# Introduction to Transverse Beam Optics

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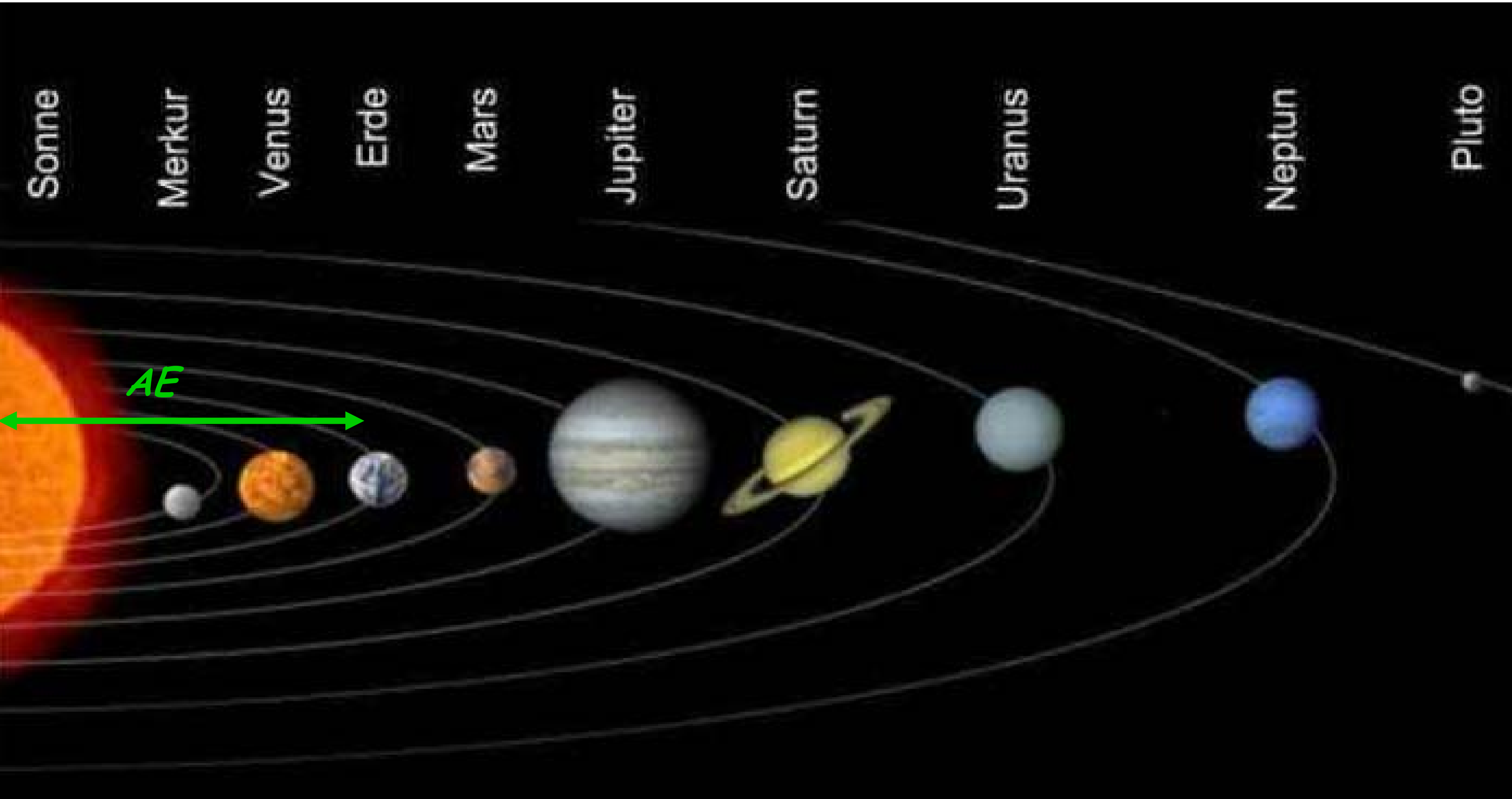
# Largest storage ring: The Solar System

**astronomical unit:** average distance earth-sun
$1AE \approx 150 * 10^6$ km
Distance Pluto-Sun $\approx$ 40 AE



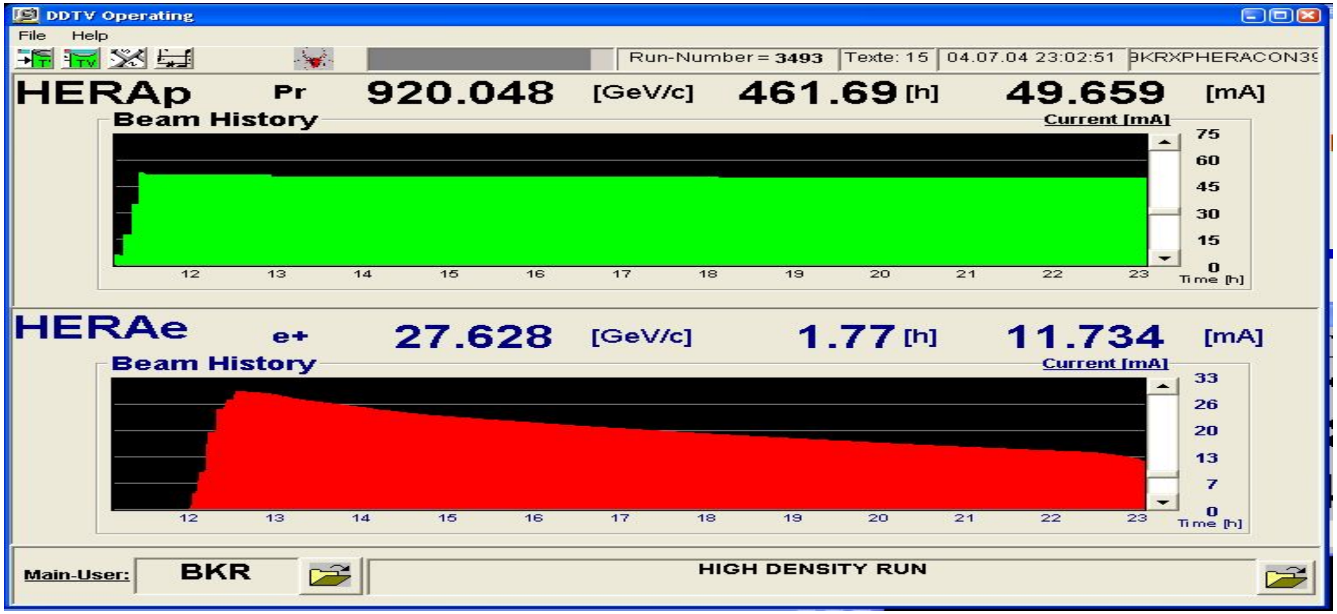
# *Luminosity Run of a typical storage ring:*

***HERA Storage Ring: Protons accelerated and stored for 12 hours***

***distance of particles travelling at about $v \approx c$***

***$L = 10^{10}$-$10^{11}$ km***

***... several times Sun - Pluto and back***




→ ***guide the particles on a well defined orbit („design orbit")***

→ ***focus the particles to keep each single particle trajectory***

***within the vacuum chamber of the storage ring, i.e. close to the design orbit.***

# *Transverse Beam Dynamics:*

## *0.) Introduction and Basic Ideas*

**„ ... in the end and after all it should be a kind of circular machine"**

**→ need transverse deflecting force**

Lorentz force $$\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$$

typical velocity in high energy machines: $$v \approx c \approx 3 * 10^8 \, m/s$$

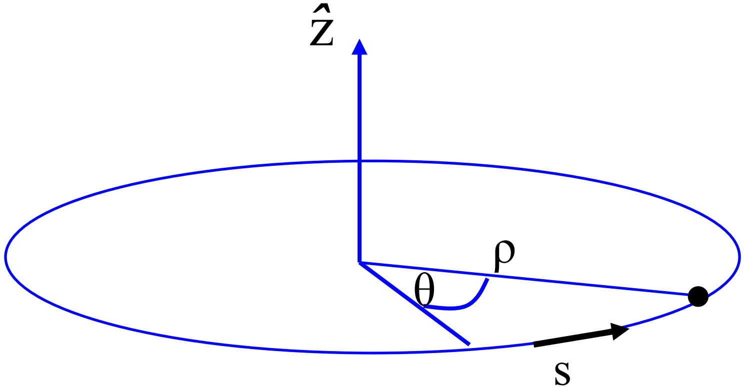
***old greek dictum of wisdom:***

***if you are clever, you use magnetic fields in an accelerator wherever it is possible.***

*But remember: magn. fields act allways perpendicular to the velocity of the particle*

*→ only bending forces, → no „beam acceleration"*

## The ideal circular orbit




*circular coordinate system*

## condition for circular orbit:

***Lorentz force*** $F_L = e * v * B$

***centrifugal force*** $F_{Zentr} = \frac{\gamma \, m_0 \, v^2}{\rho}$

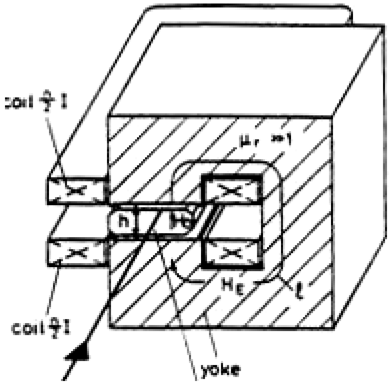
$$\frac{\gamma m_0 \cancel{v}^2}{\rho} = e * \cancel{v} * B$$

$$\boxed{\frac{p}{e} = B * \rho}$$

# *I.) The Magnetic Guide Field*

## Dipole Magnets:

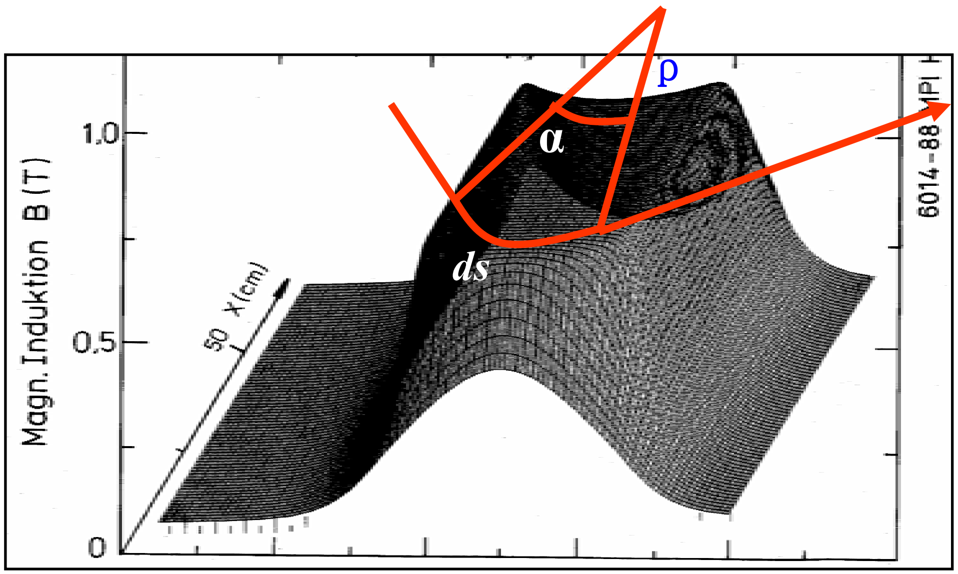
***define the ideal orbit***
***homogeneous field created by two flat pole shoes***




*court. K. Wille*

***Magnetic field of a dipole magnet:***

$$B_0 = \frac{\mu_0 n I}{h}$$




*field map of a storage ring dipole magnet*

***Normalise to momentum:***

***... remember*** $p/e = B^* \rho$

$$\frac{1}{\rho}\left[m^{-1}\right] = \frac{e \cdot B_0}{p} = 0.2998 \frac{B_0\left[T\right]}{p\left[GeV/c\right]}$$

***„radius of curvature, bending strength“***

# Quadrupole Magnets:

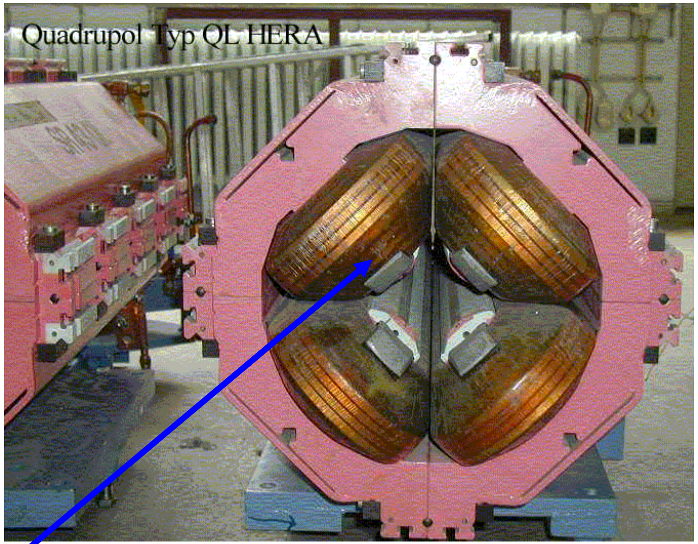
*required:* *focusing forces to keep trajectories in vicinity of the ideal orbit*
*linear increasing Lorentz force*
*linear increasing magnetic field* $B_z = -g \cdot x \quad B_x = -g \cdot z$

*at the location of the particle trajectory: no iron, no current*

$$\vec{\nabla} \times \vec{B} = 0 \quad \rightarrow \quad \vec{B} = -\vec{\nabla} V$$

*the magnetic field can be expressed as gradient of a scalar potential !*

$$V(x,z) = g \cdot xz$$




*equipotential lines (i.e. the surface of the iron contour) = hyperbolas*

## *Calculation of the Quadrupole Field:*

$$\oint \vec{H} d\vec{s} = \int_A \vec{j} d\vec{a} = N * I$$

$$B(r) = g * r$$

***gradient of a quadrupole field:***

$$g = \frac{2\mu_0 n I}{r^2}$$

**normalised quadrupole strength:**

$$k = \frac{g}{p/e}$$

**Separate Function Machines:**

**Split the magnets and optimise them according to their job:**

**bending, focusing etc**

*Example:*
*heavy ion storage ring TSR*

# *II.) The equation of motion:*

## *Linear approximation:*

*** ideal particle → design orbit**

*** any other particle → coordinates x, z small quantities**
**x,z << ρ**

**→ magnetic guide field: only linear terms in x & z of B have to be taken into account**

### *Taylor Expansion of the B field:*

$$\mathrm{B}_z(x) = B_{z0} + \frac{dB_z}{dx}x + \cancel{\frac{1}{2!}\frac{d^2B_z}{dx^2}x^2} + \cancel{\frac{1}{3!}\frac{d^3B_z}{dx^3}x^3} + \ldots$$

### *… what about the vertical plane:*

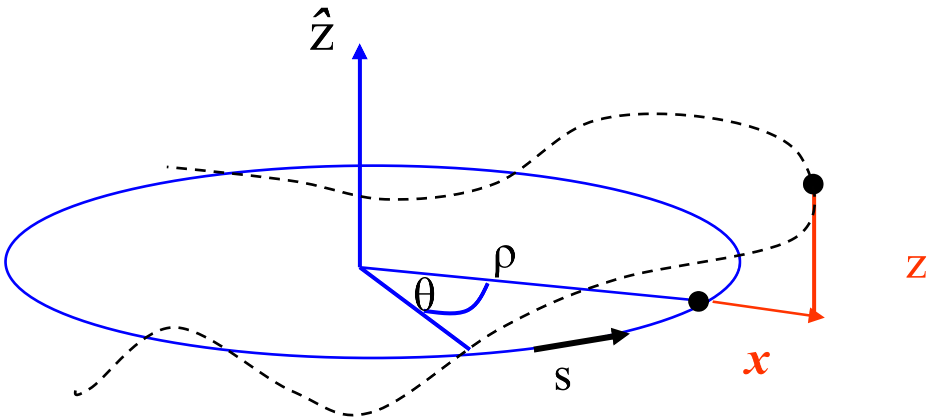
***Maxwell:***

$$\vec{\nabla} \times \vec{\mathrm{B}} = \vec{j} + \frac{\partial \vec{\mathrm{E}}}{\partial \mathrm{t}} = 0$$

$$\Rightarrow \quad \frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x}$$

$$\mathrm{B}_{\mathrm{x}}(s, x, z) = \frac{\partial B_x}{\partial z} z$$

# *Equation of Motion:*




***Consider local segment of a particle trajectory***
***... and remember the old days:***
***(Goldstein page 27)***

***radial acceleration:***

$$a_r = \frac{d^2\rho}{dt^2} - \rho\left(\frac{d\theta}{dt}\right)^2$$

***Ideal orbit:*** $\rho = const, \quad \frac{d\rho}{dt} = 0$

***Force:*** $F = m\rho\left(\frac{d\theta}{dt}\right)^2 = m\rho\omega^2$

$$F = mv^2/\rho$$

***general trajectory:***

$$F = m\frac{d^2}{dt^2}(x+\rho) - \frac{mv^2}{x+\rho} = eB_z v$$

***develop for small x:***

$$x << \rho$$

$$m\frac{d^2x}{dt^2} - \frac{mv^2}{\rho}(1 - \frac{x}{\rho}) = eB_z v$$

***guide field in linear approx.***

$$B_z = B_0 + x\frac{\partial B_z}{\partial x}$$

$$m\frac{d^2x}{dt^2} - \frac{mv^2}{\rho}(1 - \frac{x}{\rho}) = ev\left\{B_0 + x\frac{\partial B_z}{\partial x}\right\}$$

***independent variable: t → s***

$$dx\Big/dt = dx\Big/ds * ds\Big/dt$$

$$x' = dx\Big/ds$$

$$x'' - \frac{1}{\rho}(1 - \frac{x}{\rho}) = \frac{eB_0}{mv} + \frac{exg}{mv}$$

$$x'' - \frac{1}{\rho} + \frac{x}{\rho^2} = -\frac{1}{\rho} + kx$$

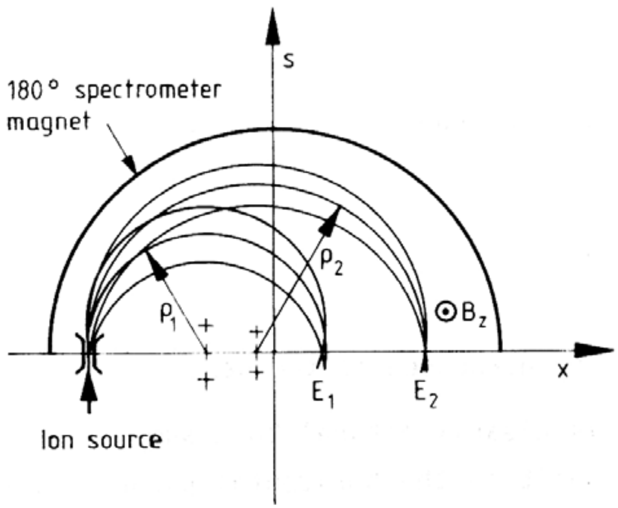
$$\boxed{x'' + x(\frac{1}{\rho^2} - k) = 0}$$

# *Remarks:*

* $$x'' + (\frac{1}{\rho^2} - k) \cdot x = 0$$

**… there seems to be a focusing even without a quadrupole gradient**

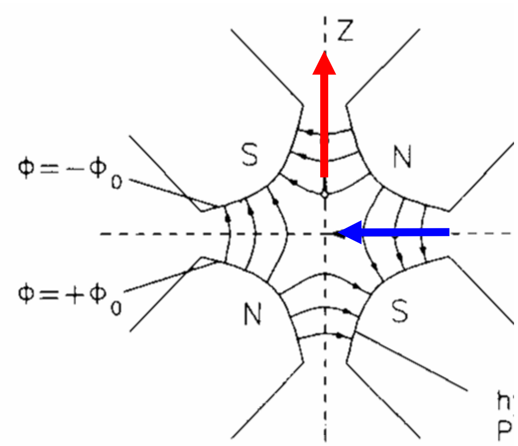
*„weak focusing of dipole magnets"*




*Mass spectrometer: particles are separated according to their energy and focused due to the 1/ρ effect of the dipole*

* **Equation for the vertical motion:**

$$z'' + k \cdot z = 0$$

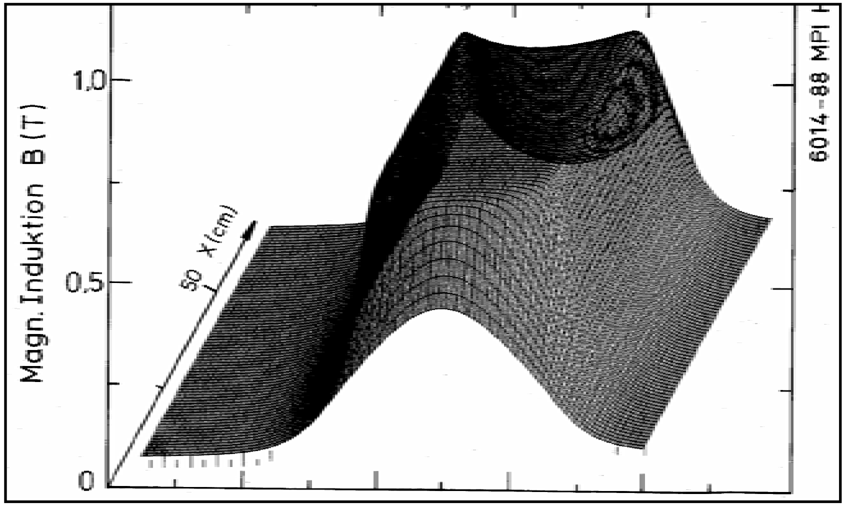


# III.) Solution of Trajectory Equations

***Define … hor. plane:*** $K = 1/\rho^2 - k$

***… vert. Plane:*** $K = k$

$$y'' + K * y = 0$$




*K = const within a magnet*

***Differential Equation of harmonic oscillator … with spring constant K***

***Ansatz:***

$$x(s) = a_1 \cdot \cos(\omega t) + a_2 \cdot \sin(\omega t)$$

***general solution: linear combination of two independent solutions***

## *Hor. Focusing Quadrupole K > 0:*

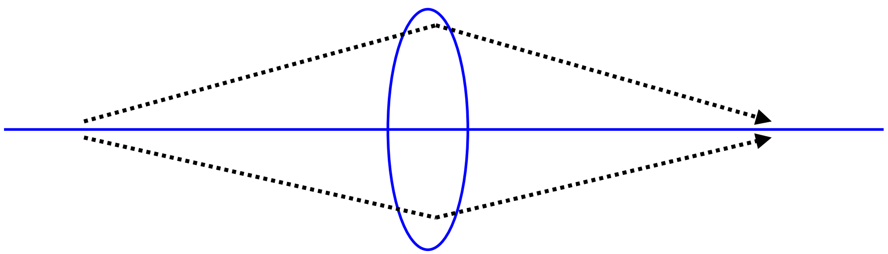
$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s)$$

$$\left.\begin{aligned} x(0) &= x_0 \\ x'(0) &= x'_0 \end{aligned}\right\} \quad \textit{starting conditions}$$

***For convenience expressed in matrix formalism:***

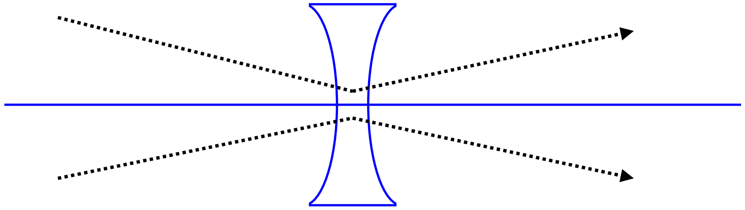
$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = M_{foc} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_0$$



$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}s) & \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s \\ -\sqrt{|K|} \sin(\sqrt{|K|}s) & \cos(\sqrt{|K|}s) \end{pmatrix}_0$$

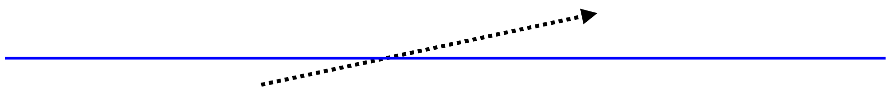
## *hor. defocusing quadrupole:* $K < 0$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$



## *drift space:* $K = 0$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$



***!*** ***with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & z is uncoupled“***

# *Thin Lens Approximation:*

***matrix of a quadrupole lens***

$$M = \begin{pmatrix} \cos\sqrt{|k|}\,l & \frac{1}{\sqrt{|k|}}\sin\sqrt{|k|}\,l \\ -\sqrt{|k|}\sin\sqrt{|k|}\,l & \cos\sqrt{|k|}\,l \end{pmatrix}$$

***in many practical cases we have the situation:***

$f = \dfrac{1}{kl_q} >> l_q$ ***... focal length of the lens is much bigger than the length of the magnet***

***limes:*** $l \to 0$ ***while keeping*** $kl = const$

$$M_x = \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix} \qquad M_z = \begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix}$$

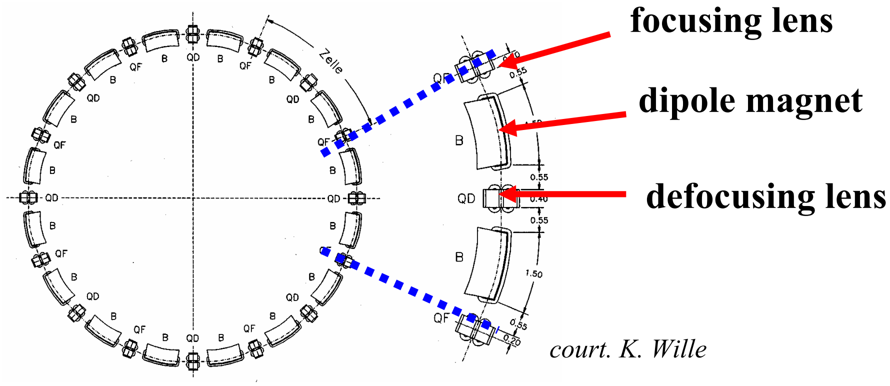
***... usefull for fast (and in large machines still quite accurate) „back on the envelope calculations“ ... and for the guided studies !***

# *Transformation through a system of lattice elements*

***combine the single element solutions by multiplication of the matrices***

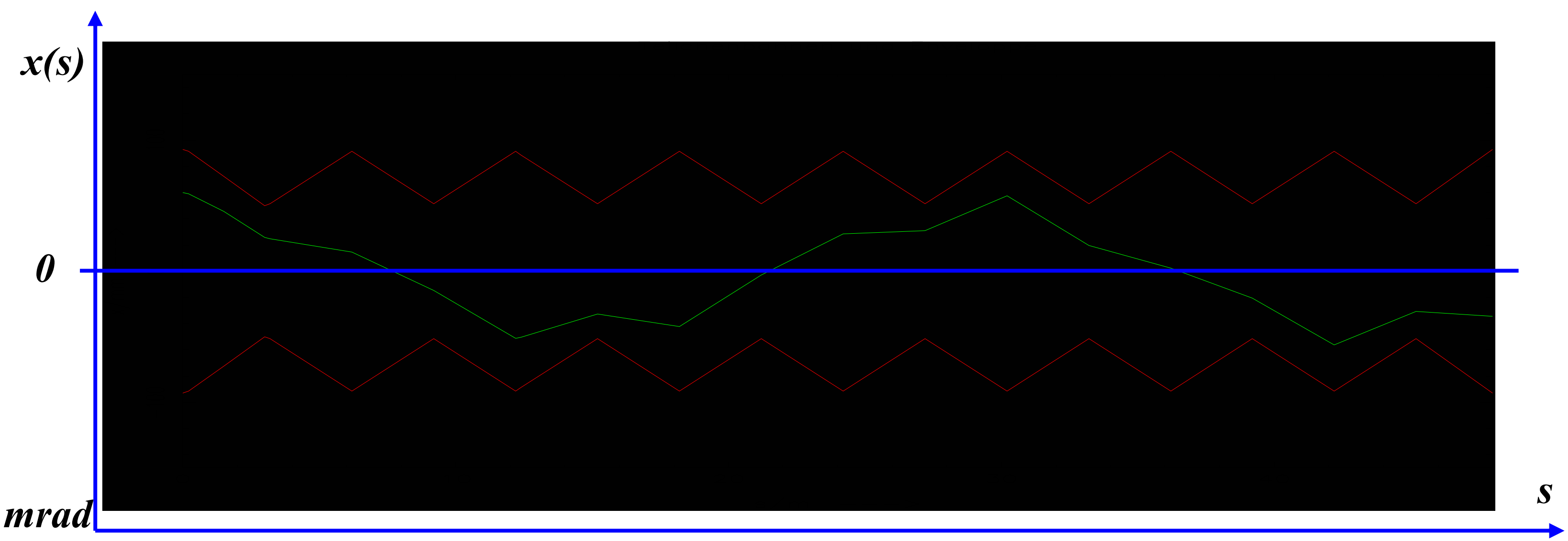
$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D^*.....}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



*court. K. Wille*

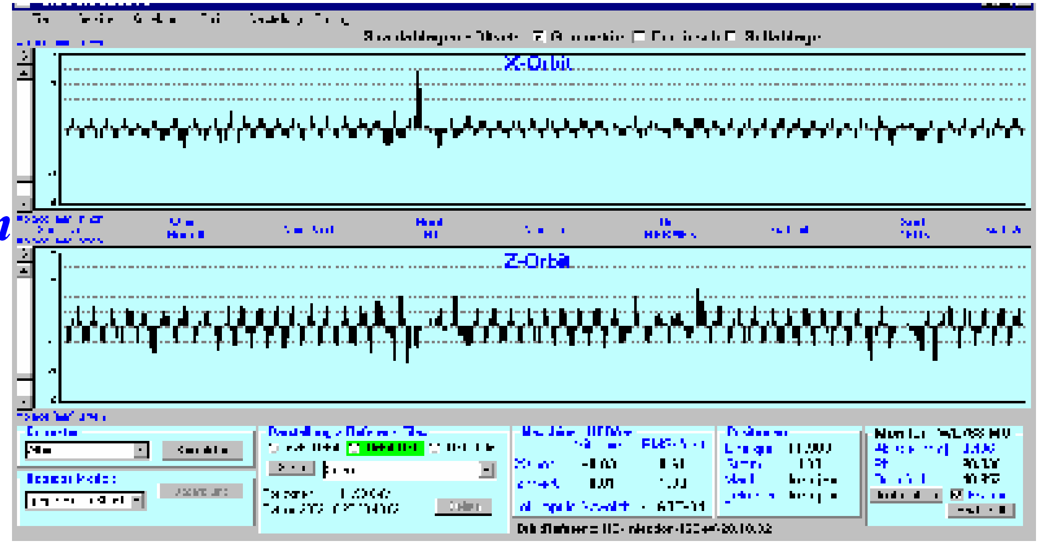
***„C" and „S" = sin- and cos- like trajectories of the lattice structure, in other words the two independent solutions of the homogeneous equation of motion***




***typical values in a strong foc. machine:***

***$x \approx mm$, $x' \leq mrad$***

# IV.) Orbit & Tune:



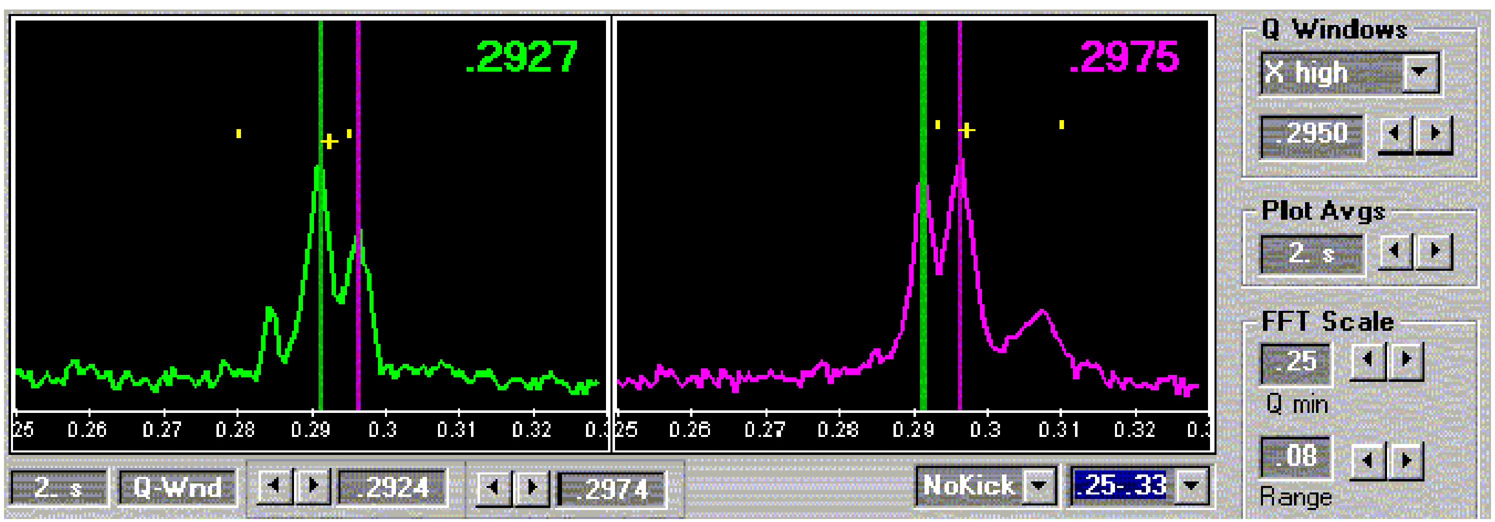

***Tune: number of oscillations per turn***

***31.292***
***32.297***

***Relevant for beam stability:***
***non integer part***

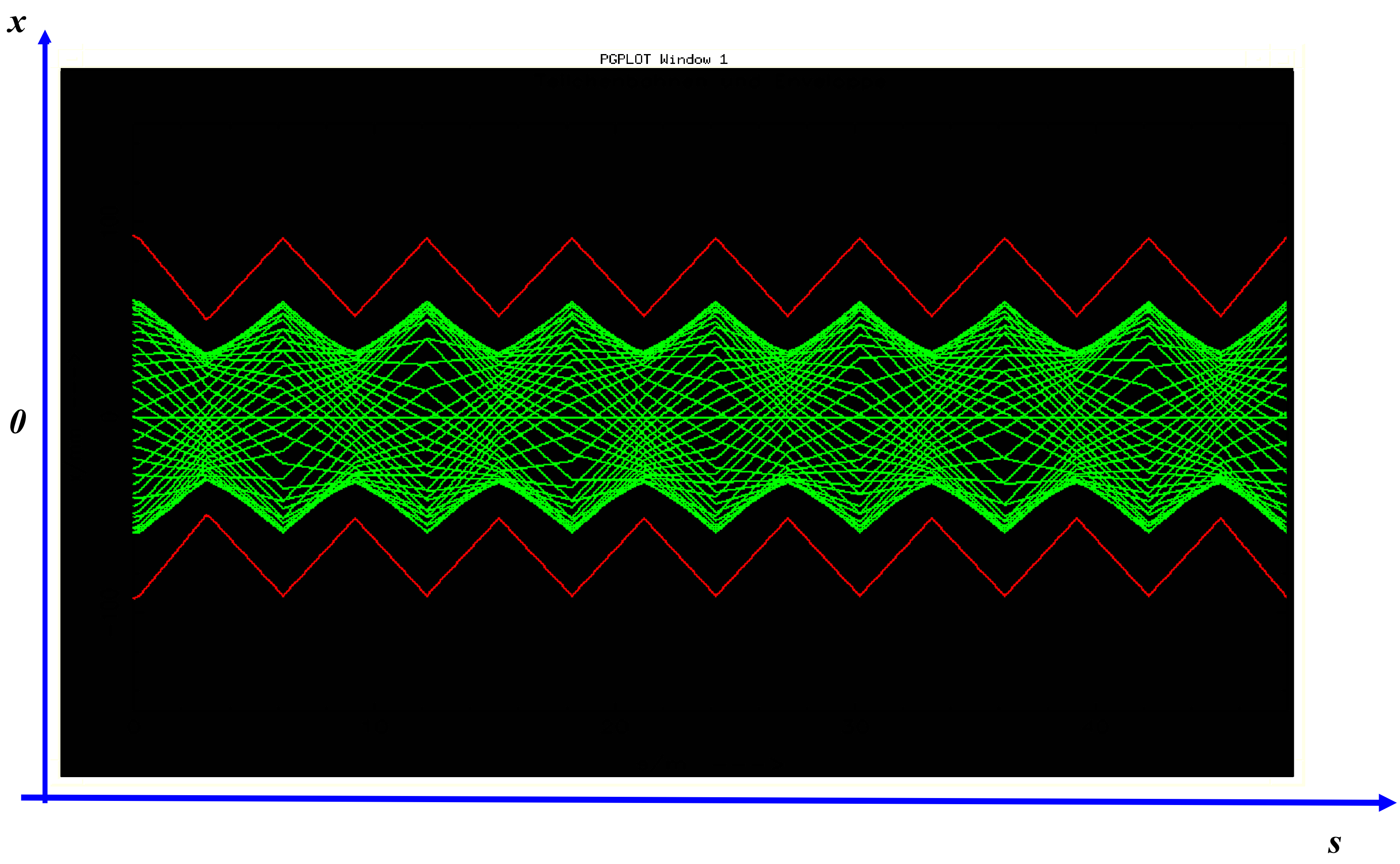
***HERA revolution frequency: 47.3 kHz***

$$0.292 * 47.3\ \ kHz = 13.81\ \ kHz$$



***Question:* *what will happen, if the particle performs a second turn ?***

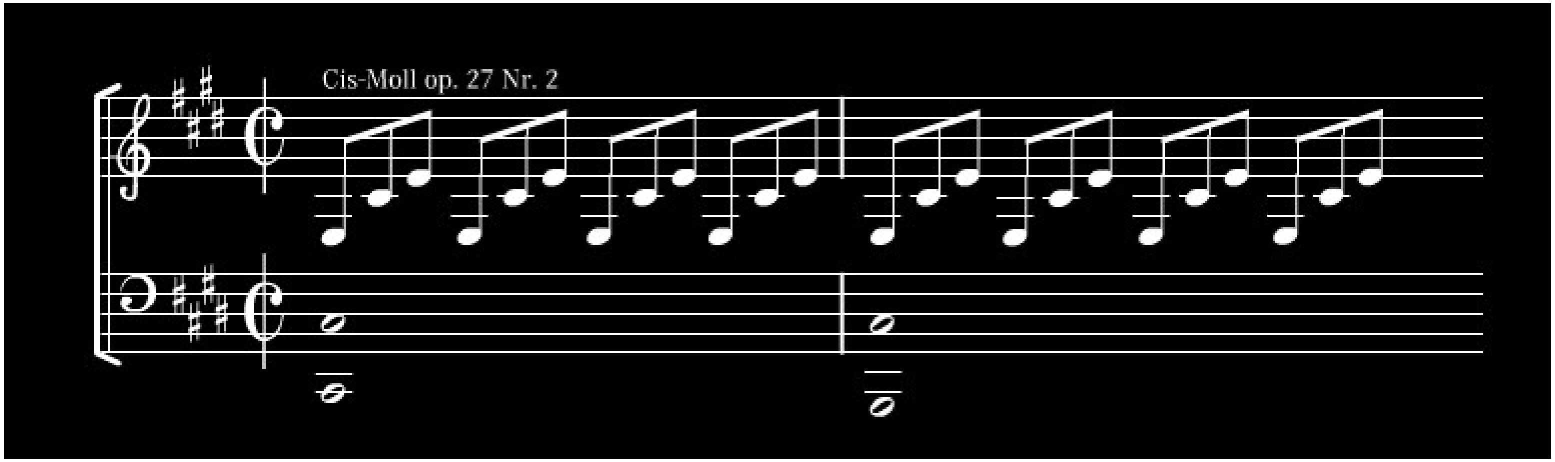
***... or a third one or ... $10^{10}$ turns***



# 19th century:

**Ludwig van Beethoven: „Mondschein Sonate“**

***Sonate Nr. 14 in cis-Moll (op. 27/II,   1801)***



## *Astronomer Hill:*

***differential equation for motions with periodic focusing properties***
***„Hill's equation"***

*Example: particle motion with periodic coefficient*

***equation of motion:*** $x''(s) - k(s)x(s) = 0$

***restoring force $\neq$ const,***
***k(s) = depending on the position s***
***k(s+L) = k(s), periodic function***

***we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.***

# V.) The Beta Function

***General solution of Hill´s equation:***

***(i)*** $$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)} \cdot \cos(\psi(s) + \phi)$$

***ε, Φ = integration constants determined by initial conditions***

***β(s) periodic function given by focusing properties of the lattice ↔ quadrupoles***

$$\beta(s+L) = \beta(s)$$

***Inserting (i) into the equation of motion …***

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

***Ψ(s) = „phase advance" of the oscillation between point „0" and „s" in the lattice.***
***For one complete revolution: number of oscillations per turn „Tune"***

$$Q_y = \frac{1}{2\pi} \cdot \oint \frac{ds}{\beta(s)}$$

# *VI.) Beam Emittance and Phase Space Ellipse*

*general solution of Hill equation*

$$(1) \quad x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \phi)$$

$$(2) \quad x'(s) = -\frac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}} * \{\alpha(s) * \cos(\psi(s) + \phi) + \sin(\psi(s) + \phi)\}$$

*from (1) we get*

$$\cos(\psi(s) + \phi) = \frac{x(s)}{\sqrt{\varepsilon} * \sqrt{\beta(s)}}$$

$$\alpha(s) = \frac{-1}{2} \beta'(s)$$

$$\gamma(s) = \frac{1 + \alpha(s)^2}{\beta(s)}$$

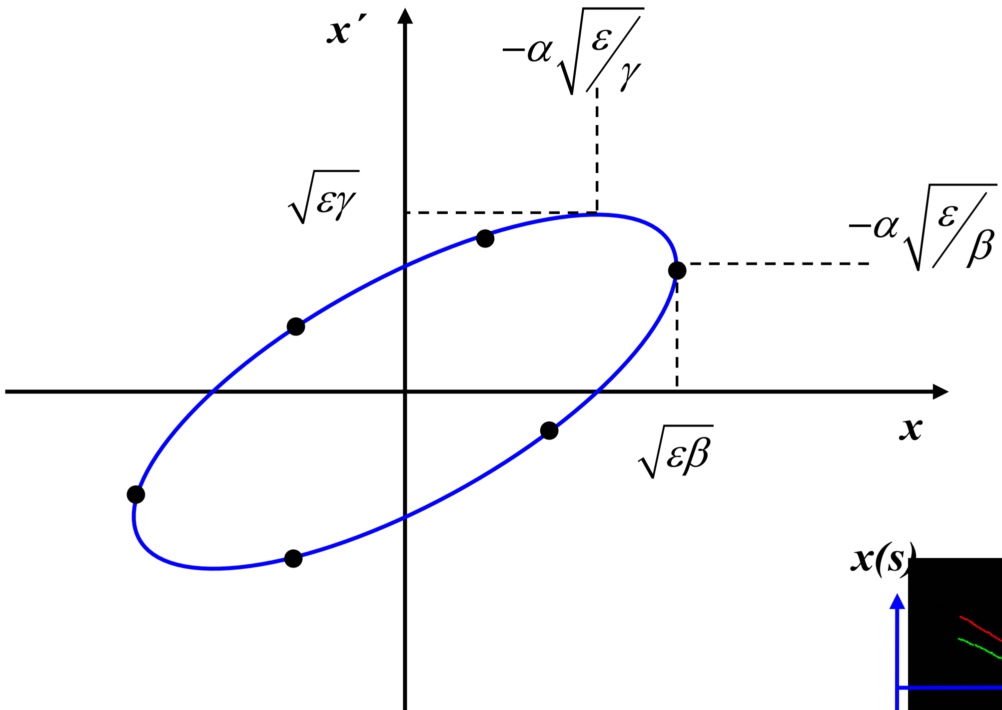
*Insert into (2) and solve for ε*

$$\boxed{\varepsilon = \gamma(s) * x^2(s) + 2\alpha(s) x(s) x'(s) + \beta(s) x'(s)^2}$$

**** ε is a constant of the motion … it is independent of „s“***
**** parametric representation of an ellipse in the x x‘ space***
**** shape and orientation of ellipse are given by α, β, γ***

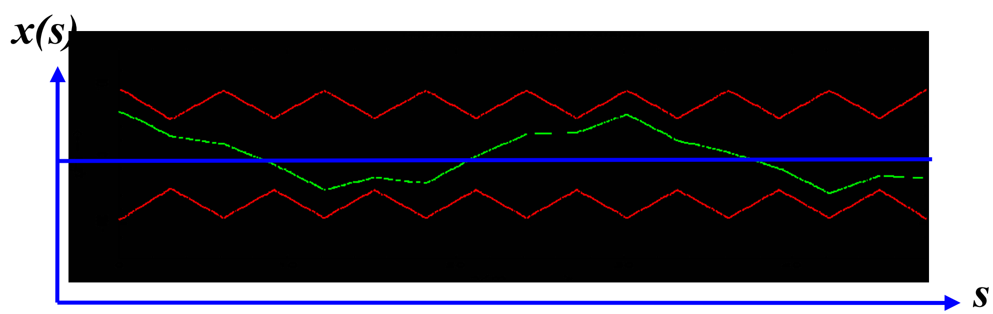
# Beam Emittance and Phase Space Ellipse

$$\varepsilon = \gamma(s) * x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$




***Liouville:* in reasonable storage rings**
***area in phase space is constant.***

***A = π*ε=const***



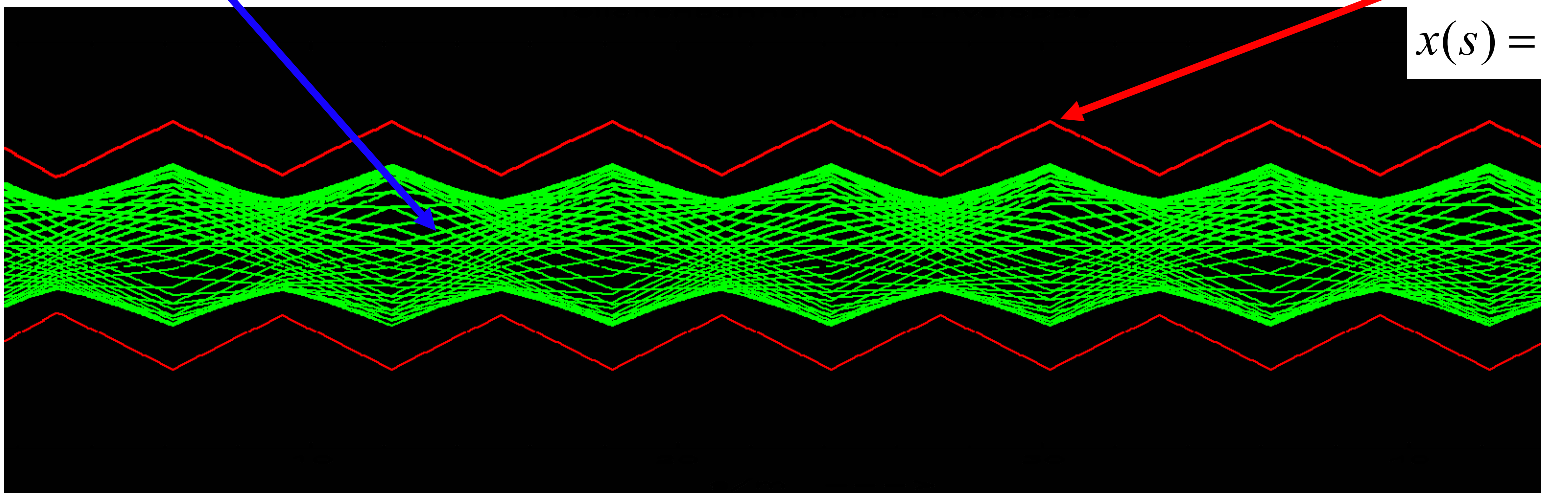

***ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter,***
***cannot be changed by the foc. properties.***
***Scientifiquely spoken: area covered in transverse x, x´ phase space … and it is constant !!!***

# Ensemble of many (...all) possible particle trajectories

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \phi)$$

***max. amplitude of all particle trajectories***

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)}$$



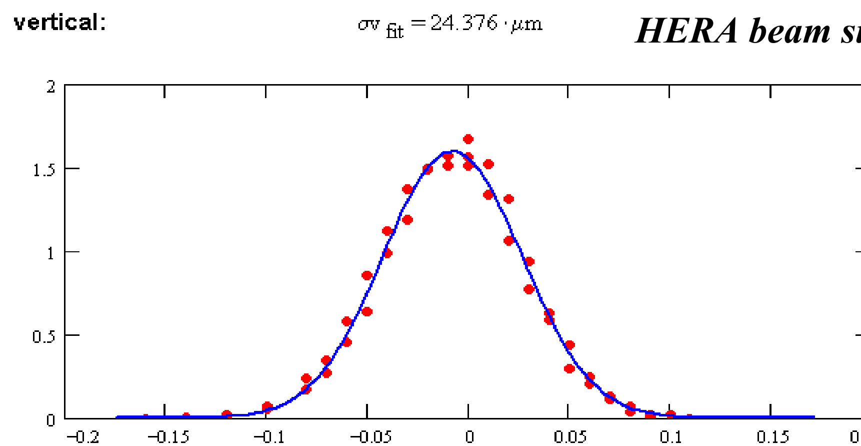
## *Beam Dimension:*

***determined by two parameters***

$$\sigma = \sqrt{\varepsilon * \beta}$$

***Example: transverse beam profile measured using a wirescanı***

***HERA beam size***



# *VII.) Transfer Matrix M* ***… yes we had the topic already***

***general solution of Hill´s equation***

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos\{\psi(s)+\phi\}$$

$$x'(s) = \frac{-\sqrt{\varepsilon}}{\sqrt{\beta(s)}}\left[\alpha(s)\cos\{\psi(s)+\phi\}+\sin\{\psi(s)+\phi\}\right]$$

***remember the trigonometrical gymnastics: sin(a+b)= … etc***

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta_s}\left(\cos\psi_s\cos\phi - \sin\psi_s\sin\phi\right)$$

$$x'(s) = \frac{-\sqrt{\varepsilon}}{\sqrt{\beta_s}}\left[\alpha_s\cos\psi_s\cos\phi - \alpha_s\sin\psi_s\sin\phi + \sin\psi_s\cos\phi + \cos\psi_s\sin\phi\right]$$

***starting at point s(0) = s₀, where we put Ψ(0) = 0***

$$\left.\begin{aligned}\cos\phi &= \frac{x_0}{\sqrt{\varepsilon\beta_0}}\quad ,\\ \sin\phi &= -\frac{1}{\sqrt{\varepsilon}}\left(x_0'\sqrt{\beta_0} + \frac{\alpha_0 x_0}{\sqrt{\beta_0}}\right)\end{aligned}\right\}$$

***inserting above …***

$$x(s)=\sqrt{\frac{\beta_s}{\beta_0}}\left\{\cos\psi_s+\alpha_0\sin\psi_s\right\}x_0+\left\{\sqrt{\beta_s\beta_0}\sin\psi_s\right\}x_0'$$

$$x'(s)=\frac{1}{\sqrt{\beta_s\beta_0}}\left\{(\alpha_0-\alpha_s)\cos\psi_s-(1+\alpha_0\alpha_s)\sin\psi_s\right\}x_0+\sqrt{\frac{\beta_0}{\beta_s}}\left\{\cos\psi_s-\alpha_s\sin\psi_s\right\}x_0'$$

***which can be expressed ... for convenience ... in matrix form*** $\begin{pmatrix} x \\ x' \end{pmatrix}_s = M \begin{pmatrix} x \\ x' \end{pmatrix}_0$

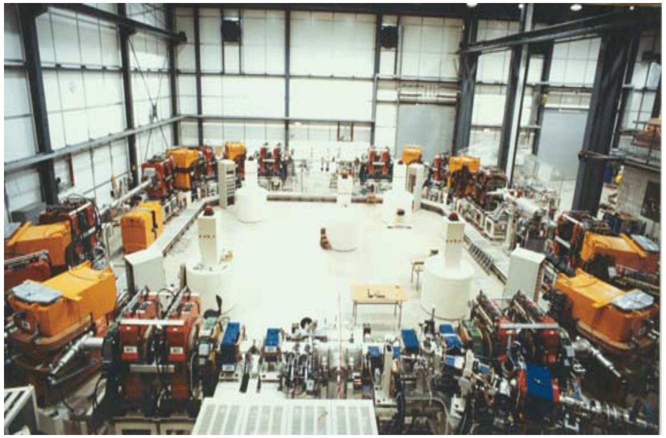
$$M=\begin{pmatrix} \sqrt{\frac{\beta_s}{\beta_0}}\left(\cos\psi_s+\alpha_0\sin\psi_s\right) & \sqrt{\beta_s\beta_0}\sin\psi_s \\ \frac{(\alpha_0-\alpha_s)\cos\psi_s-(1+\alpha_0\alpha_s)\sin\psi_s}{\sqrt{\beta_s\beta_0}} & \sqrt{\frac{\beta_0}{\beta s}}\left(\cos\psi_s-\alpha_s\sin\psi_s\right) \end{pmatrix}$$

* ***we can calculate the single particle trajectories between two locations in the ring, if we know the α β γ at these positions.***

* ***and nothing but the α β γ at these positions.***

* ***... !***

# Stability Criterion:



***Question:*** ***what will happen, if we do not make too many mistakes and your particle performs one complete turn ?***

***Matrix for 1 turn:***

$$M = \begin{pmatrix} \cos\psi_{turn} + \alpha_s \sin\psi_{turn} & \beta_s \sin\psi_{turn} \\ -\gamma_s \sin\psi_{turn} & \cos\psi_{turn} - \alpha_s \sin\psi_{turn} \end{pmatrix} = \cos\psi \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\boldsymbol{1}} + \sin\psi \underbrace{\begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}}_{\boldsymbol{J}}$$

***Matrix for N turns:***

$$M^N = \left(1 \cdot \cos\psi + J \cdot \sin\psi\right)^N = 1 \cdot \cos N\psi + J \cdot \sin N\psi$$

***The motion for N turns remains bounded, if the elements of $M^N$ remain bounded***

$$\psi = real \quad \leftrightarrow \quad \left|\cos\psi\right| < 1 \quad \leftrightarrow \quad \left|Trace(M)\right| < 2$$

# *VIII.) Transformation of α, β, γ*

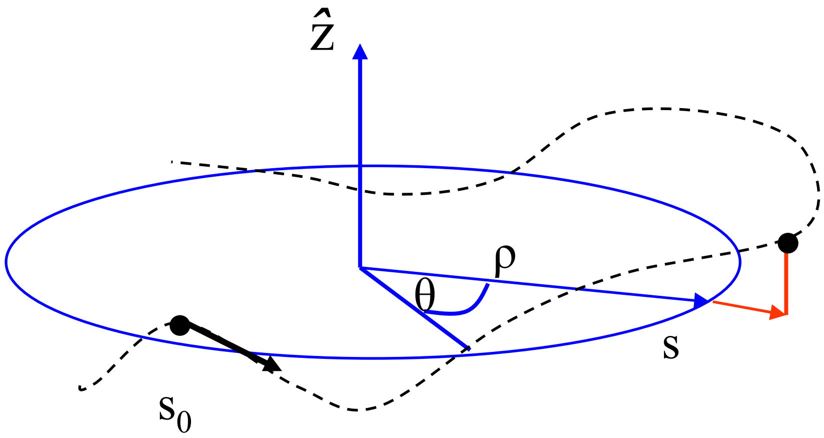
***consider two positions in the storage ring: $s_0$ , s***

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = M \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

***since ε = const:***

$$\varepsilon = \beta x'^2 + 2\alpha x x' + \gamma x^2$$

$$\varepsilon = \beta_0 x_0'^2 + 2\alpha_0 x_0 x_0' + \gamma_0 {x_0}^2$$




***express $x_0$, $x'_0$ as a function of x, x′.***

*... remember W = CS′-SC′ = 1*

$$\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{x'} \end{pmatrix}_0 = \boldsymbol{M}^{-1} * \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{x'} \end{pmatrix}_s$$

$$\boldsymbol{M} = \begin{pmatrix} \boldsymbol{C} & \boldsymbol{S} \\ \boldsymbol{C'} & \boldsymbol{S'} \end{pmatrix} \rightarrow \quad \boldsymbol{M}^{-1} = \begin{pmatrix} \boldsymbol{S'} & -\boldsymbol{S} \\ -\boldsymbol{C'} & \boldsymbol{C} \end{pmatrix}$$

$$\Rightarrow \quad x_0 = S'x - Sx'$$

$$x_0' = -C'x + Cx'$$

***inserting into ε***

$$\varepsilon = \beta x'^2 + 2\alpha x x' + \gamma x^2$$

$$\varepsilon = \beta_0 (Cx' - C'x)^2 + 2\alpha_0 (S'x - Sx')(Cx' - C'x) + \gamma_0 (S'x - Sx')^2$$

***sort via x, x′and compare the coefficients to get ....***

$$\beta(s) = C^2\beta_0 - 2SC\alpha_0 + S^2\gamma_0$$

$$\alpha(s) = -CC'\beta_0 + (SC' + S'C)\alpha_0 - SS'\gamma_0$$

$$\gamma(s) = C'^2\beta_0 - 2S'C'\alpha_0 + S'^2\gamma_0$$

***in matrix notation:***

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_s = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & SC' + CS' & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}$$

**!**

***1.) this expression is important***

***2.) given the twiss parameters α, β, γ at any point in the lattice we can transform them and calculate their values at any other point in the ring.***

***3.) the transfer matrix is given by the focusing properties of the lattice elements, the elements of M are just those that we used to calculate single particle trajectories.***

***4.) go back to point 1.)***

# IX.) Résumé:

***beam rigidity:*** $B \cdot \rho = p/q$

***bending strength of a dipole:*** $$\frac{1}{\rho}\left[m^{-1}\right] = \frac{0.2998 \cdot B_0(T)}{p(GeV/c)}$$

***focusing strength of a quadrupole:*** $$k\left[m^{-2}\right] = \frac{0.2998 \cdot g}{p(GeV/c)}$$

$$k\left[m^{-2}\right] = \frac{0.2998}{p(GeV/c)} \frac{2\mu_0 nI}{a_r^2}$$

***focal length of a quadrupole:*** $$f = \frac{1}{k \cdot l_q}$$

***equation of motion:*** $$x'' + Kx = \frac{1}{\rho}\frac{\Delta p}{p}$$

***matrix of a foc. quadrupole:*** $$x_{s2} = M \cdot x_{s1}$$

$$M = \begin{pmatrix} \cos\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sin\sqrt{|K|}l \\ -\sqrt{|K|}\sin\sqrt{|K|}l & \cos\sqrt{|K|}l \end{pmatrix} , \qquad M = \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}$$

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