RF Systems

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Outline

- Definitions and basic concepts
- On modulation
- Digital Signal Processing
- RF System & Control Loops
- RF Power Sources
- Fields in a Waveguide
- From Waveguide to Cavity
- Accelerating Gap
- Characterizing a Cavity
- Many Gaps
- Superconducting Cavities
- Some Examples of RF Systems

Definitions & basic concepts

dB

t-domain vs. ω-domain

phasors

Decibel (dB)

- Convenient logarithmic measure of a power ratio.
- A "Bel" (= 10 dB) is defined as a power ratio of 10^1. Consequently, 1 dB is a power ratio of $10^{0.1} \approx 1.259$
- If *rdb* denotes the measure in dB, we have:

$$rdb = 10\,\text{dB}\log\left(\frac{P_2}{P_1}\right) = 10\,\text{dB}\log\left(\frac{A_2^2}{A_1^2}\right) = 20\,\text{dB}\log\left(\frac{A_2}{A_1}\right)$$

$$\frac{P_2}{P_1} = \frac{A_2^2}{A_1^2} = 10^{rdb/(10\,\text{dB})} \qquad \frac{A_2}{A_1} = 10^{rdb/(20\,\text{dB})}$$

rdb	-30 dB	-20 dB	-10 dB	-6 dB	-3 dB	0 dB	3 dB	6 dB	10 dB	20 dB	30 dB
P_2/P_1	0.001	0.01	0.1	0.25	.50	1	2	3.98	10	100	1000
A_2/A_1	0.0316	0.1	0.316	0.50	.71	1	1.41	2	3.16	10	31.6

- Related: dBm (relative to 1 mW), dBc (relative to carrier)

Time domain – frequency domain (1)

- An arbitrary signal *g(t)* can be expressed in *ω*-domain using the ***Fourier transform*** (FT).

$$g(t) \circ\!\!-\!\!\bullet\ G(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(t) e^{j\omega t}\, dt$$

- The inverse transform (IFT) is also referred to as ***Fourier Integral***

$$G(\omega) \bullet\!\!-\!\!\circ\ g(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) e^{-j\omega t}\, d\omega$$

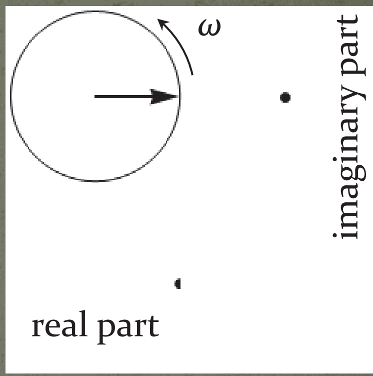
- The advantage of the *ω*-domain description is that linear time-invariant (LTI) systems are much easier described.
- The mathematics of the FT requires the extension of the definition of a *function* to allow for infinite values and non-converging integrals.
- The FT of the signal can be understood at looking at "what frequency components it is composed of".

Time domain – frequency domain (2)

- For *T*-periodic signals, the FT becomes the Fourier-Series, *dω* becomes $2\pi/T$, ∫ becomes Σ.
- The cousin of the FT is the ***Laplace transform***, which uses a complex variable (often *s*) instead of *jω*; it has generally a better convergence behaviour.
- Numerical implementations of the FT require discretisation in *t* (sampling) and in *ω*. There exist very effective algorithms (FFT).
- In digital signal processing, one often uses the related *z*-Transform, which uses the variable $z = e^{j\omega\tau}$, where *τ* is the sampling period. A delay of *kτ* becomes z^{-k}.

Fixed frequency oscillation (steady state, CW) Definition of phasors

- General: $A\cos(\omega t-\varphi) = A\cos(\omega t)\cos\varphi + A\sin(\omega t)\sin\varphi$
- This can be interpreted as the projection on the real axis of a circular motion in the complex plane: $\Re\{A(\cos\varphi + j\sin\varphi)e^{j\omega t}\}$



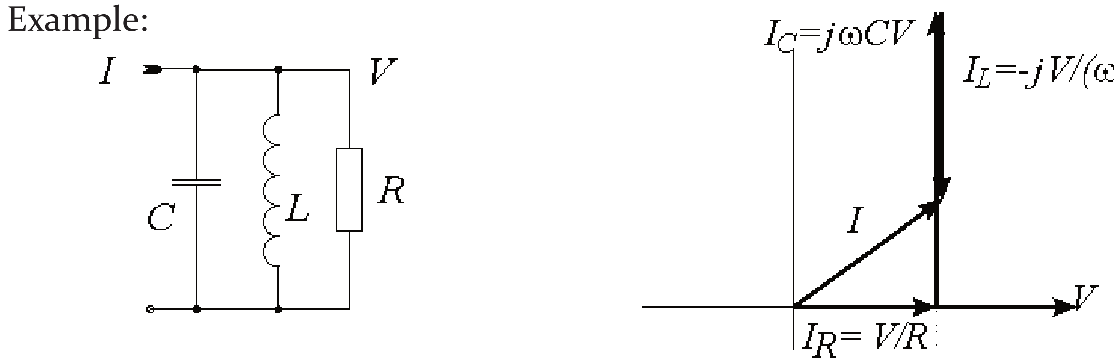
- The complex amplitude $\tilde{A}$ is called "phasor";

$$\tilde{A} \equiv A(\cos\varphi + j\sin\varphi)$$

Calculus with phasors

- Why this seeming "complication"?:
  Because things become easier!
- Using $\frac{d}{dt} \equiv j\omega$, one may now forget about the rotation with ω and the projection on the real axis, and do the complete analysis making use of complex algebra!

Example:



$$I = V\left(\frac{1}{R} + j\omega C - \frac{j}{\omega L}\right)$$

Slowly varying amplitudes

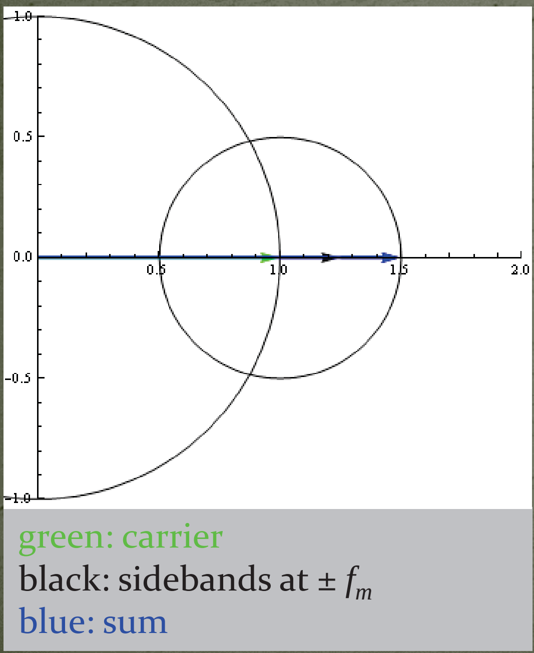
- For band-limited signals, one may conveniently use "slowly varying" phasors and a fixed frequency RF oscillation.
- So-called in-phase (I) and quadrature (Q) "baseband envelopes" of a modulated RF carrier are the real and imaginary part of a slowly varying phasor.

On Modulation

AM
PM
I-Q

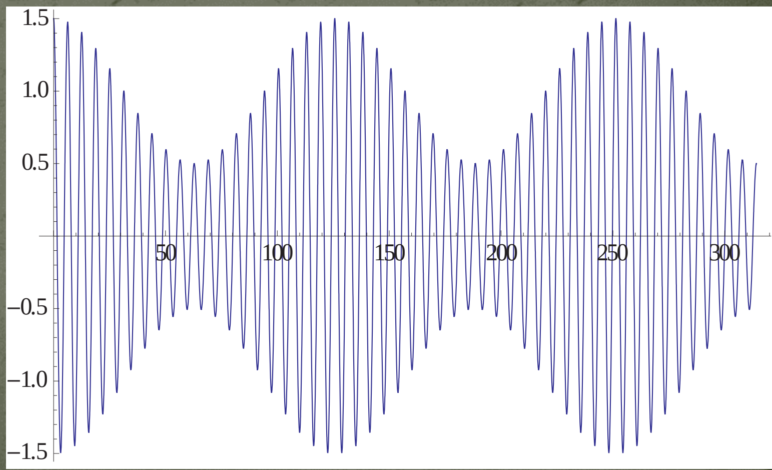
Amplitude modulation

$$(1 + m\cos(\varphi)) \cdot \cos(\omega_c t) = \Re\left\{\left(1 + \frac{m}{2}e^{j\varphi} + \frac{m}{2}e^{-j\varphi}\right)e^{j\omega_c t}\right\}$$



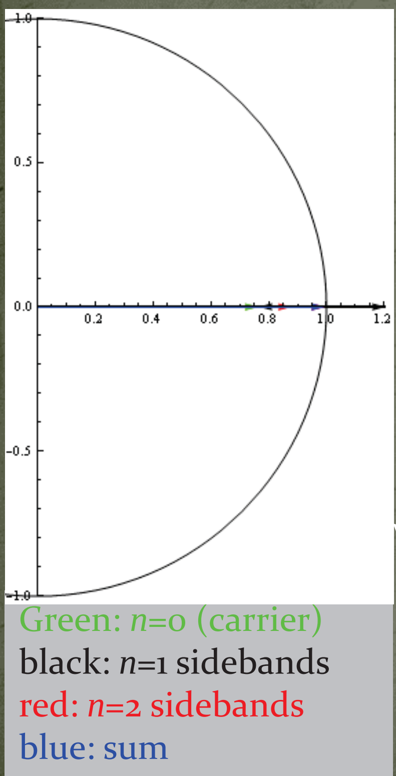
m: modulation index or modulation depth

example: $\varphi = \omega_m t = 0.05\,\omega_c t$
$m = 0.5$



Phase modulation

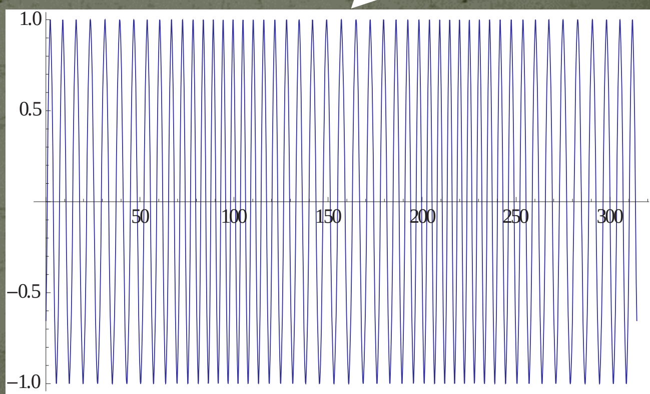
$$\Re\{e^{j(\omega_c t + M\sin\varphi)}\} = \Re\left\{\sum_{n=-\infty}^{\infty} J_n(M)e^{j(n\varphi+\omega_c t)}\right\}$$



$M = 1$

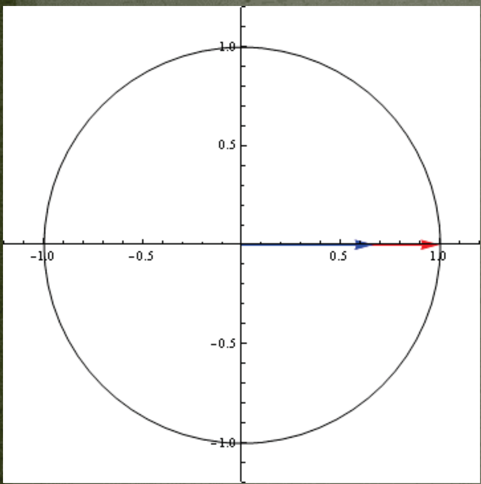
M: modulation index
(= max. phase deviation)

$\varphi = \omega_m t = 0.05\,\omega_c t, M = 4$

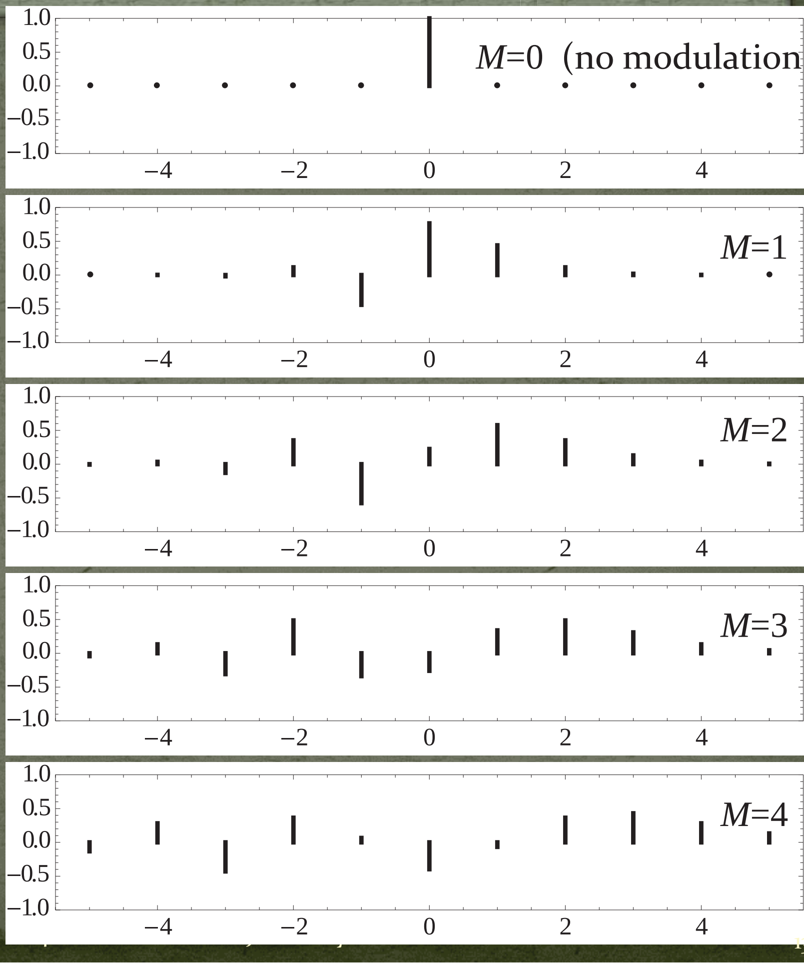


Spectrum of phase modulation

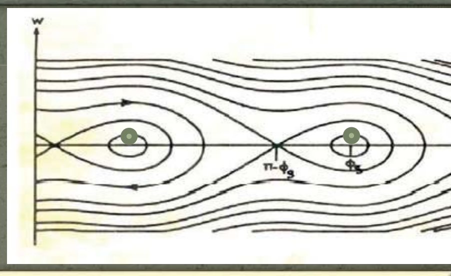
Plotted: spectral lines for sinusoidal PM at f_m
Abscissa: $(f-f_c)/f_m$

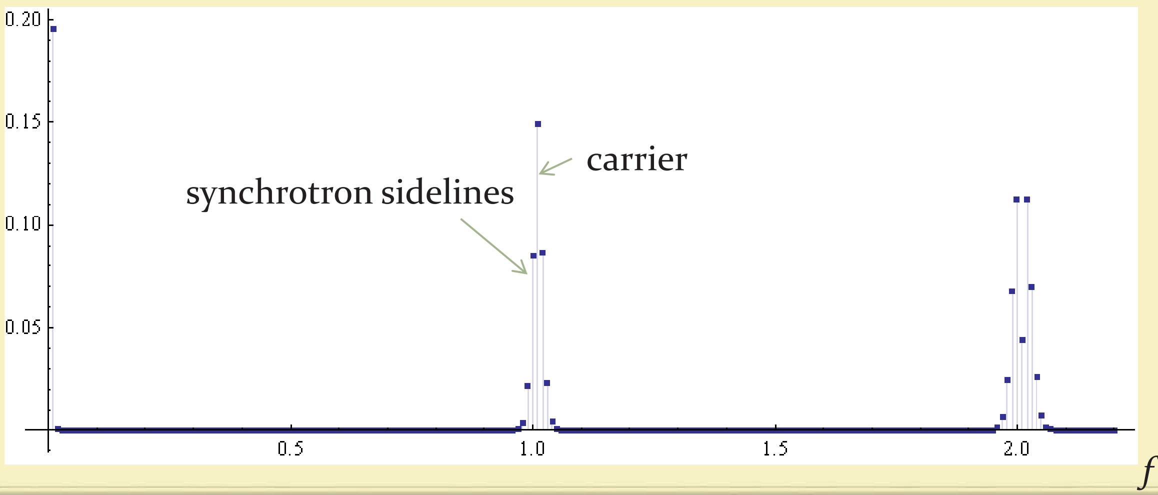


Phase modulation with $M=\pi$:
red: real phase modulation
blue: sum of sidebands $n\leq 3$

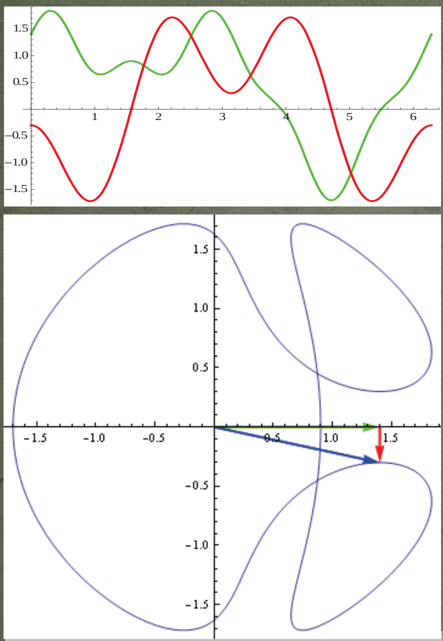


Spectrum of a beam with synchrotron oscillation, M =1 (=57°)





Vector (I-Q) modulation



I-Q modulation:
green: I component
red: Q component
blue: vector-sum

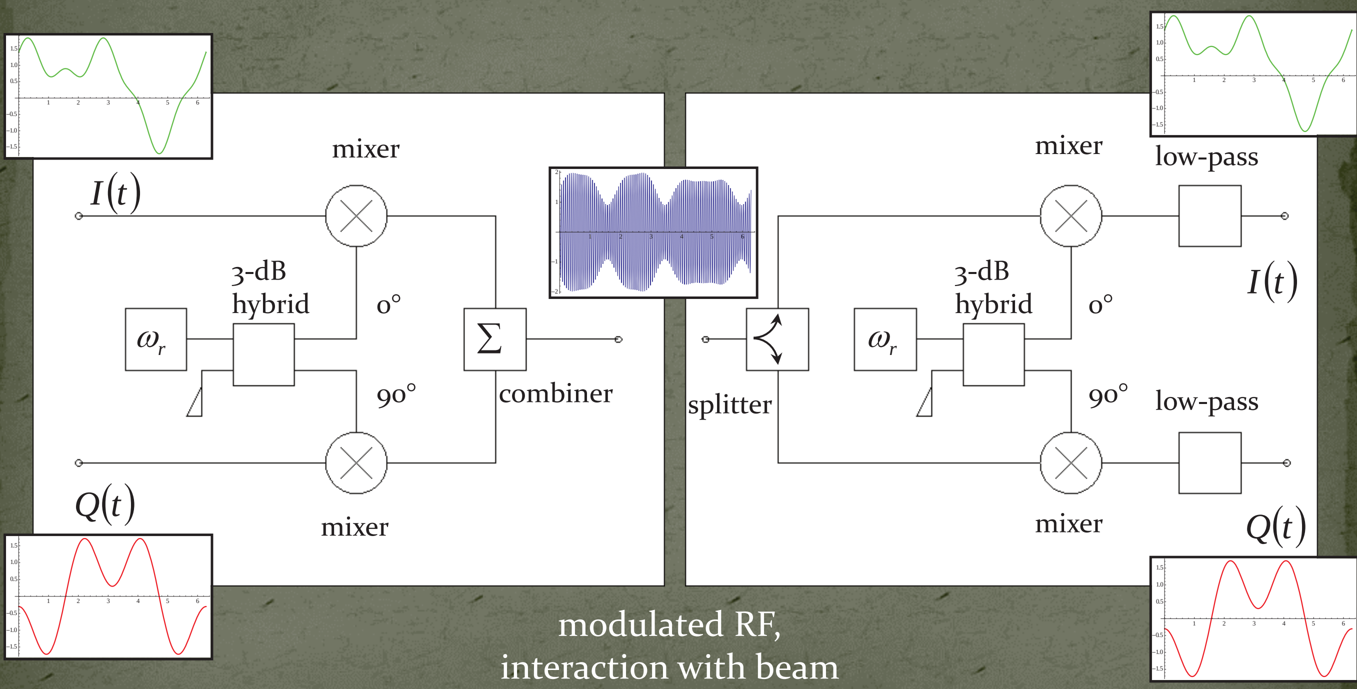
More generally, a modulation can have both amplitude and phase modulating components. They can be described as the in-phase (I) and quadrature (Q) components in a chosen reference, $\cos(\omega_r t)$. In complex notation, the modulated RF is:

$$\mathrm{Re}\left\{\left(I(t)+jQ(t)\right)e^{j\omega_r t}\right\}=$$
$$\mathrm{Re}\left\{\left(I(t)+jQ(t)\right)\left(\cos(\omega_r t)+j\sin(\omega_r t)\right)\right\}$$
$$I(t)\cos(\omega_r t)-Q(t)\sin(\omega_r t)$$

So I and Q are the cartesian coordinates in the complex "Phasor" plane, where amplitude and phase are the corresponding polar coordinates.

$$I(t)=A(t)\cdot\cos(\varphi)$$
$$Q(t)=A(t)\cdot\sin(\varphi)$$

Vector modulator/demodulator

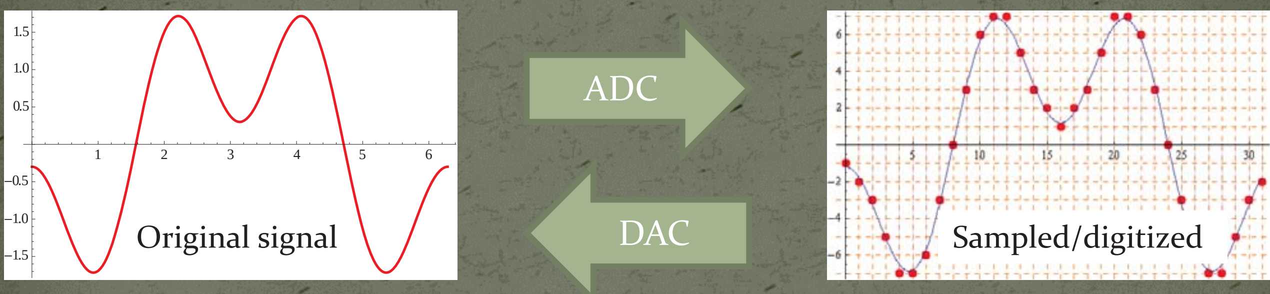


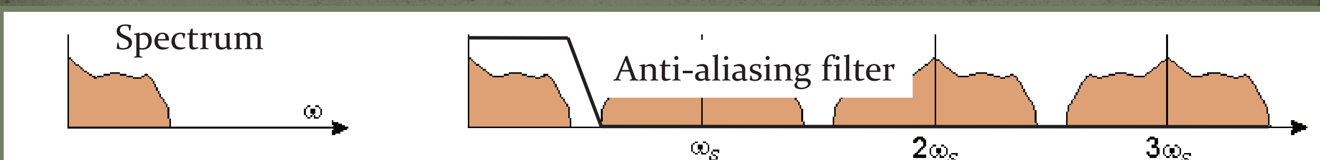
Digital Signal Processing

Just some basics

Sampling and quantization

- Digital Signal Processing is very powerful – note recent progress in digital audio, video and communication!
- Concepts and modules developed for a huge market; highly sophisticated modules available "off the shelf".
- The "slowly varying" phasors are ideal to be sampled and quantized as needed for digital signal processing.
- Sampling (at $1/\tau_s$) and quantization (n bit data words – here 4 bit):

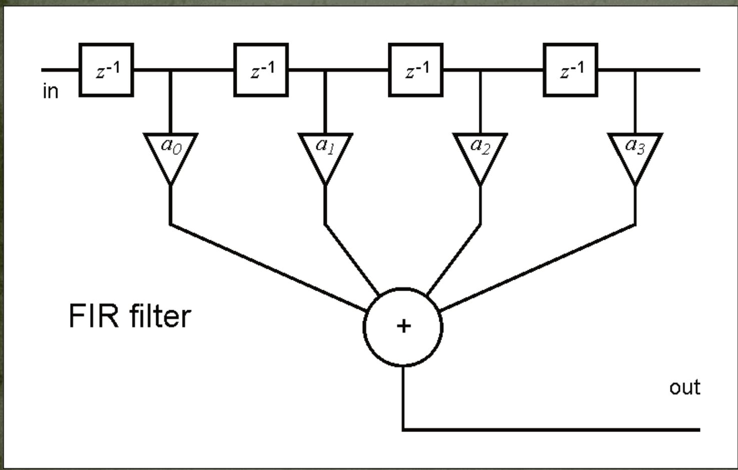


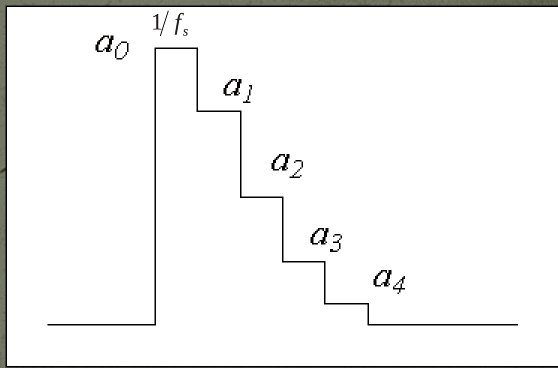


The "baseband" is limited to half the sampling rate!

Digital filters (1)

- Once in the digital realm, signal processing becomes "computing"!
- In a "finite impulse response" (FIR) filter, you directly program the coefficients of the impulse response.





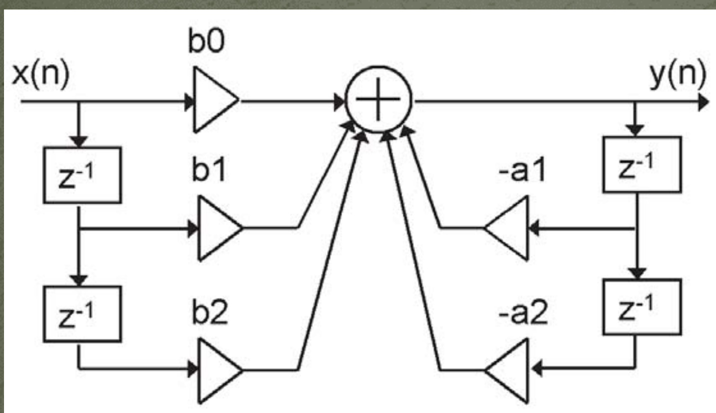
$$z = e^{j\omega\tau_s}$$

Transfer function:

$$a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}$$

Digital filters (2)

- An "infinite impulse response" (IIR) filter has built-in recursion, e.g. like

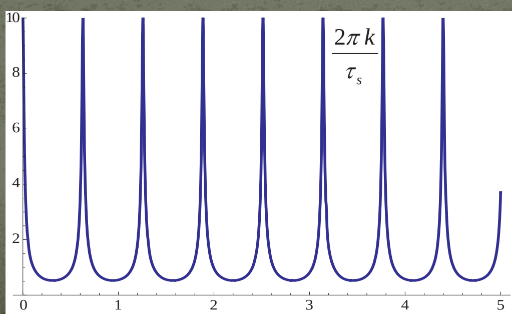


Transfer function:

$$\frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Example:

$$\frac{b_0}{1 + b_k z^{-k}}$$

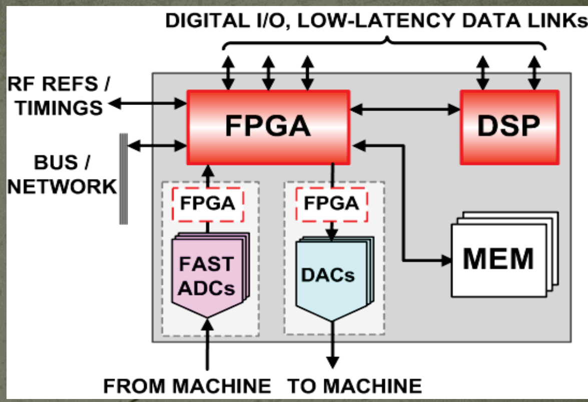


… is a comb filter

Digital LLRF building blocks – examples

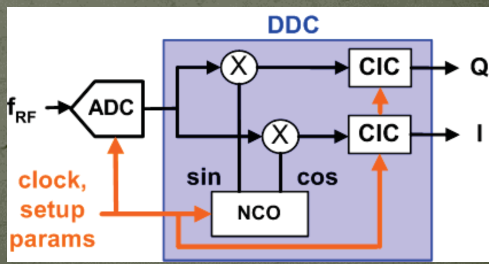
- General D-LLRF board:
  - modular!

  FPGA: Field-programmable gate array
  DSP: Digital Signal Processor



- DDC (Digital Down Converter)
  - Digital version of the I-Q demodulator

  CIC: cascaded integrator-comb
  (a special low-pass filter)



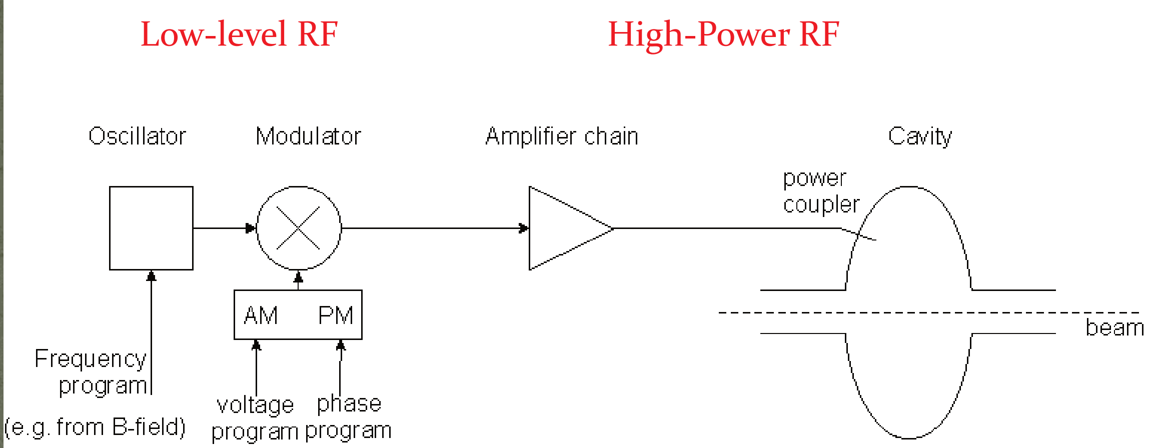
RF system & control loops

e.g.: ... for a synchrotron:

Cavity control loops

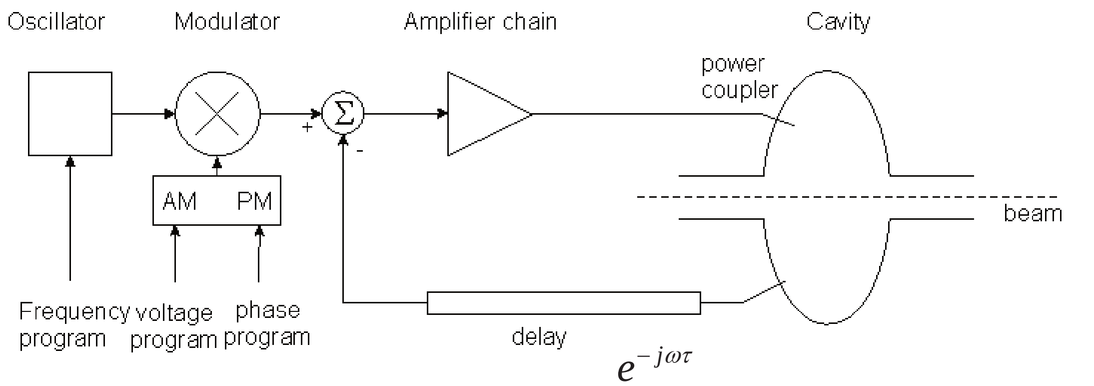
Beam control loops

Minimal RF system (of a synchrotron)



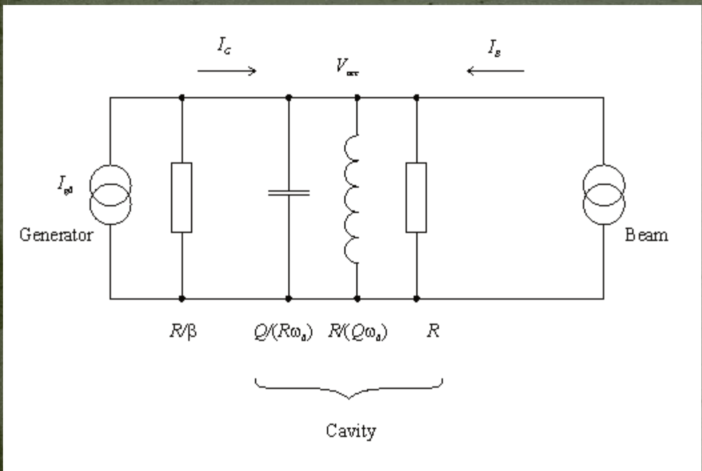
- The frequency has to be controlled to follow the magnetic field such that the beam remains in the centre of the vacuum chamber.
- The voltage has to be controlled to allow for capture at injection, a correct bucket area during acceleration, matching before ejection; phase may have to be controlled for transition crossing and for synchronisation before ejection.

Fast RF Feed-back loop



- Compares actual RF voltage and phase with desired and corrects.
- Rapidity limited by total group delay (path lengths) (some 100 ns).
- Unstable if loop gain =1 with total phase shift 180 ° – design requires to stay away from this point (stability margin)
- The group delay limits the gain·bandwidth product.
- Works also to keep voltage at zero for strong beam loading, i.e. it reduces the beam impedance.

Fast feedback loop at work



- Without feedback, $V_{acc} = (I_{G0} + I_B) \cdot Z(\omega)$

  where $$Z(\omega) = \frac{R\,(1+\beta)}{1 + jQ\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)}$$

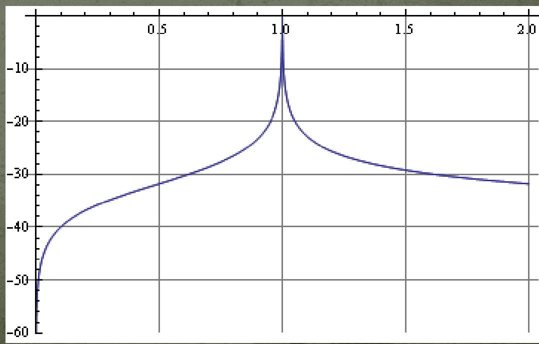
- Detect the gap voltage, feed it back to I_{G0} such that $I_{G0} = I_{\text{drive}} - G \cdot V_{acc}$

  where G is the total loop gain (pick-up, cable, amplifier chain ...)

- Result: $$V_{acc} = (I_{drive} + I_B) \cdot \frac{Z(\omega)}{1 + G \cdot Z(\omega)}$$

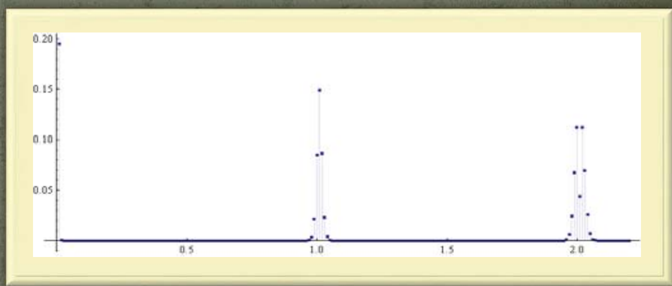
- Gap voltage is stabilised!
- Impedance seen by the beam is reduced by the loop gain!
- Plot on the right: $\dfrac{1+\beta}{R}\left|\dfrac{Z(\omega)}{1 + G \cdot Z(\omega)}\right|$ vs. ω

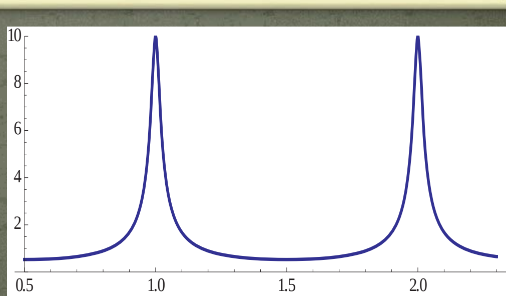
  with the loop gain varying from 0 to 50 dB



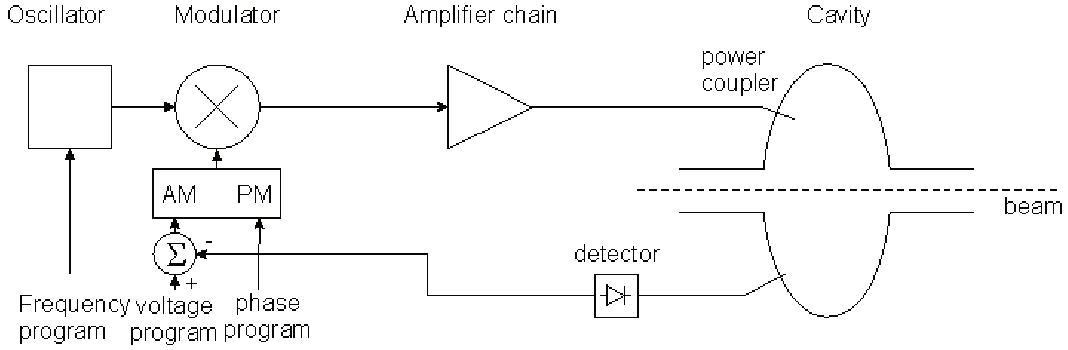
1-turn delay feed-back loop

- The speed of the "fast RF feedback" is limited by the group delay – this is typically a significant fraction of the revolution period.
- How to lower the impedance over many harmonics of the revolution frequency?
- Remember: the beam spectrum is limited to relatively narrow bands around the multiples of the revolution frequency!
- Only in these narrow bands the loop gain must be high!
- Install a comb filter! ... and extend the group delay to exactly 1 turn – in this case the loop will have the desired effect and remain stable!



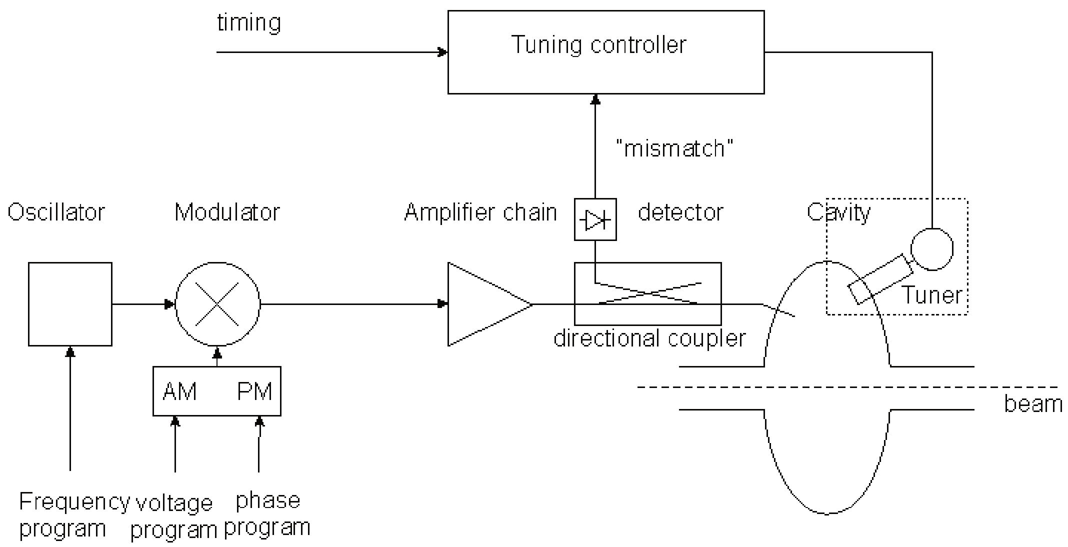


Field amplitude control loop (AVC)



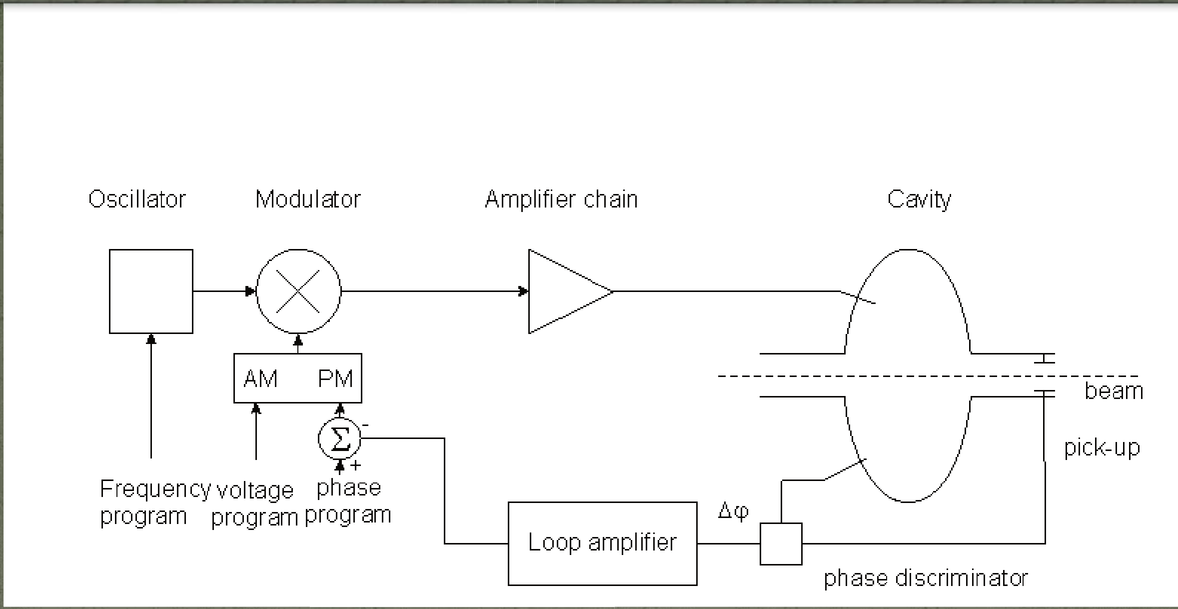
- Compares the detected cavity voltage to the voltage program. The error signal serves to correct the amplitude

Tuning loop



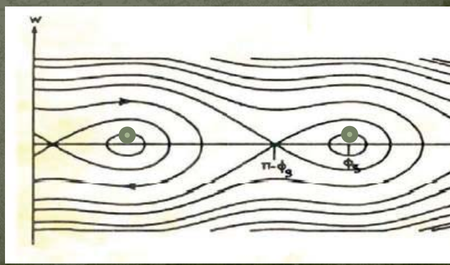
- Tunes the resonance *f* of the cavity to minimize the mismatch of the PA.
- In the presence of beam loading, this may mean $f_r \neq f$.
- In an ion ring accelerator, the tuning range might be > octave!
- For fixed *f* systems, tuners are needed to compensate for slow drifts.
- Examples for tuners:
  - controlled power supply driving ferrite bias (varying μ),
  - stepping motor driven plunger,
  - motorized variable capacitor, …

Beam phase loop



- Longitudinal motion: $\frac{d^2(\Delta\phi)}{dt^2} + \Omega_s^2(\Delta\phi)^2 = 0$
- Loop amplifier transfer function designed to damp
- synchrotron oscillation. Modified equation:

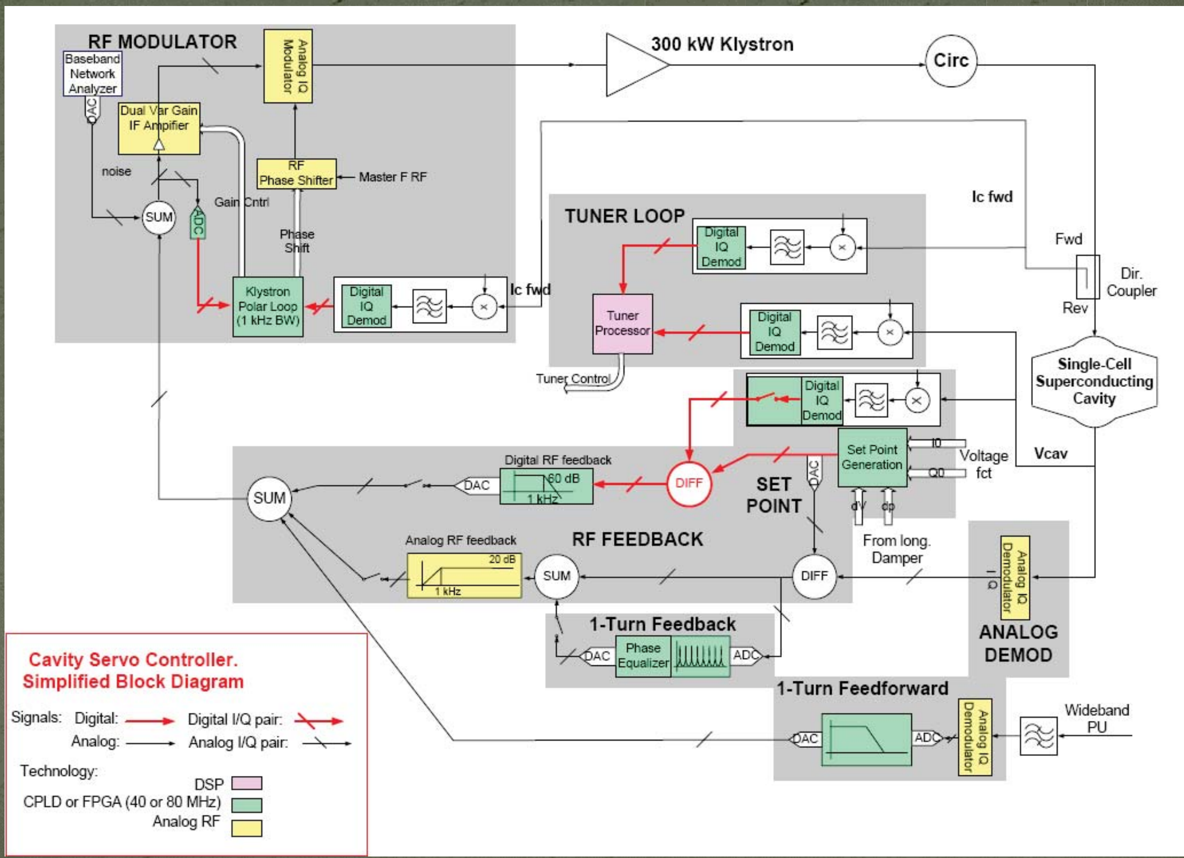
$$\frac{d^2(\Delta\phi)}{dt^2} + \alpha\frac{d(\Delta\phi)}{dt} + \Omega_s^2(\Delta\phi)^2 = 0$$



Other loops

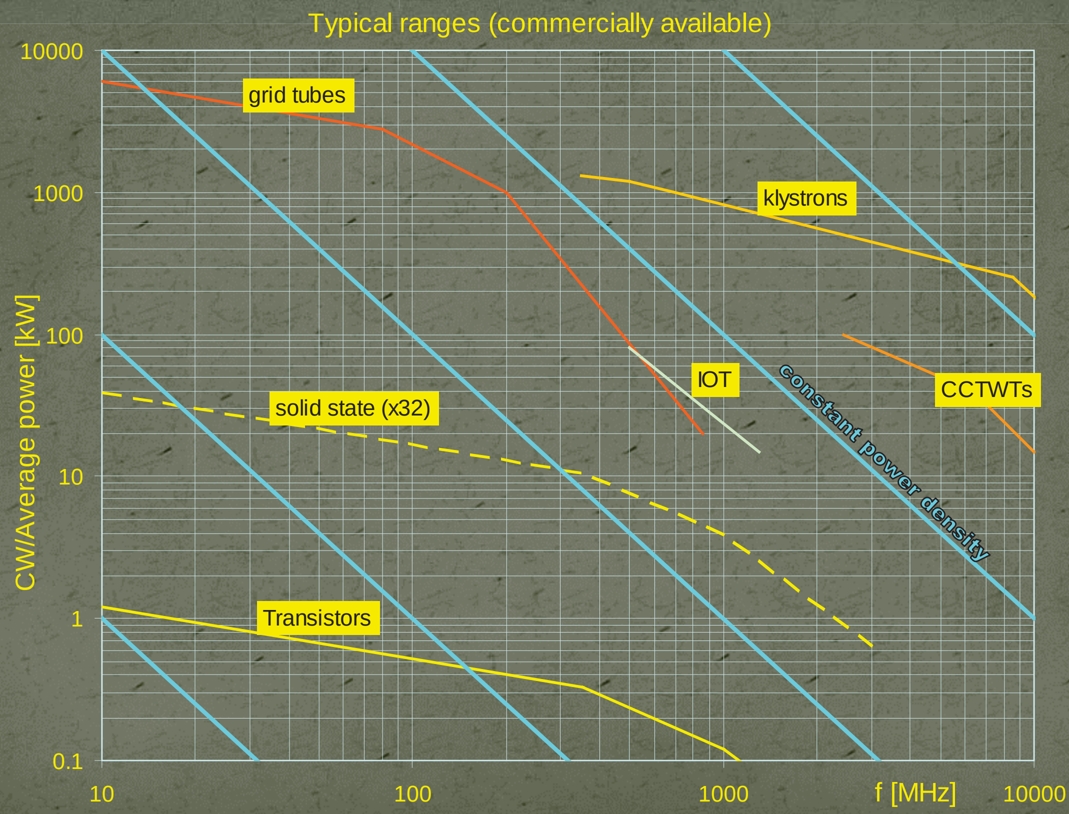
- Radial loop:
  - Detect average radial position of the beam,
  - Compare to a programmed radial position,
  - Error signal controls the frequency.
- Synchronisation loop:
  - 1st step: Synchronize f to an external frequency (will also act on radial position!).
  - 2nd step: phase loop
- …

A real implementation: LHC LLRF



RF power sources

RF power sources

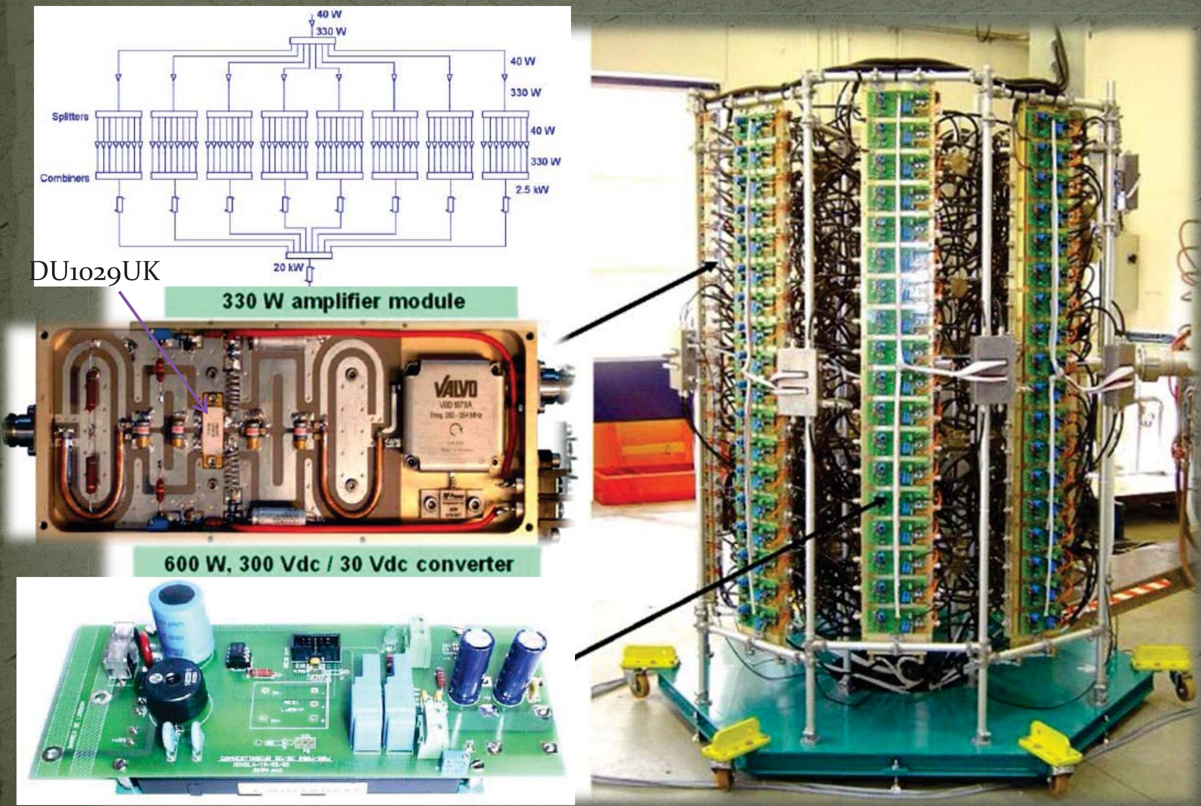


LEIR SSPA, 1 kW, 0.2 – 50 MHz

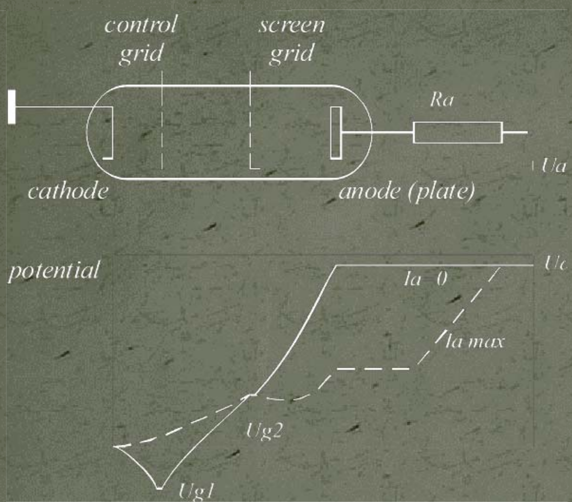
MRF151G

Soleil Booster SSPA, 40 kW, 352 MHz

147 modules



Tetrode



RS 1084 CJ (ex Siemens, now Thales),
< 30 MHz, 75 kW

4CX250B (Eimac/CPI), < 500 MHz, 600 W (Anode removed)

YL1520 (ex Philips, now Richardson),
< 260 MHz, 25 kW

High power tetrode amplifier

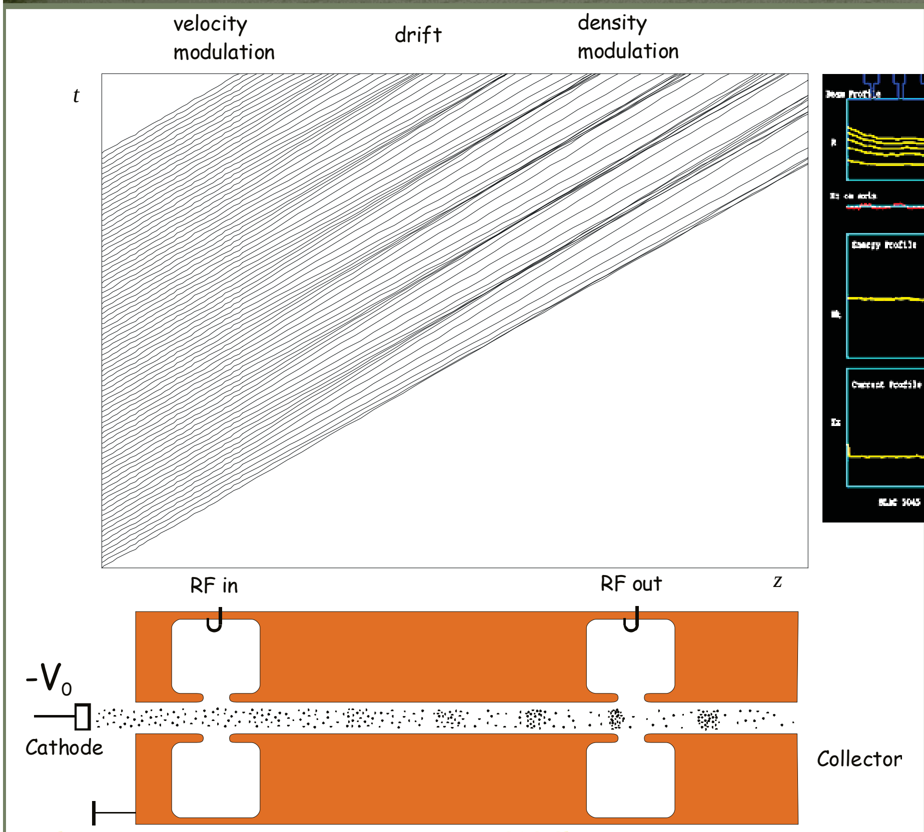
CERN Linac3: 100 MHz, 350 kW

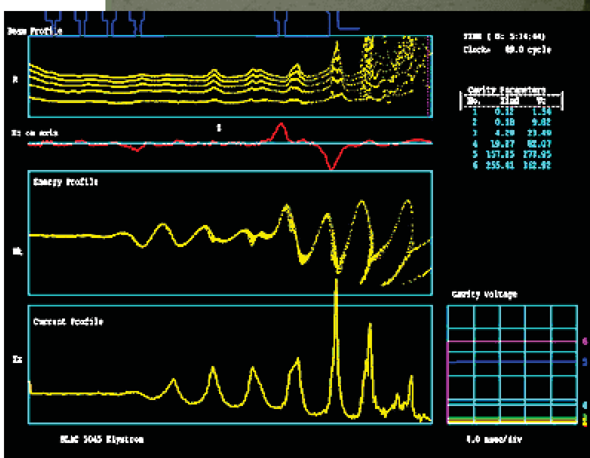
50 kW Driver: TH345, Final: RS 2054 SK

CERN PS: 13-20 MHz, 30 kW

Driver: solid state 400 W, Final: RS 1084 CJSC

Klystron principle





Klystrons

CERN CTF3 (LIL):
3 GHz, 45 MW,
4.5 μs, 50 Hz, η 45 %

CERN LHC:
400 MHz, 300 kW,
CW, η 62 %

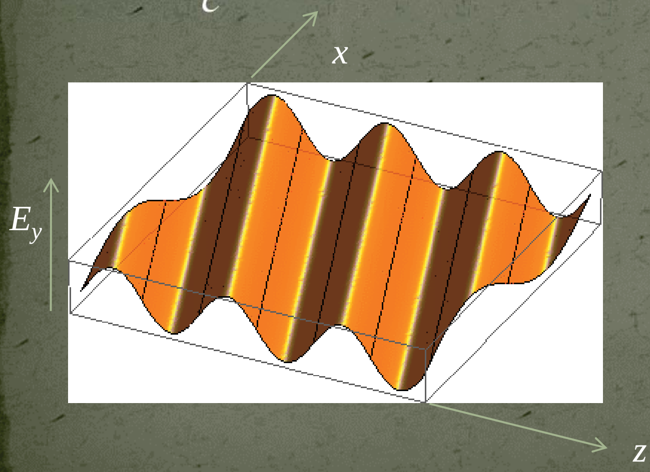
Fields in a waveguide

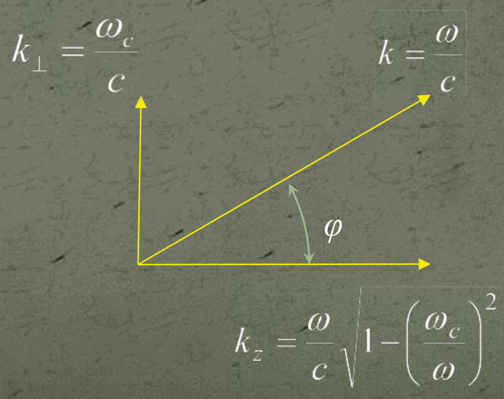
Homogeneous plane wave

$$\vec{E} \propto \vec{u}_y \cos\left(\omega t - \vec{k}\cdot\vec{r}\right)$$
$$\vec{B} \propto \vec{u}_x \cos\left(\omega t - \vec{k}\cdot\vec{r}\right)$$

$$\vec{k}\cdot\vec{r} = \frac{\omega}{c}\left(\cos(\varphi) z + \sin(\varphi) x\right)$$

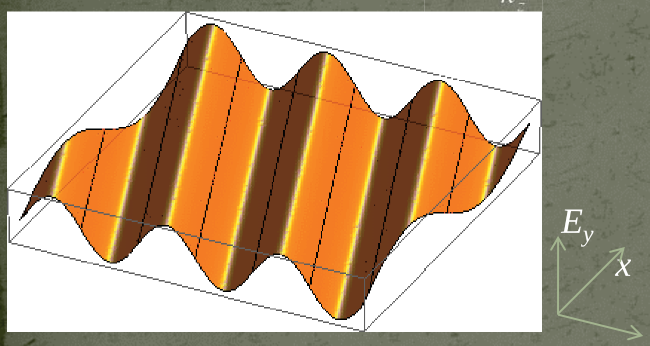
Wave vector $\vec{k}$:
the direction of $\vec{k}$ is the direction of propagation,
the length of $\vec{k}$ is the phase shift per unit length.
$\vec{k}$ behaves like a vector.

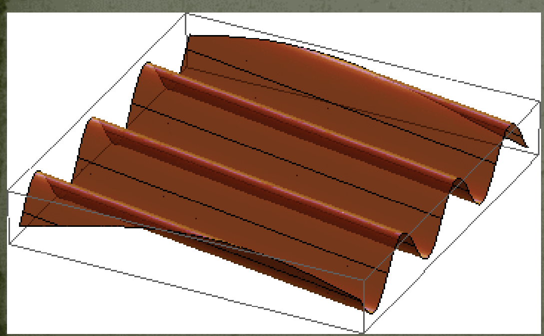


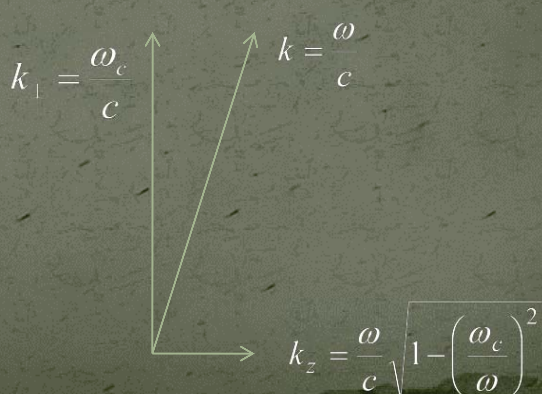


Wave length, phase velocity

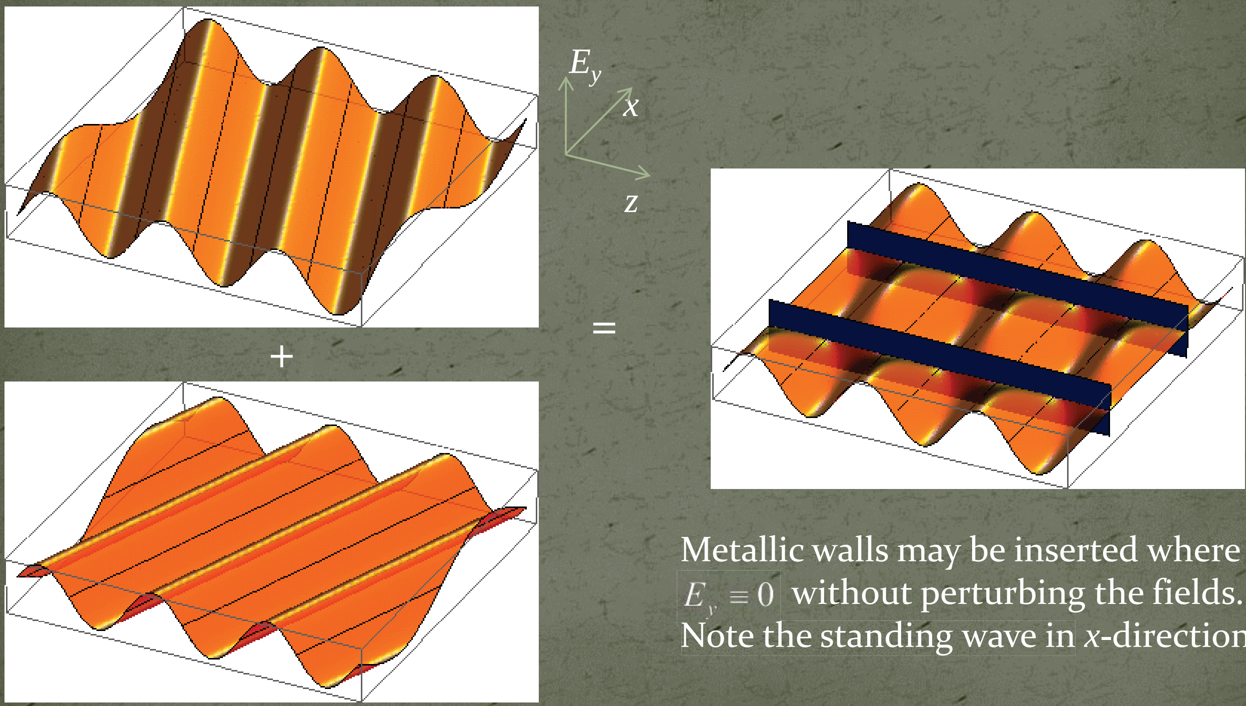
- The components of $\vec{k}$ are related to the wavelength in the direction of that component as $\lambda_z = \frac{2\pi}{k_z}$ etc. , to the phase velocity as $v_{\varphi,z} = \frac{\omega}{k_z} = f\,\lambda_z$.







Superposition of 2 homogeneous plane waves



Metallic walls may be inserted where $E_y \equiv 0$ without perturbing the fields.
Note the standing wave in x-direction!

This way one gets a hollow rectangular waveguide

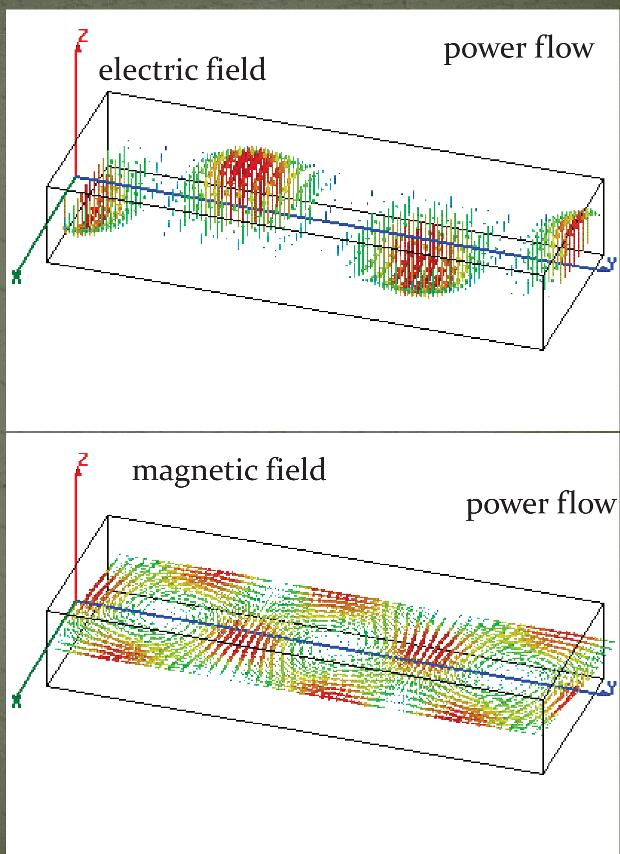
Rectangular waveguide

Fundamental (TE_{10} or H_{10}) mode
in a standard rectangular waveguide.
Example: "S-band" : 2.6 GHz ... 3.95 GHz,

Waveguide type WR284 (2.84" wide),
dimensions: 72.14 mm x 34.04 mm.

Operated at $f = 3$ GHz.

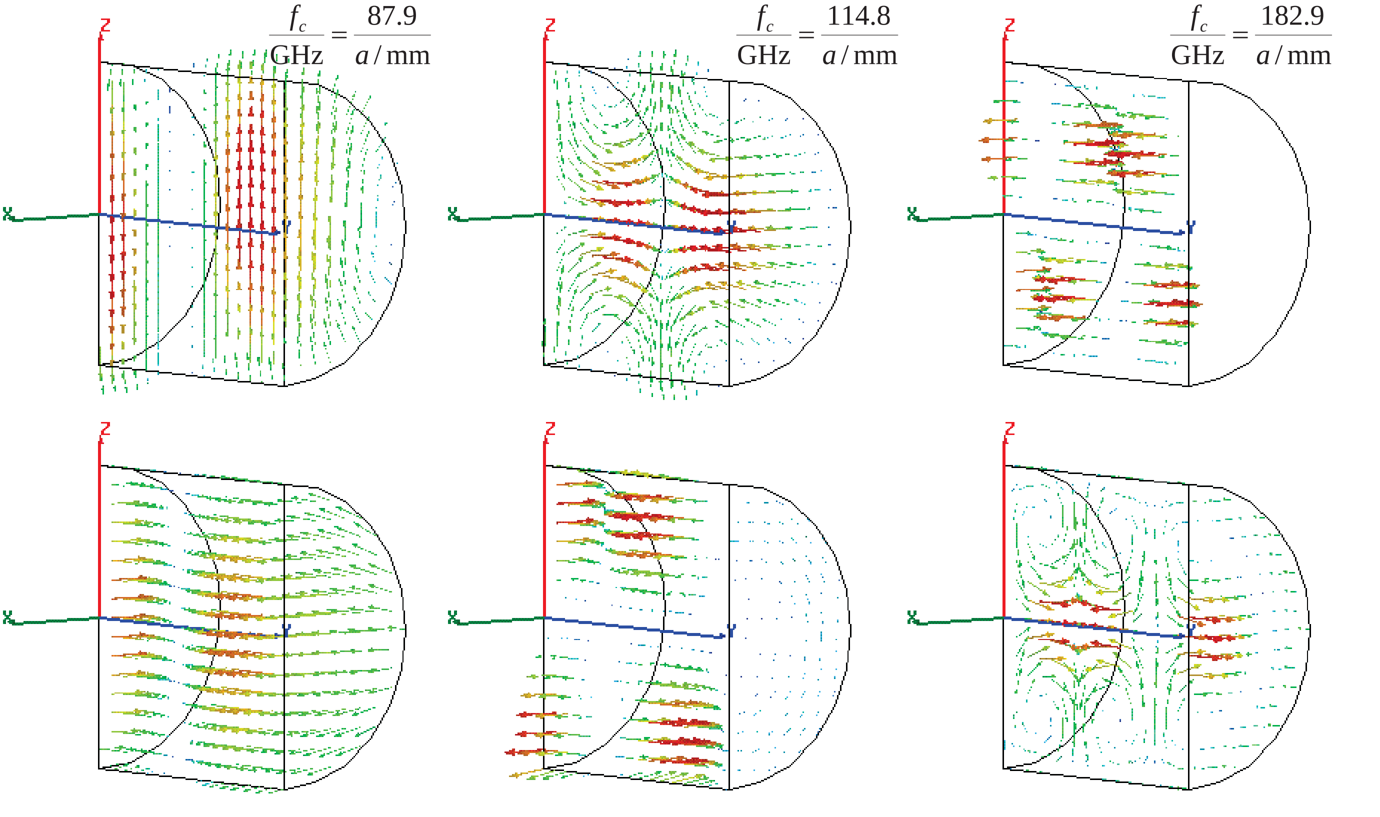
power flow: $\frac{1}{2}\mathrm{Re}\left\{\iint \vec{E} \times \vec{H}^* \cdot d\vec{A}\right\}$



Round waveguide modes

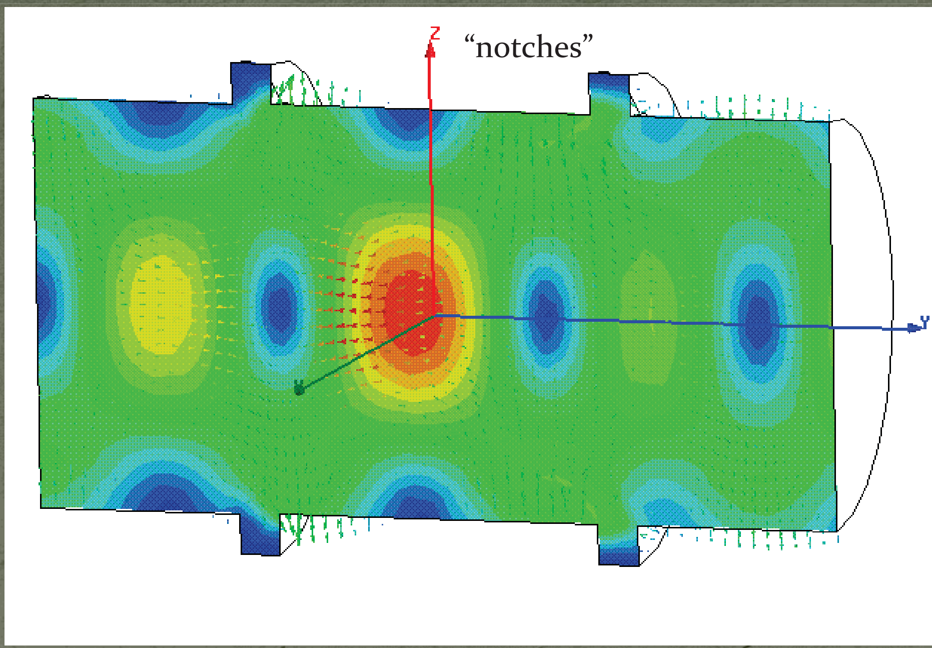
TE_{11} – fundamental | TM_{01} – axial field | TE_{01} – low loss

$$\frac{f_c}{\text{GHz}} = \frac{87.9}{a/\text{mm}} \qquad \frac{f_c}{\text{GHz}} = \frac{114.8}{a/\text{mm}} \qquad \frac{f_c}{\text{GHz}} = \frac{182.9}{a/\text{mm}}$$



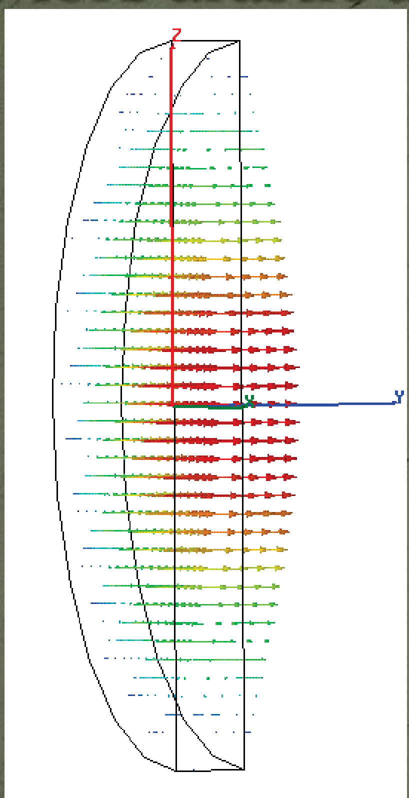
From waveguide to cavity

Waveguide perturbed by notches



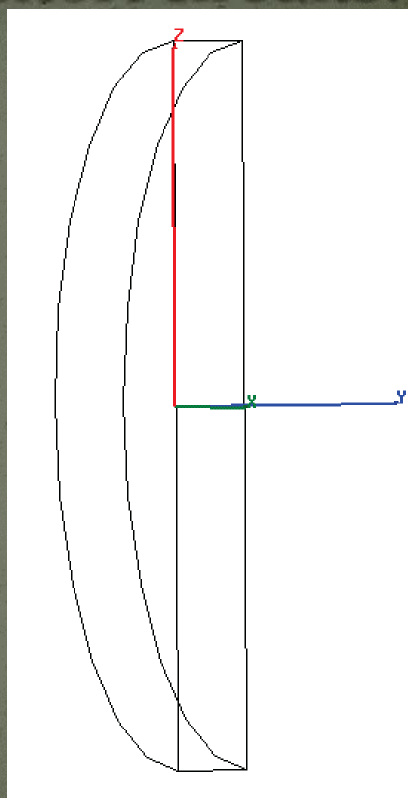
Reflections from notches lead to a superimposed standing wave pattern.
“Trapped mode”

More drastic than notches: short circuits!



TM$_{010}$-mode

This is called “Pillbox cavity”



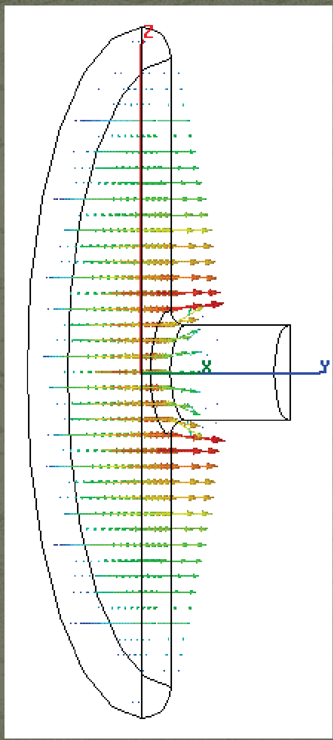
electric field (purely axial)

magnetic field (purely azimuthal)

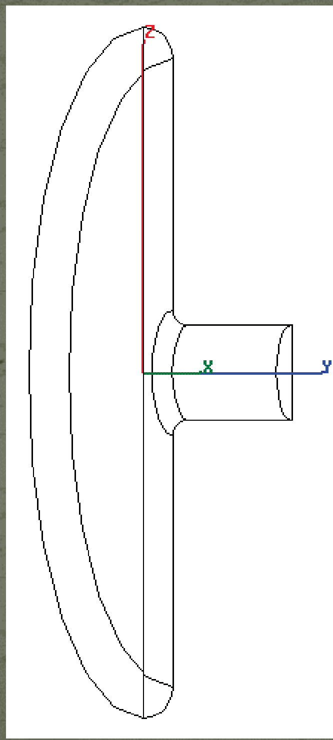
A more practical pillbox cavity

Beam pipe added,
sharp edges rounded off

TM$_{010}$-mode (only 1/4 shown)



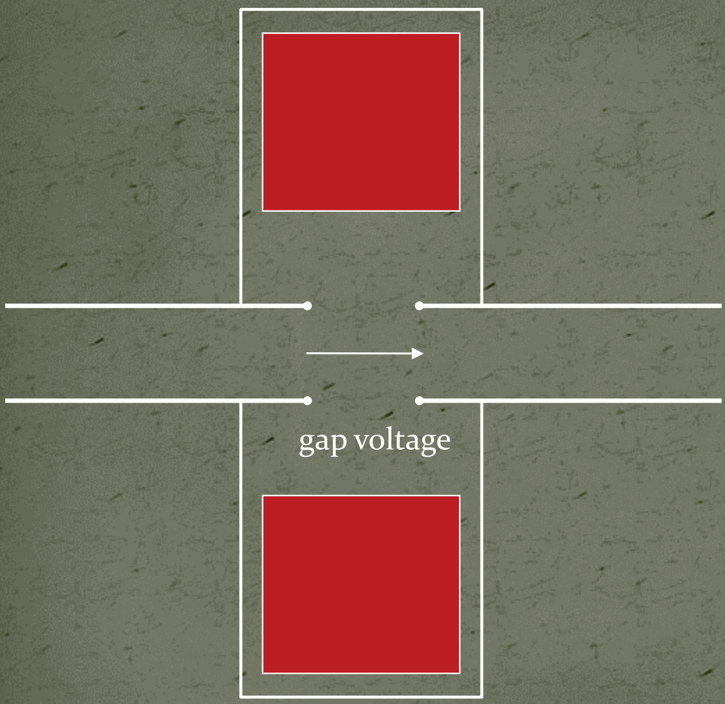
electric field



magnetic field

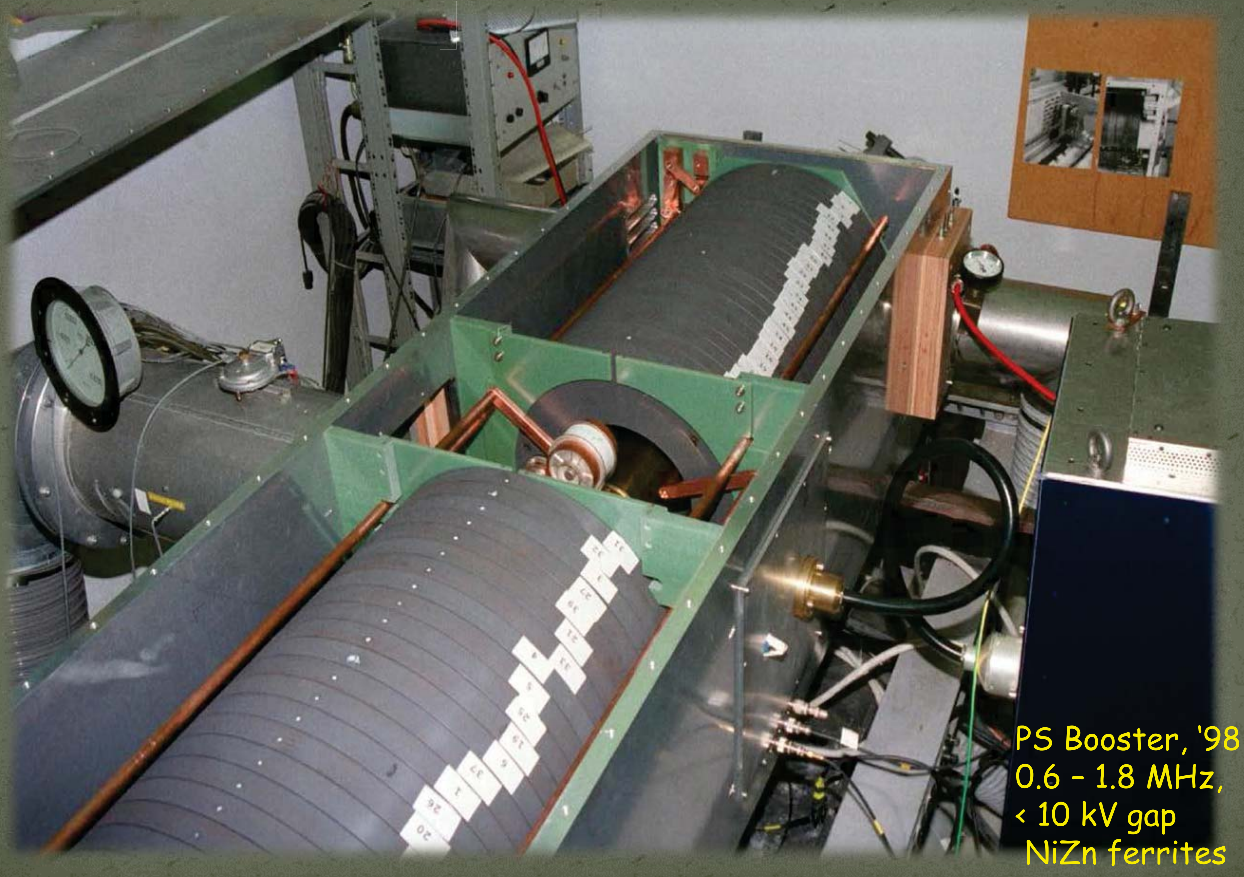
Accelerating gap

Accelerating gap

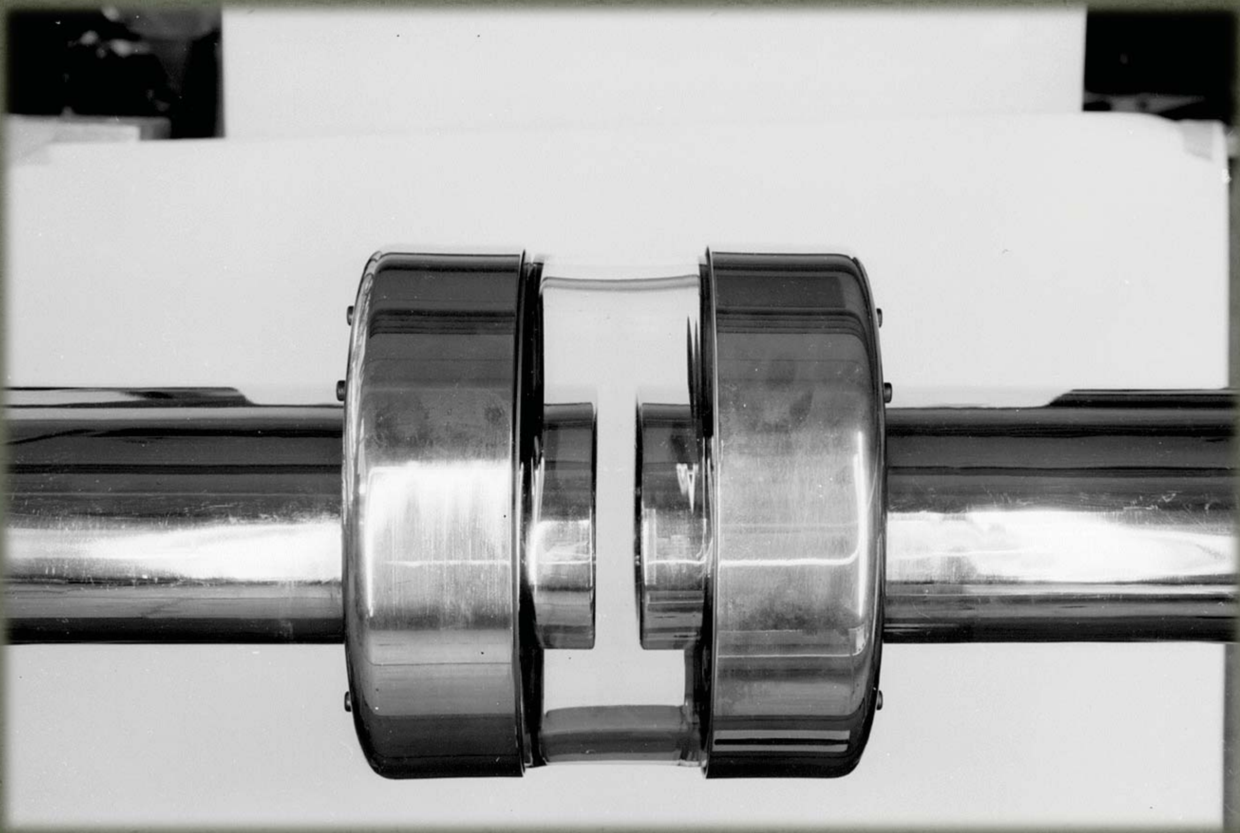


- We want a voltage across the gap!
- It cannot be DC, since we want the beam tube on ground potential.
- Use $\int \vec{E} \cdot d\vec{s} = -\iint \frac{d\vec{B}}{dt} \cdot d\vec{A}$
- The "shield" imposes a
  - upper limit of the voltage pulse duration or – equivalently –
  - a lower limit to the usable frequency.
- The limit can be extended with a material which acts as "open circuit"!
- Materials typically used:
  - ferrites (depending on *f*-range)
  - magnetic alloys (MA) like Metglas®, Finemet®, Vitrovac®…
- resonantly driven with RF (ferrite loaded cavities) – or with pulses (induction cell)

Ferrite cavity



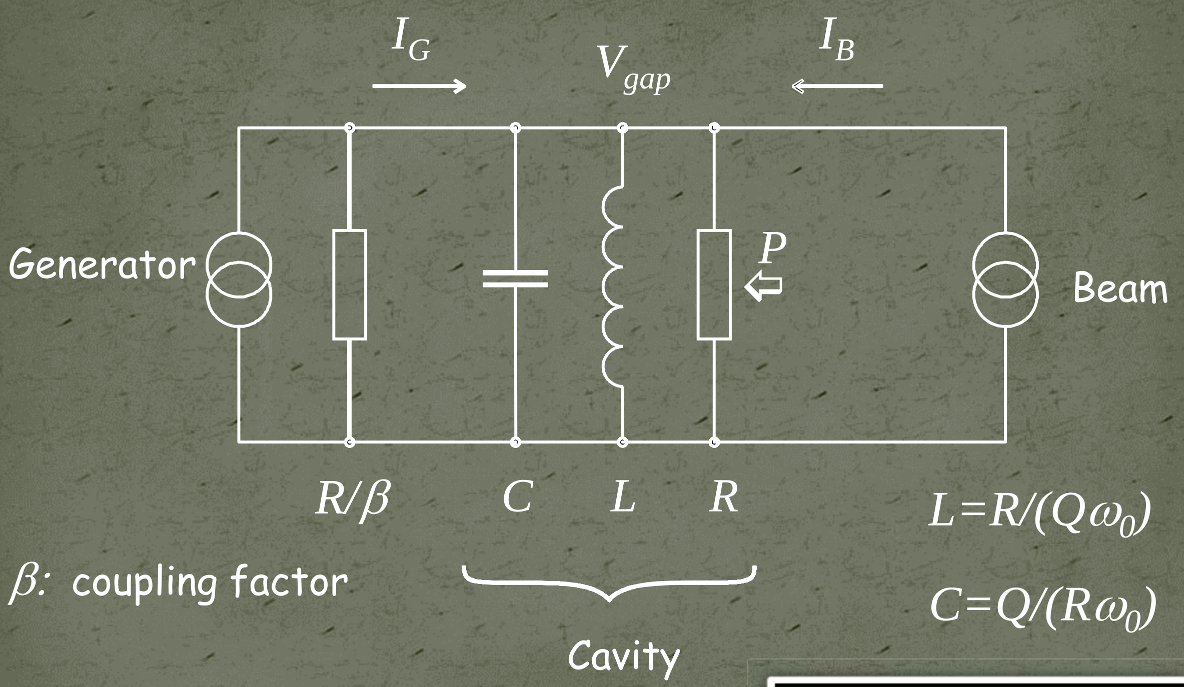
Gap of PS cavity (prototype)



Characterizing a cavity

Cavity resonator – equivalent circuit

Simplification: single mode



$L = R/(Q\omega_0)$

$C = Q/(R\omega_0)$

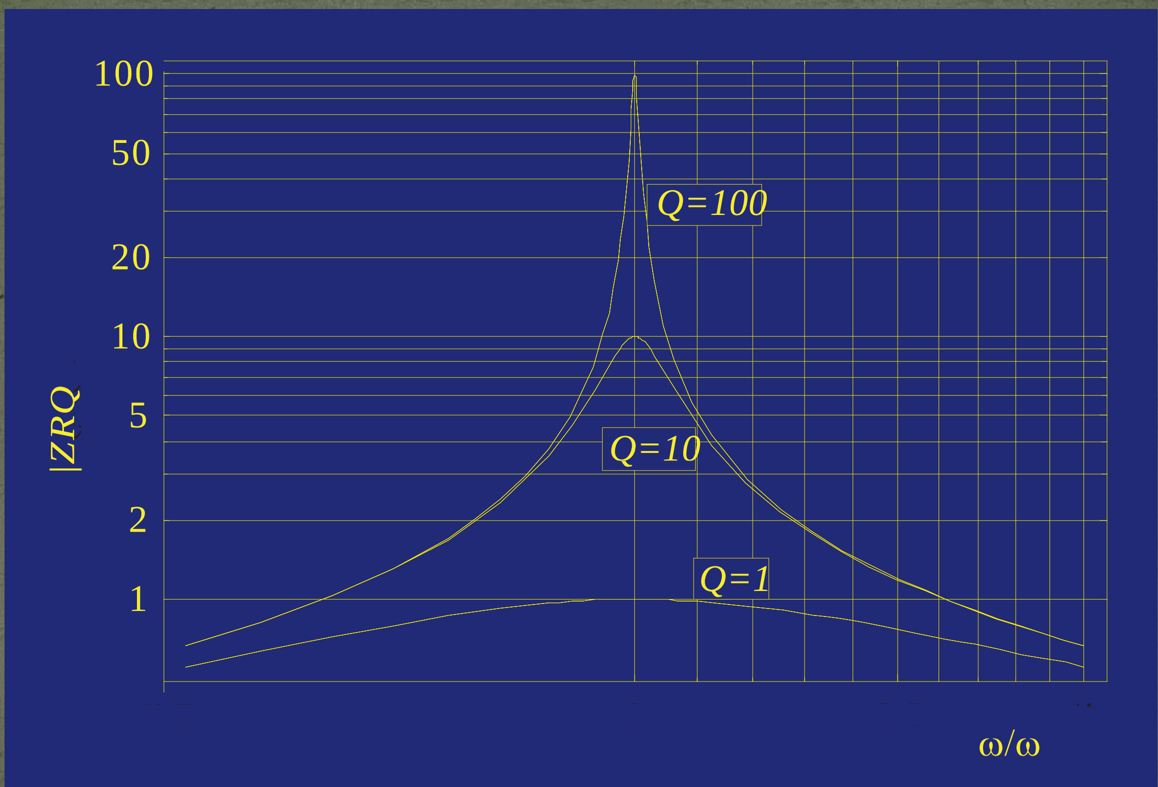
β: coupling factor

R: Shunt impedance

$\sqrt{L/C}$: R-upon-Q

We have used this before when explaining the "fast feedback"

Resonance



Summary: relations V_{gap}, W, P_{loss}

V_{gap}
gap voltage

$$\frac{R}{Q} = \frac{|V_{gap}|^2}{2\,\omega_0 W}$$

$$R_{shunt} = \frac{|V_{gap}|^2}{2\,P_{loss}}$$

W
Energy stored inside the cavity

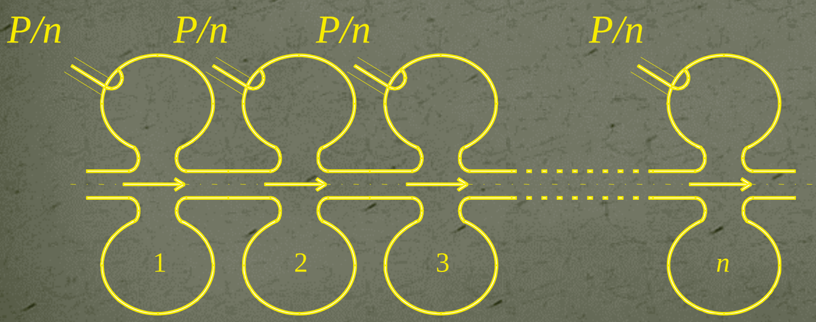
P_{loss}
Power lost in the cavity walls

$$Q = \frac{\omega_0 W}{P_{loss}}$$

Many gaps

What do you gain with many gaps?

- The R/Q of a single gap cavity is limited to some 100 Ω.
  Now consider to distribute the available power to n identical cavities: each will receive P/n, thus produce an accelerating voltage of $\sqrt{2\,R\,P/n}$.
  The total accelerating voltage thus increased, equivalent to a total equivalent shunt impedance of nR.

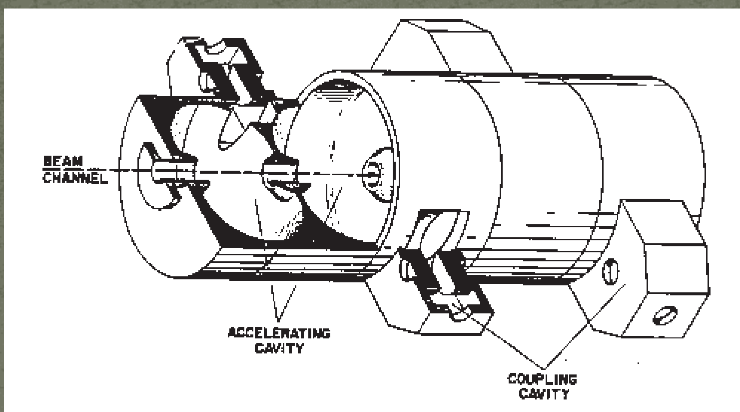


$$|V_{acc}| = n\sqrt{2R\frac{P}{n}} = \sqrt{2(nR)P}$$

Standing wave multicell cavity

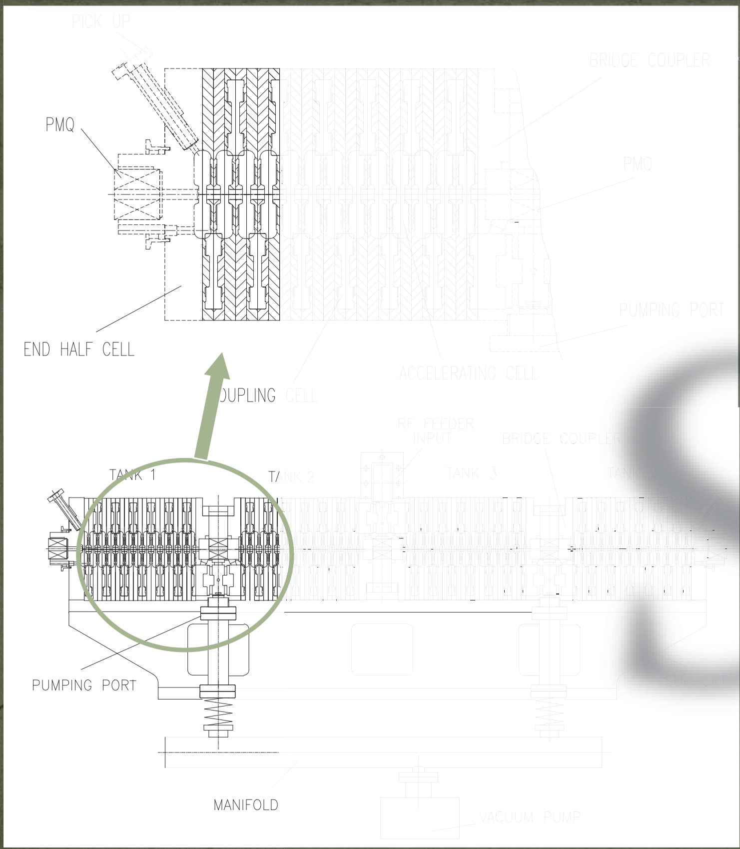
- Instead of distributing the power from the amplifier, one might as well couple the cavities, such that the power automatically distributes, or have a cavity with many gaps (e.g. drift tube linac).

- Coupled cavity accelerating structure (side coupled)



- The phase relation between gaps is important!

Side Coupled Structure : example LIBO



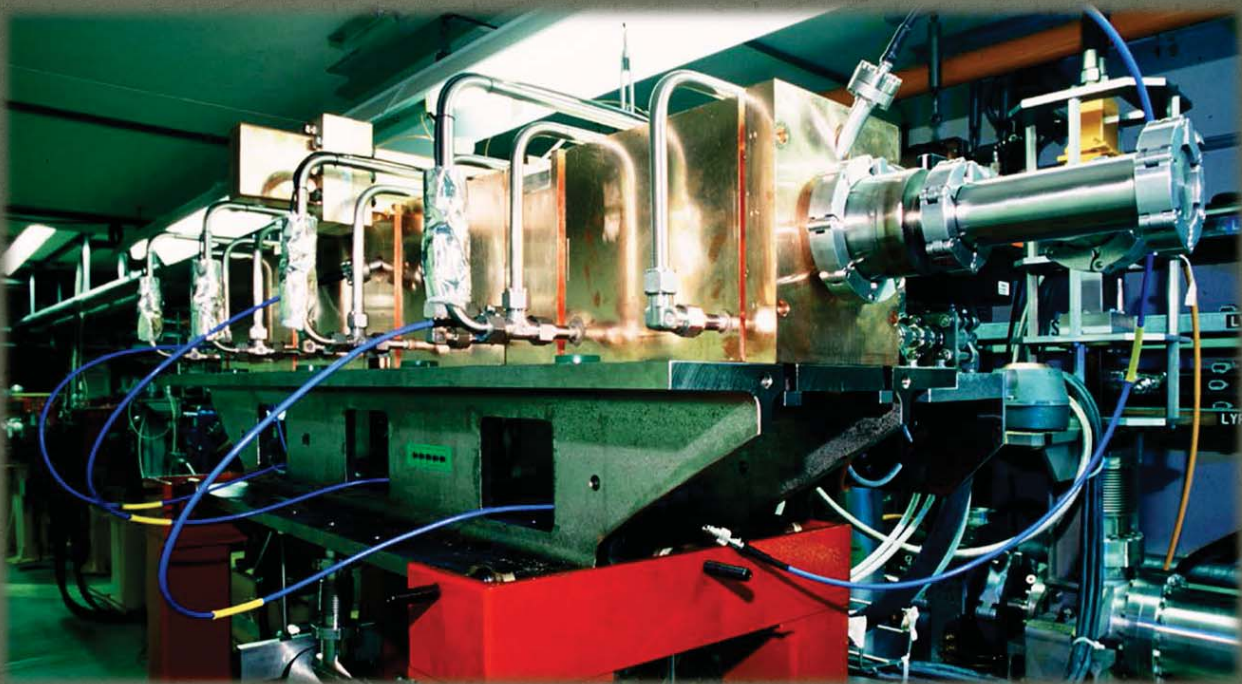
A 3 GHz Side Coupled Structure to accelerate protons out of cyclotrons from 62 MeV to 200 MeV

Medical application: treatment of tumours.

Prototype of Module 1 built at CERN (2000)

Collaboration CERN/INFN/ Tera Foundation

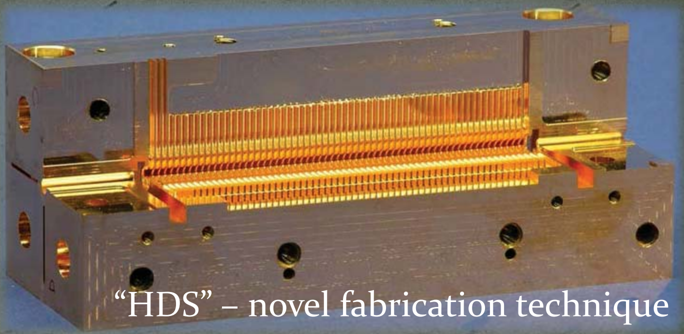
LIBO prototype



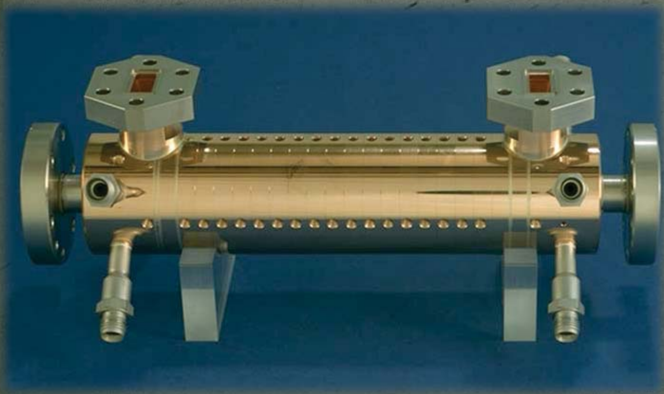
This Picture made it to the title page of CERN Courier vol. 41 No. 1 (Jan./Feb. 2001)

CLIC travelling wave structures (12 & 30 GHz)

"T18" reached 105 MV/m!



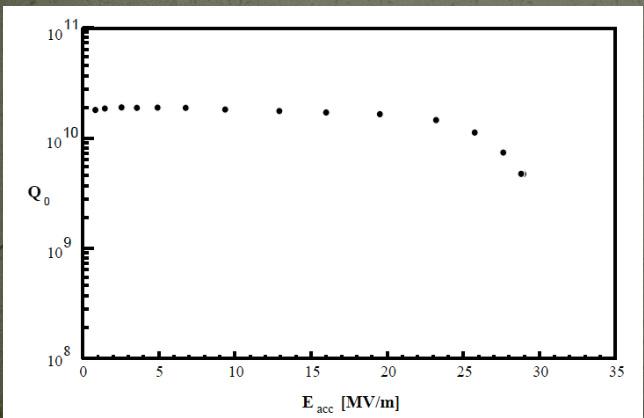
"HDS" – novel fabrication technique



Superconducting Cavities

RF Superconductivity

- Best described by BCS (Bardeen-Cooper-Schrieffer) Theory
- $R_{BCS} \propto \frac{\omega^2}{T} \exp\left(-1.76 \frac{T_c}{T}\right)$
- Surface resistance $R = R_{BCS} + R_{res}$.
- R is not zero - Q_0 is finite.
- Good values are some 10^{10}.
- Typical performance plot of a SC cavity:



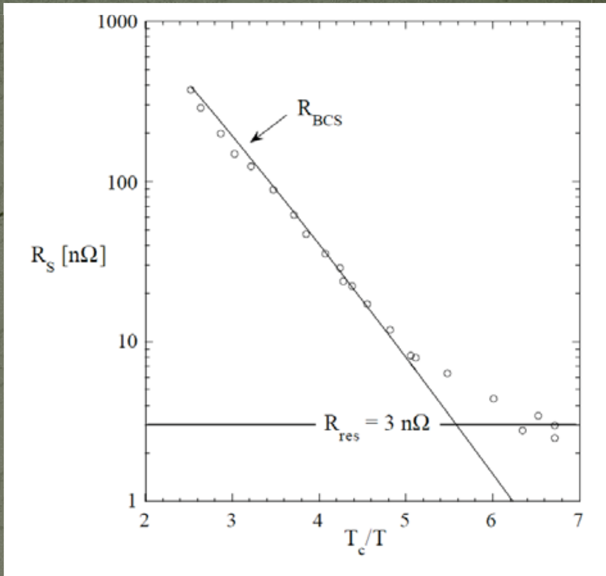
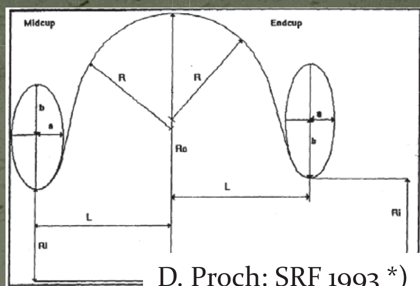


FIG. 1. The surface resistance of a 9-cell TESLA cavity plotted as a function of T_c/T. The residual resistance of 3 nΩ corresponds to a quality factor of $Q_0 = 10^{11}$.

From prst-ab.aps.org/abstract/PRSTAB/v3/i9/e092001

"Elliptical" multi-cell cavities

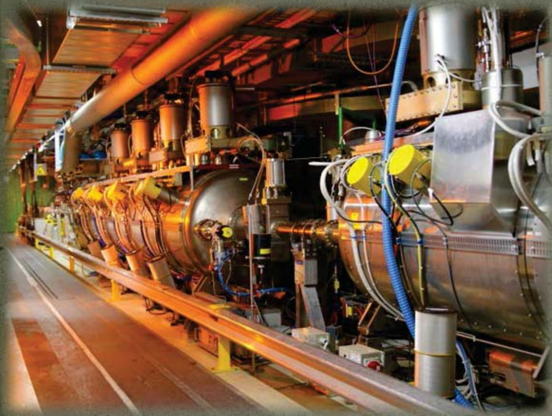
- The elliptical shape was found as optimum compromise between
  - maximum gradient (E_{acc}/E_{surf})
  - suppression of multipactor
  - mode purity
  - machinability
- Operated in π-mode, i.e. cell length is exactly $\beta\lambda/2$.
- It has become de facto standard, used for ions and leptons! E.g.:
  - ILC/X-FEL: 1.3 GHz, 9-cell cavity
  - SNS (805 MHz)
  - SPL/ESS (704 MHz)
  - LHC (400 MHz*))



D. Proch: SRF 1993 *)

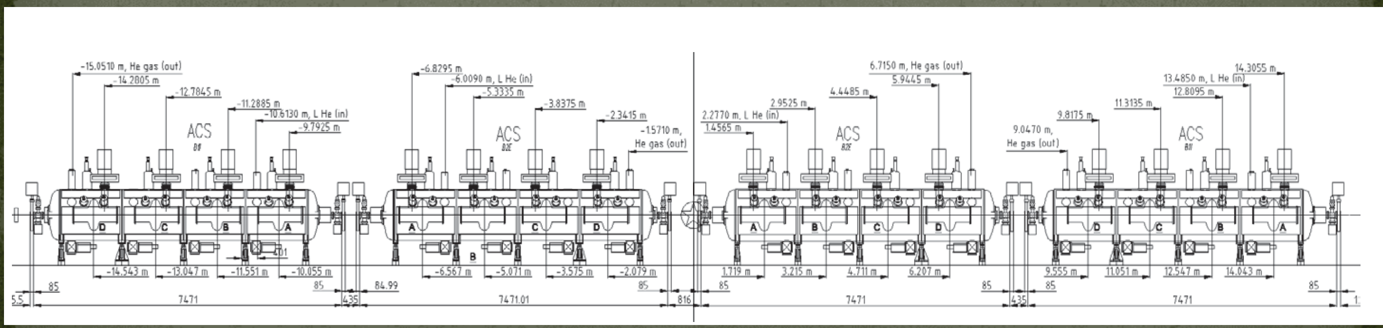
*): accelconf.web.cern.ch/accelconf/SRF93/papers/srf93g01.pdf

LHC SC RF, 4 cavity module, 400 MHz



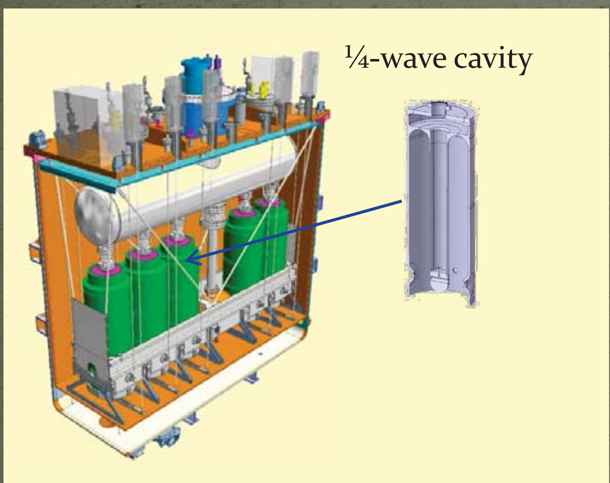
installed in LHC IP4, 2 MV/cavity

LHC spare module stored in CERN's SM18

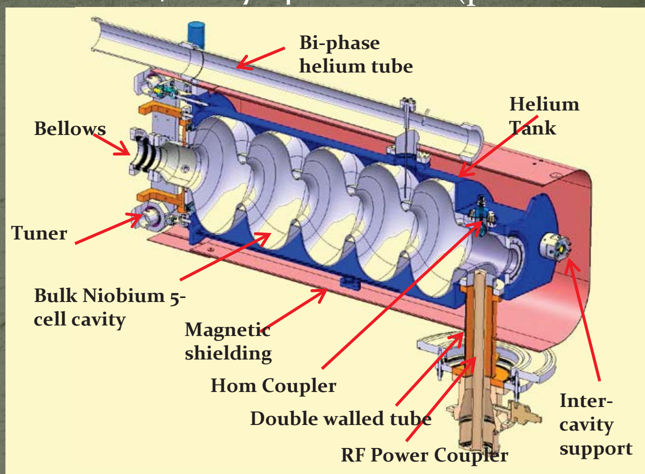


SC Cavity Cryomodules (examples)

HIE-ISOLDE (radioactive isotopes post-accelerator), 101 MHz, 5-cavity CM



SPL/ESS 704 MHz CM (partial view)



ILC/X-FEL 1.3 GHz, 8 cavity CM

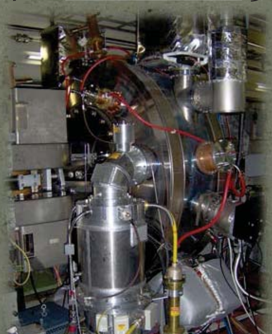
Some examples of RF Systems

CERN PS RF Systems

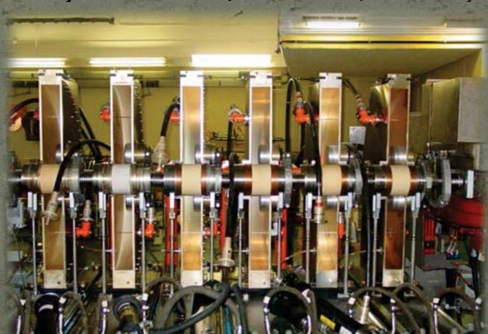
10 MHz system, h=7...21

13/20 MHz system, h=28/42

40 MHz system, h=84

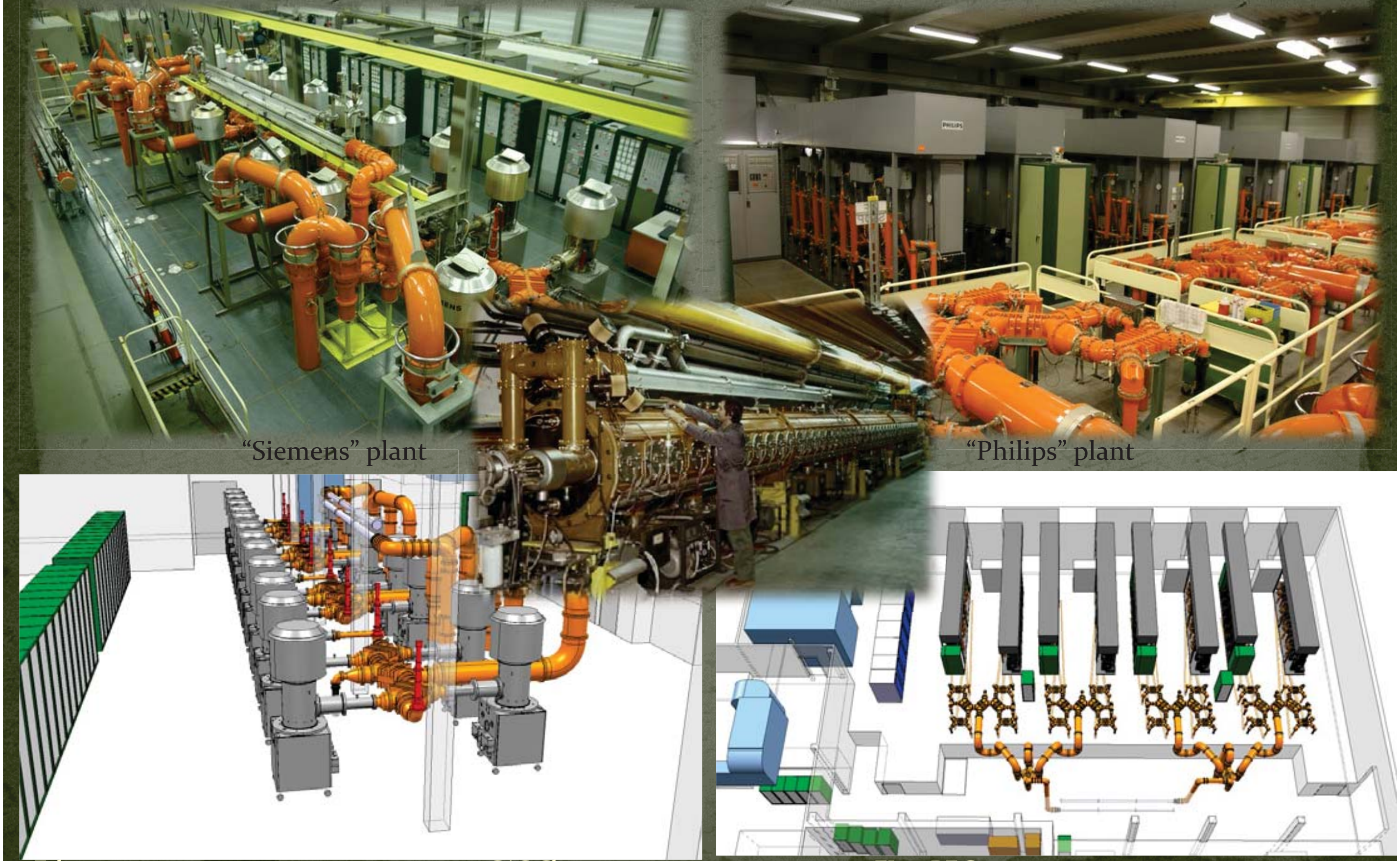


80 MHz system, h=168

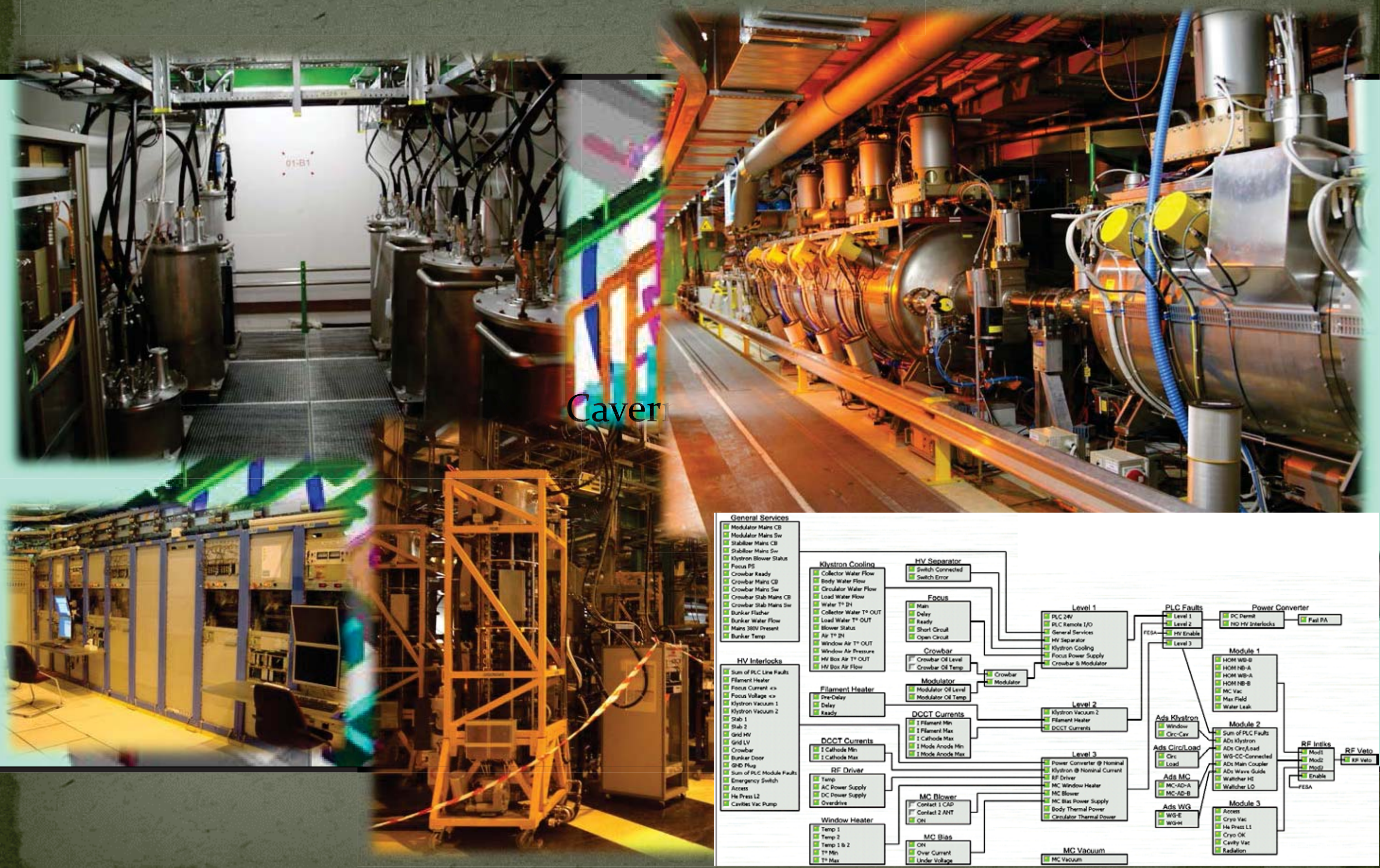


200 MHz system

CERN SPS 200 MHz system

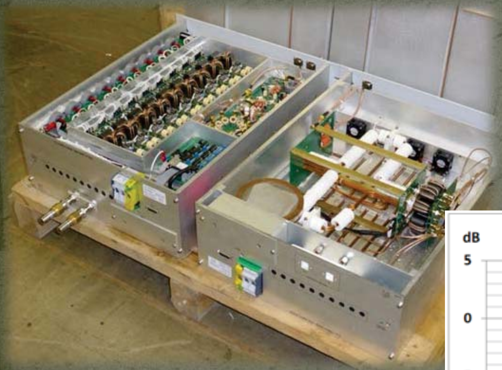


LHC RF System (ACS400)



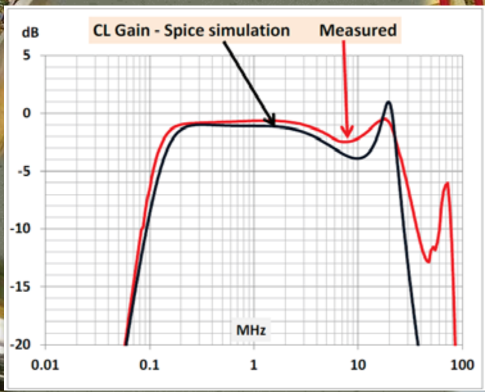
Finemet RF System (MedAustron & PSB)

(0.2 ÷ 10) MHz, 1 kW solid state amplifier



6-gap finemet cavity

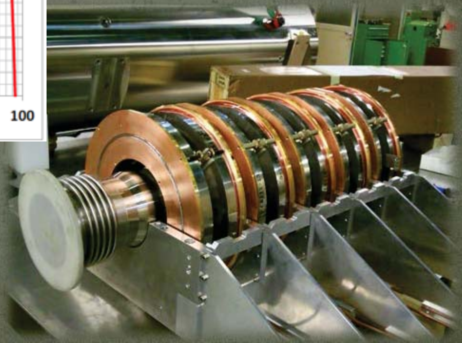
MedAustron



Large instantaneous bandwidth!

Prototype system installed in ring 4

5-gap finemet cavity



CERN PSB

Thank you for your attention!

… Questions?