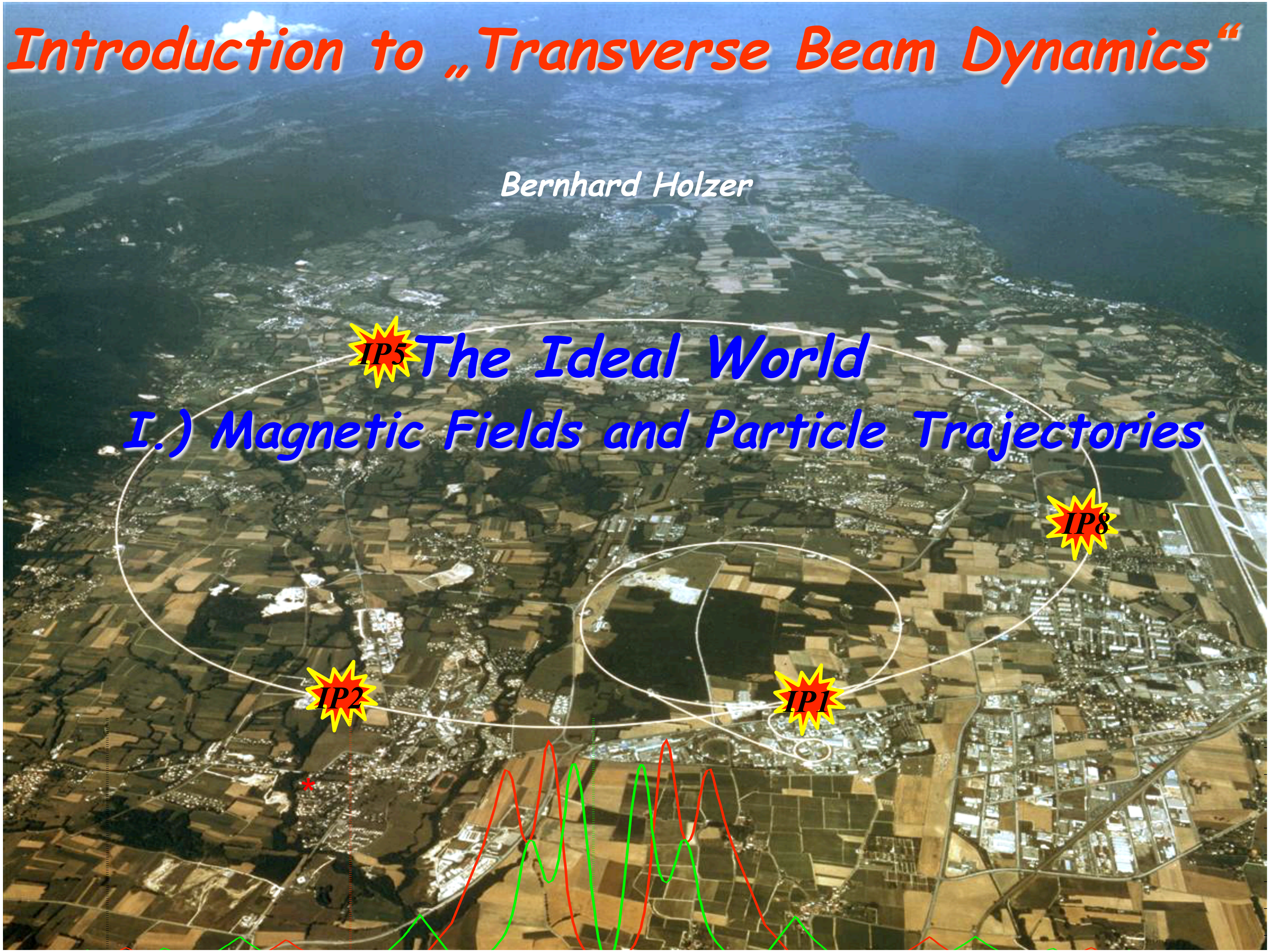
Introduction to „Transverse Beam Dynamics“

Bernhard Holzer

The Ideal World

I.) Magnetic Fields and Particle Trajectories



What we will do …

… introduce some "funny" keywords that you always wanted to understand and never really asked for.

trajectory / closed orbit / tune / resonances / chromaticity & dispersion
Higgs / structure of matter / beam emittance / adiabatic shrinking
beam size / beta function, focusing matrix / / lattice cell
mini-beta insertion / "beta-star" / dynamic aperture

… and why do the particles not follow gravity and just drop down to the bottom of the vacuum chamber (… or do they do so ?)

The „Tandem principle": Apply the accelerating voltage twice ...

... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

Example for such a „steam engine": 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

Gretchen Frage (J.W. Goethe, Faust)

Fallen die Dinger eigentlich runter ?

Antwort: JA !!

Gretchen Frage (J.W. Goethe, Faust)

Do they actually drop ?

Yes, they do !!

$$l_{VdG} = 30m$$

$$v \approx 10\% \, c \approx 3 * 10^7 m/s$$

$$\Delta t = 1 \mu s$$

Free Fall in Vacuum:

$$s = \frac{1}{2} g t^2$$

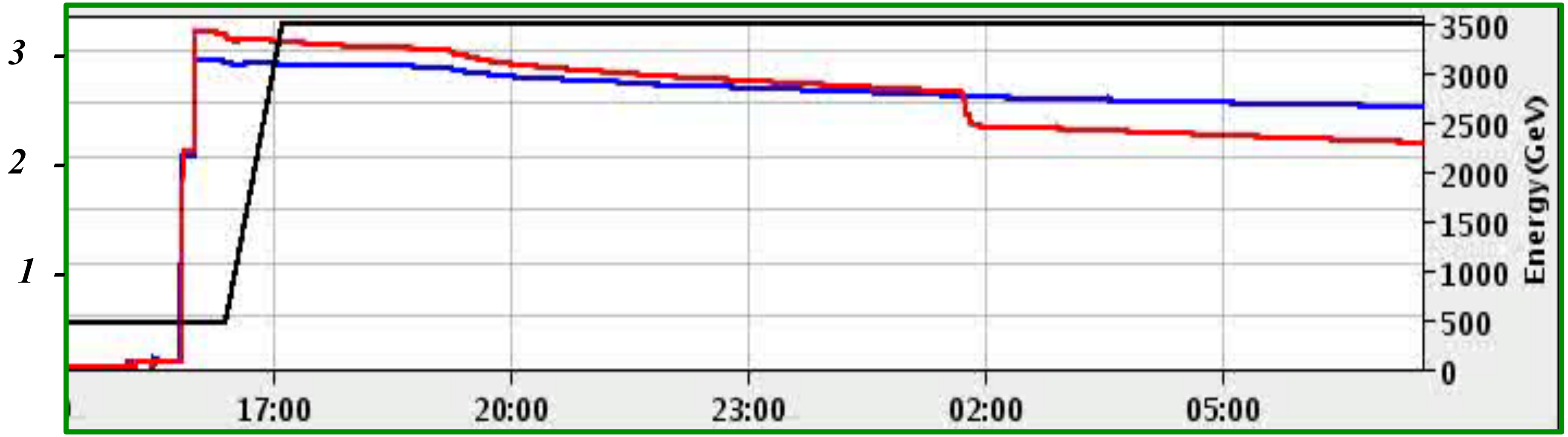
$$= \frac{1}{2} 10 \frac{m}{s^2} * (1 \mu s)^2$$

$$= 5 * 10^{-12} m = 5 pm$$

Luminosity Run of a typical storage ring:

*LHC Storage Ring: Protons **accelerated and** stored for 12 hours*
distance of particles travelling at about v ≈ c
$L = 10^{10}$-10^{11} km
... several times Sun - Pluto and back ♪

intensity (10^{11})



→ *guide the particles* ***on a well defined orbit*** *(„design orbit“)*
→ *focus the particles* ***to keep each single particle trajectory***
within the vacuum chamber of the storage ring, i.e. close to the design orbit.

1.) Introduction and Basic Ideas

„ ... in the end and after all it should be a kind of circular machine"

→ need transverse deflecting force

Lorentz force $$\vec{F} = q * (\vec{E} + \vec{v} \times \vec{B})$$

typical velocity in high energy machines: $$v \approx c \approx 3*10^8 \, m/s$$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q*3*10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

$$F = q*300 \frac{MV}{m}$$

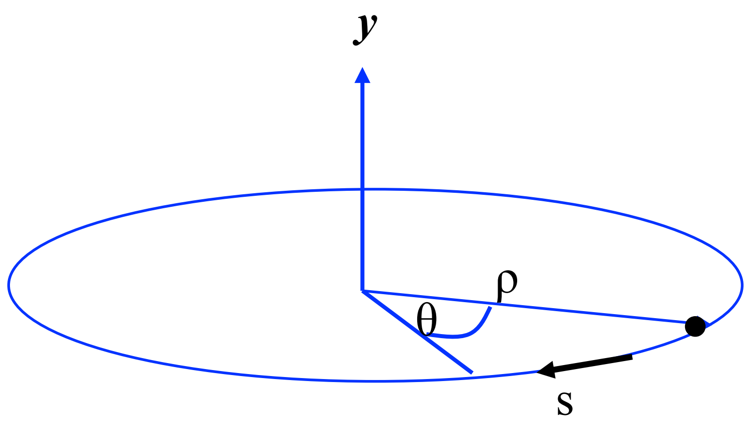
equivalent electrical field: E

Technical limit for electrical fields:

$$E \le 1 \frac{MV}{m}$$

old greek dictum of wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force $$F_L = e\,v\,B$$

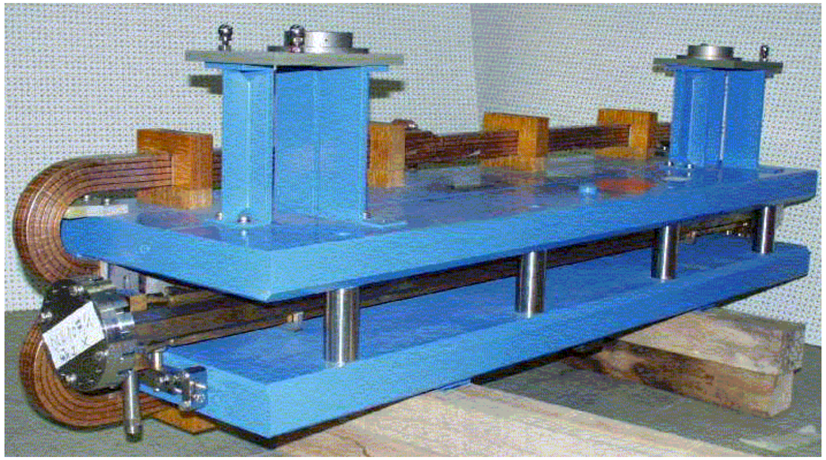
centrifugal force $$F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$$

$$\frac{\gamma\, m_0\, v^2}{\rho} = e\,v\,B$$

$$\frac{p}{e} = B\,\rho$$

B ρ = "beam rigidity"

2.) The Magnetic Guide Field



Dipole Magnets:

define the ideal orbit
homogeneous field created
by two flat pole shoes

$$B = \frac{\mu_0 \, n \, I}{h}$$

$$\frac{p}{e} = B\rho \quad \longrightarrow \quad \rho = \frac{p}{B * e}$$

The bending radius ... and so the size of the machine is determined by the dipole field and the particle momentum

convenient units:

$$B = [T] = \left[\frac{Vs}{m^2}\right] \qquad p = \left[\frac{GeV}{c}\right]$$

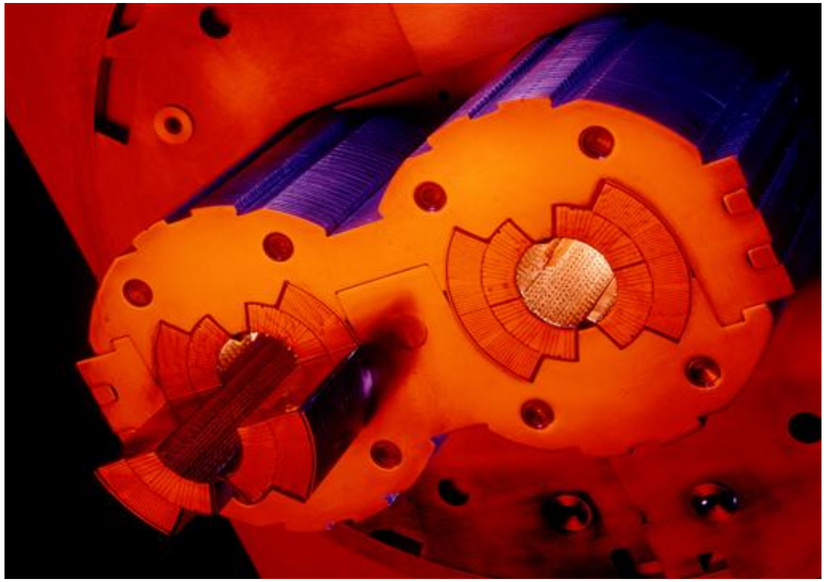
Example LHC:

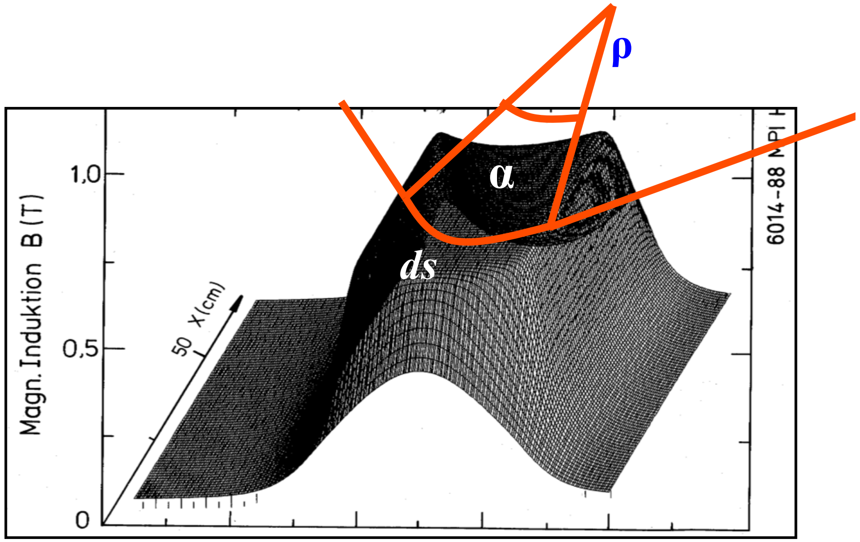
$$B = 8.3\,T$$

$$p = 7000\,\frac{GeV}{c}$$

$$\rho = 2.83\ km$$

The Magnetic Guide Field





field map of a storage ring dipole magnet

$$B \approx 1 \ldots 8 \;T$$

The dipole magnets of a storage ring (or synchrotron) create a circle (... better polygon) of circumference $2\pi\rho$ and define the maximum momentum of the particle beam.

Example LHC: → $2\pi\rho = 17.6$ *km*

$\approx 66\%$

About 1/3 of the ring size is still needed for straight sections, rf cavities, diagnostics, injection, extraction, high energy physics detectors etc etc

The Problem:

LHC Design Magnet current: I=11850 A

and the machine is 27 km long !!!

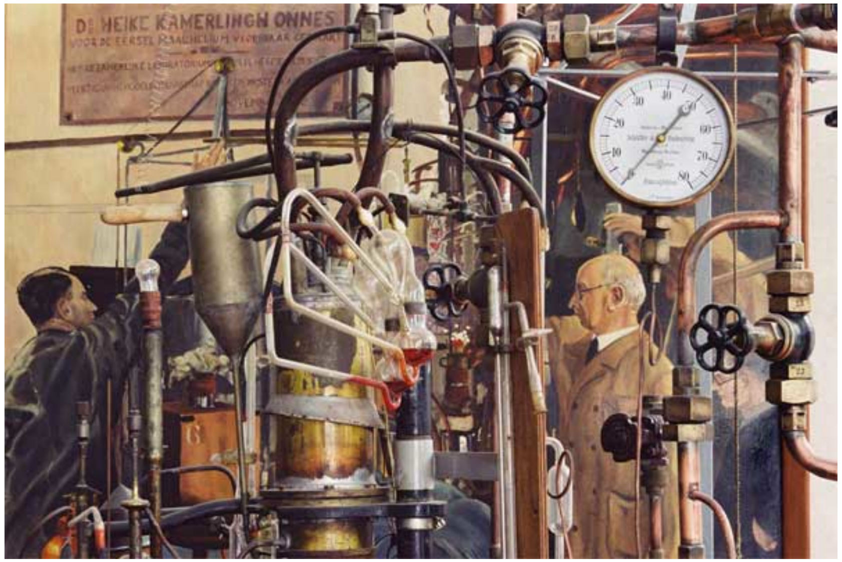
Ohm's law: $U = R * I, \quad P = R * I^2$

Problem:

reduce ohmic losses to the absolute minimum

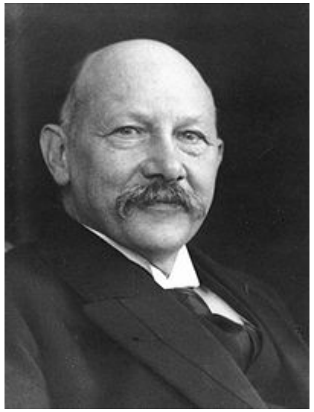
The Solution:

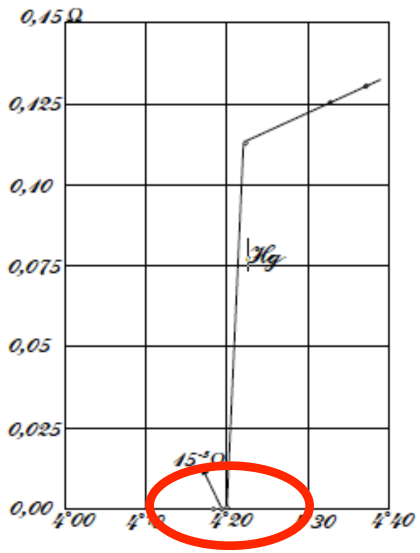
super conductivity

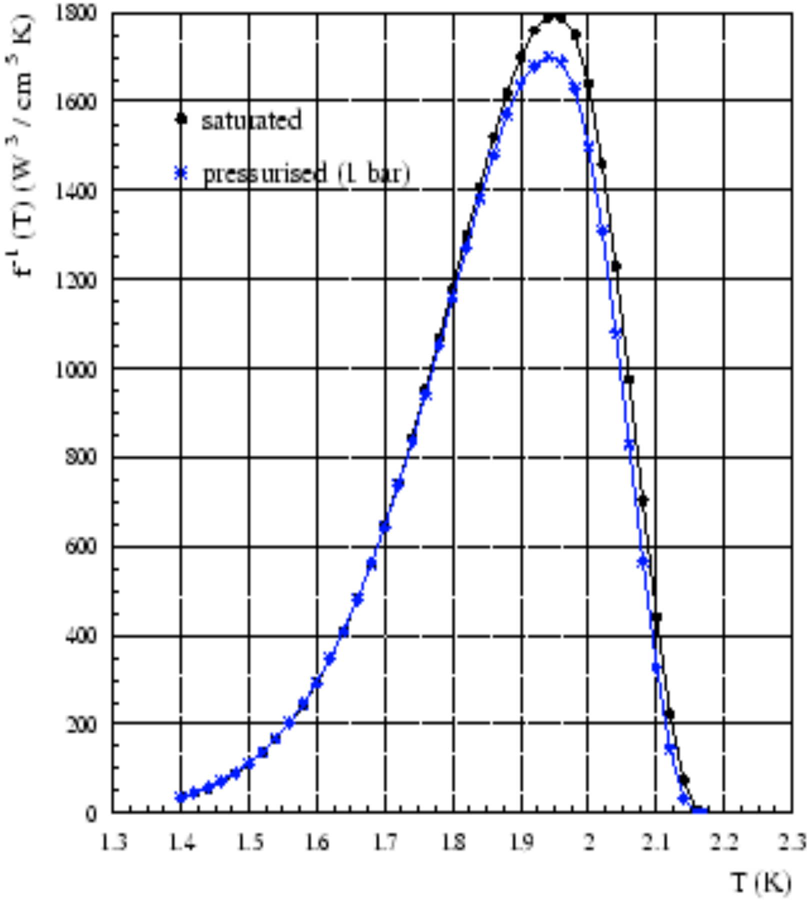


Super Conductivity ...
... and why we run at 1.9 K

discovery of sc. by H. Kammerling Onnes, Leiden 1911

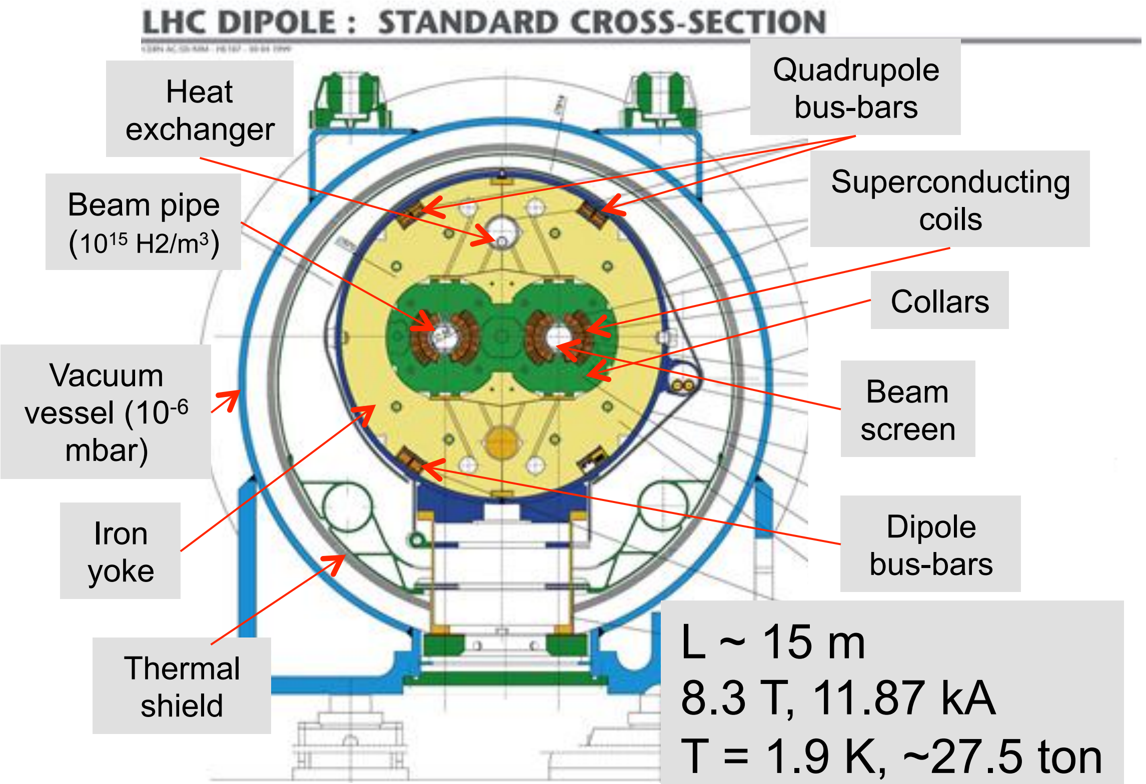




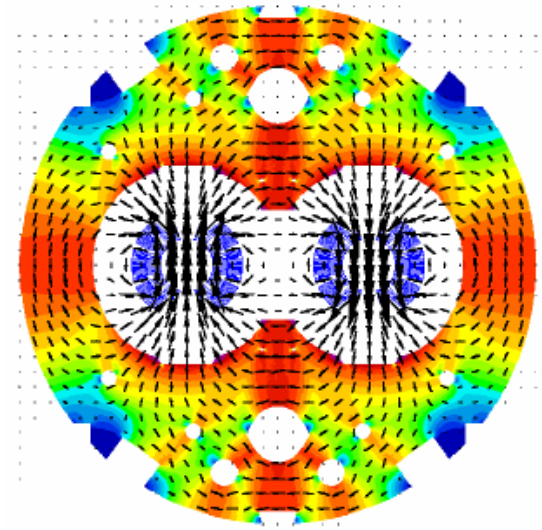


thermal conductivity of fl. Helium in supra fluid state

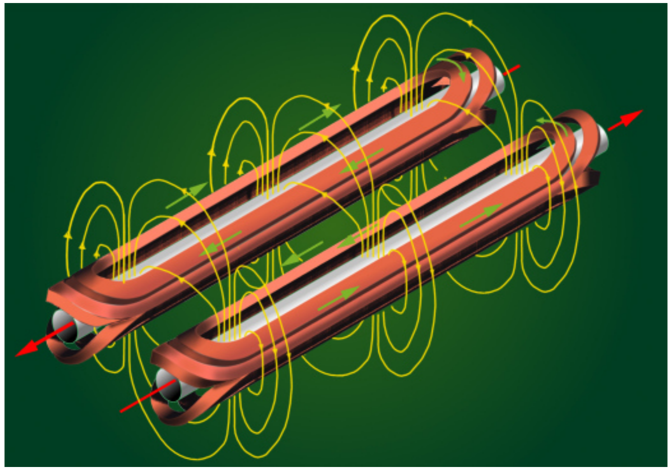
LHC: The -1232- Main Dipole Magnets

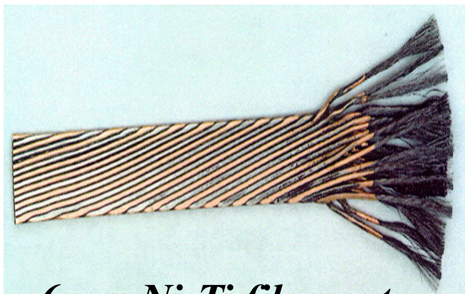


L ~ 15 m
8.3 T, 11.87 kA
T = 1.9 K, ~27.5 ton

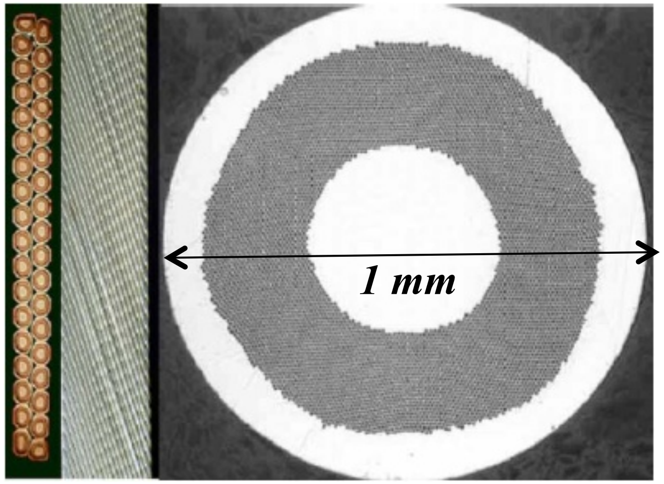


required field quality:
$\Delta B/B=10^{-4}$





6 μm Ni-Ti filament

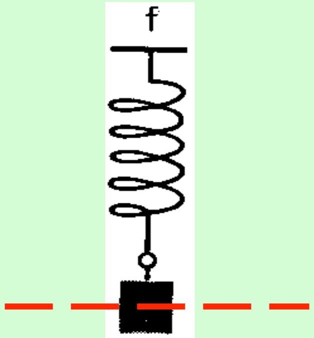


3.) Focusing Properties – Transverse Beam Optics

... keeping the flocs together:
In addition to the pure bending of the beam
we have to keep 10^{11} particles close together

focusing force

classical mechanics:
pendulum



there is a restoring force, proportional to the elongation x:

$$m * \frac{d^2 x}{dt^2} = -c * x$$

general solution: free harmonic oszillation $x(t) = A * \cos(\omega t + \varphi)$

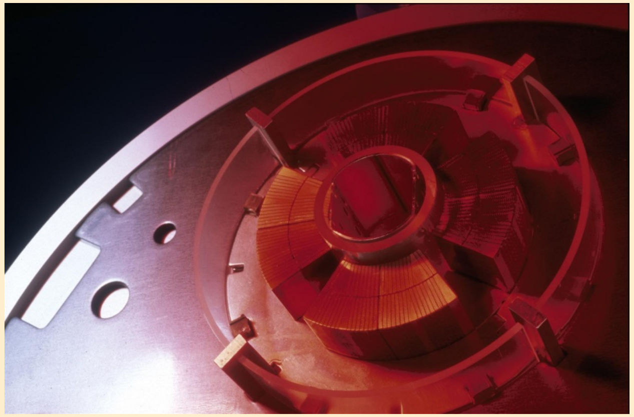
this is how grandma's Kuckuck's clock is working!!!

Quadrupole Magnets:

Storage Rings: *linear increasing Lorentz force* ***to keep trajectories in vicinity of the ideal orbit***

linear increasing magnetic field $B_y = g\,x \qquad B_x = g\,y$

$$F(x) = q * v * B(x)$$



LHC main quadrupole magnet $g \approx 25 \ldots 220 \;\; T/m$

Table 7.13: Parameter list for main quadrupole magnets (MQ) at 7.0 TeV

Integrated Gradient	690	T
Nominal Temperature	1.9	K
Nominal Gradient	223	T/m
Peak Field in Conductor	6.85	T
Temperature Margin	2.19	K
Working Point on Load Line	80.3	%
Nominal Current	11870	A
Magnetic Length	3.10	M
Beam Separation distance (cold)	194.0	mm

Focusing forces and particle trajectories:

normalise magnet fields to momentum
*(remember: **B*ρ = p / q**)*

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

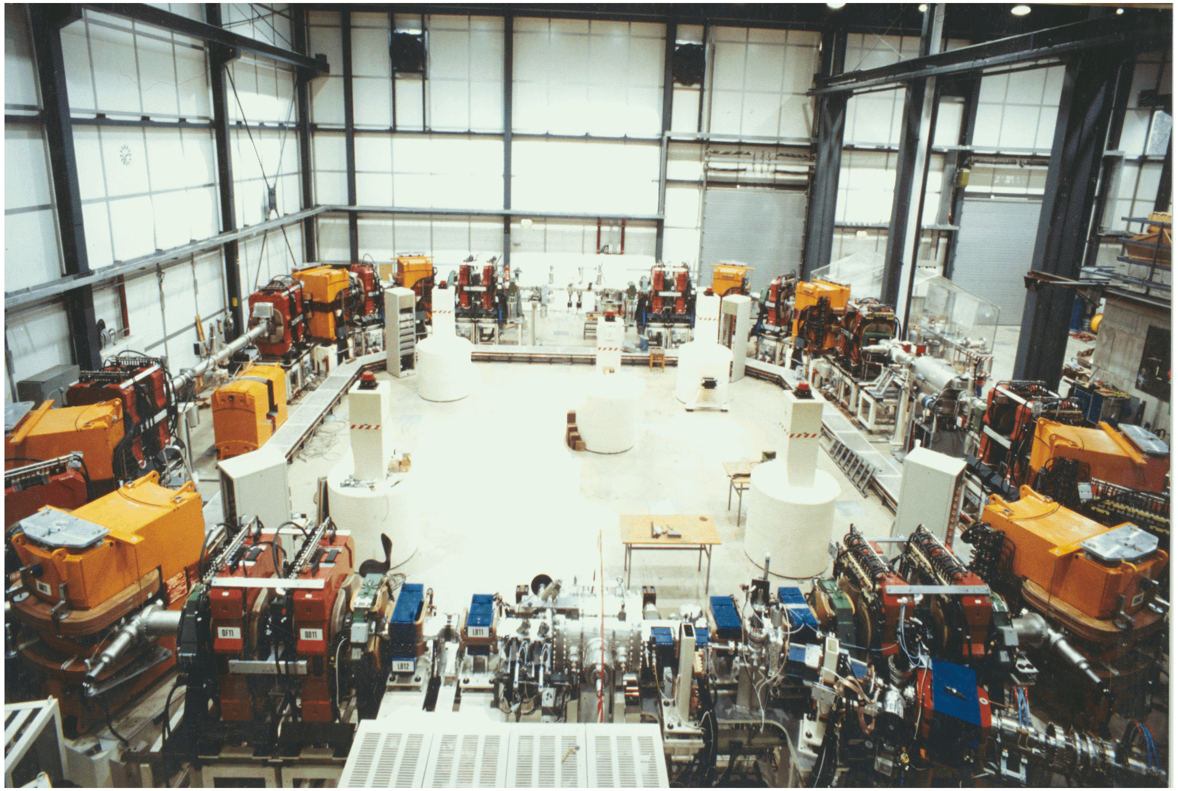
Quadrupole Magnet

$$k := \frac{g}{p/q}$$

The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \frac{1}{2!}\,m\,x^2 + \frac{1}{3!}\,n\,x^3 + \ldots$$

only terms linear in x, y taken into account *dipole fields*
quadrupole fields



Separate Function Machines:

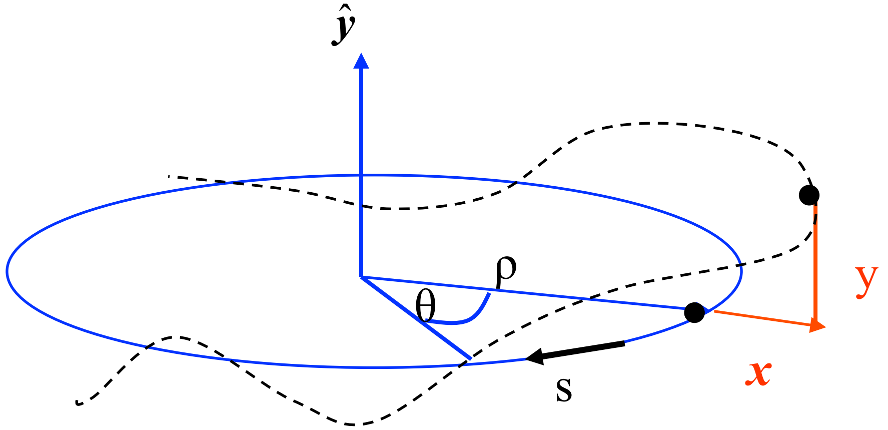
Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

* *man sieht nur dipole und quads → linear*

The Equation of Motion:



* *Equation for the **horizontal motion:***

$$x'' + x\left(\frac{1}{\rho^2} + k\right) = 0$$

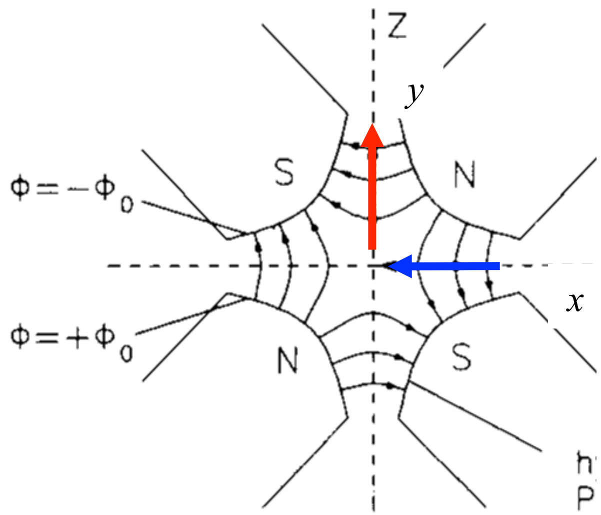
x = particle amplitude
x' = angle of particle trajectory (wrt ideal path line)

* *Equation for the **vertical motion:***

$$\frac{1}{\rho^2} = 0$$ *no dipoles … in general …*

$$k \leftrightarrow -k$$ *quadrupole field changes sign*

$$y'' - k\,y = 0$$



5.) Solution of Trajectory Equations

Define … hor. plane: K= 1/ρ² + k

… vert. Plane: K = - k

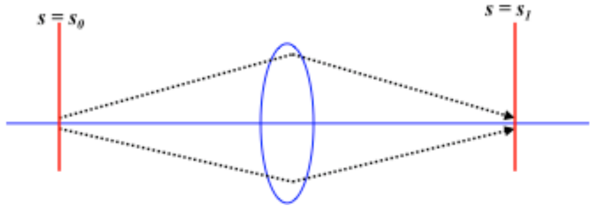
$$x'' + K\,x = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: *Hor. Focusing Quadrupole K > 0:*

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s)$$



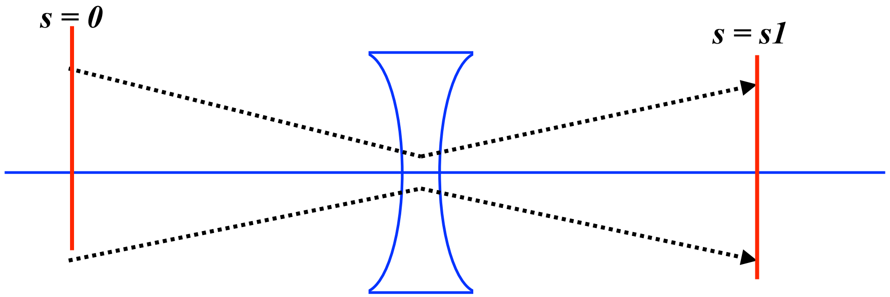
For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$M_{foc} = \begin{pmatrix} \cos\left(\sqrt{|K|}l\right) & \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}l\right) \\ -\sqrt{|K|}\sin\left(\sqrt{|K|}l\right) & \cos\left(\sqrt{|K|}l\right) \end{pmatrix}$$

hor. defocusing quadrupole:

$$x'' - K\,x = 0$$



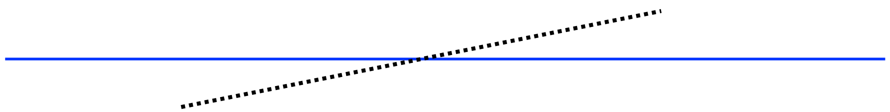
Ansatz: ***Remember from school***

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$

drift space:

$$K = 0$$



$$x(s) = x'_0 * s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

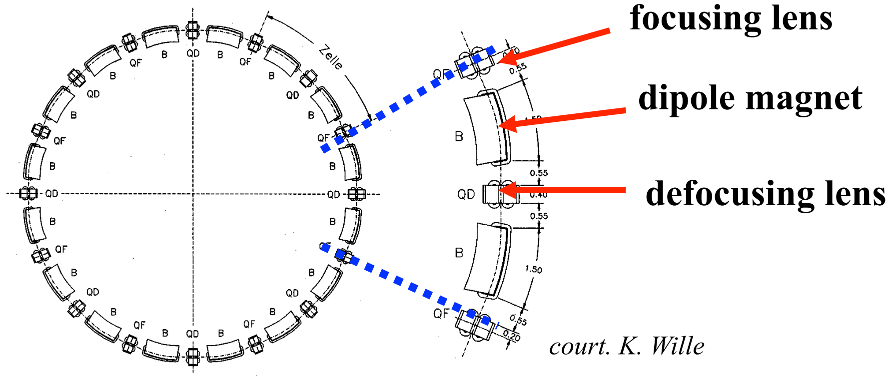
! with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“

Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

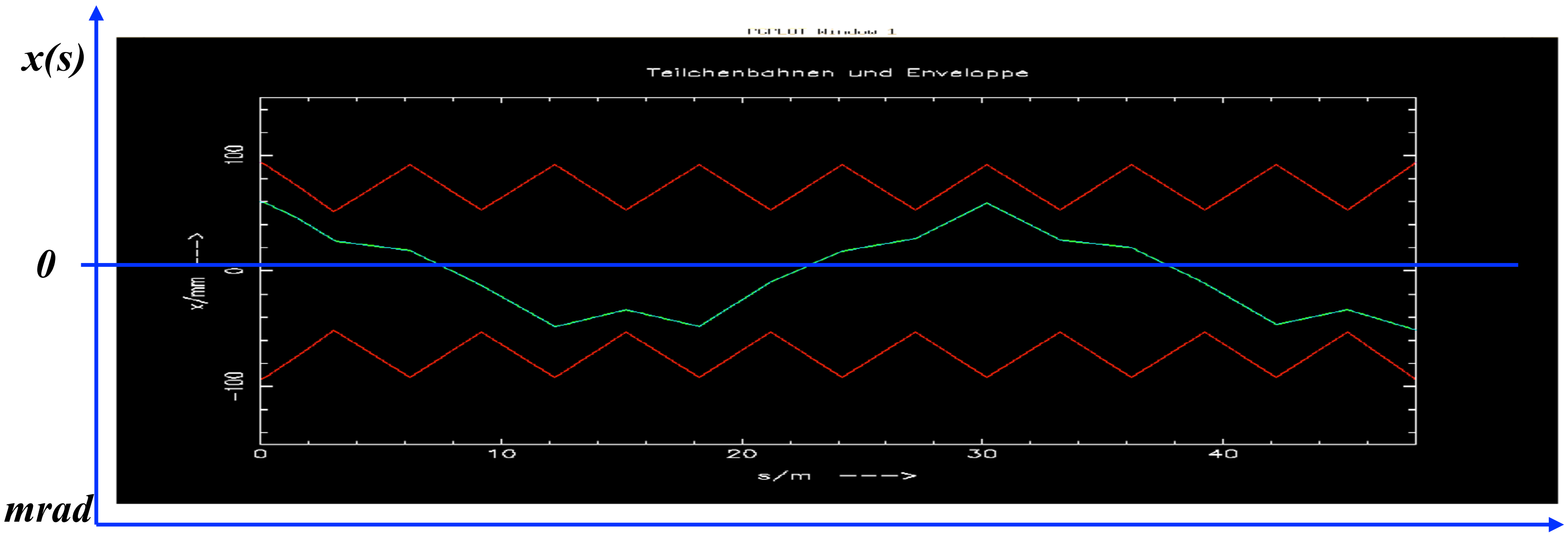
$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*}.....$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



court. K. Wille

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator ! ! !



typical values in a strong foc. machine:

$x \approx mm$, $x' \leq mrad$

Ok … ok ... it's a bit complicated and cosh and sinh and all that is a pain.
BUT ... compare ...

Weak Focusing / Strong Focusing

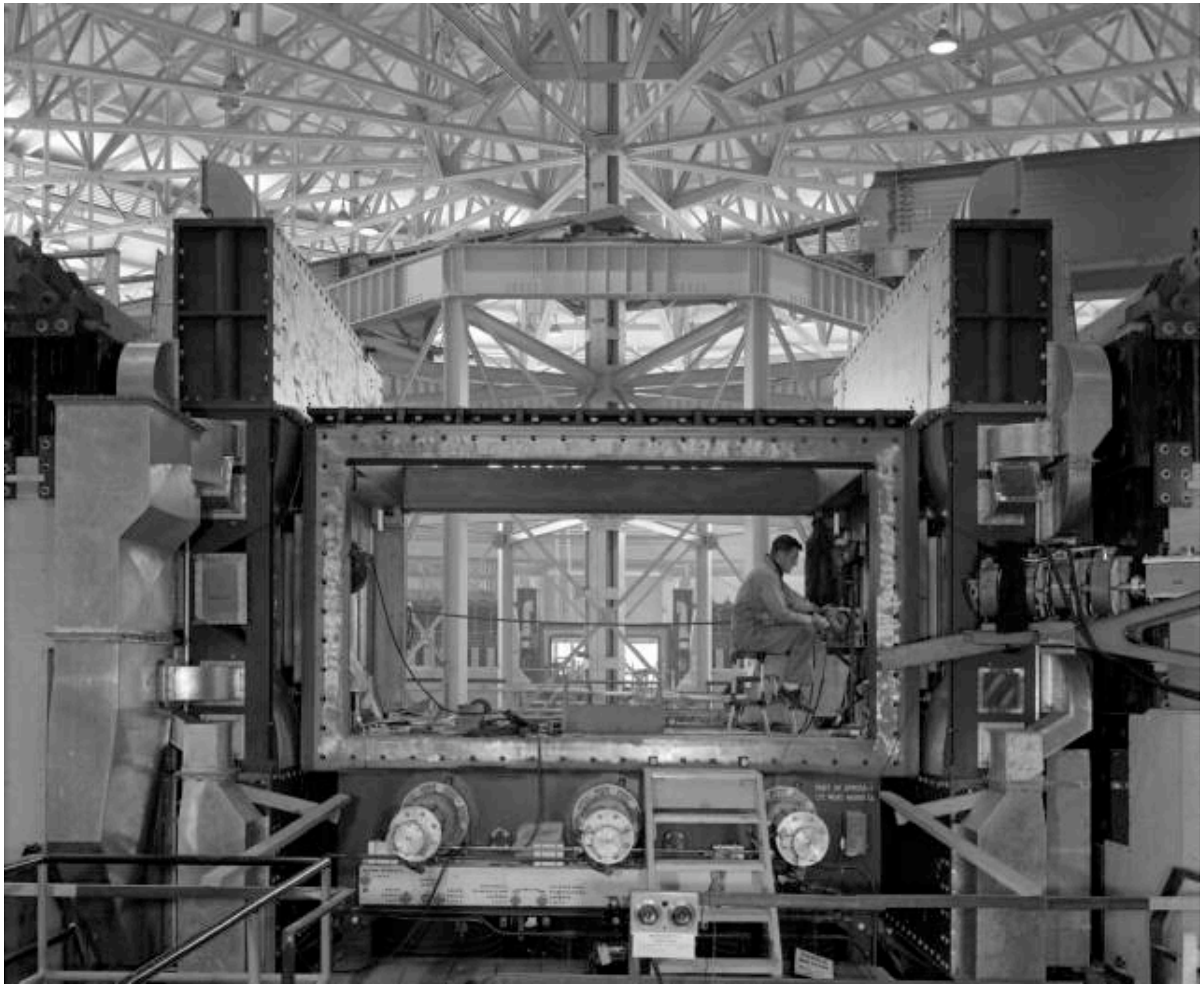
weak focusing term = $1/\rho^2$

$$x'' + x\left(\frac{1}{\rho^2} + \cancel{k}\right) = 0$$

Problem: the higher the energy, the larger the machine

The last weak focusing high energy machine … BEVATRON

→ *large apertures needed*
→ *very expensive magnets*

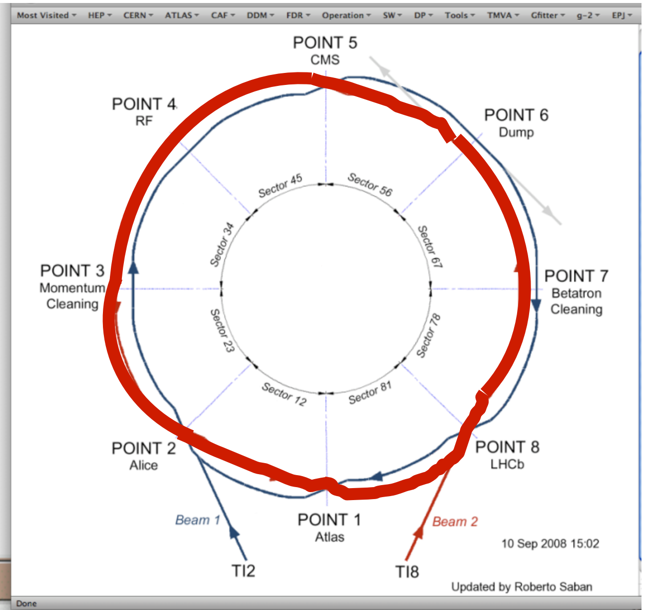


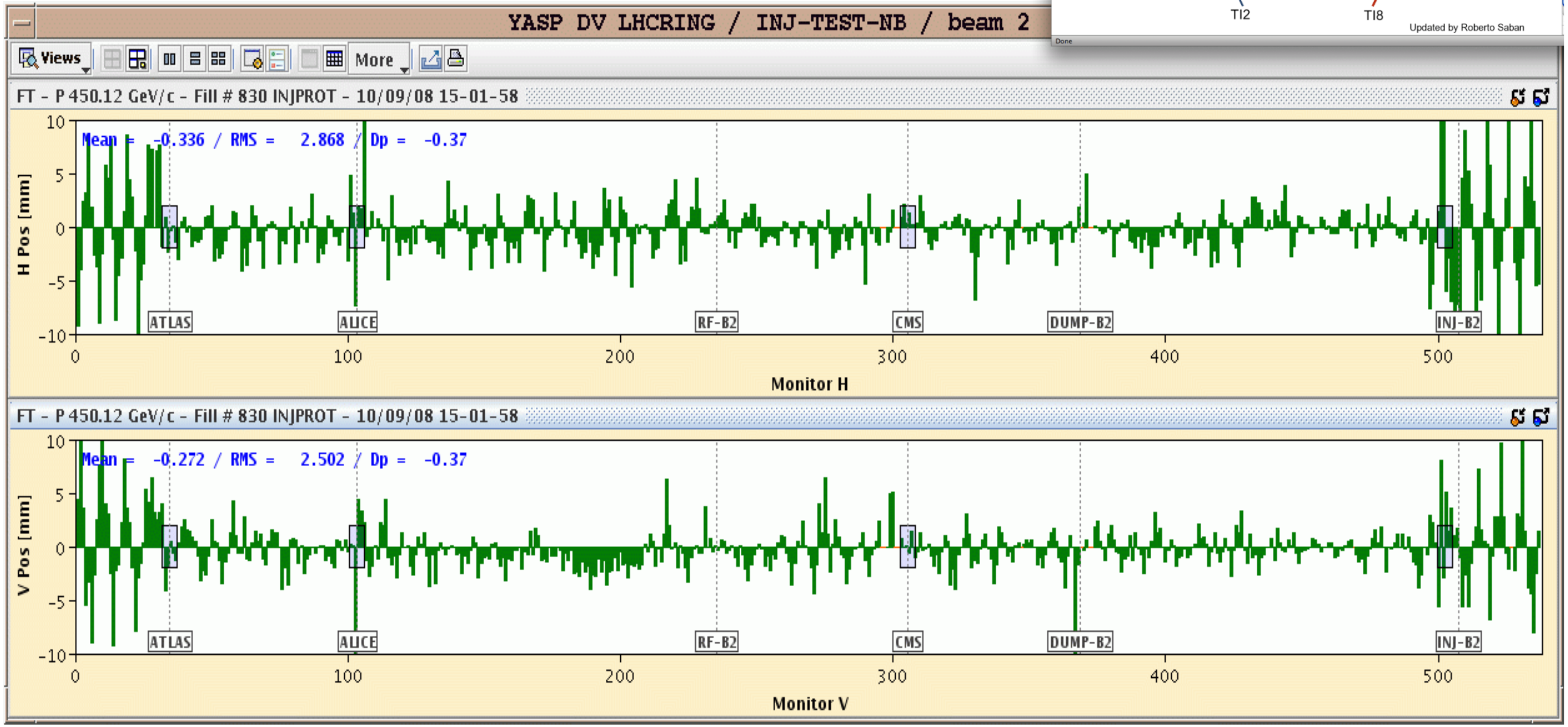
LHC Operation: Beam Commissioning

First turn steering "by sector:"

- One beam at the time
- Beam through 1 sector (1/8 ring),

correct trajectory, open collimator and move on.



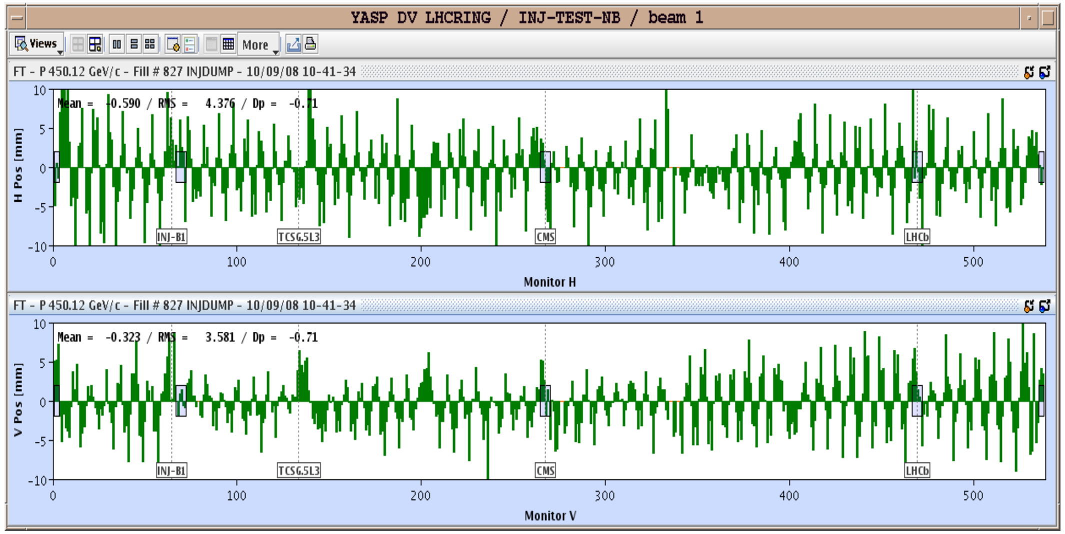


“Once more unto the breach, dear friends, once more”
(W. Shakespeare, Henry 5)

“Do they actually drop ?”

Answer: No

6.) Orbit & Tune:



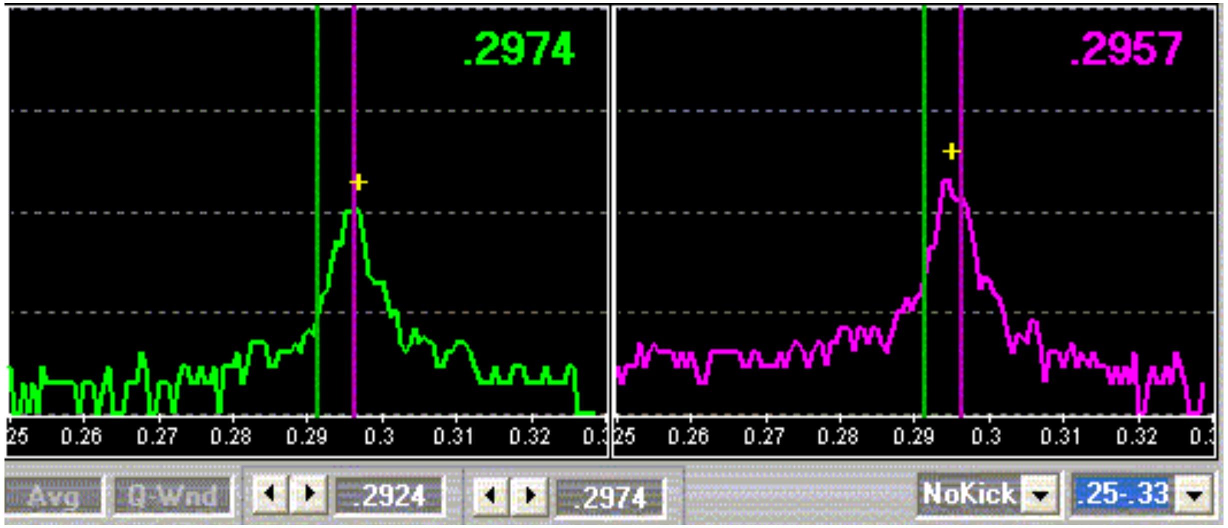
Tune: number of oscillations per turn

64.31
59.32

***Relevant** for beam stability:*
non integer part

LHC revolution frequency: 11.3 kHz

$$0.31*11.3 = 3.5kHz$$



... and the tunes in x and y are different.

i.e. we can apply different focusing forces in the two planes

i.e. we can create different beam sizes in the two planes

Dipole Magnets ...

... bend the particle trajectories onto a „polygon" (... welll kind of ring),
... define the geometry of the machine
... define the maximum momentum (… or energy) that the particle beam will have
... have a small contribution to the focusing of the beam

Quadrupole Magnets ...

... focus every single particle trajectory towards the centre of the vacuum chamber
... define the beam size
... „produce" the tune
... increase the luminosity

Trajectory ...

... under the influence of the focusing fields the particles follow a certain path along the machine. They are oscillating transversely, while moving around the "ring".

Closed Orbit ...

... There is one (!) trajectory that closes upon itself. It is given by the foc. fields and it is what we „see" when we observe the BPM readings of the stored beam.

... The single particle will perform transverse oscillations and so the single particle trajectories will oscillate (= betatron oscillations) around this closed orbit.

The Tune ...

... is the number of these transverse oscillations per turn and corresponds to the „Eigenfrequency" or sound of the particle oscilations.

There is a tune for the horizontal, the vertical and the longitudinal oscillation.

And we could even hear it ... if there were no vacuum.