Beam instabilities (I)

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in CERN Accelerator School, Intermediate Level, Chios
Monday 26.09.2011
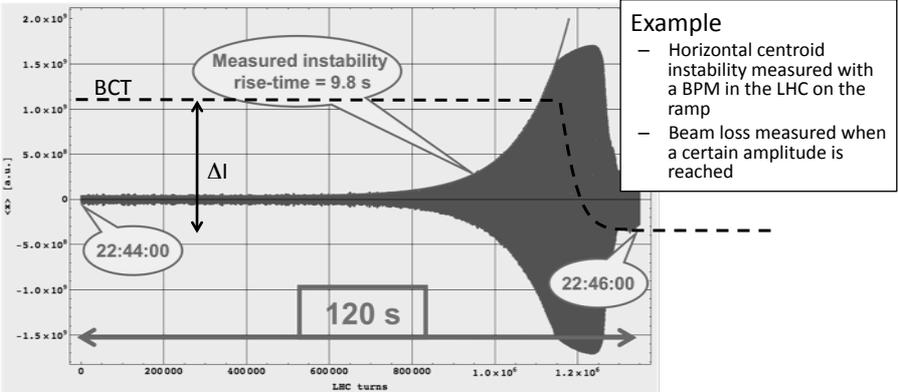Acknowledgments: H. Bartosik, H. Damerau, G. Iadarola,
K. Li, E. Métral, N. Mounet, G. Papotti, B. Salvant,
C. Zannini

CAS

Background information

- What is a beam instability?
  - A beam becomes unstable when a moment of its distribution exhibits an exponential growth (e.g. mean positions <x>, <y>, <z>, standard deviations σ_x, σ_y, σ_z, etc.) – resulting into beam loss or emittance growth!

Measured instability rise-time = 9.8 s

BCT

ΔI

22:44:00

22:46:00

120 s

Example
- Horizontal centroid instability measured with a BPM in the LHC on the ramp
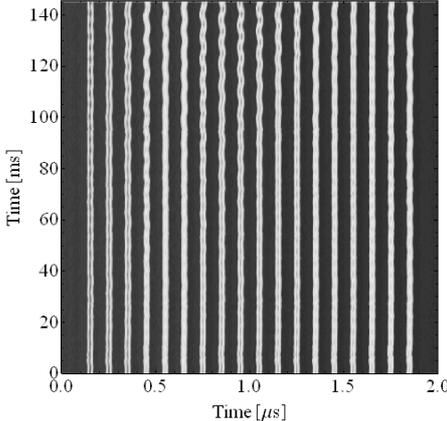- Beam loss measured when a certain amplitude is reached

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Background information

- What is a beam instability?
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Example
- The center position of the bunches in the bucket begins oscillating in the PS
- The instability can prevent further RF manipulation from being correctly applied

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Background information

- What is a beam instability?
  - A beam becomes unstable when a moment of its distribution exhibits an exponential growth (e.g. mean positions <x>, <y>, <z>, standard deviations σ_x, σ_y, σ_z, etc.) – resulting into beam loss or emittance growth!
- Why study beam instabilities?
  - The onset of a beam instability usually determines the maximum beam intensity that a machine can store/accelerate (beam dynamics limit)
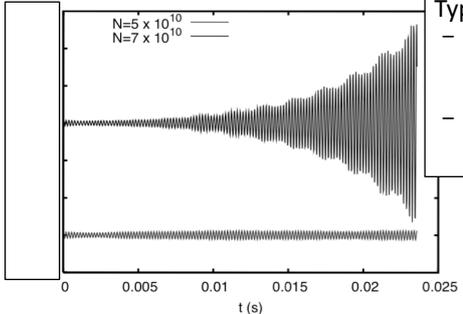
Typical situation
- A beam centroid instability appears when the intensity is ramped up above a certain threshold
- It depends also on the ensemble of the machine settings

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Background information

- What is a beam instability?
  - A beam becomes unstable when a moment of its distribution exhibits an exponential growth (e.g. mean positions <x>, <y>, <z>, standard deviations σ_x, σ_y, σ_z, etc.) – resulting into beam loss or emittance growth!
- Why study beam instabilities?
  - The onset of a beam instability usually determines the maximum beam intensity that a machine can store/accelerate (beam dynamics limit)
  - Understanding the type of instability and the underlying mechanism helps to identify the source and possibly mitigate/suppress the effect

⇒ Beam instabilities occur in both linear and circular machines
  - Longitudinal plane
  - Transverse plane
⇒ They can affect the beam on different scales
  - Cross-talk between subsequent bunches
    → The instability builds up along a bunch train and turn by turn
    → The instability is consequence of another mechanism that builds up along the bunch train
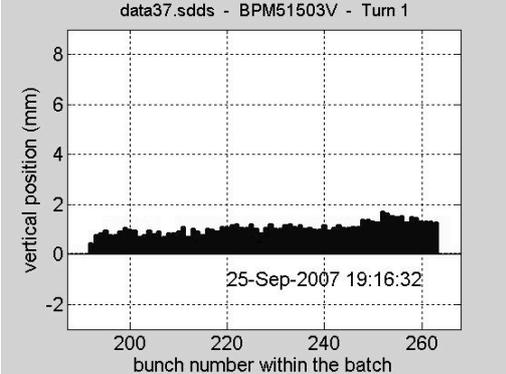 - Single bunch effect

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Background information

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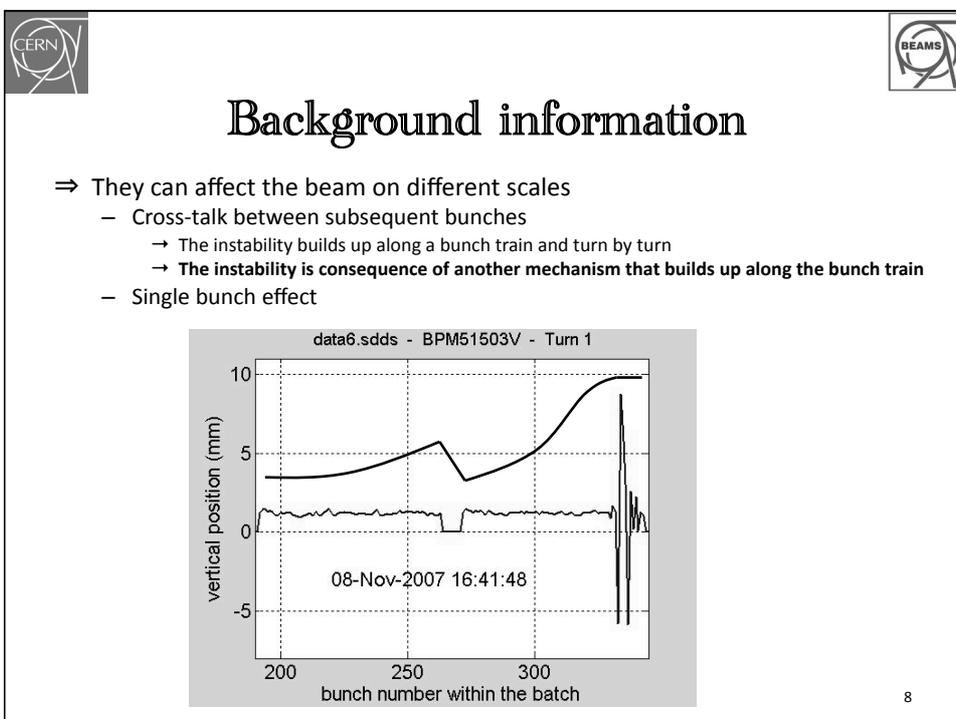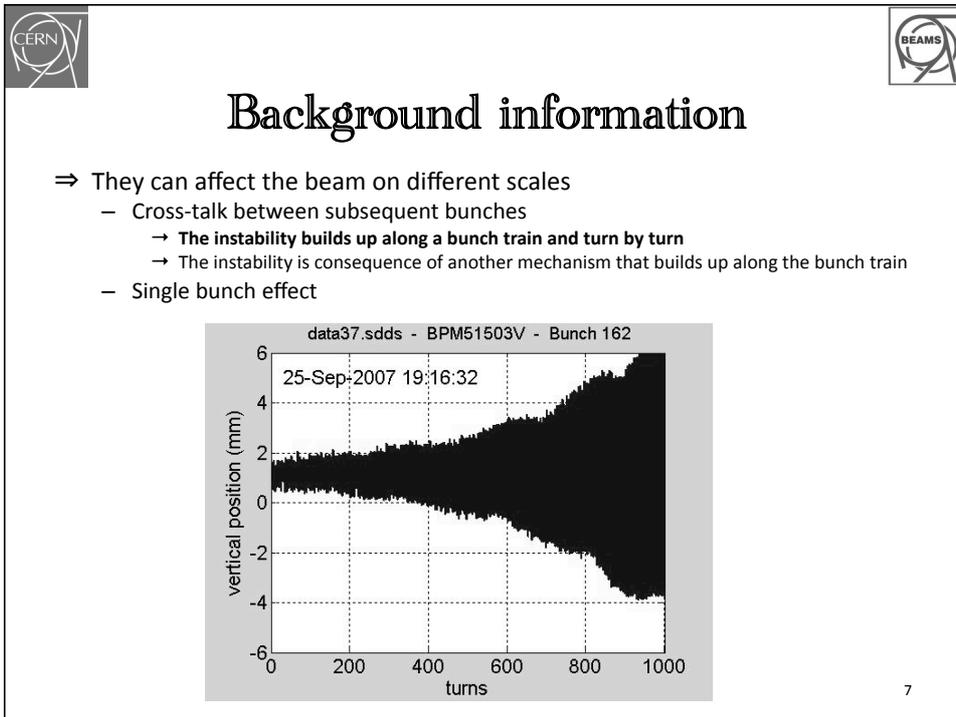 - Single bunch effect

data37.sdds - BPM51503V - Turn 1
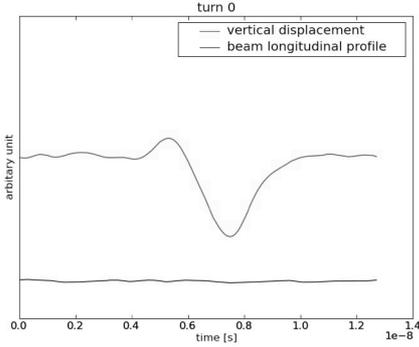
25-Sep-2007 19:16:32

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Background information

⇒ They can affect the beam on different scales
  – Cross-talk between subsequent bunches
    → **The instability builds up along a bunch train and turn by turn**
    → The instability is consequence of another mechanism that builds up along the bunch train
  – Single bunch effect

Background information

⇒ They can affect the beam on different scales
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  – Single bunch effect

Background information

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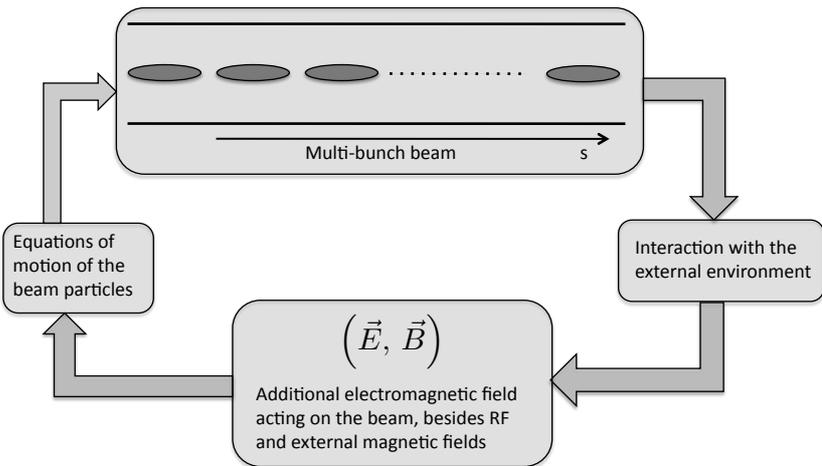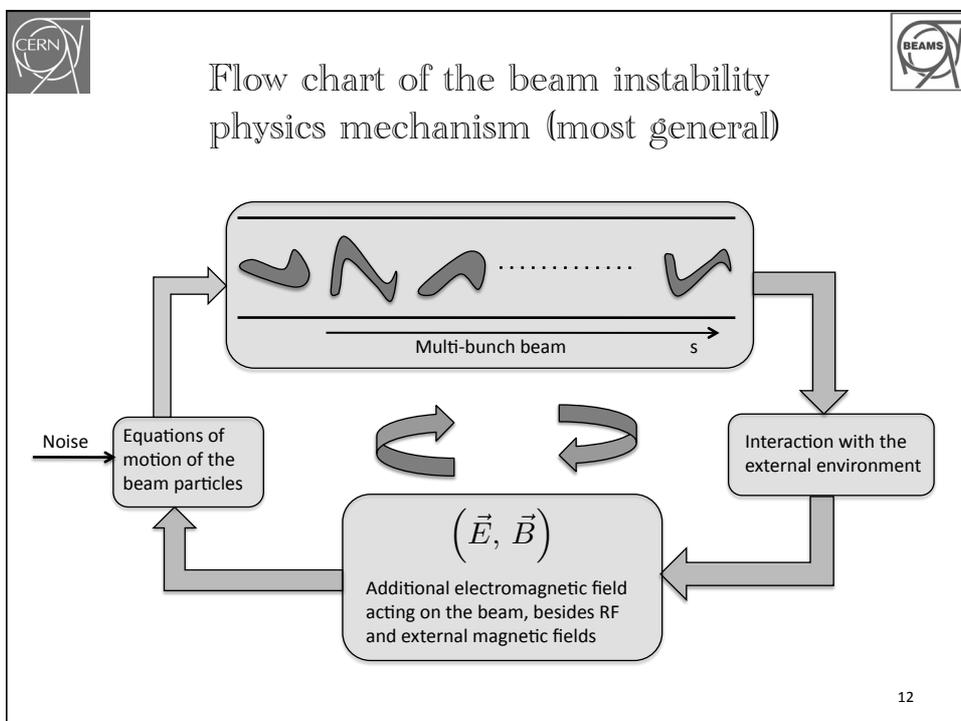 – **Single bunch effect**

Flow chart of the beam instability physics mechanism (most general)

Flow chart of the beam instability physics mechanism (most general)

Flow chart of the beam instability physics mechanism (most general)

Some examples of beam-environment interaction

Interaction of the beam with the external environment

Beam self-fields
- Set of Maxwell's equations with
  - o The beam as the source term
  - o Boundary conditions given by the accelerator component in which the beam propagates

The electron cloud
- Electron production and multipacting
- Poisson's equation with
  - o The electron cloud as the source term
  - o Boundary conditions given by the chamber in which the electron cloud builds up

Fields from a colliding beam
- Set of Maxwell's equations with
  - o The other beam as the source term
  - o Boundary conditions given by the chamber in which the beams collide

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Electro-magnetic beam-environment interaction

Interaction of the beam with the external environment

Beam self-fields
- Set of Maxwell's equations with
  - o The beam as the source term
  - o Boundary conditions given by the accelerator component in which the beam propagates

We need to assess a mathematical model to describe in a sufficiently general way the effects of the self-induced fields on the beam

Time domain:
Wake functions/ potentials

Frequency domain:
Impedances

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Wake fields (general)
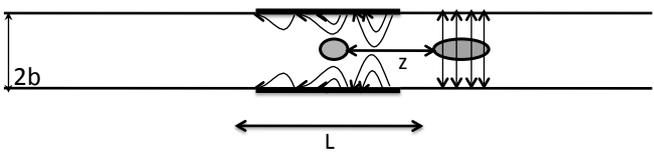
Source, q_1

Witness, q_2

2b

z z z

L

- While source and witness ($q_i \, \delta(s\text{-}ct)$), distant by $z<0$, move in a perfectly conducting chamber, the witness does not feel any force ($\gamma \gg 1$)
- When the source encounters a discontinuity, it produces an electromagnetic field, which rings behind (wake field)
  - The source loses energy
  - The witness feels a net force all along an effective length of the structure, L

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Wake fields (general)

Source, q_1

Witness, q_2

2b

z

L

- Not only geometric discontinuities cause electromagnetic fields trailing behind sources traveling at light speed.
- For example, a pipe with finite conductivity causes a delay in the image currents, which also produces delayed electromagnetic fields
  - No ringing, only slow decay
  - The witness feels a net force all along an effective length of the structure, L
- In general, also electromagnetic boundary conditions other than PEC can be the origin of wake fields.

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1. The longitudinal plane

Longitudinal wake function: definition

Source, q_1

Witness, q_2

$2b$

z

L

$$\int_0^L F_\parallel(s,z)ds = -q_1 q_2 W_\parallel(z)$$

$$F_\parallel(s,z) = q_2 E_z(s,z)$$

ΔE_2
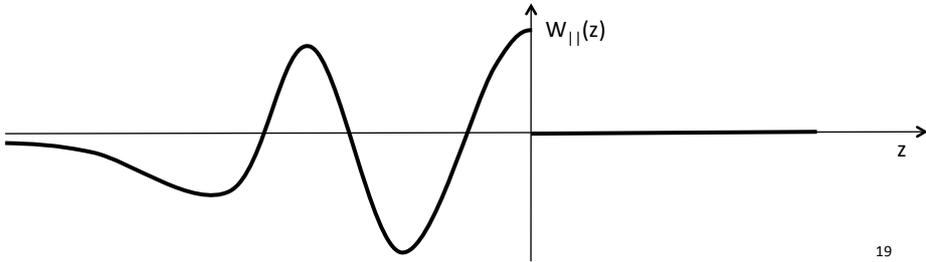
$$\frac{\Delta E_2}{E_0} \approx \frac{\Delta p_2}{p_0}$$

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Longitudinal wake function

$$W_{\parallel}(z) = -\frac{\Delta E_2}{q_1 q_2} \qquad \begin{array}{c} z \to 0 \\ q_2 \to q_1 \end{array} \qquad W_{\parallel}(0) = -\frac{\Delta E_1}{q_1^2}$$

- The value of the wake function in 0, $W_{//}(0)$, is related to the energy lost by the source particle in the creation of the wake
- $W_{//}(0)>0$ since $\Delta E_1<0$
- $W_{//}(z)$ is discontinuous in z=0 and it vanishes for all z>0 because of the ultra-relativistic approximation


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Longitudinal wake function

$$W_{\parallel}(0) = -\frac{\Delta E_1}{q_1^2}$$

- In the global energy balance, the energy lost by the source splits into
  - o Electromagnetic energy of modes that propagate down the beam chamber (above cut-off), which will be eventually lost on surrounding lossy materials
  - o Electromagnetic energy of the modes that remain trapped in the object
    - ⇒ This energy can eventually be dissipated on lossy walls or purposely designed HOM absorbers
    - ⇒ It keeps ringing forever (PEC walls), but can also be transferred to following particles (or the same over several turns), possibly feeding into an instability!
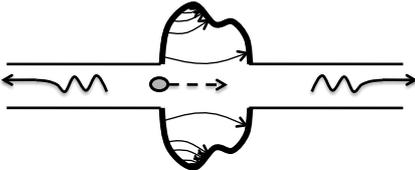
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Longitudinal impedance

- The wake function of an accelerator component is basically its Green function in time domain (i.e., its response to a pulse excitation)
    ⇒ Very useful for macroparticle models and simulations, because it can be used to describe the driving terms in the single particle equations of motion!

- We can also describe it as a transfer function in frequency domain
- This is the definition of **longitudinal beam coupling impedance** of the element under study

$$Z_\parallel(\omega) = \int_{-\infty}^{\infty} W_\parallel(z) \exp\left(-\frac{i\omega z}{c}\right) \frac{dz}{c}$$
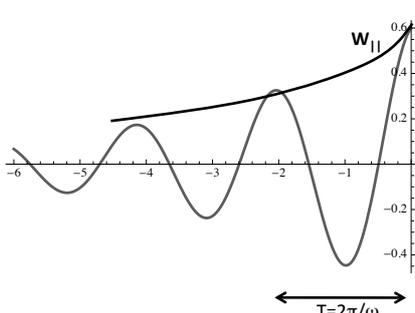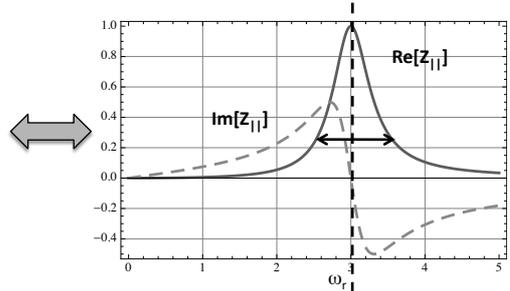
$[\Omega]$

$[\Omega/\text{s}]$

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Longitudinal impedance

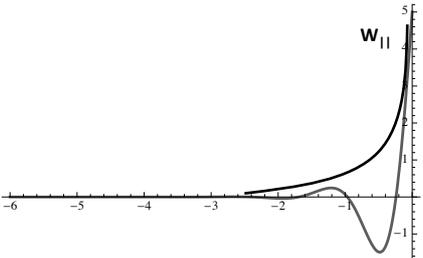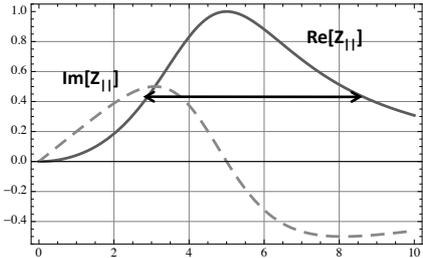$$Z_\parallel(\omega) = \int_{-\infty}^{\infty} W_\parallel(z) \exp\left(-\frac{i\omega z}{c}\right) \frac{dz}{c}$$

- The frequency ω_r is related to the oscillation of E_z, and therefore to the frequency of the mode excited in the object
- The decay time depends on how quickly the stored energy is dissipated (quantified by a quality factor Q)

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Longitudinal impedance

$$Z_{\parallel}(\omega) = \int_{-\infty}^{\infty} W_{\parallel}(z) \exp\left(-\frac{i\omega z}{c}\right) \frac{dz}{c}$$
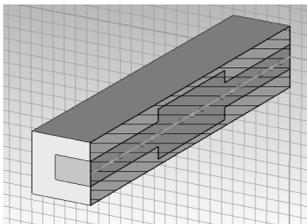

- In general, the impedance will be composed of several of these resonator peaks
- Other contributors to the global impedance can also have different shapes, e.g. the resistive wall
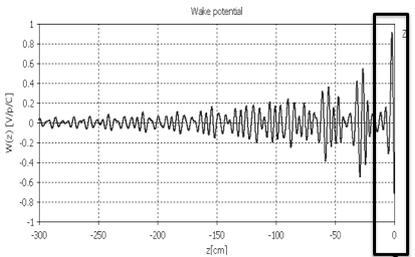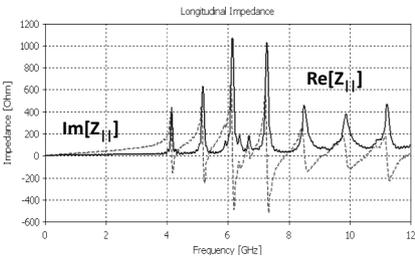
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Longitudinal impedance

- A more complex example: a simple pill-box cavity with walls having finite conductivity
- Several modes can be excited
  - Below the pipe cut-off frequency the width of the peaks is only determined by the finite conductivity of the walls
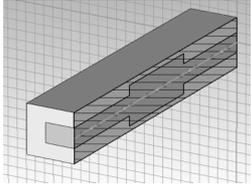 - Above, losses also come from propagation in the chamber

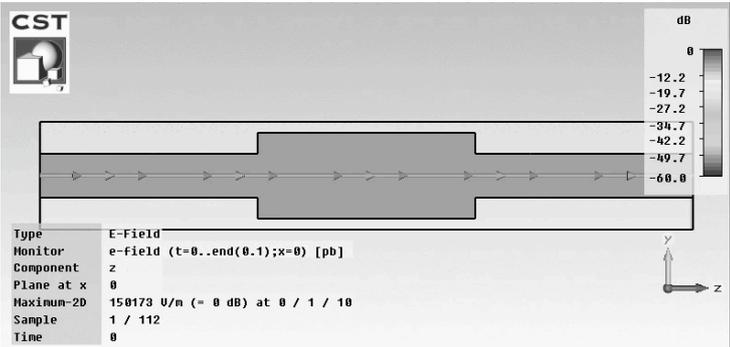Note $W_{\parallel}(0)<0$, **CST** uses opposite convention!!

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Longitudinal impedance

– Evolution of the electromagnetic fields (E_z) in the cavity while and after the beam has passed
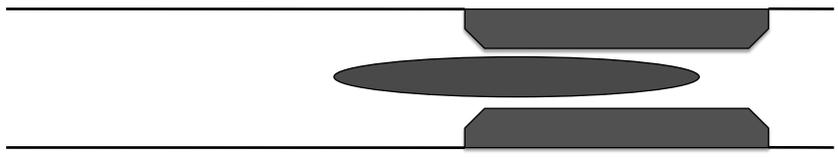
Type   E-Field
Monitor   e-field (t=0..end(0.1);x=0) [pb]
Component   z
Plane at x 0
Maximum-2D 150173 V/m (= 0 dB) at 0 / 1 / 10
Sample   1 / 112
Time   0

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Single bunch vs. Multi bunch

Im[$Z_{||}$]   Re[$Z_{||}$]

$W_{||}$
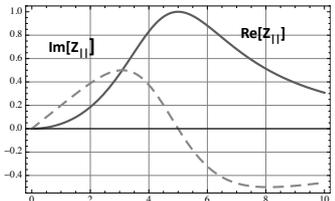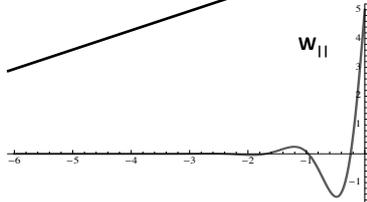
– A short-lived wake field (e.g. broad band resonator), decaying over the length of one bunch, causes intra-bunch coupling , but does not couple subsequent bunches or the same bunch over several turns
– It can be therefore responsible for single bunch instabilities

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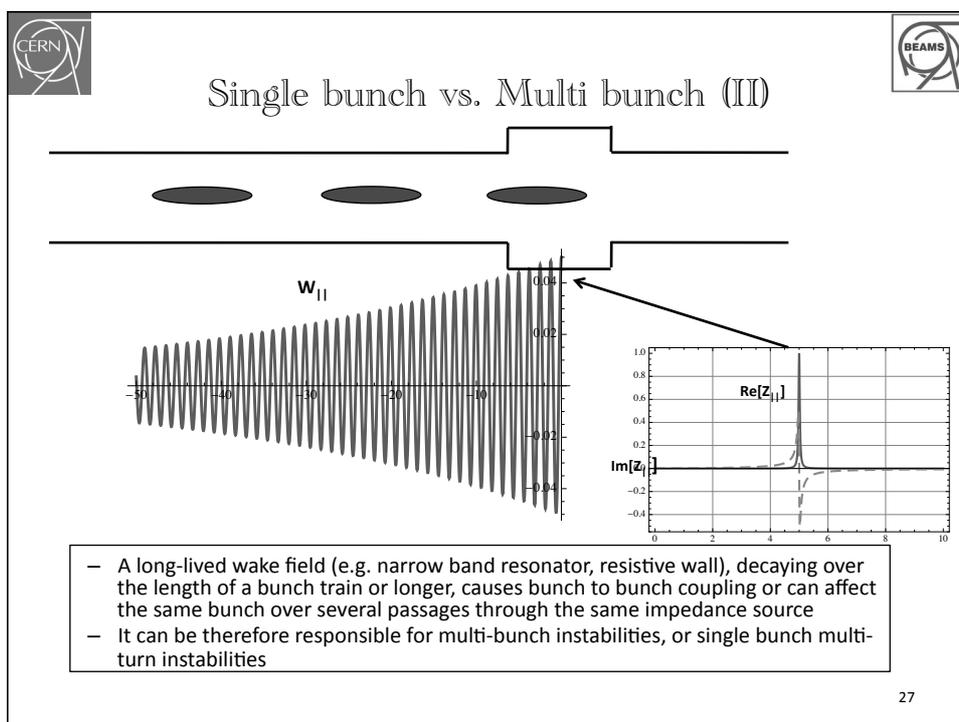
- Detailed calculations:
  - Energy loss
  - Robinson instability

- Qualitative descriptions:
  - Coupled bunch instabilities
  - Single bunch modes

[1] "Physics of Collective Beam Instabilities in High Energy Accelerators", A. W. Chao

Energy loss of a bunch (single pass)

- The energy kick ΔE(z) on the witness slice eλ(z)dz is the integral of the contributions from the wakes left behind by all the preceding eλ(z')dz slices (sources)
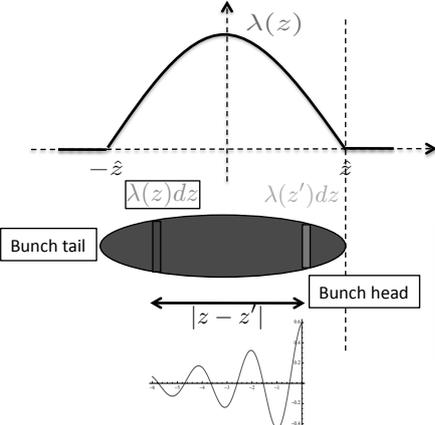- The total energy loss ΔE of the bunch can then be obtained by integrating ΔE(z) over the full bunch extension

$$\Delta E(z) = -e^2 \int_z^{\hat{z}} \lambda(z') W_\parallel(z-z') dz'$$

$$\Delta E = \int_{-\hat{z}}^{\hat{z}} \lambda(z) \Delta E(z) dz$$

$$\Delta E = -\frac{e^2}{2\pi} \int_{-\infty}^{\infty} \tilde{\lambda}(\omega)\tilde{\lambda}^*(\omega) Z_\parallel(\omega) d\omega =$$
$$-\frac{e^2}{2\pi} \int_{-\infty}^{\infty} |\tilde{\lambda}(\omega)|^2 \mathrm{Re}\left[Z_\parallel(\omega)\right] d\omega$$

Bunch tail

Bunch head

$|z - z'|$

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Energy loss of a bunch (multi-pass)

- The total energy loss ΔE of the bunch can still be obtained by integrating ΔE(z) over the full bunch extension
- ΔE(z) this time also includes contributions from all previous turns, spaced by multiples of the ring circumference C

$$\Delta E = -\frac{e^2}{2\pi} \int_{-\infty}^{\infty} \lambda(z) dz \int_{-\infty}^{\infty} dz' \lambda(z') \sum_{k=-\infty}^{\infty} W_\parallel(kC + z - z') dz'$$

$$\sum_{k=-\infty}^{\infty} W_\parallel(kC + z - z') = \frac{c}{C} \sum_{p=-\infty}^{\infty} Z_\parallel(p\omega_0) \exp\left[-\frac{ip\omega_0(z-z')}{c}\right]$$

$$\Delta E = -\frac{e^2\omega_0}{2\pi} \sum_{p=-\infty}^{\infty} Z_\parallel(p\omega_0) \underbrace{\int_{-\infty}^{\infty} \lambda(z) \exp\left(\frac{-ip\omega_0 z}{c}\right) dz}_{\tilde{\lambda}(p\omega_0)} \underbrace{\int_{-\infty}^{\infty} \lambda(z') \exp\left(\frac{ip\omega_0 z'}{c}\right) dz'}_{\tilde{\lambda}^*(p\omega_0)}$$

$$\Delta E = -\frac{e^2\omega_0}{2\pi} \sum_{p=-\infty}^{\infty} |\tilde{\lambda}(p\omega_0)|^2 \mathrm{Re}\left[Z_\parallel(p\omega_0)\right]$$
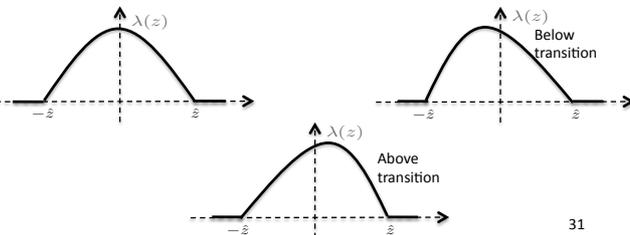
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Energy loss per turn: stable phase shift

- If we divide the total impedance of a given ring into a narrow band component $Z^{NB}_{||}(\omega)$ and a broad band component $Z^{BB}_{||}(\omega)$, then we can write a general expression for the energy loss per bunch and per turn
- To compensate for this energy loss, the equilibrium distribution of the bunch must move in the bucket from being centered to a synchronous angle $\Delta\Phi_s$

$$\Delta E_{\text{turn}} = -\frac{e^2}{2\pi}\left\{\omega_0 \sum_{p=-\infty}^{\infty}|\tilde{\lambda}(p\omega_0)|^2 \text{Re}\left[Z^{\text{NB}}_{||}(p\omega_0)\right] + \right.$$
$$\left. + \int_{-\infty}^{\infty}|\tilde{\lambda}(\omega)|^2 \text{Re}\left[Z^{\text{BB}}_{||}(\omega)\right]d\omega\right\}$$

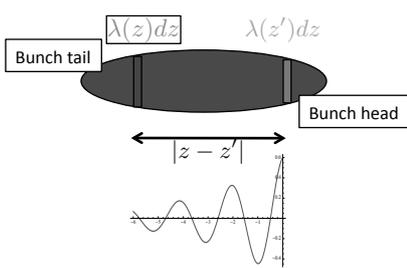$$\sin\Delta\Phi_s = \frac{\Delta E_{\text{turn}}}{eV_m}$$

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Single particle equations of the longitudinal motion in presence of wake fields

- The single particle in the witness slice λ(z)dz will feel the force from the RF, from the bunch own space charge, and that associated to the wake
- The wake contribution can extend to several turns

$$\frac{d^2z}{dt^2} + \underbrace{\frac{\eta eV_{\text{rf}}(z)}{m_0\gamma C}}_{\text{External RF}} = \underbrace{\frac{\eta e^2}{m_0\gamma C}\sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\lambda(z'+kC)W_{||}(z-z'-kC)dz'}_{\text{Wake fields}}$$

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The Robinson instability

– To illustrate the Robinson instability we will use some simplifications:
  ⇒ The bunch is point-like and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  ⇒ Space charge is neglected
  ⇒ The bunch additionally feels the effect of a multi-turn wake

$\delta p/p_0$

The perturbation also changes the oscillation amplitude

The perturbation changes ω_s

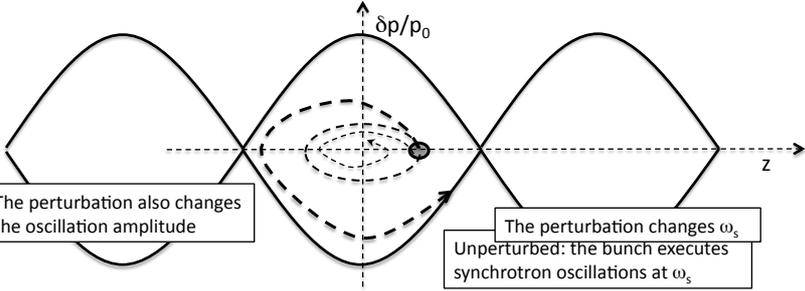Unperturbed: the bunch executes synchrotron oscillations at ω_s

z

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The Robinson instability

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  ⇒ The bunch is point-like and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  ⇒ Space charge is neglected
  ⇒ The bunch additionally feels the effect of a multi-turn wake

$$\frac{d^2 z}{dt^2} + \omega_s^2 z = \frac{Ne^2\eta}{Cm_0\gamma} \sum_{k=-\infty}^{\infty} W_\| \left[z(t) - z(t - kT_0) - kC \right]$$

We assume that the wake decays on a much slower time scale than the synchrotron period

$$W_\| \left[z(t) - z(t - kT_0) - kC \right] \approx W_\|(kC) + W_\|'(kC) \cdot \left[z(t) - z(t - kT_0) \right]$$

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The Robinson instability

$$W_\| \left[z(t) - z(t - kT_0) - kC \right] \approx W_\|(kC) + W_\|'(kC) \cdot \left[z(t) - z(t - kT_0) \right]$$

⇒ The term $\Sigma\, W_\|(kC)$ only contributes to a constant term in the solution of the equation of motion, i.e. the synchrotron oscillation will be executed around a certain z_0 and not around 0. This term represents the stable phase shift that compensates for the energy loss

⇒ The dynamic term is proportional to $z(t)\text{-}z(t\text{-}kT_0) \approx kT_0 dz/dt$, so it will introduce a "friction" term in the equation of the oscillator, which can lead to instability!

$$z(t) \propto \exp\left(-i\Omega t\right)$$

$$\Omega^2 - \omega_s^2 = -\frac{Ne^2\eta}{Cm_0\gamma} \boxed{\sum_{k=-\infty}^{\infty} \left[1 - \exp\left(-ik\Omega T_0\right) \right] \cdot W_\|'(kC)}$$

$$\boxed{i \cdot \frac{1}{C} \sum_{p=-\infty}^{\infty} \left[p\omega_0 Z_\|(p\omega_0) - (p\omega_0 + \Omega)Z_\|(p\omega_0 + \Omega) \right]}$$

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The Robinson instability

⇒ We assume a small deviation from the synchrotron tune
⇒ Re($\Omega - \omega_s$) → Synchrotron tune shift
⇒ Im($\Omega - \omega_s$) → Growth/damping rate, only depends on the dynamic term, if it is positive there is an instability!

$$\Omega^2 - \omega_s^2 \approx 2\omega_s \boxed{(\Omega - \omega_s)}$$

Complex frequency shift

$$\Delta\omega_s = \mathrm{Re}(\Omega - \omega_s) = \left(\frac{e^2}{m_0 c^2}\right) \frac{N\eta}{2\gamma T_0^2 \omega_s} \, \textsf{x}$$

$$\textsf{x} \sum_{p=-\infty}^{\infty} \left[p\omega_0 \mathrm{Im} Z_\|(p\omega_0) - (p\omega_0 + \omega_s)\mathrm{Im} Z_\|(p\omega_0 + \omega_s) \right]$$

$$\tau^{-1} = \mathrm{Im}(\Omega - \omega_s) = \left(\frac{e^2}{m_0 c^2}\right) \frac{N\eta}{2\gamma T_0^2 \omega_s} \sum_{p=-\infty}^{\infty} (p\omega_0 + \omega_s)\mathrm{Re} Z_\|(p\omega_0 + \omega_s)$$

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9/20/11

The Robinson instability

$$\tau^{-1} = \mathrm{Im}(\Omega - \omega_s) = \left(\frac{e^2}{m_0 c^2}\right) \frac{N\eta}{2\gamma T_0^2 \omega_s} \sum_{p=-\infty}^{\infty} (p\omega_0 + \omega_s)\mathrm{Re}Z_\parallel(p\omega_0 + \omega_s)$$

⇒ We assume the impedance to be peaked at a frequency ω_r close to $h\omega_0 \gg \omega_s$ (e.g. RF cavity fundamental mode or HOM)
⇒ Only two dominant terms are left in the summation at the RHS of the equation for the growth rate
⇒ Stability requires that η and $\Delta[\mathrm{Re}\,Z_\parallel(h\omega_0)]$ have different signs

$$\tau^{-1} = \left(\frac{e^2}{m_0 c^2}\right) \frac{N\eta h\omega_0}{2\gamma T_0^2 \omega_s} \underbrace{\left[\mathrm{Re}Z_\parallel(h\omega_0 + \omega_s) - \mathrm{Re}Z_\parallel(h\omega_0 - \omega_s)\right]}_{\Delta\left[\mathrm{Re}Z_\parallel(h\omega_0)\right]}$$

Stability criterion → $\quad \eta \cdot \Delta\left[\mathrm{Re}Z_\parallel(h\omega_0)\right] < 0$

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The Robinson instability

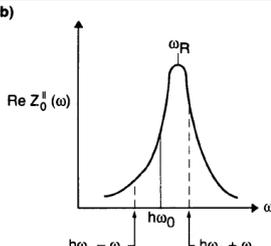Stability criterion → $\quad \eta \cdot \Delta\left[\mathrm{Re}Z_\parallel(h\omega_0)\right] < 0$
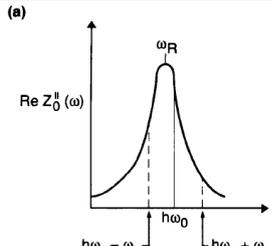
Figure 4.4. Illustration of the Robinson stability criterion. The rf fundamental mode is detuned so that ω_R is (a) slightly below $h\omega_0$ and (b) slightly above $h\omega_0$. (a) is Robinson damped above transition and antidamped below transition. (b) is antidamped above transition and damped below transition.

	$\omega_r > h\omega_0$	$\omega_r < h\omega_0$
Above transition ($\eta>0$)	unstable	stable
Below transition ($\eta<0$)	stable	unstable

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The Robinson instability

$$\tau^{-1} = \mathrm{Im}(\Omega - \omega_s) = \left(\frac{e^2}{m_0 c^2}\right)\frac{N\eta}{2\gamma T_0^2 \omega_s}\sum_{p=-\infty}^{\infty}(p\omega_0 + \omega_s)\mathrm{Re}Z_\parallel(p\omega_0 + \omega_s)$$

⇒ Other types of impedances can also cause instabilities through the Robinson mechanism
⇒ However, a smooth broad-band impedance with no narrow structures on the ω_0 scale cannot give rise to an instability
  ✓ Physically, this is clear, because the absence of structure on ω_0 scale in the spectrum implies that the wake has fully decayed in one turn time and the driving term in the equation of motion also vanishes

$$\sum_{p=-\infty}^{\infty}(p\omega_0 + \omega_s)\mathrm{Re}Z_\parallel(p\omega_0 + \omega_s) \to \frac{1}{\omega_0}\int_{-\infty}^{\infty}\omega\mathrm{Re}Z_\parallel(\omega)d\omega \to 0$$

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Other longitudinal instabilities

- The Robinson instability occurs for a single bunch under the action of a multi-turn wake field
  - It contains a term of coherent synchrotron tune shift
  - It results into an unstable rigid bunch dipole oscillation
  - It does not involve higher order moments of the bunch longitudinal phase space distribution

- Other important collective effects can affect a bunch in a beam
  - Potential well distortion (resulting in synchronous phase shift, bunch lengthening or shortening, synchrotron tune shift/spread)
  - Coupled bunch instabilities
  - High intensity single bunch instabilities (e.g. microwave instability) or negative mass instability

- To be able to study these effects we would need to resort to a more detailed description of the bunch(es)
  - Vlasov equation (kinetic model)
  - Macroparticle simulations

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Coupled bunch modes

- M bunches can exhibit M different modes of coupled rigid bunch oscillations in the longitudinal plane
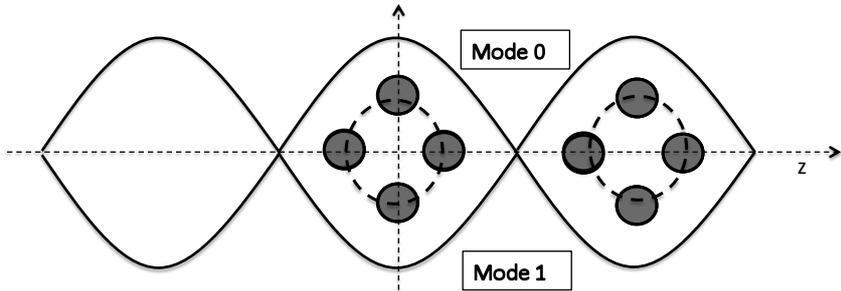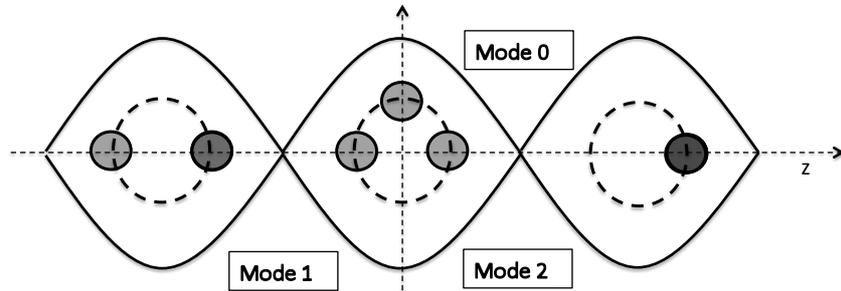

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Coupled bunch modes

- M bunches can exhibit M different modes of coupled rigid bunch oscillations in the longitudinal plane
- Any rigid coupled bunch oscillation can be decomposed into a combination of these basic modes
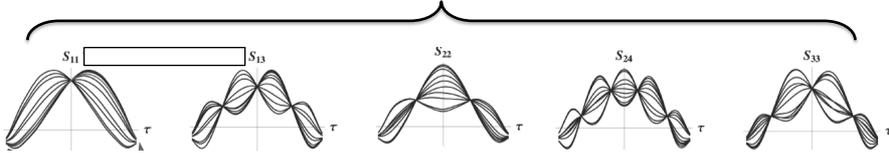- This modes can become unstable under the effect of long range wake fields


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Single bunch modes

- In a similar fashion, a single bunch exhibits a double infinity of natural modes of oscillation, with rather complicated phase space portraits.
- Whatever perturbation on the bunch phase space distribution can be expanded as a series of these modes
- One of these modes or a combination of them can become unstable under the effect of a short range wake field
- In particular, the frequencies of these modes shift with intensity, and two of the modes can merge above a certain threshold, causing a microwave instability!

Oscillation modes as observed at a wall current monitor

S_{11} S_{13} S_{22} S_{24} S_{33}

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Single bunch modes

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- One of these modes or a combination of them can become unstable under the effect of a short range wake field
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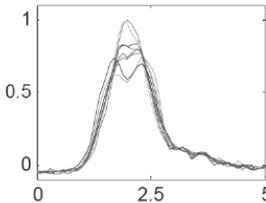### Observations in the CERN SPS in 2007
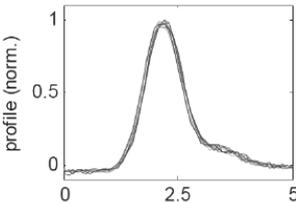
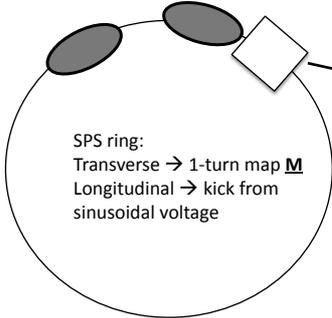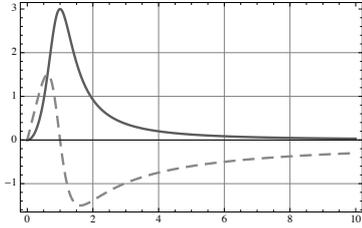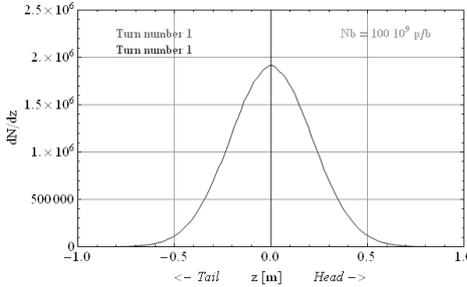stable bunch   dipole osc.   quadrupole osc.

profile (norm.)
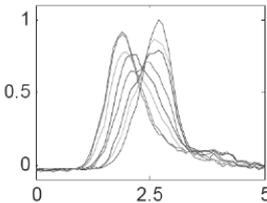
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Macroparticle simulation

- We have simulated the evolution of an SPS bunch under the effect of a longitudinal broad band impedance lumped in one point of the ring

SPS ring:
Transverse → 1-turn map **M**
Longitudinal → kick from sinusoidal voltage
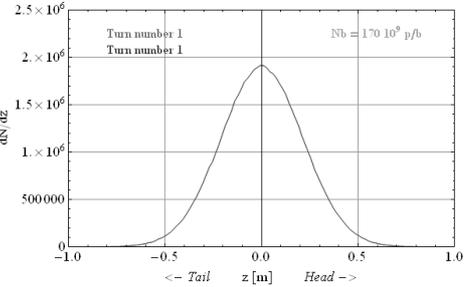
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Macroparticle simulation

- We have simulated the evolution of an SPS bunch under the effect of a longitudinal broad band impedance lumped in one point of the ring
- Two different intensity values have been simulated
  - Low intensity → below instability threshold, but potential well distortion is visible in terms of stable phase shift and bunch lengthening
  - High intensity → above microwave instability threshold

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