Introduction to Feedback Systems

Wolfgang Hofle

Wolfgang.Hofle@cern.ch

CERN

CERN CAS Darmstadt, October 2009

Feedback

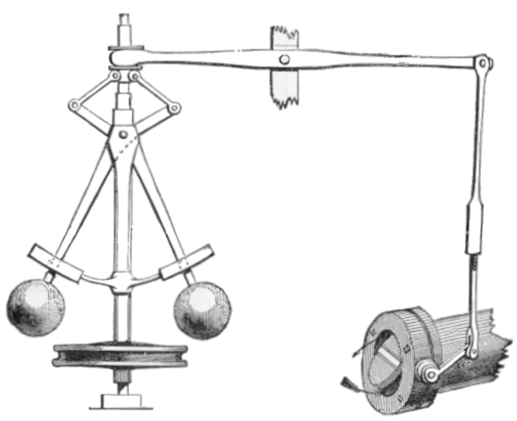
Feedback is a mechanism that influences a system by looping back an output to the input

a concept which is found in abundance in nature

and essential to regulate the processes in any form of life

Feedback systems in engineering

1788	Watt	adopted an automated regulation mechanism using a centrifugal (fly-ball) governor to control a steam engine, using the angular speed to manipulate a valve for the steam
1868	Maxwell	published an analysis of Watt's centrifugal governor mathematical analysis of a feedback system
1876	Vyshnegradskii	independently analyzed stability of steam engine governor stability



Contents and Objectives of this lecture

What is a system, what means feedback ?

What are the purposes of feedback systems ?

recap the mathematical tools to analyze system behavior

criteria for the stability of closed loop feedback systems

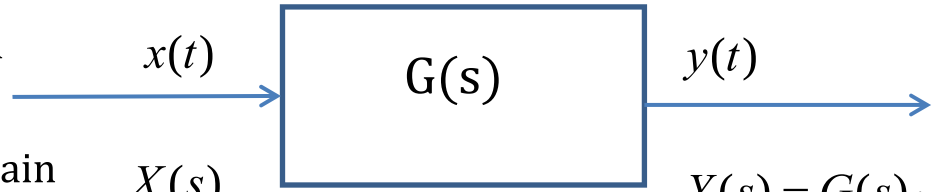
steps in designing feedback systems

System

System input

time domain $x(t)$

Laplace domain $X(s)$



System output

$y(t)$

$$Y(s) = G(s) \cdot X(s)$$

G(s) called transfer function

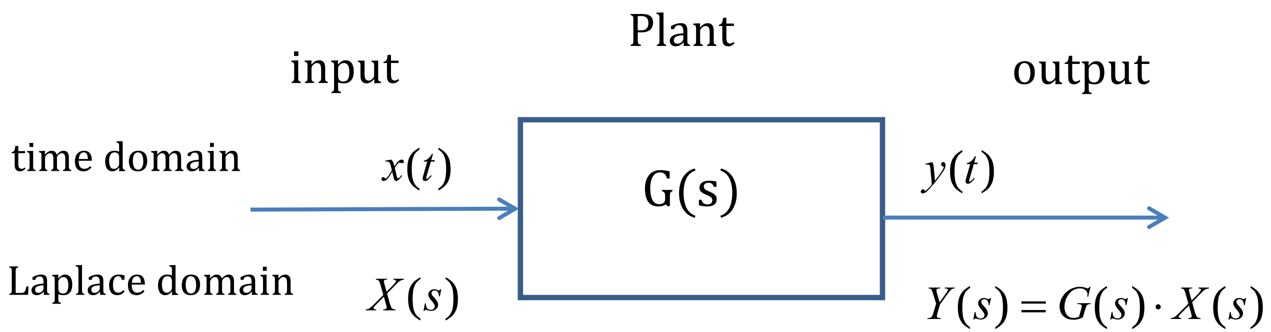
characterized by a fixed rule determining an evolution in time of the output

deterministic, output can be calculated from input and initial state

non linear systems can be linearized in the vicinity of a “working point”

Linear-Time-Invariant systems (LTI systems)

Control



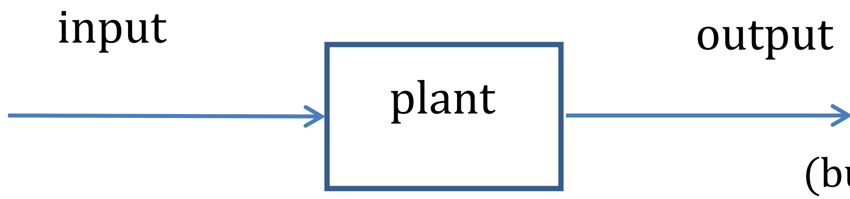
control theory tells us how

to influence the output of a system referred to as *plant*

plant can be: steam engine, living cell, your home (temperature), ship crossing a river, particle beam trajectory

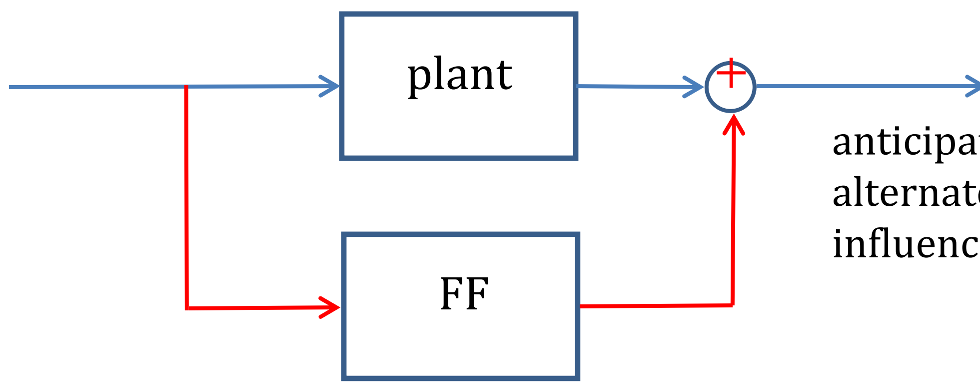
Ways to Control

open loop
(simple)



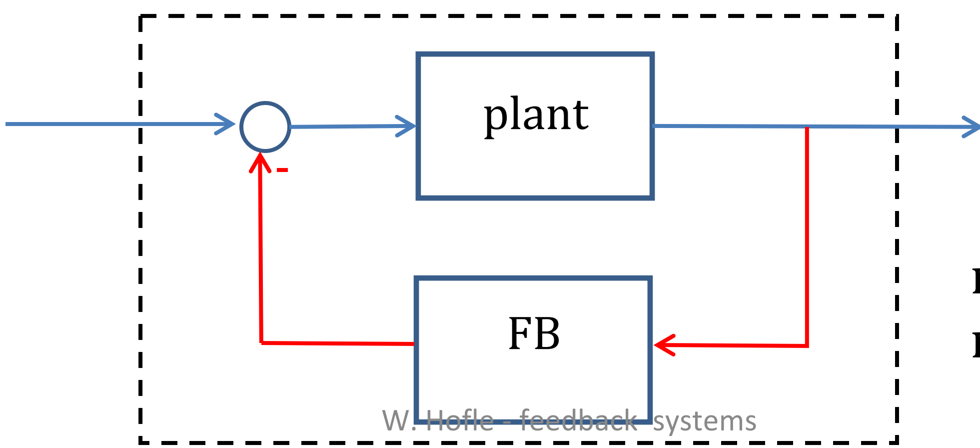
(but) requires
precise knowledge of plant

feed forward



anticipate, requires
alternate means of
influencing output

feedback

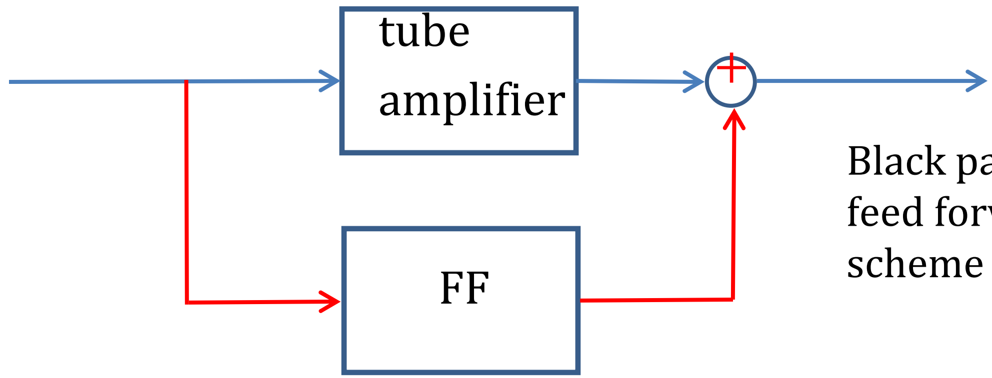


feed back means
influencing the system
output by acting back on
the input

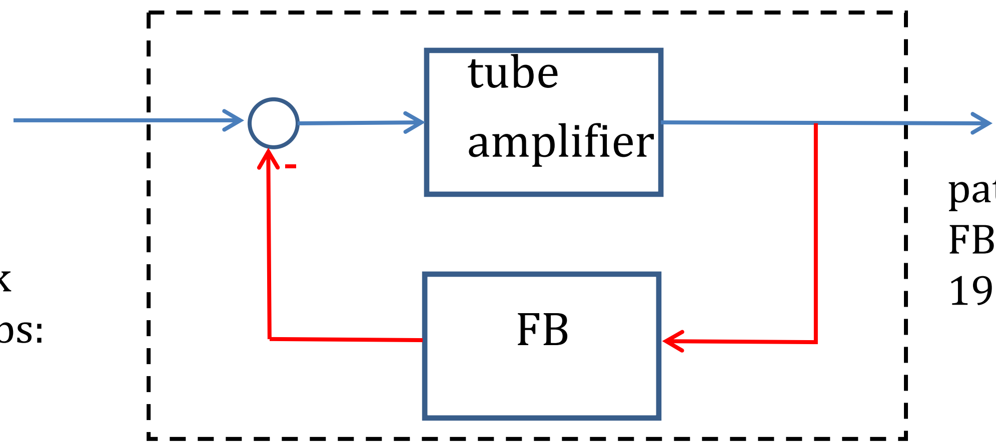
new system !
new properties ?

Inventing the negative feed back amplifier

H. S. Black, working at Bell labs in the 1920's on looking for a way to improve the linearity and bandwidth of vacuum tube amplifiers



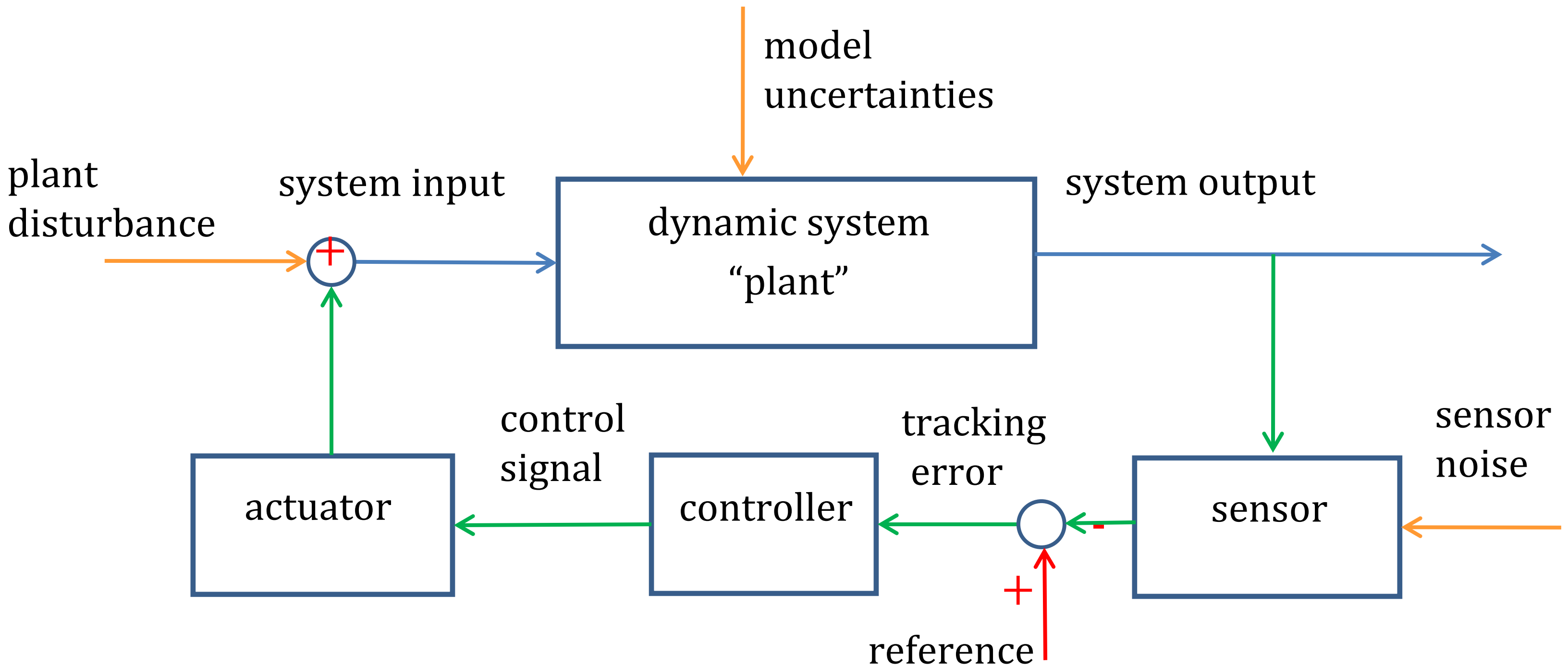
Black patented a feed forward scheme in 1928



contributing to feedback control theory at Bell labs: Nyquist, Bode

patent for negative FB amplifier in 1932 (H. S. Black)

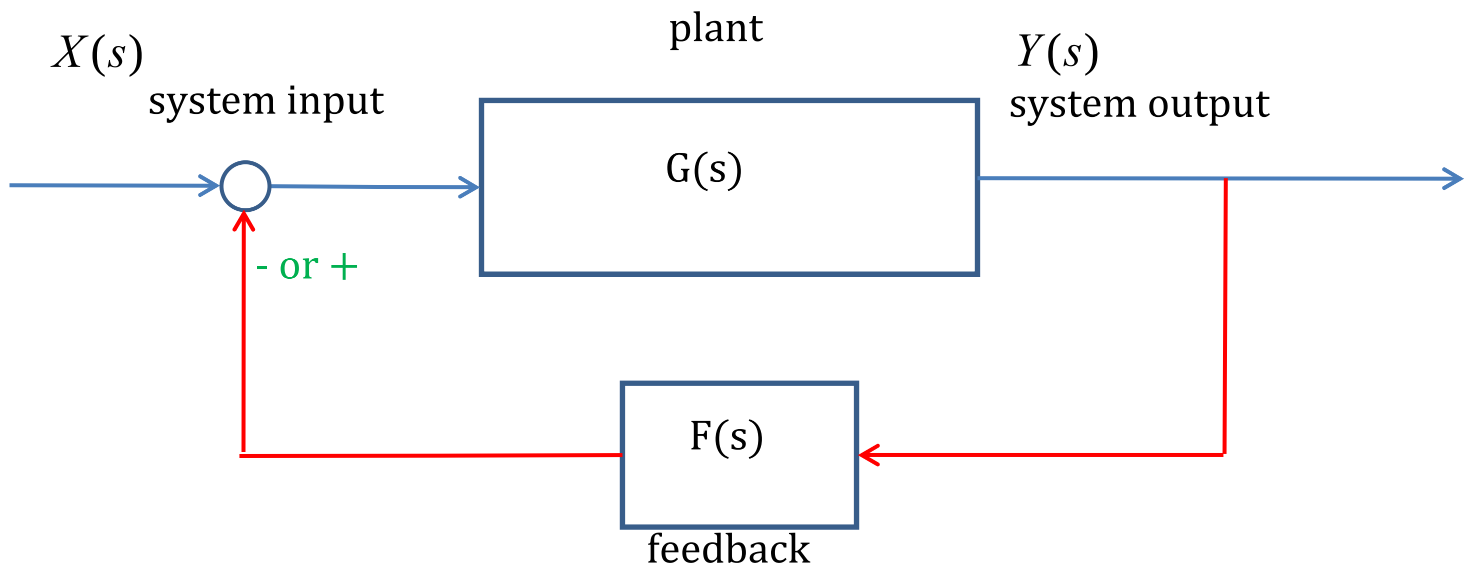
Feedback model



purpose of feedback control:

- cancel plant imperfections
- precise tracking of reference parameters
- stabilize a (potentially) unstable system
- reduce the effect of disturbances
- render output insensitive to model uncertainties

Closed loop transfer function



without feedback

$$Y(s) = G(s)X(s)$$

output of closed loop

$$Y(s) = G(s)X(s) - \underbrace{G(s)F(s)Y(s)}_{\text{feedback}}$$

closed loop transfer function

$$G_{CL}(s) = \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)F(s)}$$

open loop transfer function

$$G_{OL} = G(s)F(s)$$

negative feedback (sign convention)
- or + depending on sign convention

Let's recap the tools

LTI systems described by differential equations (continuous-time domain)
or difference equations (discrete-time domain)

solving equations subject to initial conditions using

Laplace transform, transfer function $F(s)$ in s-domain
complex frequency $s = \sigma + j\omega$, for continuous-time domain signals and systems

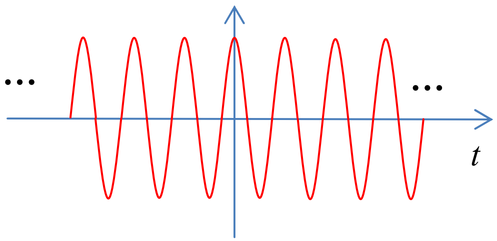
z-transform for discrete-time domain signals and systems

1952 Ragazzini & Zadeh

Fourier transform, continuous and discrete

description in frequency domain by the response to a sinusoidal input
concept of transfer function $F(\omega)$, amplitude and phase,
for purpose of system characterization by measurement

Fourier transform



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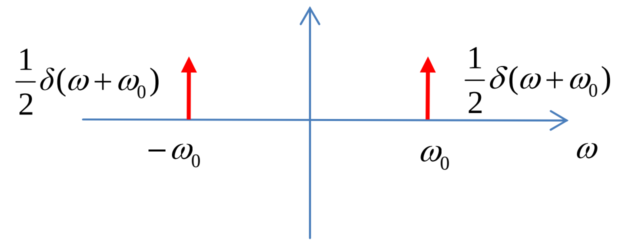


$$F(\omega)=\int_{-\infty}^{+\infty} f(t)\,\mathrm{e}^{-j\omega t}\,\mathrm{d}t$$

$$f(t)=\cos(\omega_0 t)$$

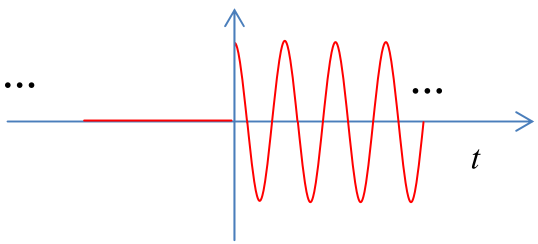
○—●

$$F(\omega)=\frac{1}{2}\left[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right]$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)\,\mathrm{e}^{+j\omega t}\,\mathrm{d}\omega$$

Laplace transforms

functions are zero for t<0



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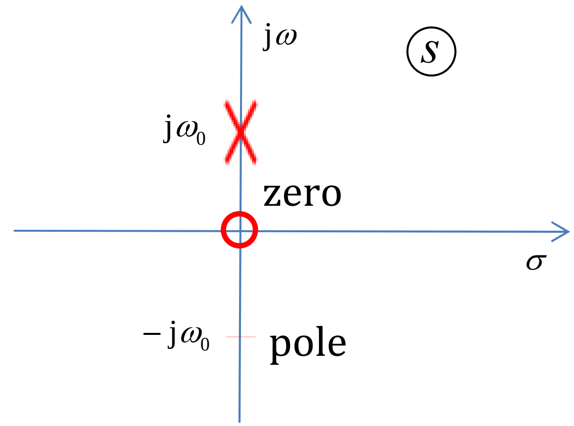


complex frequency

$$s = \sigma + j\omega$$

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$f(t) = \cos(\omega_0 t)$$

○—●

$$F(s) = \frac{s}{s^2 + \omega_0^2}$$

$t > 0$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma - j\omega}^{\sigma + j\omega} F(s)\, e^{ts}\, ds$$

Laplace transform - rules

step function	$u(t), \quad 1$	$\circ\!\!-\!\!\bullet$	$\frac{1}{s}$
delta function	$\delta(t)$	$\circ\!\!-\!\!\bullet$	1
scaling	$f(at)$	$\circ\!\!-\!\!\bullet$	$\frac{1}{a}F\left(\frac{s}{a}\right) \qquad a>0$
1^{st} shift rule (frequency)	$e^{-at} f(t)$	$\circ\!\!-\!\!\bullet$	$F(s+a)$
2^{nd} shift rule (time)	$f(t-a)$	$\circ\!\!-\!\!\bullet$	$e^{-as} F(s) \qquad a>0$
differentiation 1^{st} derivative	$\dot{f}(t)$	$\circ\!\!-\!\!\bullet$	$sF(s)-f(+0)$
differentiation 2^{nd} derivative	$\ddot{f}(t)$	$\circ\!\!-\!\!\bullet$	$s^2F(s)-f(+0)s-\dot{f}(+0)$
integration	$\int_0^t f(\tau)\,d\tau$	$\circ\!\!-\!\!\bullet$	$\frac{1}{s}F(s)$

…

Second order system - free oscillation

solving differential equations using the Laplace transform

differential equation
(free oscillation)

$$\ddot{y} - 2\alpha \cdot \dot{y} + \omega_0^2 y = 0$$

$$s^2 Y - 1 - s 2\alpha Y + \omega_0^2 Y = 0$$

$$Y = \frac{1}{s^2 - 2\alpha \cdot s + \omega_0^2}$$

$$y = \frac{1}{\sqrt{\omega_0^2 - \alpha^2}} e^{\alpha \cdot t} \sin\left(\sqrt{\omega_0^2 - \alpha^2}\, t\right)$$

un-damped oscillation for $\alpha > 0$

initial conditions (for example)

$$y(+0) = 0$$

$$\dot{y}(+0) = 1$$

!

$$\dot{f}(t) \circ\!\!-\!\!\bullet\; sF(s) - f(+0)$$

$$\ddot{f}(t) \circ\!\!-\!\!\bullet\; s^2 F(s) - s f(+0) - \dot{f}(+0)$$

Second order system – transfer function

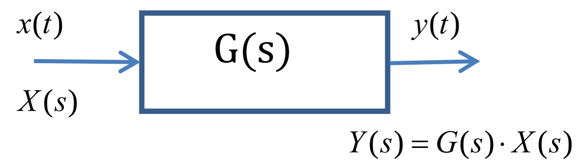
$$\ddot{y} - 2\alpha \cdot \dot{y} + \omega_0^2 y = x(t)$$

$$x(t) = \delta(t)$$

$$\delta(t) \circ\!\!-\!\!\bullet \; 1$$

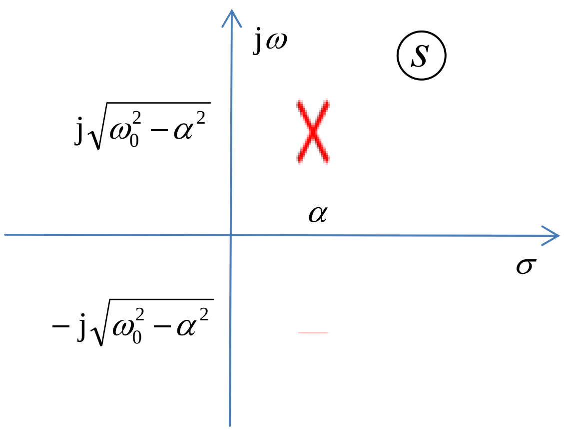
System input

System output



$$G(s) = \frac{1}{s^2 - 2\alpha \cdot s + \omega_0^2}$$

The transfer function G(s) describes
the response of the system to a δ(t) signal input

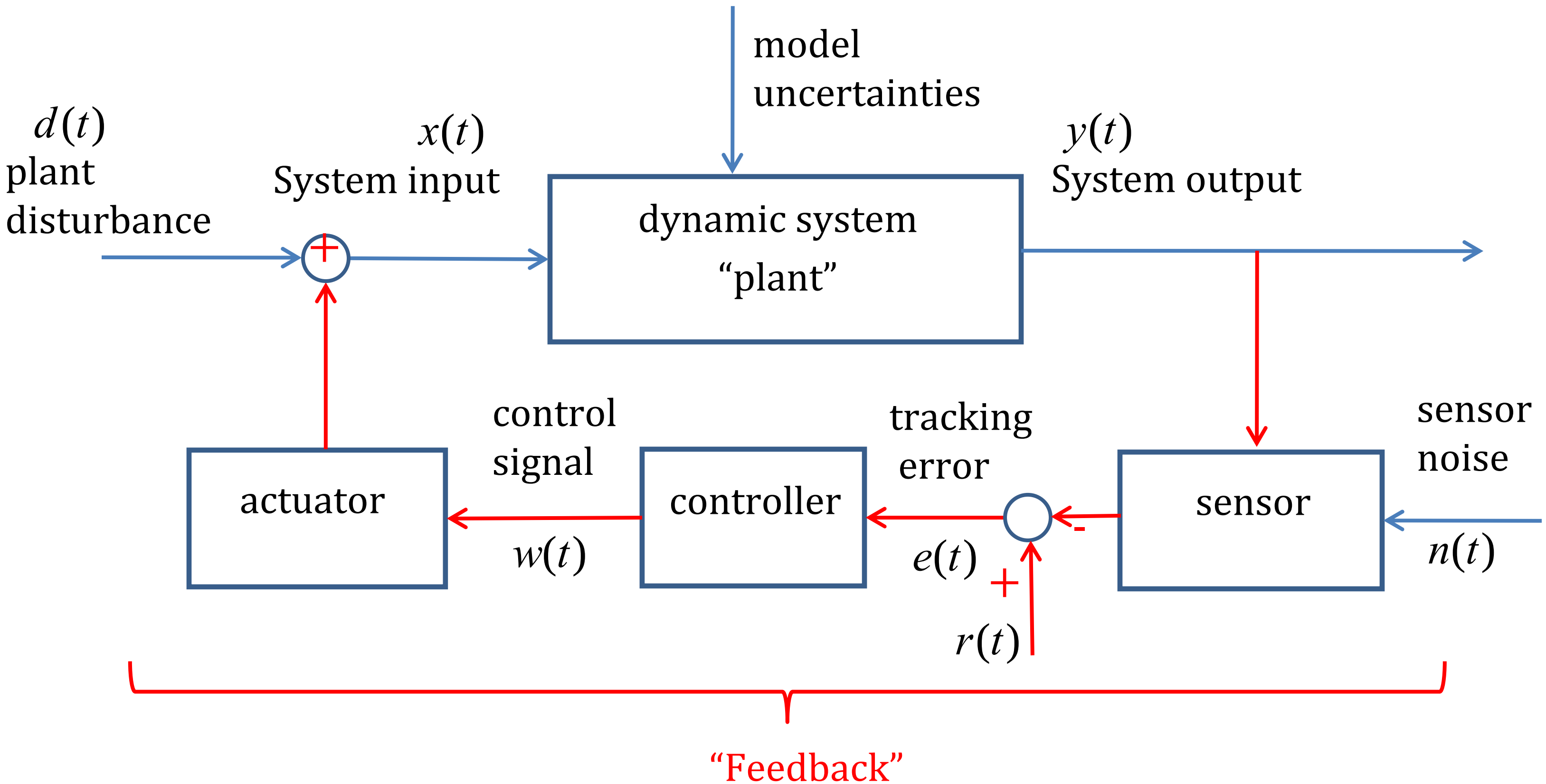


poles $\quad s_x^2 - 2\alpha \cdot s_x + \omega_0^2 = 0 \qquad s_x = \alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$

$$y = \frac{1}{\sqrt{\omega_0^2 - \alpha^2}} e^{\alpha \cdot t} \sin\left(\sqrt{\omega_0^2 - \alpha^2}\, t\right)$$

poles in right half plane -> unstable
a small disturbance will grow

Analyze and design a feedback



objective ?

identify plant

what are input & outputs ?

Design a feedback: longitudinal feedback

What do you want to achieve ? → damp longitudinal instability

identify plant

what are input & outputs ?

beam= dynamic system "plant"

beam:
synchrotron oscillation of the centroid of a bunch in a bucket

↓

How to sense output ?

sensor

phase pick-up

↓

How to act on input ?

actuator

cavity, longitudinal kicker

↓

What type of controller ?

controller

Let's try !

Restoring stability by feedback

differential equation

$$\ddot{y} - 2\alpha \cdot \dot{y} + \omega_0^2 y = x(t)$$

$$G(s) = \frac{1}{s^2 - 2\alpha \cdot s + \omega_0^2}$$

example:

beam (bunched) longitudinal instability

plant: model of beam synchrotron motion (centroid)

$\alpha > 0$ beam unstable without feedback

choose a feedback transfer function (educated guess)

$$\boxed{F(s) = Ks} \qquad K > 0$$

"velocity" feedback

feedback proportional to differentiated output

$$G_{\mathrm{CL}}(s) = \frac{G(s)}{1 + G(s)Ks} = \frac{1}{s^2 - 2\alpha \cdot s + \omega_0^2}\left[\frac{1}{1 + \frac{Ks}{s^2 - 2\alpha \cdot s + \omega_0^2}}\right] = \frac{1}{s^2 + \left(K - 2\alpha\right)\cdot s + \omega_0^2}$$

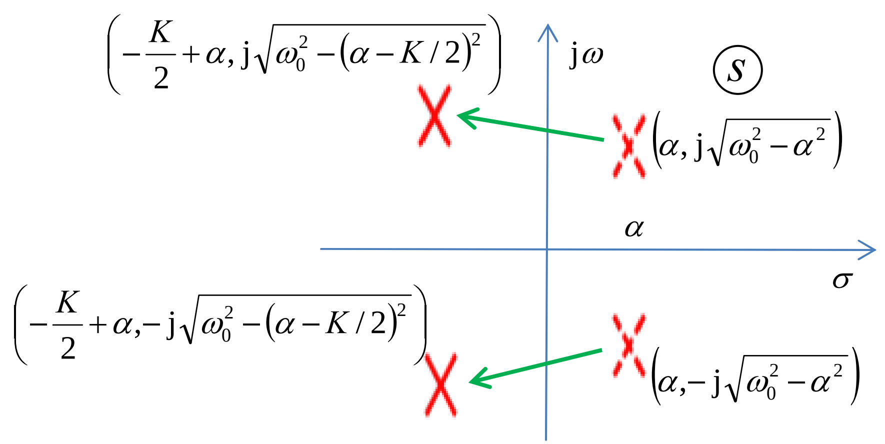
Restoring stability by feedback

$$G_{\mathrm{CL}} = \frac{1}{s^2 + (K - 2\alpha)\cdot s + \omega_0^2}$$

closed loop transfer function

poles $\quad s_x^2 + (K - 2\alpha)\cdot s_x + \omega_0^2 = 0$

$$s_x = -\frac{K}{2} + \alpha \pm \mathrm{j}\sqrt{\omega_0^2 - (\alpha - K/2)^2}$$



poles of resulting transfer function moved by feedback into left half plane for K/2>α

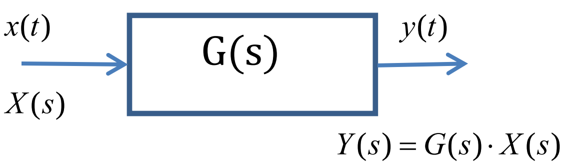
amount of damping can be adjusted by the gain of the feedback *K*

poles in left half plane if K/2 > α
stable, damped oscillation

note: feedback does not influence the zeros of the transfer function

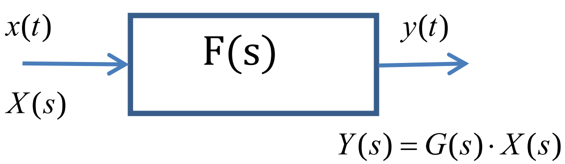
Elementary building blocks (1)

Very often plant and feedback can be characterized or approximated by transfer functions which are rational functions of s.



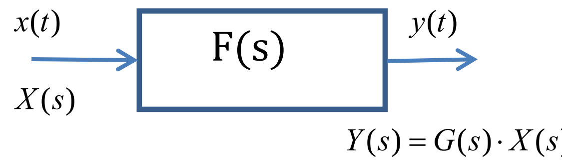
	differential equation	transfer function	impulse response	step response
P	$y(t) = K \cdot x(t)$	$G(s) = K$	$y(t) = K \cdot \delta(t)$	$y(t) = K \cdot u(t)$
Delay	$y(t) = x(t-T)$	$G(s) = \mathrm{e}^{-sT}$	$y(t) = \delta(t-T)$	$y(t) = u(t-T)$
PT	$\dot{y} + \frac{1}{T} y = x$	$G(s) = \frac{T}{sT+1}$	$y(t) = T\,\mathrm{e}^{-t/T}$	$y(t) = T^2\left(1-\mathrm{e}^{-t/T}\right)$

Elementary building blocks (2)



	differential equation	transfer function
D	$y(t) = K \cdot \dot{x}(t)$	$F(s) = sK$
PDT	$\dot{y} + \frac{1}{T} y = K \cdot \dot{x}$	$F(s) = \frac{sKT}{sT + 1}$
I	$y(t) = K \int_0^t x(\tau) \mathrm{d}\tau$	$F(s) = \frac{1}{s} K$

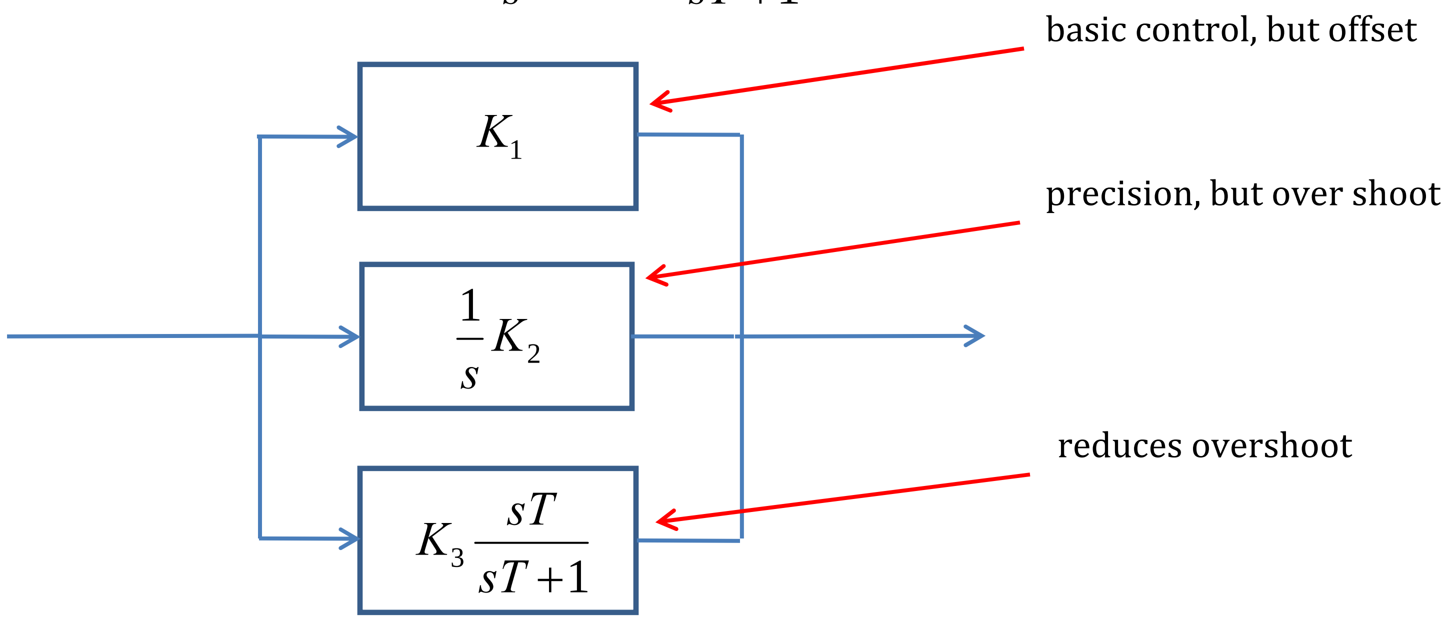
Composing building blocks



transfer function

P-I-DT

$$F(s) = K_1 + \frac{1}{s}K_2 + K_3\frac{sT}{sT+1}$$

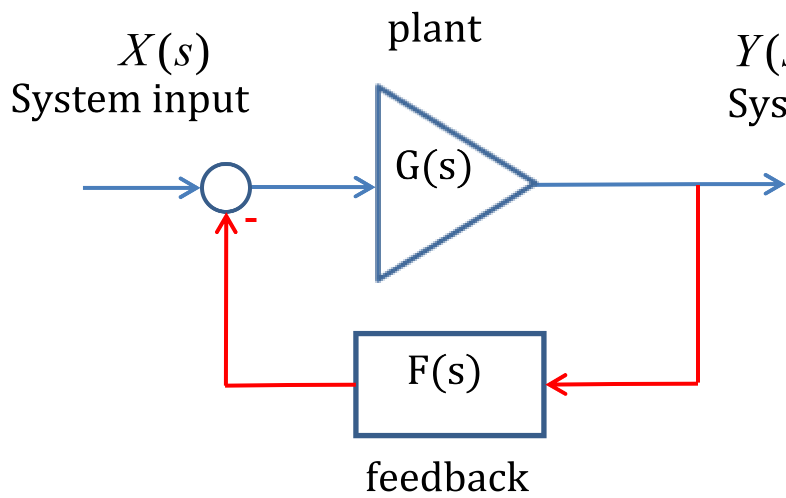


Negative feedback amplifier with delay

$G = const.$

feedback transfer function

$$F(s) = K\,\mathrm{e}^{-sT}$$



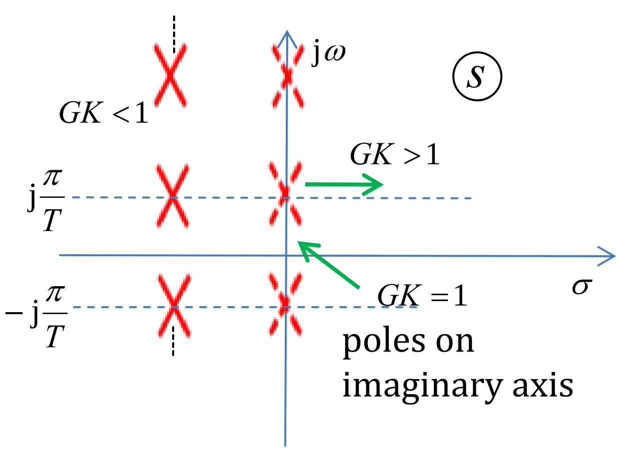
if $GK >> 1$
and $T \to 0$

$$G_{\mathrm{CL}} \to \frac{1}{K}$$

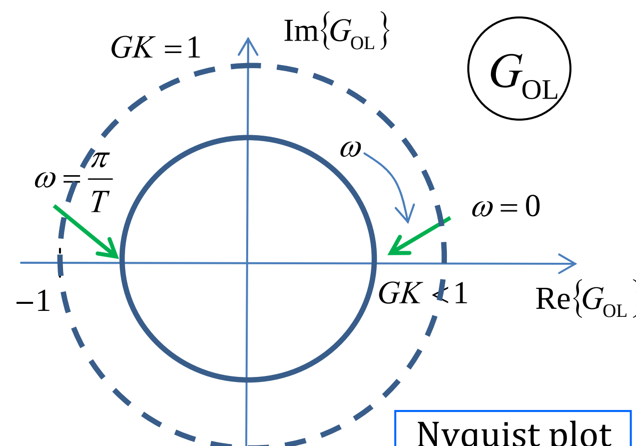
closed loop transfer function

$$G_{\mathrm{CL}} = \frac{G}{1 + G \cdot K\mathrm{e}^{-sT}}$$

poles: $$s_x = \frac{1}{T}\log(GK) + \mathrm{j}\frac{\pi(1+2n)}{T}$$



locus of open loop transfer function for s=jω



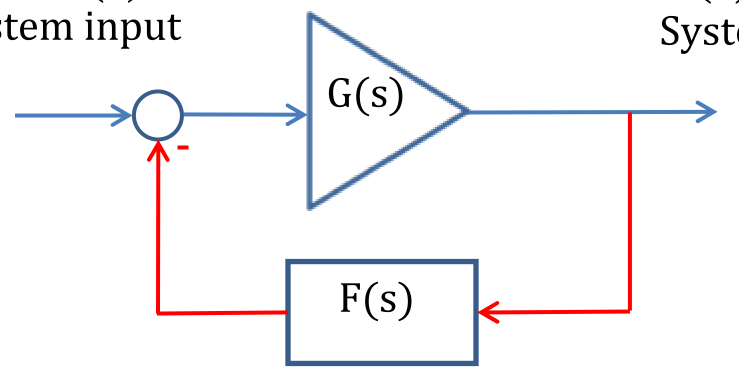
Nyquist plot

Negative feedback amplifier with delay stability restored

$G = const.$

feedback transfer function

$$F(s) = K\frac{1}{1+sT_2}\mathrm{e}^{-sT_1}$$



closed loop transfer function

$$G_{\mathrm{CL}} = \frac{G}{1+GF}$$

$$G_{\mathrm{CL}} = \frac{G}{1+\frac{1}{1+sT_2}G\cdot K\mathrm{e}^{-sT_1}}$$

poles: transcendental equation → look at plot of open loop transfer function

$$G_{\mathrm{OL}} = \frac{1}{1+sT_2}G\cdot K\mathrm{e}^{-sT_1}$$

Nyquist plot and criteria

Nyquist plot:

open loop transfer function
for s=jω

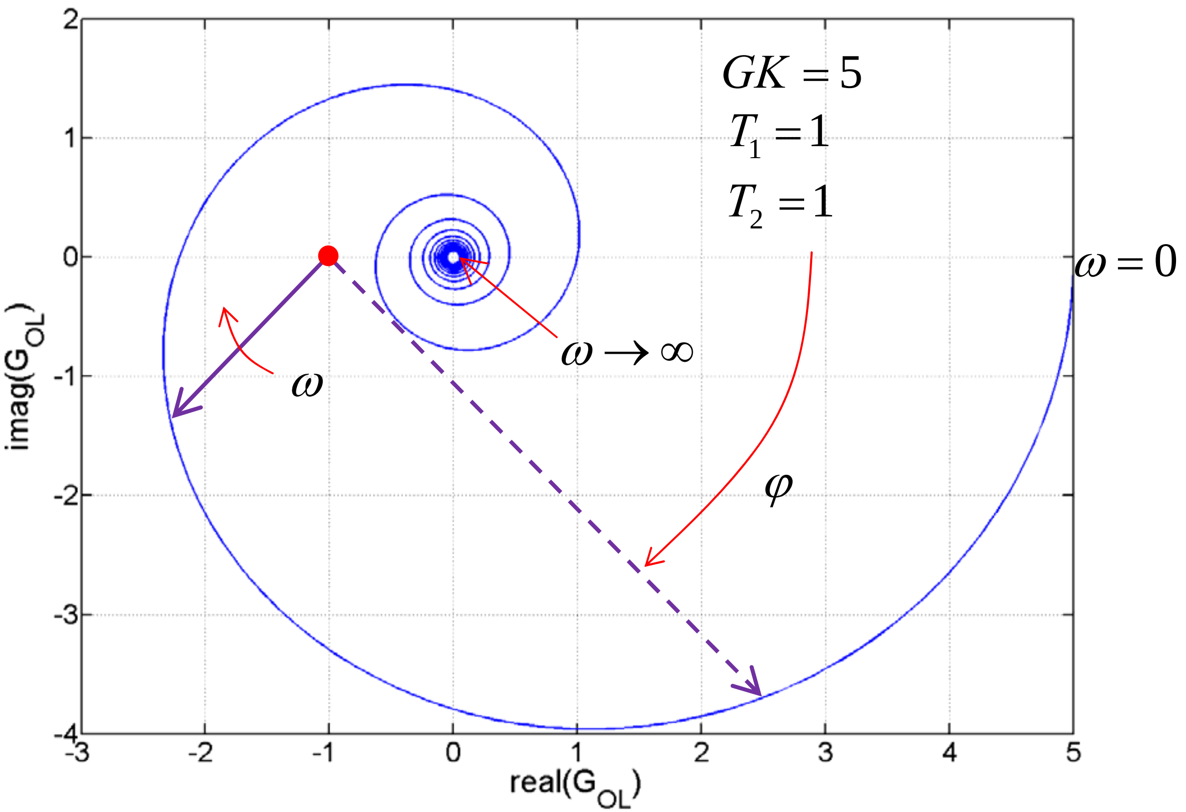
$$G_{\mathrm{OL}} = \frac{1}{1+sT_2} G \cdot K e^{-sT_1}$$

closed loop

$$G_{\mathrm{CL}} = \frac{G}{1+G_{\mathrm{OL}}}$$

$$\int_{\omega=0}^{\infty} \mathrm{d}\varphi \neq 0$$

→ unstable



Nyquist plot and criteria

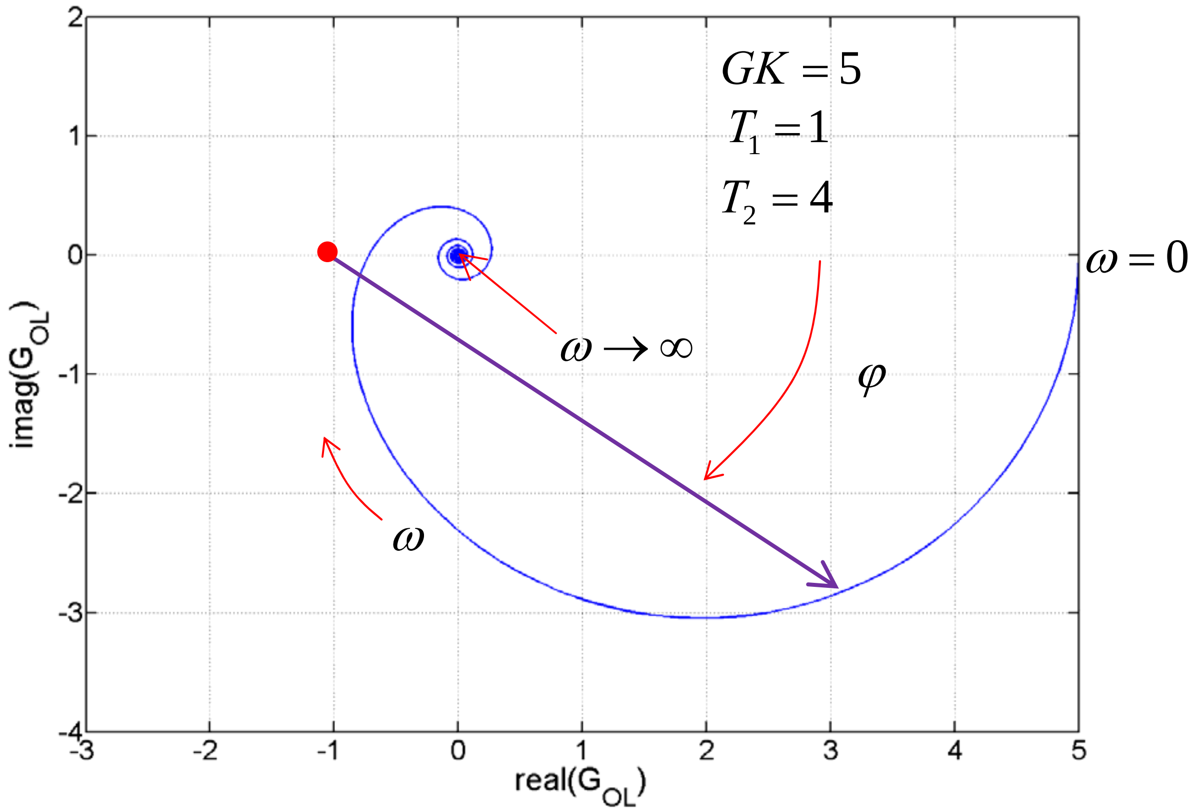
Nyquist plot:

open loop transfer function
for s=jω

$$G_{\mathrm{OL}} = \frac{1}{1+sT_2} G \cdot K e^{-sT_1}$$

closed loop

$$G_{\mathrm{CL}} = \frac{G}{1+G_{\mathrm{OL}}}$$

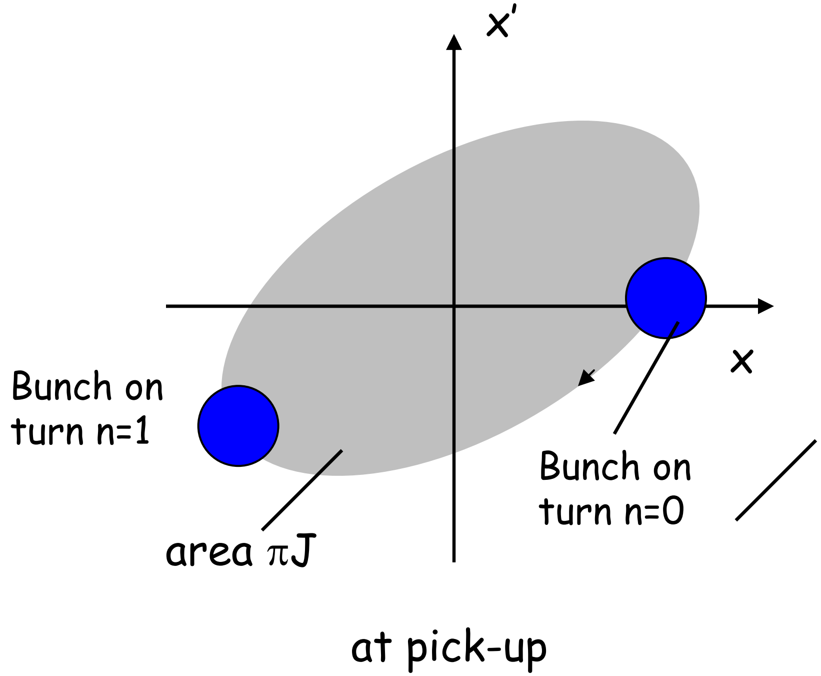


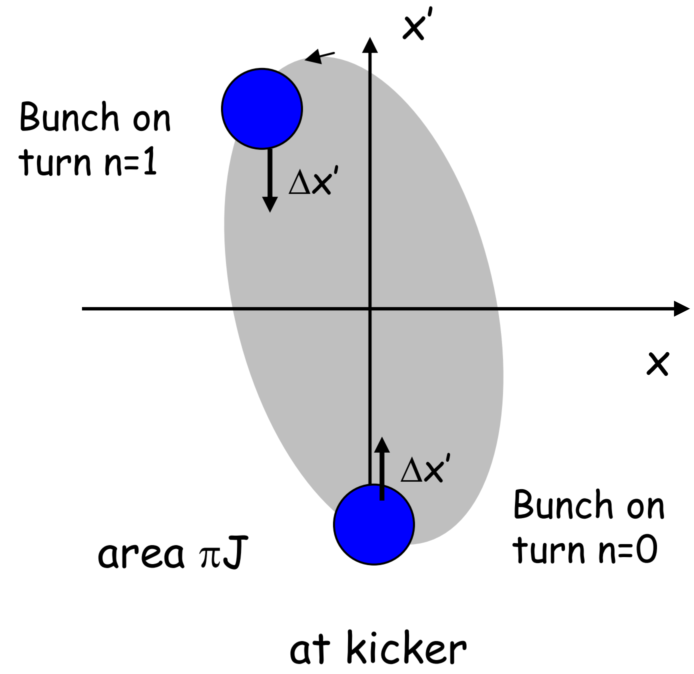
$$\int_{\omega=0}^{\infty} \mathrm{d}\varphi = 0$$

→ stable

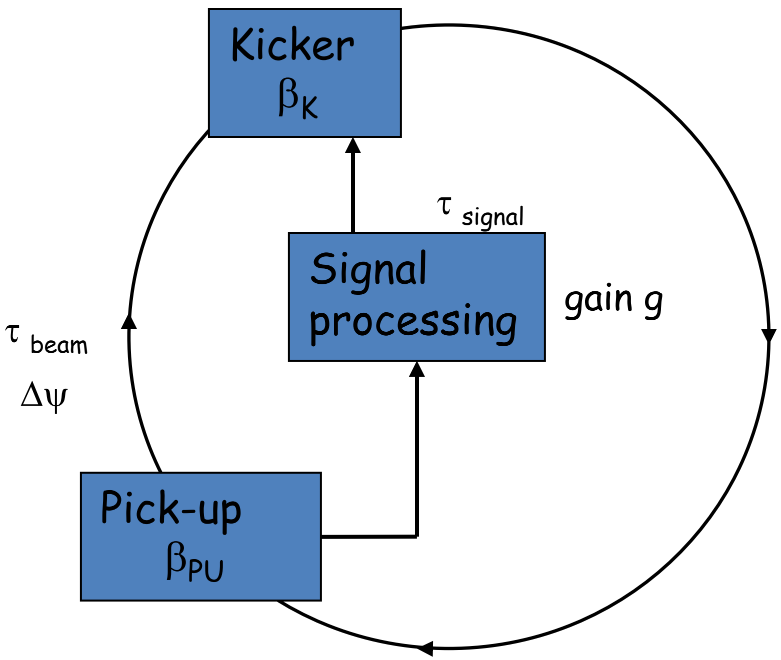
For stability the point (-1, 0) must always lie left of locus when followed $\omega \to \infty$

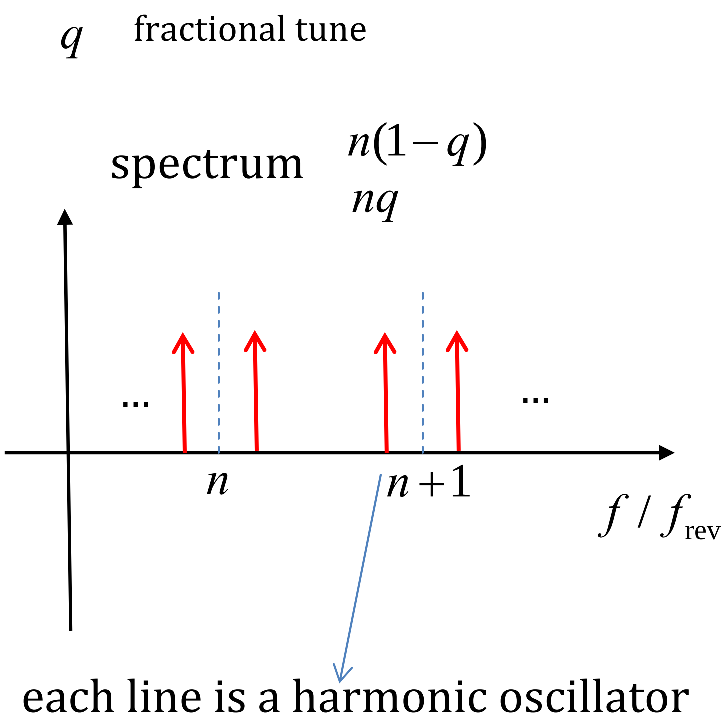
Example: Transverse feedback in SPS (coupled bunch feedback)





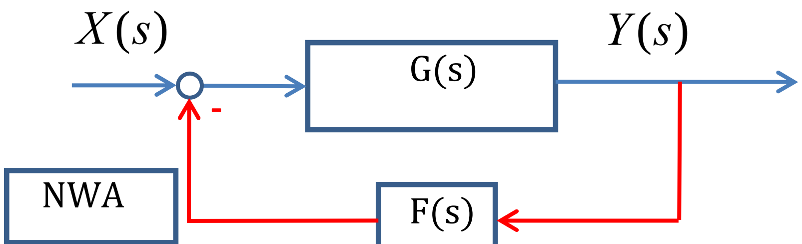
Example: Transverse feedback in SPS (coupled bunch feedback)





Example: Transverse feedback in SPS (coupled bunch feedback)

model of beam
(harmonic oscillator)



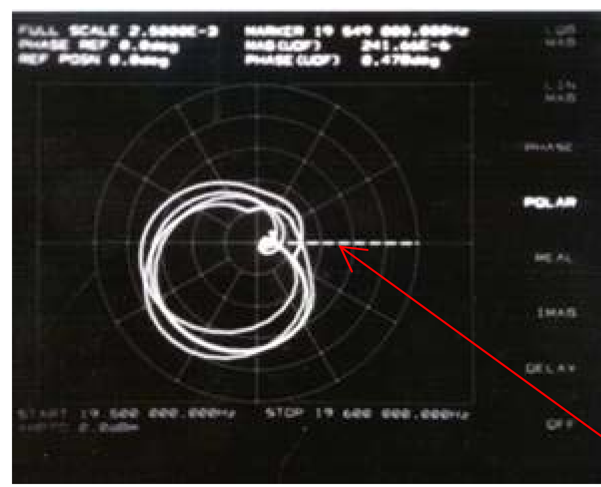
open loop, insert
network analyzer

feedback

Nyquist plot:

$$-G_{\mathrm{OL}} = -GF$$

due to sign definition in measurement the unstable point to avoid is here in right half plane



→ stable

choice of pick-up/kicker location

$$F(s) = \frac{Ks}{1+\tau \cdot s} \cdot \frac{s^2 - \omega_1 / Q_1 s + \omega_1^2}{s^2 + \omega_1 / Q_1 s + \omega_1^2} \cdot \frac{s^2 - \omega_2 / Q_2 s + \omega_2^2}{s^2 + \omega_2 / Q_2 s + \omega_2^2}$$

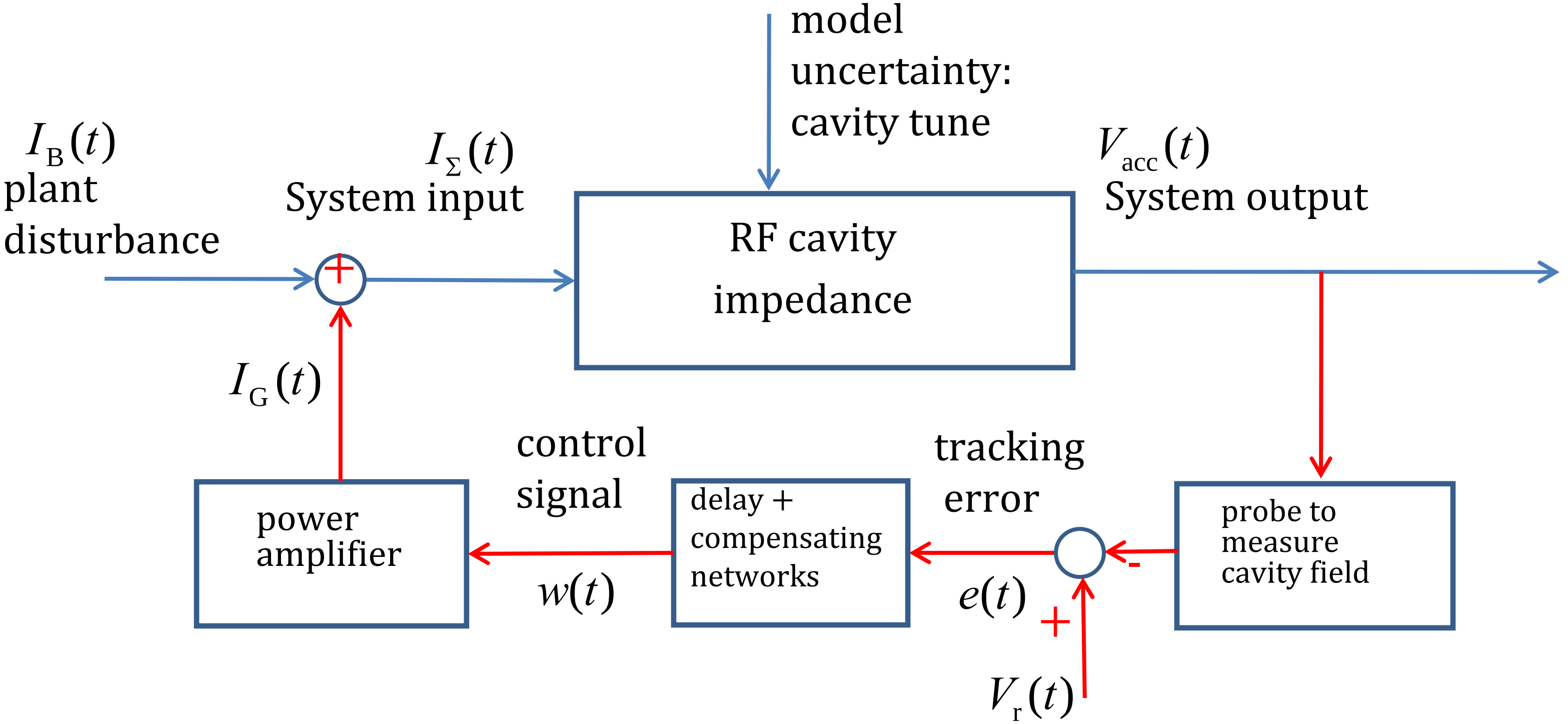
kicker transfer
function τ=35.4 ns
-3 dB @ 4.5 MHz

two second order all-passes
as correctors (electronics)
extend stable region to 20 MHz

open loop transfer function measurement by network analyzer –G(s)F(s), SPS vertical TFB
(sweep from 19.5 to 19.6 MHz)

Exact treatment for high gain requires z-transform (sampled system, once per turn)

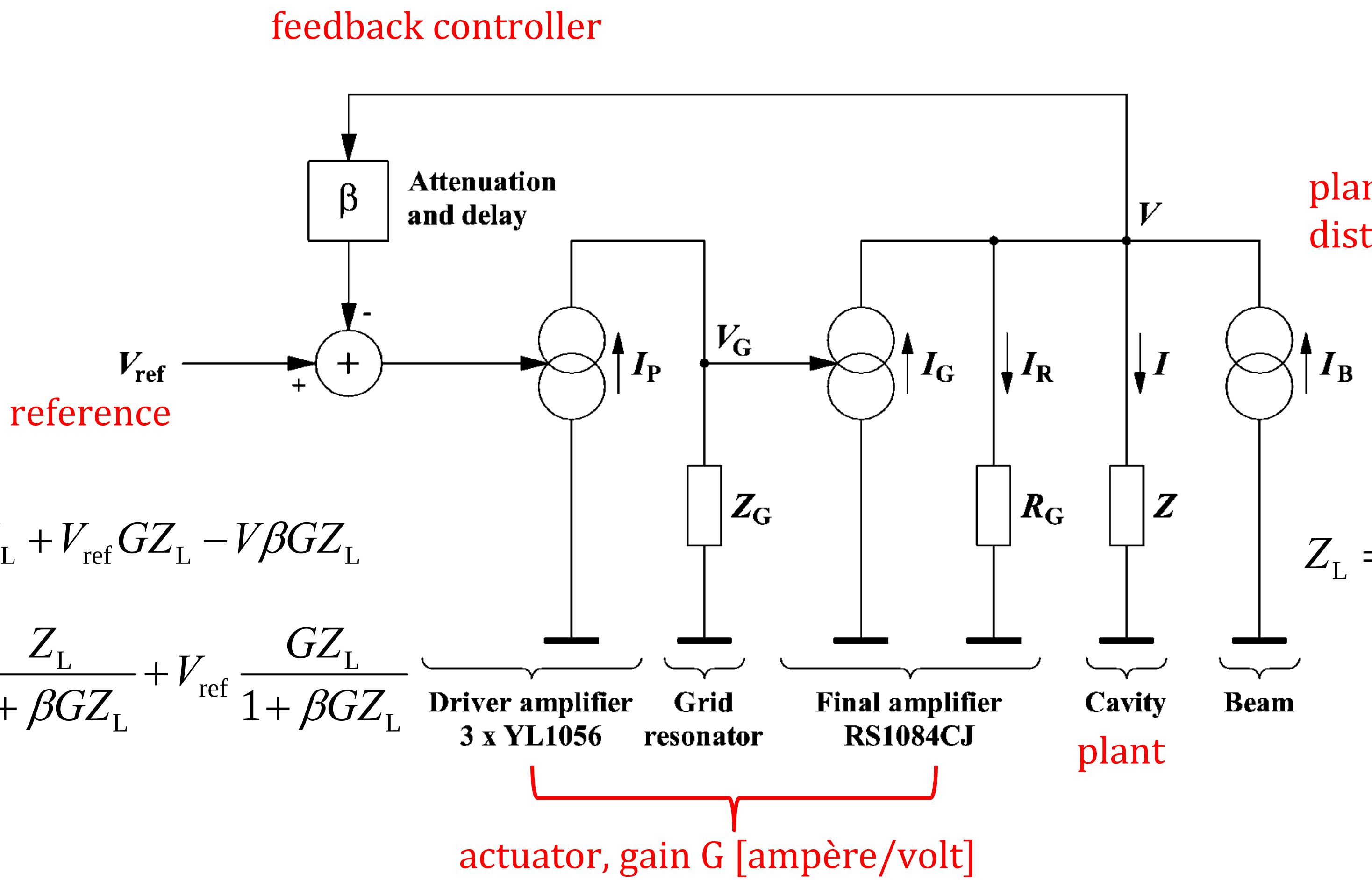
negative feedback example: RF cavity



purpose of above feedback control for cavity:

- precise setting of accelerating voltage
- reduce the effect of disturbances (beam induced voltage)
- Render the system insensitive to small model uncertainties (resonant frequency of cavity, shunt impedance)

Equivalent circuit of RF cavity feedback



$$V = I_B Z_L + V_{ref} G Z_L - V \beta G Z_L$$

$$V = I_B \frac{Z_L}{1+\beta G Z_L} + V_{ref} \frac{G Z_L}{1+\beta G Z_L}$$

$$Z_L = \frac{Z \cdot R_G}{Z + R_G}$$

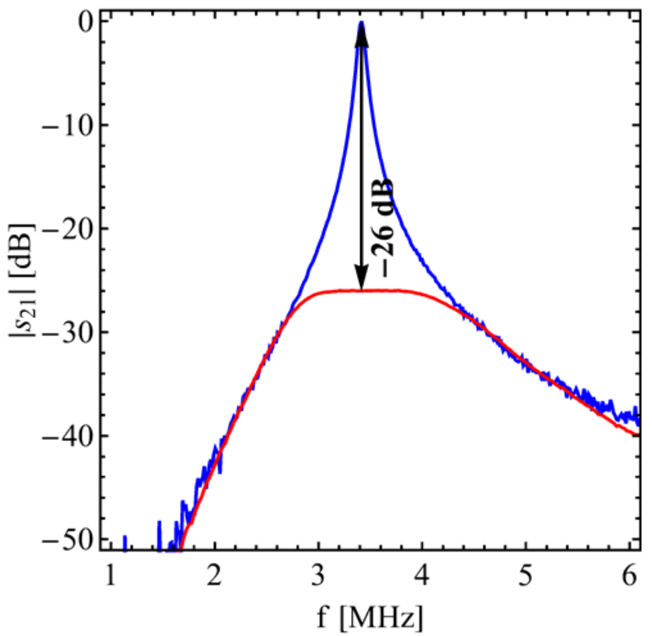
Cavity feedback equivalent circuit (CERN PS)

Reduction of effect of plant disturbance

$$V = I_B \frac{Z_L}{1+\beta G Z_L} + V_{ref} \frac{G Z_L}{1+\beta G Z_L}$$

$$\beta G Z_L >> 1$$

$$V \approx I_B \frac{1}{\beta G} + V_{ref} \frac{1}{\beta}$$



In the limiting case of large feedback gain beam impedance only depends on feedback and amplifier gain !

Beam impedance reduction by cavity feedback (CERN PS) [shown on a relative scale]

Where to find feedback systems in accelerators

magnet current regulation

direct RF feedback around accelerating cavity

DC beam current transformer

} feedback loop closed around equipment

orbit, tune, chromaticity feedbacks

RF control loops (beam phase and radial feedback loop)

transverse and longitudinal coupled bunch feedbacks

} feedback loop closed around beam

Feed forward also used to control and reduce the effect of disturbances in accelerators:

- adaptive trajectory control in a transfer line (pulse to pulse)
- RF cavity feed forward from measured beam current, turn-by-turn

Existing poles of a system response are not influenced by feed forward, only by feedback !

Summary

What is a system, what means feedback ?

What are the purposes of feedback systems ?

mathematical tools to analyze system behavior

criteria for the stability of closed loop feedback systems

steps in designing feedback systems

examples, feed forward versus feedback