Introduction to Insertion Devices

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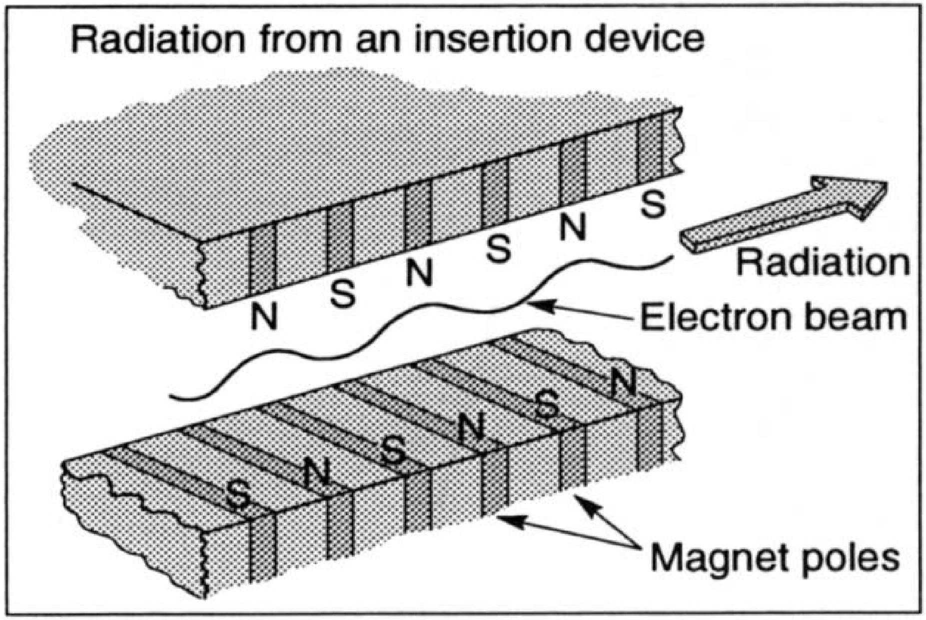
What is an Insertion Device ?



- Oscillating Magnetic field create a beam undulation
- Also called Undulators and Wigglers
- Can be 1 to 20 m long, with period 15 to 200 mm
- Operated with a small magnetic gap (5 to 15 mm)
- Use :
  - **Intense Source of Radiation in electron storage rings**
  - Control of damping times in Electron Colliders (LEP, CESR,…)
  - Reduce emittance in advanced light sources (Petra III, NSLS II)

Table of Content

- Electron beam Dynamics
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Electron beam dynamics

Electron Trajectory in an Insertion Device

Consider Ortogonal Frame Oxzs

Electron velocity: $\vec{v} = (v_x, v_z, v_s)$

Electron position: $\vec{R} = (x, z, s)$

Magnetic field: $\vec{B} = (B_x, B_z, B_s)$

Define: $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

Lorentz Force

$$\gamma m \frac{d\vec{v}}{dt} = e\vec{v} \times \vec{B}$$

$$=> \gamma m \frac{dv_x}{dt} = -e(v_s B_z - v_z B_s)$$

$$Assume: v_x, v_z << v_s \approx c$$

$$\frac{v_x(s)}{c} = -\frac{e}{\gamma mc} \int_{-\infty}^{s} B_z(s') ds'$$

$$x(s) = -\frac{e}{\gamma mc} \int_{-\infty}^{s} \int_{-\infty}^{s'} B_z(s'') ds'' ds'$$

and similar expression for $v_z(s)$ and $z(s)$

Electron Trajectory in a Planar Sinusoidal Undulator

$$Consider\ \vec{B} = (\ 0,\ B_0 \sin(2\pi \frac{s}{\lambda_0}),\ 0\ )$$

$$\frac{v_x}{c} = \frac{K}{\gamma} \cos(2\pi \frac{s}{\lambda_0})$$

$$\frac{v_z}{c} = 0$$

$$\frac{v_s}{c} = 1 - \frac{1}{2\gamma^2}(1 + K^2 \cos^2(2\pi \frac{s}{\lambda_0}))$$

$$x \cong -\frac{\lambda_0}{2\pi}\frac{K}{\gamma} \sin(2\pi \frac{s}{\lambda_0})$$

with

$$K = \frac{eB_0\lambda_0}{2\pi mc} = 0.0934\ B_0[T]\ \lambda_0[mm]$$

K is a fundamental parameter called : **Deflection Parameter**

$$Example : ESRF,\ Energy = 6GeV,\ Undulator\ \ \lambda_0 = 35\ \ mm,\ \ B_0 = 0.7\ \ T$$

$$=> \quad K = \ 2.3, \qquad \frac{K}{\gamma} = 200\ \ \mu rad, \qquad \frac{\lambda_0}{2\pi}\frac{K}{\gamma} = \ 1.1\ \ \mu m\ !!$$

$$B_x = 0$$

$$B_z = B_0 \cosh(2\pi \frac{z}{\lambda_0}) \cos(2\pi \frac{s}{\lambda_0})$$

$$B_s = -B_0 \sinh(2\pi \frac{z}{\lambda_0}) \sin(2\pi \frac{s}{\lambda_0})$$

Undulator Field Satisfying Maxwell Equation

$$\gamma m \frac{d\vec{v}}{dt} = e\vec{v} \times \vec{B}$$

Lorentz Force Equation

2nd Order in γ^{-1}

$$\frac{d^2x}{ds^2} = 0$$

$$\frac{d^2z}{ds^2} = -\frac{1}{2}\left(\frac{eB_0}{\gamma mc}\right)^2 \frac{\lambda_0}{4\pi} \sinh(4\pi \frac{z}{\lambda_0}) \cong -\frac{1}{2}\left(\frac{eB_0}{\gamma mc}\right)^2 z$$

$$K_z = \frac{1}{2}\left(\frac{eB_0}{\gamma mc}\right)^2$$

A vertical Field Undulator is Vertically Focusing !

$$\frac{1}{F_z} = \int_{ID} K_z ds = \frac{1}{2}\left(\frac{eB_0}{\gamma mc}\right)^2 L$$

Interference with the beam dynamics in the ring lattice

- An Insertion Device is the first component of a photon beamline. Its field setting is fully controlled by the users of the beamline . The change of field results in a change of beam dynamics in the whole ring.

- As far as the lattices are concerned, **Insertion devices** should ideally behave like **drift space** but the reality is different :

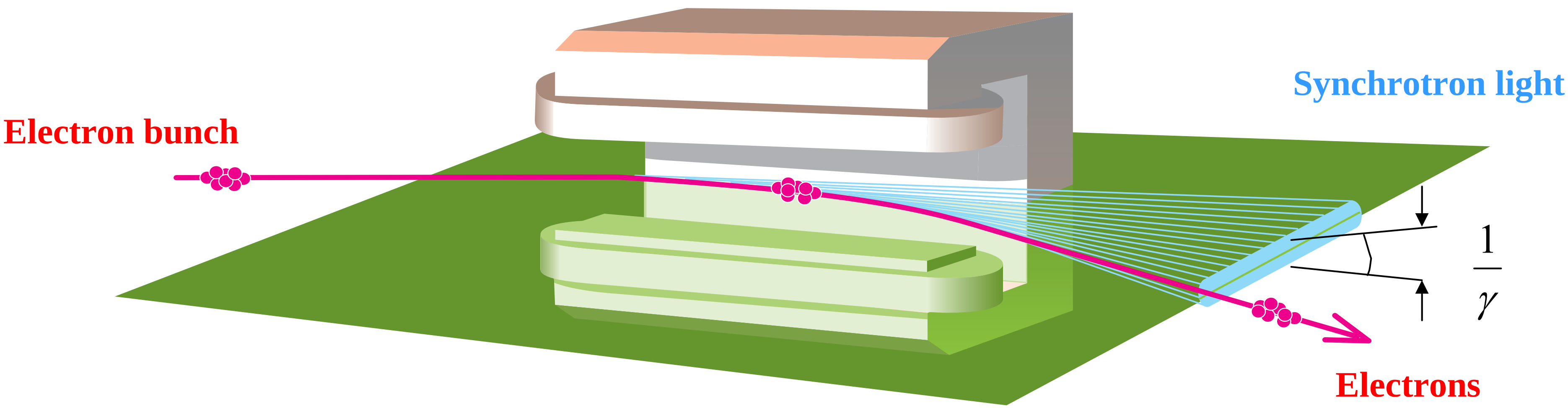
  - Closed Orbit distortion (by non zero field integrals=dipole error)
  - Betatron tune shift (by nominal field and by quadrupole errors)
  - Coupling (skew quadrupole errors)
  - Reduction of dynamic aperture (=>Lifetime reduction & reduced injection efficiency)
    - Through a break of the lattice periodicity
    - A change of the focusing versus injection point x,z
  - Very high field IDs may change the damping time, emittance , energy spread …

- By combining carefull manufacture, magnetic field shimming and local active corrections, many perturbations can be compensated.

- The problem of the reduction of dynamic aperture is most severe on low energy rings with many insertion devices.

Synchrotron Radiation from an Insertion Device

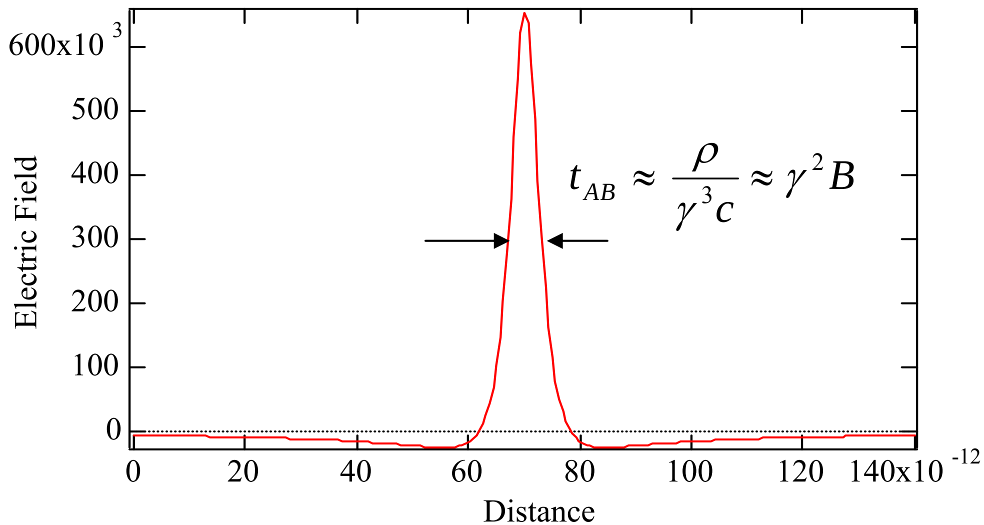
Bending Magnet



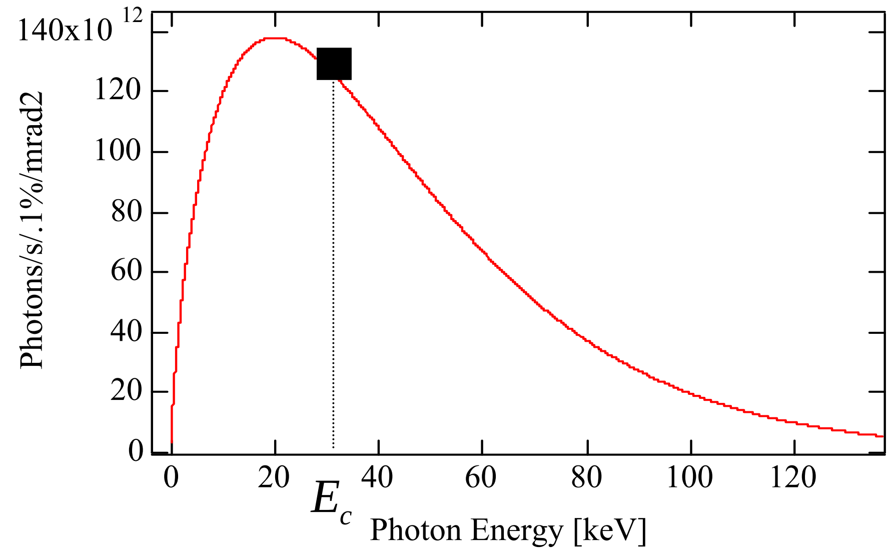
Synchrotron Radiation is emitted tangentially to the trajectory
Inside a cone of angle $1/\gamma$

Radiation by a single electron

Electric field
in the time domain



Angular spectral flux
in frequency domain

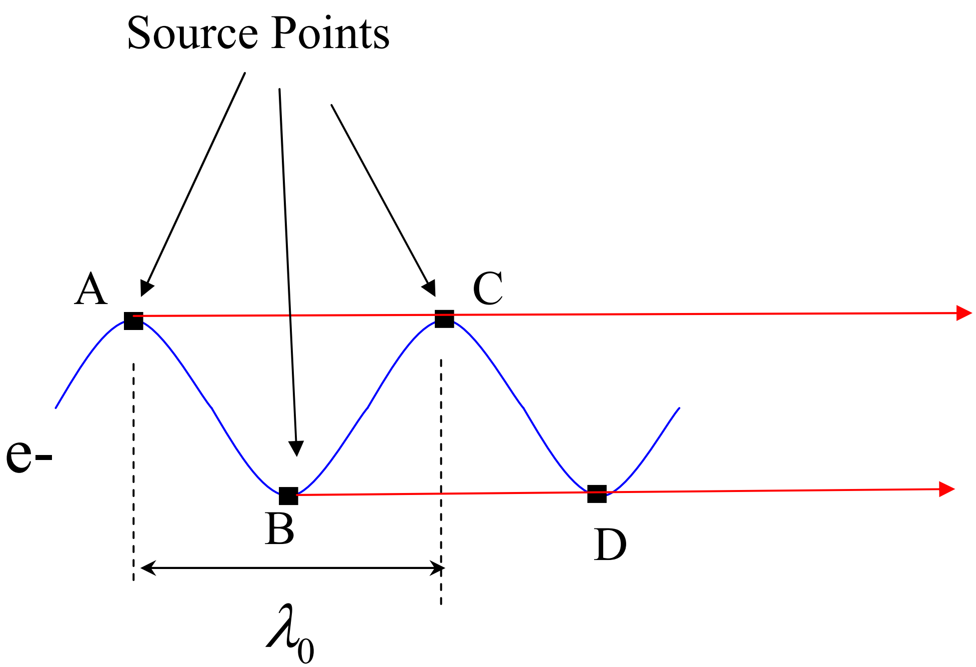


Computed for 6 GeV, I = 200 mA, B =1 tesla

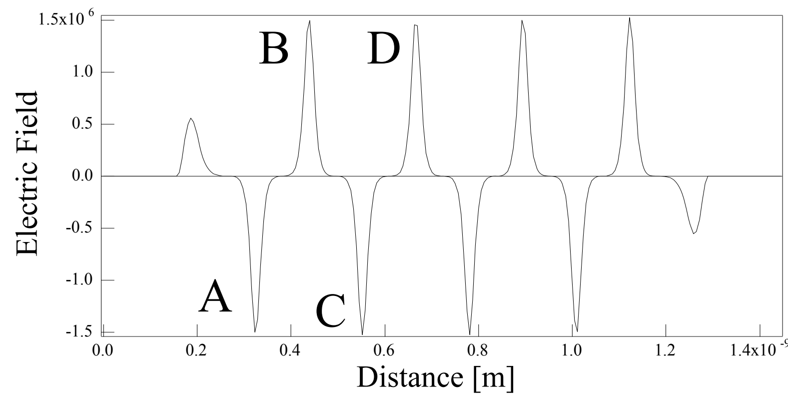
$$E_c = \frac{3hc}{4\pi}\frac{\gamma^3}{\rho} = \frac{3he}{4\pi m}\gamma^2 B$$

$$E_c[keV] = 0.665\ E^2[GeV]\ B[T]$$

Undulator

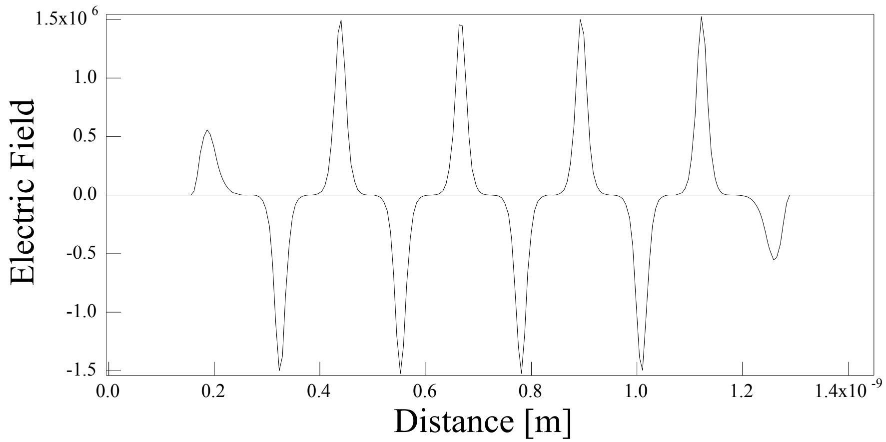


Electron trajectory

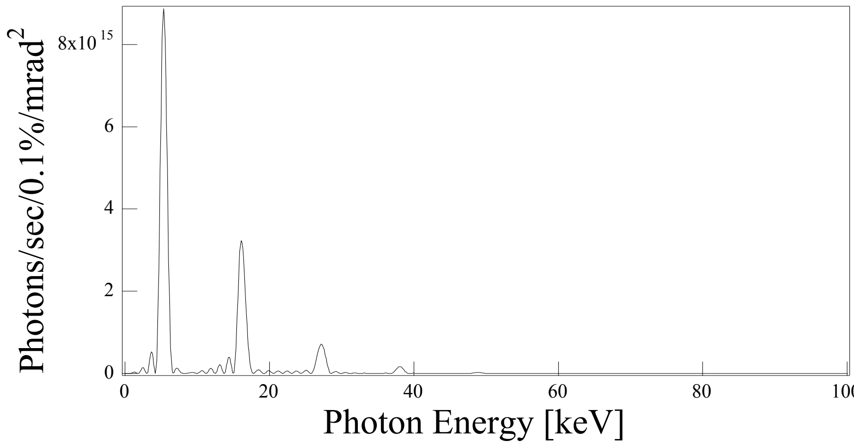


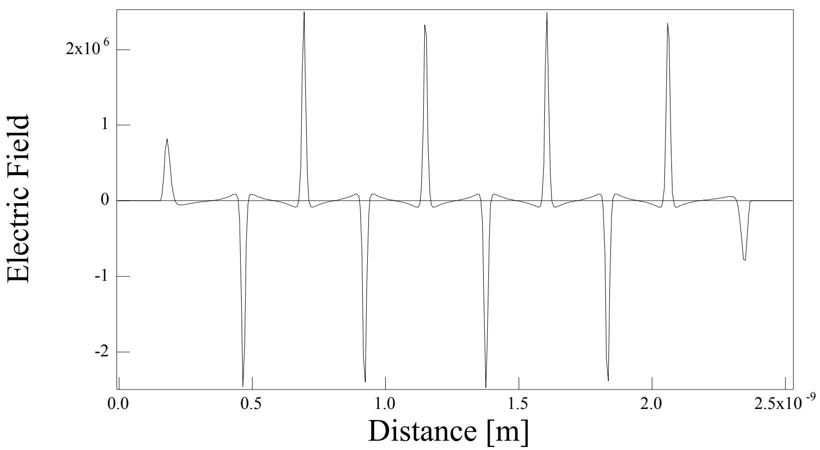
Electric field in time domain

Electric field and Spectrum vs K

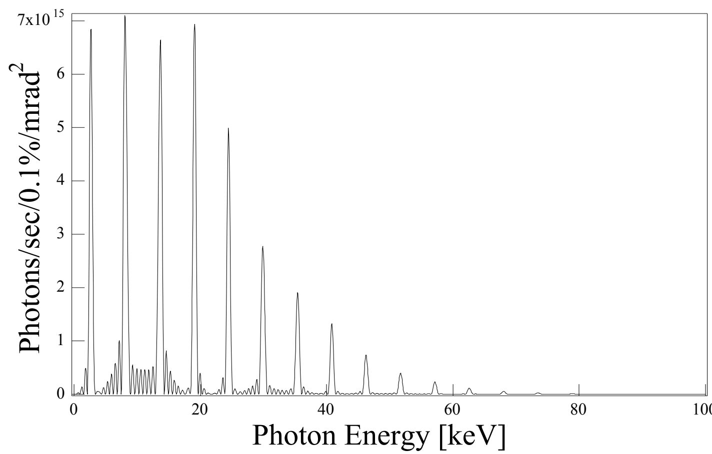


K=1



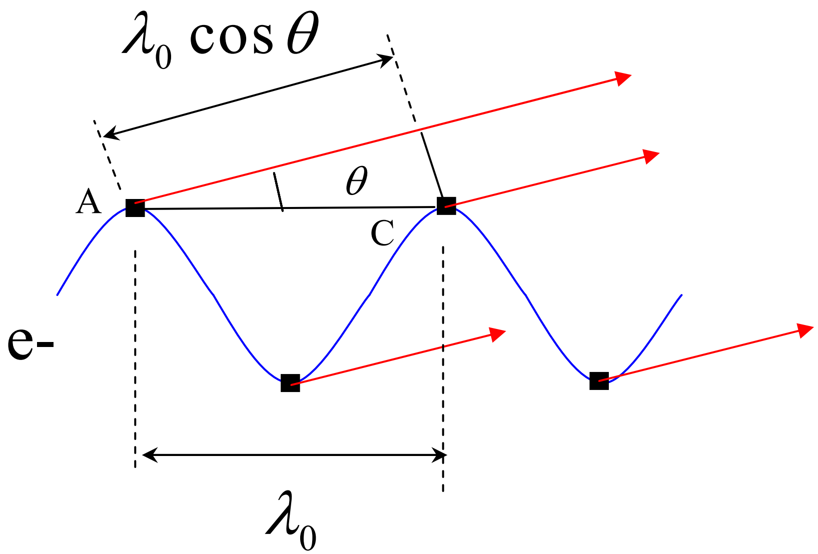


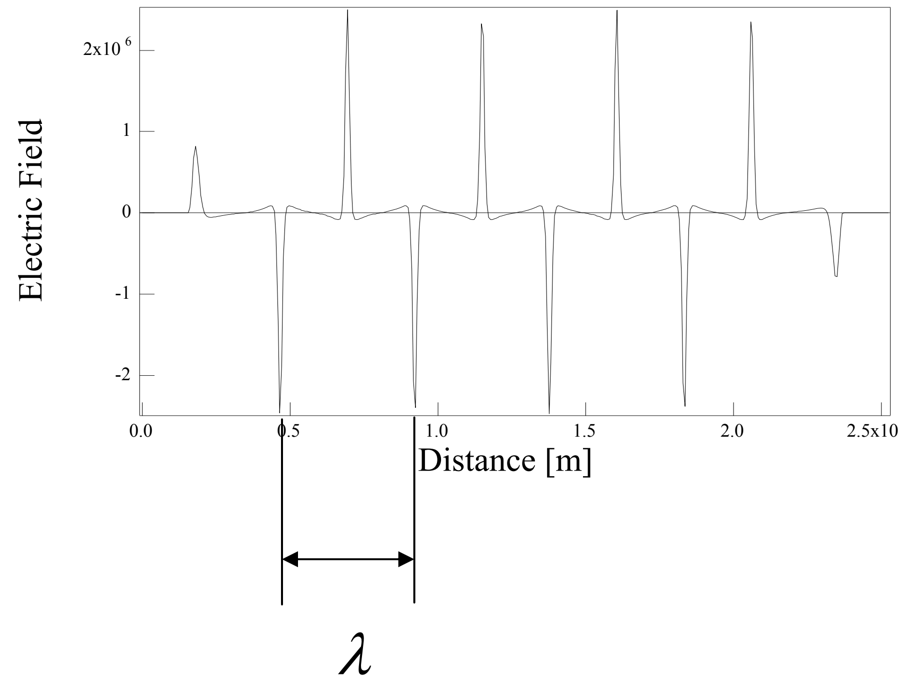
K=2



Computed for 6 GeV, I=200 mA, 35 mm period

Fundamental wavelength of the radiation field





$$t_{AC} = \frac{\lambda_0}{< v_s >} = \frac{\lambda_0}{c(1 - \frac{1}{2\gamma^2}(1 + \frac{K^2}{2}))}$$

$$\lambda = \lambda_0 \cos\theta - ct_{AC} \cong \lambda_0 (1 - \frac{\theta^2}{2}) - \frac{\lambda_0}{(1 - \frac{1}{2\gamma^2}(1 + \frac{K^2}{2}))} \cong \frac{\lambda_0}{2\gamma^2}(1 + \frac{K^2}{2} + \gamma^2\theta^2)$$

Wavelength of the Harmonics

$$\lambda_n = \frac{\lambda_0}{2n\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)$$

Equivalently, the energy E_n of the harmonics are given by

$$E_n[keV] = \frac{9.5\, n\, E^2[GeV]}{\lambda_0[mm](1+\frac{K^2}{2}+\gamma^2\theta^2)}$$

λ_n, E_n : *Wavelength, Energy of the n^{th} harmonic*

$n = 1, 2, 3, ..$: *Harmonic number*

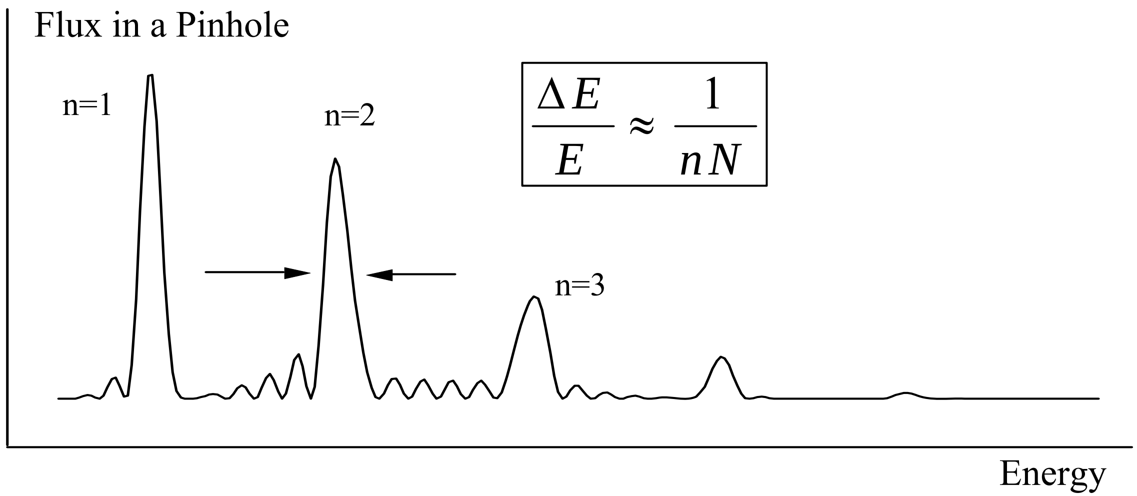
λ_0 : *Undulator period*

$E = \gamma mc^2$: *Electron Energy*

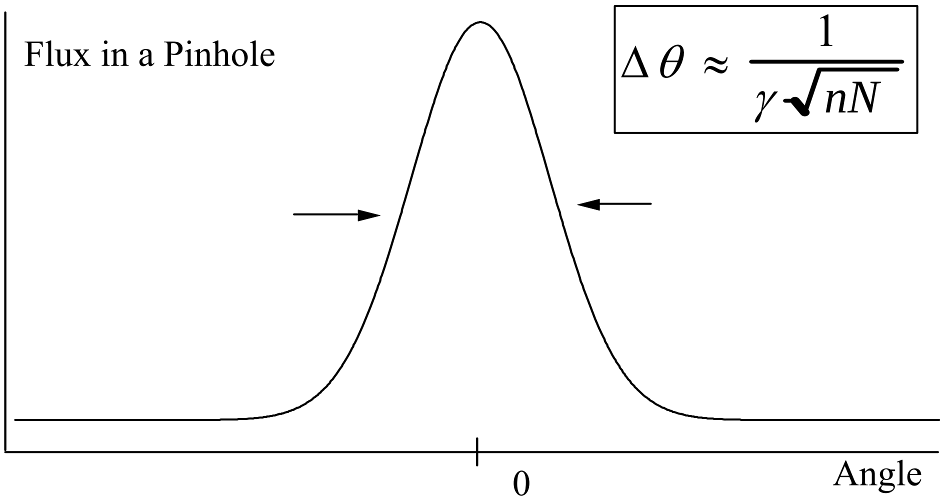
K : *Deflection Parameter* $= 0.0934\, B_0[T]\, \lambda_0[mm]$

θ : *Angle between observer direction and e – beam*

Undulator Emission by a Filament Electron Beam



$$\frac{\Delta E}{E} \approx \frac{1}{nN}$$



$$\Delta\theta \approx \frac{1}{\gamma\sqrt{nN}}$$

n : Harmonic number

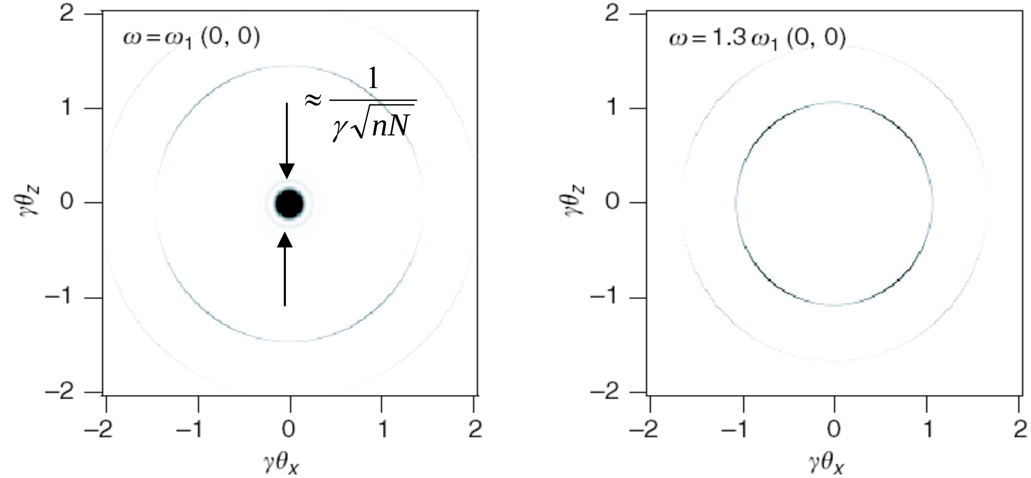
N : Number of Periods

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{E}{mc^2}$$

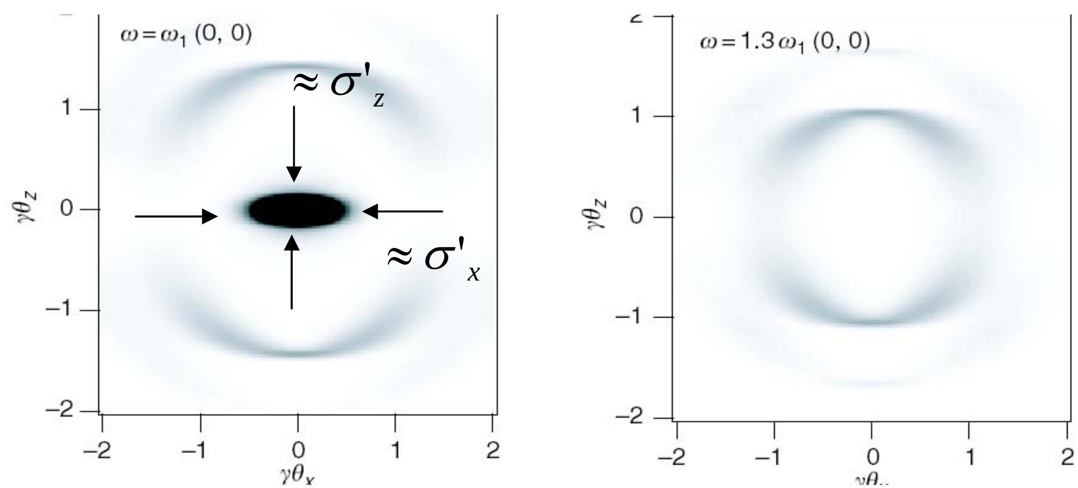
What happens if the beam presents a finite emittance (size and divergence) and finite energy spread ?

$$\lambda = \frac{\lambda_0}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

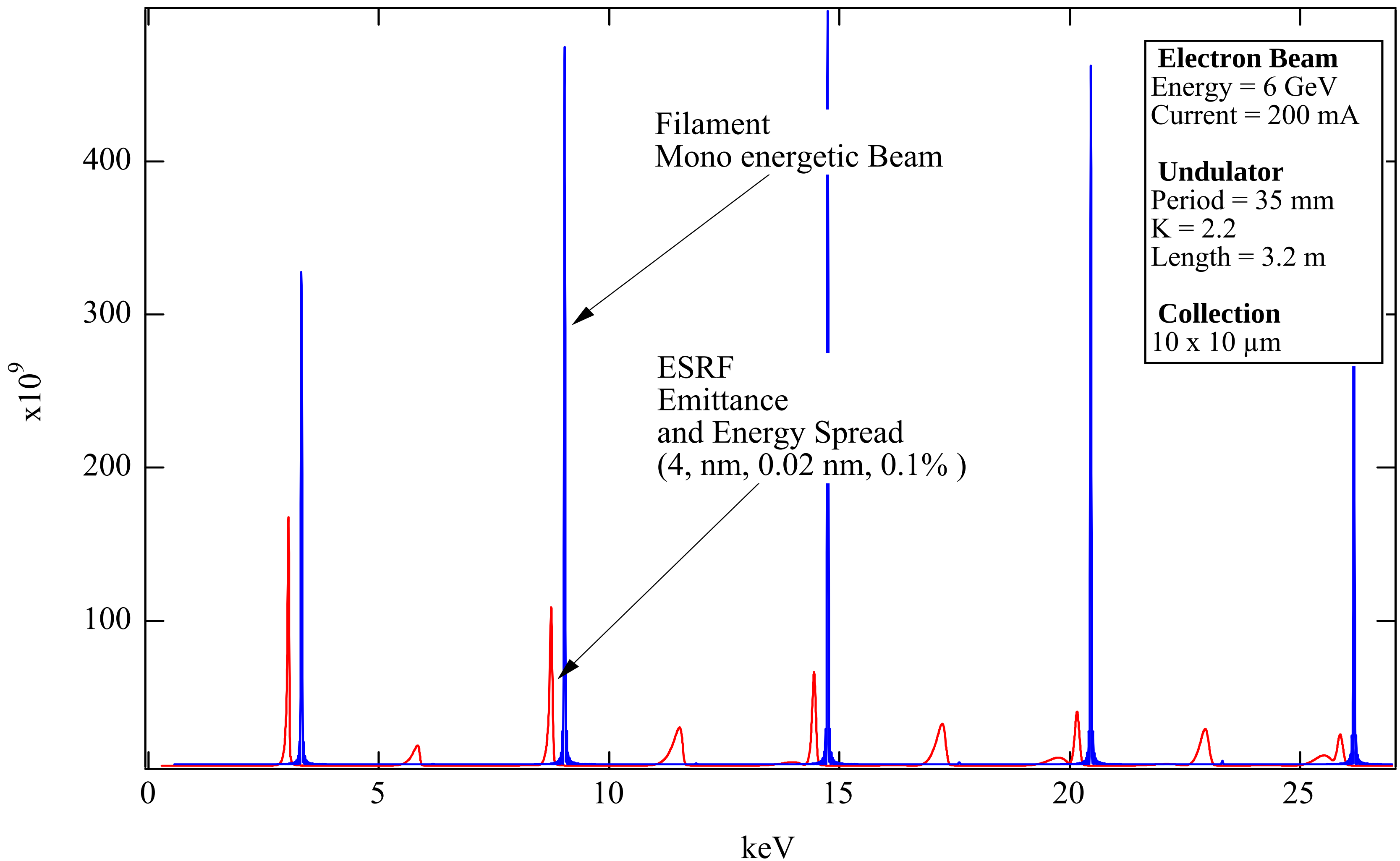
Radiation from a filament
e- Beam at a wavelength λ



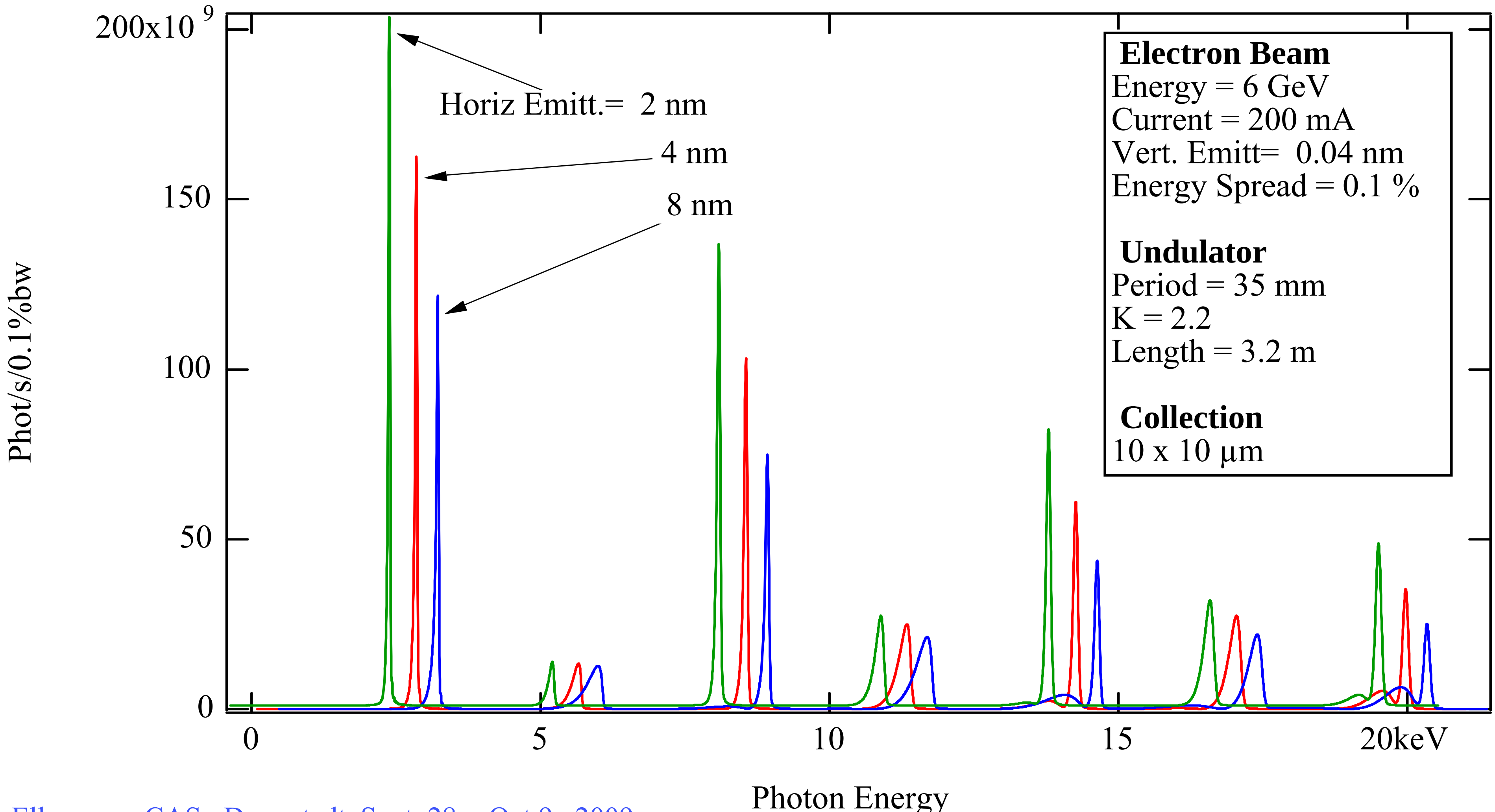
e- Beam with Finite Divergence



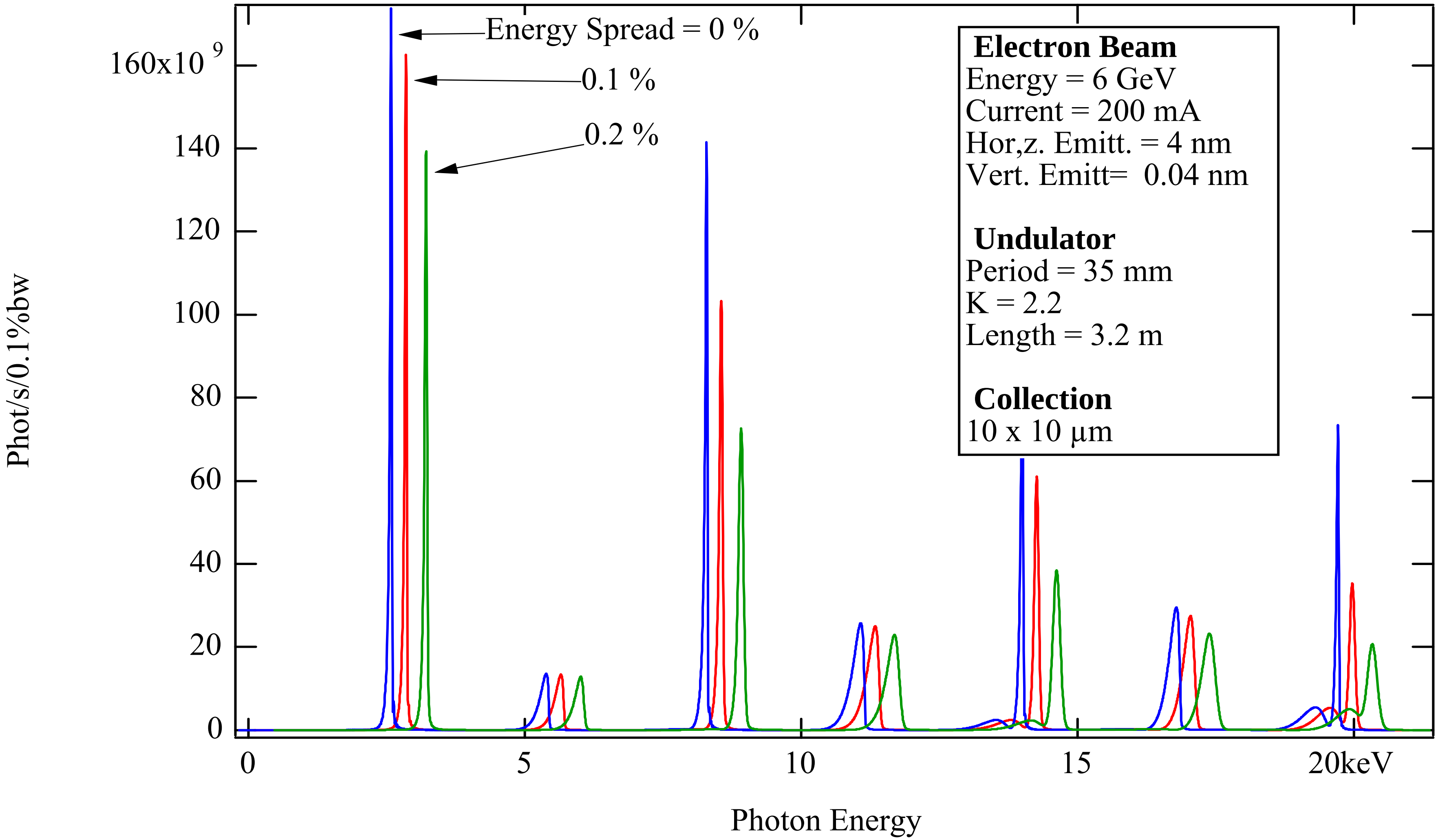
Radiation Spectrum through a narrow aperture



Broadening of the Harmonics by the Electron Emittance

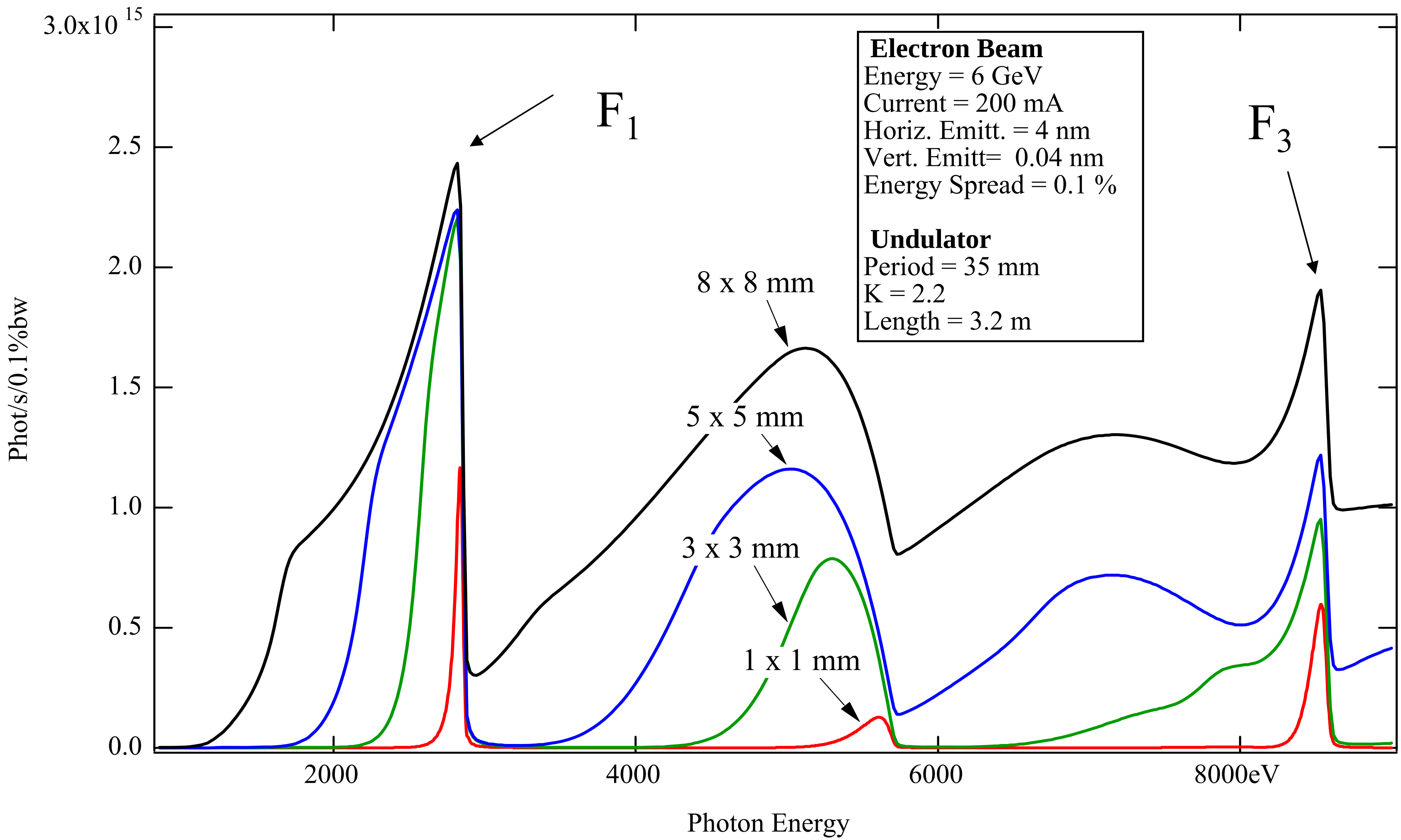


Broadening of the Harmonics by Electron Energy Spread



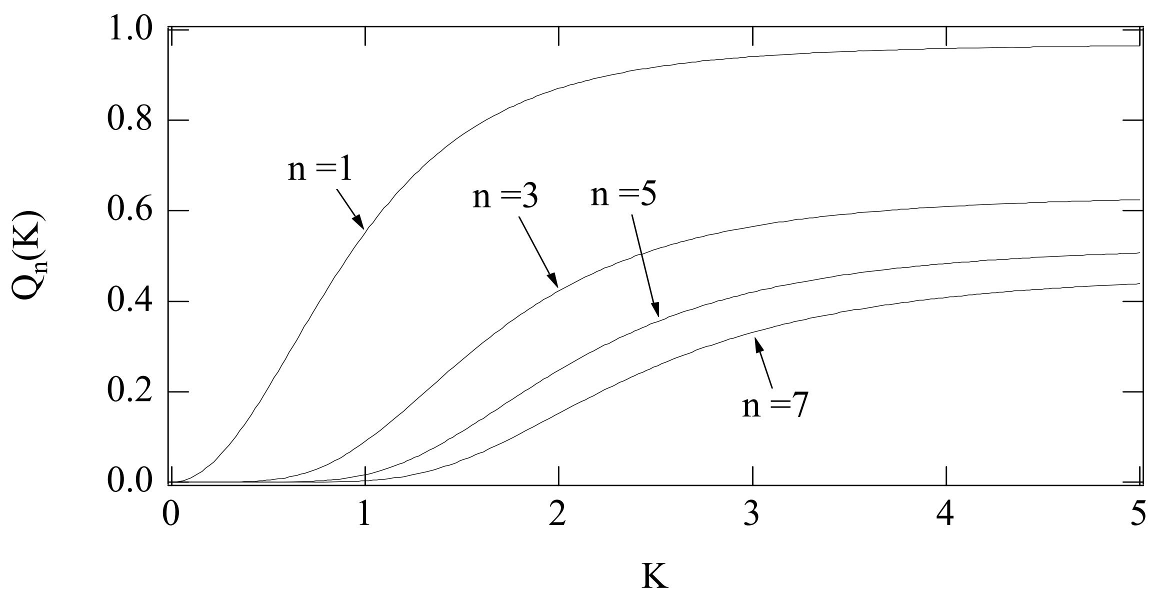
To make optimum use of the **Undulators, the magnet lattice of** synchrotron light sources should be designed to produce the **smallest emittance** and **smallest energy spread**.

Collecting Undulator Radiation in a variable Aperture



Maximum Spectral Flux On-axis on odd harmonics

$$F_n \;[Ph/\sec/\,0.1\%] \;=\; 1.431\;\;10^{14}\;\;N\;\;I[A]\;\;Q_n(K)$$



Brilliance (or Brightness)

$$B_n = \frac{F_n}{(2\pi)^2 \Sigma_x \Sigma'_x \Sigma_z \Sigma'_z}$$

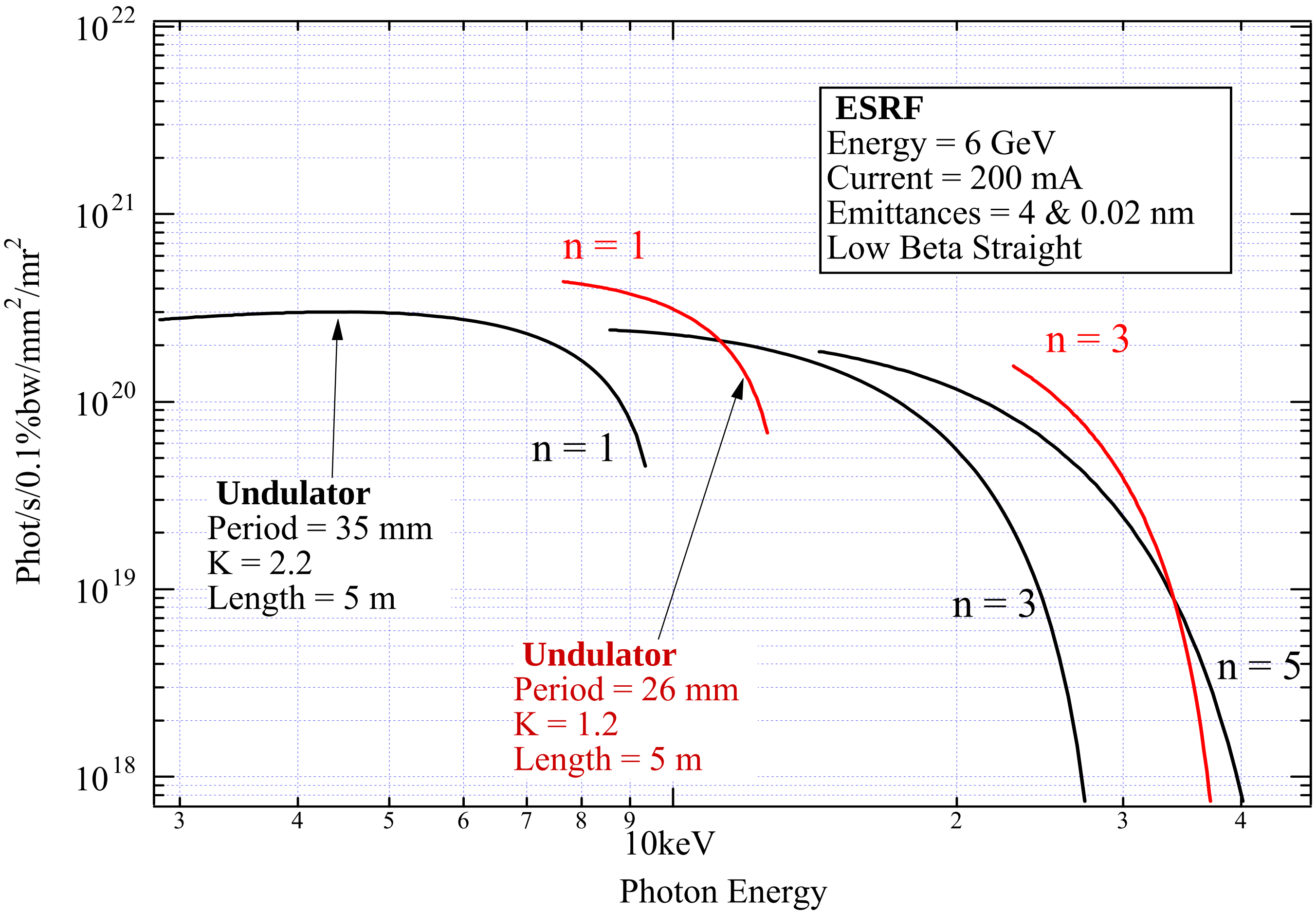
$$\Sigma_x = \sqrt{\sigma_x^2 + \frac{\lambda_n L}{(2\pi)^2}}$$

$$\Sigma'_x = \sqrt{\sigma'^2_x + \frac{\lambda_n}{2L}}$$

Electron beam — Single electron emission

$$\lambda = \frac{\lambda_0}{2n\gamma^2}(1+\frac{K^2}{2})$$

Brilliance vs Photon Energy



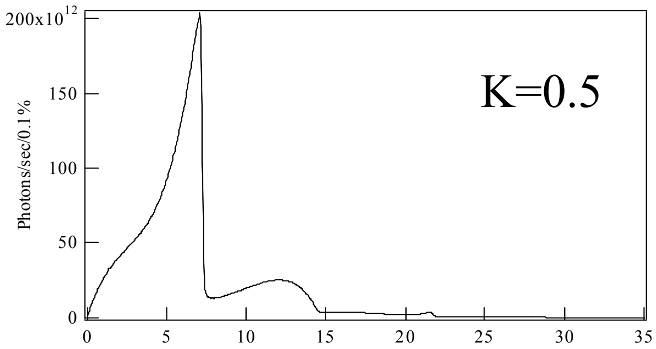
$$B_n = \frac{F_n}{(2\pi)^2 \Sigma_x \Sigma'_x \Sigma_z \Sigma'_z}$$

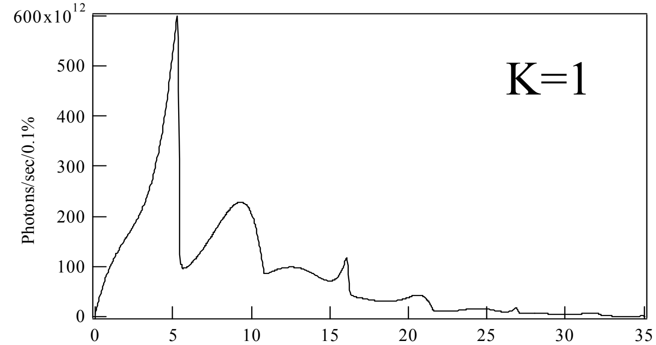
with

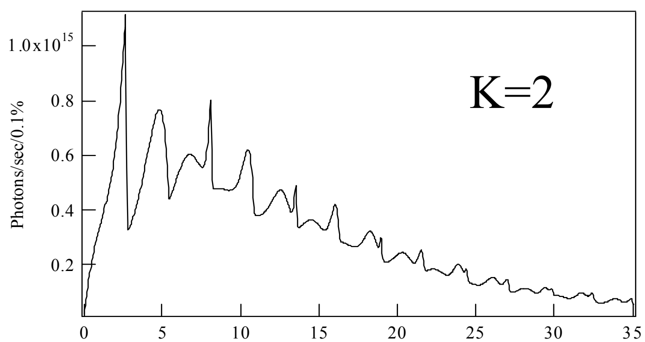
$$\Sigma_x = \sqrt{\sigma_x^2 + \frac{\lambda_n L}{(2\pi)^2}}$$

$$\Sigma'_x = \sqrt{\sigma'^2_x + \frac{\lambda_n}{2L}}$$

Angle Integrated Flux



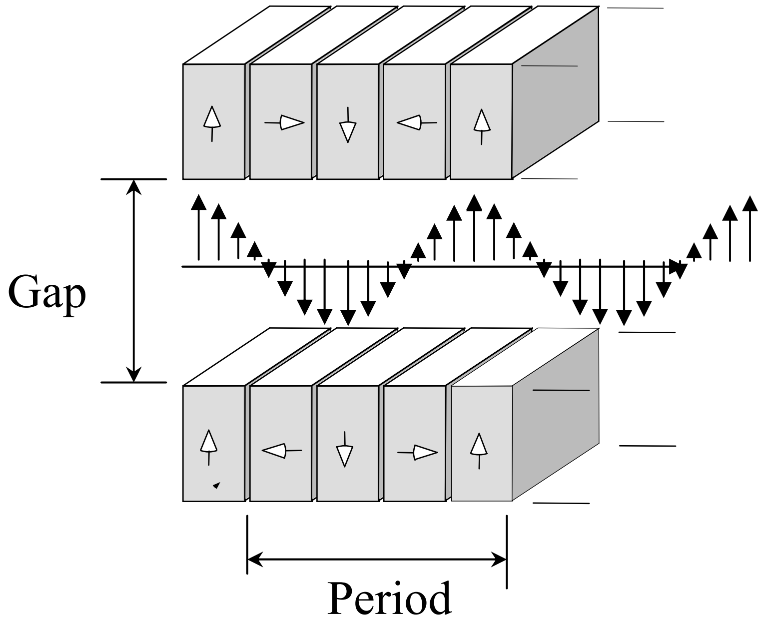




For Large K, the angle integrated spectrum from an Undulator tends toward that of a bending magnet x 2N
=> Such Devices are called **Wigglers**

Technology

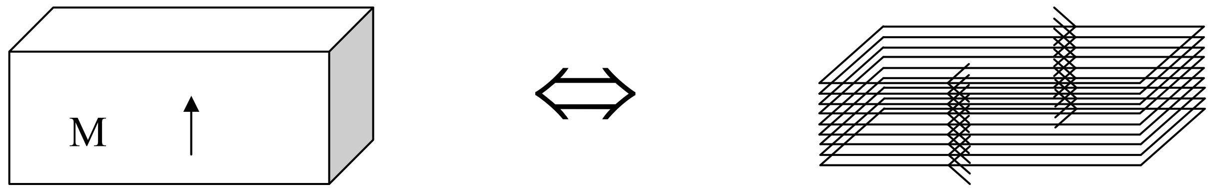
Technology of Undulators and Wigglers



$$\lambda = \frac{\lambda_0}{2n\gamma^2}(1+\frac{K^2}{2})$$

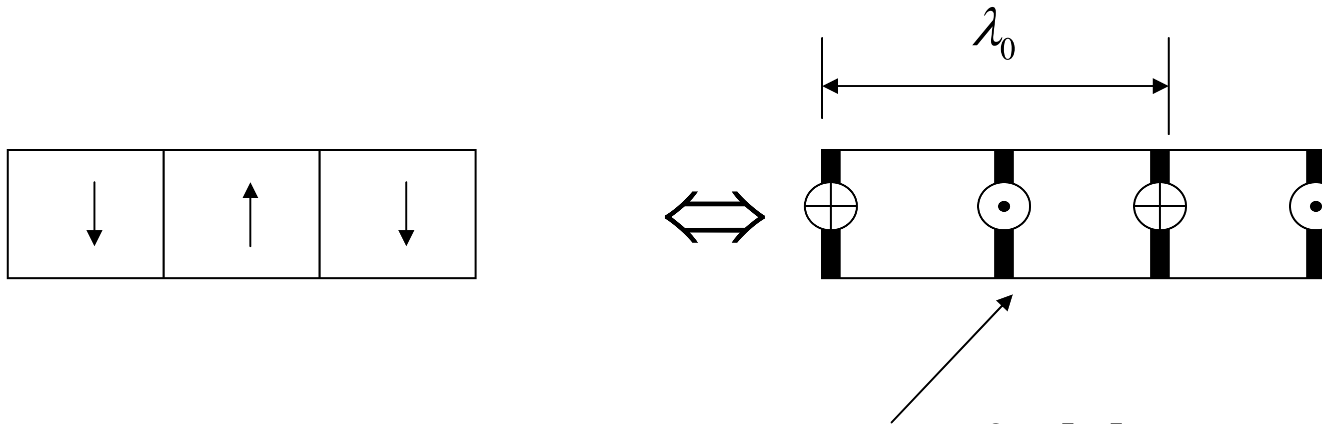
- The fundamental issue in the magnetic design of a planar undulator or wiggler is to produce a periodic field with a high peak field B and the shortest period λ_0 within a given aperture (gap).
- Three type of technologies can be used :
  - Permanent magnets (NdFeB , Sm_2Co_{17})
  - Room temperature electromagnets
  - Superconducting electromagnets

Magnetization is equivalent to a surface current



$$\text{Air coil with Surface Current Density[A/m]} \cong \frac{B_r[T]}{\mu_0}$$

Periodic Array of Magnets



$$\text{Surface Current Density[A/m]} \cong \frac{2B_r[T]}{\mu_0}$$

$$\text{or Current Density[A/m}^2\text{]} \cong \frac{4B_r[T]}{\mu_0 \lambda_0}$$

Examples:

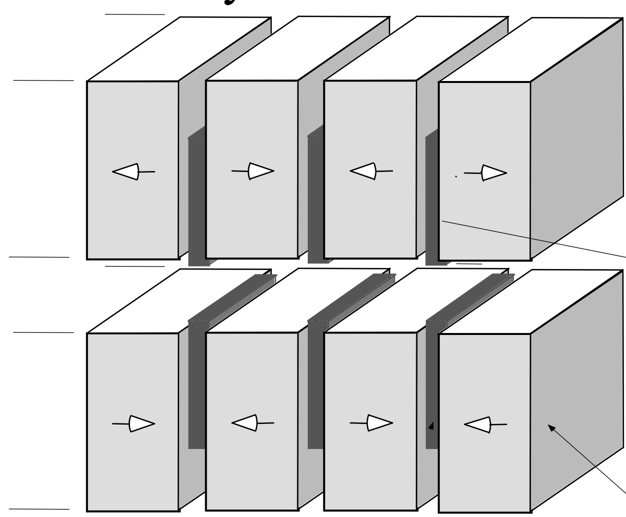
B_r =1 T , λ_0=20 mm $\Rightarrow$ Equiv. Current Density=160A/mm^2 !!

B_r =1 T , λ_0=400 mm $\Rightarrow$ Equiv. Current Density=8 A/mm^2

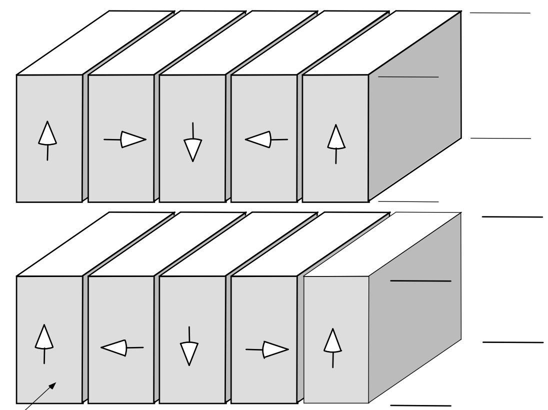
> 95 % of Insertion Devices are made of Permanent Magnets !!

Permanent Magnet Undulator

Hybrid

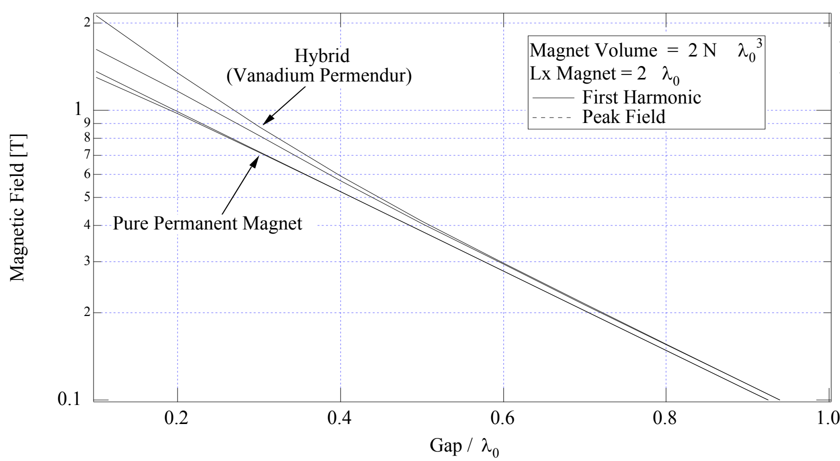


Pure Permanent Magnet



Pole (Steel)

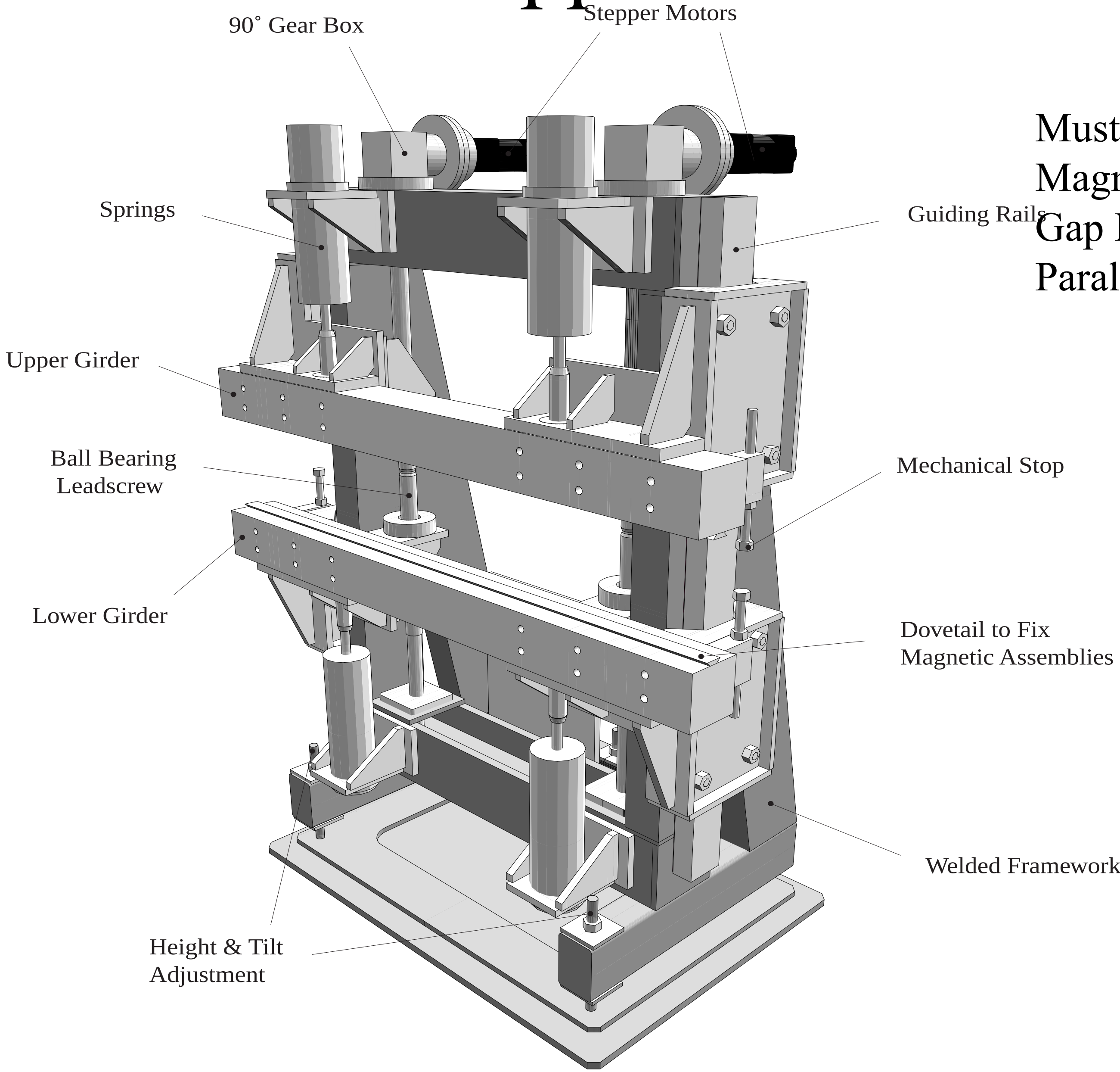
Magnet (NdFeB, Sm_2Co_{17},...)



magnet materials

Material	B_r [T]	$\mu_{r,\parallel}$	$\mu_{r,\perp}$	H_{cJ} [kA/m]	$10^{-2}/°C$
$SmCO_5$	0.9–1.01	1.05		1500–2400	−0.04
Sm_2CO_{17}	1.04–1.12	1.05–1.08		800–2000	−0.03
NdFeB	1.0–1.4	1.04–1.06	1.15–1.17	1000–3000	−0.10

Support Structure



Must Handle :

Magnetic Force : 1-20 Tons

Gap Resolution : < 1 μm

Parallellism < 20 μm

Undulators are Fundamentally Small Gap Devices

- For a permanent magnet undulator , shrinking all dimensions maintains the field unchanged.

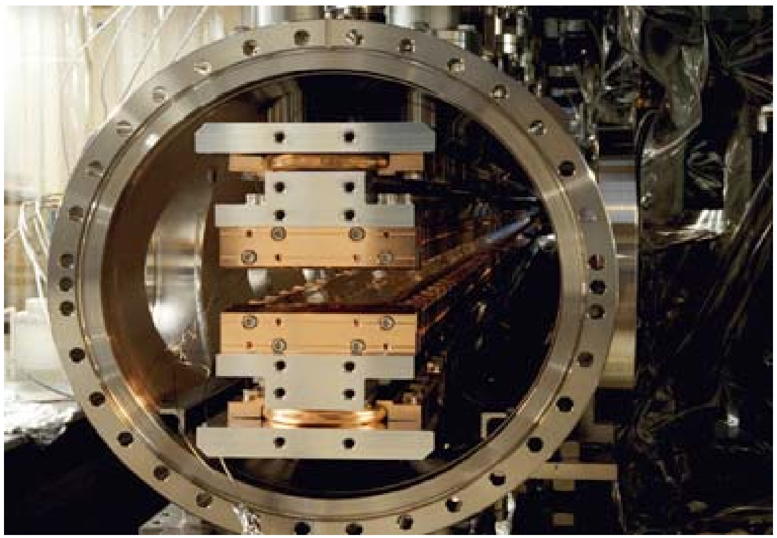
- The peak field $$B_0 \propto B_r \exp(-\pi \frac{gap}{\lambda_0})$$

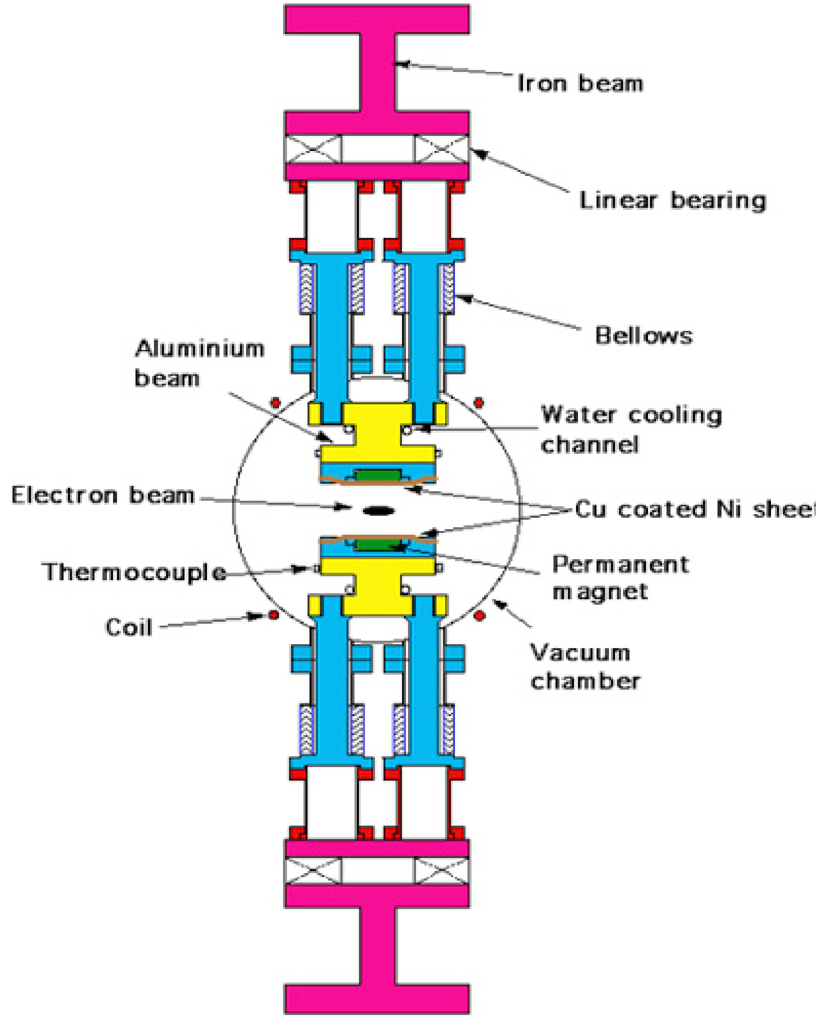
- Benefits of using small gaps Insertion Devices :
  - Decrease the volume of material (cost driving) ~ gap^3
  - The lower the gap, the higher the energy of the harmonics of the undulator emission => the lower the electron energy required to reach the same photon energy

$$\lambda = \frac{\lambda_0}{2\gamma^2}(1+\frac{K^2}{2}) \qquad with\ \ K = \frac{eB_0\lambda_0}{2\pi mc}$$

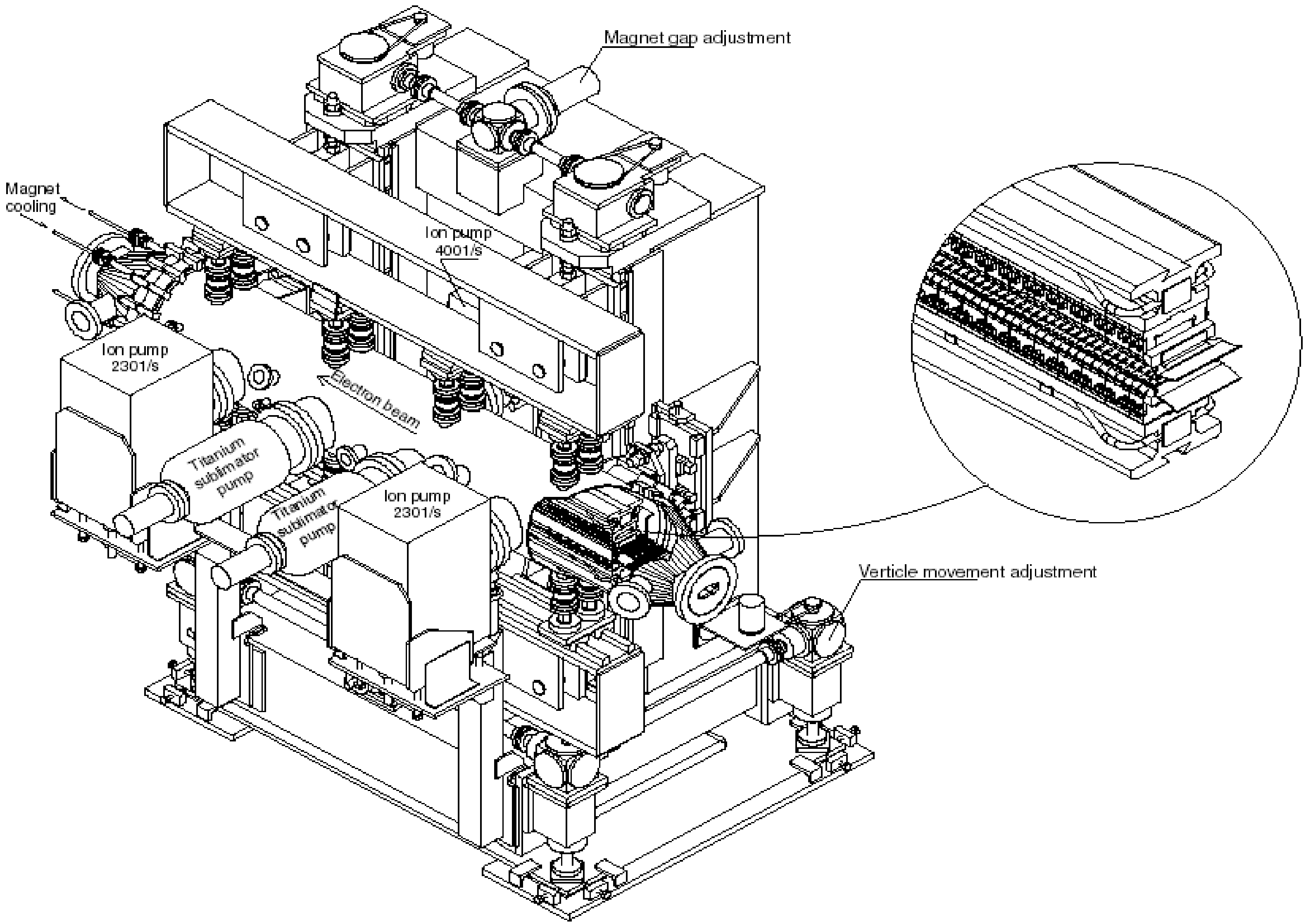
- The most advanced undulators have magnet blocks in the vacuum with an operating magnetic gap of 4-6 mm !!

In Vacuum Permanent Magnet Undulators





SPring-8 In-Vacuum Undulator

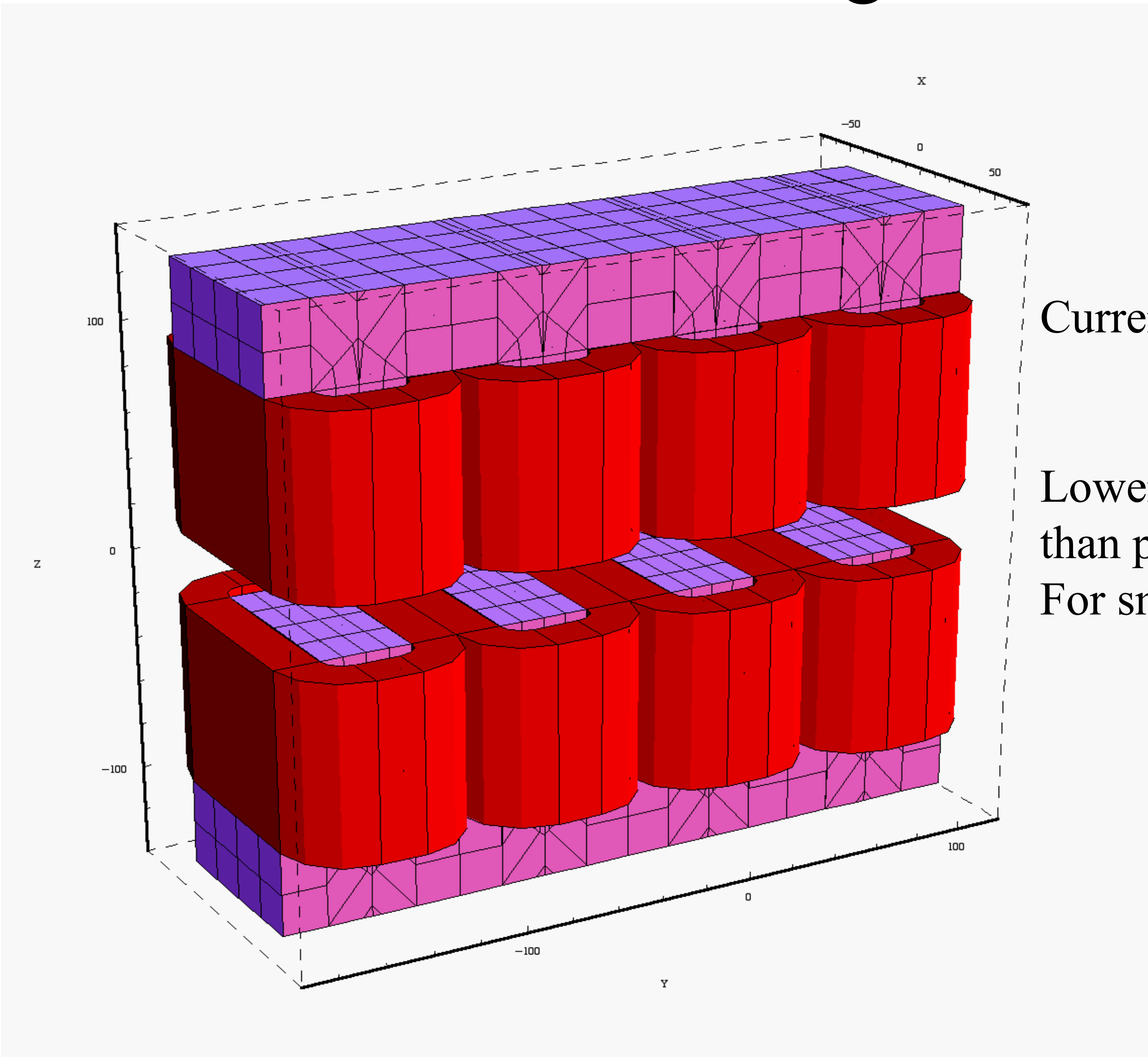

Magnet gap adjustment
Magnet cooling
Ion pump 4001/s
Ion pump 2301/s
Electron beam
Titanium sublimator pump
Titanium sublimator pump
Ion pump 2301/s
Verticle movement adjustment

Application : Build a pure permanent magnet undulator with NdFeB Magnets (B_r = 1.2 T)

Undulator with K=1 with 6 GeV energy

Gap [mm]	B [T]	Period [mm]	Fundamental [keV] @ 6 GeV	Electron Energy [GeV] Fund = 15.2 keV
5	0.72	15	15.2	6.0
10	0.49	22	10.3	7.3
15	0.38	28	8.2	8.2

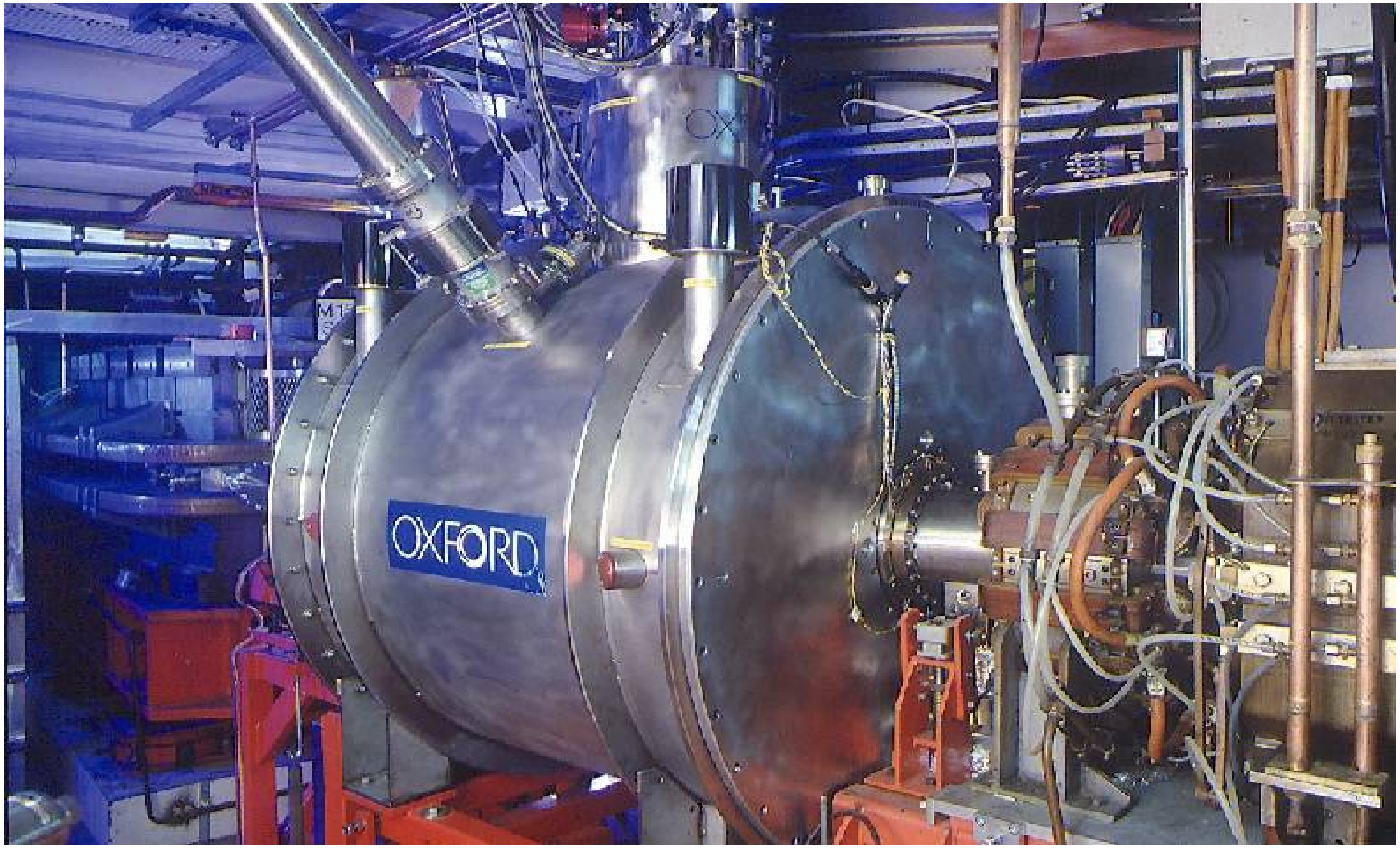
Electro-Magnet Undulator



Current Densities < 20 A/mm^2

Lower field
than permanent magnet
For small period / gap

Superconducting Wigglers



- High field : up to 10 T => Shift the spectrum to higher energies
- Complicated engineering & High costs

Magnetic Field Errors in Permanent Magnet Insertion Devices :

- Field errors originate from :
  - Non uniform magnetization of the magnet blocks (poles).
  - Dimensional and Positional errors of the poles and magnet blocks.
  - Interaction with environmental magnetic field (iron frame, earth field,…)

- Important to use highly uniform magnetized blocks
  - perform a systematic characterization of the magnetization
  - Perform a pairing of the blocks to cancel errors
  - Still insufficient …

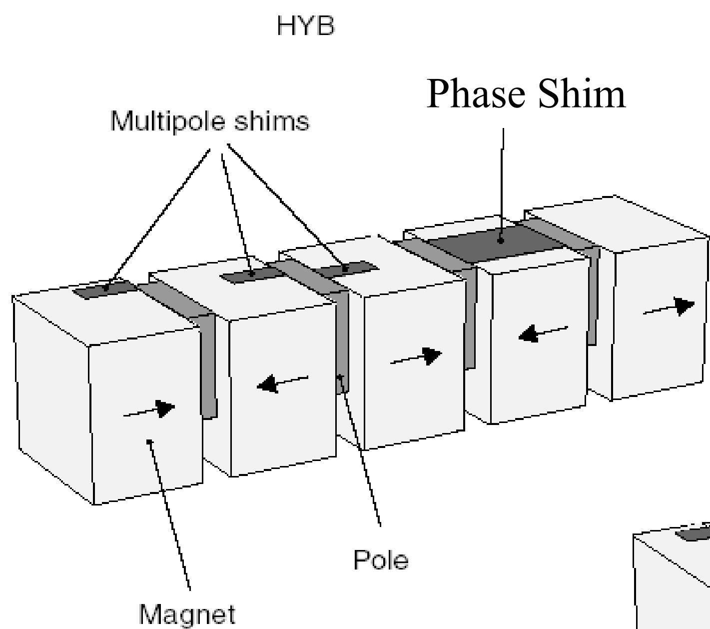
- Two main type of field errors remain
  - Multipole Field Errors (Normal and skew dipole, quadrupole, sextupole,…).
  - Phase errors which reduce the emission on the high harmonic numbers
  - Further corrections :
    - Correction magnets
    - Shimming

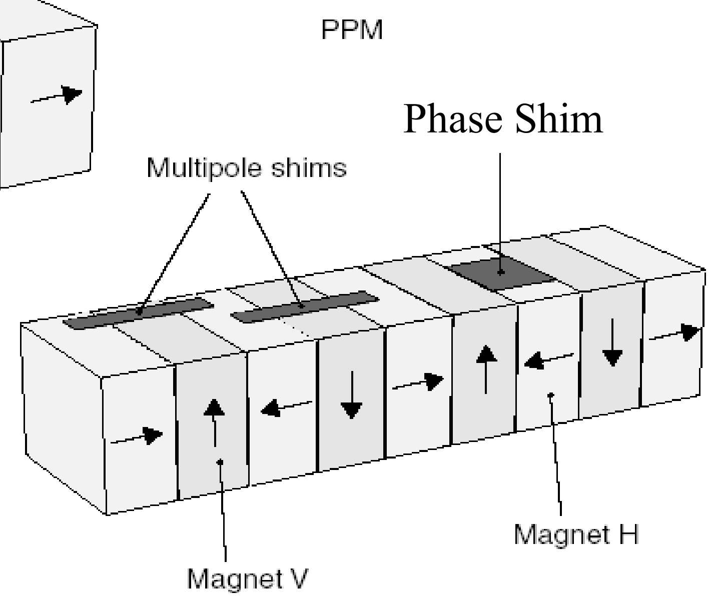
Shimming

- Mechanical Shimming :
  - Moving permanent magnet or iron pole vertically or horizontally
  - Best when free space and mechanical fixation make it possible.

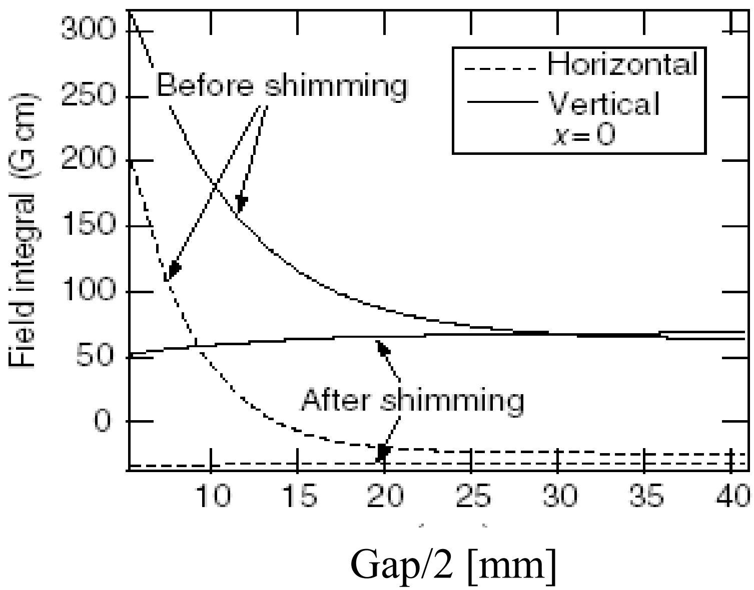
- Magnetic Shimming :
  - Add thin iron piece at the surface of the blocks
  - Reduce minimum gap and reduce the peak field

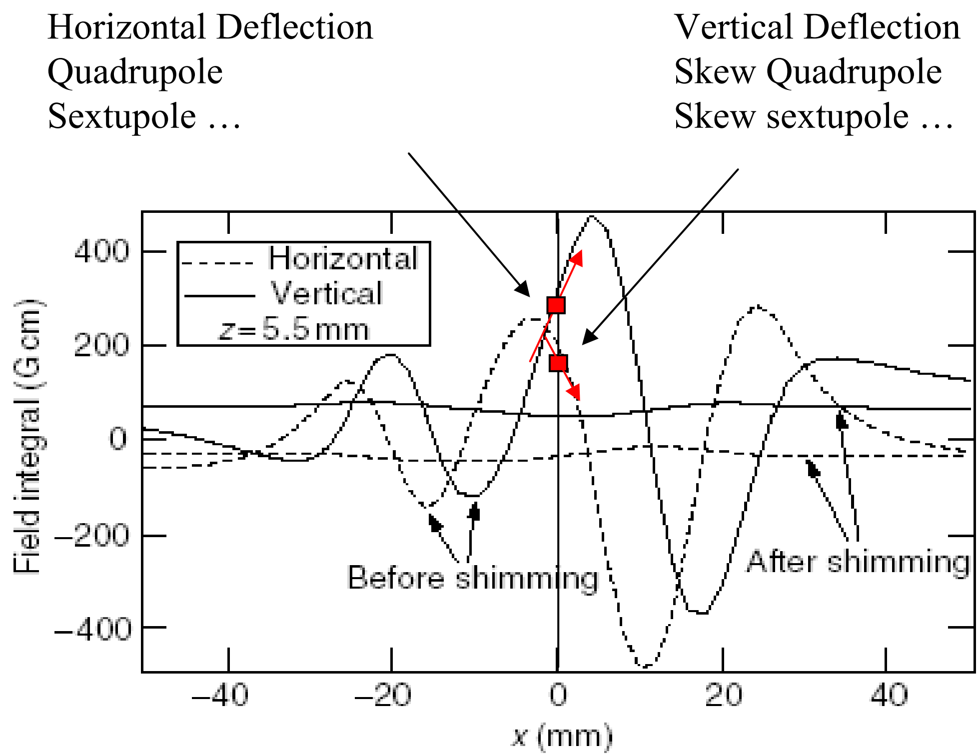
Magnetic shims





Field Integral and Multipole Shimming

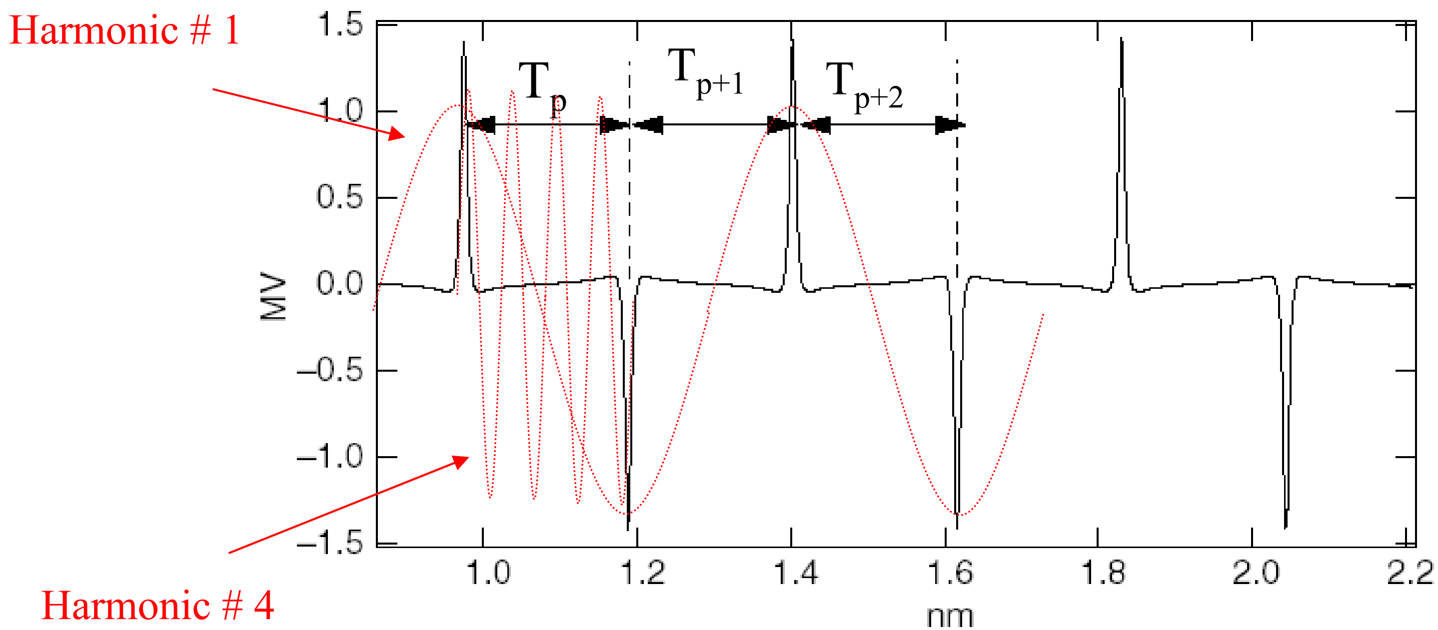




T_p : time distance between successive peaks

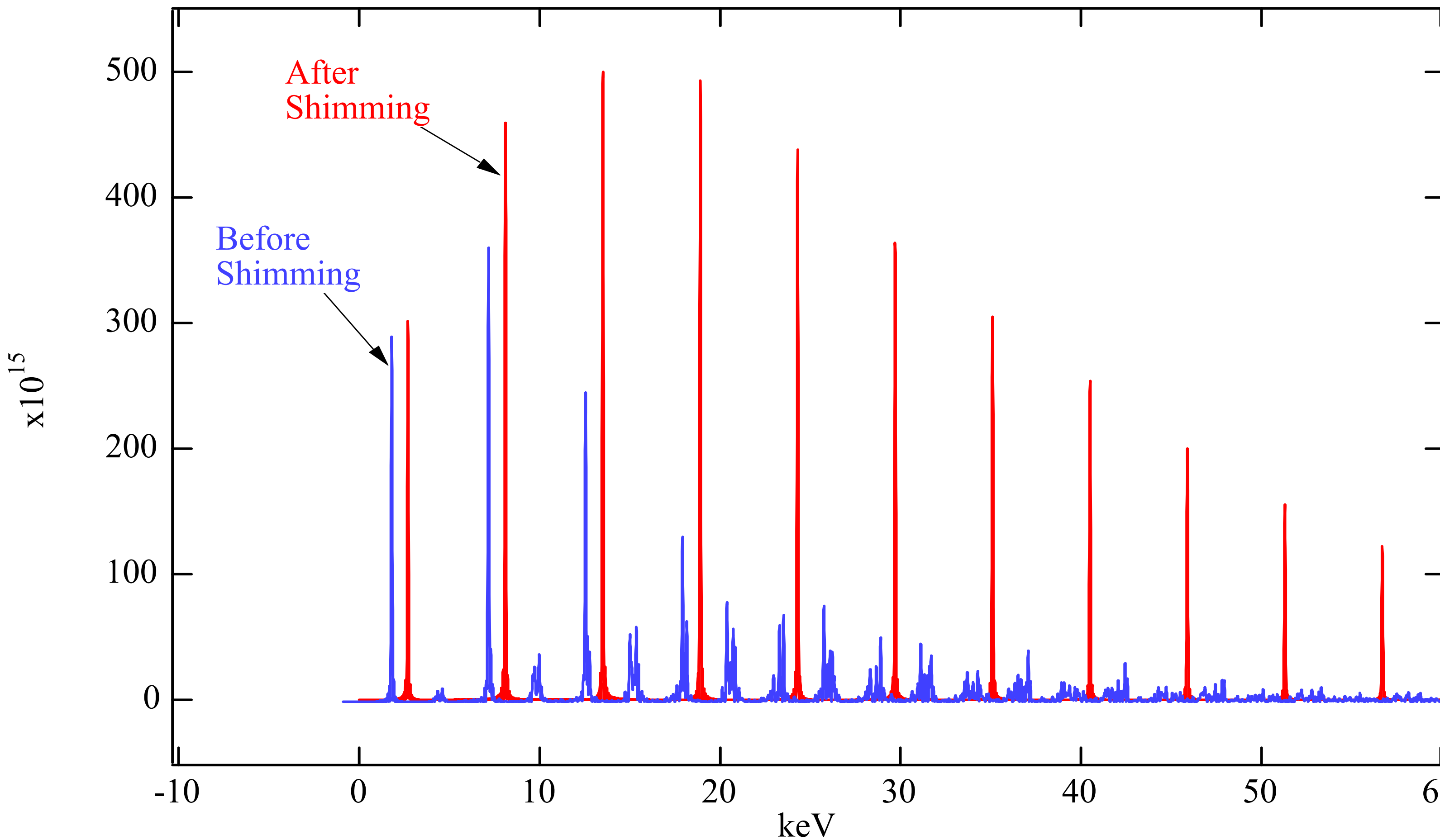
$$T_p = \frac{c\gamma^2}{\lambda_0 (1 + \frac{K^2}{2})}$$

T_p varies from one pole to the next due to period and peak field fluctuations



The Phase shimming consists of a set of local magnetic field corrections, which make T_p always identical.

Phase Shimming and the single electron spectrum



- Learning more about Insertion Devices.

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