RF Basic Concepts

Fritz Caspers, Piotr Kowina

CAS 2009, Darmstadt

Contents

- Overview
- Noise figure
- Introduction to Scattering-parameters (S-parameters)
- Properties of the S matrix of an N-port (N=1…4) and examples
- Basic properties of striplines, microstrip- and slotlines
- Application of the Smith Chart
- Modulation (AM PM)
- Beam coupling impedance

Basics of Sweep Spectrum Analyzer: superheterodyne.

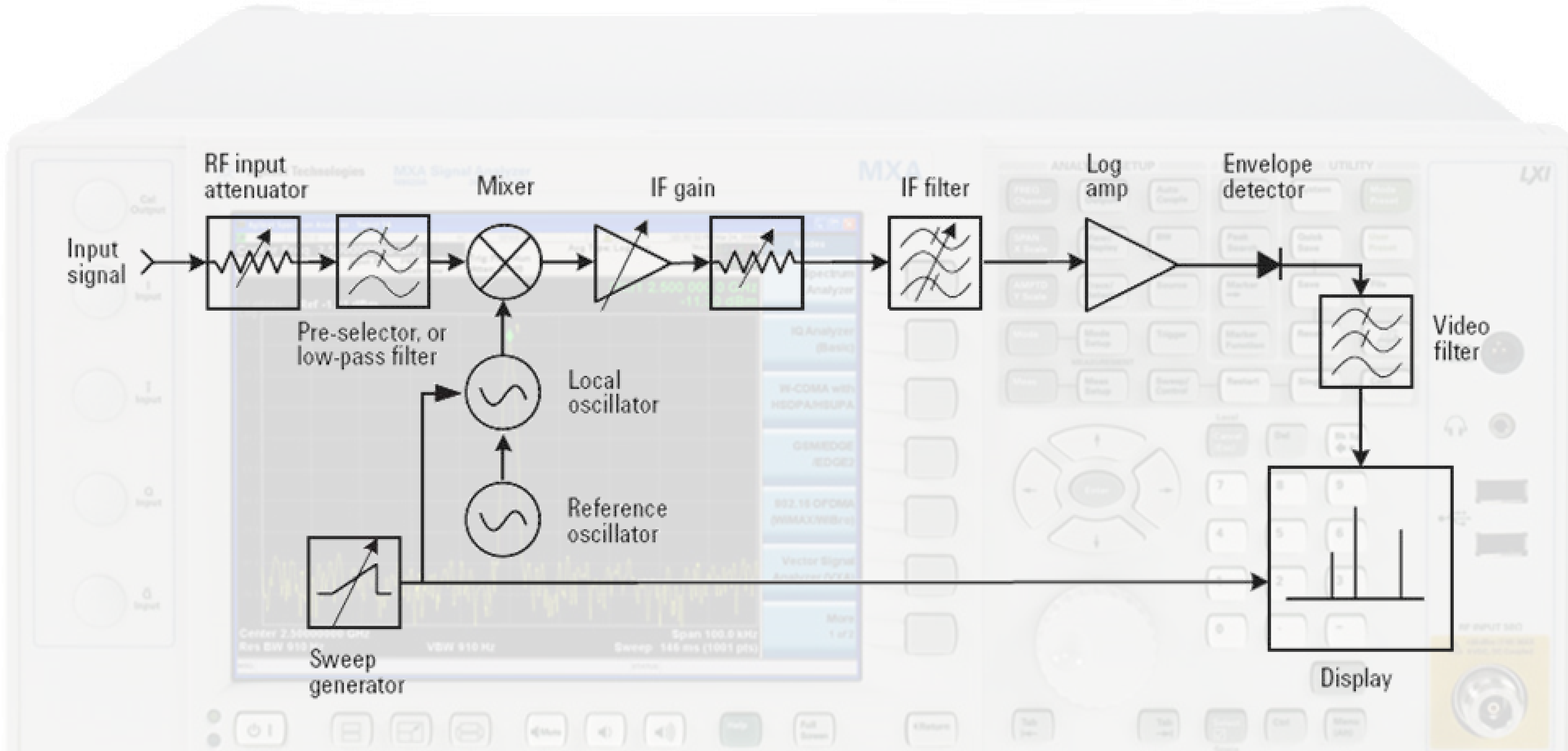


Fig.1. Block diagram of a classic superheterodyne spectrum analyzer.

Measurements using Spectrum Analyzer and oscilloscope (1)

- Measurements of several types of modulation (AM FM PM) in the time and frequency domain.
- Superposition of AM and FM spectrum (unequal carrier side bands).
- Concept of a spectrum analyzer: the superheterodyne method. Practice all the different settings (video bandwidth, resolution bandwidth etc.). Advantage of FFT spectrum analyzers.
- Measurement of the RF characteristic of a microwave detector diode (output voltage versus input power... transition between regime output voltage proportional input power and output voltage proportional input voltage).
- Concept of noise figure and noise temperature measurements, testing a noise diode, the basics of thermal noise.
- Noise figure measurements on amplifiers and also attenuators.
- The concept and meaning of ENR numbers.

Measurements using Spectrum Analyzer and oscilloscope (2)

- EMC measurements (e.g.: analyze your cell phone spectrum).
- Noise temperature of the fluorescent tubes in the room using a satellite receiver.
- Measurement of the IP3 point of some amplifier (intermodulation tests).
- Nonlinear distortion in general Concept and application of vector spectrum analyzers, spectrogram mode.
- Invent your own experiment !

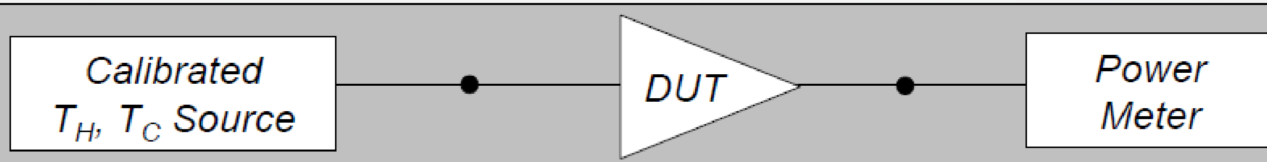
Definition of the Noise Figure

$$F = \frac{S_i / N_i}{S_o / N_o} = \frac{N_o}{GN_i} = \frac{N_o}{GkT_0B} = \frac{GN_i + N_R}{GkT_0B} = \frac{GkT_0B + N_R}{GkT_0B}$$

- *F* is the *Noise factor* of the receiver
- S_i is the available signal power at input
- $N_i = kT_0B$ is the available noise power at input
- T_0 is the absolute temperature of the source resistance
- N_o is the available noise power at the output , including amplified input noise
- N_r is the noise added by receiver
- *G* is the available receiver gain
- *B* is the effective noise bandwidth of the receiver
- If the noise factor is specified in a logarithmic unit, we use the term Noise Figure (*NF*)

$$NF = 10 \lg \frac{S_i / N_i}{S_o / N_o} \, dB$$

Measurement of Noise Figure (using a calibrated Noise Source)



$kT_HB \longrightarrow$

$kT_CB \longrightarrow$

$N_{OH}=FGkT_OB+(T_H-T_O)kBG$

$N_{OC}=FGkT_OB+(T_C-T_O)kBG$

$$\therefore Y=\frac{N_{OH}}{N_{OC}}=\frac{FT_O+T_H-T_O}{FT_O+T_C-T_O};\quad F=\frac{\left(\frac{T_H}{T_O}-1\right)-Y\left(\frac{T_C}{T_O}-1\right)}{Y-1}$$

Example:

T_H=10,290ºK (argon source), T_C=300ºK

Measured Y factor: Y=9 dB (7.94:1)

Then,

$$F=\frac{\frac{10290}{290}-1-7.94\left(\frac{300}{290}-1\right)}{7.94-1}=4.94;\ NF(dB)=10\log(4.94)=6.9dB$$

Measurements using Vector Network Analyzer (1)

- N-port (N=1…4) S-parameter measurements for different reciprocal and non-reciprocal RF-components.
- Calibration of the Vector Network Analyzer.
- Navigation in The Smith Chart.
- Application of the triple stub tuner for matching.
- Time Domain Reflectomentry using synthetic pulse →direct measurement of coaxial line characteristic impedance.
- Measurements of the light velocity using a trombone (constant impedance adjustable coax line).
- 2-port measurements for active RF-components (amplifiers):
  1 dB compression point (power sweep).
- Concept of EMC measurements and some examples.

Measurements using Vector Network Analyzer (2)

- Measurements of the characteristic cavity features (Smith Chart analysis).
- Cavity perturbation measurements (bead pull).
- Beam coupling impedance measurements with the wire method (some examples).
- Beam transfer impedance measurements with the wire (button PU, stripline PU.)
- Self made RF-components: Calculate build and test your own attenuator in a SUCO box (and take it back home then).
- Invent your own experiment!

S-parameters (1)

- The abbreviation *S* has been derived from the word *scattering*.
- For high frequencies, it is convenient to describe a given network in terms of *waves* rather than voltages or currents. This permits an easier definition of reference planes.
- For practical reasons, the description in terms of in- and outgoing waves has been introduced.
- Now, a 4-pole network becomes a 2-port and a 2n-pole becomes an n-port. In the case of an odd pole number (e.g. 3-pole), a common reference point may be chosen, attributing one pole equally to two ports. Then a 3-pole is converted into a (3+1) pole corresponding to a 2-port.
- As a general conversion rule for an odd pole number one more pole is added.

S-parameters (2)

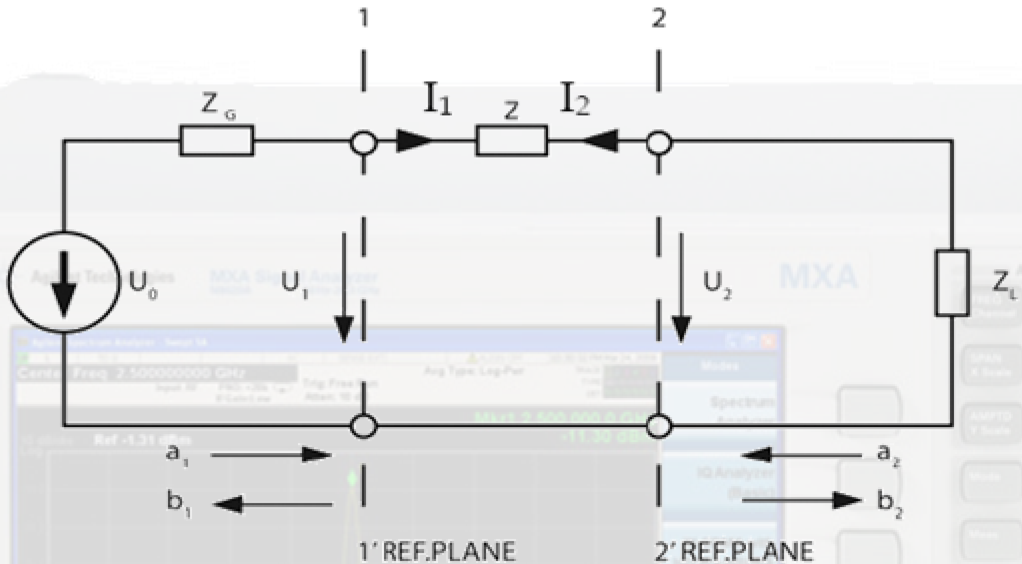


Fig.2. 2-port network

- Let us start by considering a simple 2-port network consisting of a single impedance Z connected in series (Fig.2). The generator and load impedances are Z_G and Z_L, respectively. If $Z = 0$ and $Z_L = Z_G$ (for real Z_G) we have a matched load, i.e. *maximum available power* goes into the load and $U_1 = U_2 = U_0/2$.
- Please note that *all the voltages and currents are peak values*. The lines connecting the different elements are supposed to have zero electrical length. Connections with a finite electrical length are drawn as double lines or as heavy lines. Now we need to relate U_0, U_1 and U_2 with a and b.

Definition of "power waves"(1)

- The *waves* going *towards* the n-port are a = (a_1, a_2, ..., a_n), the *waves* travelling *away* from the n-port are b = (b_1, b_2, ..., b_n). By definition currents going *into* the n-port are counted positively and currents flowing out of the n-port negatively. The wave a_1 is going into the n-port at port 1 is derived from the voltage wave going into a matched load.
- In order to make the definitions consistent with the conservation of energy, the voltage is normalized to . Z_0 is in general an arbitrary reference impedance, but usually the characteristic impedance of a line (e.g. Z_0 = 50 Ω) is used and very often $Z_G = Z_L = Z_0$. In the following we assume Z_0 to be real. The definitions of the waves a_1 and b_1 are

$$a_1 = \frac{U_0}{2\sqrt{Z_0}} = \frac{\textit{incident voltage wave}\,(\textit{port}\ 1)}{\sqrt{Z_0}} = \frac{U_1^{inc}}{\sqrt{Z_0}}$$

$$b_1 = \frac{U_1^{refl}}{\sqrt{Z_0}} = \frac{\textit{reflected voltage wave}\,(\textit{port}\ 1)}{\sqrt{Z_0}}$$

Note that a and b have the dimension $\sqrt{\text{power}}$.

Definition of "power waves"(2)

- The power travelling towards port 1, P_1^{inc}, is simply the available power from the source, while the power coming out of port 1, P_1^{refl}, is given by the reflected voltage wave.

$$P_1^{inc} = \frac{1}{2}\left|a_1\right|^2 = \frac{\left|U_1^{inc}\right|^2}{2Z_0} = \frac{\left|I_1^{inc}\right|^2}{2} Z_0$$

$$P_1^{refl} = \frac{1}{2}\left|b_1\right|^2 = \frac{U_1^{refl}}{2Z_0} = \frac{\left|I_1^{refl}\right|^2}{2} Z_0$$

- Please note the factor 2 in the denominator, which comes from the definition of the voltages and currents as peak values ("European definition"). In the "US definition" effective values are used and the factor 2 is not present, so for power calculations it is important to check how the voltages are defined. For most applications, this difference does not play a role since ratios of waves are used.
- In the case of a mismatched load Z_L there will be some power reflected towards the 2-port from Z_L

$$P_2^{inc} = \frac{1}{2}\left|a_2\right|^2$$

The S-Matrix

- The relation between a_i and b_i (i = l...n) can be written as a system of n linear equations (a_i being the independent variable, b_i the dependent variable)

$$\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned}$$

Or in matrix formulation $\boxed{\mathbf{b} = \mathbf{S}\mathbf{a}}$

- The physical meaning of S_{11} is the input reflection coefficient with the output of the network terminated by a matched load ($a_2 = 0$). S_{21} is the forward transmission (from port 1 to port 2), S_{12} the reverse transmission (from port 2 to port 1) and S_{22} the output reflection coefficient.
- When measuring the S parameter of an n-port, *all* n ports must be terminated by a matched load (not necessarily equal value for all ports), including the port connected to the generator (matched generator).
- The reflection coefficient of a single impedance Z_L connected to a generator of source impedance Z_0 (Fig. 1, case $Z_G = Z_0$ and $Z = 0$)

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{U_1 - I_1 Z_0}{U_1 + I_1 Z_0} = \frac{Z_L - Z_0}{Z_L + Z_0} = \rho = \frac{(Z_L/Z_0) - 1}{(Z_L/Z_0) + 1}$$

- which is the familiar formula for the reflection coefficient ρ (often also denoted Γ).

Properties of the S matrix of an N-port

- In general the S parameters are complex and frequency dependent.
- Their phases change when the reference plane is moved
  ➔a clear definition the reference planes is crucial!.
- For a given structure, often the S parameters can be determined from considering mechanical symmetries and, in case of lossless networks, from energy conservation.
- A generalized n-port has n^2 scattering coefficients. While the S_{ij} may be all independent. However, symmetries reduce the number of independent coefficients.
- An n-port is *reciprocal* when $S_{ij} = S_{ji}$ for all *i* and *j*. Most passive components are reciprocal, active components such as amplifiers are generally non-reciprocal.
- A two-port is *symmetric*, when it is reciprocal ($S_{21} = S_{12}$) and when the input and output reflection coefficients are equal ($S_{22} = S_{11}$).
- An N-port is *passive and lossless* if is S matrix is *unitary*, i.e. $S^\dagger S = 1$, where $x^\dagger = (x^*)^T$ is the conjugate transpose of x.

Examples of S matrices: 1-ports

- Simple lumped elements are 1-ports, but also cavities with one test port, long transmission lines or antennas can be considered as 1-ports
- 1-ports are characterized by their reflection coefficient ρ, or in terms of S parameters, by S_{11}.
- Ideal short: $S_{11} = -1$
- Ideal termination: $S_{11} = 0$
- Active termination (reflection amplifier): $|S11| > 1$

Examples of S matrices: 2-ports

- Ideal, reciprocal attenuator

$$\mathbf{S} = \begin{pmatrix} 0 & e^{-\alpha} \\ e^{-\alpha} & 0 \end{pmatrix}$$

with the attenuation α in Neper. The attenuation in Decibel is given by A = -20*log$_{10}$(S_{21}), 1 Np = 8.686 dB. An attenuator can be realized e.g. with three resistors in a T circuit or with resistive material in a waveguide.

- Ideal isolator

$$\mathbf{S} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

The isolator allows transmission in one directly only, it is used e.g. to avoid reflections from a load back to the generator.

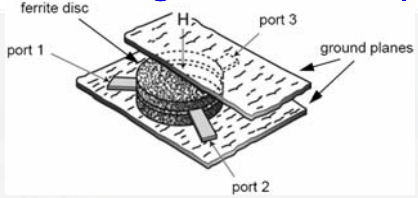
- Ideal amplifier

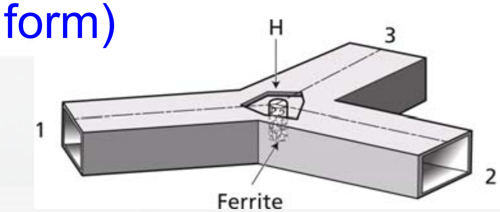
$$\mathbf{S} = \begin{pmatrix} 0 & 0 \\ G & 0 \end{pmatrix}$$

with the voltage gain $G > 1$. Please note the similarity between an ideal amplifier and an ideal isolator!

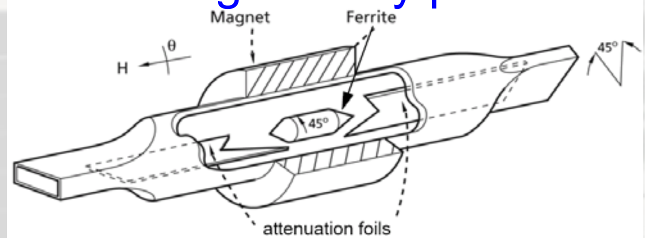
Other examples of N-port RF elements

- Circulators (in wave guide or stripline form)

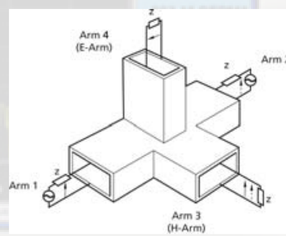




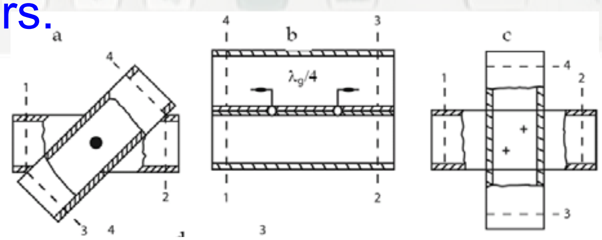
- Isolators (e.g. Faraday isolator based on the rotation of a polarized wave in the presence of a magnetically polarized ferrite.

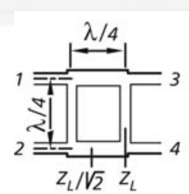


- A "magic" T: A very special combination of a E-plane and H-plane waveguide.



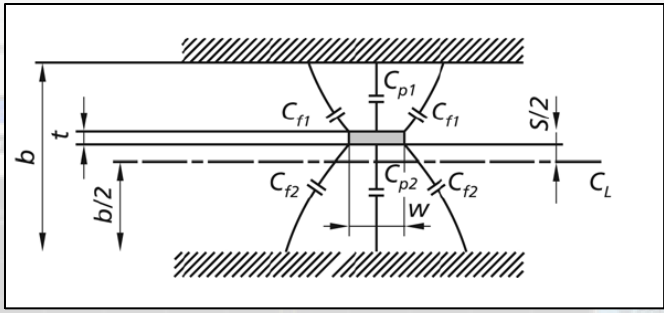
- Directional couplers.





Striplines

- A stripline is a flat conductor between a top and bottom ground plane. The space around this conductor is filled with a homogeneous dielectric material. This line propagates a pure TEM mode.



$$C_{tot} = C_{p1} + C_{p2} + 2C_{f1} + 2C_{f2}$$

Fig.8. Design, dimensions and characteristics for offset center-conductor strip transmission line [14]

- For a mathematical treatment, the effect of the fringing fields may be described in terms of static capacities [14]. The total capacity is the sum of the principal and fringe capacities C_p and C_f.
- For striplines with an homogeneous dielectric the phase velocity is the same, and frequency independent, for all TEM-modes.
- The coupled striplines are commonly used as directional couplers.

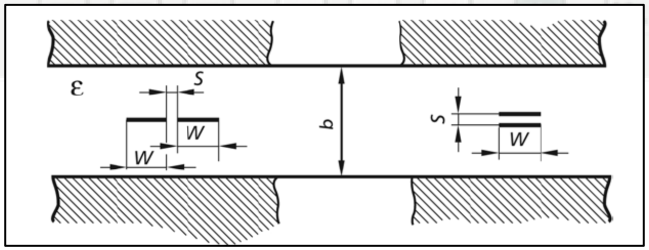


Fig.9. Types of coupled striplines [14]: left: side coupled parallel lines, right: broad-coupled parallel lines

Microstrips

- A microstripline may be visualized as a stripline with the top cover and the top dielectric layer taken away (Fig.10). It is thus an asymmetric open structure, and only part of its cross section is filled with a dielectric material. Since there is a transversely inhomogeneous dielectric, only a quasi-TEM wave exists.
- Broadly used due to cheap production and easy access to the surface for the integration of active elements.

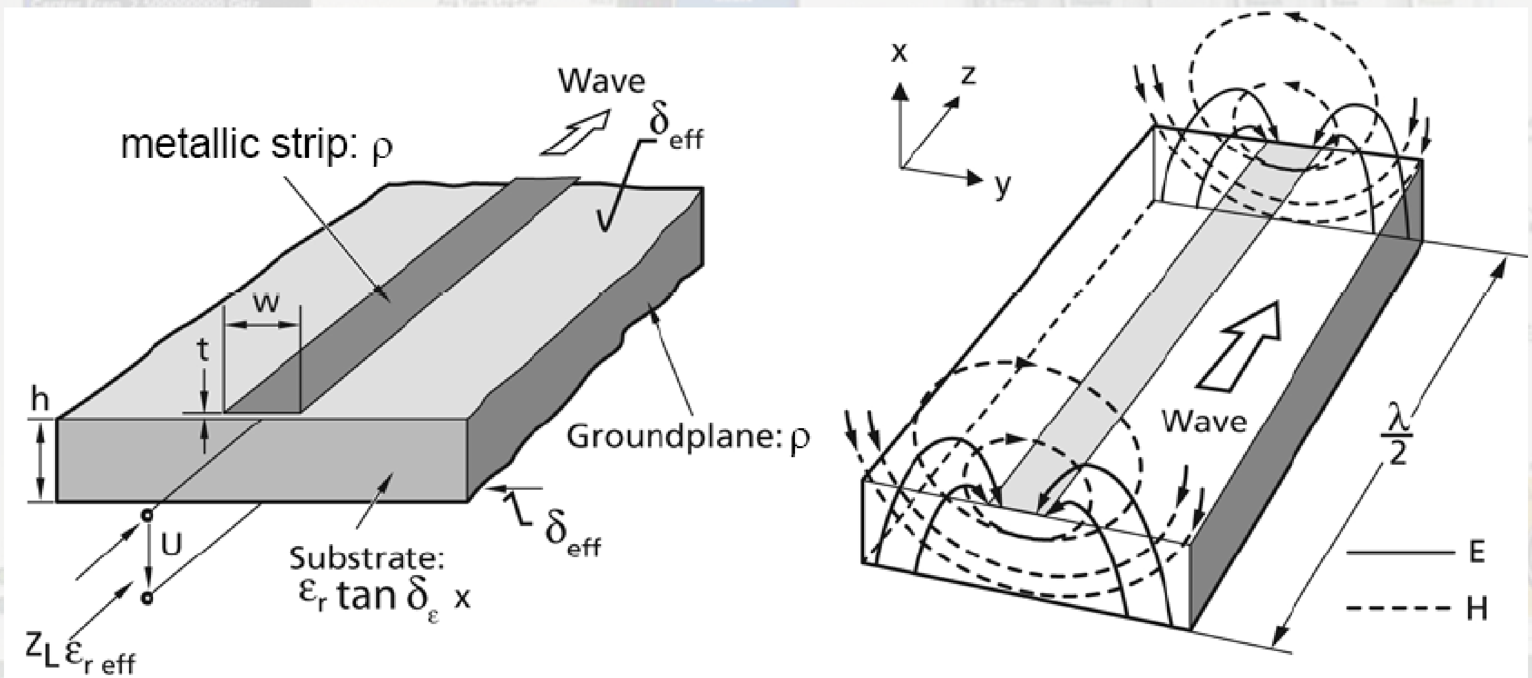


Fig.10 Microstripline: left: Mechanical construction, right: static field approximation [16].

Slotlines

- The slotline may be considered as the dual structure of the microstrip. It is essentially a slot in the metallization of a dielectric substrate.

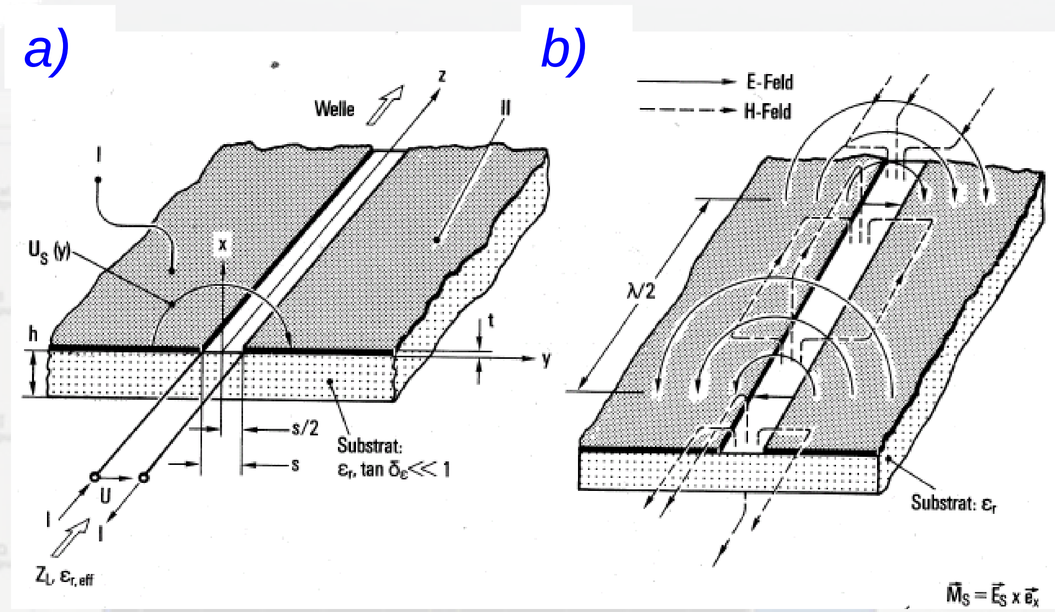


Fig.11. Slotlines
a) Mechanical construction,
b) Field pattern (TE-approximation)

- The characteristic impedance and the effective dielectric constant exhibit similar dispersion properties to those of the microstrip line.
- A unique feature of the slotline is that it may be combined with microstrip lines on the same substrate. This, in conjunction with through holes, permits interesting topologies such as pulse inverters in sampling heads (e.g. for sampling scopes), or……

.... Fritz Bones (Slotline)

- Fig.12 shows a broadband (decade bandwidth) pulse inverter. Assuming the upper microstrip to be the input, the signal leaving the circuit on the lower microstrip is inverted since this microstrip ends on the opposite side of the slotline compared to the input. Printed slotlines are also used for broadband pickups in the GHz range, like these applied in the GSI/Fair stochastic cooling systems*.

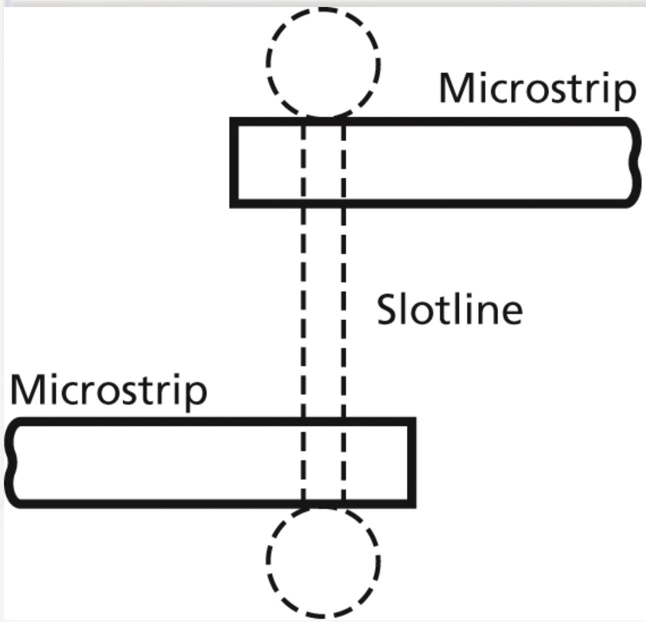


Fig 12. Two microstrip-slotline transitions connected back to back for 180° phase change [15]

***) There are 2 Fritz: a thin one and a thick one; both are involved.**

The Smith Chart - definition

- The Smith Chart is a very handy tool for visualizing and solving RF problems
- It indicates in the plane of the complex reflection coefficient ρ the corresponding value of the complex impedance $Z = R + jX$. Often the normalized impedance z is used, with $z = Z/Z_0$, where Z_0 is a an arbitrary characteristic impedance, e.g. $Z_0 = 50\ \Omega$. The reflection coefficient is given as

$$\rho = \frac{Z - Z_0}{Z + Z_0} = \frac{z - 1}{z + 1}$$

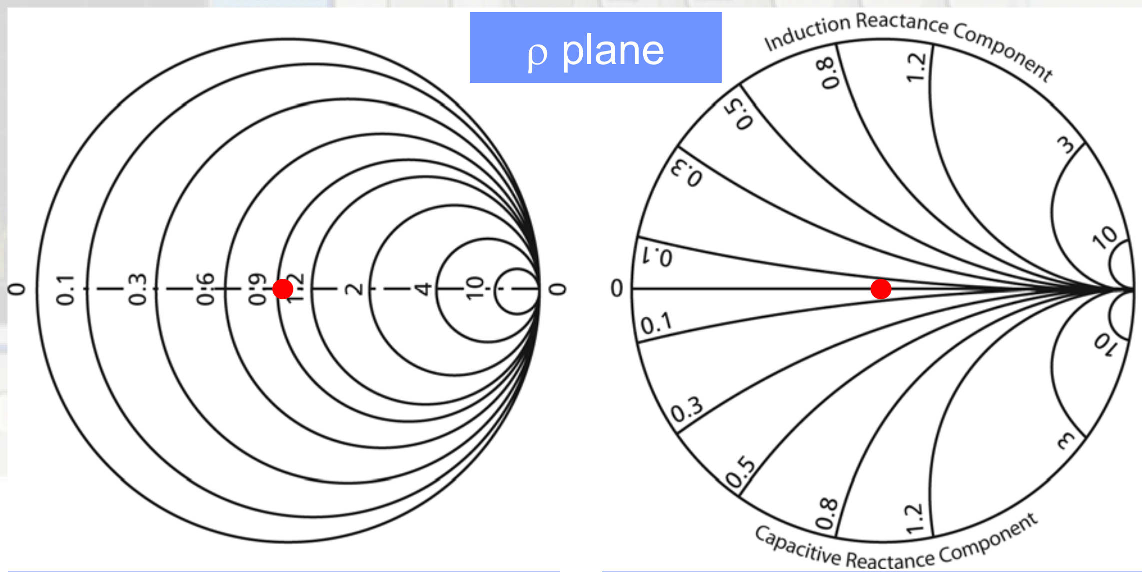
or in terms of admittance $Y = 1/Z = G + jB$, with the normalized admittance $y = Y/Y_0$, where $Y_0 = 1/Z_0$

$$\rho = -\frac{Y - Y_0}{Y + Y_0} = -\frac{y - 1}{y + 1}$$

These equations define *conformal mappings* from the z and the y plane to the ρ plane.

The Smith Chart - construction

- A very basic use of the Smith Chart is to graphically convert values of ρ into z and vice versa. To this purpose in the ρ plane a grid is drawn that allows to find the value of z at a given point ρ.
- An important property of conformal mappings is that general circles are mapped to general circles. Straight lines are considered as circle with infinite radius
- Below the loci of constant resistance and constant reactance are drawn in the ρ plane
- The origin of the ρ plane is marked with a red dot, the diameter of the largest circle is $|\rho|=1$

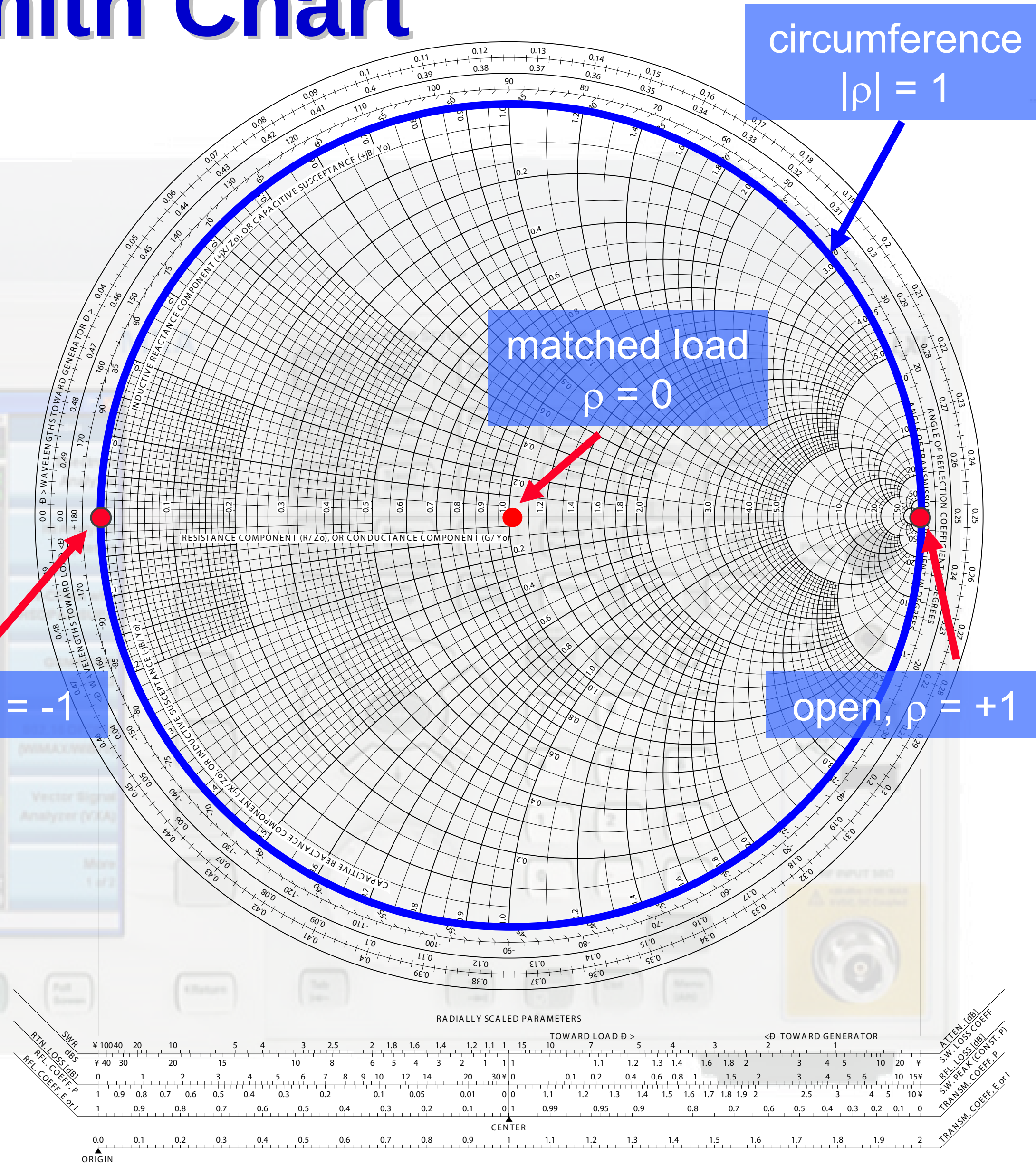


constant resistance circles

constant reactance circles

The Smith Chart

- The Smith Chart is simply the ρ plane with overlaid circles that help to find the z for each point. The radius in is general $\rho = 1$.
- Important points:
  - Center of the Smith Chart: matched load. $\rho = 0$, $z = 1$
  - Open circuit: $\rho = +1$, $z = \infty$
  - Short circuit: $\rho = -1$, $z = 0$
- Lossless elements lie on the circle $|\rho|=1$; active elements, such as reflection amplifiers, lie outside this circle.



The quality factor

- The quality (Q) factor of a resonant circuit is defined as the ratio of the stored energy *W* over the energy dissipated *P* in one cycle.

$$Q = \frac{\omega W}{P}$$

- Q_0: *Unloaded Q factor* of the unperturbed system, e.g. a closed cavity
- Q_L: *Loaded Q factor* with measurement circuits etc connected
- Q_{ext}: *External Q factor* of the measurement circuits etc
- These Q factors are related by

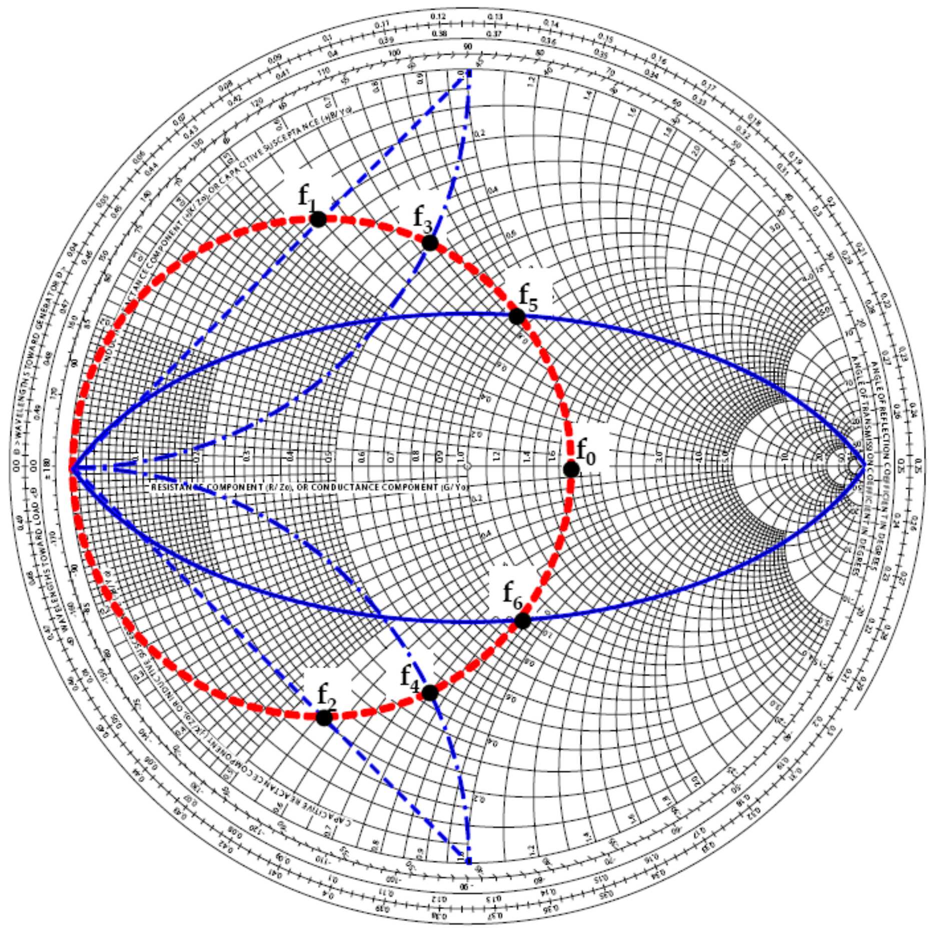
$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

- The Q factor of a resonance peak or dip can be calculated from the center frequency f_0 and the 3 dB bandwidth Δf as

$$Q = \frac{f_0}{\Delta f}$$

Q factor measurement in the Smith Chart

- The typical locus of a resonant circuit in the Smith chart is illustrated as the dashed red circle
- From the different marked frequency points the 3 dB bandwidth and thus the quality factors Q_0, Q_L and Q_{ext} can be determined (see lecture notes)
- The larger the circle, the stronger the coupling
- In practice, the circle may be rotated around the origin due to transmission lines between the resonant circuit and the measurement device



Transmission line model for beam coupling impedance bench measurements

- DUT + Coaxial Wire = TEM transmission line (with distributed parameters)
- The DUT coupling impedance is modeled as a series impedance of an ideal REFerence line.
- Coupling impedance is obtained from the REF and DUT characteristic impedances and propagation constants.
- Transmission line are characterized via S-parameters with Vector Network Analyzers (transmission measurements preferred).
- In the framework of the transmission line model, the DUT impedance can be computed from S-parameters.

Practical approximated formulae

Beam coupling impedance (formulae)

Improved log formula (Vaccaro, 1994)

$$Z_{LOG} = -Z_c \ln\left(\frac{S_{21}^{DUT}}{S_{21}^{REF}}\right)\left[1 + \frac{\ln\left(S_{21}^{DUT}\right)}{\ln\left(S_{21}^{REF}\right)}\right]$$

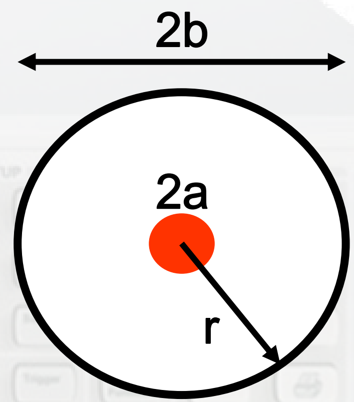
Small Impedance wrt to Zc

Log formula (Walling et al, 1989)

$$Z_{log} = -2Z_c \ln\left(\frac{S_{21}^{DUT}}{S_{21}^{REF}}\right)$$

Hahn-Pedersen formula (1978)

$$Z_{HP} = -2Z_c \frac{S_{21}^{DUT} - S_{21}^{REF}}{S_{21}^{DUT}}$$



$$Z_c = \frac{Z_0}{2\pi}\ln\left(\frac{b}{a}\right)$$

Lumped element: DUT length << λ

Already implemented in the conversion formula of modern VNA

Systematic errors are discussed in: E. Jensen, PS-RF-Note 2000-00 (2001)
H. Hahn, PRST-AB 3 122001 (2001)

You will have a lot of fun...

Course 1 “RF Measurements Techniques”