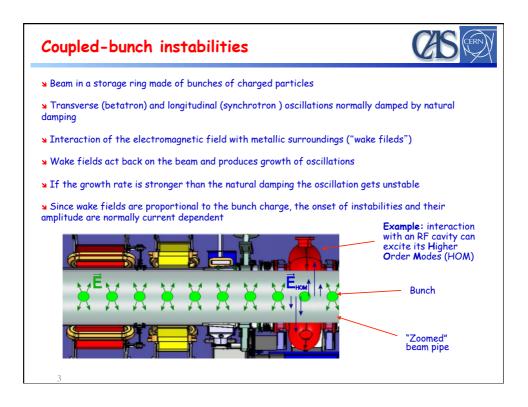
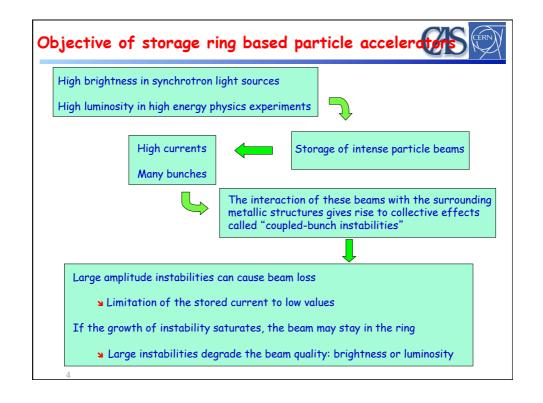


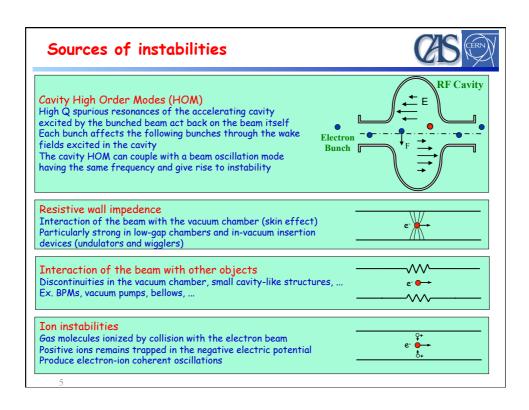
Outline

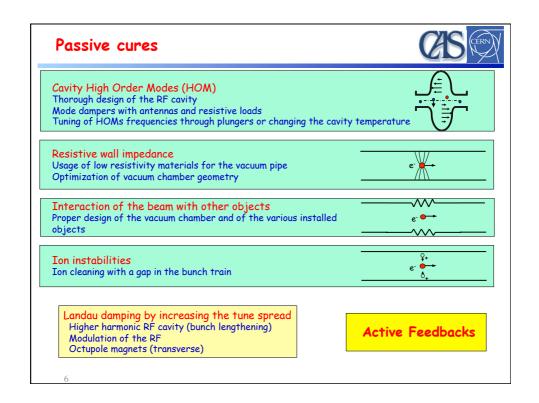


- Coupled-bunch instabilities
- Basics of feedback systems
- Feedback system components
- Digital signal processing
- Integrated diagnostic tools
- Conclusions





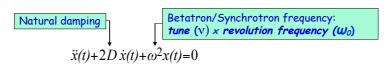




Equation of motion of one particle: harmonic oscillator an



"x" is the oscillation coordinate (transverse or longitudinal displacement)



$$x(t) = e^{-\frac{t}{\tau_D}} \sin(\omega t + \varphi)$$

where $\tau_{\rm D}$ = 1/D is the "damping time constant" (D is called "damping rate")

Excited oscillations (ex. by quantum excitation) are damped by natural damping (ex. due to synchrotron radiation damping). The oscillation of individual particles is uncorrelated and shows up as an emittance growth

Coherent Bunch Oscillations



Coupling with other bunches through the interaction with surrounding metallic structures addd a "driving force" term F(t) to the equation of motion:

$$\ddot{x}(t) + 2D \dot{x}(t) + \omega^2 x(t) = F(t)$$

Under given conditions the oscillation of individual particles becomes correlated and the centroid of the bunch oscillates giving rise to coherent bunch (coupled bunch) oscillations

Each bunch oscillates according to the equation of motion:

$$\ddot{x}(t) + 2(D - G)\dot{x}(t) + \omega^2 x(t) = 0$$

where $\tau_G = 1/G$ is the "growth time constant" (G is called "growth rate")

If $D \ge G$ the oscillation amplitude decays exponentially



If $D \le G$ the oscillation amplitude grows exponentially

as:
$$x(t) = e^{-\frac{t}{\tau}} \sin(\omega t + \varphi)$$
 where $\frac{1}{\tau} = \frac{1}{\tau_D} - \frac{1}{\tau_D}$

Since ${\it G}$ is proportional to the beam current, if the latter is lower than a given current threshold the beam remains stable, if higher a coupled bunch instability is excited

Feedback Damping Action

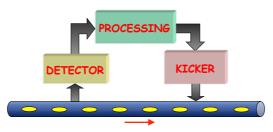


The feedback action adds a damping term D_{th} to the equation of motion

$$\ddot{x}(t) + 2(D - G + D_{th}) \dot{x}(t) + \omega^2 x(t) = 0$$

Such that D-G+ D_{fb} > 0

A multi-bunch feedback detects an instability by means of one or more Beam Position Monitors (BPM) and acts back on the beam by applying electromagnetic 'kicks' to the bunches



In order to introduce damping, the feedback must provide a kick proportional to the derivative of the bunch oscillation

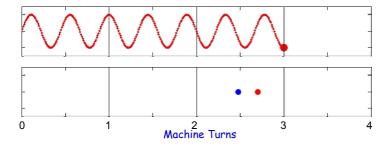
Since the oscillation is sinusoidal, the kick signal for each bunch can be generated by shifting by $\pi/2$ the oscillation signal of the same bunch when it passes through the kicker

0

Multi-bunch modes



Typically, betatron tune frequencies (horizontal and vertical) are higher than the revolution frequency, while the synchrotron tune frequency (longitudinal) is lower than the revolution frequency



Ex.
Vertical
Tune = 2.25

Longitudinal
Tune = 0.5

Although each bunch oscillates at the tune frequency, there can be different modes of oscillation, called multi-bunch modes depending on how each bunch oscillates with respect to the other bunches

Multi-bunch modes



Let us consider ${\it M}$ bunches equally spaced around the ring

Each multi-bunch mode is characterized by a bunch-to-bunch phase difference of:

$$\Delta \Phi = m \frac{2\pi}{M}$$
 $m = \text{multi-bunch mode number } (0, 1, ..., M-1)$

Each multi-bunch mode is associated to a characteristic set of frequencies:

$$\omega = pM \omega_0 \pm (m+v)\omega_0$$

Where:

p is and integer number $-\infty$

 ω_0 is the revolution frequency

 $M\omega_0$ = ω_{rf} is the RF frequency (bunch repetition frequency)

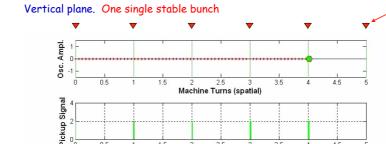
v is the tune

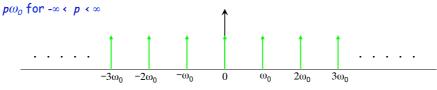
Two sidebands at $\pm (m+v)\omega_0$ for each multiple of the RF frequency

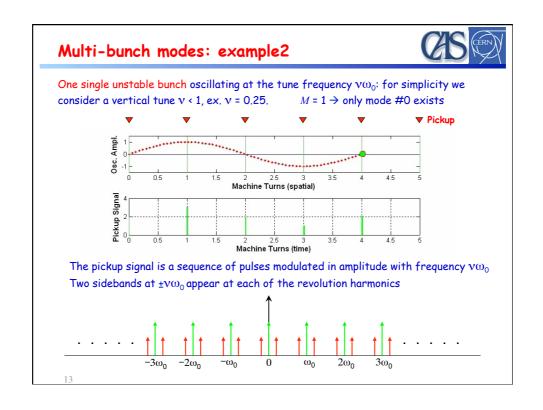
4.4

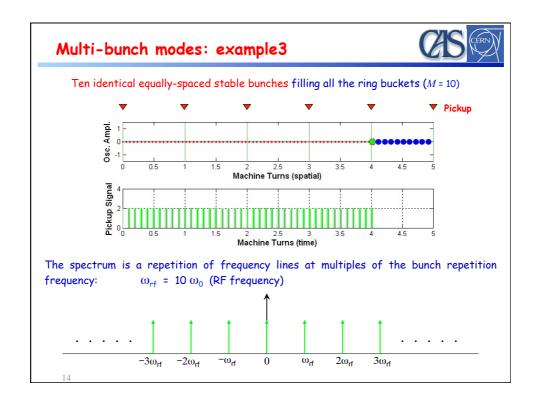
Multi-bunch modes: example1









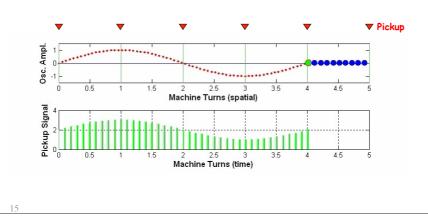


Multi-bunch modes: example4



Ten identical equally-spaced unstable bunches oscillating at the tune frequency $\nu\omega_0$ (ν = 0.25) $M = 10 \rightarrow$ there are 10 possible modes of oscillation m = 0, 1, ..., M-1

Ex.: mode #0 (m = 0) $\Delta\Phi$ =0 all bunches oscillate with the same phase

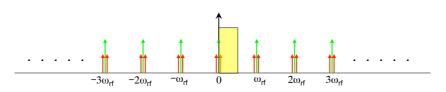


Multi-bunch modes: example4

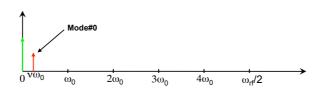


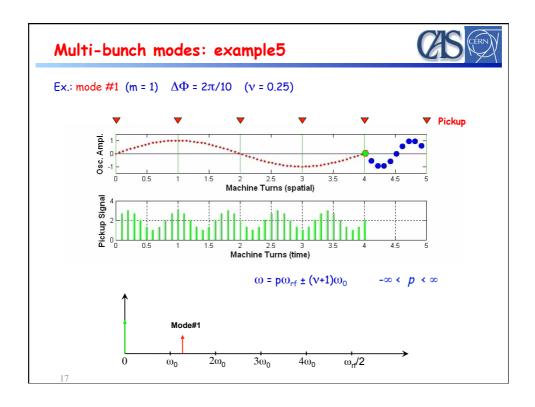


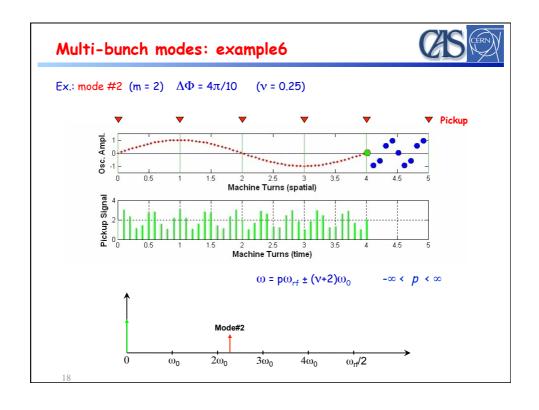
The spectrum is a repetition of frequency lines at multiples of the bunch repetition frequency with sidebands at $\pm v\omega_0$: $\omega = p\omega_{rf} \pm v\omega_0$ $-\infty (<math>v = 0.25$)

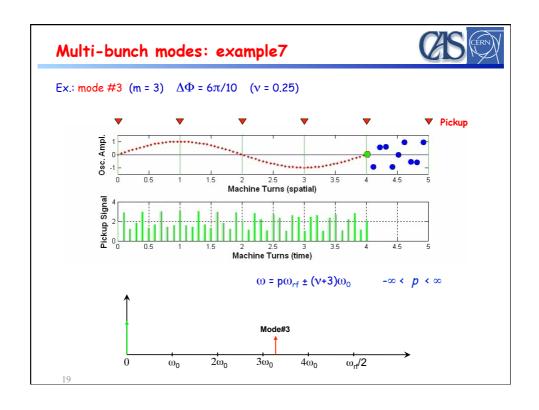


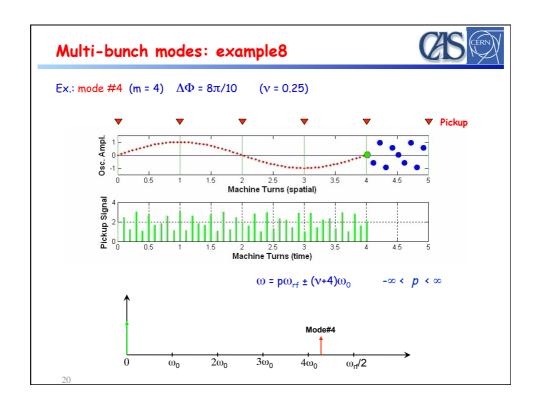
Since the spectrum is periodic and each mode appears twice (upper and lower side band) in a ω_{rf} frequency span, we can limit the spectrum analysis to a 0- $\!\omega_{rf}$ /2 frequency range

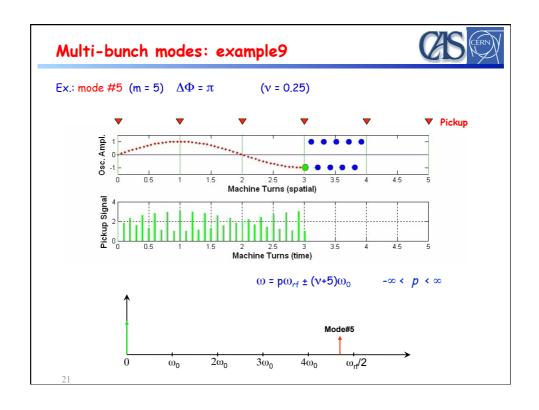


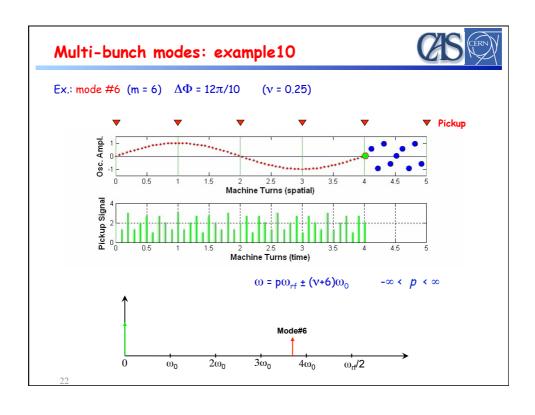


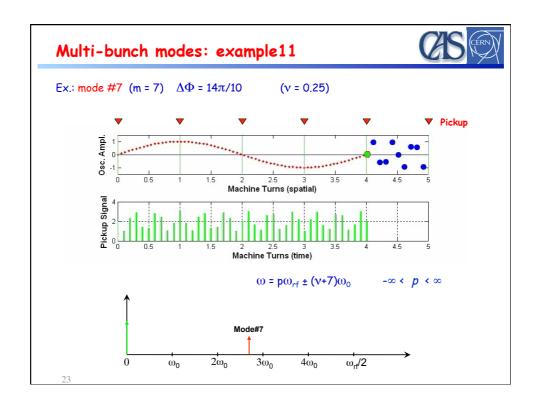


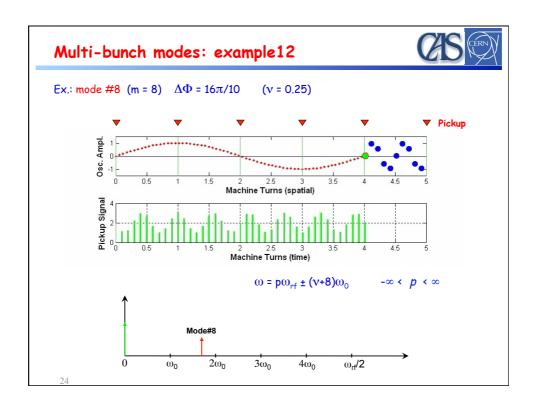


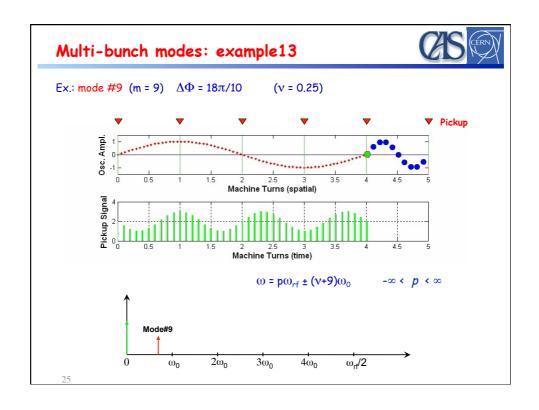


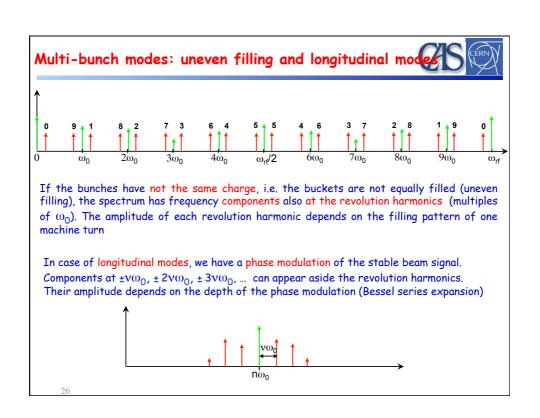












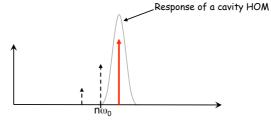
Multi-bunch modes: coupled-bunch instability



One multi-bunch mode can become unstable if one of its sidebands overlaps, for example, with the frequency response of a cavity high order mode (HOM). The HOM couples with the sideband giving rise to a coupled-bunch instability, with consequent increase of the sideband amplitude



Synchrotron Radiation Monitor showing the transverse beam shape



Effects of coupled-bunch instabilities:

- increase of the transverse beam dimensions
- increase of the effective emittance
- beam loss and max current limitation
- increase of lifetime due to decreased Touschek scattering (dilution of particles)

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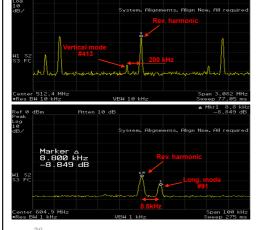
Real example of multi-bunch modes



ELETTRA Synchrotron: f_{rf} =499.654 Mhz, bunch spacing \approx 2ns, 432 bunches, f_0 = 1.15 MHz

 ν_{hor} = 12.30(fractional tune frequency=345kHz), ν_{vert} =8.17(fractional tune frequency=200kHz)

 v_{long} = 0.0076 (8.8 kHz)



 $\omega = p M \omega_0 \pm (m+v) \omega_0$

Spectral line at 512.185 MHz

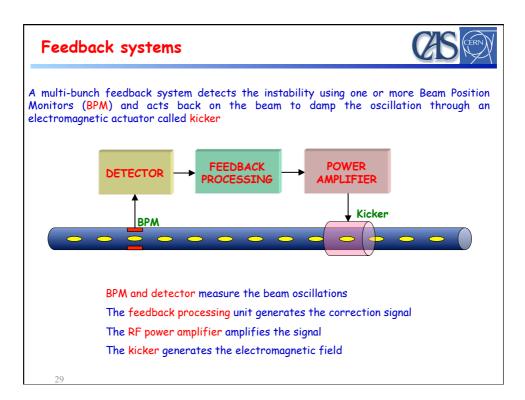
Lower sideband of $2f_{\rm rf}$, 200 kHz apart from the $443^{\rm rd}$ revolution harmonic

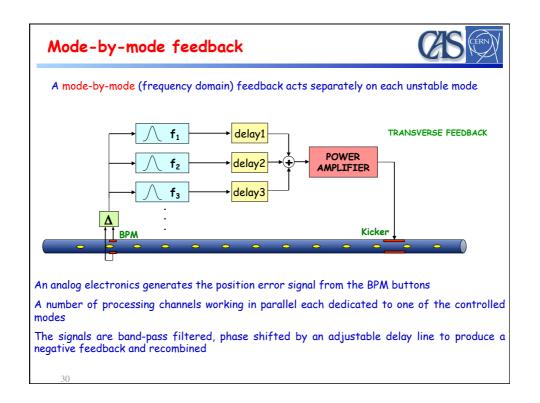
→ vertical mode #413

Spectral line at 604.914 MHz

Upper sideband of $f_{\rm rf},\,8.8 kHz$ apart from the $523^{\rm rd}$ revolution harmonic

→ longitudinal mode #91



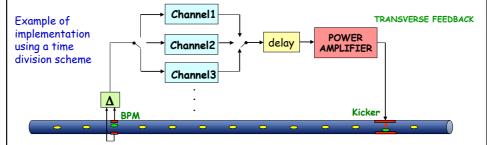


Bunch-by-bunch feedback



A bunch-by-bunch (time domain) feedback individually steers each bunch by applying small electromagnetic kicks every time the bunch passes through the kicker: the result is a damped oscillation lasting several turns

The correction signal for a given bunch is generated based on the motion of the same bunch



Every bunch is measured and corrected at every machine turn but, due to the delay of the feedback chain, the correction kick corresponding to a given measurement is applied to the bunch one or more turns later

Damping the oscillation of each bunch is equivalent to damping all multi-bunch modes

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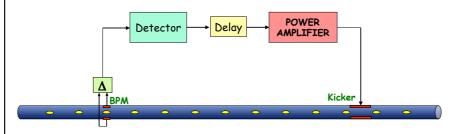
Analog bunch-by-bunch feedback: one-BPM feedback



Transverse feedback

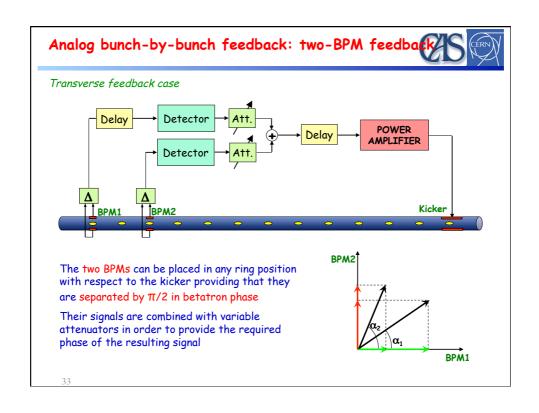
The correction signal applied to a given bunch must be proportional to the derivative of the bunch oscillation at the kicker, thus it must be a sampled sinusoid shifted $\pi/2$ with respect to the oscillation of the bunch when it passes through the kicker

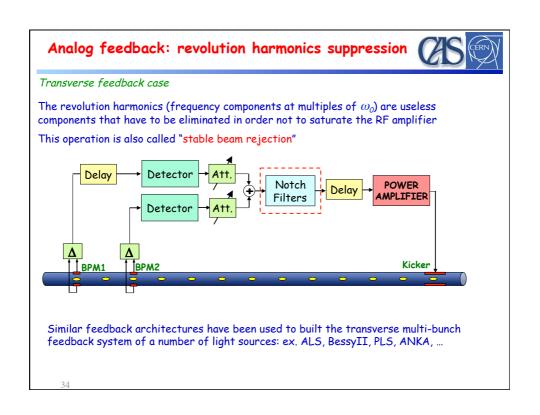
The signal from a BPM with the appropriate betatron phase advance with respect to the kicker can be used to generate the correction signal



The detector down converts the high frequency (typically a multiple of the bunch frequency $f_{\rm rf}$) BPM signal into base-band (range 0 - $f_{\rm rf}/2$)

The delay line assures that the signal of a given bunch passing through the feedback chain arrives at the kicker when, after one machine turn, the same bunch passes through it





Digital bunch-by-bunch feedback Transverse and longitudinal case Digital RF POWER Stable Bed Signal Detector Modulator Delay AMPLIFIER Processing Combiner Kicker The combiner generates the X, Y or Σ signal from the BPM button signals The detector (RF front-end) demodulates the position signal to base-band "Stable beam components" are suppressed by the stable beam rejection module The resulting signal is digitized, processed and re-converted to analog by the digital processor The modulator translates the correction signal to the kicker working frequency (long. only) The delay line adjusts the timing of the signal to match the bunch arrival time The RF power amplifier supplies the power to the kicker

Digital vs. analog feedbacks



ADVANTAGES OF DIGITAL FEEDBACKS ...



- reproducibility: when the signal is digitized it is not subject to temperature/ environment changes or aging
- » programmability: the implementation of processing functionalities is usually made using DSPs or FPGAs, which are programmable via software/firmware
- performance: digital controllers feature superior processing capabilities with the possibility to implement sophisticated control algorithms not feasible in analog
- additional features: possibility to combine basic control algorithms and additional useful features like signal conditioning, saturation control, down sampling, etc.
- w implementation of diagnostic tools, used for both feedback commissioning and machine physics studies
- w easier and more efficient integration of the feedback in the accelerator control system for data acquisition, feedback setup and tuning, automated operations, etc.

DISADVANTAGE OF DIGITAL FEEDBACKS



▶ High delay due to ADC, digital processing and DAC

BPM and Combiner

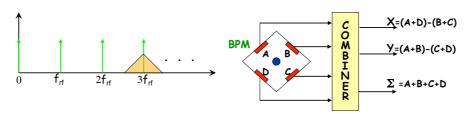


The four signals from a standard four-button BPM can be opportunely combined to obtain the wide-band X, Y and Σ signals used respectively by the horizontal, vertical and longitudinal feedbacks

Any $f_{\rm rf}/2$ portion of the beam spectrum contains the information of all potential multi-bunch modes and can be used to detect instabilities and measure their amplitude

Usually BPM and combiner work around a multiple of $f_{\rm rf},$ where the amplitude of the overall frequency response of BPM and cables is maximum

Moreover, a higher $f_{\rm rf}$ harmonic is preferred for the longitudinal feedback because of the better sensitivity of the phase detection system



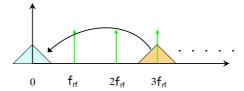
The SUM (Σ) signal contains only information of the phase (longitudinal position) of the bunches, since the sum of the four button signals has almost constant amplitude

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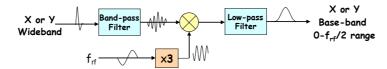
Detector: transverse feedback

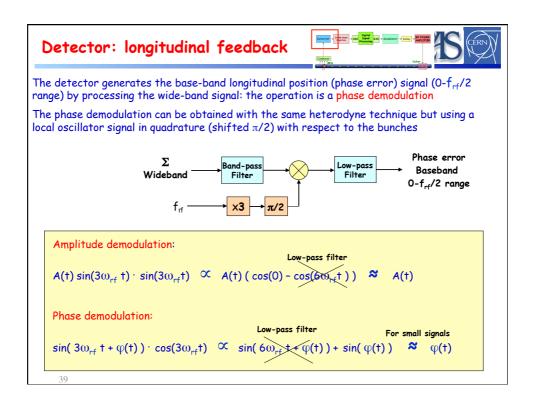


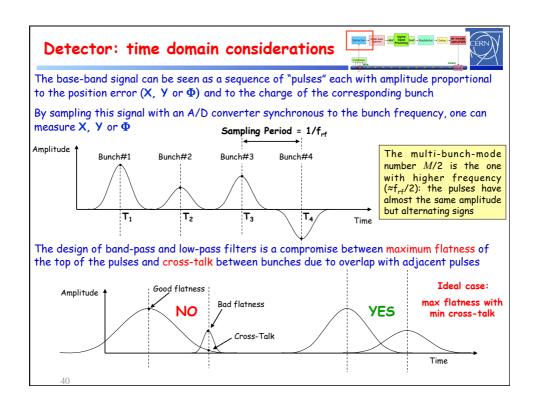
The detector (or RF front-end) translates the wide-band signal to base-band (0- $f_{\rm rf}/2$ range): the operation is an amplitude demodulation



Heterodyne technique: the "local oscillator" signal is derived from the RF by multiplying its frequency by an integer number corresponding to the chosen harmonic of $f_{\rm rf}$







Rejection of stable beam signal The turn-by-turn pulses of each bunch can have a constant offset (stable beam signal) due to: u transverse case: off-centre beam or unbalanced BPM electrodes or cables ulongitudinal case: beam loading, i.e. different synchronous phase for each bunch In the frequency domain, the stable beam signal carries non-zero revolution harmonics These components have to be suppressed because don't contain information about multi-bunch modes and can saturate ADC, DAC and amplifier Examples of used techniques:

Balancing of BPM buttons: variable attenuators on the electrodes to equalize the amplitude of the To the detector signals (transverse feedback) Comb filter using delay lines and combiners: the detector frequency response is a series of notches at multiple of $\omega_{\text{O}},\,\text{DC}$ included From the Digital DC rejection: the signal is sampled at $f_{\rm rf}$, the turn-by-turn signal is integrated for each bunch, recombined with the other bunches, converted to detector Delay DAC FPGA ADC analog and subtracted from the original signal

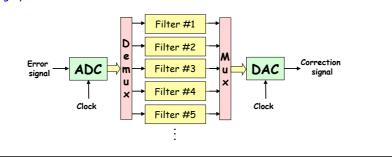
Digital processor The A/D converter samples and digitizes the signal at the bunch repetition frequency: each sample corresponds to the position (X, Y or Φ) of a given bunch. Precise synchronization of

the sampling clock with the bunch signal must be provided The digital samples are then de-multiplexed into M channels (M is the number of bunches):

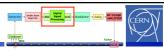
in each channel the turn-by-turn samples of a given bunch are processed by a dedicated digital filter to calculate the correction samples The basic processing consists in DC component suppression (if not completely done by the

After processing, the correction sample streams are recombined and eventually converted to analog by the D/A converter

external stable beam rejection) and phase shift at the betatron/synchrotron frequency



Digital processor implementation



ADC: existing multi-bunch feedback systems usually employ 8-bit ADCs at up to 500 Msample/s; some implementations use a number of ADCs with higher resolution (ex. 14 bits) and lower rate working in parallel. ADCs with enhanced resolution have some advantages:

- ≥ lower quantization noise (crucial for low-emittance machines)
- higher dynamic range (external stable beam rejection not necessary)

DAC: usually employed DACs convert samples at up to 500 Msample/s and 14-bit resolution

Digital Processing: the feedback processing can be performed by discrete digital electronics (obsolete technology), DSPs or FPGAs

	Pros	Cons
DSP	Easy programmingFlexible	 Difficult HW integration Latency Sequential program execution A number of DSPs are necessar
FP <i>GA</i>	 Fast (only one FPGA is necessary) Parallel processing Low latency 	> Trickier programming > Less flexible

Examples of digital processors





- ≥ PETRA transverse and longitudinal feedbacks: one ADC, a digital processing electronics made of discrete components (adders, multipliers, shift registers, ...) implementing a FIR filter, and a DAC
- ALS/PEP-II/DAΦNE longitudinal feedback (also adopted at SPEAR, Bessy II and PLS): A/D and D/A conversions performed by VXI boards, feedback processing made by DSP boards hosted in a number of VME crates
- ▶ PEP-II transverse feedback: the digital part, made of two ADCs, a FPGA and a DAC, features a digital delay and integrated diagnostics tools, while the rest of the signal processing is made analogically
- 🛚 KEKB transverse and longitudinal feedbacks: the digital 🕅 processing unit, made of discrete digital electronics and banks of memories, performs a two tap FIR filter featuring stable beam rejection, phase shift and delay
- Elettra/SLS transverse and longitudinal feedbacks: the digital processing unit is made of a VME crate equipped with one ADC, one DAC and six commercial DSP boards (Elettra only) with four microprocessors each







Examples of digital processors



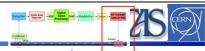
- w CESR transverse and longitudinal feedbacks: they employ VME digital processing boards equipped with ADC, DAC, FIFOs and PLDs
- w HERA-p longitudinal feedback: it is made of a processing chain with two ADCs (for I and Q components), a FPGA and two DACs
- Spring-8 transverse feedback (also adopted at TLS, KEK Photon Factory and Soleil): fast analog de-multiplexer that distributes analog samples to a number of slower ADC FPGA channels. The correction samples are converted to analog by one DAC
- ESRF transverse/longitudinal and Diamond transverse feedbacks: commercial product 'Libera Bunch by Bunch' (by Instrumentation Technologies), which features four ADCs sampling the same analog signal opportunely delayed, one FPGA and one DAC
- ${f u}$ HLS tranverse feedback: the digital processor consists of two ADCs, one FPGA and two DACs
- DAONE transverse and KEK-Photon-Factory longitudinal feedbacks: commercial product called 'iGp' (by Dimtel), featuring an ADC-FPGA-DAC chain





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Amplifier and kicker



The kicker is the feedback actuator. It generates a transverse/longitudinal electromagnetic field that steers the bunches with small kicks as they pass through the kicker. The overall effect is damping of the betatron/synchrotron oscillations

The amplifier must provide the necessary RF power to the kicker by amplifying the signal from the DAC (or from the modulator in the case of longitudinal feedbacks)

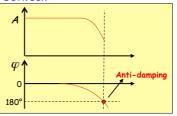
A bandwidth of at least $f_{\rm rf}/2$ is necessary: from ~DC (all kicks of the same sign) to $\sim f_{\rm rf}/2$ (kicks of alternating signs)

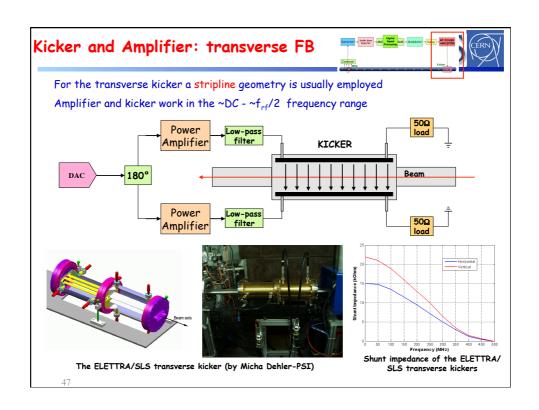
The bandwidth of amplifier-kicker must be sufficient to correct each bunch with the appropriate kick without affecting the neighbour bunches. The amplifier-kicker design has to maximize the kick strength while minimizing the cross-talk between corrections given to adjacent bunches

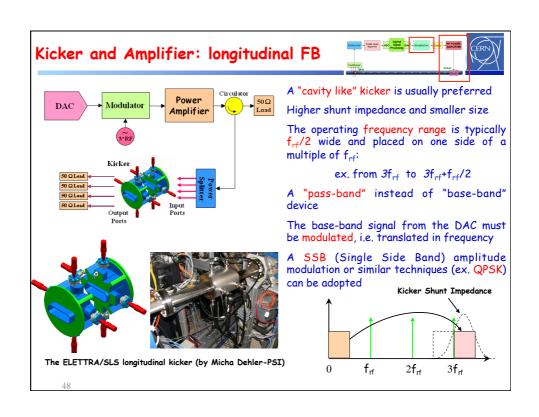
Important issue: the group delay of the amplifier must be as constant as possible, i.e. the phase response must be linear, otherwise the feedback efficiency is reduced for some modes and the feedback can even become positive

Shunt impedance, ratio between the squared voltage seen by the bunch and twice the power at the kicker input:

$$R = \frac{V^2}{2P_{IN}}$$



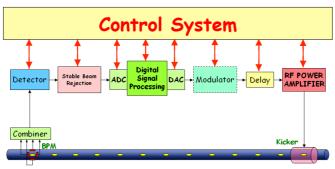




Control system integration



- 🛚 It is desirable that each component of the feedback system that needs to be configured and adjusted has a control system interface
- Any operation must be possible from remote to facilitate the system commissioning and the optimization of its performance
- An effective data acquisition channel has to provide fast transfer of large amounts of data for analysis of the feedback performance and beam dynamics studies
- It is preferable to have a direct connection to a numerical computing environment and/or a script language (ex. Matlab, Octave, Scilab, Python, IGOR Pro, IDL, ...) for quick development of measurement procedures using scripts as well as for data analysis and visualization



RF power requirements: transverse feedback



The transverse motion of a bunch of particles not subject to damping or excitation can be described as a pseudo-harmonic oscillation with amplitude proportional to the square root of the β -function

$$x(s) = a\sqrt{\beta(s)}\cos\varphi(s)$$
, where $\varphi(s) = \int_{0}^{s} \frac{d\overline{s}}{\beta(s)}$

The derivative of the position, i.e. the angle of the trajectory is:

$$x' = -\frac{a}{\sqrt{\beta}}\sin\varphi + \frac{a\beta'}{2\sqrt{\beta}}\cos\varphi$$
, with $\varphi' = \frac{1}{\beta}$

By introducing $\alpha = -\frac{\beta'}{2}$

$$\alpha = -\frac{\beta'}{2}$$

we can write:
$$x' = \frac{a}{\sqrt{\beta}} \sqrt{1 + \alpha^2} \sin(\varphi + \arctan \alpha)$$

At the coordinate $s_{\rm k}$, the electromagnetic field of the kicker deflects the particle bunch which varies its angle by k: as a consequence the bunch starts another oscillation

 $x_1 = a_1 \sqrt{\beta} \cos \varphi_1$

which must satisfy the following constraints:

$$\begin{cases} x(s_k) = x_1(s_k) \\ x'(s_k) = x_1'(s_k) + k \end{cases}$$

By introducing

$$A = a\sqrt{\beta}$$
, $A_1 = a_1\sqrt{\beta}$

 $A=a\sqrt{eta}\,,\;A_{\scriptscriptstyle \parallel}=a_{\scriptscriptstyle \parallel}\sqrt{eta}$ the two-equation two-unknown-variables system becomes:

$$\begin{cases} A\cos\varphi = A_{i}\cos\varphi_{i} \\ A\frac{\sqrt{1+\alpha^{2}}}{\beta}\sin(\varphi + arctg(\alpha)) = A_{i}\frac{\sqrt{1+\alpha^{2}}}{\beta}\sin(\varphi_{i} + arctg(\alpha)) + k \end{cases}$$

The solution of the system gives amplitude and phase of the new oscillation:

$$\begin{cases} A_{1} = \sqrt{(A\sin\varphi - k\beta)^{2} + A^{2}\cos^{2}\varphi} \\ \varphi_{1} = \arccos(\frac{A}{A_{1}}\cos\varphi) \end{cases}$$

RF power requirements: transverse feedback



From $A_1 = \sqrt{\left(A\sin\varphi - k\beta\right)^2 + A^2\cos^2\varphi}$ if the kick is small $\left(k < \frac{A}{\beta}\right)$ then $\frac{\Delta A}{A} = \frac{A - A_1}{A} \cong \frac{\beta}{A} k \sin\varphi$

In the linear feedback case, i.e. when the turn-by-turn kick signal is a sampled sinusoid proportional to the bunch oscillation amplitude, in order to maximize the damping rate the kick signal must be in-phase with $\sin \varphi$, that is in quadrature with the bunch oscillation $k = g \frac{A}{\beta} \sin \varphi$ with 0 < g < 1

The optimal gain g_{opt} is determined by the maximum kick value k_{max} that the kicker is able to generate. The feedback gain must be set so that k_{max} is generated when the oscillation amplitude A at the kicker location is

set so that n_{\max} is generated when the oscillation amplitude A is $g_{opt} = \frac{k_{\max}}{A_{\max}} \beta$ Therefore $k = \frac{k_{\max}}{A_{\max}} A \sin \varphi$ $\frac{\Delta A}{A} \cong \frac{k_{\max}}{A} \beta \sin^2 \varphi$ The relative amplitude decrease is monotonic and its average is: For small kicks

The average relative decrease is therefore constant, which means that, in average, the amplitude decrease is exponential with time constant $\boldsymbol{\tau}$ (damping time) given by:

 $\frac{1}{\tau} = \left\langle \frac{\Delta A}{A} \right\rangle \frac{1}{T_0} = \frac{\beta k_{\text{max}}}{2 A_{\text{max}} T_0}$ where T_0 is the revolution period.

By referring to the oscillation at the BPM location: $\frac{1}{\tau} = \frac{k_{\text{max}}}{2 T_{\text{n}} A_{\text{num}}} \sqrt{\beta_{\text{x}} \beta_{\text{s}}} \qquad A_{\text{Bmax}} \text{ is the max oscillation amplitude at the BPM}$

$$\frac{1}{\tau} = \frac{k_{\text{max}}}{2 T_{0} A_{\text{Bmax}}} \sqrt{\beta_{\text{K}} \beta_{\text{B}}}$$

RF power requirements: transverse feedback



For relativistic particles, the change of the transverse momentum \boldsymbol{p} of the bunch passing through the kicker can be expressed by:

$$\Delta p = \frac{e}{c} V_{\perp} \qquad \text{where} \qquad V_{\perp} = \int_{a}^{L} (\overline{E} + c \times \overline{B})_{\perp} dz \qquad \text{is the kick voltage and} \qquad p = \frac{E_{\scriptscriptstyle B}}{c}$$

e = electron charge, c = light speed, \overline{E} , \overline{B} = fields in the kicker, L = length of the kicker, E_B = beam energy

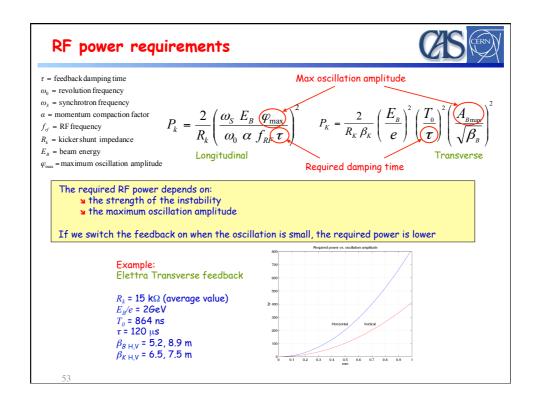
 V_{\perp} can be derived from the definition of kicker shunt impedance: $R_{k} = \frac{V_{\perp}^{2}}{2P}$

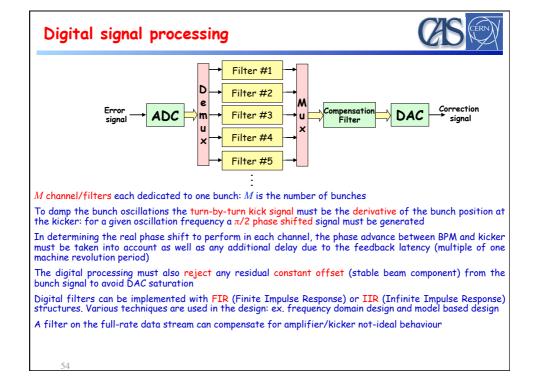
The max deflection angle in the kicker is given by:

$$k_{\text{max}} = \frac{\Delta p}{p} = e \frac{V_{\perp}}{E_{\scriptscriptstyle B}} = \left(\frac{e}{E_{\scriptscriptstyle B}}\right) \sqrt{2P_{\scriptscriptstyle K}R_{\scriptscriptstyle K}}$$

From the previous equations we can obtain the power required to damp the bunch oscillation with time constant τ :

$$P_{K} = \frac{2}{R_{K} \beta_{K}} \left(\frac{E_{B}}{e}\right)^{2} \left(\frac{T_{0}}{\tau}\right)^{2} \left(\frac{A_{B \max}}{\sqrt{\beta_{B}}}\right)^{2}$$



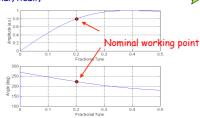


Digital filter design: 3-tap FIR filter



The minimum requirements are:

- DC rejection (coefficients sum = 0)
- Given amplitude response at the tune frequency
- Given phase response at the tune
- frequency
 A 3-tap FIR filter can fulfil these requirements: the filter coefficients can be calculated analytically



Example:

- Tune ω /2 π = 0.2
- Amplitude response at tune $|H(\omega)| = 0.8$
- Phase response at tune α = 222°

$$H(z) = -0.63 + 0.49 z^{-1} + 0.14 z^{-2}$$

Z transform of the FIR filter response

In order to have zero amplitude at DC, we must put a "zero" in z=1. Another zero in z=c is added to fulfill the phase requirements.

c can be calculated analytically:

$$H(z) = k(1 - z^{-1})(1 - cz^{-1})$$

 $H(z) = k(1 - (1 + c)z^{-1} + cz^{-2})$ $z = e^{j\omega}$

$$H(z) = k(1 - (1+c)z^{-1} + cz^{-2}) \qquad z = e^{j\omega}$$

$$H(\omega) = k(1 - (1+c)e^{-j\omega} + ce^{-2j\omega})$$

$$e^{-j\omega} = \cos \omega - j \sin \omega$$
, $\alpha = ang(H(\omega))$

$$tg(\alpha) = \frac{c(\sin(\omega) - \sin(2\omega)) + \sin(\omega)}{c(\cos(2\omega) - \cos(\omega)) + 1 - \cos(\omega)}$$

$$c = \frac{tg(\alpha)(1 - \cos(\omega)) - \sin(\omega)}{(\sin(\omega) - \sin(2\omega)) - tg(\alpha)(\cos(2\omega) - \cos(\omega))}$$

k is determined given the required amplitude response at tune $|H(\omega)|\colon$

$$k = \frac{|H(\omega)|}{\sqrt{(1-(1+c)\cos(\omega)+c\cos(2\omega))^2 + ((1+c)\sin(\omega)-c\sin(2\omega))^2}}$$

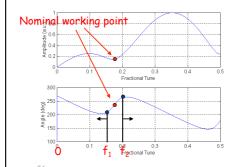
Digital filter design: 5-tap FIR filter



With more degrees of freedom additional features can be added to a FIR filter

Ex.: transverse feedback. The tune frequency of the accelerator can significantly change during machine operations. The filter response must guarantee the same feedback efficiency in a given frequency range by performing automatic compensation of phase changes.

In this example the feedback delay is four machine turns. When the tune frequency increases, the phase of the filter must increase as well, i.e. the phase response must have a positive slope around the working point.



The filter design can be made using the Matlab function invfreqz()

This function calculates the filter coefficients that best fit the required frequency response using the least squares method

The desired response is specified by defining amplitude and phase at three different frequencies: 0, f1 and f2

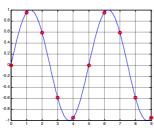
Digital filter design: selective FIR filter



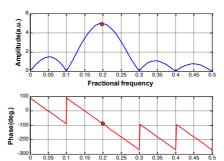
A filter often employed in longitudinal feedback systems is a selective FIR filter which impulse response (the filter coefficients) is a sampled sinusoid with frequency equal to the synchrotron tune

The filter amplitude response has a maximum at the tune frequency and linear phase

The more filter coefficients we use the more selective is the filter



Samples of the filter impulse response (= filter coefficients)



Amplitude and phase response of the filter

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Advanced filter design



More sophisticated techniques using longer FIR or IIR filters enable a variety of additional features exploiting the potentiality of digital signal processing:

- $\ensuremath{\mathbf{u}}$ enlarge the working frequency range with no degradation of the amplitude response
- \mathbf{y} enhance filter selectivity to better reject unwanted frequency components (noise)
- minimize the amplitude response at frequencies that must not be fed back
- stabilize different tune frequencies simultaneously by designing a filter with two separate working points (for example when horizontal and vertical as well as dipole and quadrupole instabilities have to be addressed by the same feedback system)
- improve the robustness of the feedback under parametric changes of accelerator or feedback components (ex. optimal control, robust control, etc.)

Down sampling (longitudinal feedback)



The synchrotron frequency is usually much lower than the revolution frequency: one complete synchrotron oscillation is accomplished in many machine turns

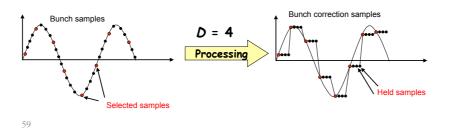
In order to be able to properly filter the bunch signal down sampling is usually carried out

One out of D samples is used: D is the dawn sampling factor

The processing is performed over the down sampled digital signal and the filter design is done in the down sampled frequency domain (the original one enlarged by \mathcal{D})

The turn-by-turn correction signal is reconstructed by a **hold buffer** that keeps each calculated correction value for D turns

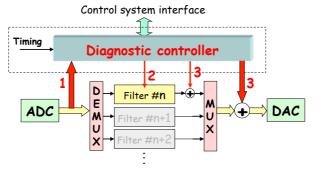
The reduced data rate allows for more time available to perform filter calculations and more complex filters can therefore be implemented



Integrated diagnostic tools



- A feedback system can implement a number of diagnostic tools useful for commissioning and optimization of the feedback system as well as for machine physics studies:
- ADC data recording: acquisition and recording, in parallel with the feedback operation, of a large number of samples for off-line data analysis
- Modification of filter parameters on the fly with the required timing and even individually for each bunch: switching ON/OFF the feedback, generation of grow/damp transients, optimization of feedback performance, ...
- 3. Injection of externally generated digital samples: for the excitation of single/multi bunches

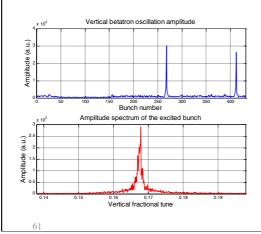


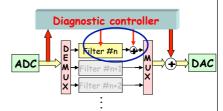
Diagnostic tools: excitation of individual bunches



The feedback loop is switched off for one or more selected bunches and the excitation is injected in place of the correction signal. Excitations can be:

- white (or pink) noise
- sinusoids





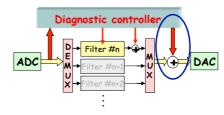
In this example two bunches are vertically excited with pink noise in a range of frequencies centered around the tune, while the feedback is applied on the other bunches. The spectrum of one excited bunch reveals a peak at the tune frequency

This technique is used to measure the betatron tune with almost no deterioration of the beam quality

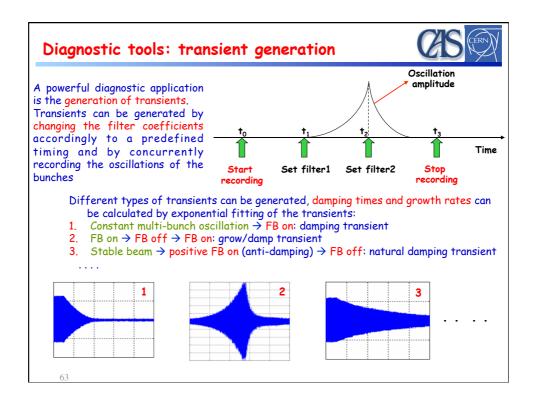
Diagnostic tools: multi-bunch excitation

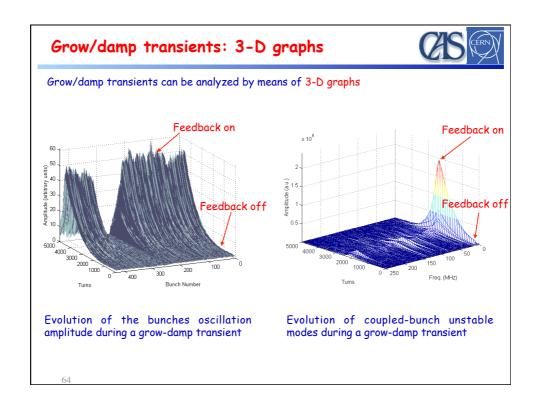


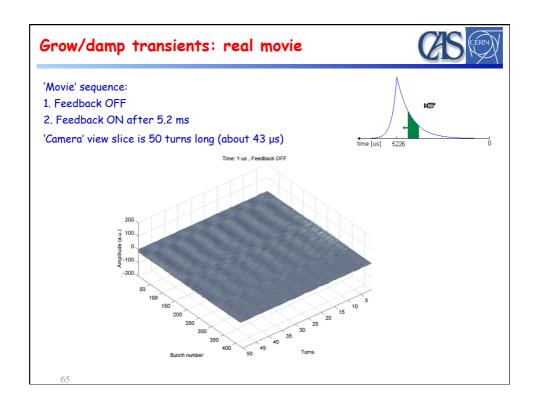
Interesting measurements can be performed by adding pre-defined signals in the output of the digital processor



- By injecting a sinusoid at a given frequency, the corresponding beam multi-bunch mode can be excited to test the performance of the feedback in damping that mode
- 2. By injecting an appropriate signal and recording the ADC data with filter coefficients set to zero, the beam transfer function can be calculated
- 3. By injecting an appropriate signal and recording the ADC data with filter coefficients set to the nominal values, the closed loop transfer function can be determined





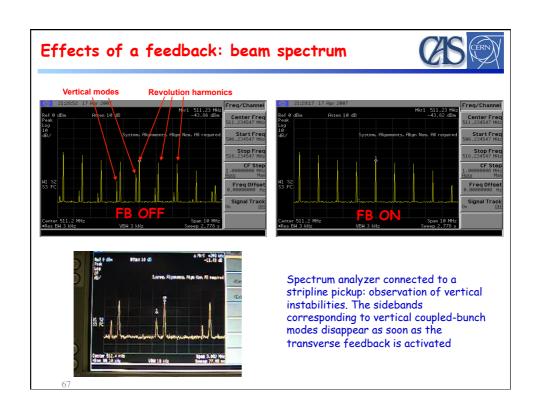


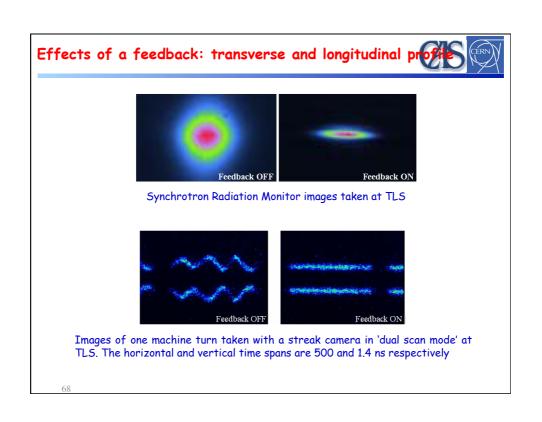
Applications using diagnostic tools



Diagnostic tools are helpful to tune feedback systems as well as to study coupled-bunch modes and beam dynamics. Here are some examples of measurements and analysis:

- ▶ Feedback damping times: can be used to characterize and optimize feedback performance
- » Resistive and reactive response: a feedback not perfectly tuned has a reactive behavior (induces a tune shift when switched on) that has to be minimized
- w Modal analysis: coupled-bunch mode complex eigenvalues, i.e. growth rates (real part) and oscillation frequency (imaginary part)
- \mathbf{y} Accelerator impedance: analysis of complex eigenvalues and bunch synchronous phases can be used to evaluate the machine impedance
- Stable modes: coupled-bunch modes below the instability threshold can be studied to predict their behavior at higher currents
- **Bunch train studies:** analysis of different bunches in the train give information on the sources of coupled-bunch instabilities
- ▶ Phase space analysis: phase evolution of unstable coupled-bunch modes for beam dynamics studies

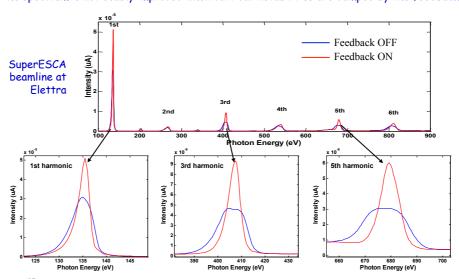




Effects of a feedback: photon beam spectra



Effects on the synchrotron light: spectrum of photons produced by an undulator The spectrum is noticeably improved when vertical instabilities are damped by the feedback



Conclusions



- y Feedback systems are indispensable tools to cure multi-bunch instabilities in storage rings
- Technology advances in digital electronics allow implementing digital feedback systems using programmable devices
- Digital signal processing theory widely used to design and implement filters as well as to analyze data acquired by the feedback
- $\ensuremath{\mathbf{v}}$ Feedback systems not only for closed loop control but also as powerful diagnostic tools for:
 - v optimization of feedback performance
 - beam dynamics studies
- Many potentialities of digital feedback systems still to be discovered and exploited

References and acknowledges



- ▶ Marco Lonza (Elletra) for leaving his splendid slides
- ▶Herman Winick, "Synchrotron Radiation Sources", World Scientific
- Many papers about coupled-bunch instabilities and multi-bunch feedback systems (PETRA, KEK, SPring-8, DaΦne, ALS, PEP-II, SPEAR, ESRF, Elettra, SLS, CESR, HERA, HLS, DESY, PLS, BessyII, SRRC, ...)
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