Beam-beam effects

(an introduction)

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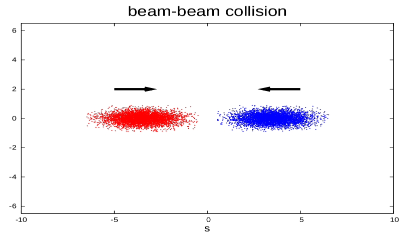
http://cern.ch/Werner.Herr/CAS2011/lectures/Chios_beambeam.pdf
http://cern.ch/Werner.Herr/CAS2011/proceedings/bb_proc.pdf

What are beam-beam effects ?

- They occur when two beams collide
- Two types of beam-beam effects:
  - High energy collisions between two particles (wanted)
  - Distortions of beams by electromagnetic forces (unwanted)
- Unfortunately: usually both go together ...

Beam-beam collision



Typically:

- 0.001% (or less) of particles collide
- 99.999% (or more) of particles are distorted

Beam-beam effects

- In circular colliders: interactions happen (at least) once per turn !
  - Many different effects and problems
  - Try to understand some of them
- In linear collider: VERY different problems
- Two main questions:
  - What happens to a single particle ?
  - What happens to the whole beam?

BEAMS: moving charges

- Beam is a collection of charges
- Represent electromagnetic potential for other charges
- Forces on itself (space charge) and opposing beam (beam-beam effects)
- Main limit in past, present and future colliders
- Important for high density beams, i.e. high intensity and/or small beams:
  for high luminosity !

Beam-beam effects

Remember:

$$\mathcal{L} = \frac{N_1 N_2 f n_B}{4\pi\sigma_x\sigma_y} \quad = \quad \frac{N_1 N_2 f n_B}{4\pi \cdot \ \sigma_x\sigma_y}$$

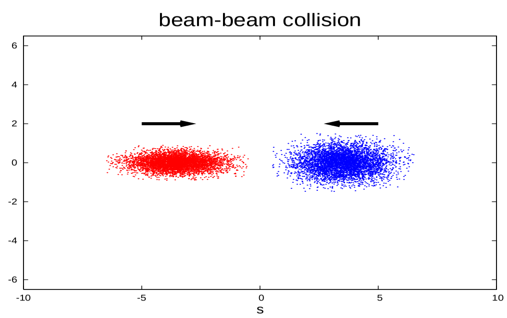
- Overview: which effects are important for present and future machines (LEP, PEP, Tevatron, RHIC, LHC, ...)
- Qualitative and physical picture of the effects
- Mathematical derivations in:

http://cern.ch/Werner.Herr/CAS2011/proceedings/bb_proc.pdf

Beam-beam effects

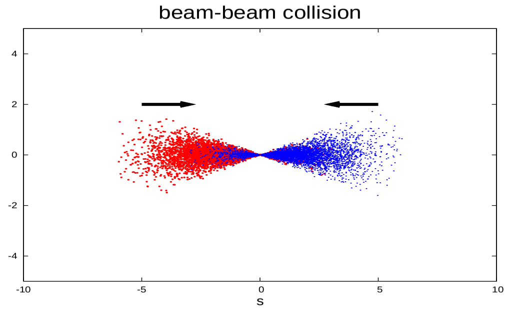
- A beam acts on particles like an electromagnetic lens, but:
  - Does not represent simple form, i.e. well defined multipoles
  - Very non-linear form of the forces, depending on distribution
  - Can change distribution as result of interaction (time dependent forces ..)

Beam-beam collision



- Two beams can have different parameters (**I**, σ ..)
- Very detrimental effects on the beams

Beam-beam collision



- They can change as a result of the beam-beam interaction
- Very detrimental effects on the beams

Studying beam-beam effects

- Need knowledge of the forces
- Apply concepts of non-linear dynamics
- Apply concepts of multi-particle dynamics
- Analytical models and simulation techniques well developed in last 10 years

Fields and Forces (I)

- **Need fields $\vec{E}$ and $\vec{B}$ of opposing beam with a particle distribution $\rho(x, y, z)$**
- **In rest frame only electrostatic field: $\vec{E}'$, $\vec{B}' \equiv 0$**
- **Derive potential $U(x, y, z)$ from Poisson equation:**

$$\Delta U(x, y, z) = -\frac{1}{\epsilon_0}\rho(x, y, z)$$

- **The electrostatic fields become:**

$$\vec{E}' = -\nabla U(x, y, z)$$

Fields and Forces (II)

- **Transform into moving frame and calculate Lorentz force $\vec{F}$ on particle with charge $q = Z_2$ e**

$$E_{\parallel} = E'_{\parallel}, \quad E_{\perp} = \gamma \cdot E'_{\perp} \quad \text{with :} \quad \vec{B} = \vec{\beta} \times \vec{E}/c$$

$$\vec{F} = q(\vec{E} + \vec{\beta} \times \vec{B})$$

- **Example Gaussian distribution:**

$$\rho(x, y, z) = \frac{N Z_1 e}{\sigma_x \sigma_y \sigma_z \sqrt{2\pi}^3} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right)$$

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Simple example: Gaussian

- For 2D case the potential becomes (see proceedings):

$$U(x, y, \sigma_x, \sigma_y) = \frac{NZ_1 e}{4\pi\epsilon_0} \int_0^\infty \frac{\exp(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q})}{\sqrt{(2\sigma_x^2+q)(2\sigma_y^2+q)}} dq$$

- Can derive $\vec{E}$ and $\vec{B}$ fields and therefore forces
- For arbitrary distribution (non-Gaussian): difficult (or impossible, numerical solution required)

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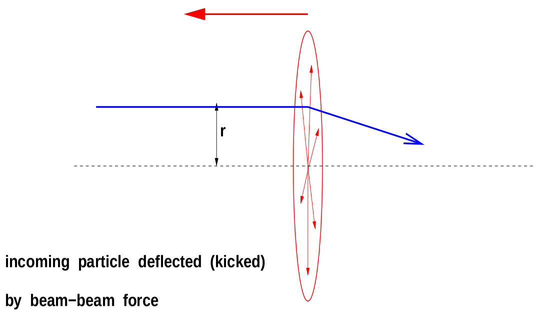
Simple example: round Gaussian beams

- **Assumption 1:** $\sigma_x = \sigma_y = \sigma$, $Z_1 = -Z_2 = 1$
- **Assumption 2:** very relativistic → $\beta \approx 1$
- Only components E_r and B_Φ are non-zero
- Force has only radial component, i.e. depends only on distance **r** from bunch centre *(where: $r^2 = x^2 + y^2$)* (see proceedings)

$$F_r(r) = -\frac{Ne^2(1+\beta^2)}{2\pi\epsilon_0 \cdot r}\left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

Beam-beam kick:

- We use (x, x', y, y') as coordinates
- We need the deflections (kicks $\Delta x'$, $\Delta y'$) of the particles:



Beam-beam kick:

- Kick ($\Delta r'$): angle by which the particle is deflected during the passage
- Integration of force over the collision, i.e. time of passage Δt (assuming: m_1=m_2 and Z_1=$-Z_2$= 1):

$$\text{Newton's law :} \qquad \Delta r' = \frac{1}{mc\beta\gamma} \int_{-\frac{\Delta t}{2}}^{+\frac{\Delta t}{2}} F_r(r, s, t) dt$$

with:

$$F_r(r, s, t) = -\frac{Ne^2(1+\beta^2)}{\sqrt{(2\pi)^3}\epsilon_0 r \sigma_s} \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right] \cdot \left[\exp(-\frac{(s+vt)^2}{2\sigma_s^2})\right]$$

Beam-beam kick:

- Using the classical particle radius (implies $Z_1 = \pm Z_2$):

$$r_0 = e^2/4\pi\epsilon_0 mc^2$$

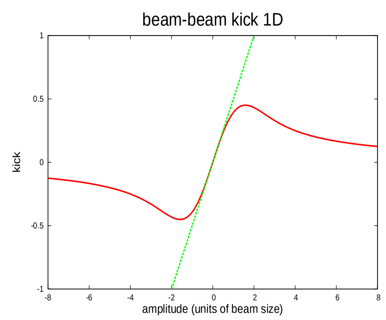
we have (radial kick and in Cartesian coordinates):

$$\Delta r' = -\frac{2Nr_0}{\gamma} \cdot \frac{r}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

$$\Delta x' = -\frac{2Nr_0}{\gamma} \cdot \frac{x}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

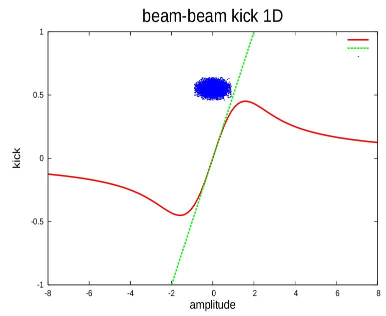
$$\Delta y' = -\frac{2Nr_0}{\gamma} \cdot \frac{y}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

Beam-beam force/kick



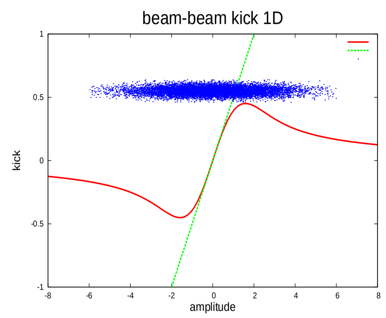
- **For small amplitude: linear force (like quadrupole)**
- **For large amplitude: very non-linear force**

Beam-beam force/kick



- For small amplitude: tune shift

Beam-beam force/kick



- For small amplitude: tune shift
- For large amplitude: amplitude dependent tune shift

Can we quantify the beam-beam strength ?

- Try the slope of force (kick $\Delta r'$) at zero amplitude
- This defines: beam-beam parameter ξ
- For head-on interactions and round beams ($\beta^* = \beta_x^* = \beta_y^*$) we get:

$$\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N \cdot r_o \cdot \beta^*}{4\pi\gamma\sigma^2}$$

LEP - LHC

	LEP (e^+e^-)	LHC (pp)
Beam sizes	160 - 200μm · 2 - 4μm	16.6μm · 16.6μm
Intensity N	4.0 · 10^{11}/bunch	1.15 · 10^{11}/bunch
Energy	100 GeV	7000 GeV
$\epsilon_x \cdot \epsilon_y$	($\approx$) 20 nm · 0.2 nm	0.5 nm · 0.5 nm
$\beta_x^* \cdot \beta_y^*$	($\approx$) 1.25 m · 0.05 m	0.55 m · 0.55 m
Crossing angle	0.0	285 μrad
Beam-beam parameter(ξ)	0.0700	0.0037

Can we quantify the beam-beam strength ?

- In general for non-round beams ($\beta_x^* \neq \beta_y^*$):

$$\xi_{x,y} = \frac{N \cdot r_o \cdot \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x+\sigma_y)}$$

- Proportional to (linear) tune shift ΔQ_{bb} from beam-beam interaction: $\Delta Q_{bb} \propto \pm \xi$
- Good measure for strength of beam-beam interaction
- BUT: does not describe
  - → changes to optical functions
  - → non-linear part of beam-beam force

Linear beam-beam tune shift

- For small amplitudes linear force like a quadrupole with focal length f

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \left[\frac{\xi \cdot 4\pi}{\beta^*}\right]$$

- Transformation matrix over the interaction becomes:

$$\begin{pmatrix} 1 & 0 \\ \frac{1}{-f} & 1 \end{pmatrix}$$

Linear beam-beam tune shift

- **Full turn matrix including the tune shift ΔQ computed from unperturbed full turn matrix plus interaction**

$$\begin{pmatrix} cos(2\pi(Q+\Delta Q)) & \beta^* sin(2\pi(Q+\Delta Q)) \\ -\frac{1}{\beta^*} sin(2\pi(Q+\Delta Q)) & cos(2\pi(Q+\Delta Q)) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{1}{-2f} & 1 \end{pmatrix} \circ \begin{pmatrix} cos(2\pi Q) & \beta_0^* sin(2\pi Q) \\ -\frac{1}{\beta_0^*} sin(2\pi Q) & cos(2\pi Q) \end{pmatrix} \circ \begin{pmatrix} 1 & 0 \\ \frac{1}{-2f} & 1 \end{pmatrix}$$

Linear beam-beam tune shift

- **Solving this equation gives us:**

$$cos(2\pi(Q + \Delta Q)) = cos(2\pi Q) - \frac{\beta_0^*}{2f} sin(2\pi Q)$$

and

$$\frac{\beta^*}{\beta_0^*} = sin(2\pi Q)/sin(2\pi(Q + \Delta Q))$$

- **Tune is changed by ΔQ**
- **β-function is changed (β-beating)**

Linear beam-beam tune shift

- For small ξ and Q not too close to 0.0 and 0.5 we have:

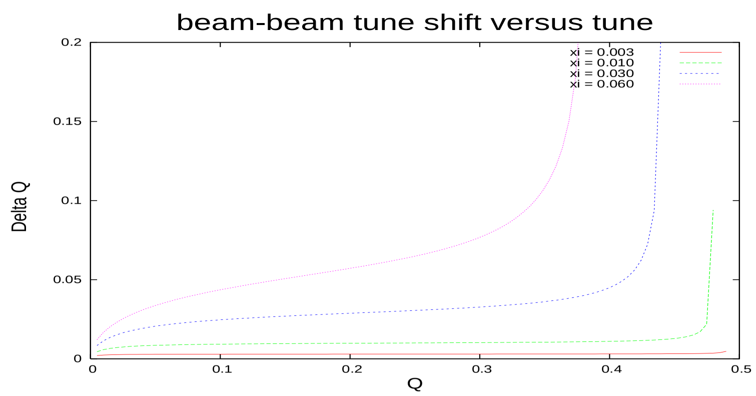
$$\Delta Q \approx \xi$$

and

$$\frac{\beta^*}{\beta^*_0} = \frac{sin(2\pi Q)}{sin(2\pi(Q+\Delta Q))} = \frac{\beta_0}{\sqrt{1+4\pi\xi cot(2\pi Q) - 4\pi^2\xi^2}}$$

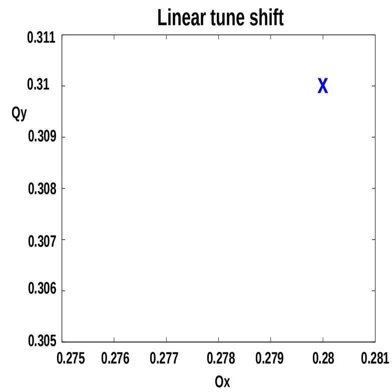
- β can become smaller or larger at interaction point (dynamic β)

Tune dependence of tune shift



- Strong dependence on Q for larger ξ (dynamic β)

Linear tune shift - two dimensions



- Start with standard working point
- LHC (equally charged beams)
- Beam–beam shifts tune in both planes

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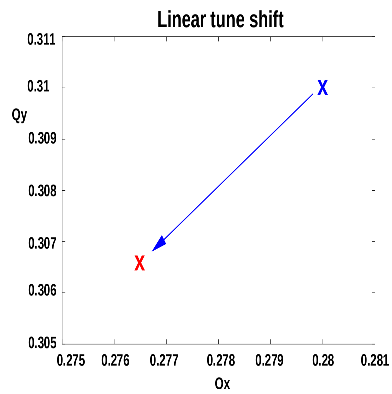
Linear tune shift - two dimensions



- Start with standard working point
- LHC (equally charged beams)
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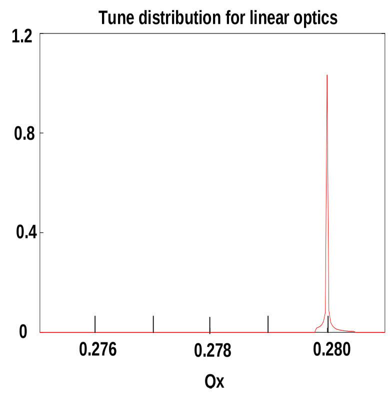
Tune measurement: linear optics



- Linear force: →
- all particles have same tune
- ▶ Only one frequency (tune) visible

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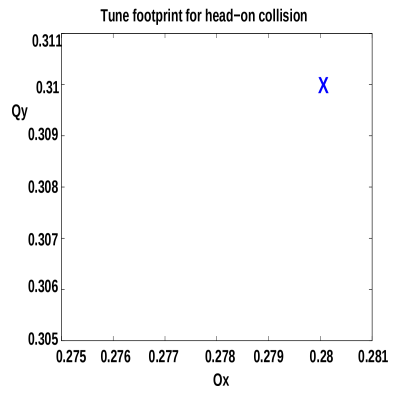
Non-linear tune shift - two dimensions



- Tunes depend on x **and** y amplitudes
- No single tune in the beam
- Compute and plot for every amplitude (pair) the tunes in both planes
- In 2 dimensions: plotted as **footprint**

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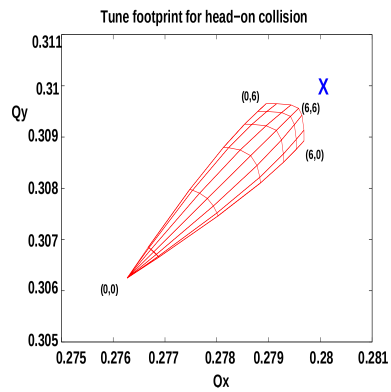
Non-linear tune shift - two dimensions



- Tunes depend on x **and** y amplitudes
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- In 2 dimensions:
  plotted as **footprint**

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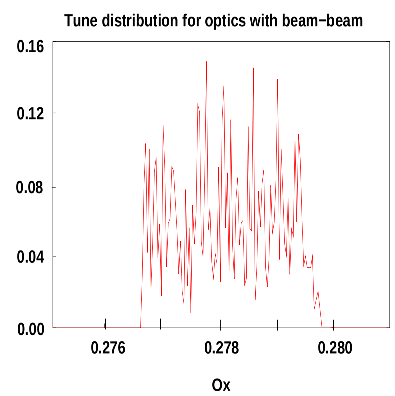
Tune measurement: with beam-beam



- Non−linear force: →
- particles with different amplitudes have different frequencies (tunes)
- ▶ We get frequency (tune) **spectra**
- ▶ Width of the spectra: about ξ

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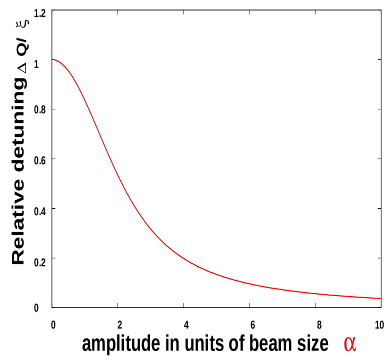
Amplitude detuning

Detuning with amplitude – round beams



- Non–linear force: → tune depends on amplitude
- Largest effect for small amplitudes
- Calculation in the proceedings

→ with $\alpha = \frac{a}{\sigma_*}$ we get: $\Delta Q/\xi = \frac{4}{\alpha^2}\left[1 - I_0(\frac{\alpha^2}{4}) \cdot e^{\frac{-\alpha^2}{4}}\right]$

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Weak-strong and strong-strong

- Both beams are very strong (strong-strong):
  - Both beam are affected and change due to beam-beam interaction
  - Examples: LHC, LEP, RHIC, ...
  - Evaluation of effects challenging
- One beam much stronger (weak-strong):
  - Only the weak beam is affected and changed due to beam-beam interaction
  - Examples: SPS collider, Tevatron, ...

Incoherent effects

(single particle effects)

- Single particle dynamics: treat as a particle through a static electromagnetic lens
- Basically non-linear dynamics
- All single particle effects observed:
  - Unstable and/or irregular motion
  - Beam blow up
  - Bad lifetime, particle loss

Observations hadrons

- Non-linear motion can become chaotic
  - reduction of ”dynamic aperture”
  - particle loss and bad lifetime
- Strong effects in the presence of noise or ripple
- Very bad: unequal beam sizes (studied at SPS, HERA)
- Evaluation is done by simulation

Observations leptons

Remember:

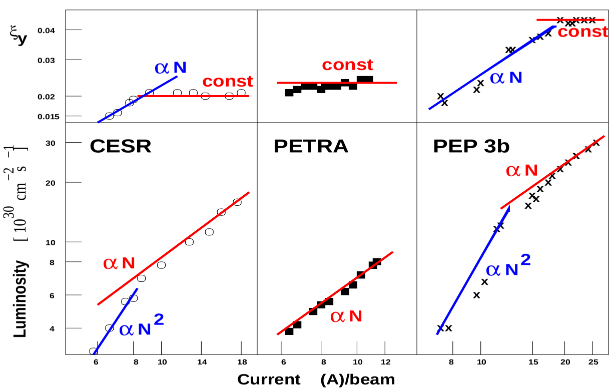
$$\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f n_B}{4\pi\sigma_x\sigma_y}$$

- Luminosity should increase $\propto N_1 N_2$

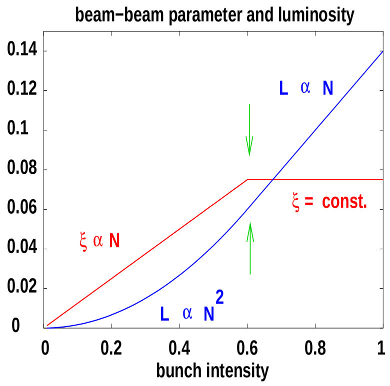
  → for: $N_1 = N_2 = N$ → $\propto N^2$

- Beam-beam parameter should increase $\propto N$
- But:

Examples: beam-beam limit



Beam-beam limit (schematic)



- Beam−beam parameter increases linearly with intensity
- Saturation above some intensity
- Luminosity increases linearly
- So−called **beam−beam limit**

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What is happening ?

we have
$$\xi_y = \frac{N r_0 \beta_y}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)} \overset{(\sigma_x \gg \sigma_y)}{\approx} \frac{r_0\beta_y}{2\pi\gamma(\sigma_x)} \cdot \frac{N}{\sigma_y}$$

and
$$\mathcal{L} = \frac{N^2 f n_B}{4\pi\sigma_x\sigma_y} = \frac{N f n_B}{4\pi\sigma_x} \cdot \frac{N}{\sigma_y}$$

- **Above beam-beam limit:** σ_y **increases when** N **increases to keep** ξ **constant** ➔ **equilibrium emittance !**
- **Therefore:** $\mathcal{L} \propto N$ **and** $\xi \approx$ **constant**
- ξ_{limit} **is NOT a universal constant !**
- **Difficult to predict**

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What is happening ?

- Where does it come from ?
  - From synchrotron radiation: vertical plane damped, horizontal plane excited
  - Horizontal beam size usually (much) larger
  - Vertical beam-beam effect depends on horizontal (large) amplitude
  - Coupling from horizontal to vertical plane
- Equilibrium between this excitation and damping determines ξ_{limit}

Lesson: **Keep the coupling small !**

The next problem

Remember:

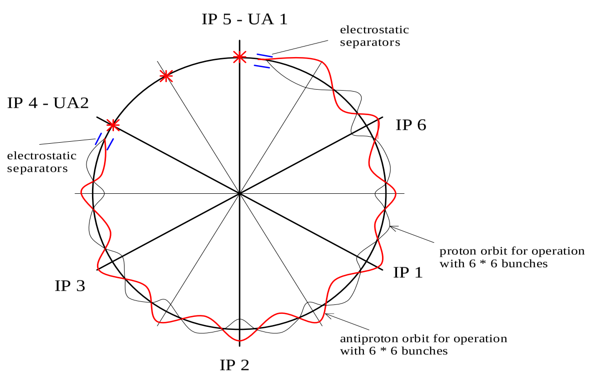
$$\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f \cdot n_B}{4\pi\sigma_x\sigma_y}$$

- How to collide many bunches (for high $\mathcal{L}$) ??
- Must avoid unwanted collisions !!
- Separation of the beams:
  - Pretzel scheme (SPS,LEP,Tevatron)
  - Bunch trains (LEP,PEP)
  - Crossing angle (LHC)

Separation: SPS

- Few equidistant bunches
  (6 against 6)
- Beams travel in same beam pipe
  (12 collision points !)
  - Two experimental areas
  - Need global separation
  - Horizontal pretzel around most of the circumference

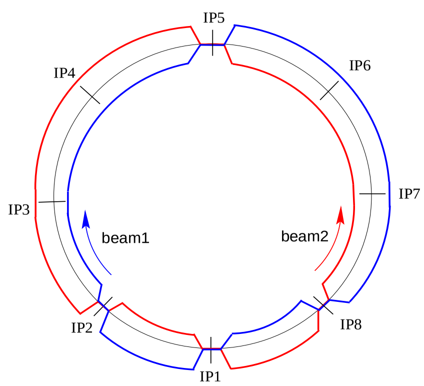
Separation: SPS



Separation: LHC

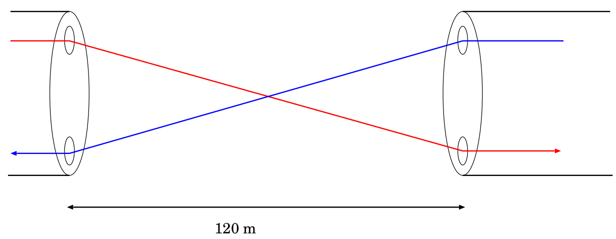
- Many equidistant bunches (2808 per beam)
- Two beams already separated in two separate beam pipes except:
  - Four experimental areas
  - Need local separation
- Two horizontal and two vertical crossing angles

Layout of LHC



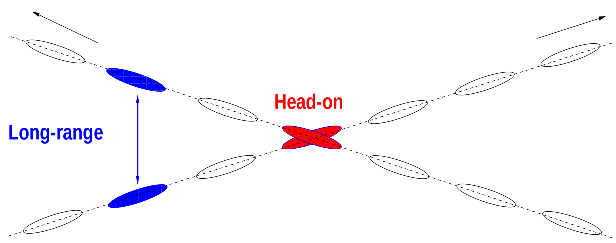
Example: LHC

- Two beams, 2808 bunches each, every 25 ns
- In common chamber around experiments



- Over 120 m: about 30 parasitic interactions

Crossing angles (example LHC)



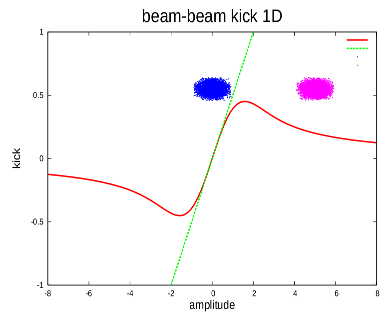
- Particles experience distant (weak) forces
- Separation typically 6 - 12 σ

➡ We get so-called long range interactions

What is special about them ?

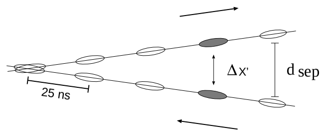
- Break symmetry between planes, stronger resonance excitation
- Mostly affect particles at large amplitudes
- Cause effects on closed orbit
- PACMAN effects
- Tune shift has opposite sign in plane of separation

Why opposite tuneshift ???



- **Local** slope has opposite sign for large separation
- **Opposite** sign for focusing !

Long range interactions (LHC)

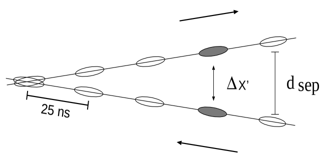


→ **For horizontal separation d:**

$$\Delta x'(x+d,y,r) = -\frac{2Nr_0}{\gamma}\cdot\frac{(x+d)}{r^2}\left[1-\exp(-\frac{r^2}{2\sigma^2})\right]$$

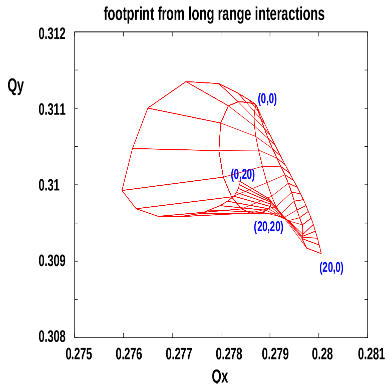
(with: $r^2 = (x+d)^2 + y^2$***)***

Long range interactions (LHC)



- **Number of long range interactions depends on spacing and length of common part**
- **In LHC 15 collisions on each side, 120 in total !**
- **Effects depend on separation:** $\Delta Q \propto -\frac{N}{d^2}$ **(for large enough d !)** **footprints ??**

Footprints



- Large for largest amplitudes where non−linearities are strong
- Size proportional to $\frac{1}{d^2}$
- Must expect problems at small separation
- Footprint very asymmetric

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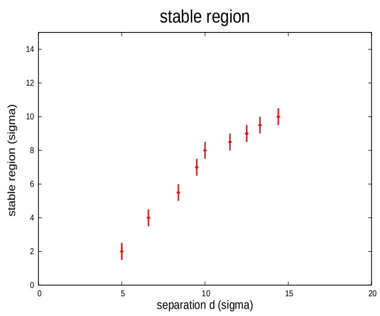
Particle losses

- Small crossing angle $\Longleftrightarrow$ small separation
- Small separation: particles become unstable and get lost



- Minimum separation for LHC: $\approx 10\ \sigma$

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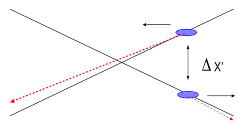
Closed orbit effects

$$\Delta x'(x+d, y, r) = -\frac{2Nr_0}{\gamma} \cdot \frac{(x+d)}{r^2}\left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

For well separated beams ($d \gg \sigma$) the force (kick) has an amplitude independent contribution: ➡ **orbit kick**

$$\Delta x' \quad = \quad \underbrace{\frac{const.}{d} \cdot [\ 1}\ \ - \ \frac{x}{d} \ + \ O\left(\frac{x^2}{d^2}\right) \ + \ ...$$



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Closed orbit effects

- **Beam-beam kick from long range interactions changes the orbit**
  - **Has been observed in LEP with bunch trains**
  - **Self-consistent calculation necessary**
  - **Effects can add up and become important**
  - **The two beams separate, more than 1σ not unusual !**

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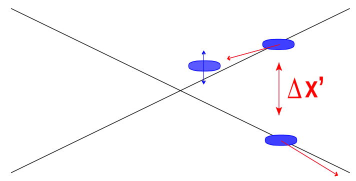
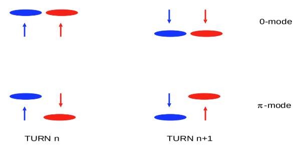
Coherent beam-beam effect



- Whole bunch sees a kick as an entity (coherent kick)
- The coherent kick of separated beams can excite coherent dipole oscillations
- All bunches couple because each bunch "sees" many opposing bunches: many coherent modes possible !

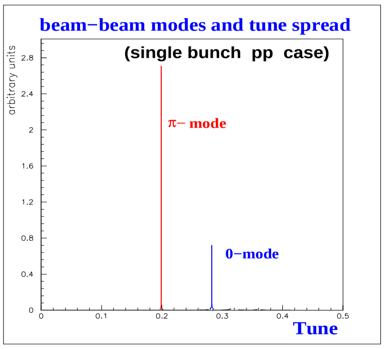
Coherent beam-beam effect

Simplest case: one bunch per beam:



- Coherent mode: two bunches are "locked" in a coherent oscillation
- 0-mode is stable (Mode with NO tune shift)
- π-mode can become unstable (Mode with LARGEST tune shift)

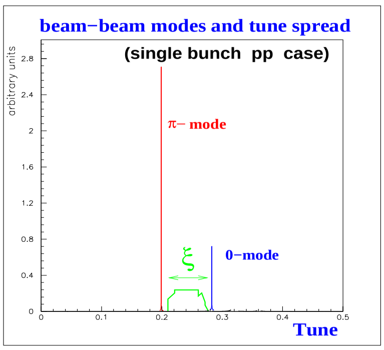
Coherent beam-beam frequencies (schematic)



- 0−mode is at unperturbed tune
- π−mode is shifted by 1.1 − 1.3 · ξ

- Two separate modes visible
- But we have many particles and tune spread ... !

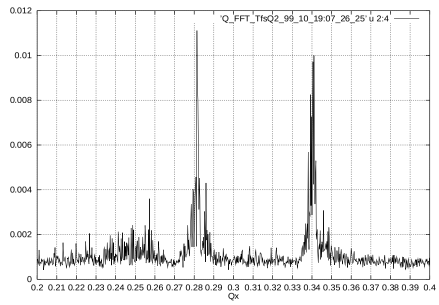
Coherent beam-beam frequencies (schematic)



- 0−mode is at unperturbed tune
- π−mode is shifted by 1.1 − 1.3 · ξ
- Incoherent spread between [0.0,1.0] · ξ

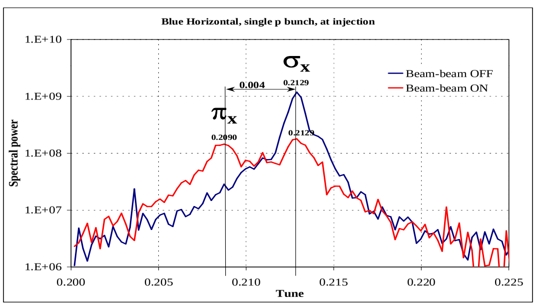
- Strong-strong case: π-mode shifted outside tune spread
- No Landau damping possible

What we measure: LEP



- Two modes clearly visible
- Can be distinguished by phase relation, i.e. sum and difference signals

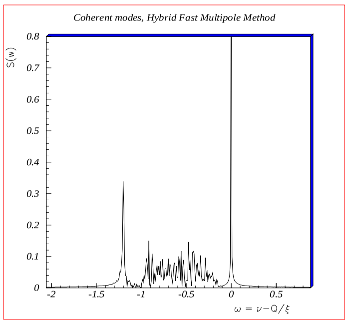
What we measure: RHIC



Courtesy W. Fischer (BNL)

➡ Compare spectra with and without beams : two modes visible with beams

Simulation of coherent spectra



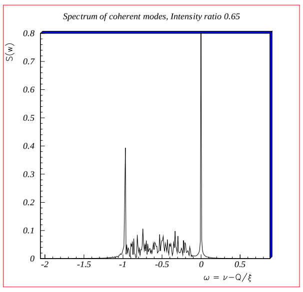
- Full simulation of both beams required
- Use up to 10^8 particles in simulations
- Must take into account changing fields
- Requires computation of arbitrary fields

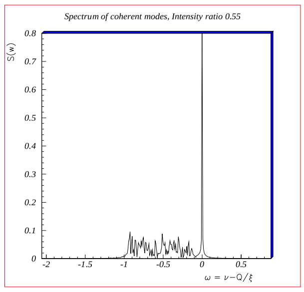
- Time consuming for many particles ..

What can be done to avoid problems ?

- Coherent motion requires 'organized' motion of many particles
- Therefore high degree of symmetry required
- Possible countermeasure: (symmetry breaking)
  - Different bunch intensity
  - Different tunes in the two beams

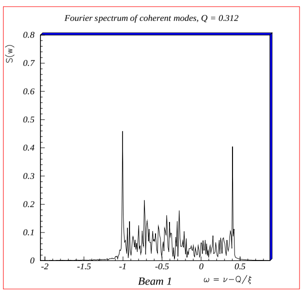
Beams with different intensity

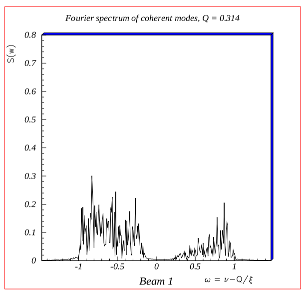




- Bunches with different intensities cannot maintain coherent motion
- Landau damping restored

Beams with different tunes





- Bunches with different tunes cannot maintain coherent motion
- Landau damping restored

Can we suppress beam-beam effects ?

- Find 'lenses' to correct beam-beam effects
  - Head on effects:
    - Linear "electron lens" to shift tunes
    - Non-linear "electron lens" to reduce spread
    - Tests in progress at Tevatron and RHIC
  - Long range effects:
    - At very large distance: force is 1/r
    - Same force as a wire !
- So far: mixed success with active compensation

Others: Möbius lattice

- Principle:
  - Interchange horizontal and vertical plane each turn
- Effects:
  - Round beams (even for leptons)
  - Some compensation effects for beam-beam interaction
  - First test at CESR at Cornell

Not mentioned:

- **Effects in linear colliders**
- **Asymmetric beams**
- **Coasting beams**
- **Beamstrahlung**
- **Synchrobetatron coupling**
- **Monochromatization**
- **Beam-beam experiments**
- **... and many more**

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