Tune and Chromaticity Diagnostics

Part I

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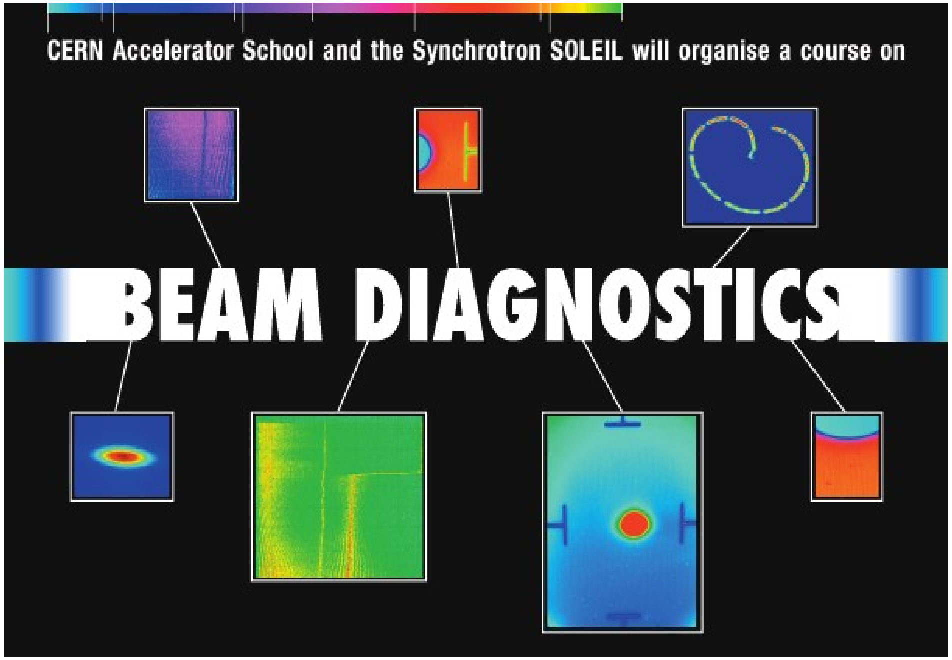
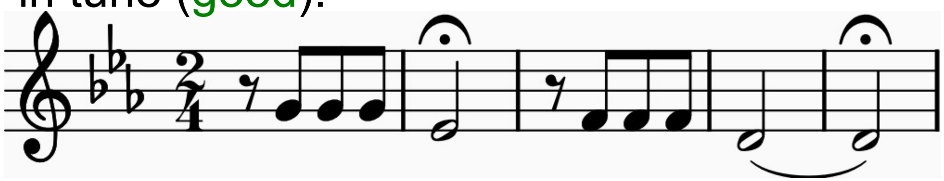
Tune Diagnostics - Primer

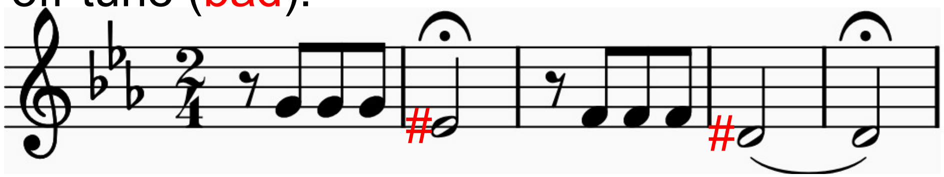


- Laymen/Musician's view (Beethoven's 5th):

in tune (good):



off-tune (bad):



- Audience will leave the concert

↔ Beam will leave the vacuum pipe

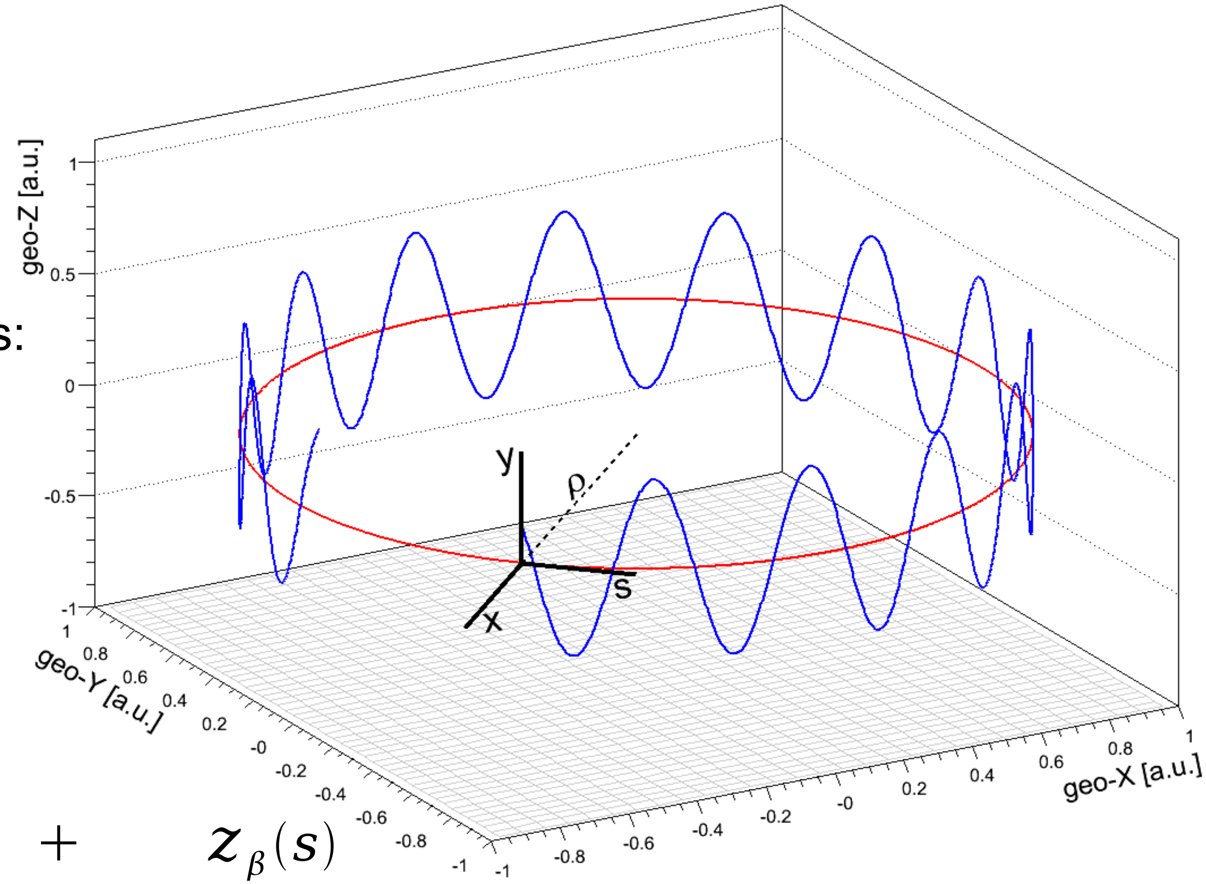
- Importance of tune:
  - defines beam life-time
  - strong impact on beam physics experiments:

"I don't think we've quite repeated the experiment - last time we did it, the glass gave out a middle 'c'."

- Hill's equation
  ... the mother of all accelerator physics:

$$z'' + k(s)\cdot z = f(s,t)$$

  - k(s): focusing strength, defines:
    - phase advance μ(s)
    - betatron function β(s)
  - f(s,t): driving force



- first-order solution:

$$z(s) = \underbrace{z_{co}(s)}_{closed\ orbit} + \underbrace{D(s)\cdot\frac{\Delta p}{p}}_{dispersion\ orbit} + \underbrace{z_\beta(s)}_{betatron\ oscillations}$$

  - D(s): dispersion function [m] → typically: few cm to a few meters
  - Δp/p: relative momentum offset w.r.t. c.o. → typically: 10^{-3} ...10^{-4}

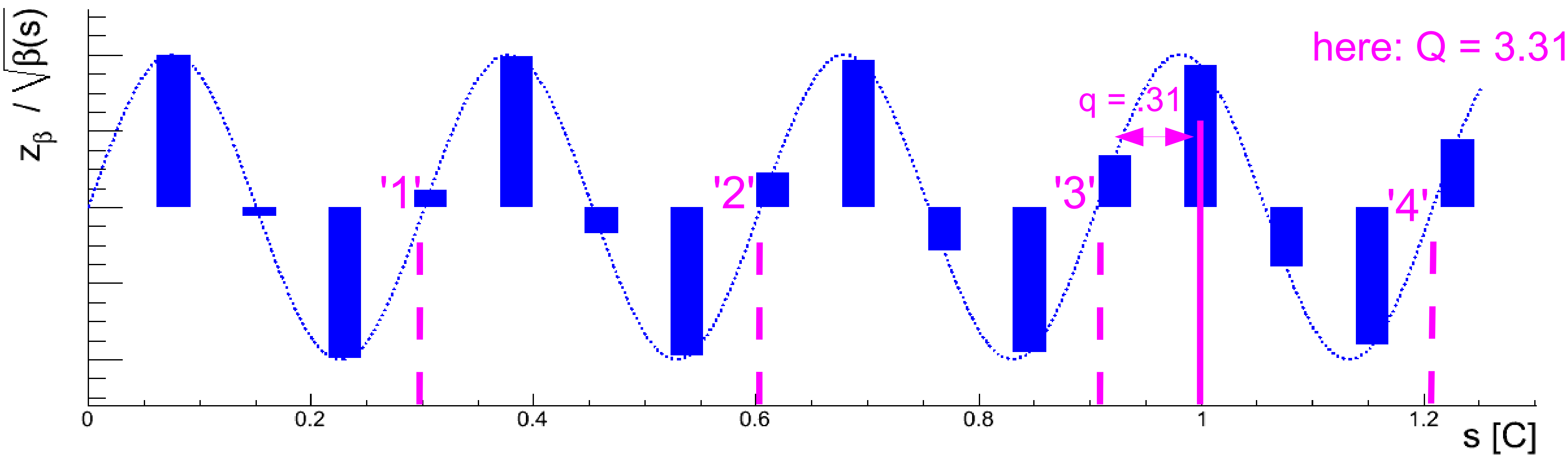
- Main tune dependent part:

$$z_\beta(s) = \sqrt{\epsilon_i \beta(s)}\cdot\sin(\mu(s)+\phi_i)$$

ε_i, Φ_i *: initial particle state*

→ particle describe sinusoidal oscillations in a circular accelerator

- Free Betatron Oscillations: $$z_\beta(s)=\sqrt{\epsilon_i \beta(s)}\cdot\sin(\mu(s)+\phi_i)$$



- Betatron Phase Advance: $\mu(s)$

- *Tune* defined as betatron phase advance over one turn:

$$Q := \frac{1}{2\pi}\oint_C \mu(s)\,ds$$

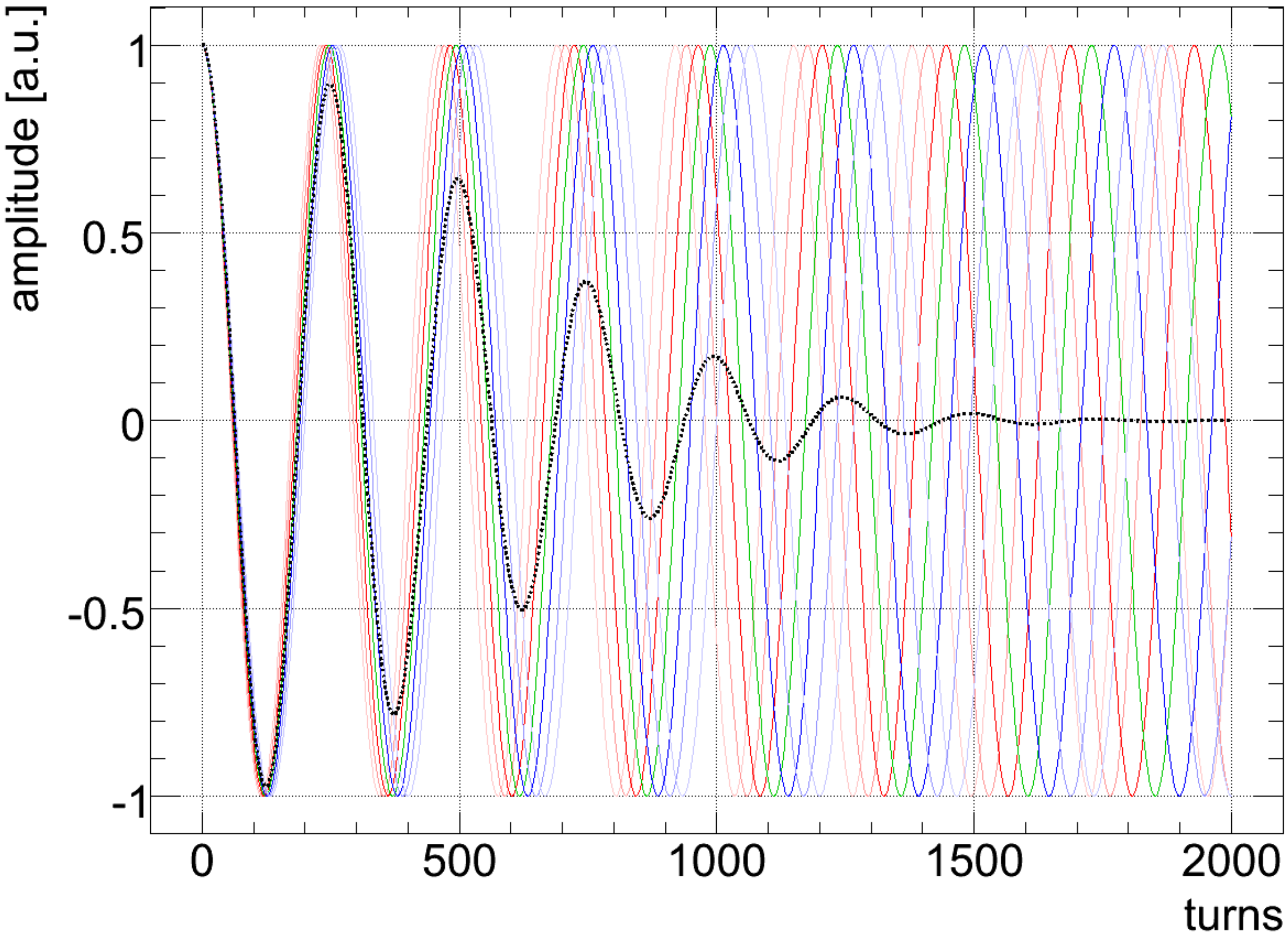
common: $Q = \underbrace{Q_{int}}_{integer\ tune} + \underbrace{q_{frac}}_{fractional\ tune}$

- Tune measurement options:
  1. Single-turn: 'count oscillations along circumference' (usually while threading 'first turn')
  2. Turn-by-turn: pick and observe the oscillation at a given single BPM

$$\Delta z_\beta=\sqrt{\epsilon_i\beta}\cdot\sin(\mu+\phi_i+2\pi Q\cdot n)$$

→ FFT analysis returns q_{frac}

- Individual bunch particles usually differ slightly w.r.t. their individual tune
  → Literature: "Landau Damping" (Historic misnomer: particle energy is preserved!)



  – E.g. if f(ΔQ) is a narrow Gaussian distribution with with σ_Q << Q:

$$\overline{z}(t)=\overline{z}_0\cdot e^{-\frac{1}{2}\cdot\sigma_Q^2 n^2}\cdot\cos(2\pi Q\cdot n)$$

dampening — tune oscillations

→ large tune spread ↔ fast damping of e.g. head-tail instabilities

→ Tune oscillations are usually damped

Part I:

- Recap: What the .... is 'Q', Oscillations Dampening → just done
  - Perturbation Sources, Requirements
- Tune Diagnostics
  - Classic Fourier-Transform Based
    - Detectors: BPMs, Diode-Peak-Detection, (Schottky → F. Casper)
  - Phase-Locked-Loop (PLL) Systems
- Advanced Topic → your choice

Part II: → in about an hour

- Recap: Definitions, Requirements & Constraints
- Classic Chromaticity Diagnostics
  - Momentum shift Δp/p based Q' tracking methods → LHC examples
- Collective Effects
  - Head-tail phase shift
  - De-coherence based methods: PLL Side-Exciter

Tune Perturbation Sources I/II – Quadrupole Driven

- Why do we need to measure the tune at all? Does it change?
- Quadrupole strength (hor. focusing): $k(s) = \frac{q}{p}\frac{\partial B}{\partial x}$
- Quadrupole gradient errors: $k(s) \rightarrow k_0(s) + \Delta k(s)$
  - saturation of iron yoke, magnet calibration errors, power converter ripple, etc.

$$\Delta Q = \frac{1}{4\pi}\,\beta(s)\cdot\Delta k(s)$$

→ watch out for quadrupole errors at large beta functions (e.g. final focus)!

- Energy perturbation $p \rightarrow p_0 + \frac{\Delta p}{p_0}$
  - Main dipoles vs. quadrupoles mismatch → *natural chromaticity* Q'_{nat}

$$\Delta Q = -\frac{1}{4\pi}\,\beta(s)\cdot\left(k(s)\cdot\frac{\Delta p}{p_0}\right) \sim Q'_{nat.}\cdot\frac{\Delta p}{p_0}$$

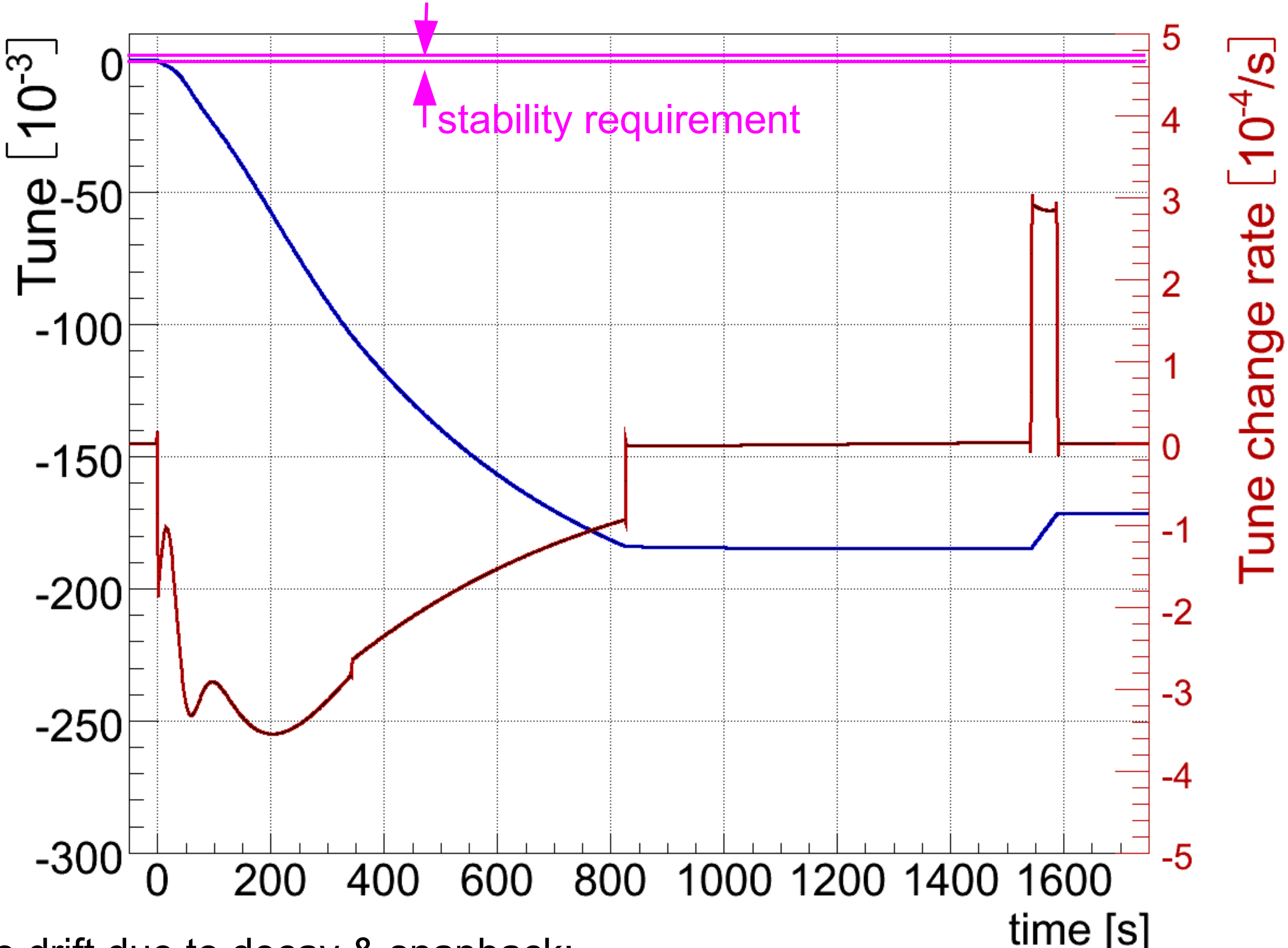
  - RF frequency change (aka. radial steering)

subtle but important difference: LHC: $Q'_{nat} \approx -140$ but $Q' \approx 1$ → next lecture

$$\Delta Q := Q'\cdot\frac{\Delta p}{p_0}$$

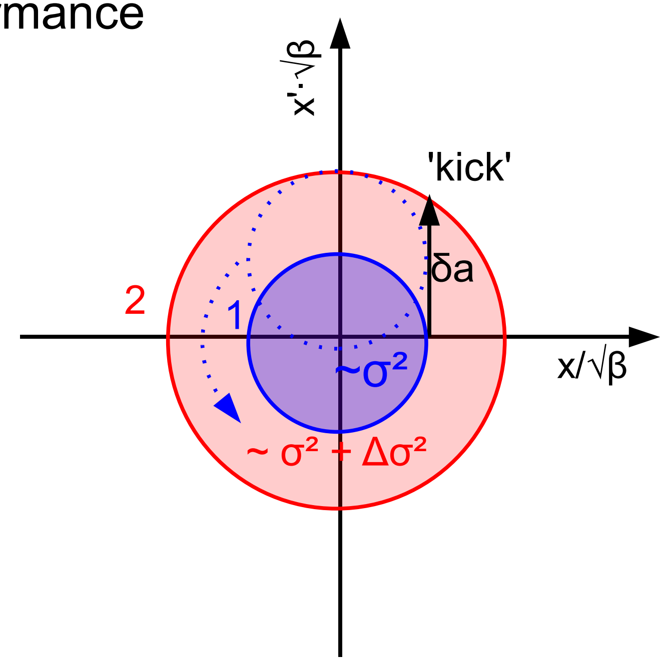
→ defines machine's *chromaticity* Q'

→ bottom line: tune is usually not a constant



- LHC Tune drift due to decay & snapback:
  - effect intrinsic to superconducting magnets
  - Tune drift (without b_3 effects): $\Delta Q \approx 0.1$
  - Tune change rate: $\Delta Q/\Delta t|_{max} < 10^{-3}\ s^{-1}$

- Transverse beam size as an impact on accelerator performance
  - smaller beam-sizes σ favourable
    - HEP colliders: higher luminosity
    - Light Sources: higher brightness



- beam size increases quadratically with angular kick δa

$$\frac{\Delta \sigma}{\sigma} \approx \frac{1}{2}\left(\frac{\delta a}{\sigma}\right)^2$$

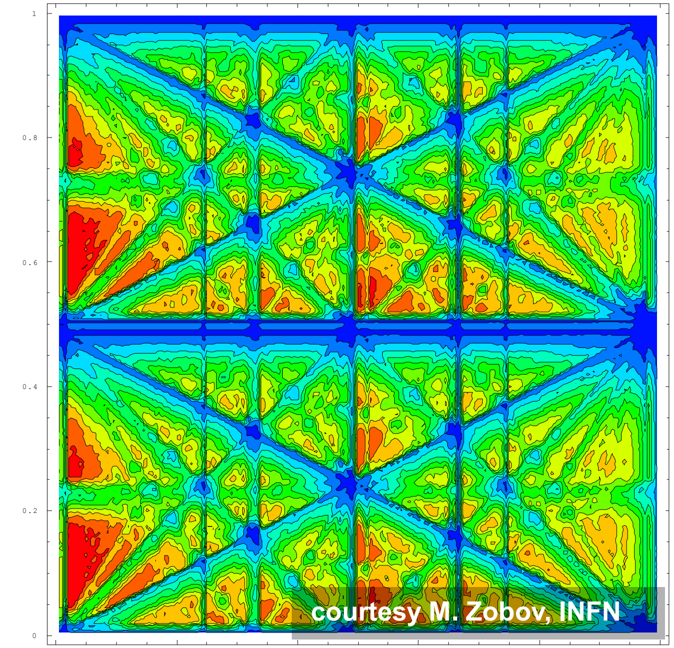
  - N.B. for electrons, esp. synchrotron light sources, this is partially compensated by energy losses due to synchrotron light radiation.
  - Protons: memory effect – the beam does not forgive...!
    - LHC limit: δa << 10 μm = ~1/20 σ !!
- Further constraints on kick amplitudes: aperture limitations due to functional insertion, machine protection systems, ..

→ Limit excitation to necessary minimum, favours passive/sensitive systems

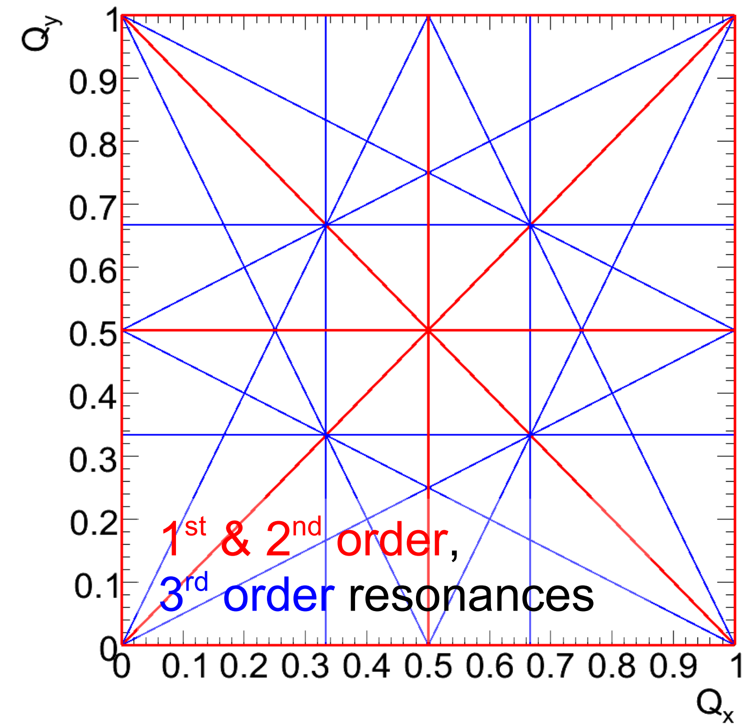
- Unstable particle motion reduces beam-lifetime (~dynamic aperture) if resonance condition is met:

$$p = m \cdot Q_x + n \cdot Q_y \quad \wedge \quad m, n, p \in \mathbb{Z}$$

  - similar relation also in between Q_x & Q_s (important for lepton accelerators)
- Resonance order: $O = |m| + |n|$

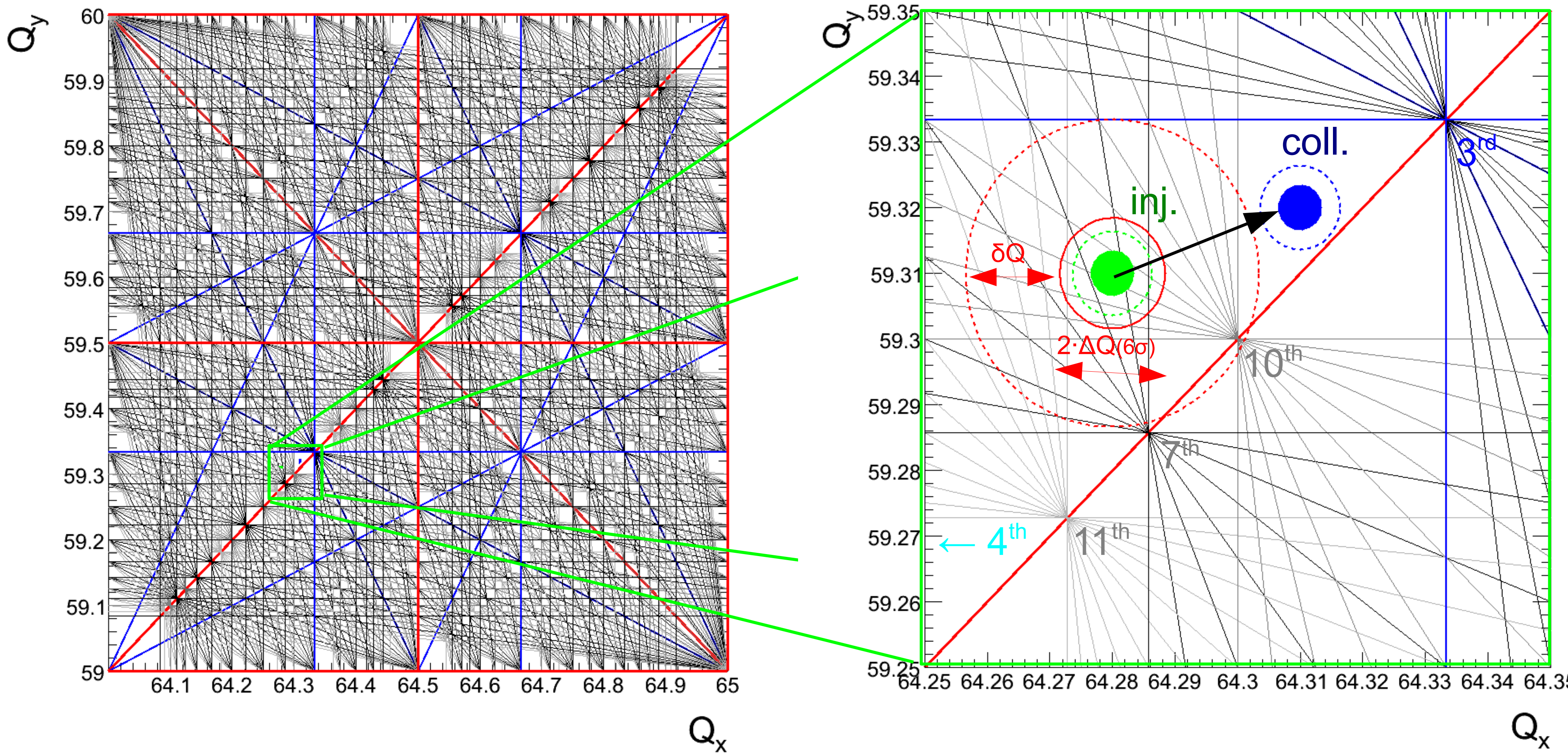


- Lepton accelerator: avoid up to ~ 3rd order
- Hadron colliders:
  - negligible synchrotron radiation damping
  - need often to avoid up to the 12th order



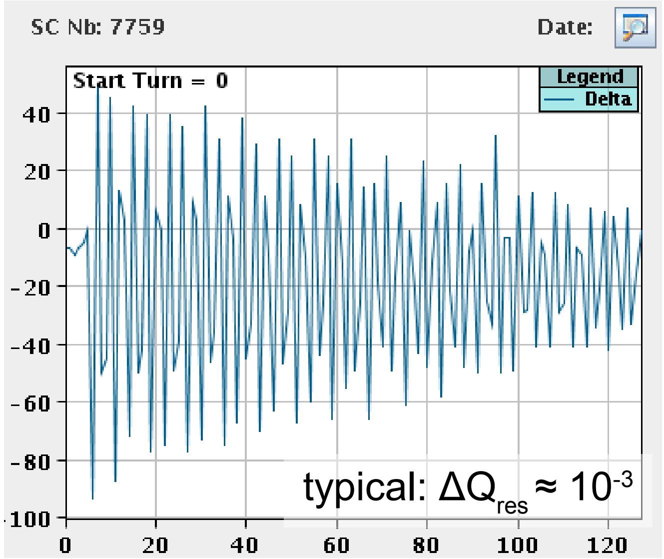
"Hadron beams are like elephants – treat them bad and they'll never forgive you!"

- Example LHC: Tune stability requirement: $\Delta Q \approx 0.001$ vs. exp. drifts ~ 0.06

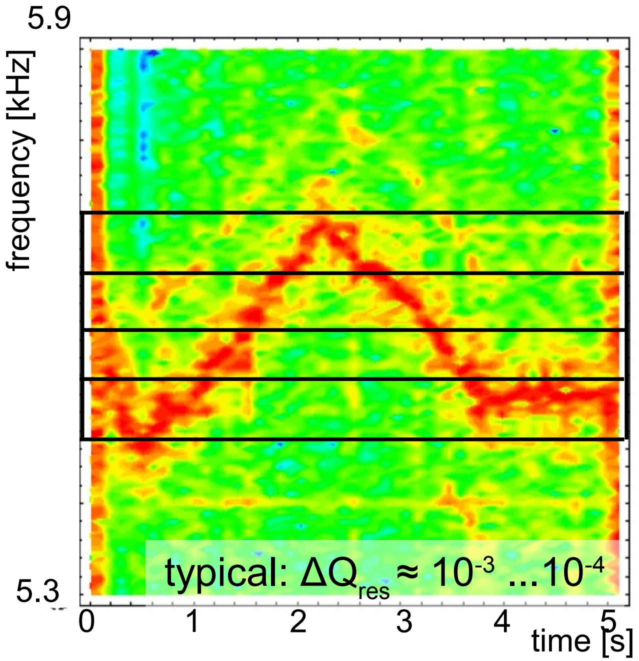


- N.B. need to stay much further off these resonance lines due to
  – finite tune width: chromaticity, space charge, momentum spread, detuning with amplitude and resonance's stop band itself

- Classic, using BPMs with 'kick' or 'chirp' excitation
  - limited by aperture constraints
    - Performance reduction
      - typically: $\Delta z \leqslant 0.1\sigma$
    - Loss of particles & protection
      - LHC: $\Delta z \leq 25$ µm & $\Delta p/p \leq 5 \cdot 10^{-5}$
  - limited by emittance blow-up



- Passive monitoring of residual oscillations:
  - Schottky monitors
  - Diode-Detection based Base-Band-Q (BBQ) meter



- Active Phase-Locked-Loop (PLL) systems
  - In combination with RF modulation
    → chromaticity tracking

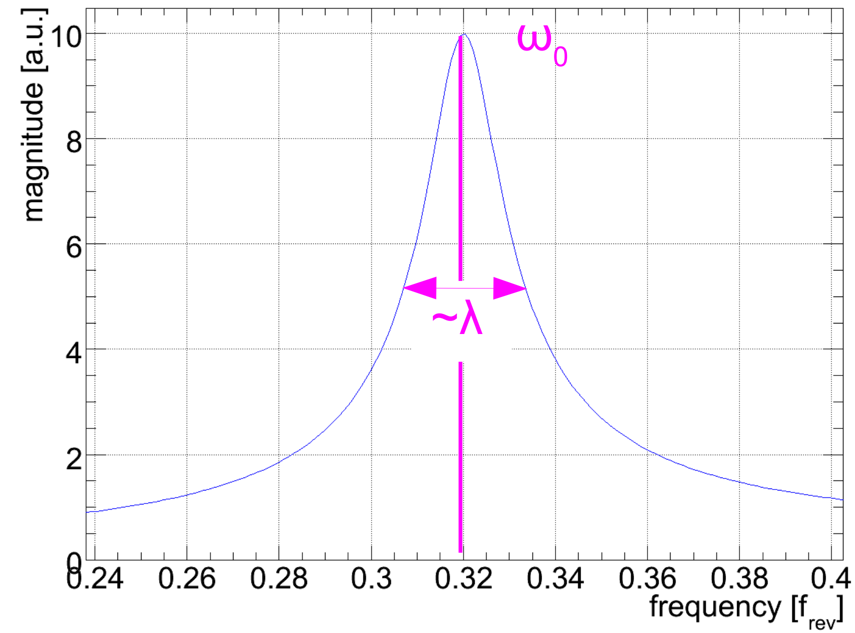
typical: $\Delta Q_{res} \approx 10^{-3} \ldots 10^{-5}$

Tune Diagnostics Principle

- Control Theory → System Identification

- Example (first order) beam response ≈ damped harmonic oscillator resonance
  (ω_0: resonant frequency (Q), λ: tune resonance width (σ_Q),
  ω: driving frequency)

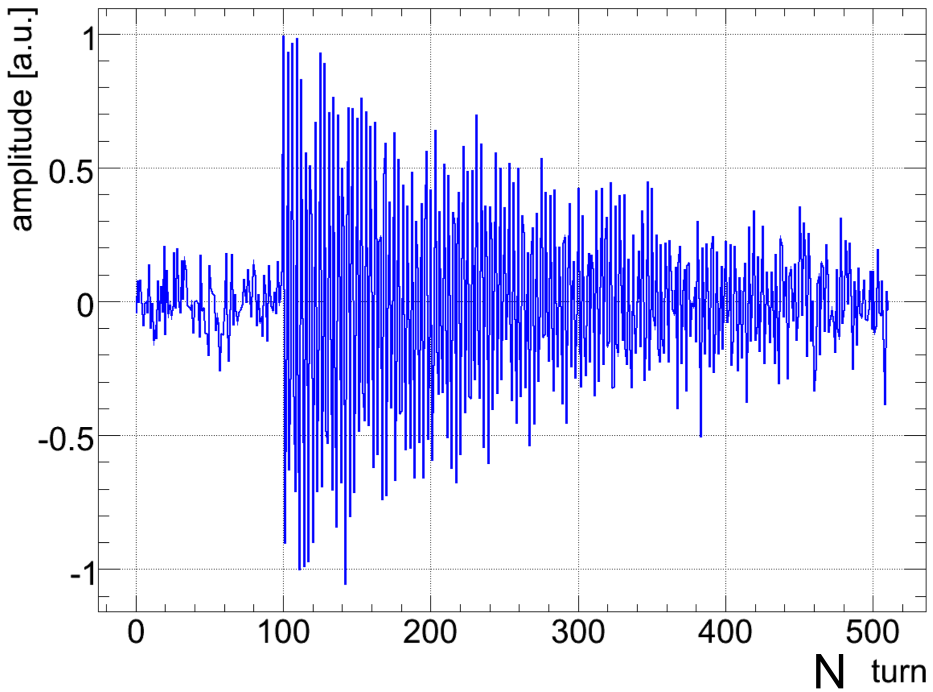
$$\left|G(\omega)\right| := \left|\frac{X(s)}{E(s)}\right| \approx \frac{\omega_0}{\sqrt{\left(\omega^2-\omega_0^2\right)^2+\left(2\lambda\omega_0\omega\right)^2}}$$



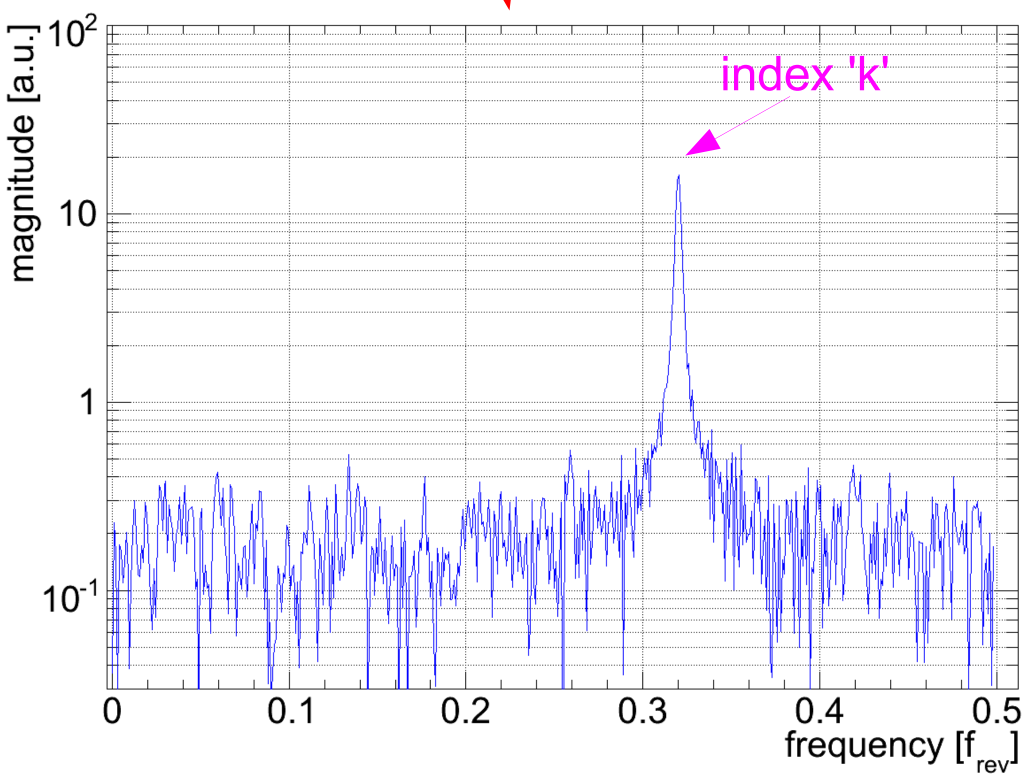
- Excitation choices:
  - White or remnant noise
    - no information on signal phase
  - Single-turn transverse kick (classic)
  - Frequency Sweep aka. 'Chirp'
    - focuses excitation power on frequency range of interest → less ε-blow-up, constant power
  - Phase-Locked-Loop Systems = resonant excitation on the Tune

- Note: Exciter and pickup have additional non-beam related responses!

- .... how an kick-induced beam oscillation usually looks like (no sync. beating)



FFT



- Fourier analysis of turn-by-turn data:
  - magnitude peaks at q_{frac}

$$q_{frac} \approx \frac{k}{N}$$

  - N.B. no information on Q_{int} !
  - improve resolution by fitting central bin width → additional topic

Underlying measurement related to BPM design:

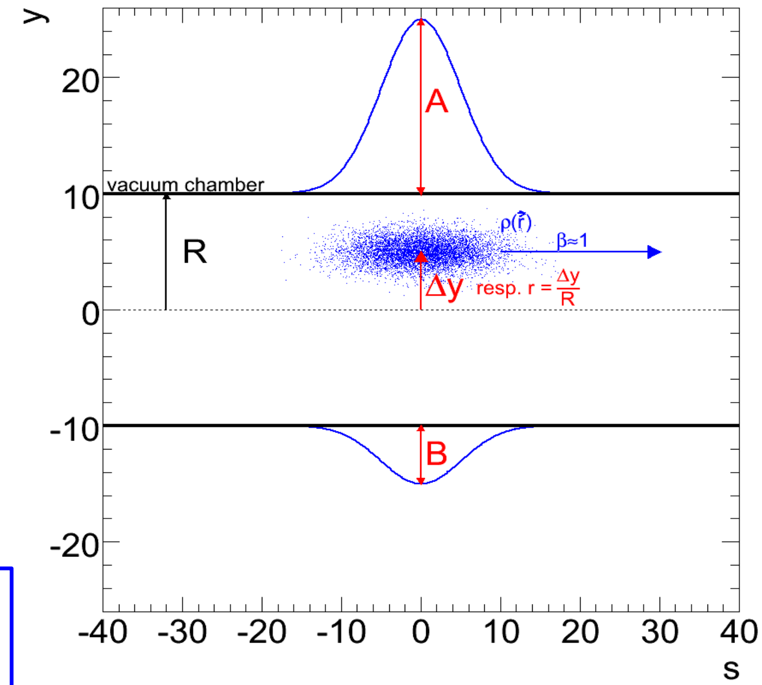
- Usual choices:
  - wall-current, button, shoebox, strip-line pickup (→ P. Fork lecture)
  - resonant pickups (e.g. Schottky → F. Caspers)

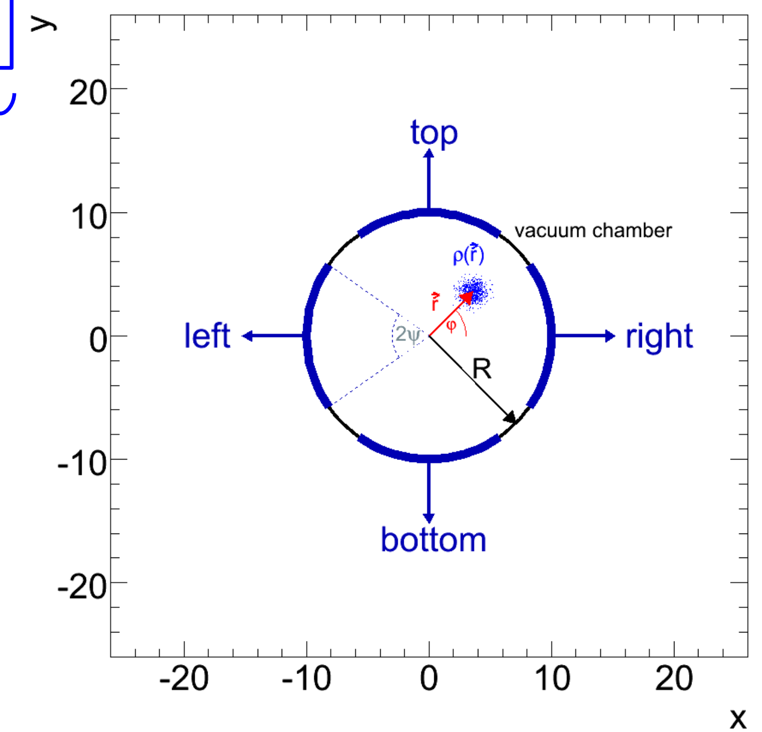
- Single charge image density on pickup segment[1]:

$$I_{L/R}(t)=\frac{I_{\omega}(t)}{2\pi}\left[2\psi\mp 2\frac{x}{R}\sin(\psi)+\frac{x^2-y^2}{R^2}\sin(2\psi)+h.o.\right]$$

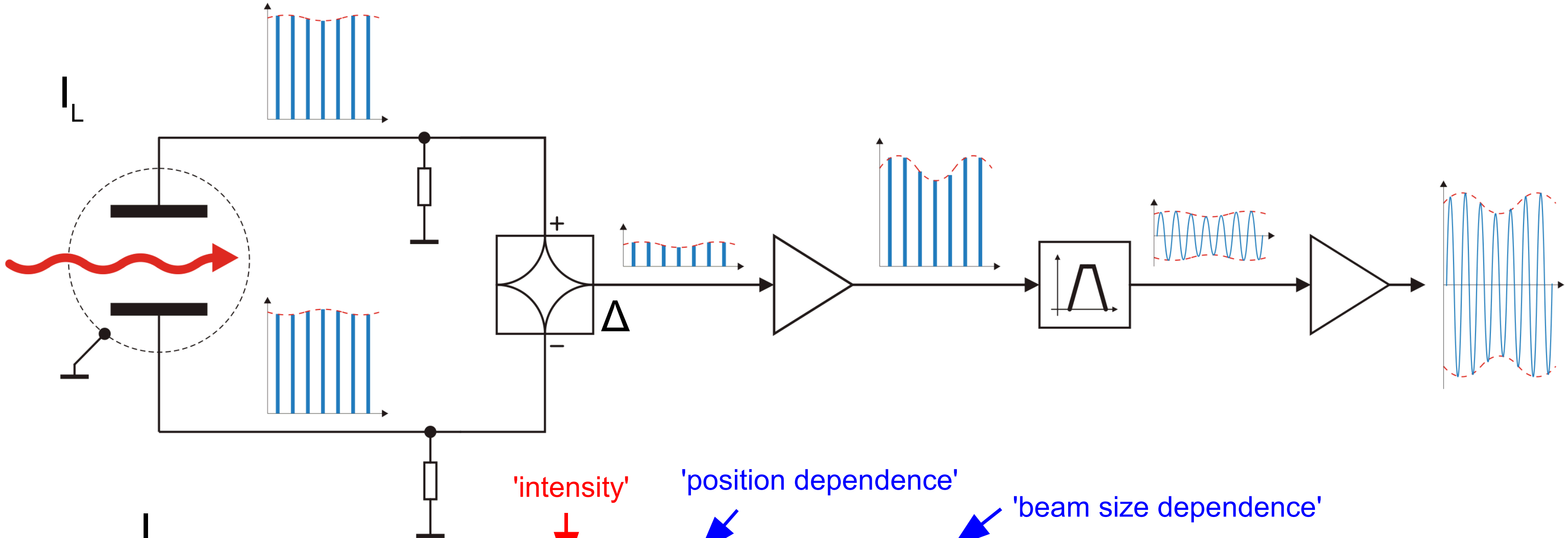
longitudinal beam signal (first factor) — transverse beam signal (bracket)

  - real-life signal is usually further convoluted with pickup and acquisition electronics response[2,3]!
  - will elaborate a bit more on above equation





[1]R. Littauer, *“Beam Instrumentation”*, *SLAC Summer School*, 1982. (p.902)
[2]D. McGinnis, *“The Design of Beam Pickup and Kickers”*, BIW'94, 1994
[3]G. Vismara, *“Signal Processing for Beam Position Monitors”*, CERN-SL-2000-056-BI



'intensity' 'position dependence' 'beam size dependence'

$$I_{L/R}(t)=\frac{I_\omega(t)}{2\pi}\left[2\psi \mp 2\frac{x}{R}\sin(\psi)+\frac{x^2-y^2}{R^2}\sin(2\psi)+h.o.\right]$$

- Classic detection approach: Σ-Δ hybrid (or direct pickup signal sampling)

$$\rightarrow \boxed{\frac{x}{R}\approx\frac{\Delta}{\Sigma}=\frac{I_L-I_R}{I_L+I_R}}$$

R: pickup half-aperture

  – Eliminates most 'common mode' signal (e.g. intensity),
  – However ADC needs still to accommodate 'common mode' signals due to:
    - Closed orbit offset
    - 2^{nd} order: intensity bleed-trough intrinsic to any Σ-Δ hybrid

- A little bit in more detail:

$$I_{L/R}(t) = \underbrace{\frac{I_\omega(\sigma_s,t)}{2\pi}}_{\text{longitudinal beam signal (PM)}} \cdot \underbrace{\left[2\psi \mp 2\frac{x}{R}\sin(\psi) + \frac{x^2-y^2}{R^2}\sin(2\psi) + h.o.\right]}_{\text{transverse beam signal (AM)}}$$

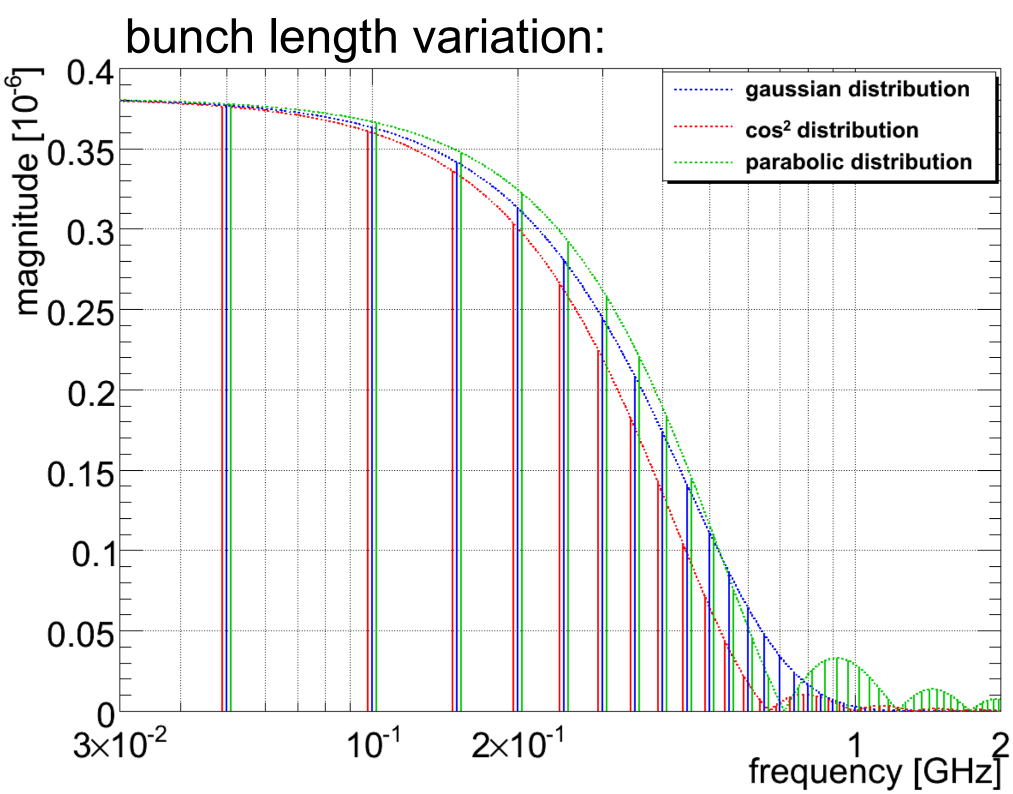
- N.B. multiplication in time-domain ↔ convolution in frequency domain
- Some important observations:
  1. Transverse pickups are also sensitive to modulation of the longitudinal carrier signal
  2. For tune measurement important beam-observable is x_β:

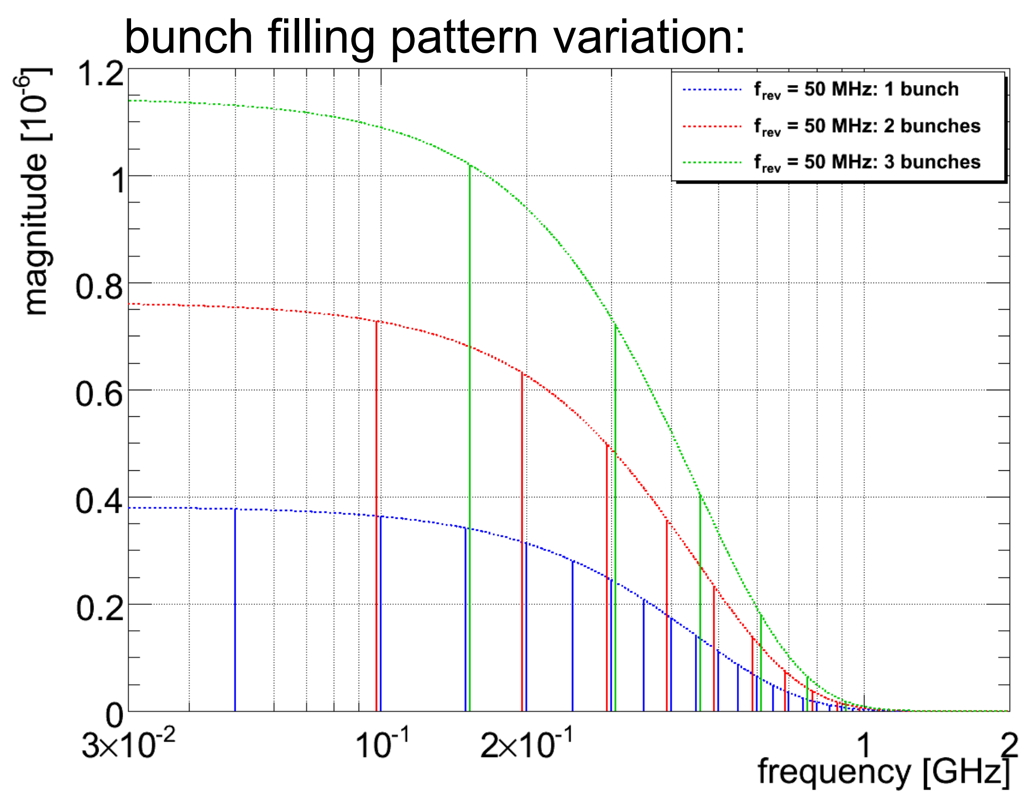
$$x \rightarrow x_{co} + D\cdot\frac{\Delta p}{p} + x_\beta \rightarrow I_{L/R}(t) \sim I_{CM} + \Delta I(x_{beta})$$

     - 'Common-mode' signal I_{CM} limits dynamic range and ADC resolution
     - Example: R ≈ 44 mm & nm resolution → required sensitivity $\Delta I/I_{CM} \sim 10^{-8}$
       - most BPM systems: $\Delta I/I_{CM} \sim 10^{-3}$
       - with e.g. good Σ-Δ hybrid: $\Delta I/I_{CM} \sim 10^{-5}$

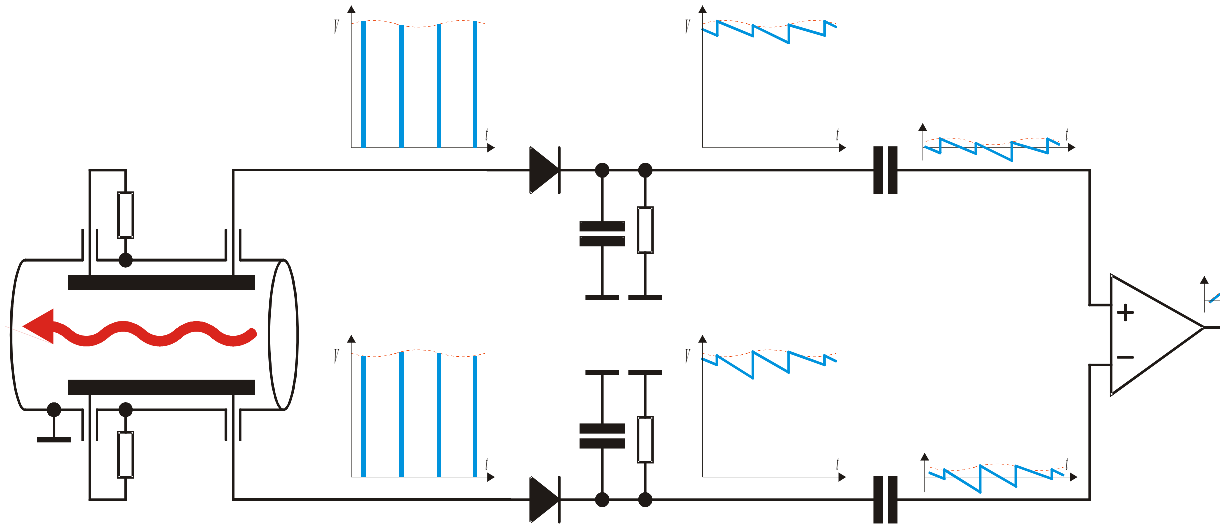
       → need something different for 'nm' resolution
  3. Higher Order term 'x^2-y^2': $I_{L/R}(t)$ sensitive to beam size → a.k.a. 'quadrupolar pickup'

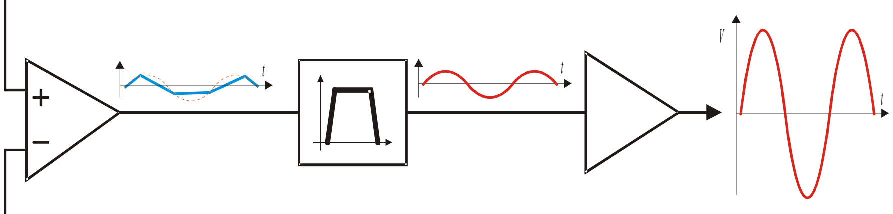
Longitudinal Bunch Spectrum Variation





- Longitudinal carrier signal changes with shape, arrival time (synchrotron oscillations) and number of circulating bunches:
  - processing chain has to accommodate this through e.g. multiple gain stages
  - optimise for one bandwidth → in-/less sensitive if number of bunches change

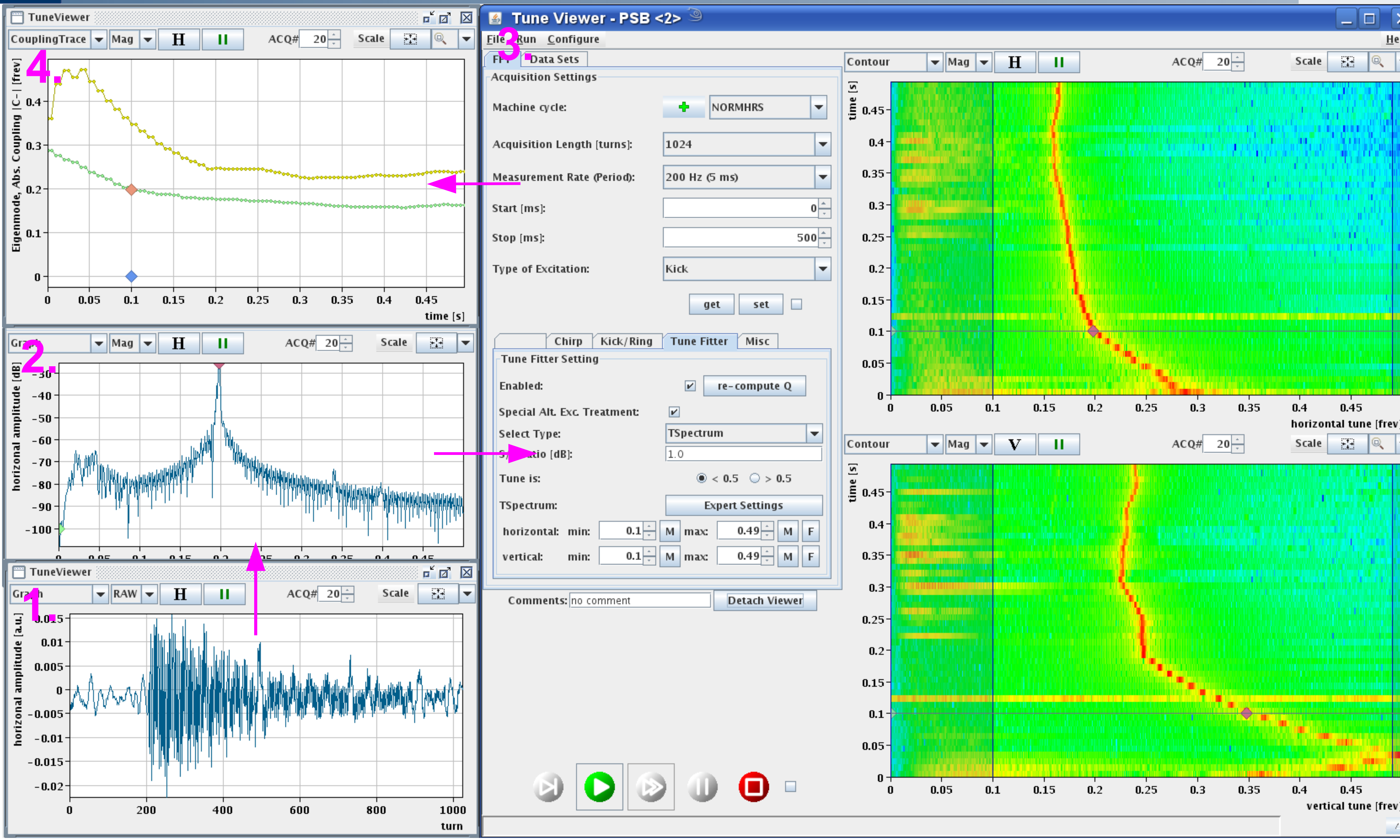




- Basic principle: AC-coupled peak detector[1]
  - intrinsically down samples spectra: ... GHz → kHz (independent on filling pattern)
    - thus 'Base-Band-Tune Meter' (aka. BBQ)
    - Base-band operation: very high sensitivity/resolution ADC available
    - Measured resolution estimate: < 10 nm → ε blow-up is a non-issue
  - AC-coupling removes common-mode → only relative changes play a role
    - capacitance keeps the "memory" of the to be rejected signal
  - no saturation, self-triggered, no gain changes to accommodate single vs. multiple bunches or low vs. high intensity beam
- However: no specific bunch-by-bunch information (unless using gating)

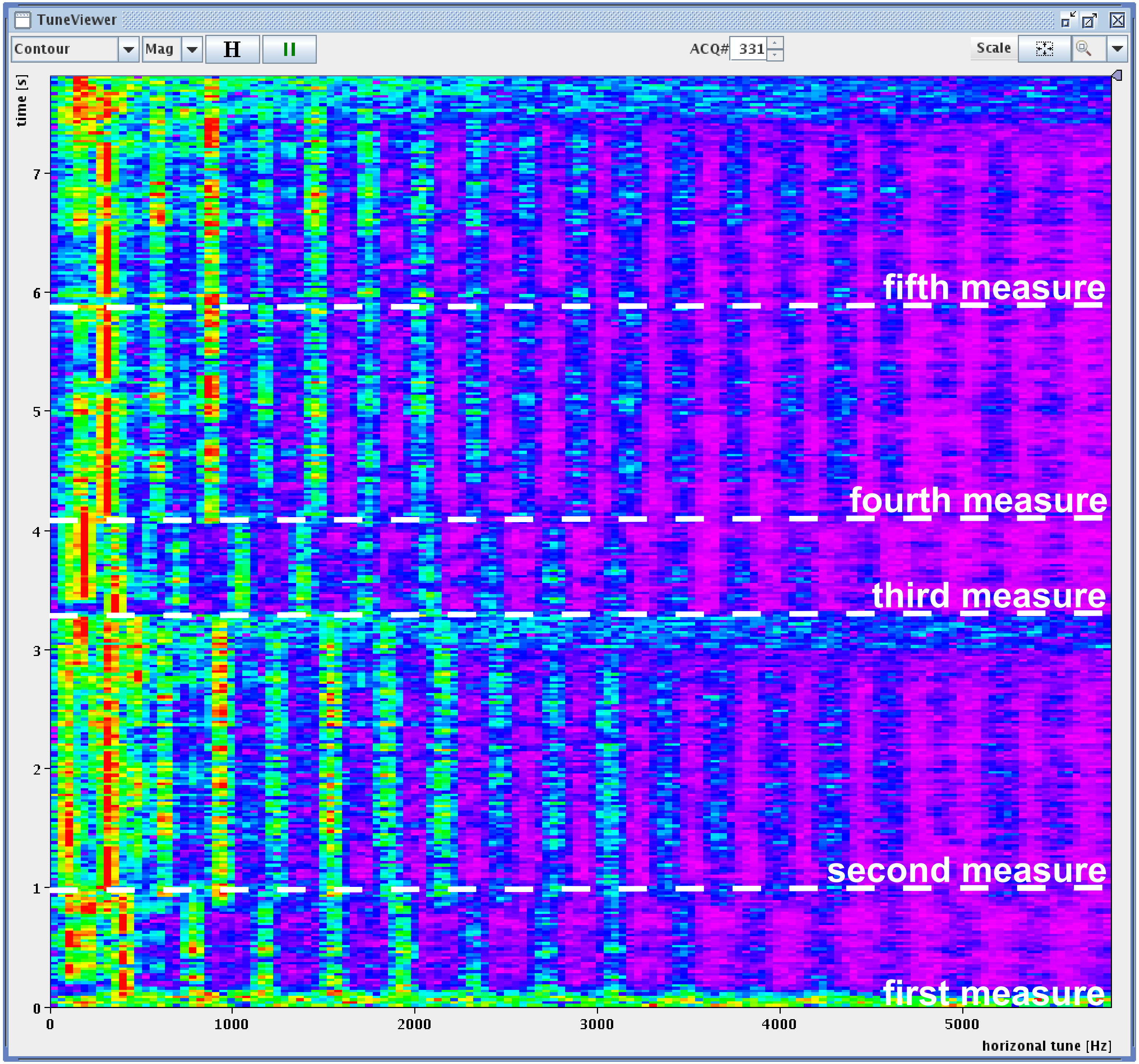
[1]M. Gasior, *"The principle and first results of betatron tune measurement by direct diode detection"*, CERN-LHC-Project-Report-853, 2005

BBQ Example Spectra
CERN-PSB, $f_{rev} \approx 2$ MHz

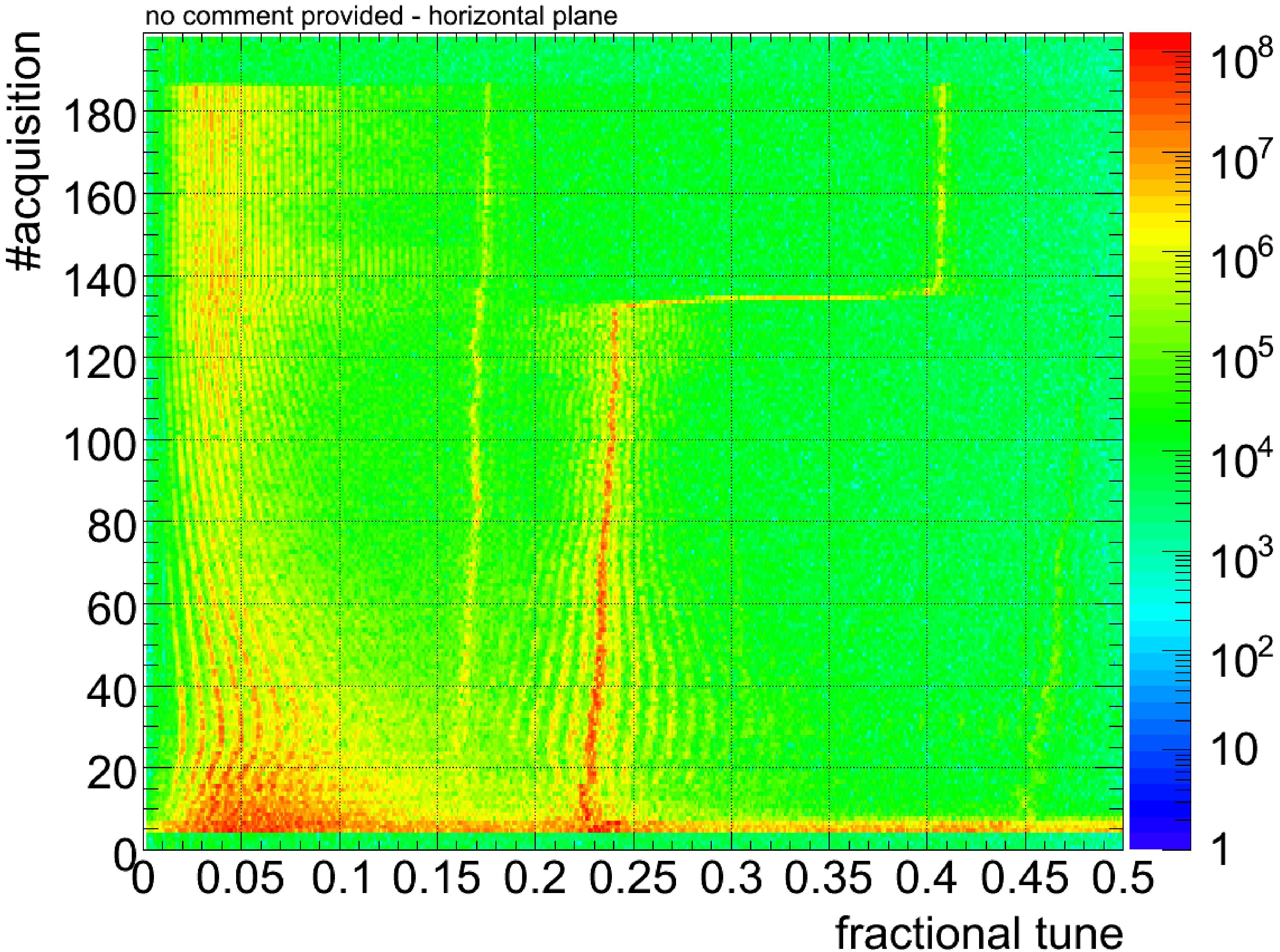


- BBQ → fast ADC → FPGA based digital signal processing chain, FFTs @ 500 – 1 kHz!
  - provides real-time Q diagnostics for operation

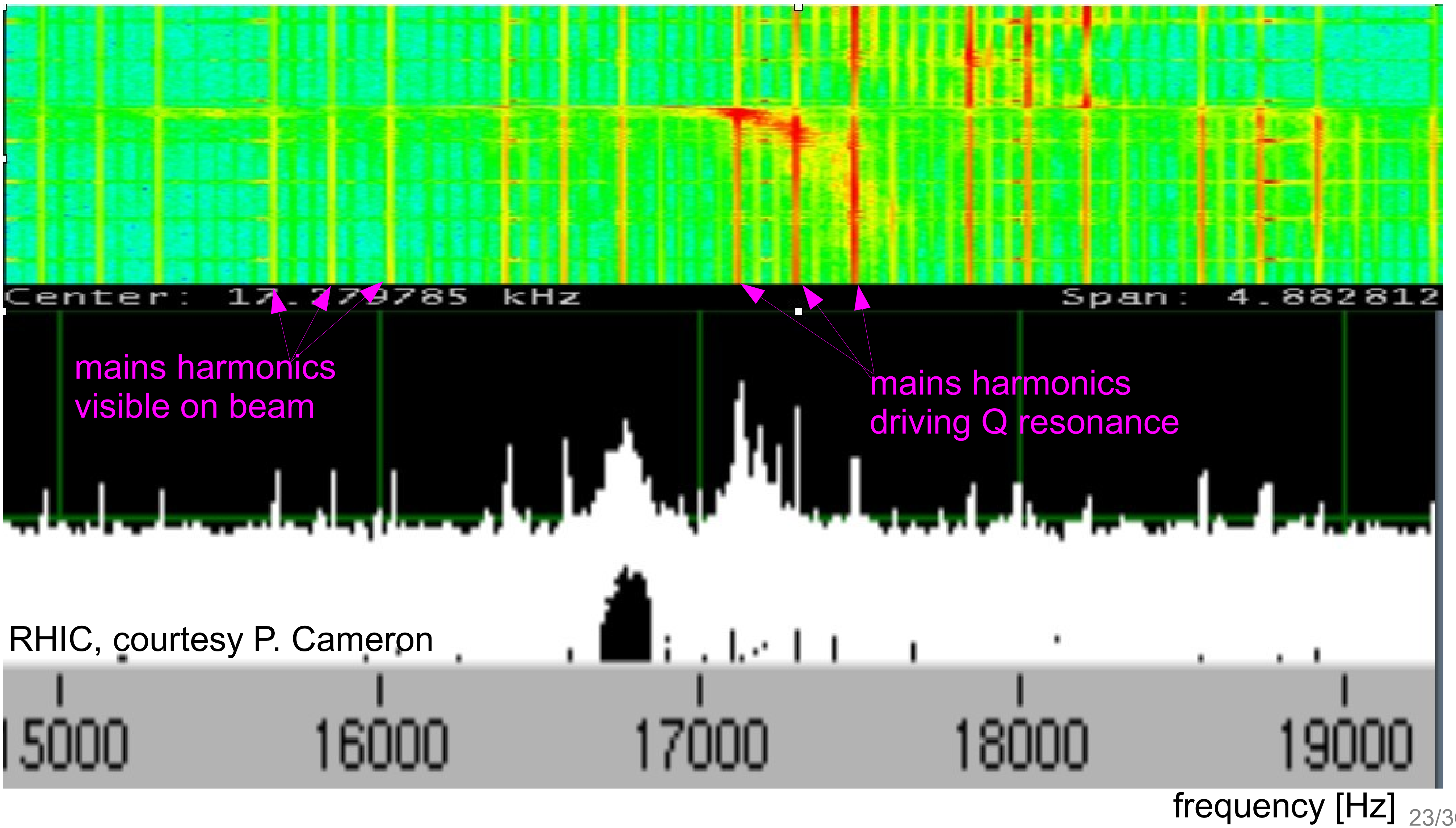
Reference Spectra
Beethoven's 5th, First Five Measures

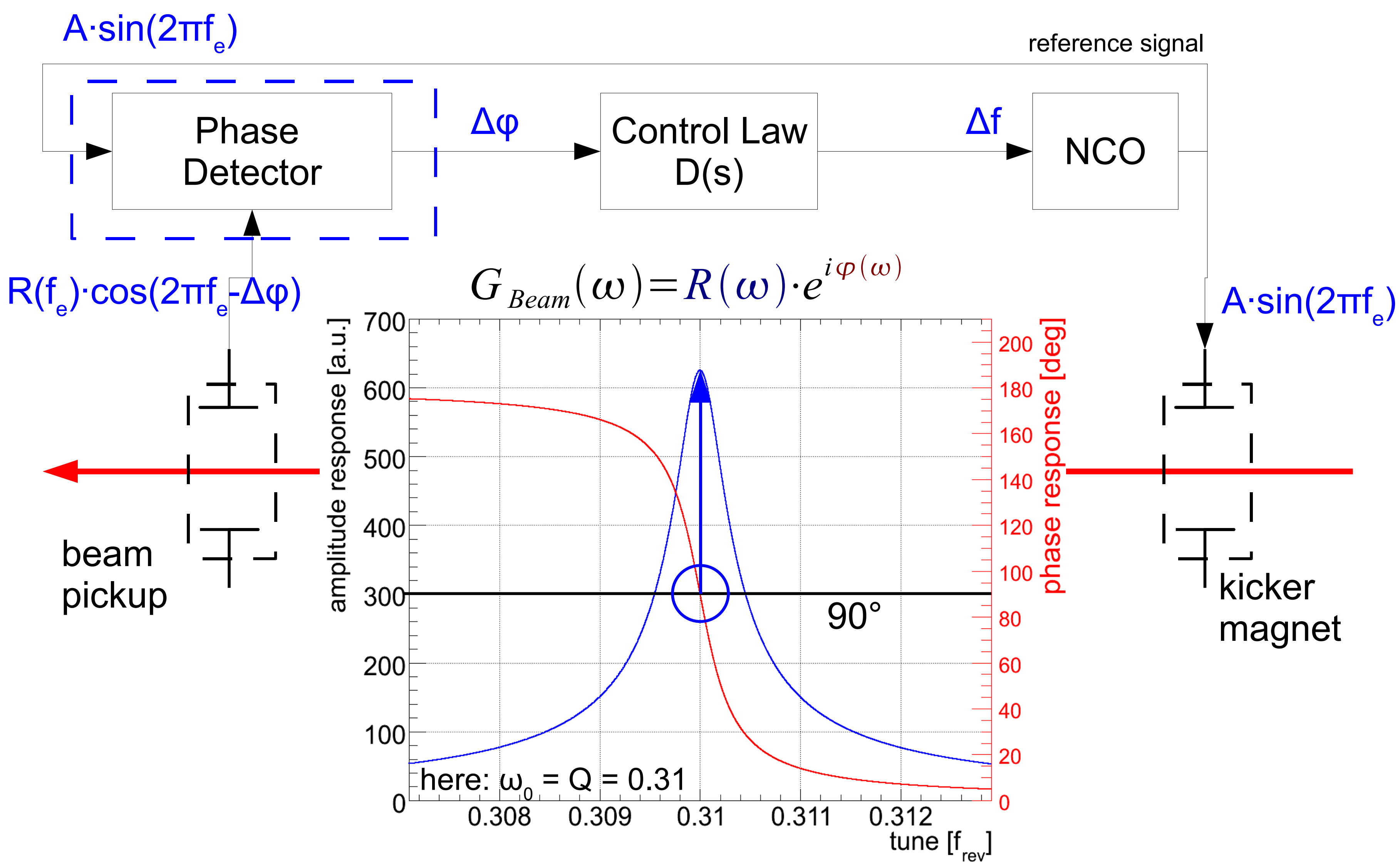


BBQ Example Spectra – without Excitation
LHC Testbeds: CERN-SPS $f_{rev} \approx 43$ kHz, LHC Beam



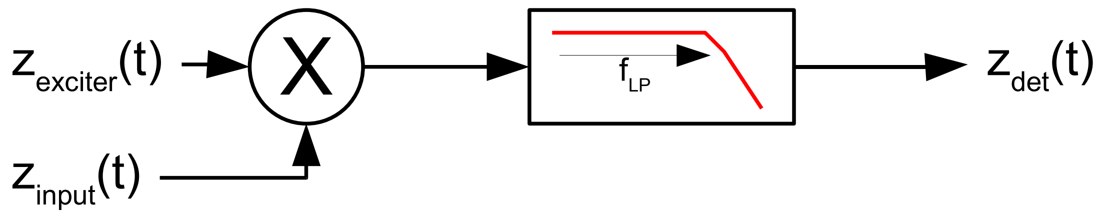
- BBQ system's high sensitivity revealed mains harmonic at RHIC and Tevatron
  - drives beam at tune resonance → emittance blow-up, particle loss





- BTF provides also information on collective effects (landau → spread distribution):
  - impedance, stability diagram, lattice non-linearities (Q', Q''), etc.

Classic PLL Detector



$$z_{det}(t) = LP\left(z_{input}(t)\cdot z_{exciter}(t)\right)$$
$$= LP\left(R(f_e)\cdot\cos(2\pi f_e - \Delta\varphi(t))\cdot A\sin(2\pi f_e)\right)$$

$$= \frac{AR}{2}\sin(\Delta\varphi(t)) + \frac{AR}{2}\sin(4\pi f_e - \Delta\varphi(t))$$

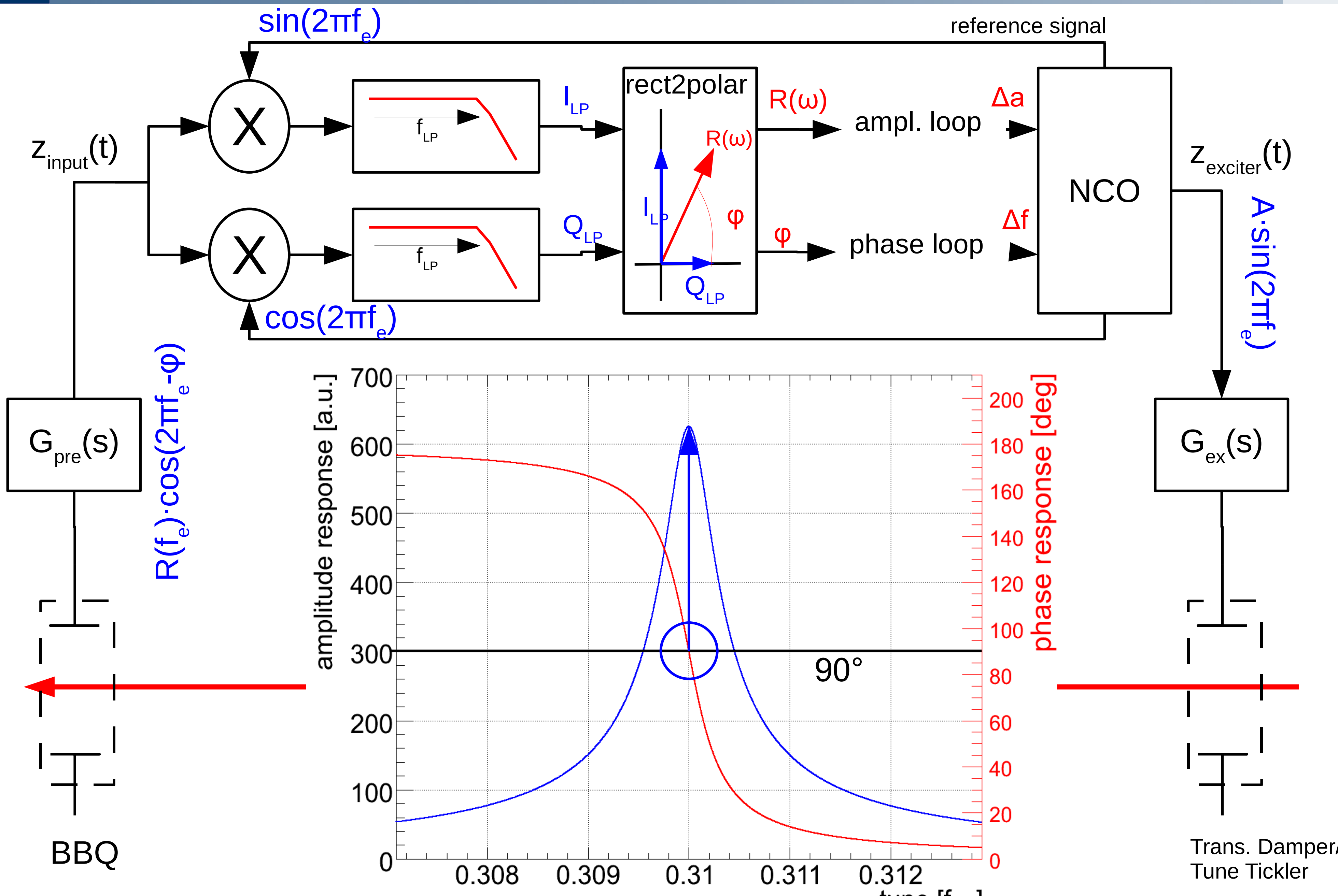
for small phases

$$\approx \Delta\varphi(t)$$

removed by low-pass filter

- Pro: robust analogue circuit implementation possible
- Con:
  - non-linear control signal for large phase difference Δφ
  - Control signal depends on beam response's amplitude $R(f_e)$

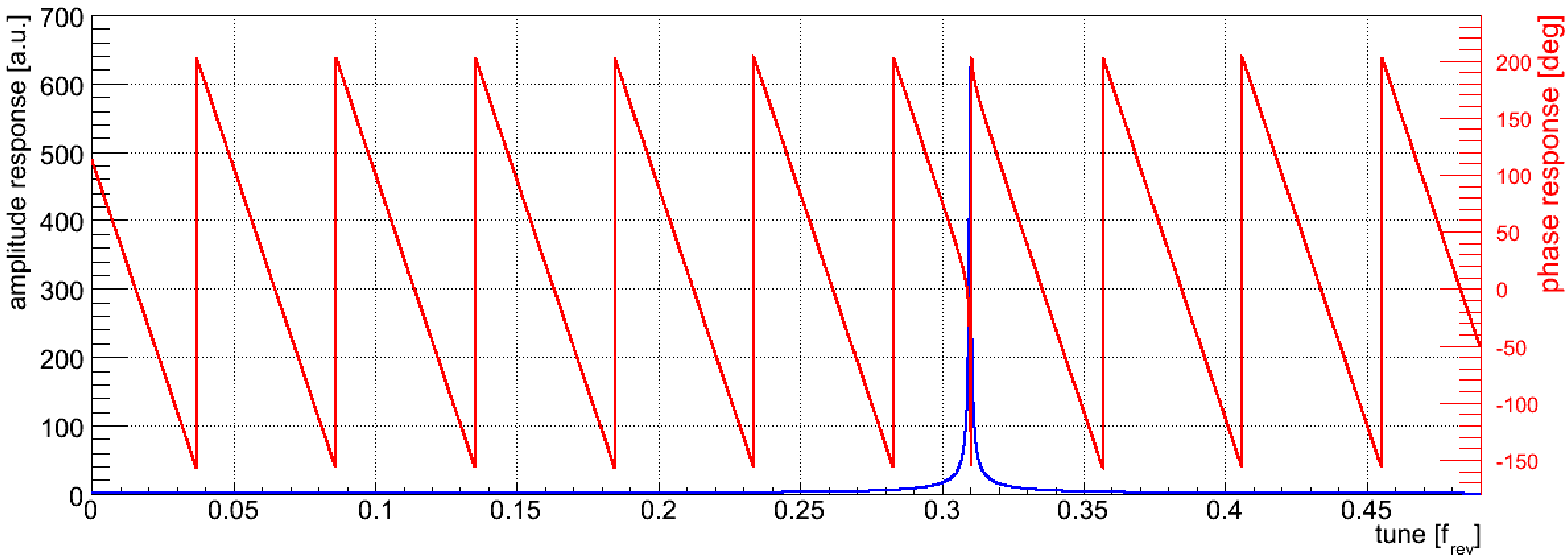
Advanced Phase-Locked-Loop Scheme



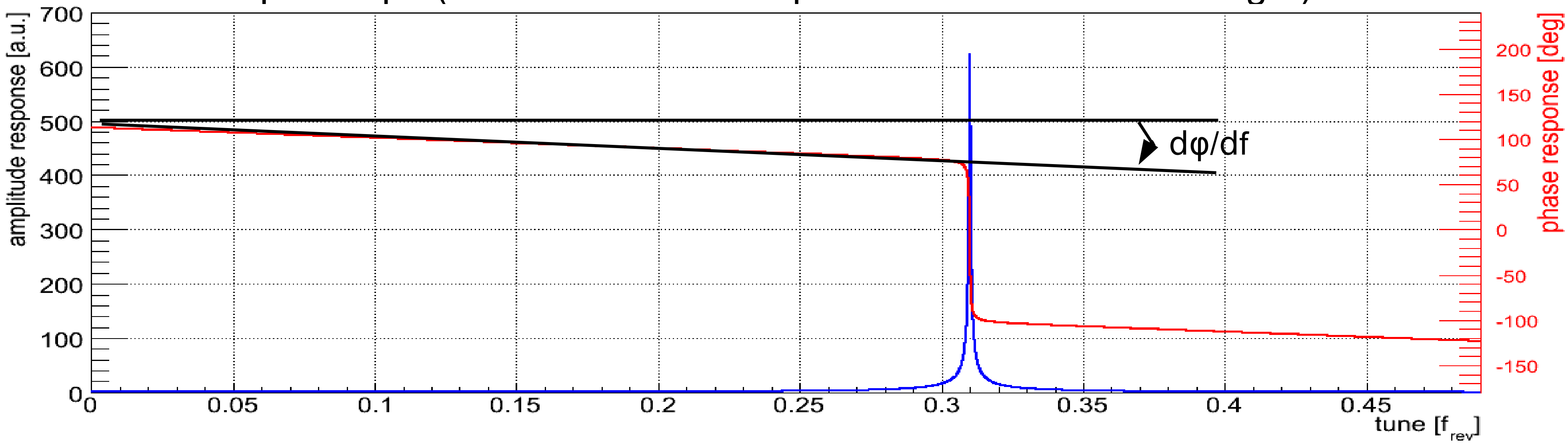
$$G_{Beam}(\omega) = R(\omega) \cdot e^{i\varphi(\omega)}$$

Example: PLL Setup – Step I (HW lag compensation)

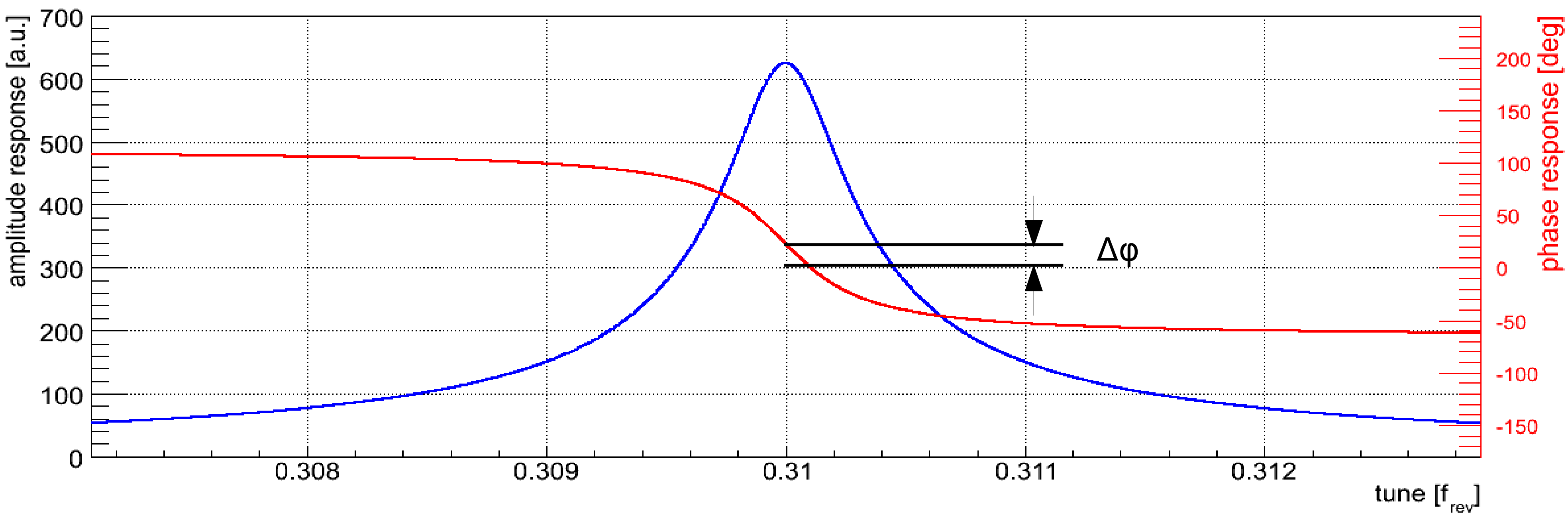
- BTF functions do not always look always as pretty as reports suggests or claim – an insider view on the real story:
- BTF and compensation consists of the adjustment of four parameters, preferably with stable beam condition ('chicken-egg' problem)
  - 1st step: verify necessary excitation amplitude and plane mapping (obvious?)
  - 2nd step: verify long sample delay (once per installation, constant)
    - full range BTF and count ±π wrap-around → number of delayed samples



- Measure dφ/df slope (~ front-end non-lin. phase and kicker cable length)

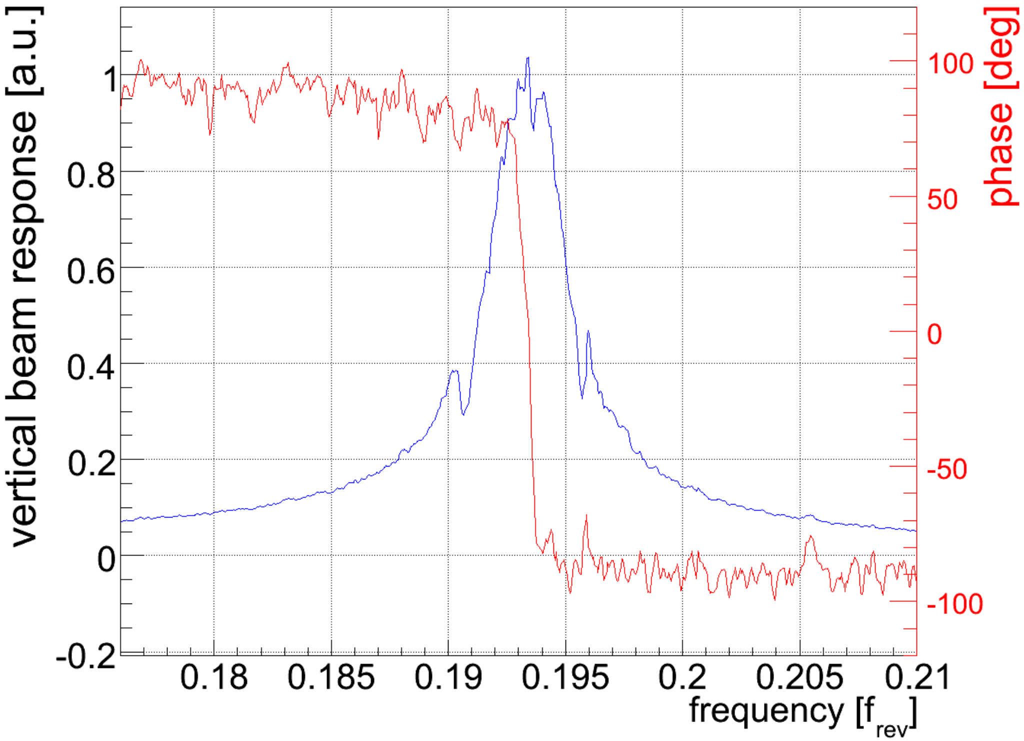


- Adjustments of the locking phase (tune-peak – phase matching)

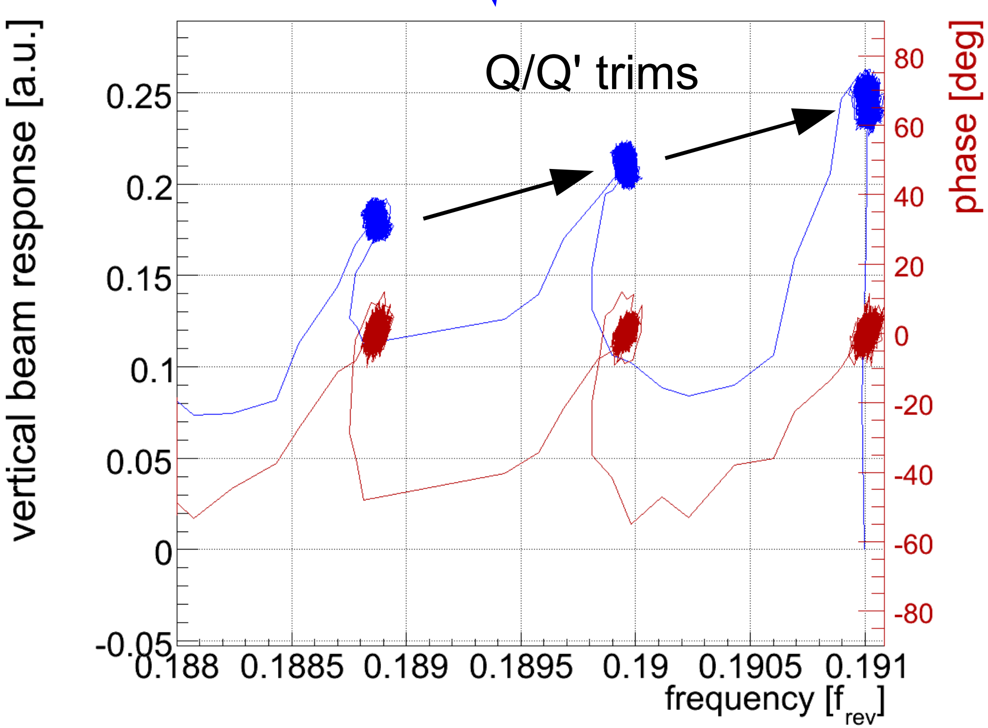


Example: PLL Setup – Step III → Ready for Q/C⁻/Q' Tracking

- What's published in papers and CAS reports:

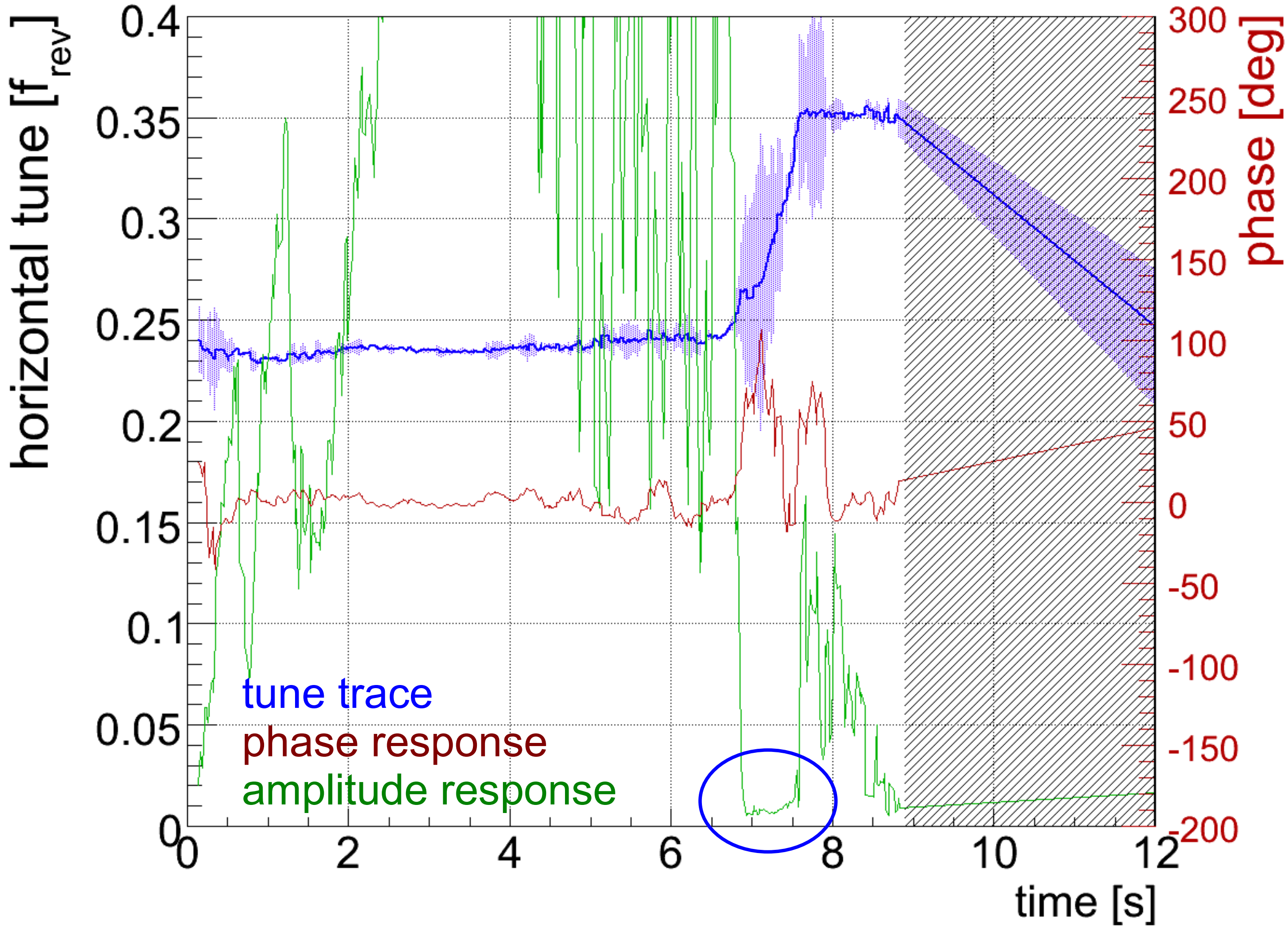


switch on PLL

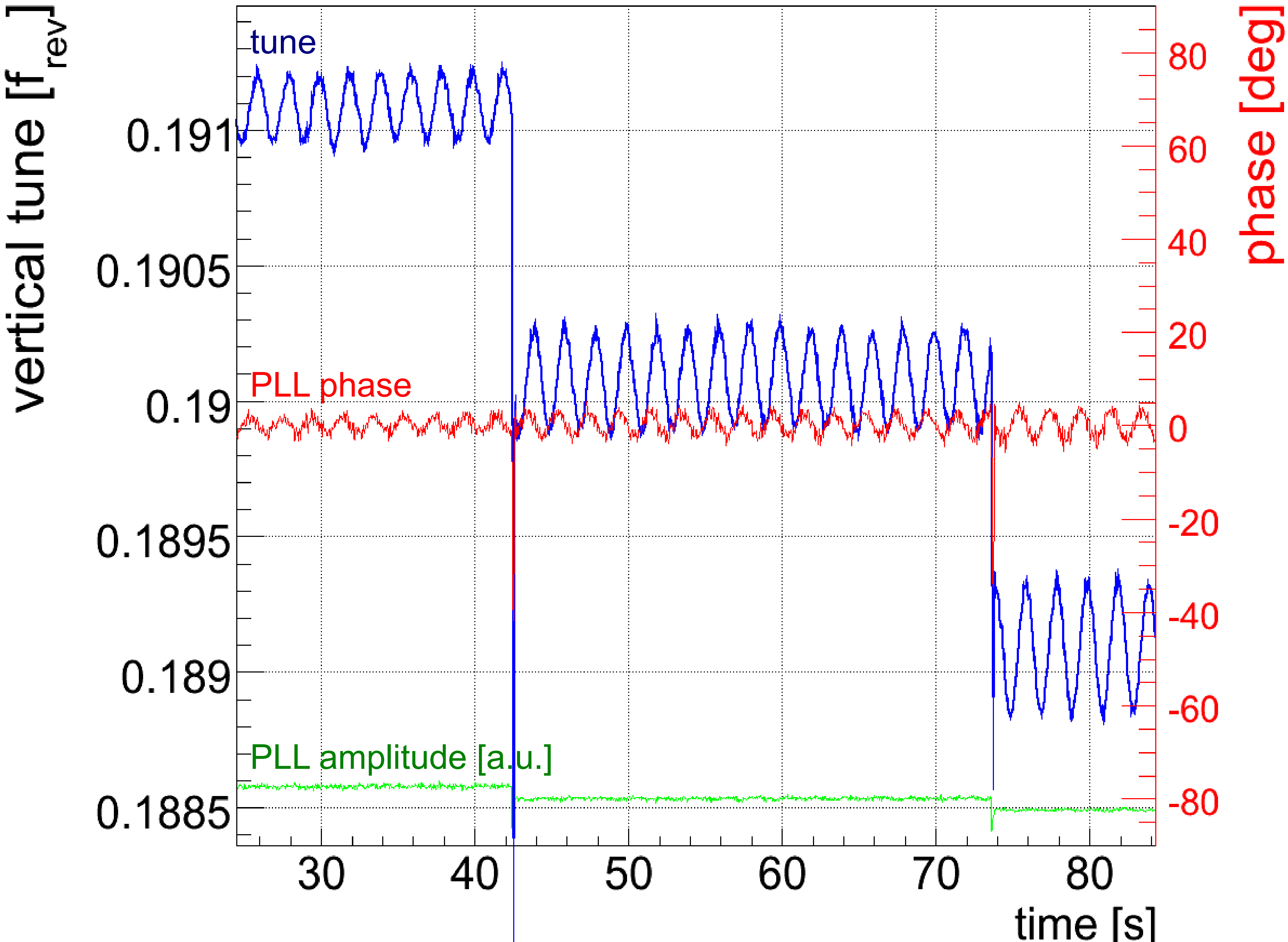


Tune-PLL Tracking Example:
CERN-SPS PLL Tune Tracking – fast tracking

- Two domains of tracking, either slow and very precise (low loop bandwidth) or fast:



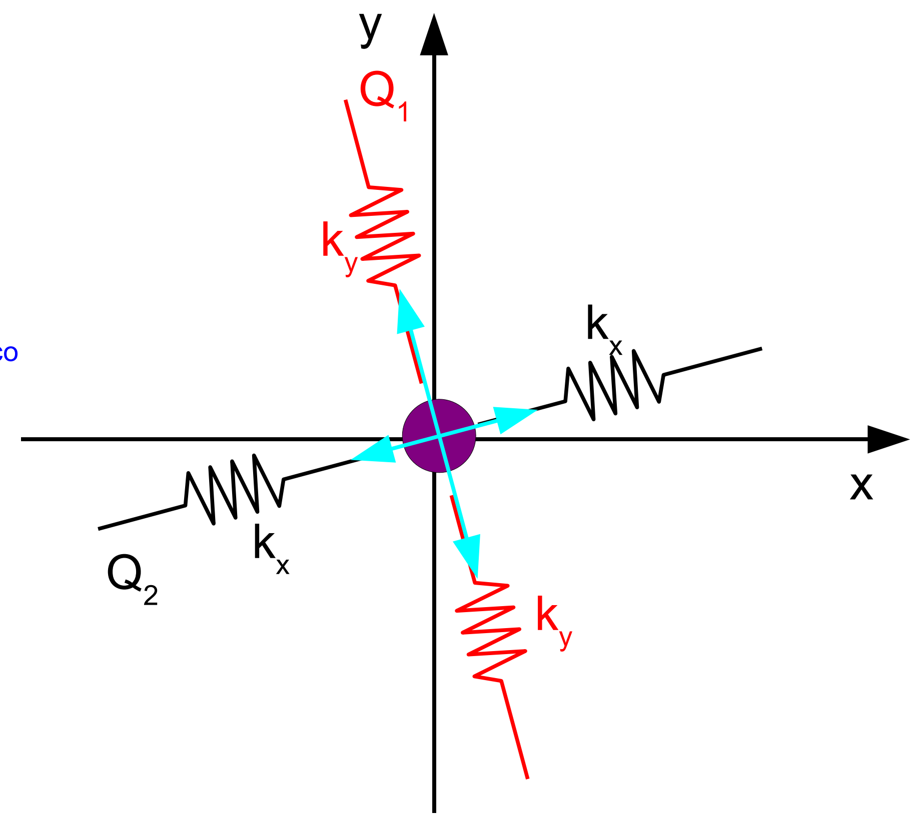
- Phase error and non-vanishing amplitude indicates lock
- here: $\Delta Q/\Delta t|_{max} \approx 0.3$ within 300 ms $f_{rev} \approx 43$ kHz



here: PLL-Tune resolution: $\Delta f_{res} \approx 10^{-6}$

→ more during the second part

- Feed-down due to systematic closed orbit offset Δx_{co}:
  - horizontal plane:
    → add. quadrupole → tune shift ~ Δx_{co}
    + small dipole kick ~ $(\Delta x_{co})^2$
  - vertical plane:
    → add. skew-quadrupole → coupling ~ Δy_{co}
    + small dipole kick ~ $(\Delta y_{co})^2$
    - first order: rotates oscillation plane



- Feed-down due to closed orbit + change of sextupolar field:
  - important for superconducting accelerators: large changes of persistent currents (decay & snapback phenomena)
    - also visible while changing (trimming) Q'
    - Higher order effects: space charge, beam-beam, ...

Betatron Coupling I/II

- In the presence of coupling (solenoids, skew-quadrupoles):

$$\begin{aligned} x'' + k(s)\cdot x &= \kappa(s)\cdot y \\ y'' + k(s)\cdot y &= \kappa(s)\cdot x \end{aligned}$$

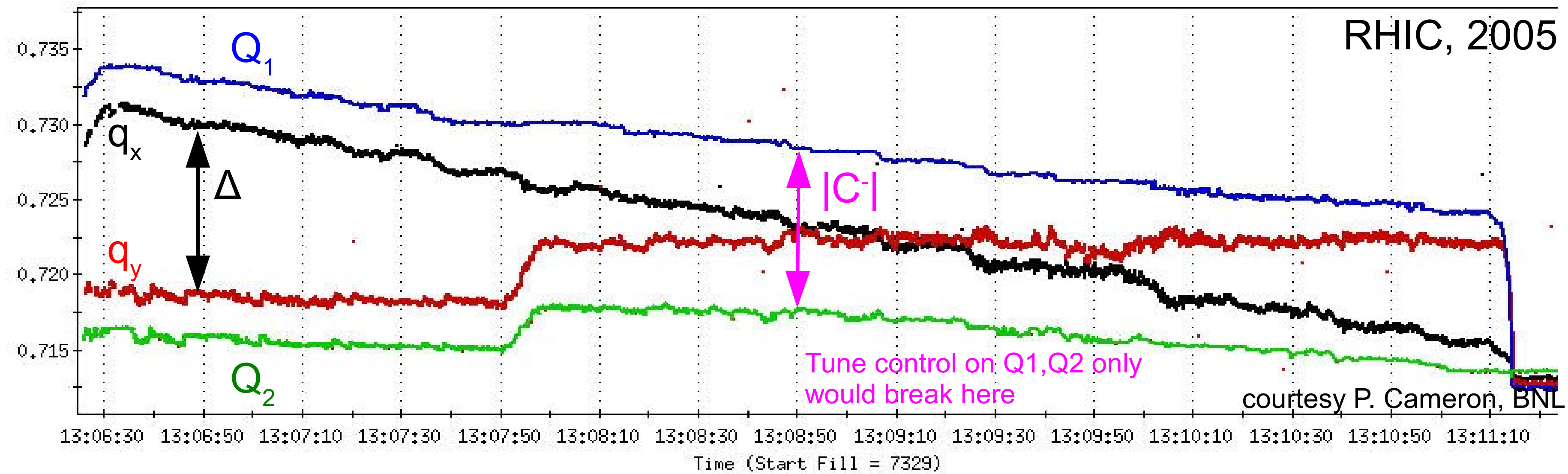
classic harmonic oscillator, defines unperturbed tunes: q_x, q_y

coupling terms

$$\kappa(s) = \frac{q}{2p}\left(\frac{\partial B}{\partial y} - \frac{\partial B}{\partial x}\right)$$

  - assuming weak coupling, eigenmodes (Q_1, Q_2) may be rotated w.r.t. unperturbed tunes (q_x, q_y, $\Delta = |q_y - q_y|$)

$$Q_{1,2} = \frac{1}{2}\left(q_x + q_y \pm \sqrt{\Delta^2 + |C^-|^2}\right)$$

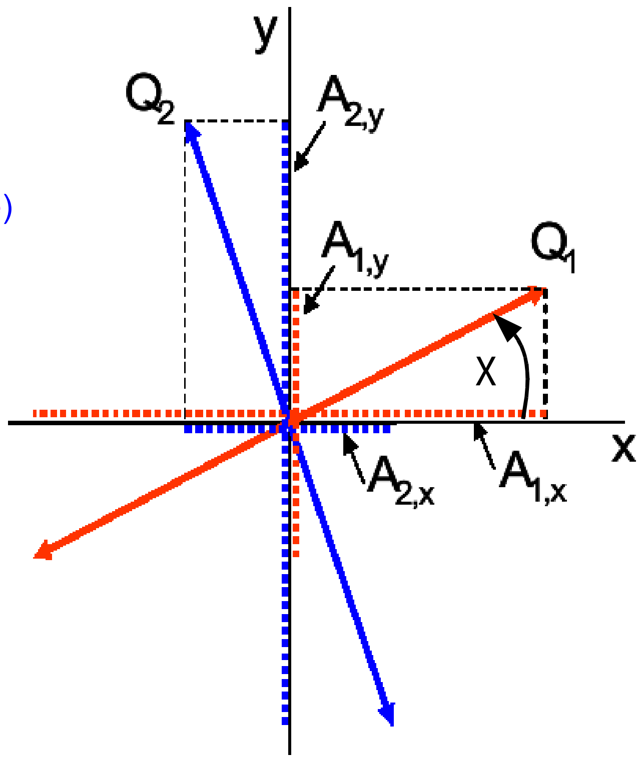


Betatron Coupling II/II

Possible improvement:

- Optimise tune working point (larger tune-split),
- Vertical orbit stabilisation in lattice sextupoles (Orbit FB → M. Böge)
- Active compensation and correction of coupling
  - ratio between regular and cross-term:
    - $A_{1,x}$: eigenmode amplitude '1' in vert. plane
    - $A_{1,y}$: eigenmode amplitude '1' in hor. plane

$$r_1 = \frac{A_{1,y}}{A_{1,x}} \wedge r_2 = \frac{A_{2,x}}{A_{2,y}}$$

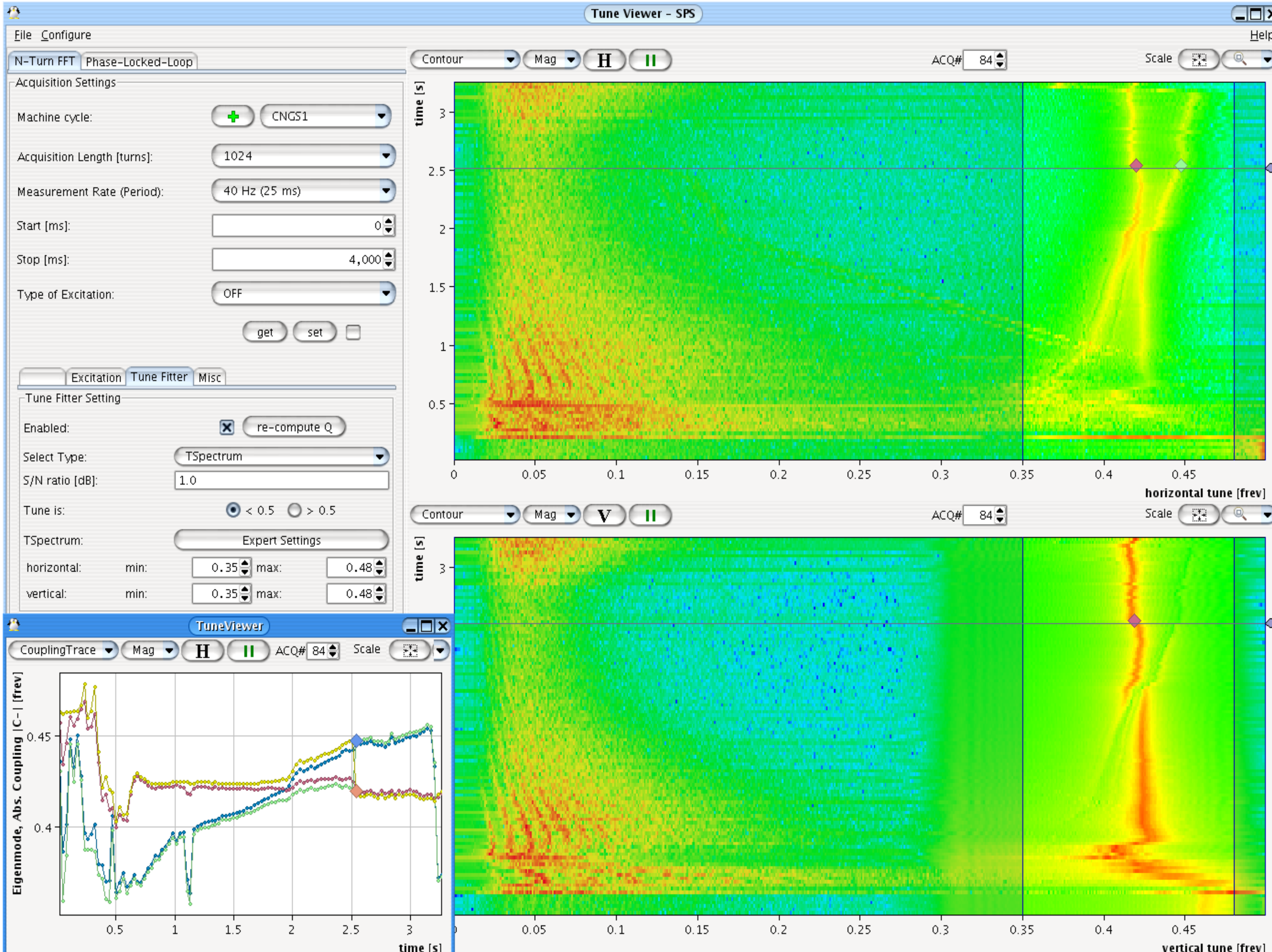


$$\Rightarrow \quad |C^-| = |Q_1 - Q_2| \cdot \frac{2\sqrt{r_1 r_2}}{(1 + r_1 r_2)} \wedge \Delta = |Q_1 - Q_2| \cdot \frac{(1 - r_1 r_2)}{(1 + r_1 r_2)}$$

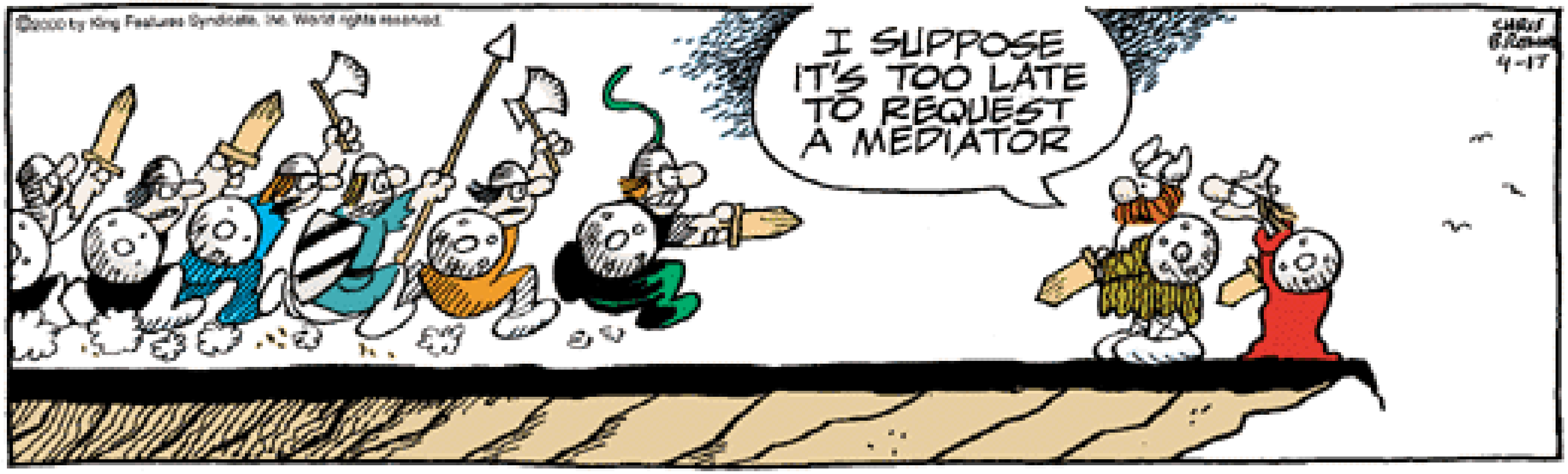
  - decouples beam feedback control
    - q_x, q_y → quadrupole circuits strength
    - $|C^-|$, χ → skew-quadrupole circuits strength

R. Jones e.al., "Towards a Robust Phase Locked Loop Tune Feedback System", DIPAC'05, Lyon, France, 2005

Betatron Coupling Detection
Example: CERN-SPS



- That's all – questions?

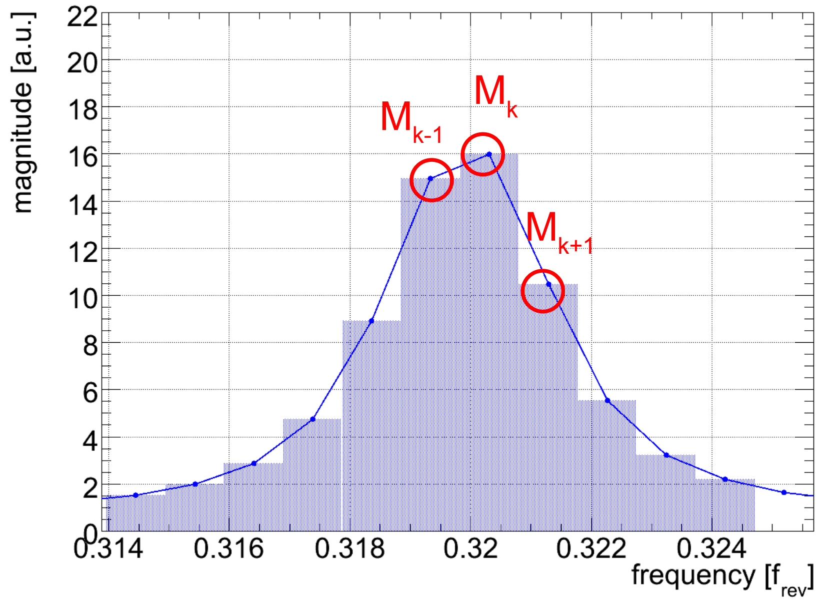


- If interested: some additional advanced topics not covered so far (see Appendix):
  - Classic Tune Frequency Analysis
    - Improving Frequency Resolution of FFT based Spectra
  - Tune Phase-Locked-Loop Locking issues in the presence of:
    - Coupled Bunch Instabilities
    - Synchrotron Side-bands
    - Changing Tune Width (Q' dependence, amplitude detuning, impedance, ...)
  - Feedback on Tune, Chromaticity and Coupling

Additional Slides

Additional Topic I:
Improving Frequency Resolution
of Fast-Fourier-Transform based Spectra

- Tune frequency resolution can be improved through FFT based Interpolation algorithms
(*k*: index of highest bin, *N*: total number of turns, M_k: magnitude of bin k)



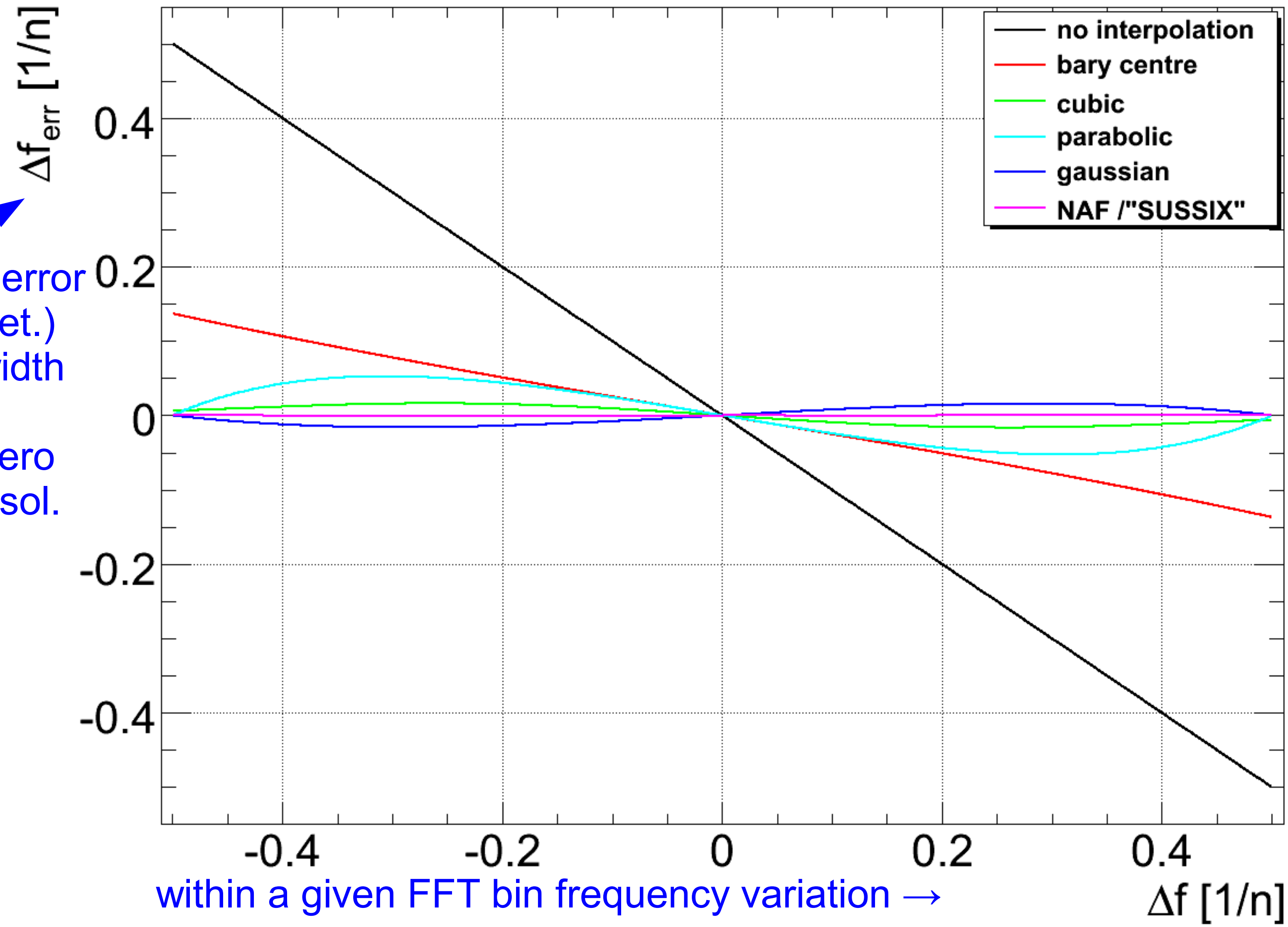
- Some common approaches:
  - No interpolation: $q \approx \frac{k}{N}$
  - Barycentre (n=1) & cubic (n=3) fit: $q \approx \frac{M_{k-1}^n (k-1) + M_k^n (k) + M_{k+1}^n (k+1)}{N\left(M_{k-1}^n + M_k^n + M_{k+1}^n\right)}$
  - Parabolic fit: $q \approx \frac{k}{N} + 0.5 \cdot \frac{M_{k+1} - M_{k-1}}{2M_k - M_{k-1} - M_{k+1}}$
  - Gaussian fit: $q \approx \frac{k}{N} + 0.5 \cdot \frac{\log(M_{k+1}/M_{k-1})}{\log(M_k^2/(M_{k-1} M_{k+1}))}$
  - NAFF/"SUSSIX": $q \approx \frac{k}{N} \pm \frac{1}{\pi} \cdot atan\left(\frac{|M_{k\pm1}| \sin(\frac{\pi}{N})}{|M_k| + |M_{k\pm1}| \cos(\frac{\pi}{N})}\right)$

- Test case: controlled oscillation at a given frequency which is varied within one bin, normalised to sampling frequency

- 1024 turns: perfect sinusoidal oscillation & within one bin varying frequency
  – introducing some

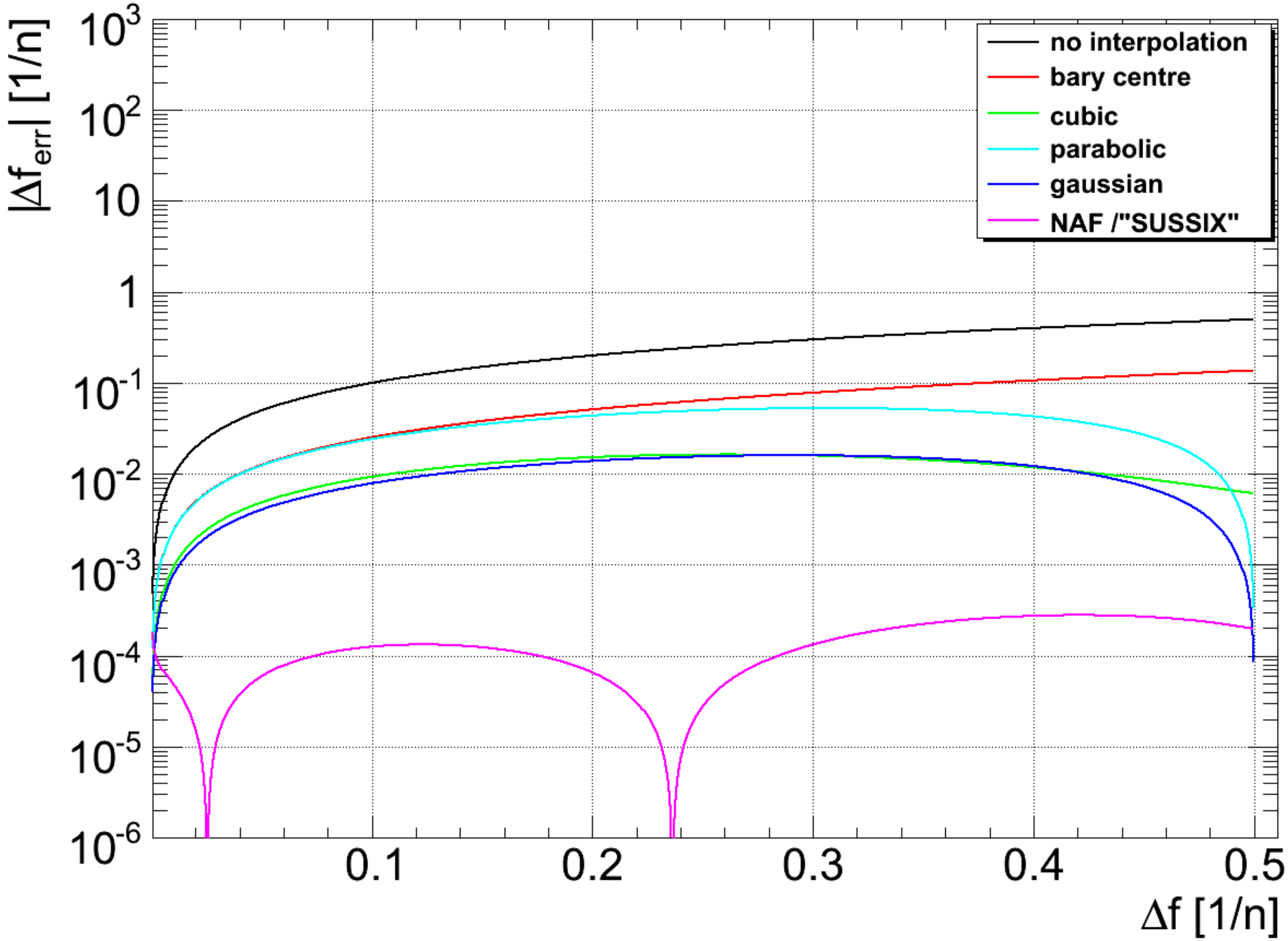
frequency error (sim. vs. det.) w.r.t. bin width

closer to zero = better resol.



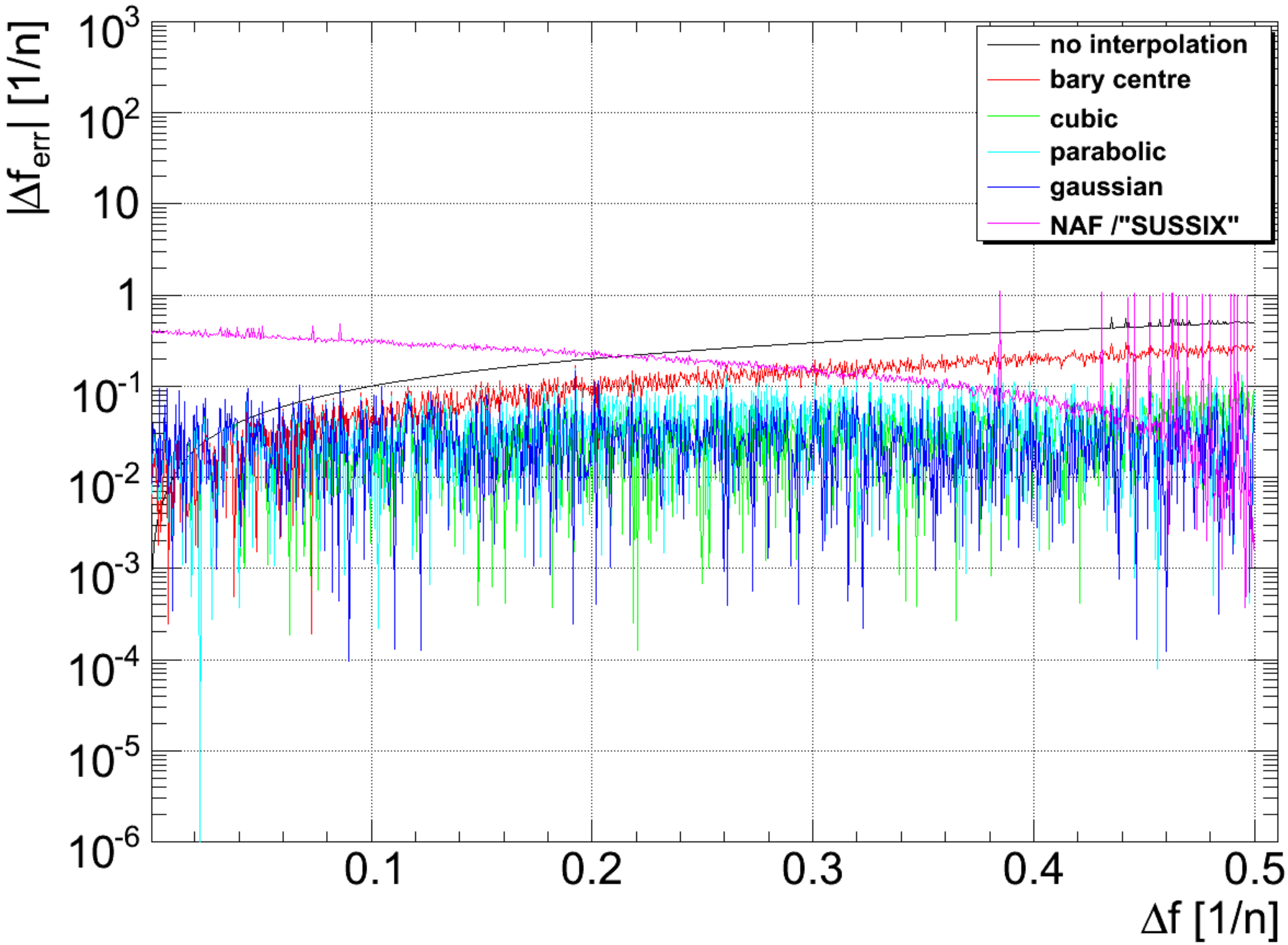
within a given FFT bin frequency variation →

- same plot as before but: absolute error, logarithmic scale and considering frequency only within half a bin width (symmetry!)



  - ... what about more realistic signals with damping, noise ...?

- same as before + 0.1 r.m.s. noise vs. kick amplitude of '1'



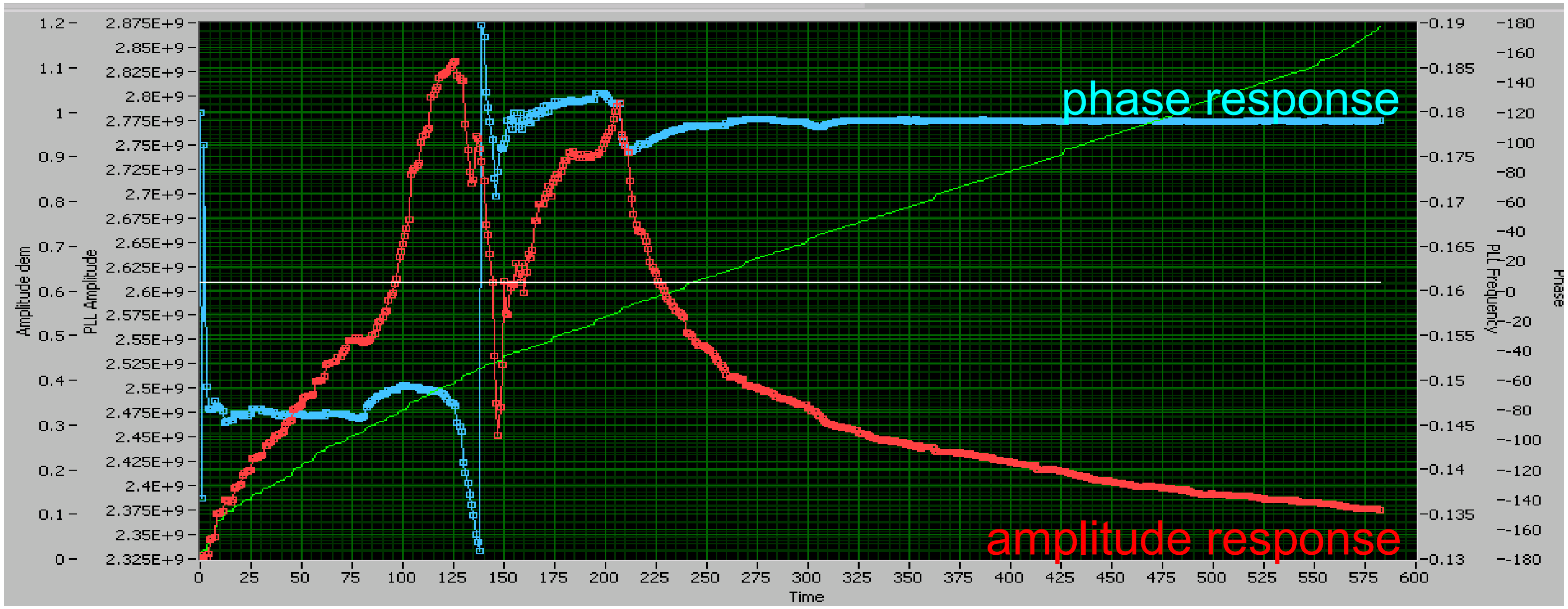
  – Measurement noise is the limiting the resolution, cubic, barycentre, parabolic and Gaussian interpolation seem to yield similar performance. → Gaussian-fit of central peak gives good results im most cases.

Additional Topic II:

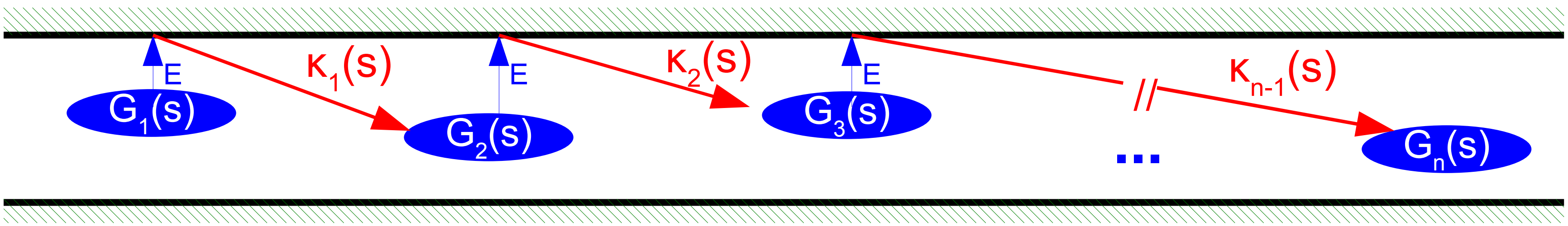
Phase-Locked-Loop Locking in the Presence

Coupled Bunch Instabilities, Synchrotron Side Bands and Tune Width Dependence

- Coupled bunch effects can hamper look became more pronounced during later MDs
  - possible causes: impedance driven wake fields, e-cloud, beam-beam, ...

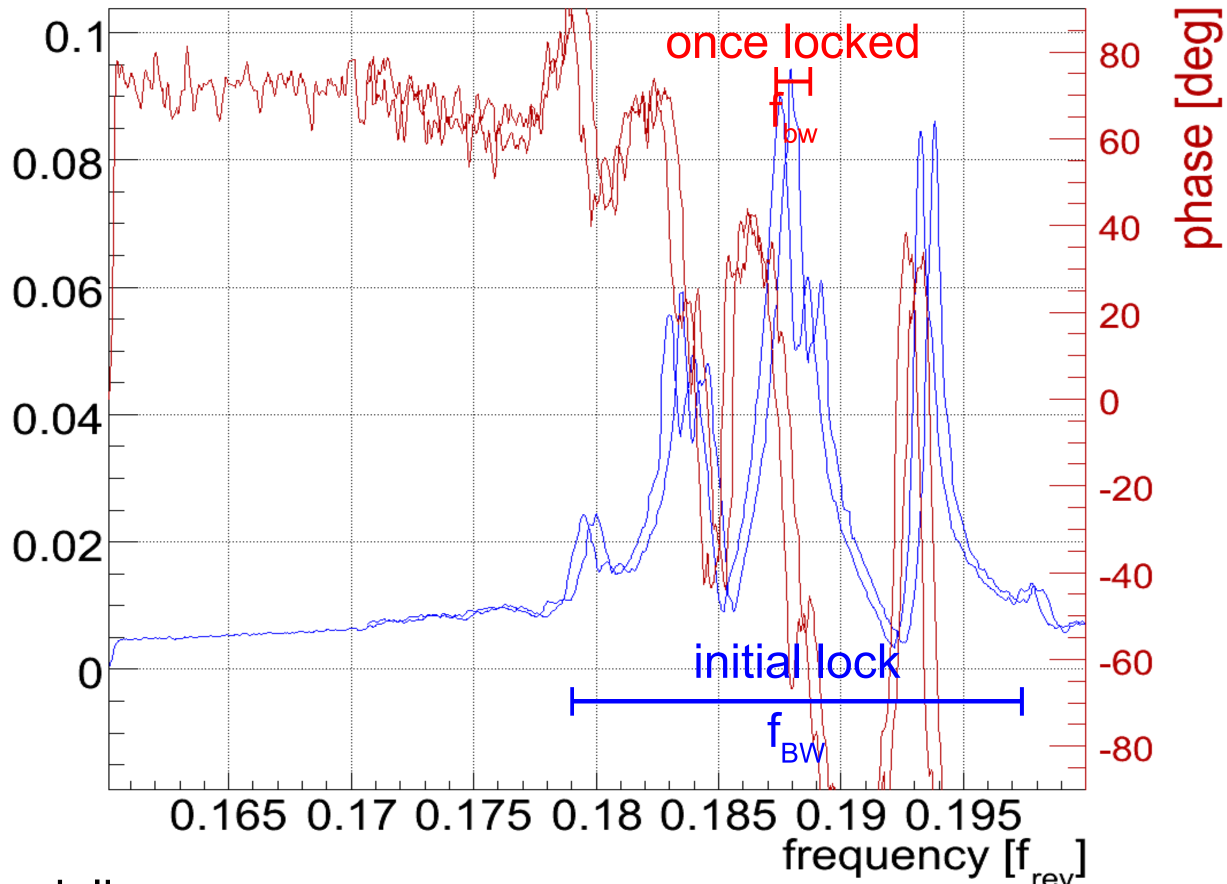


- Mechanism (impedance):



- Possible remedy:
  - Detector selects and measures only one (/first) representative bunch

Synchrotron Sidebands: PLL locks on the largest peak



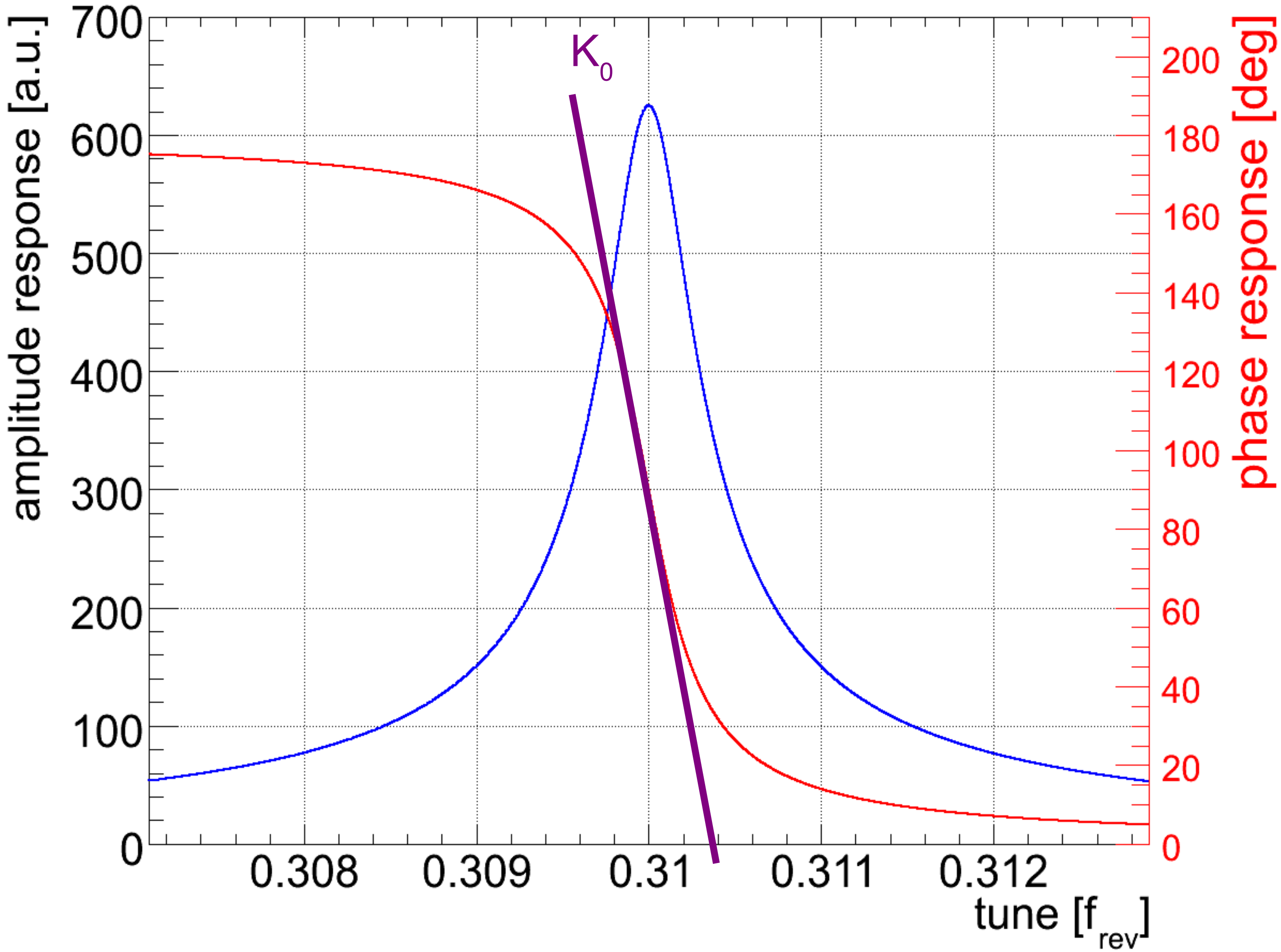
Option I: gain scheduling

initial lock: open bandwidth to cover more than one side band (PLL noise ~ chirp)

- side-bands "cancel out", strongest resonance prevails

once locked: reduce bandwidth for better stability/resolution

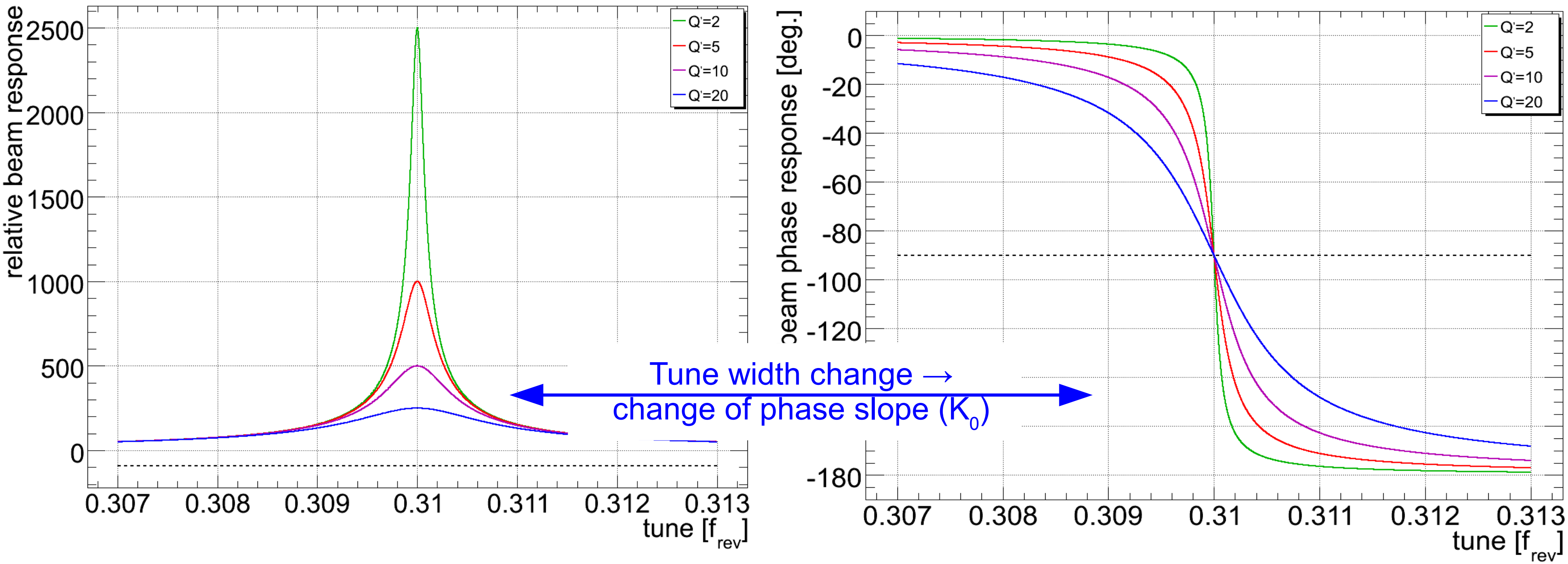
Option II: larger excitation bandwidth, multiple exciter or broadband excitation(FNAL)



- Reminder:
  - optimal PLL Settings (1/α ~ PLL bandwidth/tracking speed):

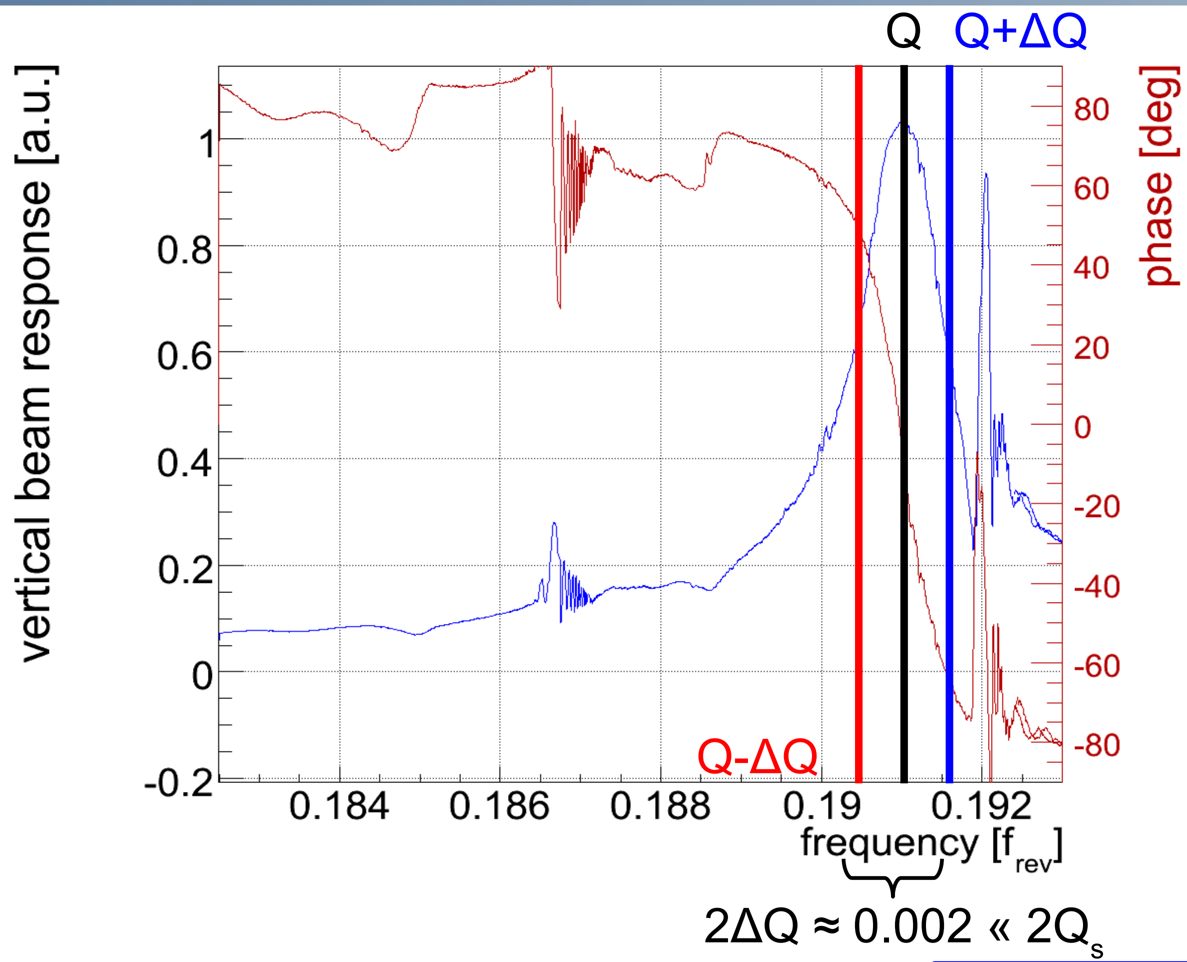
$$D(s) = K_P + K_i \frac{1}{s} \quad with \quad K_p = K_0 \frac{\tau}{\alpha} \wedge K_i = K_0 \frac{1}{\alpha}$$

Advanced PLL Lock Issues
Tune Width Dependence II/III



- Optimal PLL parameters (tracking speed, etc.) depend - beside measurement noise – on the effective tune width.
- Intrinsic trade-off:
  - Optimal PI for large ΔQ ↔ sensitivity to noise (unstable loop) for small ΔQ
  - Optimal PI for small ΔQ ↔ slow tracking speed for large ΔQ
- Can be improved by putting knowledge into the system: "gain scheduling"

Exploitation: Tune Width Measurement using PLL Side Exciter



- Resonant phase change ↔ tune width change

  → “free” real-time tune footprint measurement

  → measurable dependence of ΔQ ~ Q'

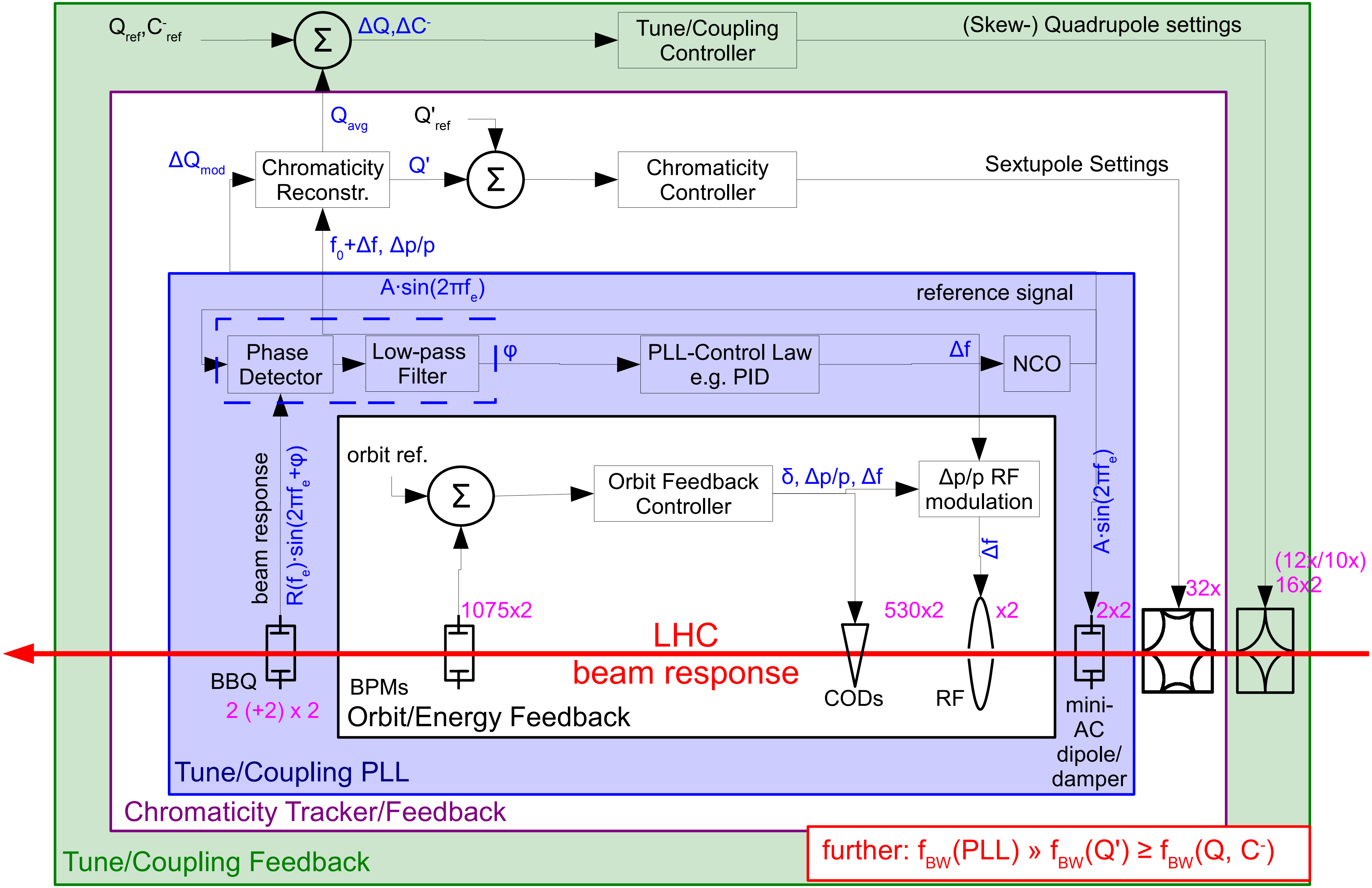
driven resonance:

$$\tan(\varphi) \approx \frac{\Delta Q \cdot \omega_Q \omega_D}{\omega_Q^2 - \omega_D^2}$$

Additional Topic III:

Feed-Backs on Tune, Coupling and Chromaticity

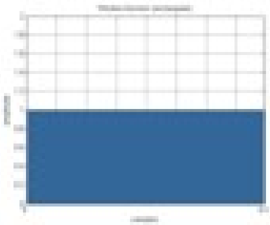
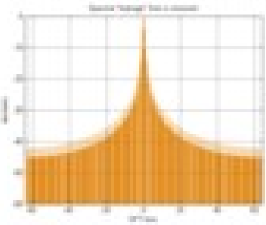
Integration of Q/Q' Measurements for Q/Q' Control
Full LHC Beam-Based Feedback Control Scheme



LHC FBs: 2158 input devices, 1136 output devices → total: ~3300 devices!

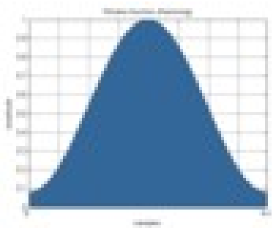
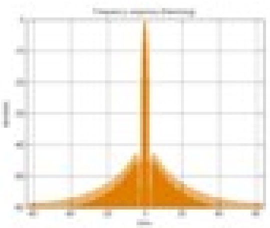
Frequency Resolution of FFT based Data Apodisation - "Windowing Function"

- rectangular, B=1.0

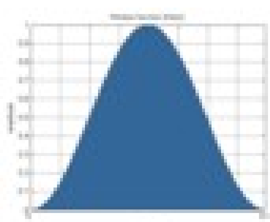
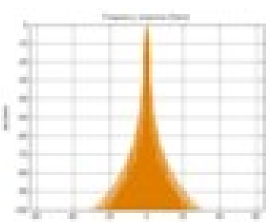
$$\omega(n) = 1$$

- Hamming, B = 1.37

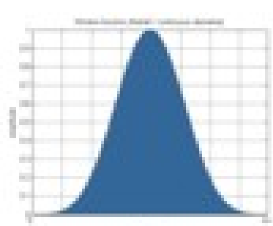
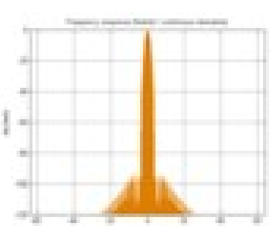
$$\omega(n) = 0.53836 - 0.46164\cos\left(\frac{2\pi n}{N-1}\right)$$

- Von Hann, B = 1.5

$$\omega(n) = 0.5\cdot\left[1-\cos\left(\frac{2\pi n}{N-1}\right)\right]$$

- Nuttall, B = 2.01

$$\omega(n) = a_0 - a_1\cos\left(\frac{2\pi n}{N-1}\right) + a_2\cos\left(\frac{4\pi n}{N-1}\right) - a_3\cos\left(\frac{6\pi n}{N-1}\right)$$

$$a_0=0.35875,\ a_1=0.48829,\ a_2=0.14128,\ a_3=0.01168$$

- See wikipedia article http://en.wikipedia.org/wiki/Window_function for details