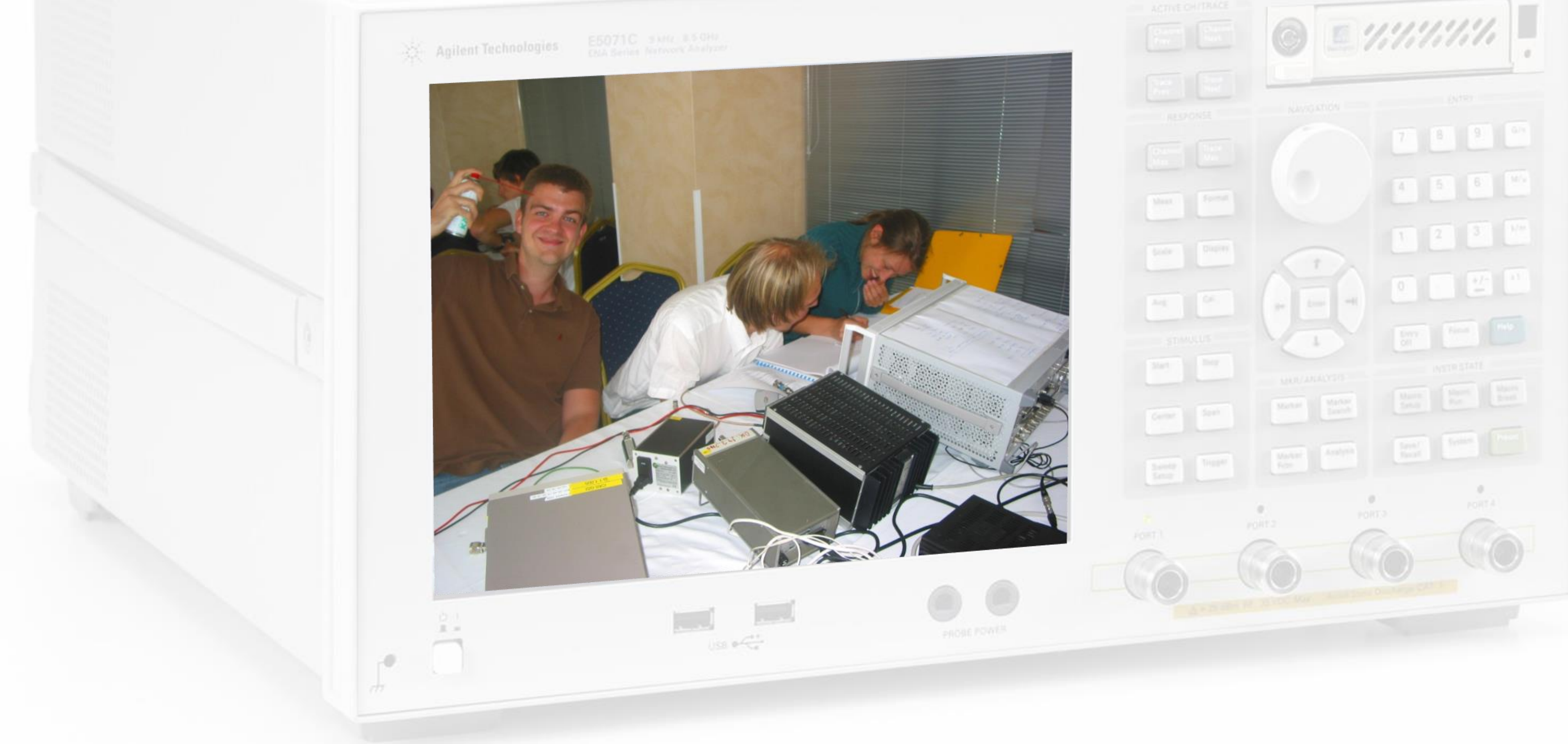
You will have enough time to think…

…and have contact with hardware and your colleagues





We hope you will have a lot of fun…!



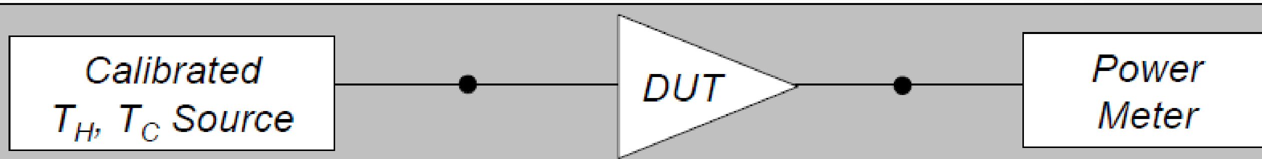
Appendix A: Definition of the Noise Figure

$$F = \frac{S_i / N_i}{S_o / N_o} = \frac{N_o}{GN_i} = \frac{N_o}{GkT_0B} = \frac{GN_i + N_R}{GkT_0B} = \frac{GkT_0B + N_R}{GkT_0B}$$

- *F* is the *Noise factor* of the receiver
- S_i is the available signal power at input
- $N_i = kT_0B$ is the available noise power at input
- T_0 is the absolute temperature of the source resistance
- N_o is the available noise power at the output , including amplified input noise
- N_r is the noise added by receiver
- *G* is the available receiver gain
- *B* is the effective noise bandwidth of the receiver
- If the noise factor is specified in a logarithmic unit, we use the term Noise Figure (*NF*)

$$NF = 10 \lg \frac{S_i / N_i}{S_o / N_o} dB$$

Measurement of Noise Figure (using a calibrated Noise Source)



$kT_HB \rightarrow$

$kT_CB \rightarrow$

$N_{OH}=FGkT_OB+(T_H-T_O)kBG$

$N_{OC}=FGkT_OB+(T_C-T_O)kBG$

$$\therefore Y=\frac{N_{OH}}{N_{OC}}=\frac{FT_O+T_H-T_O}{FT_O+T_C-T_O};\quad F=\frac{\left(\frac{T_H}{T_O}-1\right)-Y\left(\frac{T_C}{T_O}-1\right)}{Y-1}$$

Example:

T_H=10,290°K (argon source), T_C=300°K

Measured Y factor: Y=9 dB (7.94:1)

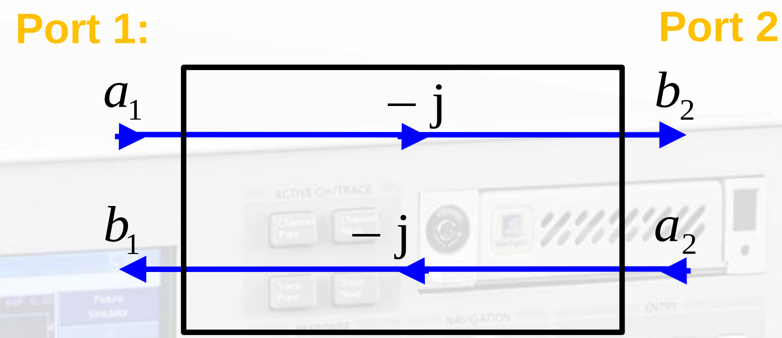
Then,

$$F=\frac{\frac{10290}{290}-1-7.94\left(\frac{300}{290}-1\right)}{7.94-1}=4.94;\quad NF(dB)=10\log(4.94)=6.9dB$$

Appendix B: Examples of 2-ports (1)

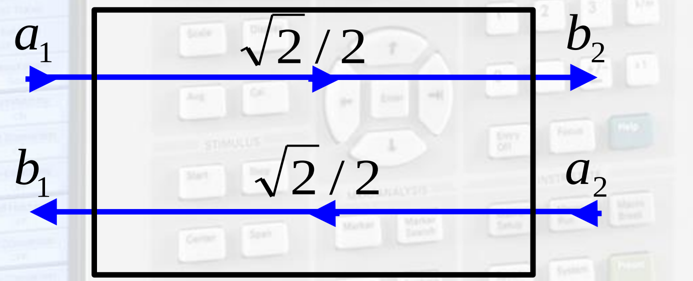
Line of Z=50Ω, length l=λ/4

$$(S) = \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix} \qquad \begin{aligned} b_1 &= -j a_2 \\ b_2 &= -j a_1 \end{aligned}$$



Attenuator 3dB, i.e. half output power

$$(S) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \begin{aligned} b_1 &= \frac{1}{\sqrt{2}} a_2 = 0.707 a_2 \\ b_2 &= \frac{1}{\sqrt{2}} a_1 = 0.707 a_1 \end{aligned}$$

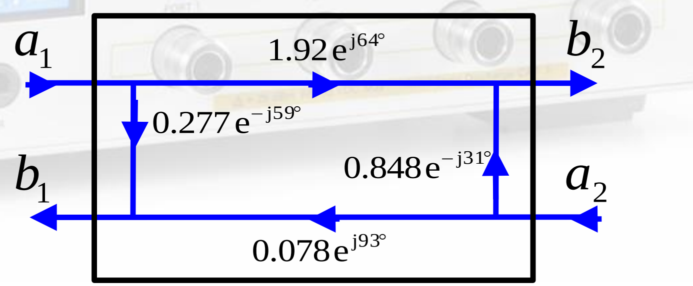


RF Transistor

backward transmission

$$(S) = \begin{bmatrix} 0.277\,e^{-j59°} & 0.078\,e^{j93°} \\ 1.92\,e^{j64°} & 0.848\,e^{-j31°} \end{bmatrix}$$

forward transmission



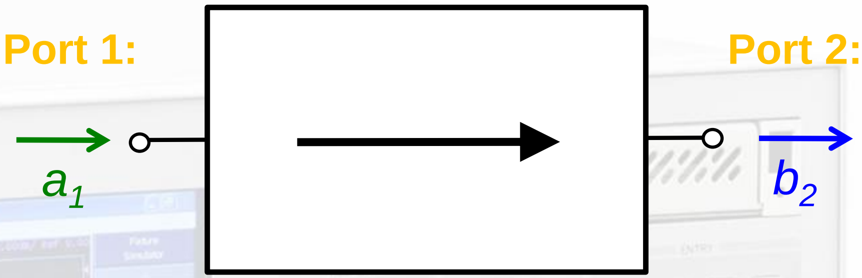
non-reciprocal since $S_{12} \neq S_{21}$!
=different transmission forwards and backwards

Examples of 2-ports (2)

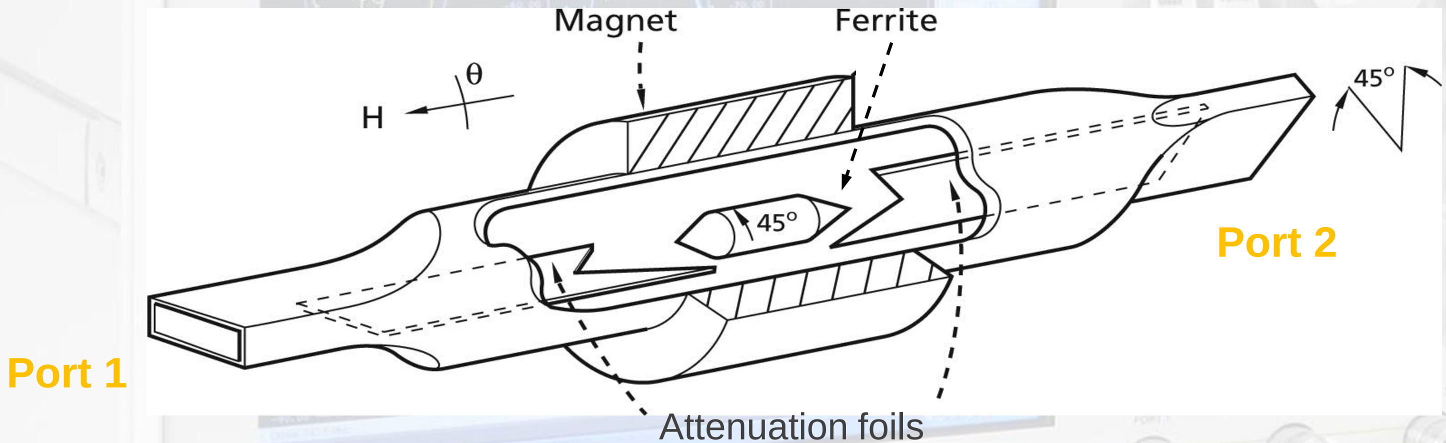
Ideal Isolator

$$(S)=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix} \qquad b_2 = a_1$$

only forward transmission



Faraday rotation isolator



The left waveguide uses a TE_{10} mode (=vertically polarized H field). After transition to a circular waveguide, the polarization of the mode is rotated counter clockwise by 45° by a ferrite. Then follows a transition to another rectangular waveguide which is rotated by 45° such that the forward wave can pass unhindered. However, a wave coming from the other side will have its polarization rotated by 45° clockwise as seen from the right hand side.

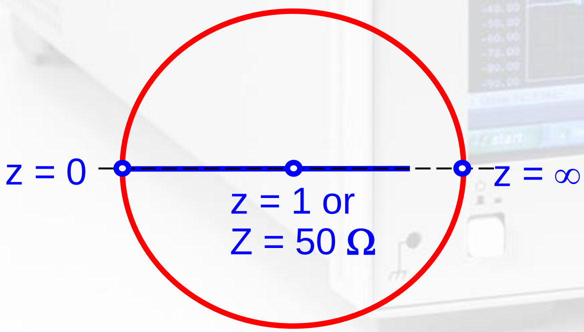
Pathing through a 2-port (1)

In general:

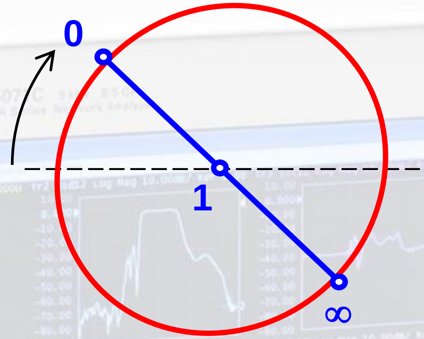
$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

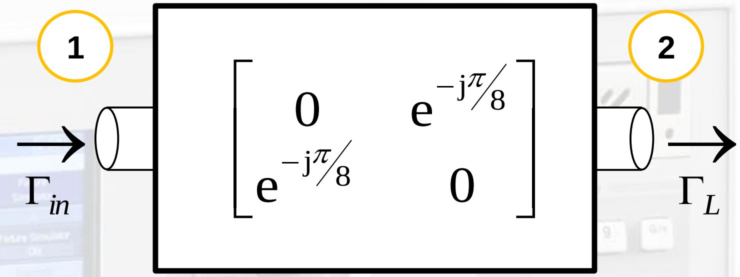
were Γ_{in} is the reflection coefficient when looking through the 2-port and Γ_{load} is the load reflection coefficient.

The outer circle and the real axis in the simplified Smith diagram below are mapped to other circles and lines, as can be seen on the right.



Line λ/16:

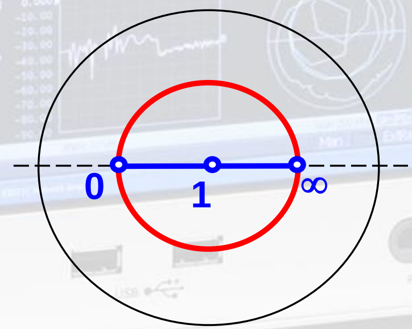


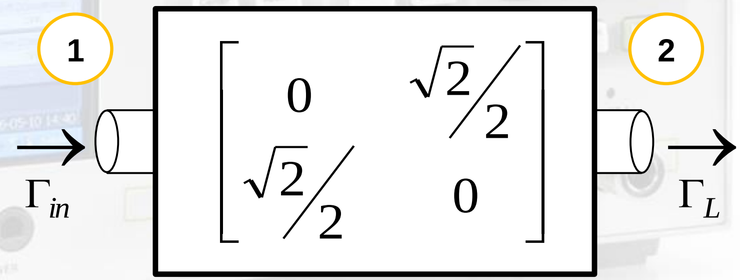


$$\begin{bmatrix} 0 & e^{-j\pi/8} \\ e^{-j\pi/8} & 0 \end{bmatrix}$$

$$\Rightarrow \Gamma_{in} = \Gamma_L\, e^{-j\pi/4}$$

Attenuator 3dB:



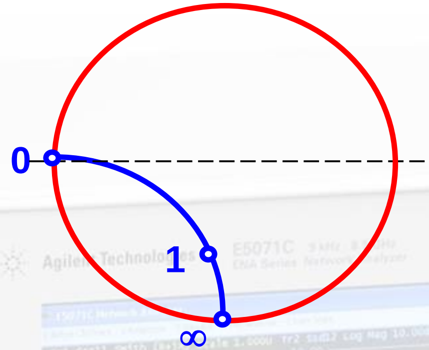


$$\begin{bmatrix} 0 & \sqrt{2}/2 \\ \sqrt{2}/2 & 0 \end{bmatrix}$$

$$\Rightarrow \Gamma_{in} = \Gamma_L/2$$

Pathing through a 2-port (2)

Lossless Passive Circuit



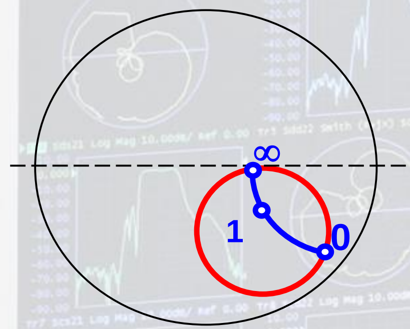
1 — 2

If S is unitary

$$S^{*}S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

→ Lossless Two-Port

Lossy Passive Circuit



1 — 2

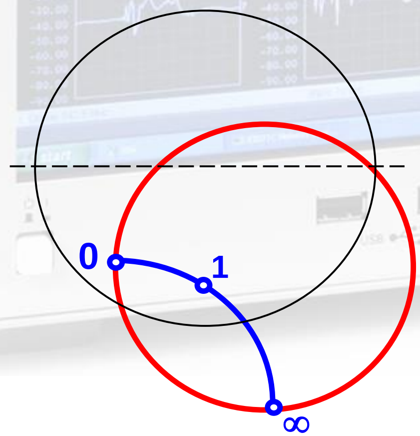
Lossy Two-Port:

If $K_{LINVILL} < 1$

$K_{ROLLET} > 1$

unconditionally stable

Active Circuit



1 — 2

Active Circuit:

If $K_{LINVILL} \geq 1$

$K_{ROLLET} \leq 1$

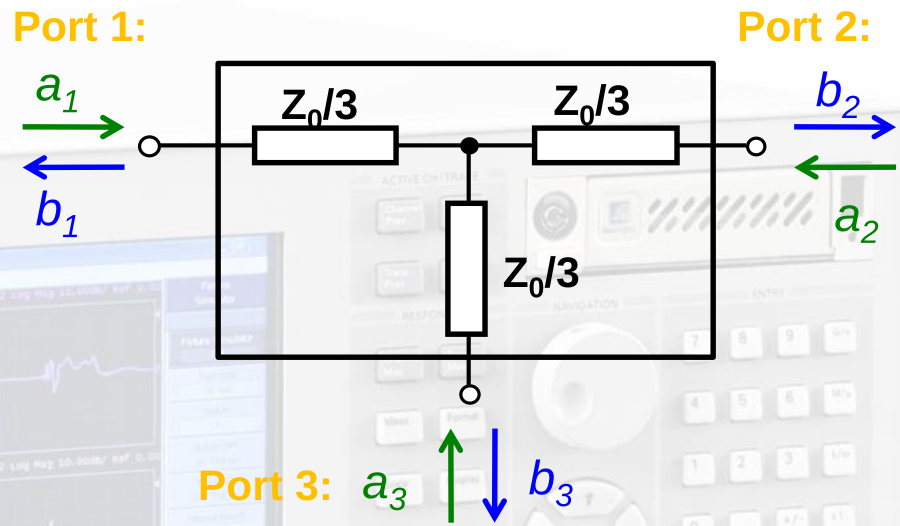
potentially unstable

Examples of 3-ports (1)

Resistive power divider

$$(S) = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

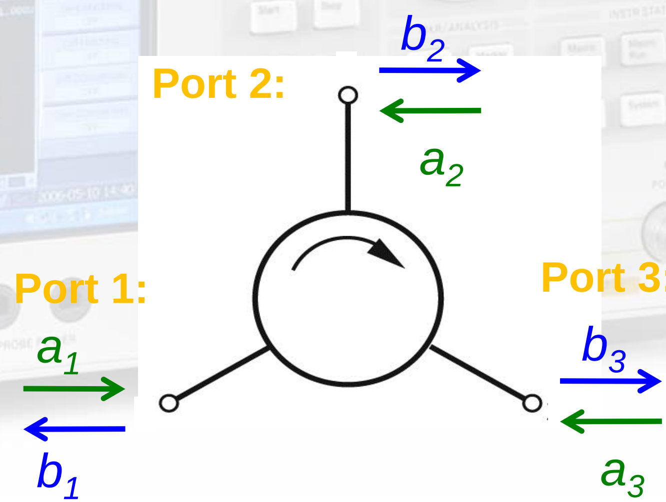
$$b_1 = \frac{1}{2}(a_2 + a_3)$$
$$b_2 = \frac{1}{2}(a_1 + a_3)$$
$$b_3 = \frac{1}{2}(a_1 + a_2)$$



3-port circulator

$$(S) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

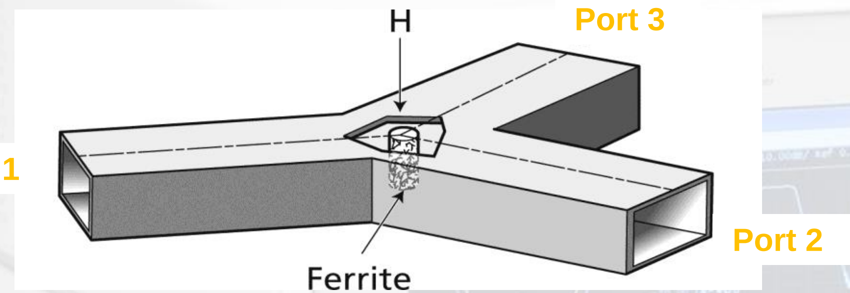
$$b_1 = a_3$$
$$b_2 = a_1$$
$$b_3 = a_2$$



The ideal circulator is lossless, matched at all ports, but not reciprocal. A signal entering the ideal circulator at one port is transmitted exclusively to the next port in the sense of the arrow.

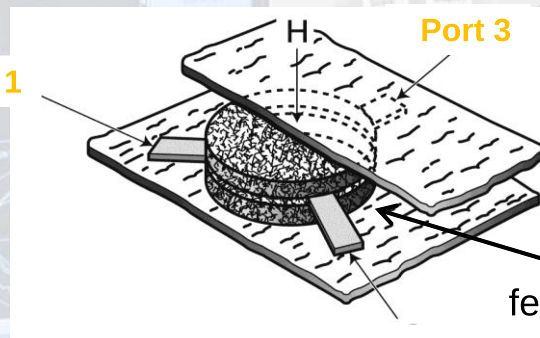
Examples of 3-ports (2)

Practical implementations of circulators:



Waveguide circulator

Stripline circulator



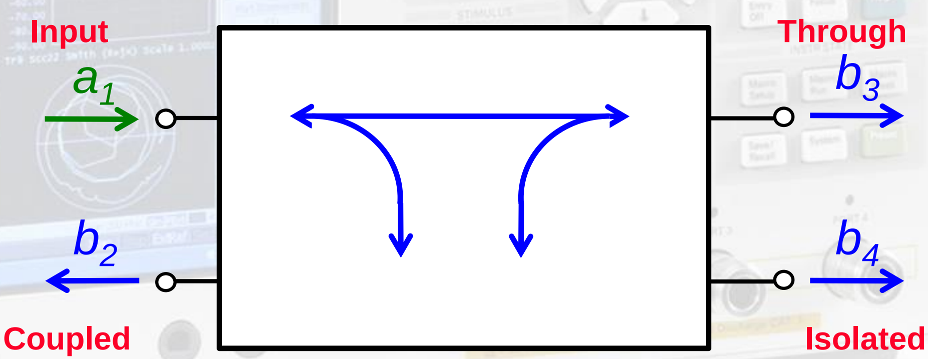
A circulator contains a volume of ferrite. The magnetically polarized ferrite provides the required non-reciprocal properties, thus power is only transmitted from port 1 to port 2, from port 2 to port 3, and from port 3 to port 1.

Examples of 4-ports (1)

Ideal directional coupler

$$(S)=\begin{bmatrix} 0 & jk & \sqrt{1-k^2} & 0 \\ jk & 0 & 0 & \sqrt{1-k^2} \\ \sqrt{1-k^2} & 0 & 0 & jk \\ 0 & \sqrt{1-k^2} & jk & 0 \end{bmatrix} \text{ with } k=\left|\frac{b_2}{a_1}\right|$$

To characterize directional couplers, three important figures are used:



the coupling $C=-20\log_{10}\left|\frac{b_2}{a_1}\right|$

the directivity $D=-20\log_{10}\left|\frac{b_4}{b_2}\right|$

the isolation $I=-20\log_{10}\left|\frac{a_1}{b_4}\right|$

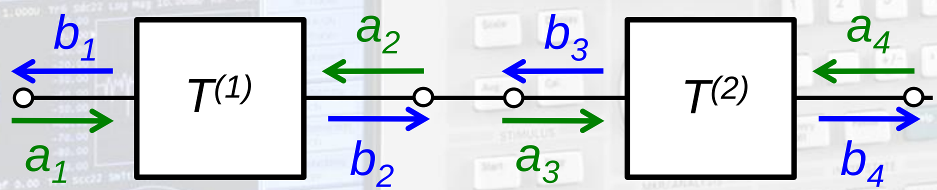
Appendix C: T matrix

The T-parameter matrix is related to the incident and reflected normalised waves at each of the ports.

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

T-parameters may be used to determine the effect of a cascaded 2-port networks by simply multiplying the individual T-parameter matrices:

$$[T] = \left[T^{(1)}\right]\left[T^{(2)}\right]\ldots\left[T^{(N)}\right] = \prod_N \left[T^{(i)}\right]$$



T-parameters can be directly evaluated from the associated S-parameters and vice versa.

From S to T:

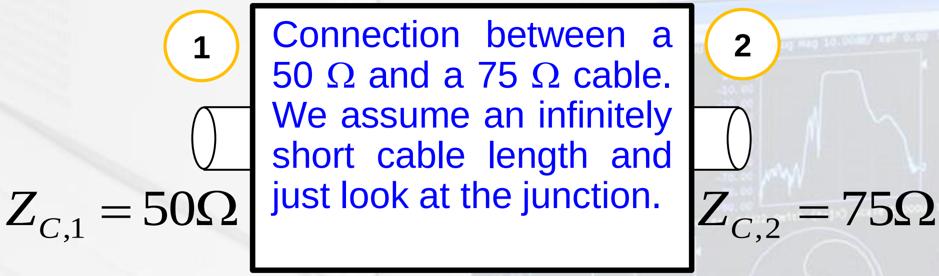
$$[T] = \frac{1}{S_{21}} \begin{bmatrix} -\det(S) & S_{11} \\ -S_{22} & 1 \end{bmatrix}$$

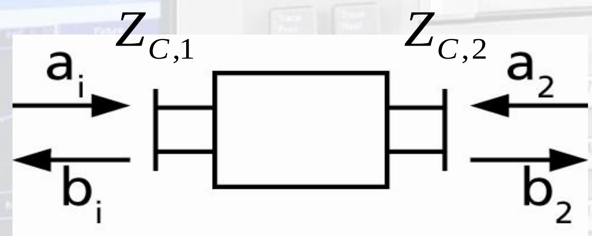
From T to S:

$$[S] = \frac{1}{T_{22}} \begin{bmatrix} T_{12} & \det(T) \\ 1 & -T_{21} \end{bmatrix}$$

Appendix D: A Step in Characteristic Impedance (1)

Consider a connection of two coaxial cables, one with $Z_{C,1}$ = 50 Ω characteristic impedance, the other with $Z_{C,2}$ = 75 Ω characteristic impedance.





Step 1: Calculate the reflection coefficient and keep in mind: all ports have to be terminated with their respective characteristic impedance, i.e. 75 Ω for port 2.

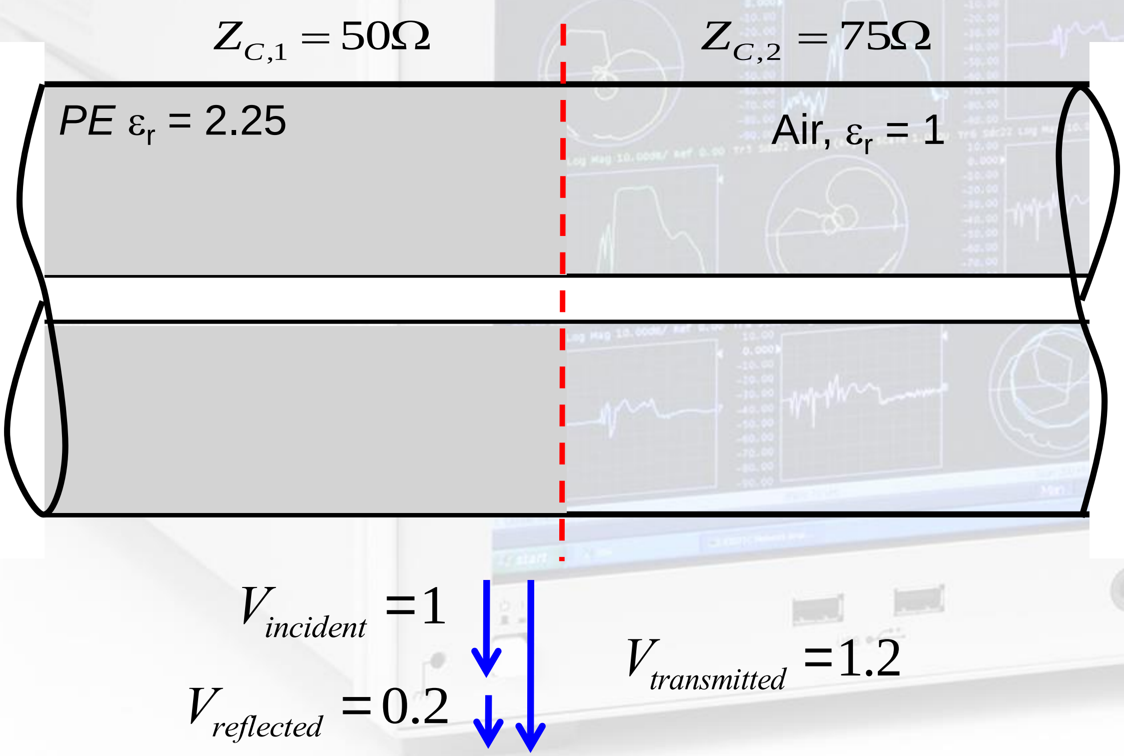
$$\Gamma_1 = \frac{Z - Z_{C,1}}{Z + Z_{C,1}} = \frac{75 - 50}{75 + 50} = 0.2$$

Thus, the voltage of the reflected wave at port 1 is 20% of the incident wave, and the reflected power at port 1 (proportional Γ^2) is 0.2^2 = 4%. As this junction is lossless, the transmitted power must be 96% (conservation of energy). From this we can deduce b_2^2 = 0.96. But: how do we get the voltage of this outgoing wave?

Example: a Step in Characteristic Impedance (2)

Step 2: Remember, *a* and *b* are power-waves, and defined as voltage of the forward- or backward traveling wave normalized to $\sqrt{Z_C}$.

The tangential electric field in the dielectric in the 50 Ω and the 75 Ω line, respectively, must be continuous.



t = voltage transmission coefficient, in this case: $t = 1 + \Gamma$

This is counterintuitive, one might expect 1-Γ. Note that the voltage of the transmitted wave is higher than the voltage of the incident wave. But we have to normalize to $\sqrt{Z_C}$ to evaluate the corresponding S-parameter. $S_{12} = S_{21}$ via reciprocity! But $S_{11} \neq S_{22}$, i.e. the structure is NOT symmetric.

Example: a Step in Characteristic Impedance (3)

Once we have determined the voltage transmission coefficient, we have to normalize to the ratio of the characteristic impedances, respectively. Thus we get for

$$S_{12} = 1.2\sqrt{\frac{50}{75}} = 1.2 \cdot 0.816 = 0.9798$$

We know from the previous calculation that the reflected power (proportional Γ^2) is 4% of the incident power. Thus 96% of the power are transmitted.

Check done $S_{12}{}^2 = 1.44\frac{1}{1.5} = 0.96 = (0.9798)^2$

$$S_{22} = \frac{50-75}{50+75} = -0.2$$ To be compared with S11 = +0.2!

Example: a Step in Characteristic Impedance (4)

Visualization in the Smith chart:

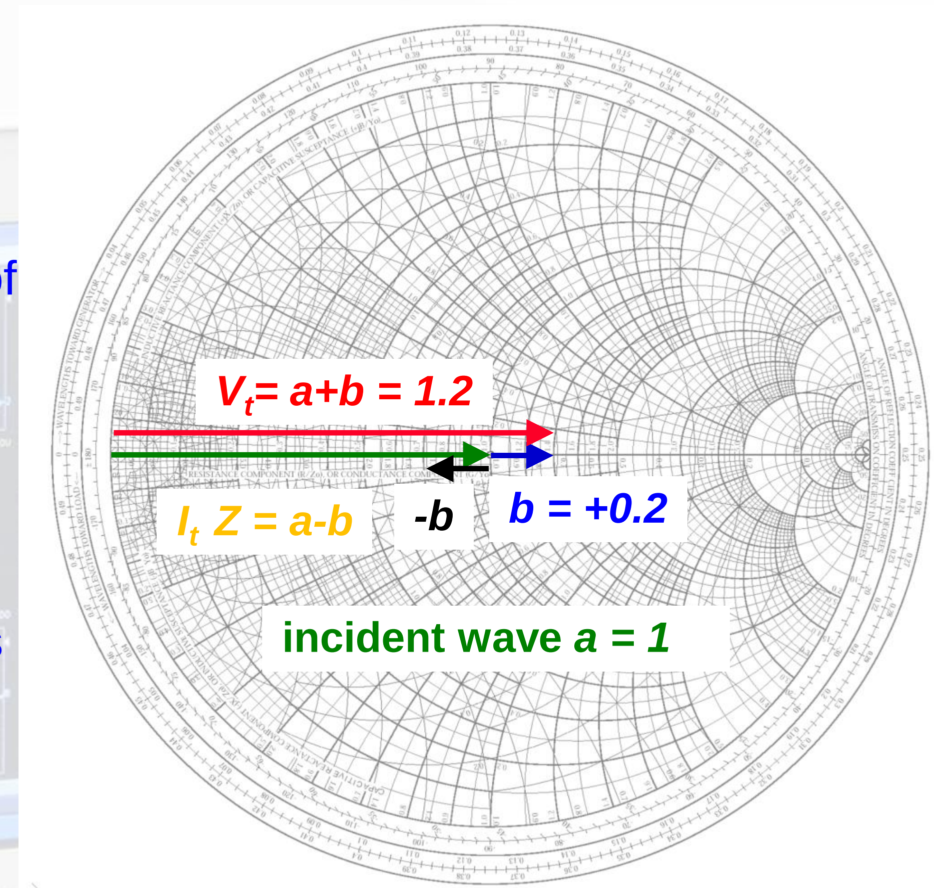
As shown in the previous slides the voltage of the transmitted wave is

$V_t = a + b$, with $t = 1 + \Gamma$

and subsequently the current is

$I_t Z = a - b$.

Remember: the reflection coefficient Γ is defined with respect to voltages. For currents the sign inverts. Thus a positive reflection coefficient in the normal definition leads to a subtraction of currents or is negative with respect to current.

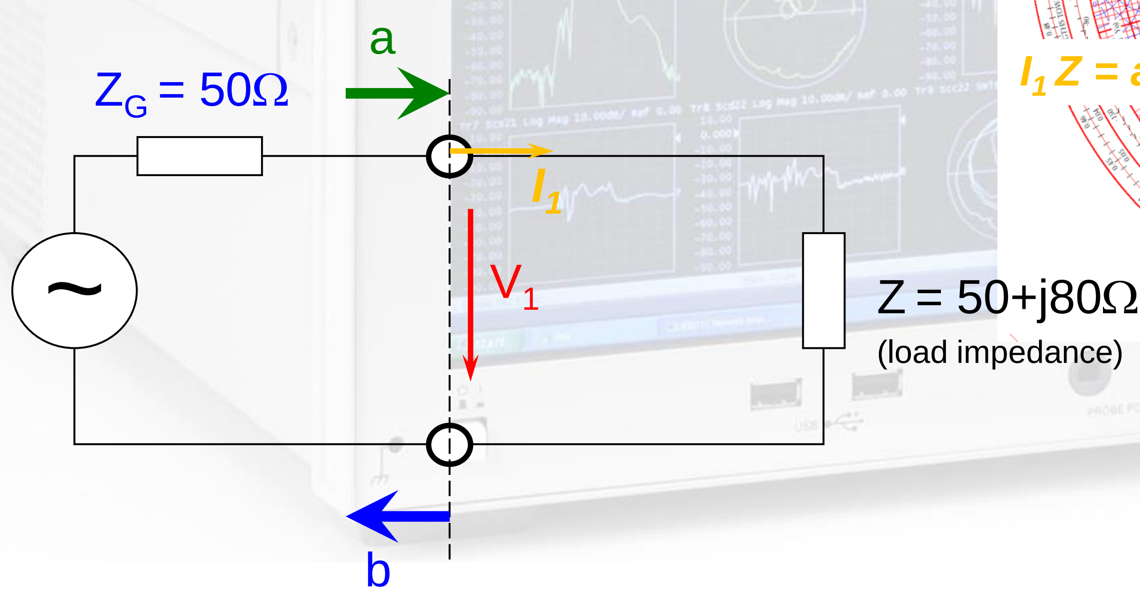


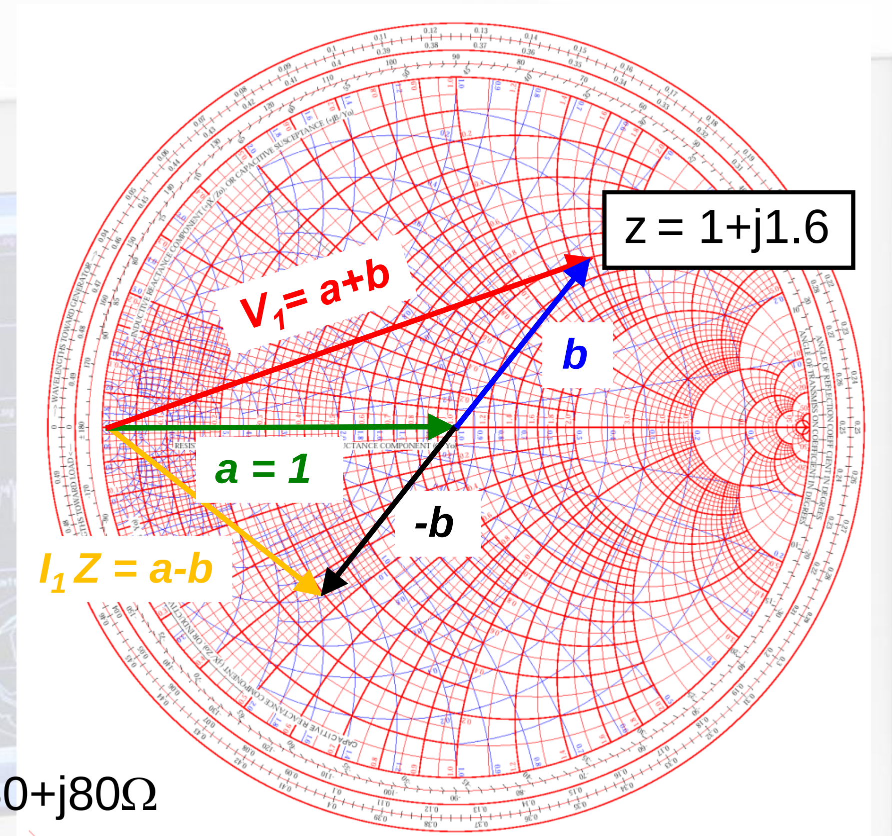
Note: here Z_{load} is real

Example: a Step in Characteristic Impedance (5)

General case:

Thus we can read from the Smith chart immediately the amplitude and phase of voltage and current on the load (of course we can calculate it when using the complex voltage divider).





Appendix E: Navigation in the Smith Chart (1)

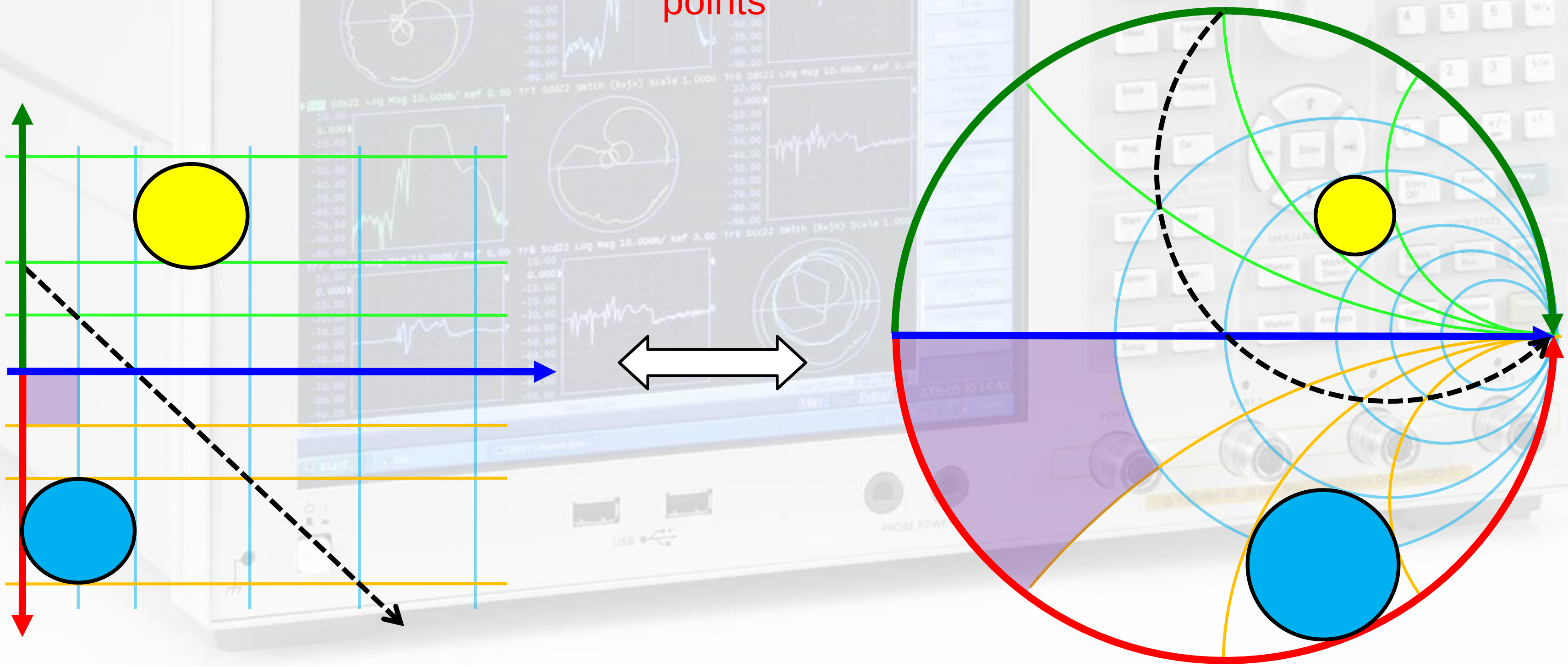
This is a “bilinear” transformation with the following properties:

- generalized circles are transformed into generalized circles
  - circle → circle
  - straight line → circle
  - circle → straight line
  - straight line → straight line
- angles are preserved locally

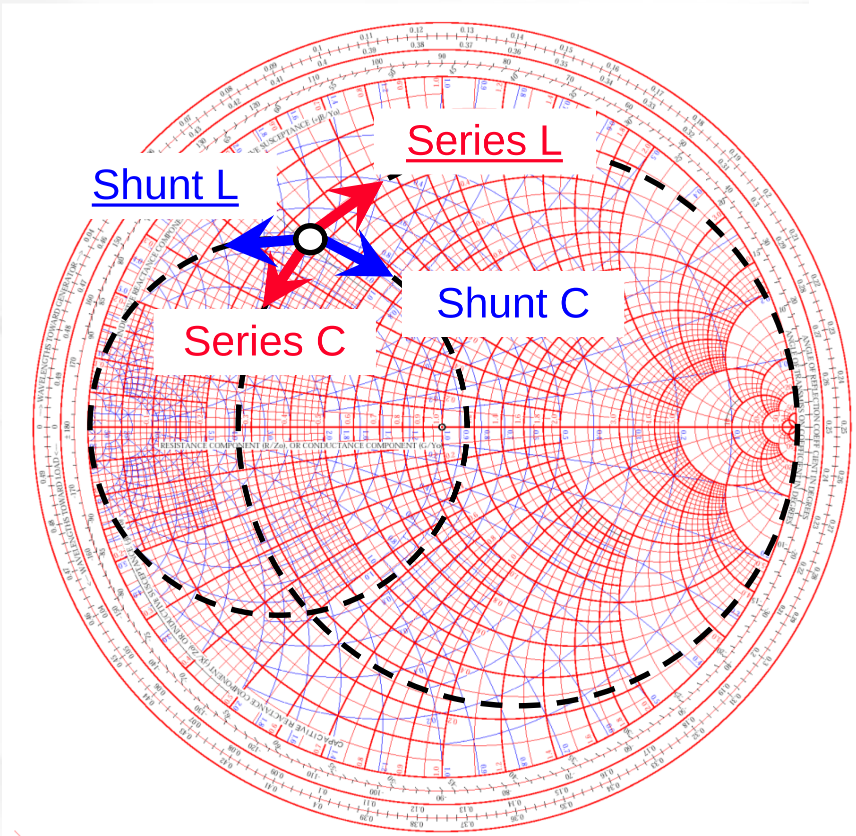
a straight line is nothing else than a circle with infinite radius

a circle is defined by 3 points

a straight line is defined by 2 points



Navigation in the Smith Chart (2)

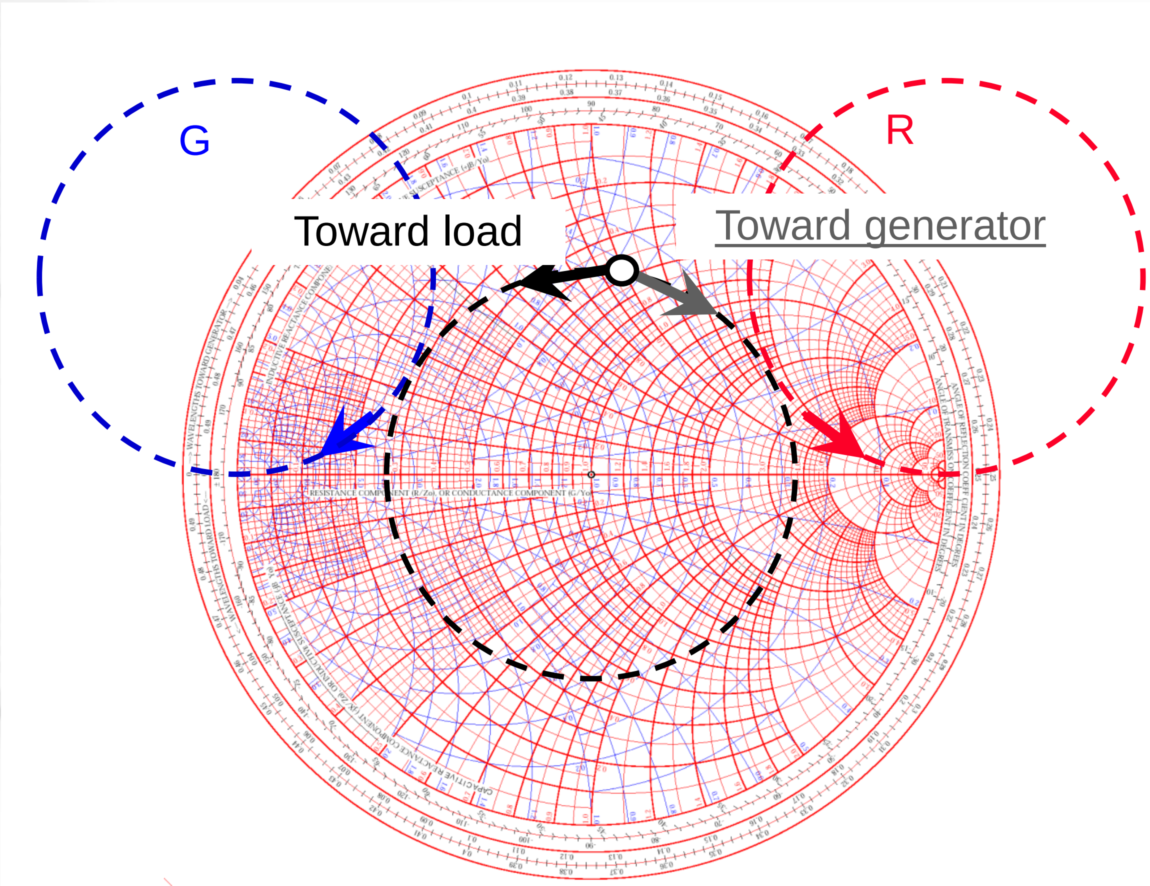


in blue: Impedance plane (=Z)

in red: Admittance plane (=Y)

	Up	Down
Red circles	**Series L**	**Series C**
Blue circles	**Shunt L**	**Shunt C**

Navigation in the Smith Chart (3)



Red arcs	**Resistance R**
Blue arcs	**Conductance G**
Con-centric circle	**Transmission line going Toward load Toward generator**

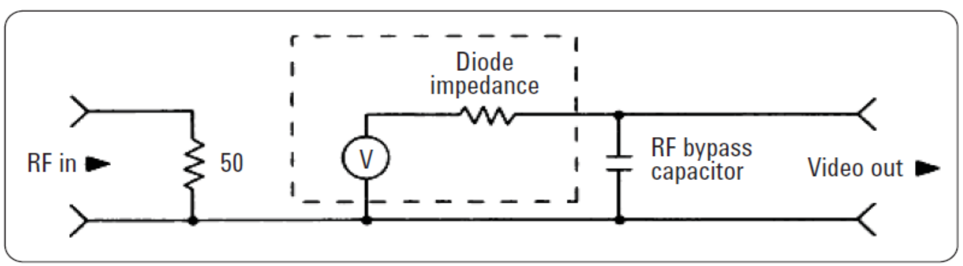
Appendix F: The RF diode (1)

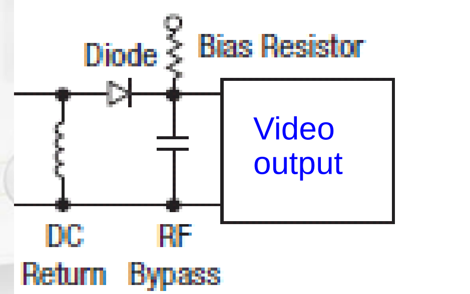
- We are not discussing the generation of RF signals here, just the detection
- Basic tool: fast RF* diode (= Schottky diode)
- In general, Schottky diodes are fast but still have a voltage dependent junction capacity (metal – semi-conductor junction)

A typical RF detector diode

Try to guess from the type of the connector which side is the RF input and which is the output

Equivalent circuit:





*Please note, in this lecture we will use RF (radio-frequency) for both, the RF and the microwave range, since there is no defined borderline between the RF and microwave regime.

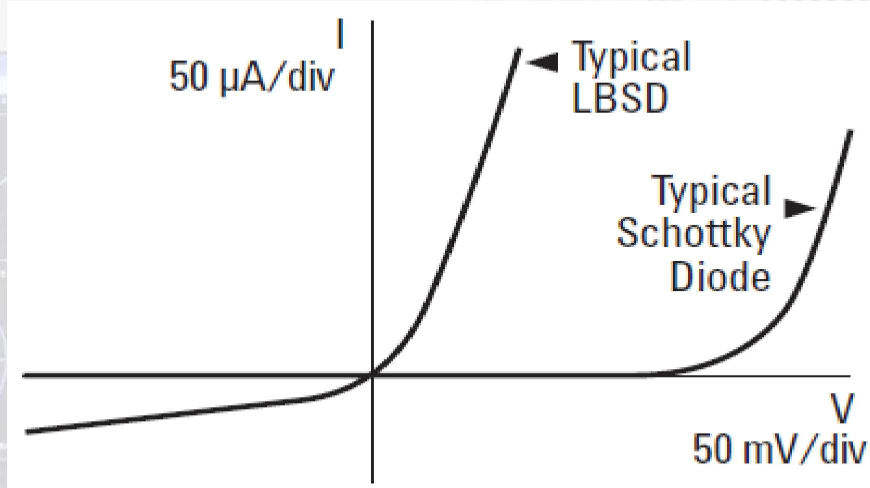
The RF diode (2)

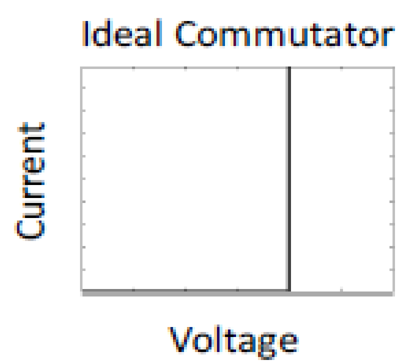
- Characteristics of a diode:

  The current as a function of the voltage for a barrier diode can be described by the Richardson equation:

$$I = AA^{**}_2 \exp\left(-\frac{q\phi B}{kT}\right)\left[\exp\left(\frac{qv}{NkT}\right) - 1\right]$$

where
A = area (cm^2)
A** = modified Richardson constant $(amp/oK)^2/cm^2$
k = Boltzman's Constant
T = absolute temperature (°K)
ϕB = barrier heights in volts
V = external voltage across the depletion layer (positive for forward voltage) - V - IR_S
R_S = series resistance
I = diode current in amps (positive forward current)
n = ideality factor





- The RF diode is NOT an ideal commutator for small signals! We cannot apply big signals otherwise burnout

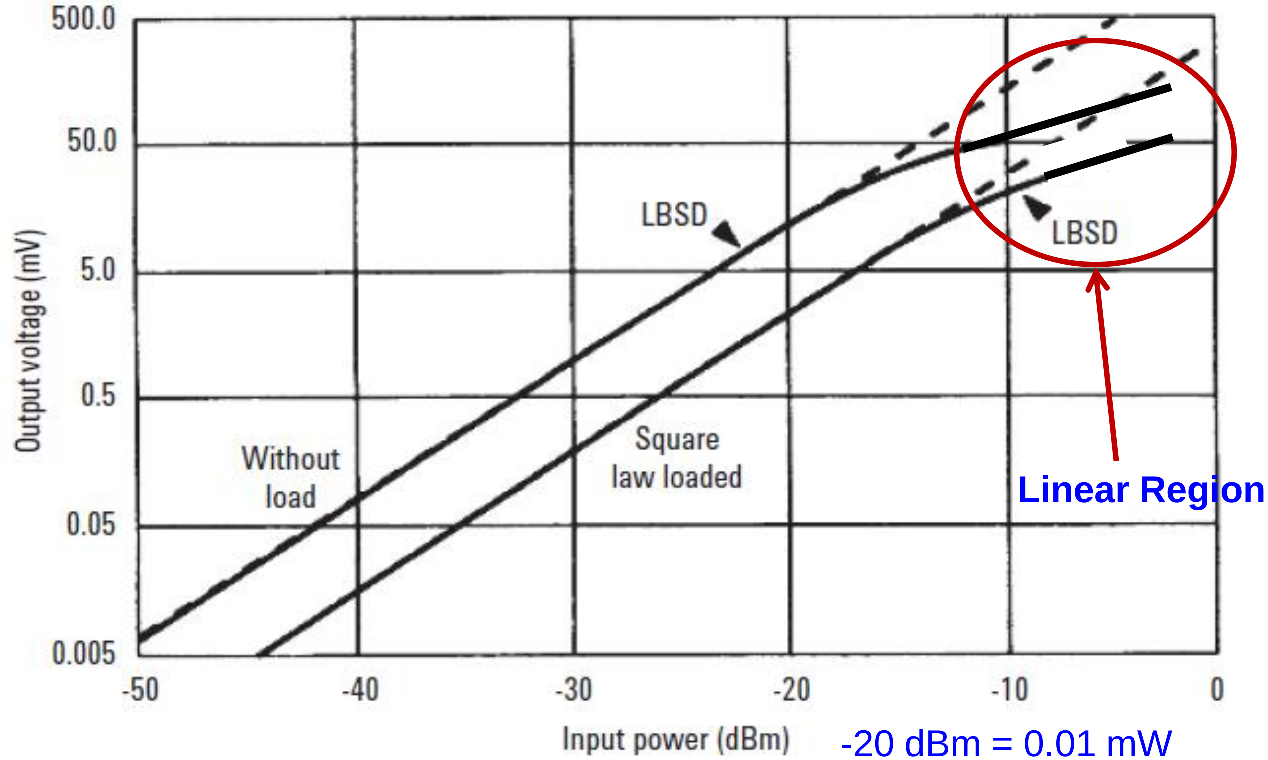
The RF diode (3)

- This diagram depicts the so called square-law region where the output voltage (V_{Video}) is proportional to the input power

Since the input power is proportional to the square of the input voltage (V_{RF}^2) and the output signal is proportional to the input power, this region is called square- law region.

In other words:

$$V_{Video} \sim V_{RF}^2$$



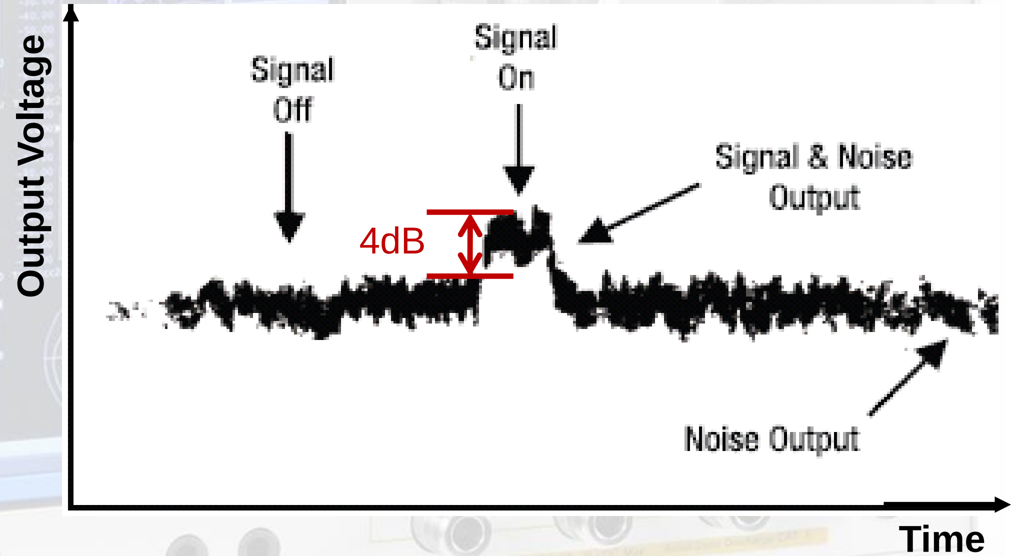
-20 dBm = 0.01 mW

- The transition between the linear region and the square-law region is typically between -10 and -20 dBm RF power (see diagram).

The RF diode (5)

- Due to the square-law characteristic we arrive at the thermal noise region already for moderate power levels (-50 to -60 dBm) and hence the V_{Video} disappears in the thermal noise

- This is described by the term *tangential signal sensitivity* (TSS) where the detected signal (Observation BW, usually 10 MHz) is 4 dB over the thermal noise floor



Appendix G: The RF mixer (1)

- For the detection of very small RF signals we prefer a device that has a linear response over the full range (from 0 dBm (= 1mW) down to thermal noise = -174 dBm/Hz = $4 \cdot 10^{-21}$ W/Hz)
- It is called “RF mixer”, and uses 1, 2 or 4 diodes in different configurations (see next slide)
- Together with a so called LO (local oscillator) signal, the mixer works as a signal multiplier, providing a very high dynamic range since the output signal is always in the “linear range”, assuming the mixer is not in saturation with respect to the RF input signal (For the LO signal the mixer should always be in saturation!)
- The RF mixer is essentially a multiplier implementing the function

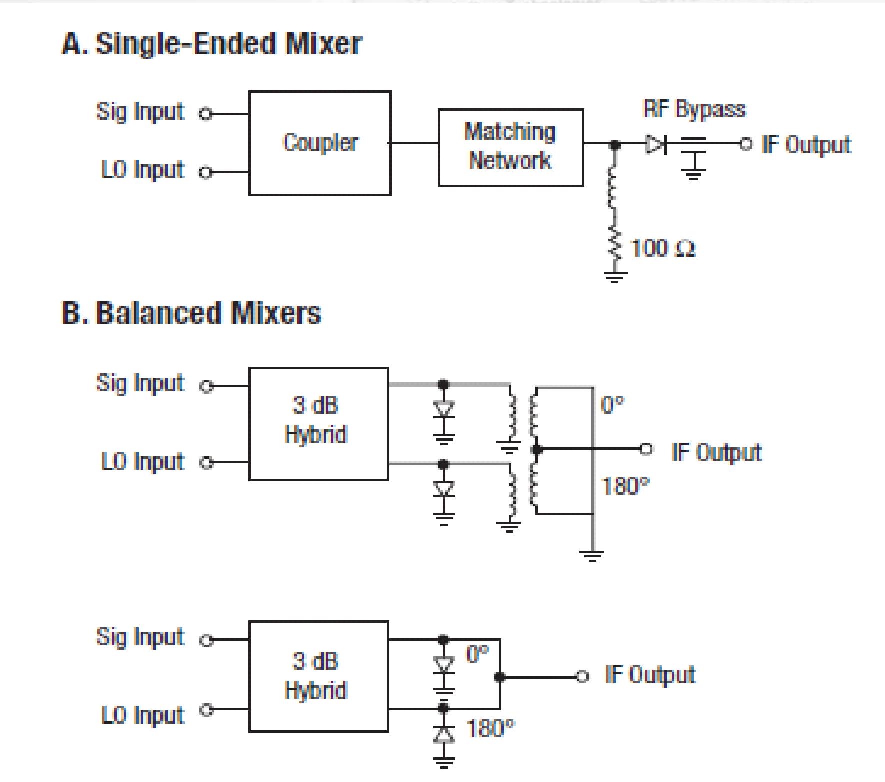
$f_1(t) \cdot f_2(t)$ with $f_1(t)$ = RF signal and $f_2(t)$ = LO signal

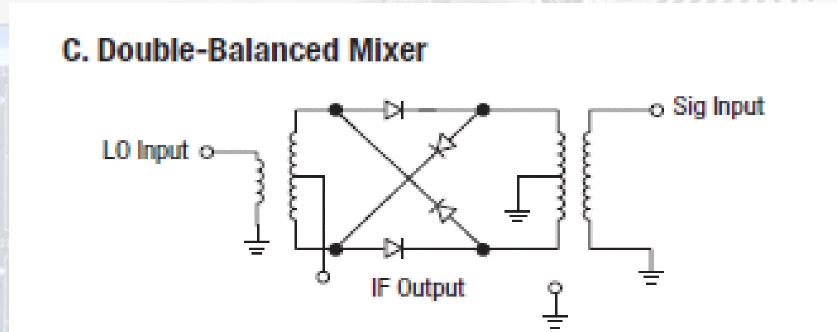
$$a_1 \cos(2\pi f_1 t + \varphi) \cdot a_2 \cos(2\pi f_2 t) = \frac{1}{2} a_1 a_2 [\cos((f_1 + f_2)t + \varphi) + \cos((f_1 - f_2)t + \varphi)]$$

- Thus we obtain a response at the IF (intermediate frequency) port as sum and difference frequencies of the LO and RF signals

The RF mixer (2)

- **Examples of different mixer configurations**



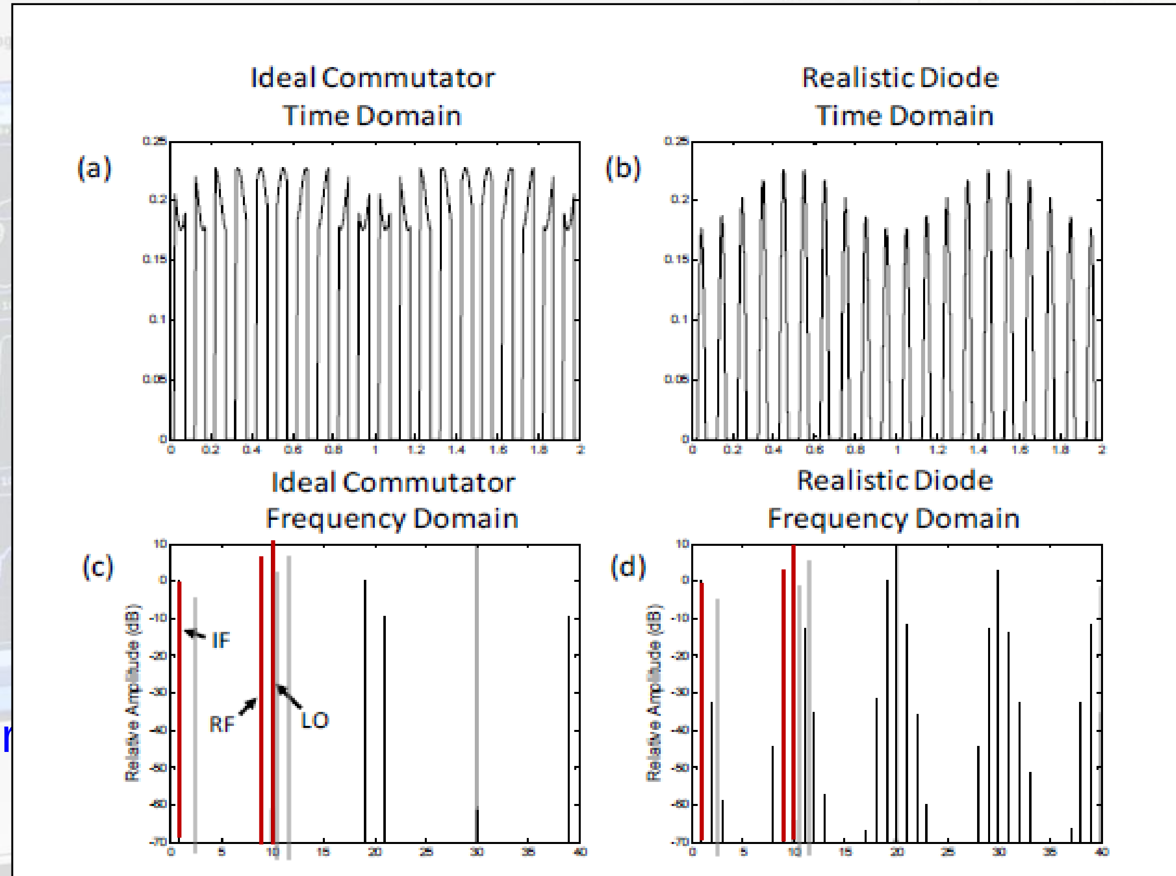


- A typical coaxial mixer (SMA connector)

The RF mixer (3)

- Response of a mixer in time and frequency domain:

- Input signals here:
- LO = 10 MHz
- RF = 8 MHz



- Mixing products at
2 and 18 MHz and
higher order terms at higher frequencies

The RF mixer (4)

Dynamic range and IP3 of an RF mixer

- The abbreviation IP3 stands for *third order intermodulation point*, where the two lines shown in the right diagram intersect. Two signals (f_1,f_2 > f_1) which are closely spaced by Δf in frequency are simultaneously applied to the DUT. The intermodulation products appear at + Δf above f_2 and at – Δf below f_1.

- This intersection point is usually not measured directly, but extrapolated from measurement data at much lower power levels to avoid overload and/or damage of the DUT.