Free-Electron Lasers

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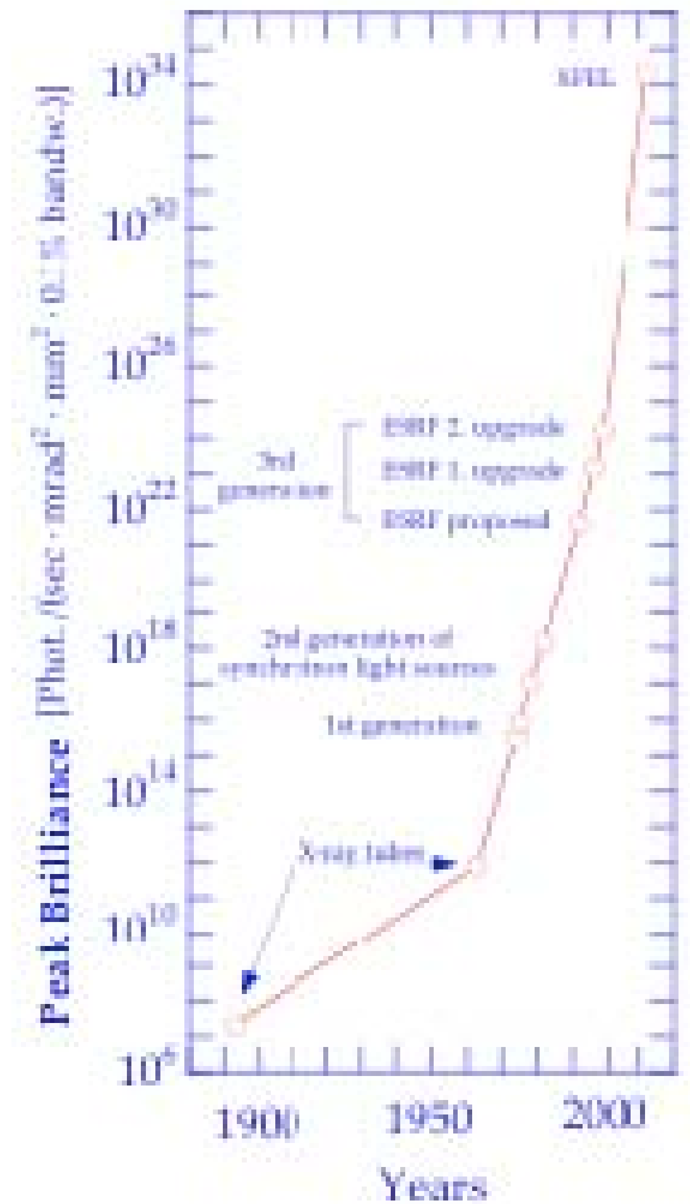
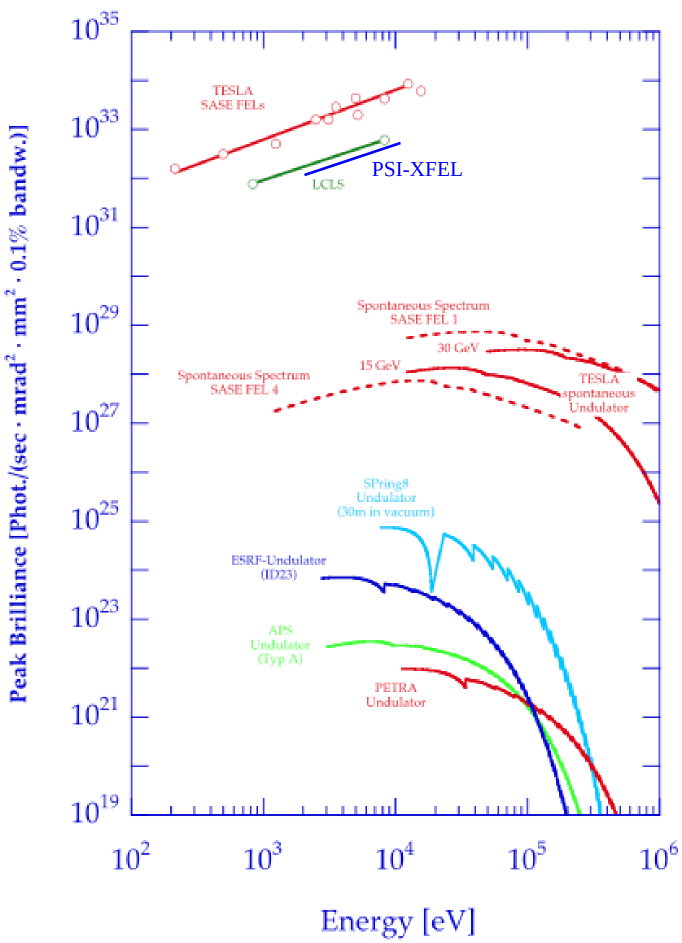
Light Sources

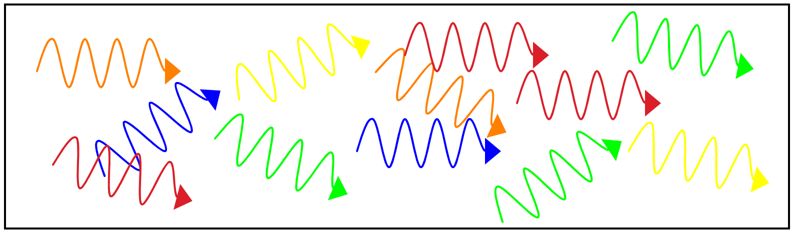
- 1st Generation: Synchrotron radiation from bending magnets in high energy physics storage rings
- 2nd Generation: Dedicated storage rings for synchrotron radiation
- 3rd Generation: Dedicated storage rings with insertion devices (wigglers/undulators)
- 4th Generation: XFELs



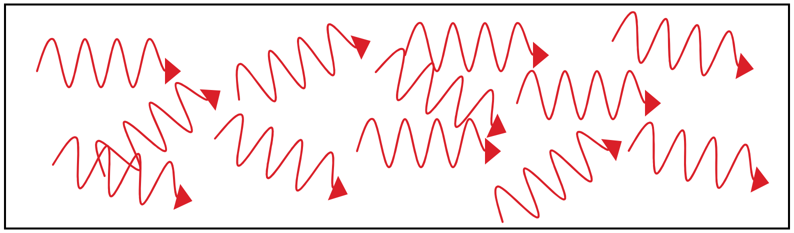
FEL as a Brilliant Light Source



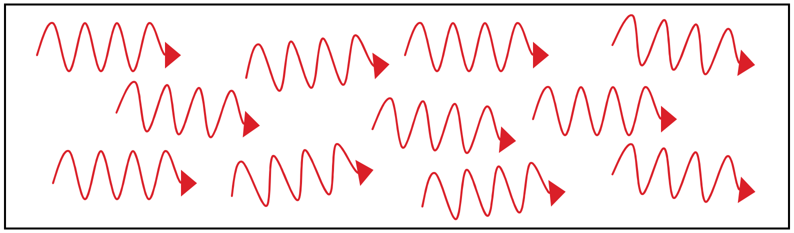
High photon flux



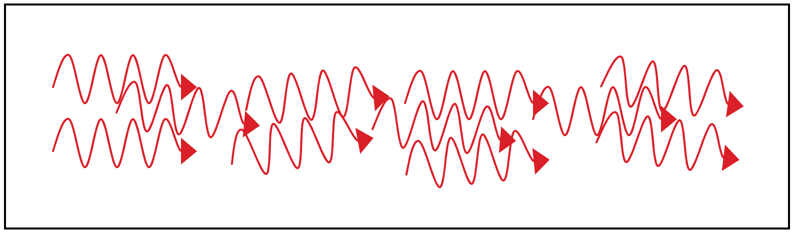
Small freq. bandwidth



Low divergence



Small source size



4th Generation Light Sources

- Tunable wavelength, down to 1 Ångstroem
- Pulse Length less then 100 fs
- High Peak Power above 1 GW
- Fully Transverse Coherence
- Transform limited Pulses

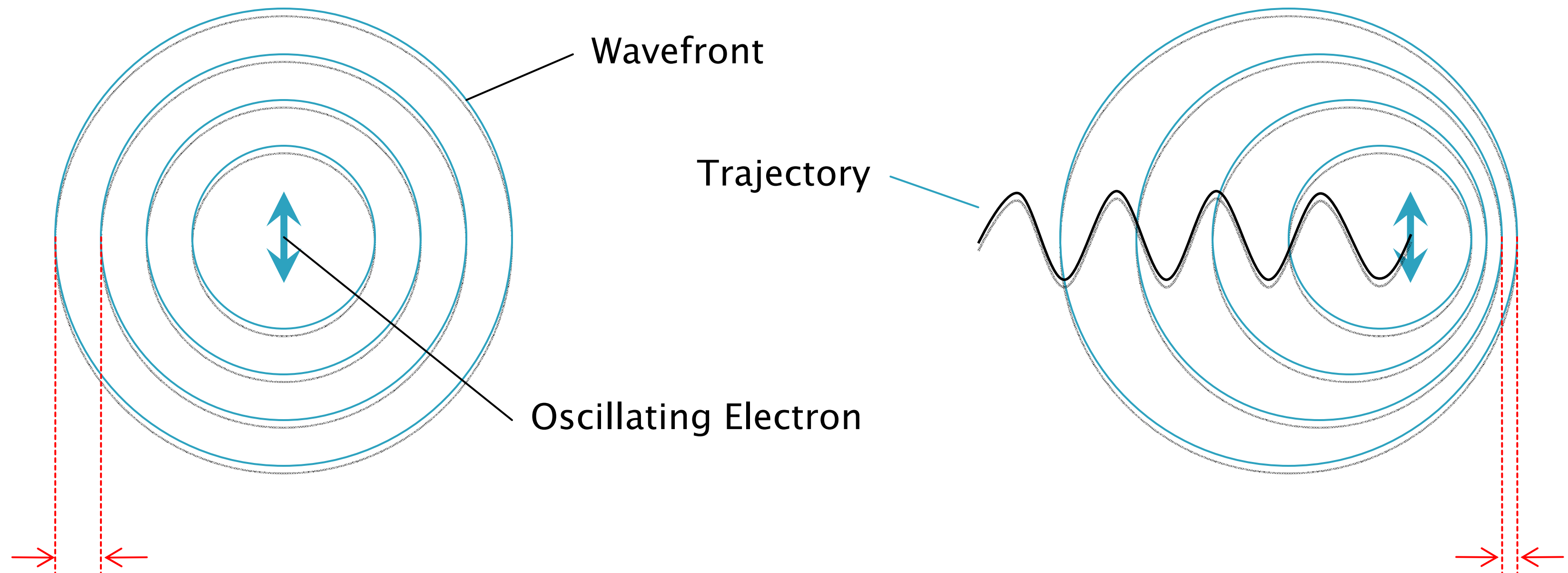
XFELs fulfill all criteria except for the longitudinal coherence (but we are working on it ☺)

X-ray/VUV FEL Projects Around the World

Controlling the Wavelength – The Idea

Dipole Radiation (Antenna)

Dipole Radiation + Doppler Shift



$$\lambda = c \cdot T$$

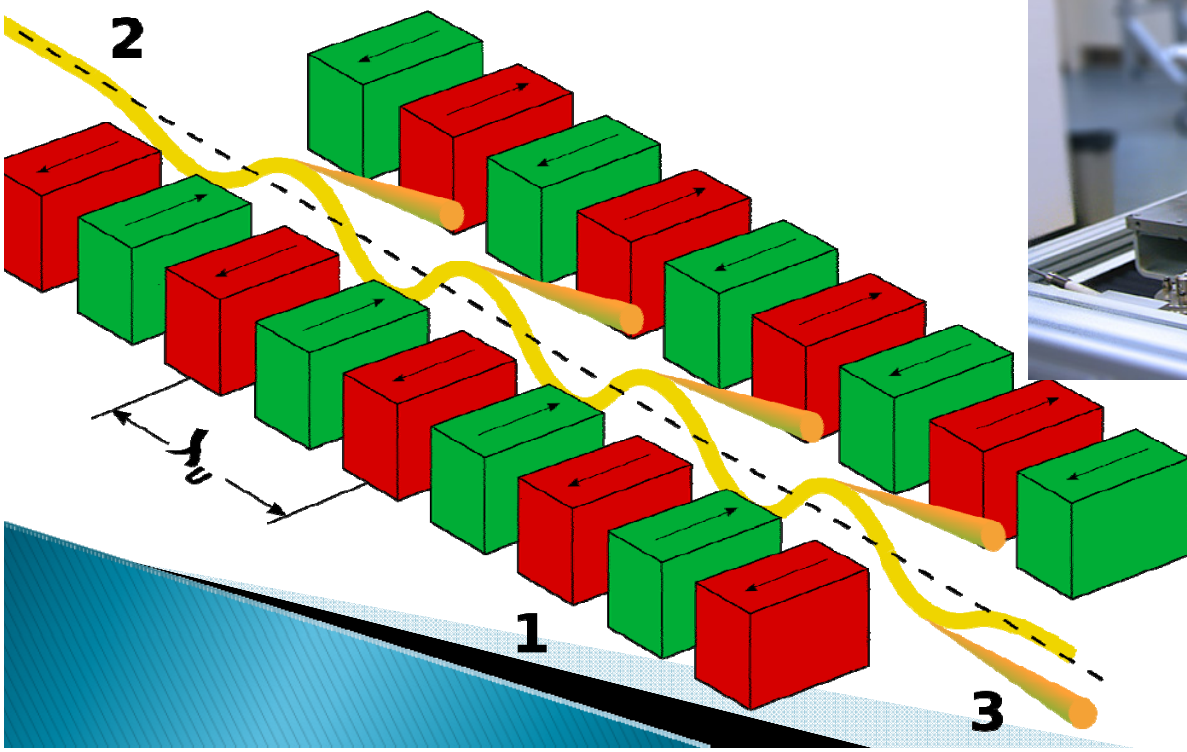
$$\lambda = (c - v_z) \cdot T$$

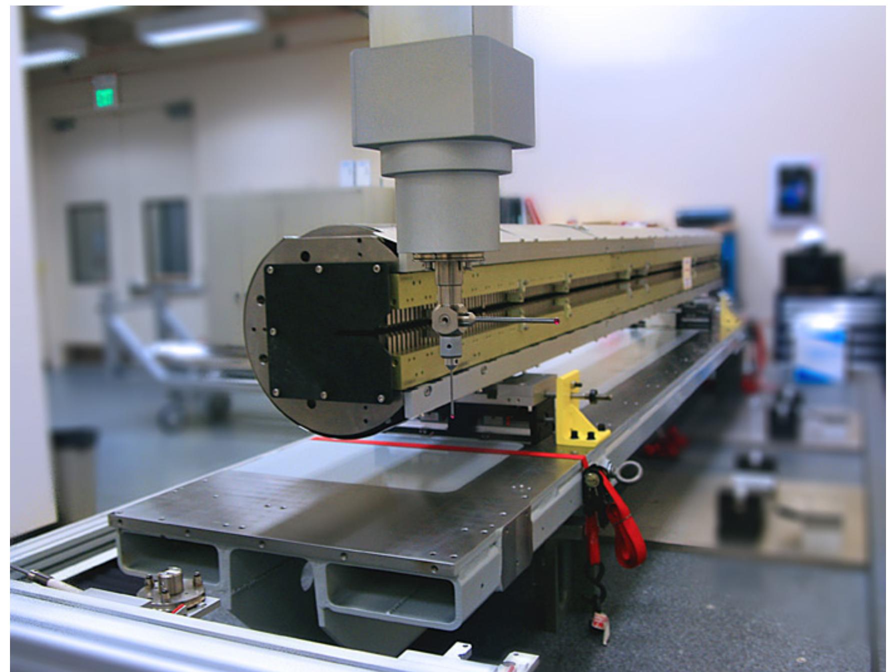
(forward direction)

For relativistic electrons the longitudinal velocity v_z is close to c, resulting in very short wavelength (blue shift of photon energy)

Forcing the Electrons to Wiggle…

- … by injecting them into a period field of an wiggler magnet (also often called undulator).





Wiggler module from the LCLS XFEL

Motion in a Wiggler

- Periodic Field of the Wiggler: $\vec{B} = \vec{e}_y \cdot B_0 \sin(k_u z) \qquad k_u = \frac{2\pi}{\lambda_u}$
- Lorentz-Force: $\vec{F} = \frac{d}{dt}\vec{p} = e \cdot \vec{v} \times \vec{B}$
- Simplifications +Relativistic Approximations:
  - Velocity in *z* is dominant, thus force acts mainly in *x*-direction.
  - Energy (thus γ) is preserved for motion in magnetic field.
  - Average motion in longitudinal direction is $z = \beta_z c t$ (Allows us to integrate Lorentz-Force equation).

$$\frac{d}{dt}\beta_x = \beta_z \frac{eB_0}{\gamma m_0} \sin(k_u \beta_z c t) \quad \Rightarrow \quad \beta_x = -\frac{1}{\gamma} \cdot \boxed{\frac{eB_0}{m_0 c k_u}} \cdot \cos(k_u z)$$

Undulator Parameter: $K \approx 0.93 \cdot B_0[\mathrm{T}] \cdot \lambda_u[\mathrm{cm}]$

The Undulator Wavelength

- We got everything now to calculate the wavelength:

$$\lambda = cT(1-\beta_z)$$

Period Length $\quad cT = \beta_z \lambda_u \approx \lambda_u$

Long. Velocity $\quad \beta_z = \sqrt{1-\frac{1}{\gamma^2}-\beta_x^2} \approx 1-\frac{1}{2\gamma^2}-\frac{K^2}{2\gamma^2}\sin^2(k_u z)$

- For the average velocity, the square of the sine function is ½. The emitted wavelength is:

$$\boxed{\lambda = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)}$$

The Resonant Wavelength

- The wavelength can be controlled by
  - Changing the electron beam energy,
  - Varying the magnetic field (requires K significantly larger than 1)

- To reach 1 Å radiation, it requires an undulator period of 15 mm, a K-value of 1.2 and an energy of 5.8 GeV (γ=11000)

The Free-Electron Lasers are based on undulator radiation and have the same resonant wavelength

'Free-Electron' refers to the fact that unlike quantum lasers the electrons are unbound in the periodic 'potential' of the undulator.

FEL from a Quantum Mechanic Point of View.

- The emission of photons allows also for absorption.
- Electrons ‘communicate’ through photons. Phase of the photon is linked to electron position.
- Electrons, absorbing more photons than emitting, become faster and tend to group with electrons, which are emitting more photons than absorbing.

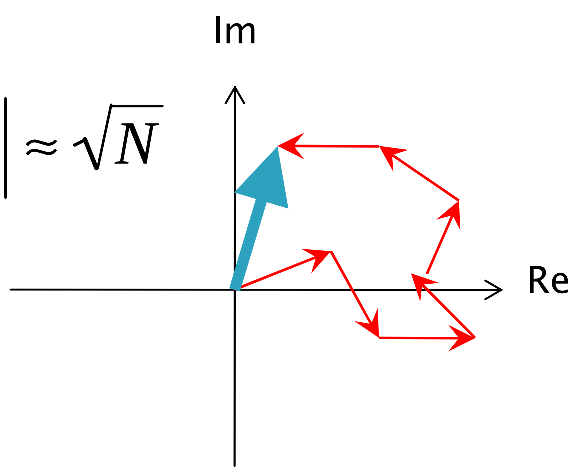
The FEL exploits a collective process, which ends with an almost fully coherent emission at the resonant wavelength.

Coherence

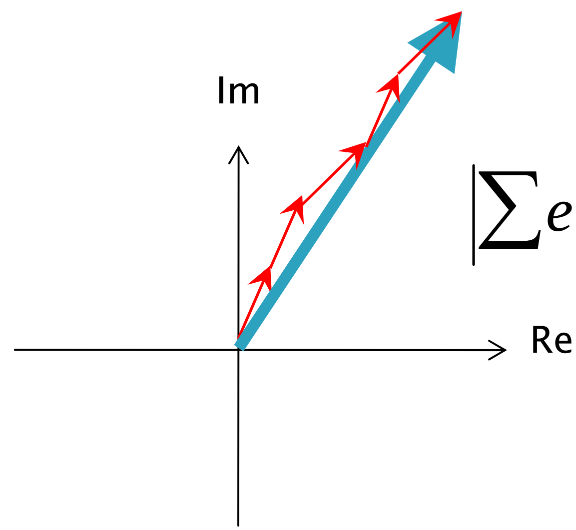
- The electrons are spread out over the bunch length with its longitudinal position δz_j. The position adds a phase $\phi_j = k\delta z_j$ to the emission of the photon.
- The total signal is: $$E(t) \propto \sum_j e^{i(kz_j - \omega t)} = e^{i(k\langle z\rangle - \omega t)} \cdot \sum_j e^{ik\delta z_j}$$

Electrons spread over wavelength:
Phasor sum = random walk in 2D

$$\left|\sum e^{i\phi}\right| \approx \sqrt{N}$$



Electrons bunched within wavelength:
Phasor sum = Add up in same direction

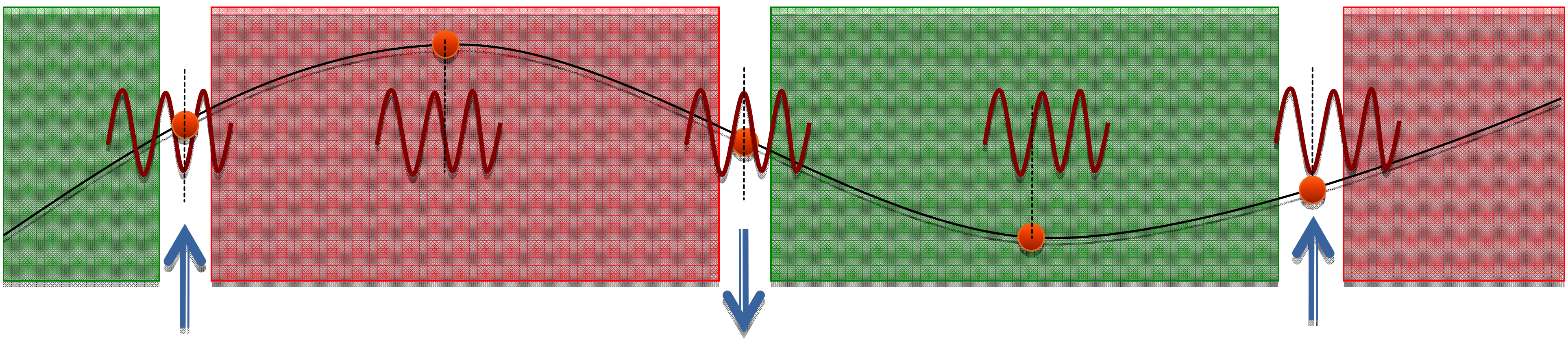


$$\left|\sum e^{i\phi}\right| \approx N$$

Power ~ $|E|^2$ -> Possible Enhancement: N

Co-Propagation of Electrons and Field

- The electron moves either with or against the field line of a co-propagating wave.
- After half undulator period the radiation field has slipped half wavelength. Transverse velocity remains in same direction as radiation field.



There is a net energy change of the electron, depending on the injection phase ϕ

Step I : Energy Modulation

- Energy change, when electron runs along electric field lines:

$$\frac{d}{dt}\gamma = \vec{v} \bullet \frac{\vec{E}}{mc^2} = c\frac{K}{\gamma}\sin(k_u z)\cdot\frac{E_x}{mc^2}\cos(kz - \omega t + \phi)$$

- Beat wave has two spatial frequencies: ($k + k_u$) and ($k - k_u$). Only the first one allows for a steady energy gain or loss over many periods:

$$\left\langle \frac{d}{dz}\gamma \right\rangle = -k\frac{f_c K}{2\gamma}\left(ue^{i\theta} + c.c.\right)$$

Averaging over undulator period. Longitudinal oscillation gives reduced coupling (f_c <1)

$$\left(u = \frac{E_x e^{i\phi}}{mc^2 k} \right)$$

$$\theta = (k + k_u)z - \omega t = \underbrace{\left[(k + k_u)\beta_z - k\right]}_{=0} ct + \theta_0$$

(ponderomotive phase)

$$\beta_z = \frac{k}{k + k_u} \quad \Rightarrow \quad \lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

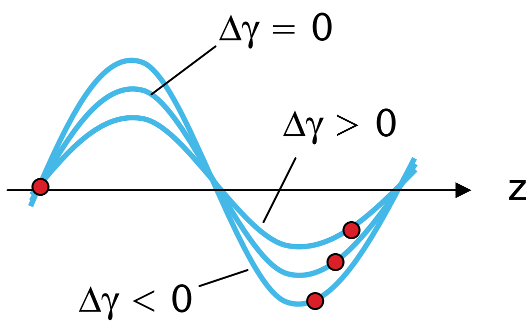
Resonance Condition

Step II: Longitudinal Motion

- For a given wavelength λ there is one energy γ_r, where the electron stays in phase with radiation field.

- Electrons with energies above the resonant energy, move faster ($d\theta/dt > 0$), while energies below will make the electrons fall back ($d\theta/dt < 0$).

- For small energy deviation from the resonant energy, the change in phase is linear with $\Delta\gamma = (\gamma - \gamma_r)$.



$$\frac{d}{dz}\theta = (k + k_u)\beta_0 - k$$

$$= k_u - k\frac{1 + K^2/2}{2(\gamma_r + \Delta\gamma)^2} = 2k_u\frac{\Delta\gamma}{\gamma_r}$$

Analogy to Pendulum

FEL Equations

$$\frac{d}{dz}\theta \propto \frac{\Delta\gamma}{\gamma_r} \qquad \frac{d}{dz}\frac{\Delta\gamma}{\gamma_r} \propto \frac{KE_x}{\gamma_r^2}\sin(\theta+\phi)$$

Frequency

$$\Omega \propto \frac{\sqrt{KE_x}}{\gamma_r}$$

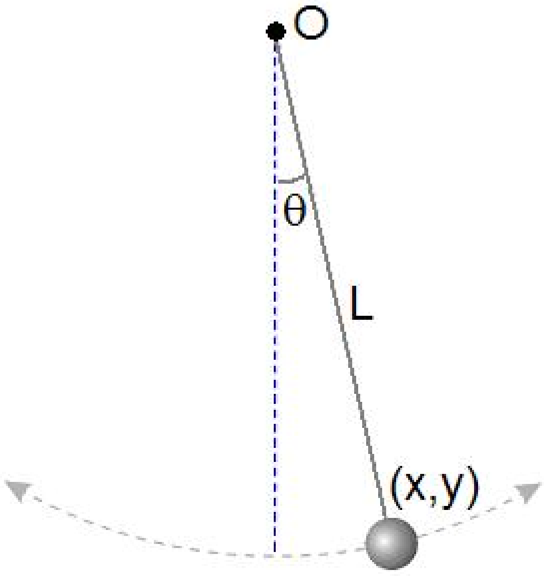
Pendulum Equations

$$\frac{d}{dt}\theta = I \qquad \frac{d}{dt}I = \frac{g}{L}\sin\theta$$

Frequency

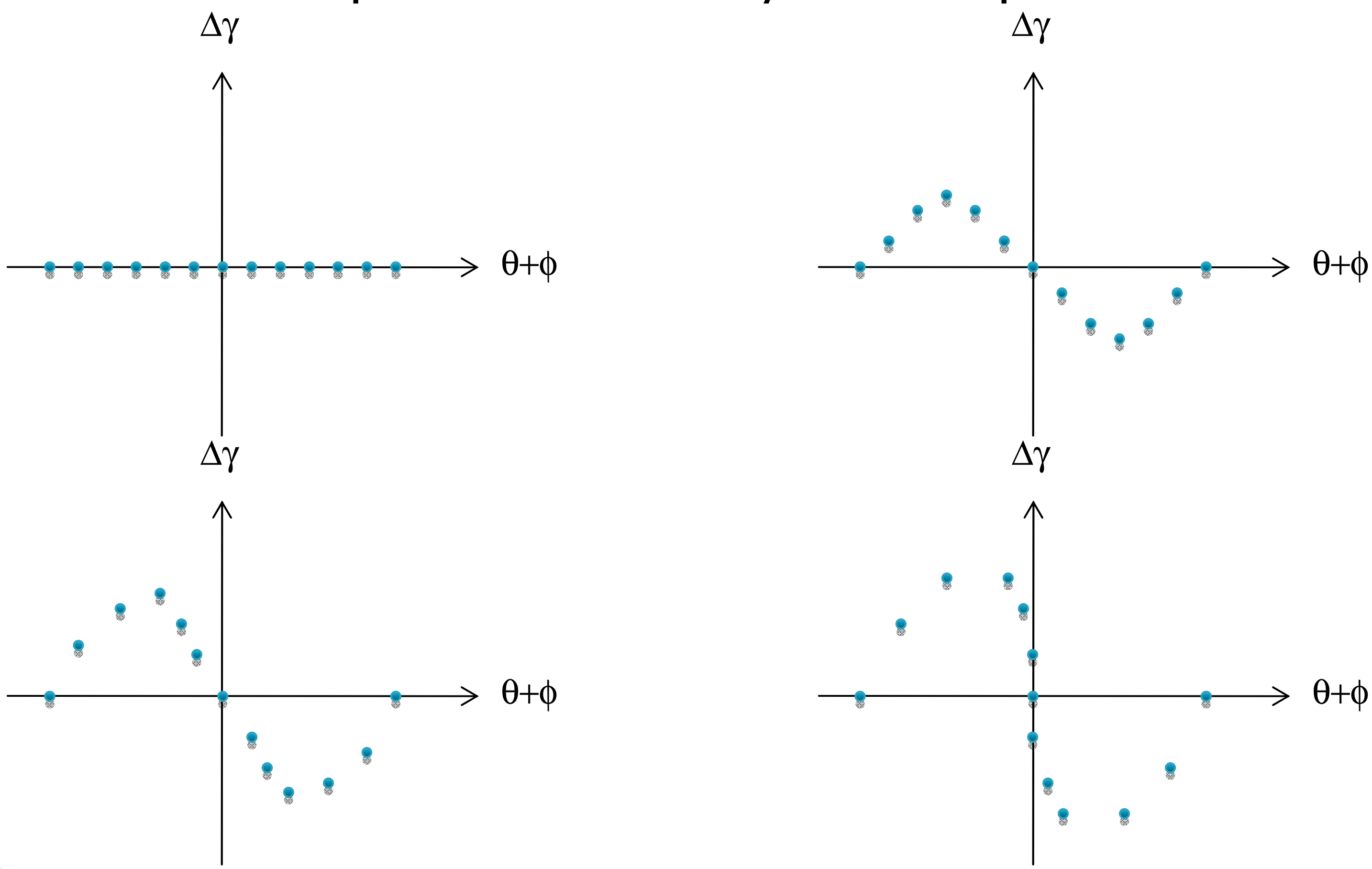
$$\Omega \propto \sqrt{\frac{g}{L}}$$

- Stable Fix Point: $\Delta\gamma=0$, $\theta+\phi=0$
- Instable Fix Point: $\Delta\gamma=0$, $\theta+\phi=\pi$
- Oscillation gets faster with growing E-Field (Pendulum: shorter length L)



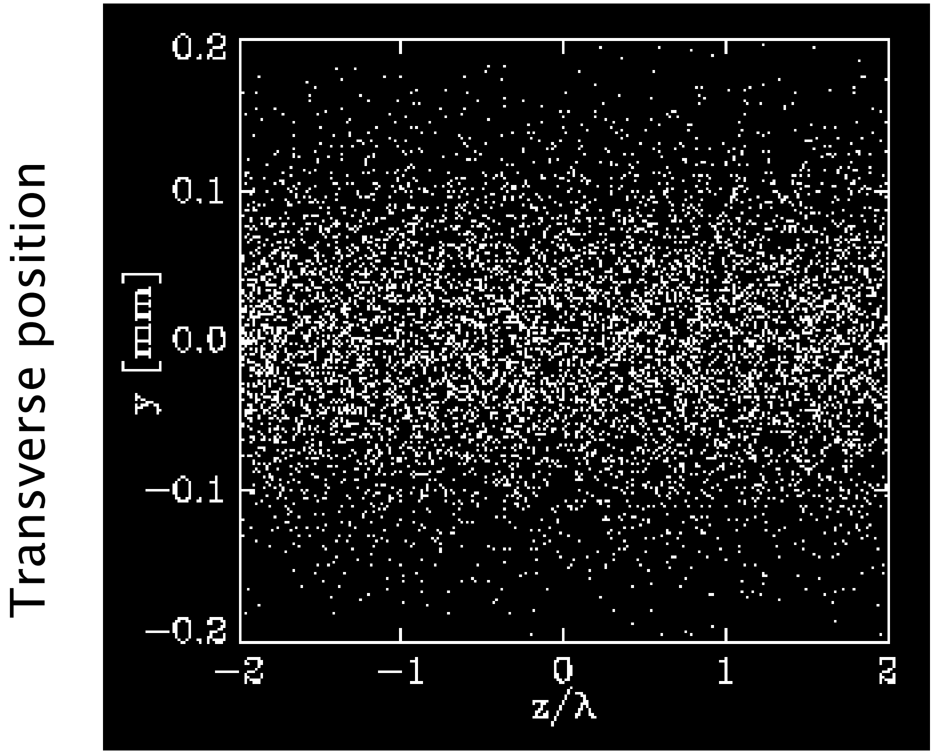
Motion in Phasespace

- Wavelength typically much smaller than bunch length.
- Electrons are spread out initially over all phases.



Electrons are bunched on same phase after quarter rotation

Microbunching



Slice of electron bunch (4 wavelengths)

3D Simulation for FLASH FEL over 4 wavelengths

Frame moving with electron beam through 15 m undulator

Wiggle motion is too small to see. The 'breathing' comes from focusing to keep beam small.

Microbunching has periodicity of FEL wavelength. All electrons emit coherently.

Evolution of the Radiation Field

- Solving Maxwell Equation, but using the paraxial approximation ($E = u \cdot \exp(ikz - i\omega t)$):

$$\left[\nabla_\perp^2 + 2ik\left(\frac{\partial}{\partial z} + \frac{\partial}{c\partial t}\right)\right]u = i\frac{e^2\mu_0}{m}\sum_j \frac{f_c K}{\gamma_j} e^{-i\theta_j}$$

Diffraction

Evolution along Undulator

Evolution along Bunch (Slippage)

Source Term

$$\left(u = \frac{E_0}{mc^2 k} e^{i\phi}\right)$$

1D Model: Neglect time derivative and transverse Laplace operator

Scaling of 1D FEL Equations

$$\frac{d}{dz}\theta = 2k_u \frac{\Delta\gamma}{\gamma_r} \quad\Longrightarrow\quad \frac{d}{d\hat{z}}\theta = \Delta + \eta$$

$$\frac{d}{dz}\gamma = -k\frac{f_c K}{2\gamma_0}\left(ue^{i\theta} + c.c.\right) \quad\Longrightarrow\quad \frac{d}{d\hat{z}}\eta = -\left(Ae^{i\theta} + c.c.\right)$$

$$\frac{d}{dz}u = \frac{e^2\mu_0 n_e}{2km}\frac{f_c K}{\gamma_0}\left\langle e^{-i\theta}\right\rangle \quad\Longrightarrow\quad \frac{d}{d\hat{z}}A = \rho^{-3}\left[\frac{1}{\gamma_0^3}\left(\frac{f_c K}{4k_u\sigma_x}\right)^2 \frac{I}{I_A}\right]\left\langle e^{-i\theta}\right\rangle$$

$$\hat{z} = 2k_u\rho z \qquad \eta = \frac{\gamma - \gamma_0}{\rho\gamma_0}$$

$$A = \frac{kf_c K}{4\gamma_0^2 k_u \rho^2}u \qquad \Delta = \frac{\gamma_0 - \gamma_r}{\rho\gamma_0}$$

(Δ=Detuning)

choose ρ to yield 1

$$\rho = \frac{1}{\gamma_0}\left[\left(\frac{f_c K}{4k_u\sigma_x}\right)^2 \frac{I}{I_A}\right]^{\frac{1}{3}}$$

Solving the Coupled FEL Equations (1D)

- Assume that the FEL interaction can be derived from a Hamiltonian.
- Phase space distribution *f* has to obey Liouville's Theorem, resulting in the Vlasov equation:

$$\frac{d}{dz} f(\theta,\eta,z) = \left[\frac{\partial}{\partial z} + \theta' \frac{\partial}{\partial \theta} + \eta' \frac{\partial}{\partial \eta}\right] f(\theta,\eta,z) = 0$$

- Fourier series expansion of *f*:

$$f(\theta,\eta,z) = \underbrace{f_0(\eta)}_{\text{Initial distribution}} + \underbrace{f_1(\eta,z)e^{i\theta}}_{\text{Current modulation}} + \ldots$$

Solving the FEL Equations

- Sorting all terms proportional to the term exp(iθ)

$$\frac{\partial}{\partial z} f_1 + i(\Delta + \eta) f_1 - A \frac{\partial}{\partial \eta} f_0 = 0$$

- The formal solution is:

$$f_1(z) = \int_0^z A \frac{\partial f_0}{\partial \eta} e^{-i(\Delta+\eta)(z-z')} dz'$$

- The solution is then used to solve the field equation (still formal):

$$\frac{d}{dz} A = \left\langle e^{-i\theta} \right\rangle = \iint f(\theta,\eta,z) e^{-i\theta} d\eta d\theta = \int f_1(\eta,z) d\eta$$

Solving the FEL Equations

- Laplace transformation: (Fourier transformation does not work) :

$$G(p) = \int_0^\infty g(z) e^{-pz} dz$$

- FEL Equation:

$$\int_0^\infty \left(\frac{d}{dz} A(z) \right) e^{-pz} dz = \int_{-\infty}^\infty d\eta \frac{\partial f_0}{\partial \eta} \int_0^\infty dz\, e^{i(ip-\eta-\Delta)z} \int_0^z dz' A(z') e^{i(\eta+\Delta)z'}$$

- Integration by parts:

$$A(0) + p\tilde{A}(p) = -\int_{-\infty}^\infty d\eta \frac{\partial f_0}{\partial z} \int_0^\infty \frac{A(z)e^{-pz}}{i(ip-\eta-\Delta)} dz$$

- Formal Solution:

$$\tilde{A}(p) = \frac{A(0)}{p - \int_{-\infty}^\infty \frac{\partial f_0}{\partial \eta} \frac{d\eta}{p + i\eta + i\Delta}}$$

Dispersion Equation

- Laplace Transformation:

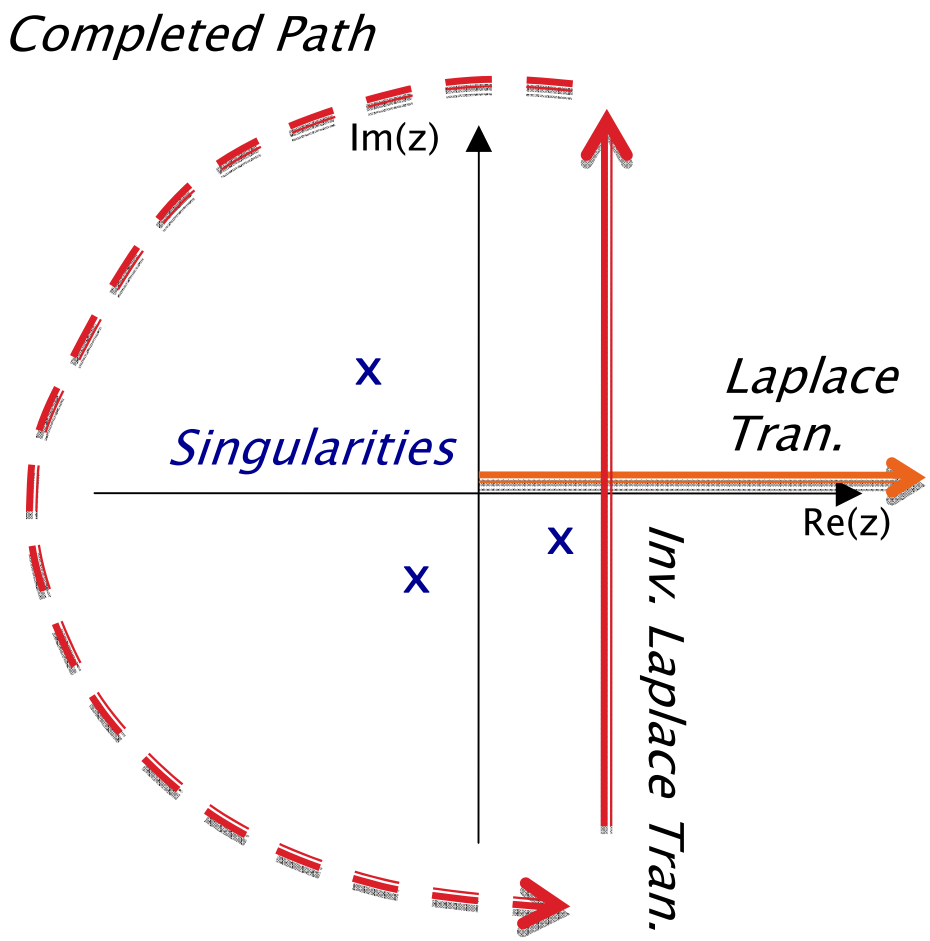
$$\tilde{A}(p) = \frac{A(0)}{p - \int_{-\infty}^{\infty} \frac{\partial f_0}{\partial \eta} \frac{d\eta}{p + i\eta + i\Delta}}$$

- Inverse Laplace Transformation:

$$A(z) = \frac{1}{2\pi i} \int_{p_0 - i\infty}^{p_0 + i\infty} \tilde{A}(p) e^{pz} dp$$

Dispersion Equation

$$p - \int_{-\infty}^{\infty} \frac{\partial f_0}{\partial \eta} \frac{d\eta}{p + i\eta + i\Delta} = 0$$



The 1D Solution

- No detuning ($\Delta=0$) and no energy spread ($f_0=\delta(\eta)$)

$$p-\int_{-\infty}^{\infty}\frac{\partial f_0}{\partial\eta}\frac{d\eta}{p+i\eta+i\Delta}=0 \quad\Rightarrow\quad p^3=i$$

- 3 Solutions to $A(z)\sim A(0)\cdot\exp(pz)$:
  - Oscillating Mode: $p=-i$
  - Exponential Growing Mode: $\boxed{p=\frac{\sqrt{3}}{2}+\frac{i}{2}}$ ***FEL Mode***
  - Exponential Decaying Mode: $p=-\frac{\sqrt{3}}{2}+\frac{i}{2}$

The 1D Solution

- The characteristic growth of the radiation power:

$$P(z) \propto |A(z)|^2 \propto e^{2p\hat{z}} = e^{2\sqrt{3}k_u \rho z}$$

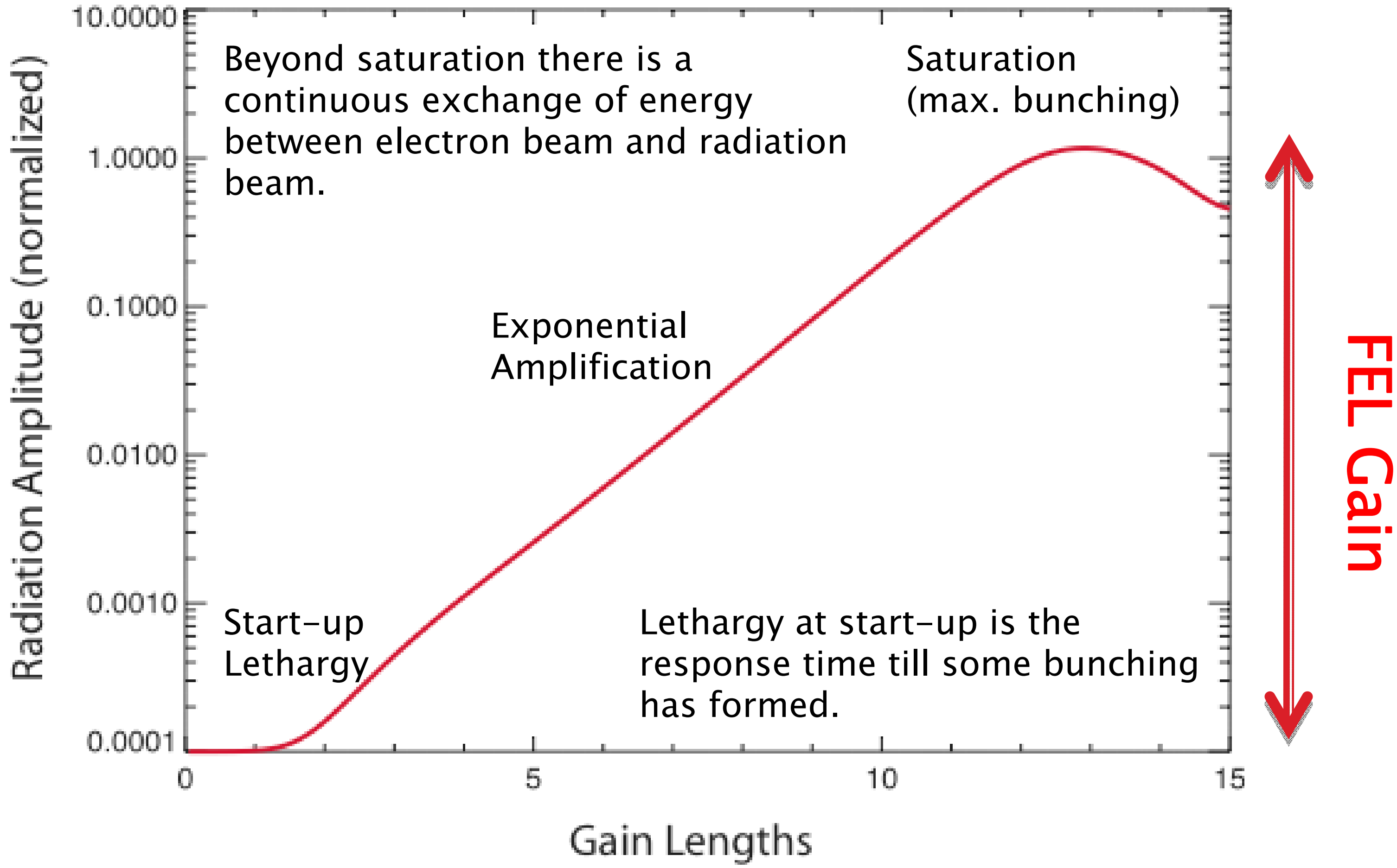
- Gain Length (e-folding length):

$$\boxed{L_g = \frac{\lambda_u}{4\pi\sqrt{3}\cdot\rho}}$$

- Amplification stops at maximum bunching. Energy loss of electron is $<\eta>\sim 1$:

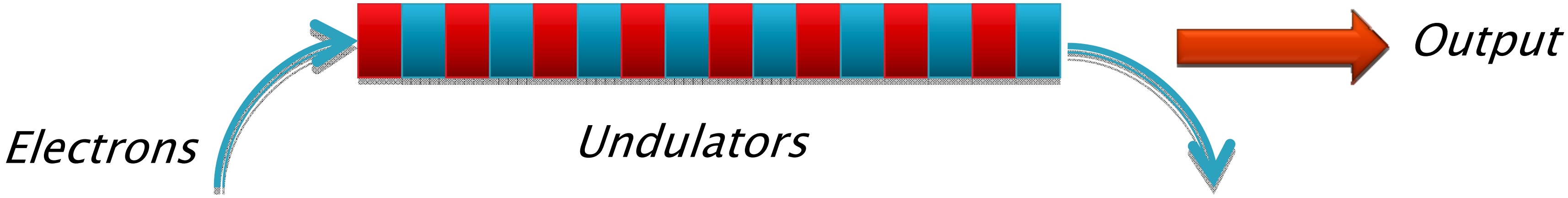
$$\boxed{P_{FEL} = \rho P_{beam}} \qquad P_{beam}[\mathrm{GW}] = E[\mathrm{MeV}]\cdot I[\mathrm{kA}]$$

The Generic Amplification Process



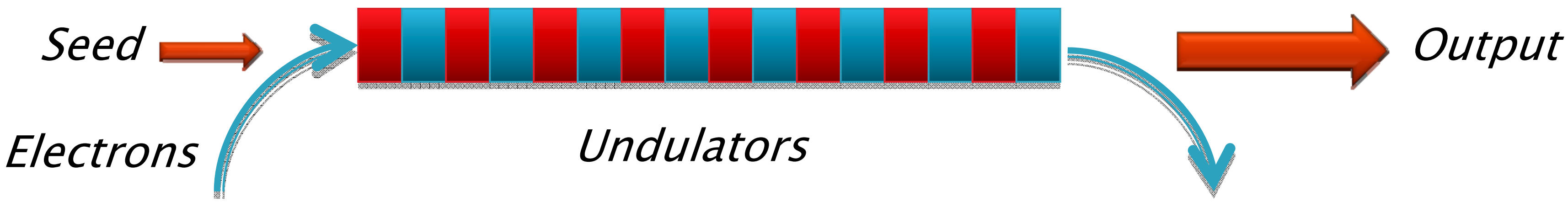
FEL Modes

- SASE FEL (Self-Amplified Spontaneous Emission)



Disadvantage: Output is noisy

- FEL Amplifier (starts with an input signal)

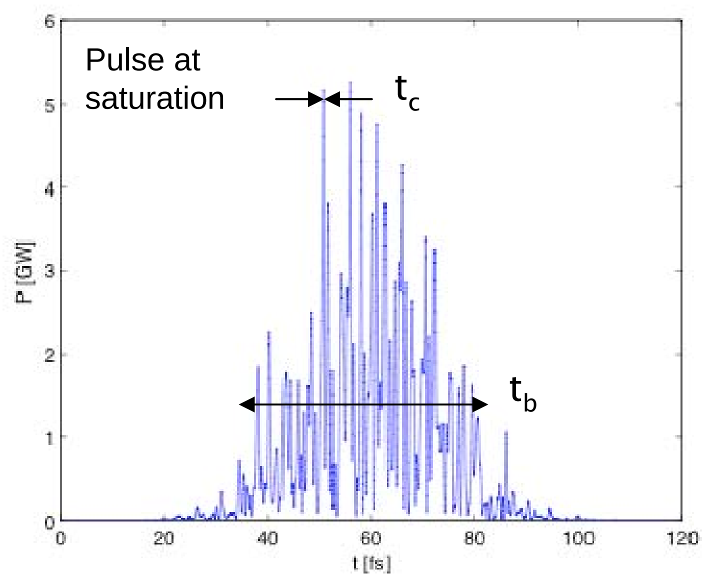


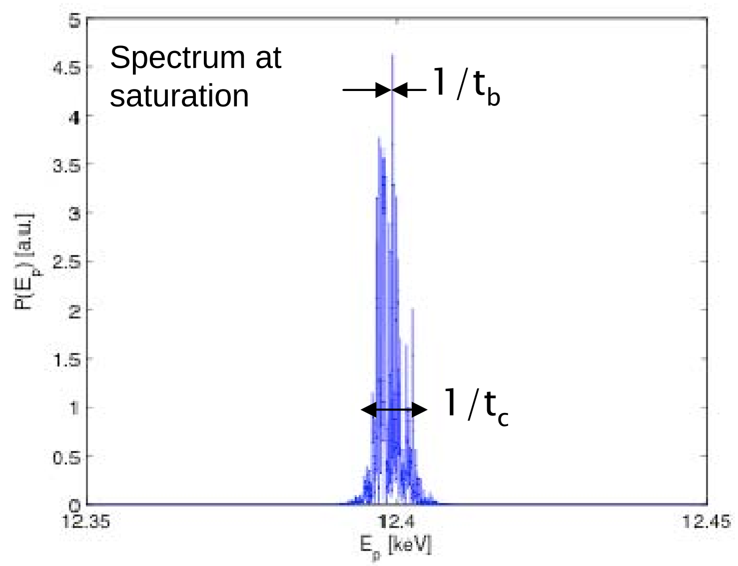
Disadvantage: Seed source may not exist

SASE FELs

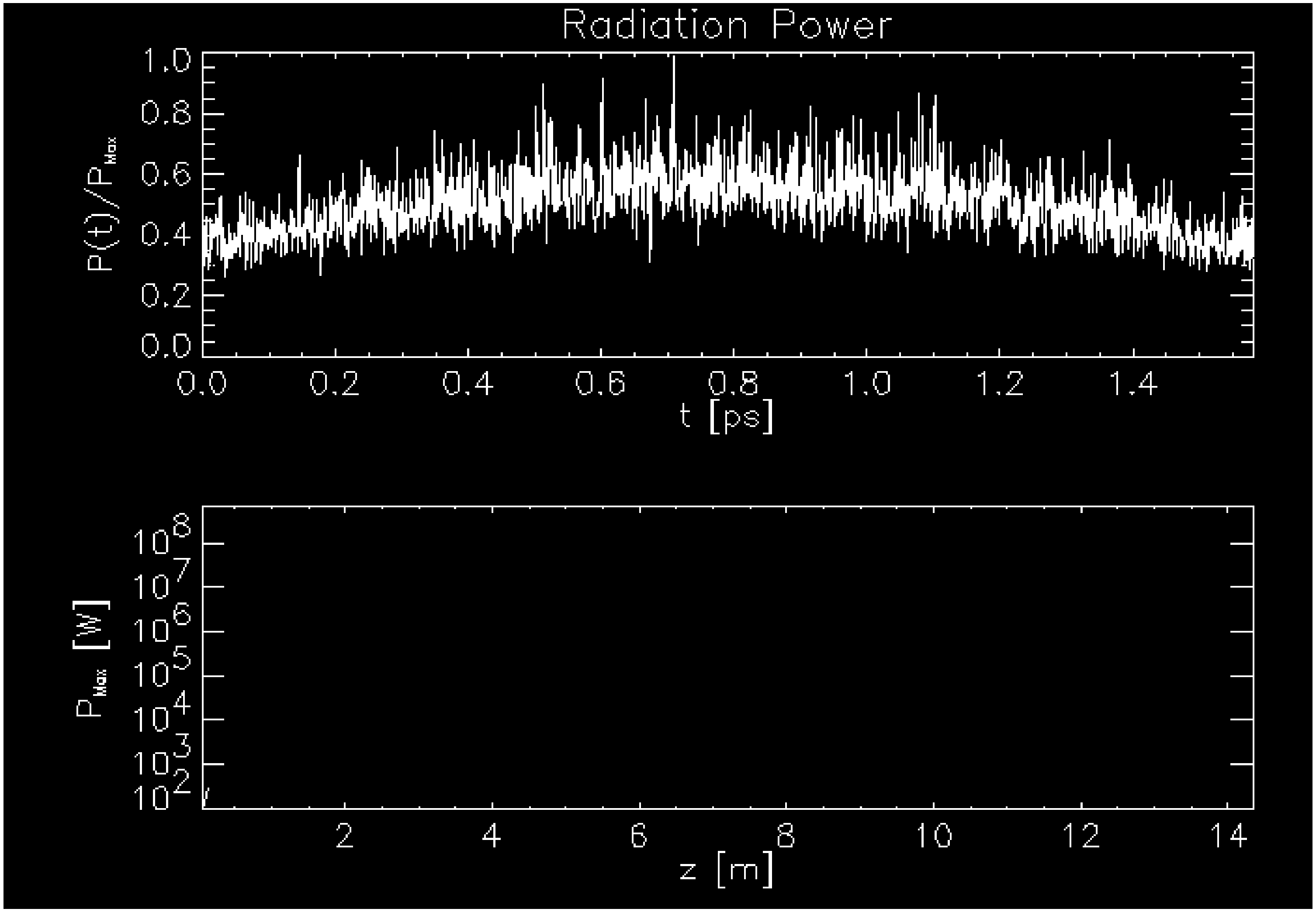
- All FELs can operate at SASE FELs, starting from the spontaneous radiation.
- Spontaneous radiation is noisy/incoherent ☹
- SASE FEL starts with zero coherence. Slippage (one wavelength/ undulator period) propagates phase information.
- X-ray FEL: Maximum coherence t_c about 1 fs.

SwissFEL: Simulation for 1 Angstrom radiation





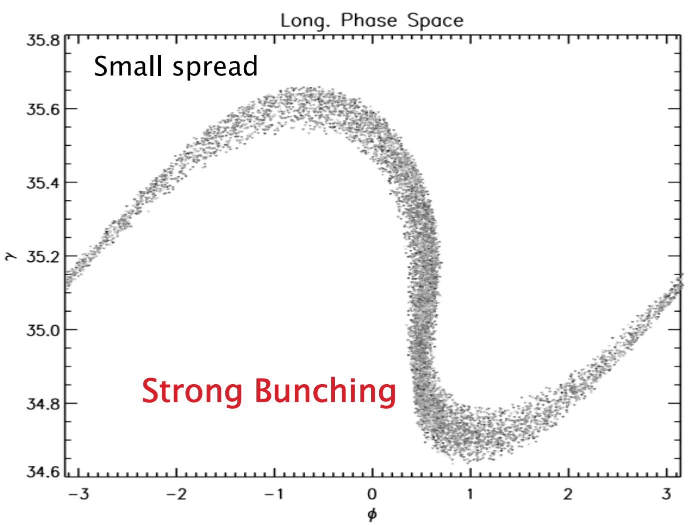
Typical Growth of SASE Pulse

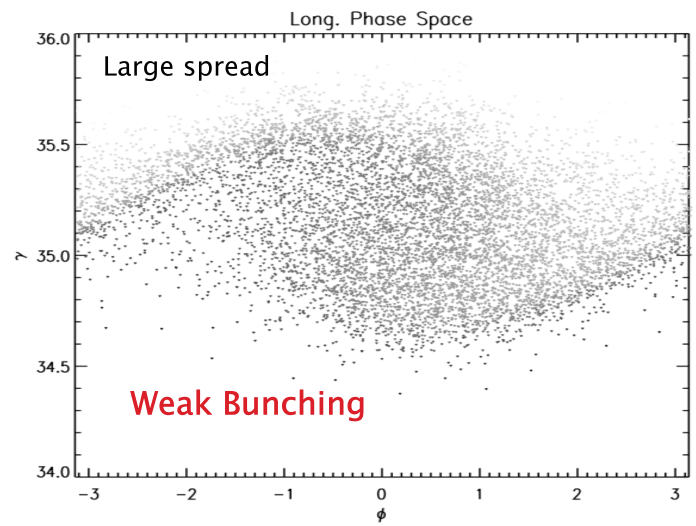


Simulation for FLASH FEL

Electron Beam Requirements: Energy Spread

- Only electrons within the FEL bandwidth can contribute to FEL gain.
- FEL process is a quarter rotation in the separatrix of the FEL. If separatrix is filled homogeneously, no bunching and thus coherent emission can be achieved.



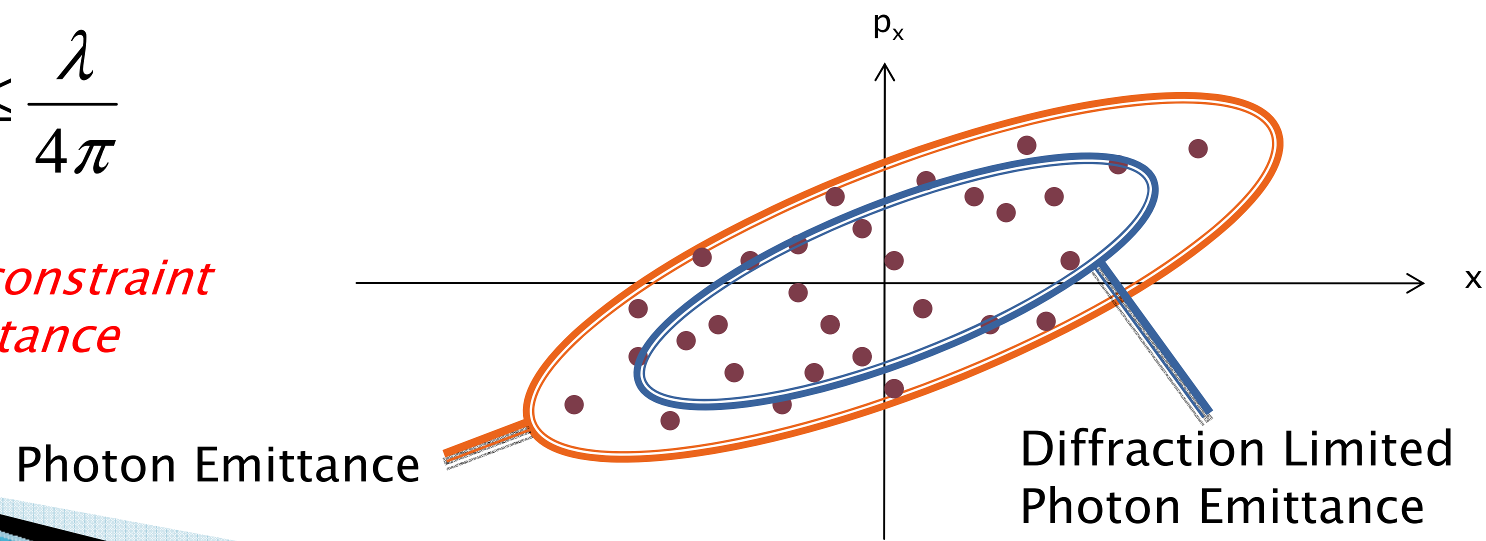


Electron Beam Requirements: Emittance

- Simple model: Electron emits photon in the forward direction.
- These photons spawn up the same emittance for the radiation field as the electron beam.
- All these photons should fall within the minimum photon emittance of a diffraction limited laser field.

$$\frac{\varepsilon_N}{\gamma} \leq \frac{\lambda}{4\pi}$$

Strong constraint for emittance



Providing the Electron Beam

- To meet the requirements for an XFEL, today's most brilliant electron beams have to be generated.

Brilliance = 6D Phasespace Density

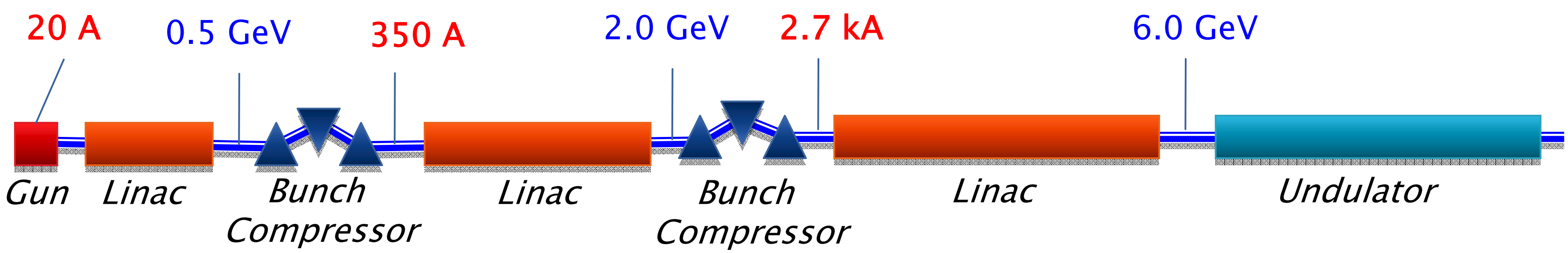
- Additional Requirements:
  - High peak current to reduce undulator length and increase FEL power
  - Beam energies of about 1-10 GeV.
- Electron source is the most important component, defining the brilliance
- Acceleration and compression to minimize the dilution of the brilliance.

Design Goals for 1 Angstrom FEL

Compact FEL	Maximum Photon Count
Design for smallest emittance	Design for high charge
Requires lower bunch charges	Yields higher emittance values
Low emittance allows for lower beam energy	Higher emittance needs to be compensated by higher beam energy
Shorter accelerator	Longer accelerator
Lower beam energy makes beam more sensitive (less compression)	Higher energy makes beam more rigid (higher compression)
Lower energy allows for weaker undulator field and shorter period	Higher energy requires longer periods and stronger field
Example: **SwissFEL** •5.8 GeV •2.7 kA •500 m linac •70 m undulator •<900 m total length	Example: **European XFEL** •18.5 GeV •5 kA •2 km linac •250m undulator •3.5 km total length

Typical Layout for Electron Linac

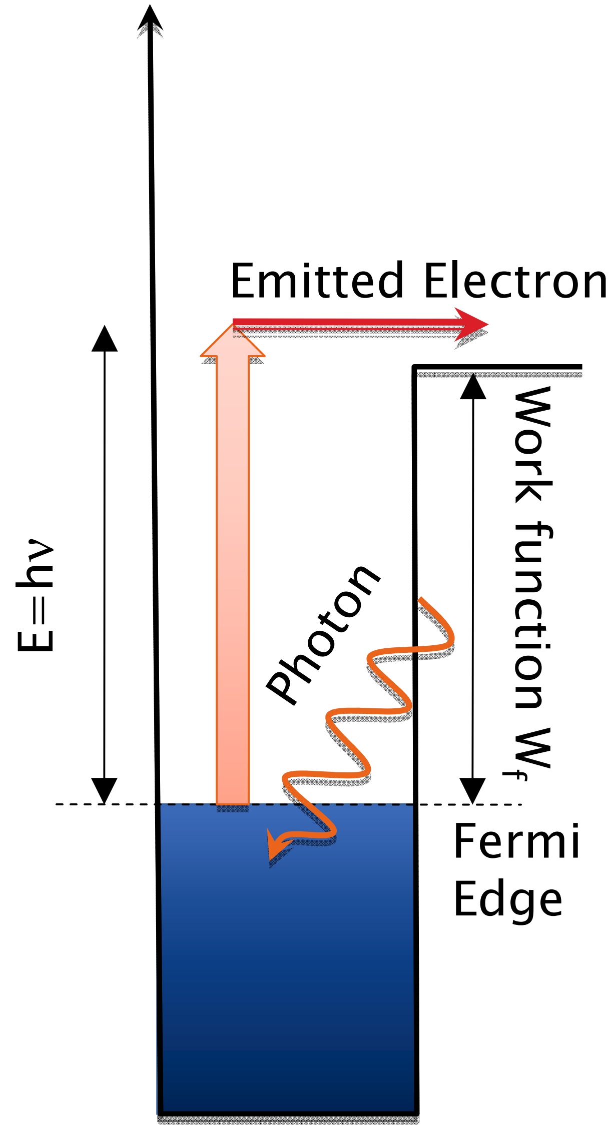
Example:SwissFEL



- Key components are:
  - Electron Source (Gun), typically RF-Photoelectron gun
  - Linear Accelerator (Linac)
  - Bunch Compressor (Chicane with 4 dipole magnets)
- Each component are optimized to minimize the degradation of the electron brilliance
- Alternative concepts (e.g. Plasma gun + acceleration) are investigated but not yet feasible

RF Photo Gun

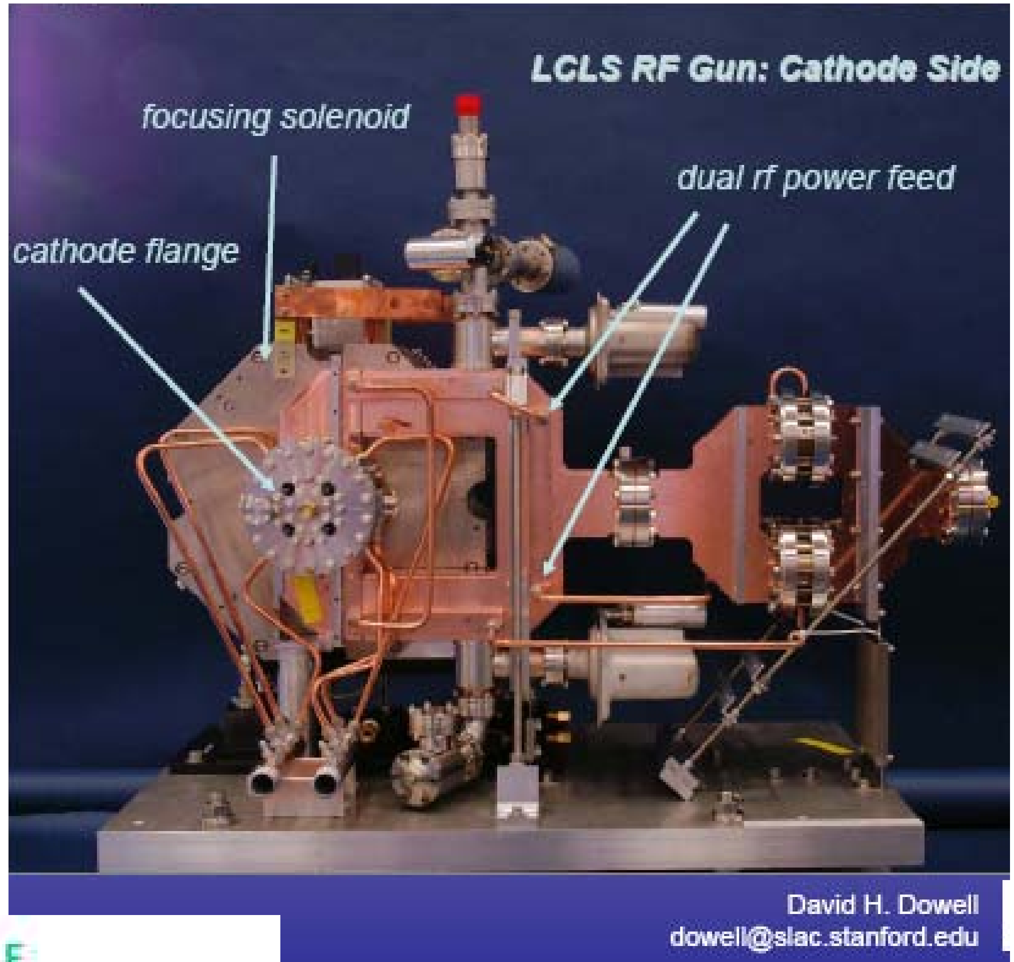
- Electrons are generated by the photo effect on a metallic cathode, when a laser pulse hits the surface.
- Spread in transverse momentum (and thus emittance) is given by maximum kinetic energy $h\nu - W_f$
- Lower photon energy has smaller electron yield (quantum efficiency).
- High power laser is required to achieve up to 1 nC bunch charge ($6 \cdot 10^9$ electrons)

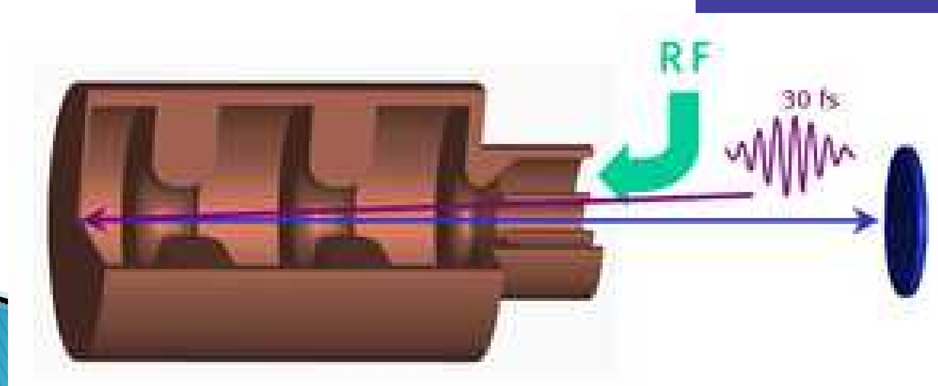


RF Photo Gun

- Photon electrons are rapidly accelerated to dampen the impact of the space charge fields (repulsive Coulomb field of electron bunch)

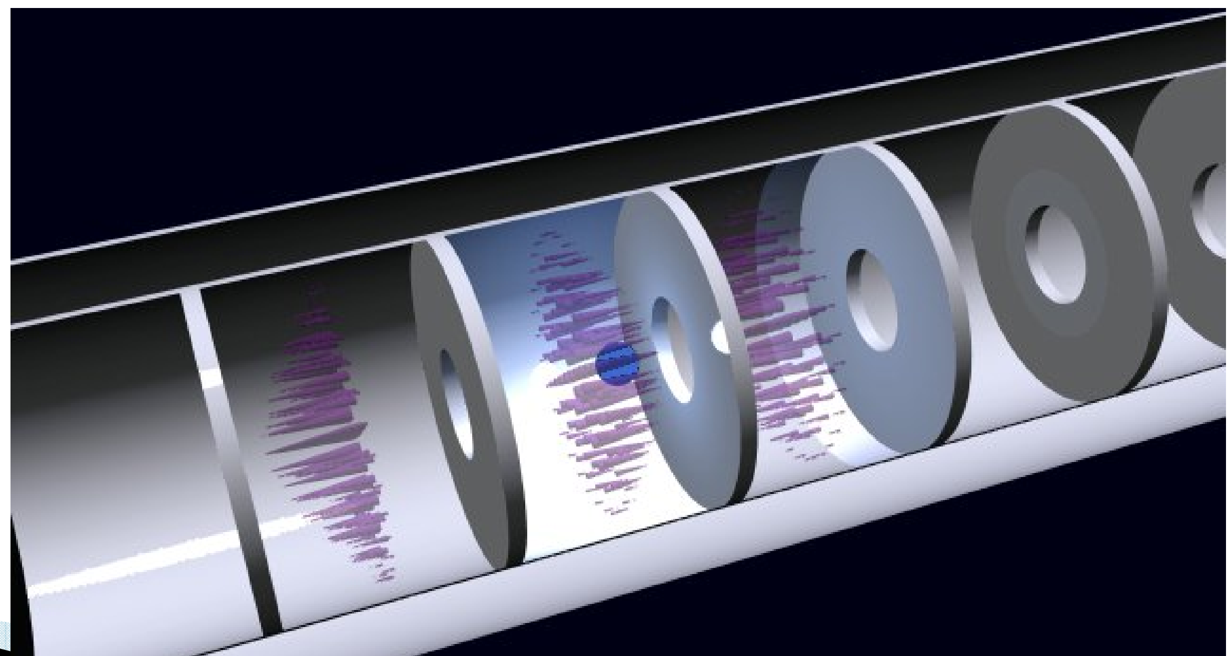
- Acceleration done by longitudinal RF field in a multi-cell cavity.





RF Accelerator

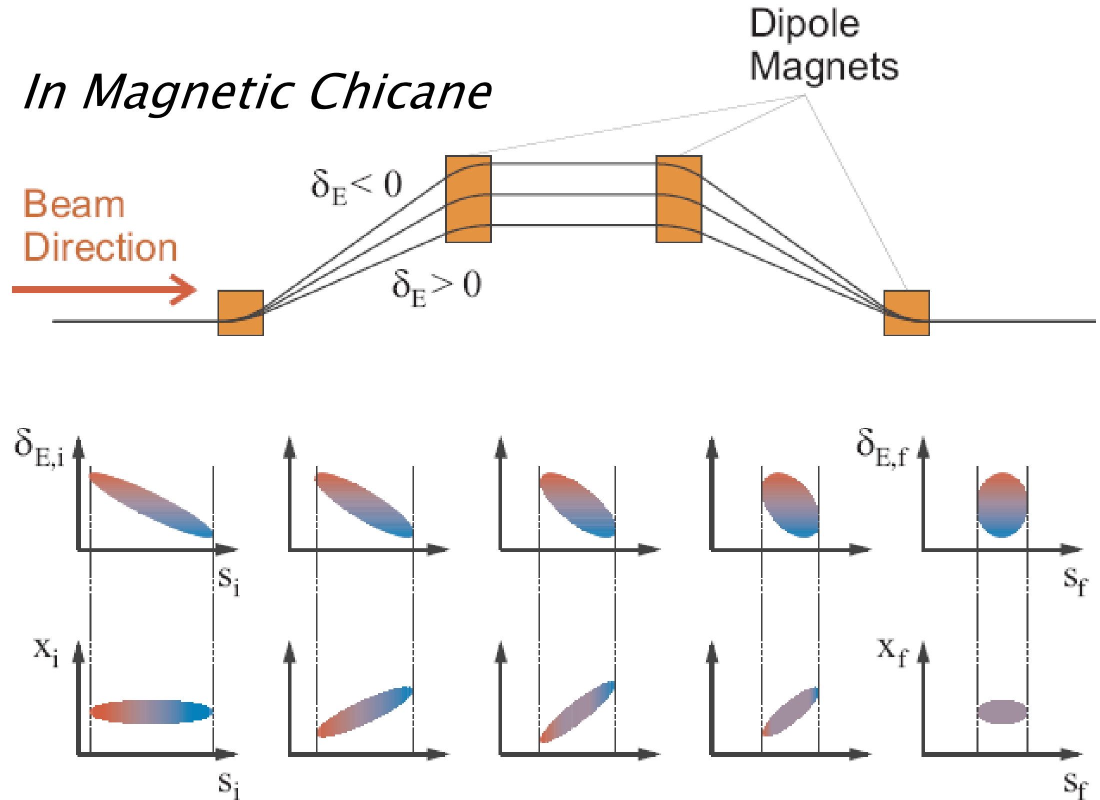
- Electrons ride on an RF wave through a chain of cavity cells
- The cells produces a longitudinal field, which accelerate the electrons
- Typical gradient: 20-30 MeV/m (about 300 m for 6 GeV)



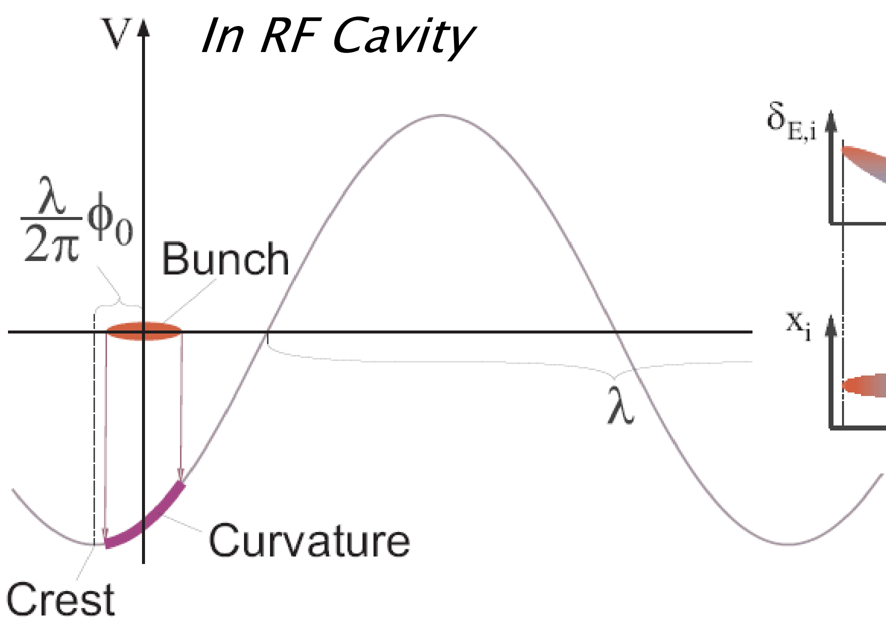
Bunch Compression

Head particles have a lower energy than tail particles.

In Magnetic Chicane



In RF Cavity



The tail electrons move on a shorter path which allows them to overtake the leading electrons.

The Challenges: Beam Generation

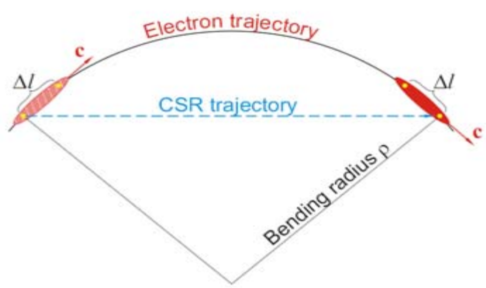
- Producing electron beams with low emittances but sufficient charges
  - Current record at the rf gun for the LCLS X-ray FEL at SLAC

- Optimum charges can range between 1 to 1000 pC. Low charges has the best efficiency (photons per electrons) and shortest pulse length though high charges yield more photons in total.

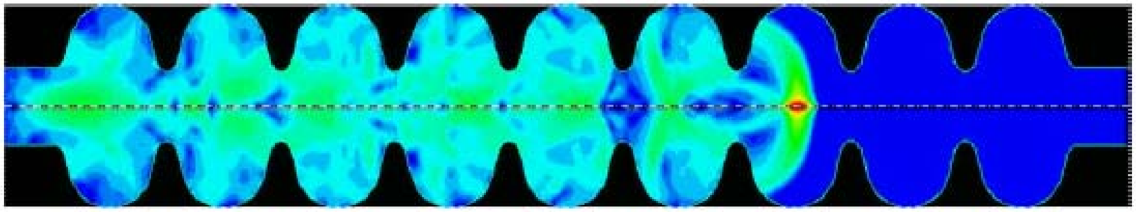
- Novel electron sources are research (Field emitter arrays, plasma injectors) which could provide a significant improvement in beam brightness

The Challenges: Beam Preservation

- Space Charges:
  - Active in low energy part of injector
  - Non-linear fields degrades emittance and spoils energy spread
  - Solution: Rapid acceleration and pulse shaping
- Coherent Synchrotron Radiation:
  - Active in bunch compressor
  - Emitted radiation interacts back
  - Multiple, gentle compression



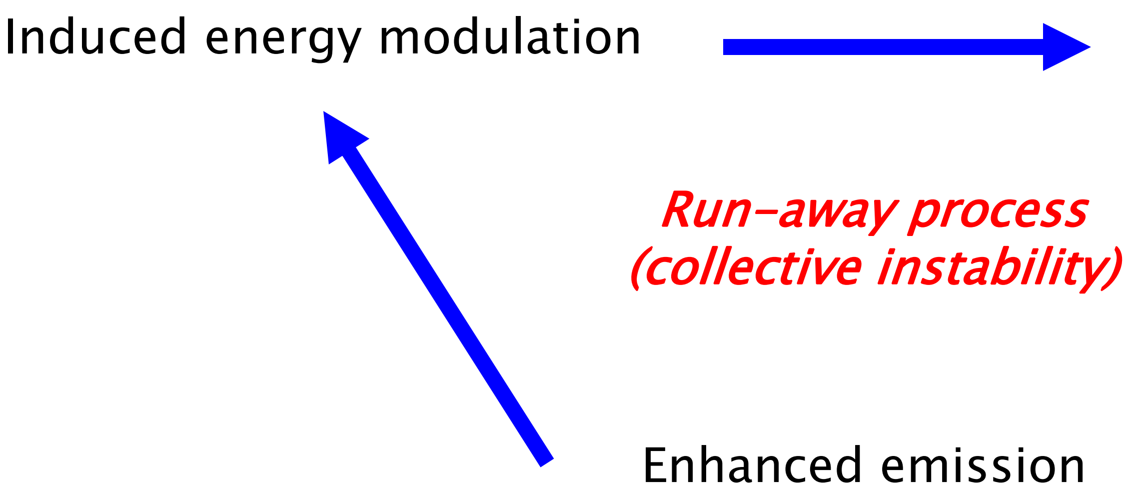
- Wakefields
  - Active in RF Cavities and undulator
  - Non homogenous energy loss and transverse kicks of bunch
  - Large cavities



M. Dohlus

Summary

FEL Process



- FEL utilized the strong coherent emission in the collective instability with the tuning ability of the wavelength.
- Instability can only occur with a high brightness electron beam.
- RF Photo gun + RF Linac + Bunch Compression is currently the best way to provide the beam.