RF Cavity Design

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Overview

- DC versus RF
 - Basic equations: Lorentz & Maxwell, RF breakdown
- Some theory: from waveguide to pillbox
 - rectangular waveguide, waveguide dispersion, standing waves ... waveguide resonators, round waveguides, Pillbox cavity
- Accelerating gap
 - Induction cell, ferrite cavity, drift tube linac, transit time factor
- Characterizing a cavity
 - resonance frequency, shunt impedance,
 - beam loading, loss factor, RF to beam efficiency,
 - transverse effects, Panofsky-Wenzel, higher order modes, PS 80 MHz cavity (magnetic coupling)
- More examples of cavities
 - PEP II, LEP cavities, PS 40 MHz cavity (electric coupling),
- RF Power sources
- Many gaps
 - Why?
 - Example: side coupled linac, LIBO
- Travelling wave structures
 - Brillouin diagram, iris loaded structure, waveguide coupling
- Superconducting Accelerators
- RFQ's

DC versus RF

DC accelerator



RF accelerator



Lorentz force

A charged particle moving with velocity \vec{v} through an electro-magnetic field experiences a force

$$\frac{\mathrm{d}\,\vec{p}}{\mathrm{d}\,t} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \qquad \vec{v} = \frac{\vec{p}}{m\gamma}$$

The energy of the particle is $W = \sqrt{(mc^2)^2 + (pc)^2} = \gamma mc^2$ $W_{kin} = mc^2(\gamma - 1)$

Change of W due to the this force (work done); differentiate: $WdW = c^2 \vec{p} \cdot d\vec{p} = qc^2 \vec{p} \cdot (\vec{E} + \vec{v} \times \vec{B}) dt = qc^2 \vec{p} \cdot \vec{E} dt$ $dW = q\vec{v} \cdot \vec{E} dt$

Note: no work is done by the magnetic field.

Maxwell's equations (in vacuum)

$$\nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial}{\partial t} \vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$$
(source-free)
$$\nabla \times \vec{E} + \frac{\partial}{\partial t} \vec{B} = 0 \qquad \nabla \cdot \vec{E} = 0$$

why not DC?

1) DC ($\frac{\partial}{\partial t} \equiv 0$): $\nabla \times \vec{E} = 0$ which is solved by $\vec{E} = -\nabla \Phi$ Limit: If you want to gain 1 MeV, you need a potential of 1 MV!

2) Circular machine: DC acceleration impossible since $\oint \vec{E} \cdot d\vec{s} = 0$

With time-varying fields:

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B} \qquad \oint \vec{E} \cdot d\vec{s} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

Maxwell's equation in vacuum (contd.)

$$\nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial}{\partial t} \vec{E} = 0 \quad \nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} + \frac{\partial}{\partial t} \vec{B} = 0 \quad \nabla \cdot \vec{E} = 0$$

curl of 3rd and $\frac{\partial}{\partial t}$ of 1st equation:

$$\nabla \times \nabla \times \vec{E} + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = 0$$

vector identity:

 $\nabla\times\nabla\times\vec{E}=\nabla\nabla\cdot\vec{E}-\Delta\vec{E}$

with 4th equation:

$$\Delta \vec{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = 0$$

i.e. Laplace in 4 dimensions

Another reason for RF: breakdown limit

in vacuum, Cu surface, room temperature



Some theory: from waveguide to pillbox



2 superimposed plane waves



 $|\vec{E}|$

Waveguides

-y Fundamental (TE_{10} or H_{10}) mode in a standard rectangular waveguide. E.g. forward wave electric field Z**power flow:** $\frac{1}{2} \operatorname{Re} \left\{ \iint_{\substack{\text{cross}\\\text{section}}} \vec{E} \times \vec{H}^* \cdot d\vec{A} \right\}$ power flow X colour coding magnetic field 1.0000e+00 9.0000e-01 8.0000e-01 7.0000e-01 6.0000e-01 5.0000e-01 Z 4.0000e-01 3.0000e-01 2.0000e-01 1.0000e-01 0.0000e+00 X

Waveguide dispersion



Waveguide dispersion (continued)



General waveguide equations:

TE (or H) modes

 Z_0

 $\vec{e} =$

 $H_z = \left(\frac{\omega_c}{c}\right)$

Transverse wave equation (membrane equation):

$$\Delta T + \left(\frac{\omega_c}{c}\right)^2 T = 0$$

TM (or E) modes

boundary condition:

longitudinal wave equations (transmission line equations):

propagation constant:

characteristic impedance:

ortho-normal eigenvectors:

transverse fields:

longitudinal field:

$$\vec{n} \cdot \nabla T = 0 \qquad T = 0$$

$$\frac{\mathrm{d}U(z)}{\mathrm{d}z} + \gamma Z_0 I(z) = 0$$

$$\frac{\mathrm{d}I(z)}{\mathrm{d}z} + \frac{\gamma}{Z_0} U(z) = 0$$

$$\gamma = \mathrm{j}\frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

$$Z_0 = \frac{\mathrm{j}\omega\mu}{\gamma} \qquad Z_0 = \frac{\gamma}{\mathrm{j}\omega\varepsilon}$$

$$\vec{e} \, \vec{u}_z \times \nabla T \qquad \vec{e} = -\nabla T$$
$$\vec{E} = U(z)\vec{e}$$
$$\vec{H} = I(z)\vec{u}_z \times \vec{e}$$
$$E_z = \left(\frac{\omega_c}{c}\right)^2$$

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TI(z)

Rectangular waveguide : transverse eigenfunctions

$$TE (H) \text{ modes:} \qquad T_{mn}^{(H)} = \frac{1}{\pi} \sqrt{\frac{ab\varepsilon_m \varepsilon_n}{(mb)^2 + (na)^2}} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right)$$

$$TM (E) \text{ modes:} \qquad T_{mn}^{(E)} = \frac{2}{\pi} \sqrt{\frac{ab}{(mb)^2 + (na)^2}} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right)$$

$$\frac{\omega_c}{c} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \xrightarrow{a}$$

$$Round \text{ waveguide : transverse eigenfunctions}$$

$$TE (H) \text{ modes:} \qquad T_{mn}^{(H)} = \sqrt{\frac{\varepsilon_m}{\pi (\chi_{mn}^{\prime \prime 2} - m^2)}} \frac{J_m \left(\chi_{mn}^{\prime \prime m}\frac{\rho}{a}\right)}{J_m (\chi_{mn}^{\prime \prime m})} \left\{ \begin{array}{c} \cos(m\varphi) \\ \sin(m\varphi) \\ \sin(m\varphi) \end{array} \right\}$$

$$TM (E) \text{ modes:} \qquad T_{mn}^{(E)} = \sqrt{\frac{\varepsilon_m}{\pi}} \frac{J_m \left(\chi_{mn}\frac{\rho}{a}\right)}{\chi_{mn} J_{m-1}(\chi_{mn})} \left\{ \begin{array}{c} \sin(m\varphi) \\ \cos(m\varphi) \\ \cos(m\varphi) \\ \end{array} \right\}$$

$$where \qquad \varepsilon_i = \begin{cases} 1 & for \quad i = 0 \\ 2 & for \quad i \neq 0 \end{cases}$$

Standing wave - resonator





114.74

a/mm

 f_c

GHz

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 \vec{E}

 \vec{B}

87.85

 f_c _

 $GHz \quad a/mm$

 $\frac{f_c}{\text{GHz}} = \frac{334.74}{a/\text{mm}}$

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electric field

magnetic field

Pillbox cavity field (w/o beam tube)

$$T(\rho, \varphi) = \sqrt{\frac{1}{\pi}} \frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{\chi_{01} J_1\left(\frac{\chi_{01}}{a}\right)}$$

The only non-vanishing field components :

$$E_{z} = \frac{1}{j\omega\varepsilon_{0}} \frac{\chi_{01}}{a} \sqrt{\frac{1}{\pi}} \frac{J_{0}\left(\frac{\chi_{01}\rho}{a}\right)}{aJ_{1}\left(\frac{\chi_{01}}{a}\right)}$$
$$B_{\varphi} = \mu_{0} \sqrt{\frac{1}{\pi}} \frac{J_{1}\left(\frac{\chi_{01}\rho}{a}\right)}{aJ_{1}\left(\frac{\chi_{01}}{a}\right)}$$





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 $\chi_{01} = 2.40483...$

dielectric guide - transversely damped wave



Accelerating gap

Accelerating gap



- We want a voltage across the gap!
- It cannot be DC, since we want the beam tube on
- $\oint \vec{E} \cdot d\vec{s} = -\iint \frac{d\vec{B}}{dt} \cdot d\vec{A}$
- - a lower limit to the usable frequency.
 - The limit can be extended with a material which
 - Materials typically used:
 - ferrites (depending on *f*-range)
 - magnetic alloys (MA) like Metglas®, Finemet®,
 - resonantly driven with RF (ferrite loaded cavities) – or with pulses (induction cell)

Linear induction accelerator



Ferrite cavity



Gap of PS cavity (prototype)



Drift Tube Linac (DTL) - how it works

For slow particles protons @ few MeV e.g. - the drift tube lengths can easily be adapted.





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electric field



Drift tube linac - practical implementations





Transit time factor

If the gap is small, the voltage $\int E_z dz$ is small.

If the gap large, the RF field varies notably while the particle passes.



Characterizing a cavity

Cavity resonator - equivalent circuit





Resonance



Reentrant cavity

Nose cones increase transit time factor, round outer shape minimizes losses.

Nose cone example Freq = 500.003







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Summary: relations V_{gap}, W, P_{loss} gap voltage $\frac{R}{Q} = \frac{\left|V_{gap}\right|^{2}}{2\omega_{0}W}$ $k_{loss} = \frac{\omega_{0}}{2Q} \frac{R}{Q} = \frac{\left|V_{gap}\right|^{2}}{4W}$ $R_{shunt} = \frac{\left|V_{gap}\right|^2}{2 P_t}$ Energy stored inside Power lost in the cavity the cavity walls $Q = \frac{\omega_0 W}{P_{loss}}$

Beam loading - RF to beam efficiency

- The beam current "loads" the generator, in the equivalent circuit this appears as a resistance in parallel to the shunt impedance.
- If the generator is matched to the unloaded cavity, beam loading will cause the accelerating voltage to decrease.
- The power absorbed by the beam is $-\frac{1}{2} \operatorname{Re} \{ V_{gap} \ I_B^* \}$, the power loss $P = \frac{|V_{gap}|^2}{2R}$.
- For high efficiency, beam loading shall be high.
- The RF to beam efficiency is $\eta = \frac{1}{1 + \frac{V_{gap}}{R |I|}} = \frac{|I_B|}{|I_G|}$.

Characterizing cavities

- Resonance frequency
- Transit time factor field varies while particle is traversing the gap

$$\omega_{0} = \frac{1}{\sqrt{L \cdot C}}$$
$$\left| \int E_{z} e^{j \frac{\omega}{c} z} dz \right|$$
$$\left| \int E_{z} dz \right|$$

Shunt impedance gap voltage - power relation

- $\cdot \quad Q$ factor
 - *R/Q* independent of losses - only geometry!

loss factor

$$\left|V_{gap}\right|^2 = 2 R_{shunt} P_{loss}$$

Linac definition

$$\left|V_{gap}\right|^2 = R_{shunt} P_{loss}$$

 V_{gap}

 $\omega_0 W$

 $\omega_0 R$

R

k_{loss}

$$\omega_0 W = Q P_{loss}$$

$$\frac{R}{Q} = \frac{\left|V_{gap}\right|^2}{2\,\omega_0 W} = \sqrt{\frac{L}{C}}$$

$$\sigma_{ss} = \frac{\omega_0}{2} \frac{R}{Q} = \frac{\left|V_{gap}\right|^2}{4W}$$

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k

35

4W

gap

Example Pillbox:
Higher order modes



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. . .

Higher order modes (measured spectrum)



Pillbox: Dipole mode (TM₁₁₀)



electric field (@ 0°)

magnetic field (@ 90°)

Panofsky-Wenzel theorem

For particles moving virtually at v=c, the integrated transverse force (kick) can be determined from the transverse variation of the integrated longitudinal force!

$$j\frac{\omega}{c}\vec{F}_{\perp} = \nabla_{\perp}F_{\parallel}$$

Pure TE modes: No net transverse force !

Transverse modes are characterized by

- the transverse impedance in ω -domain
- the transverse loss factor (kick factor) in t-domain !

W.K.H. Panofsky, W.A. Wenzel: "Some Considerations Concerning the Transverse Deflection of Charged Particles in Radio-Frequency Fields", RSI 27, 1957]

CERN/PS 80 MHz cavity (for LHC)





Higher order modes

Example shown: 80 MHz cavity PS for LHC. Color-coded:













255.6 MHz, m=0



292 MHz, m=2







357.9 MHz. m=3

439.2 MHz, m=1

376.8 MHz, m=2

462.2 MHz, m=2

387.8 MHz, m=1

418.5 MHz, m=4

422.9 MHz, m=3

337.5 MHz, m=1

437.6 MHz, m=0

344.5 MHz, m=0









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More examples of cavities

PS 19 MHz cavity (prototype, photo: 1966)



Examples of cavities



PEP II cavity 476 MHz, single cell, 1 MV gap with 150 kW, strong HOM damping,



LEP normal-conducting Cu RF cavities, 350 MHz. 5 cell standing wave + spherical cavity for energy storage, 3 MV



CERN/PS 40 MHz cavity (for LHC)



RF power sources

RF Power sources

> 200 MHz: Klystrons



Thales TH1801, Multi-Beam Klystron (MBK), 1.3 GHz, 117 kV. Achieved: 48 dB gain, 10 MW peak, 150 kW average, η = 65 %

dB: $\frac{output \ power}{input \ power} = 10^{4.8}$

< 1000 MHz: grid tubes



pictures from http://www.thales-electrondevices.com

RF power sources



Example of a tetrode amplifier (80 MHz, CERN/PS)



400 kW, with fast RF feedback

18 Ω coaxial output (towards cavity)

22 kV DC anode voltage feed-through with $\lambda/4$ stub

tetrode cooling water feed-throughs



Many gaps

What do you gain with many gaps?

• The R/Q of a single gap cavity is limited to some 100 Ω . Now consider to distribute the available power to nidentical cavities: each will receive P/n, thus produce an accelerating voltage of $\sqrt{2RP/n}$. The total accelerating voltage thus increased, equivalent to a total equivalent shunt impedance of nR.



Standing wave multicell cavity

- Instead of distributing the power from the amplifier, one might as well couple the cavities, such that the power automatically distributes, or have a cavity with many gaps (e.g. drift tube linac).
- Coupled cavity accelerating structure (side coupled)



The phase relation between gaps is important!

An example of Side Coupled Structure : LIBO (= Linac Booster)



A 3 GHz Side Coupled Structure to accelerate protons out of cyclotrons from 62 MeV to 200 MeV

Medical application: treatment of tumours.

Prototype of Module 1 built at CERN (2000)

Collaboration CERN/INFN/ Tera Foundation

LIBO prototype



This Picture made it to the title page of CERN Courier vol. 41 No. 1 (Jan./Feb. 2001)

Travelling wave structures



Iris loaded waveguide



Disc loaded structure with strong HOM damping "choke mode cavity"



Waveguide coupling ¹/₄ geometry shown



3 GHz Accelerating structure (CTF3)



Superconducting Linacs

LEP was not a linac, but still ...

LEP Superconducting cavities

SUPERCONDUCTING CAVITY WITH ITS CRYOSTAT



LHC SC RF, 4 cavity module, 400 MHz



Small β superconducting cavities (example RIA, Argonne)

115 MHz split-ring cavity,

172.5 MHz β = 0.19 "lollipop" cavity



pictures from Shepard et al.: "Superconducting accelerating structures for a multi-beam driver linac for RIA", Linac 2000, Monterey

More superconducting cavities for linacs with $\beta < 1$ (proton driver - heavy ion)

Need to standardise construction of cavities: only few different types of cavities are made for some β 's more cavities are grouped in cryostats





ILC high gradient SC Linac at 1.3 GHz

Technology has made big progress, > 40 MV/m accelerating gradient have been obtained. The plot below illustrates the effect of "buffered chemical polishing" and "electropolishing".



ILC 9-cell Niobium cavity



More on the ILC at http://www.linearcollider.org/cms/



Old preinjector 750 kV DC , CERN Linac 2 before 1990



All this was replaced by the RFQ ...

RFQ of CERN Linac 2



The Radio Frequency Quadrupole (RFQ)

Minimum Energy of a DTL: 500 keV (low duty) - 5 MeV (high duty) At low energy / high current we need strong focalisation Magnetic focusing (proportional to β) is inefficient at low energy. Solution (Kapchinski, 70's, first realised at LANL):

Electric quadrupole focusing + bunching + acceleration





RFQ electrode modulation

The electrode modulation creates a longitudinal field component that creates the "bunches" and accelerates the beam.




A look inside CERN AD's "RFQD"



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