# Introduction to „Transverse Beam Dynamics"

Bernhard Holzer

## The Ideal World

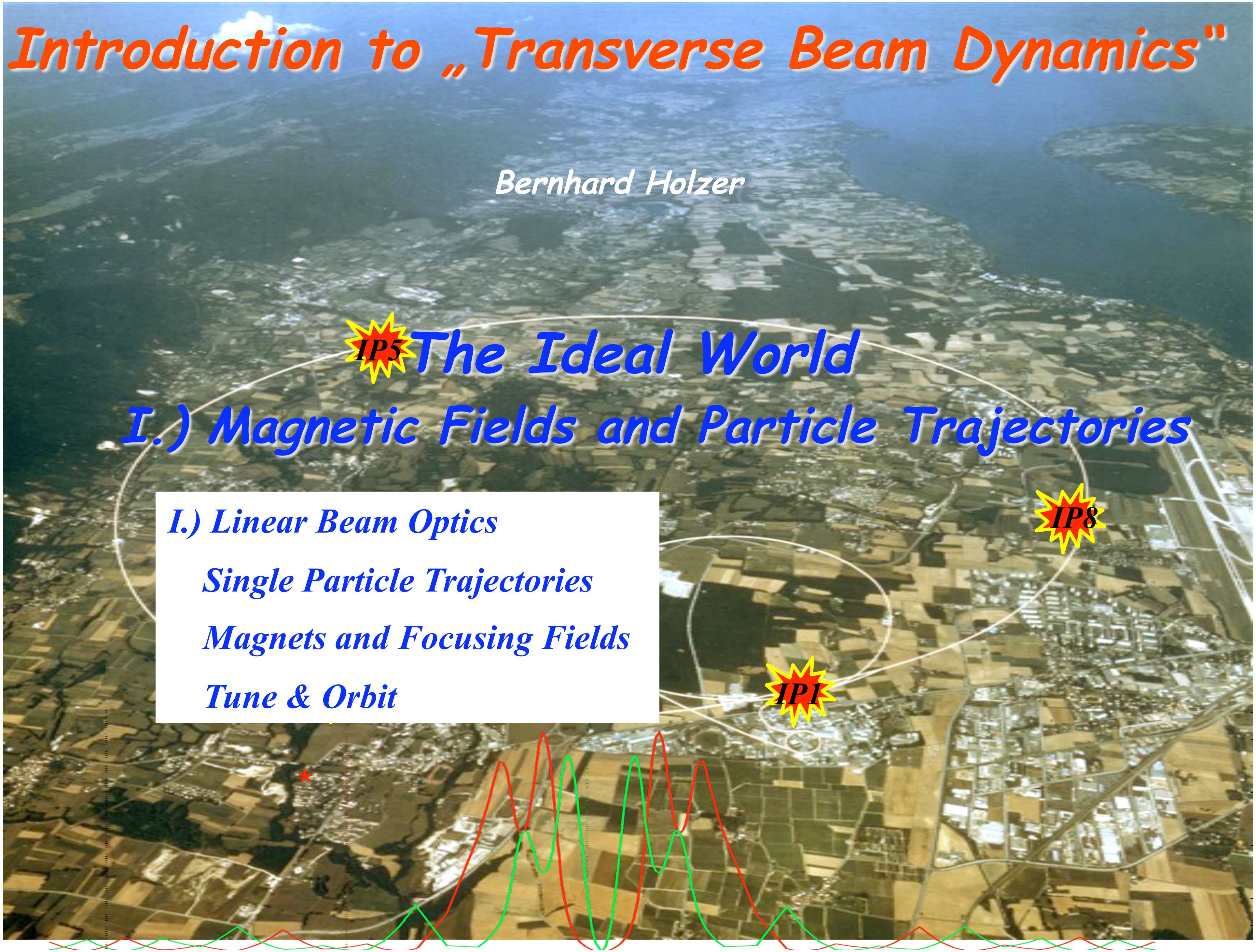
## I.) Magnetic Fields and Particle Trajectories

*I.) Linear Beam Optics*

*Single Particle Trajectories*

*Magnets and Focusing Fields*

*Tune & Orbit*



# *Luminosity Run of a typical storage ring:*

*LHC Storage Ring: Protons* ***accelerated and*** *stored for 12 hours*

***distance of particles travelling at about*** $v \approx c$

$L = 10^{10}$*-*$10^{11}$ *km*

*... several times Sun - Pluto and back* ♪


→ *guide the particles* ***on a well defined orbit*** *(„design orbit“)*

→ *focus the particles* ***to keep each single particle trajectory within the vacuum chamber of the storage ring, i.e. close to the design orbit.***

# *1.) Introduction and Basic Ideas*

***„ … in the end and after all it should be a kind of circular machine"***

***→ need transverse deflecting force***

***Lorentz force*** $\vec{F} = q * (\vec{E} + \vec{v} \times \vec{B})$

***typical velocity in high energy machines:*** $v \approx c \approx 3*10^8\ m/s$

*Example:*

$$B = 1\,T \quad \rightarrow \quad F = q*3*10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

$$F = q*300 \frac{MV}{m}$$

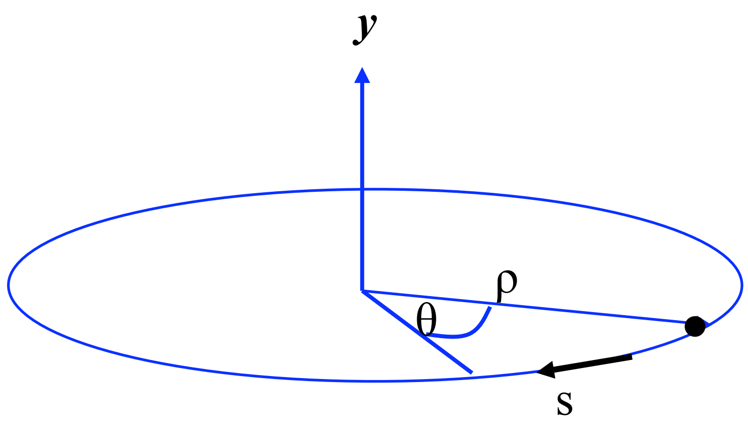
***equivalent el. field*** … $E$

*technical limit for el. field:*

$$E \le 1 \frac{MV}{m}$$

***old greek dictum of wisdom:***
***if you are clever, you use magnetic fields in an accelerator wherever it is possible.***

***The ideal circular orbit***




*circular coordinate system*

***condition for circular orbit:***

***Lorentz force*** $F_L = e\,v\,B$

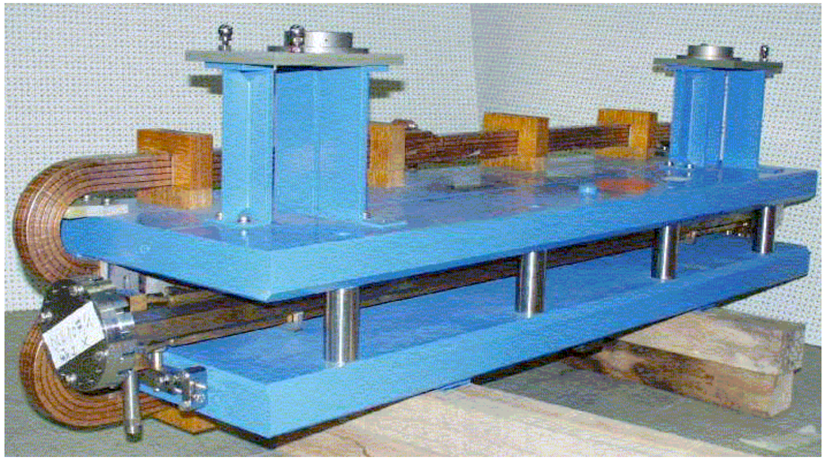
***centrifugal force*** $F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$

$$\frac{\gamma\, m_0\, v^2}{\rho} = e\,v\,B$$

$$\frac{p}{e} = B\,\rho$$

***$B\,\rho$ = "beam rigidity"***

# 2.) The Magnetic Guide Field



## *Dipole Magnets:*

***define the ideal orbit***
***homogeneous field created***
***by two flat pole shoes***

$$B = \frac{\mu_0 \, n \, I}{h}$$

## *Normalise magnetic field to momentum:*

$$\frac{p}{e} = B\rho \quad \longrightarrow \quad \frac{1}{\rho} = \frac{e\,B}{p}$$

## *convenient units:*

$$B = [T] = \left[\frac{Vs}{m^2}\right] \qquad p = \left[\frac{GeV}{c}\right]$$

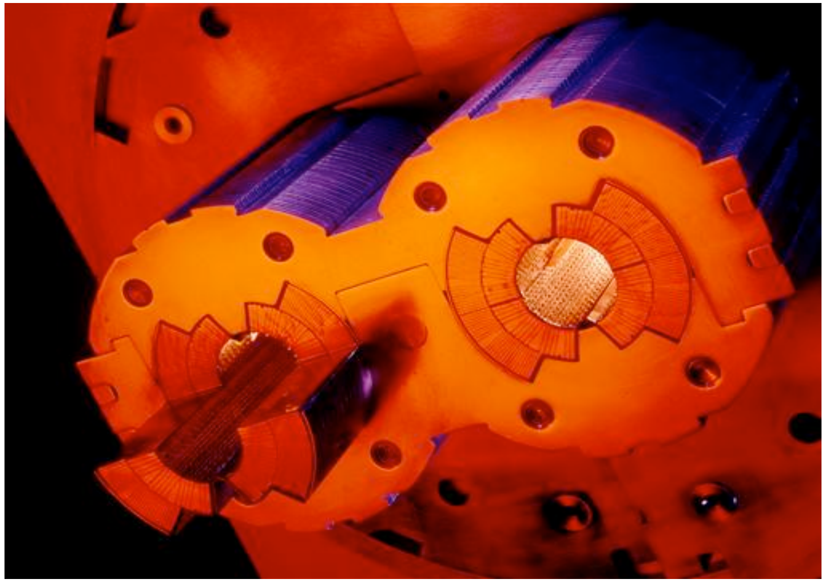
## *Example LHC:*

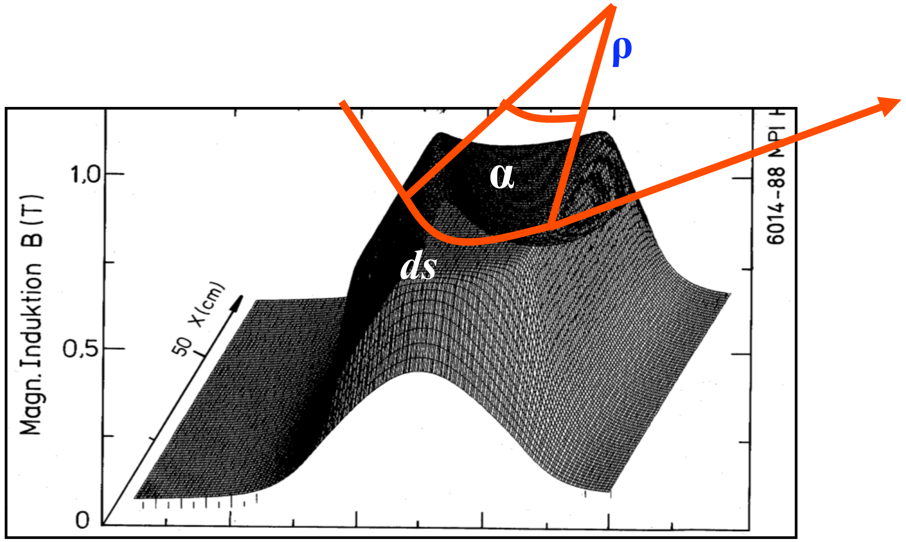
$$\left.\begin{array}{l} B = 8.3\,T \\ p = 7000\,\dfrac{GeV}{c} \end{array}\right\}$$

$$\frac{1}{\rho} = e\,\frac{8.3\,Vs/m^2}{7000*10^9\,eV/c} = \frac{8.3\,s*3*10^8\,m/s}{7000*10^9\,m^2}$$

$$\frac{1}{\rho} = 0.333\,\frac{8.3}{7000}\,1/m$$

# The Magnetic Guide Field





*field map of a storage ring dipole magnet*

$\rho = 2.53\ \boldsymbol{km}$ → $2\pi\rho = 17.6\ km$
$\approx 66\%$

$B \approx 1 \ldots 8\ \boldsymbol{T}$

*rule of thumb:*

$$\frac{1}{\rho} \approx 0.3 \frac{B[T]}{p[GeV/c]}$$

*„normalised bending strength“*

# The Problem:


***LHC Design Magnet current: I=11850 A***

***and the machine is 27 km long !!!***

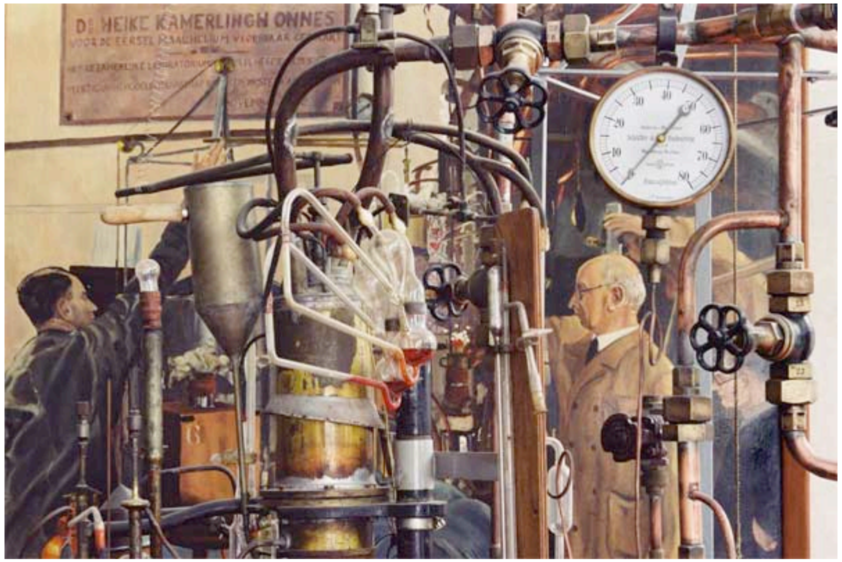
***Ohm's law:*** $U = R * I, \qquad P = R * I^2$

***Problem:***

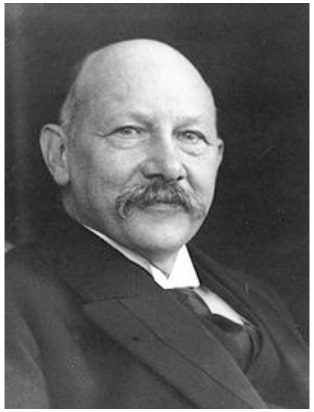
***reduce ohmic losses to the absolute minimum***

# The Solution:

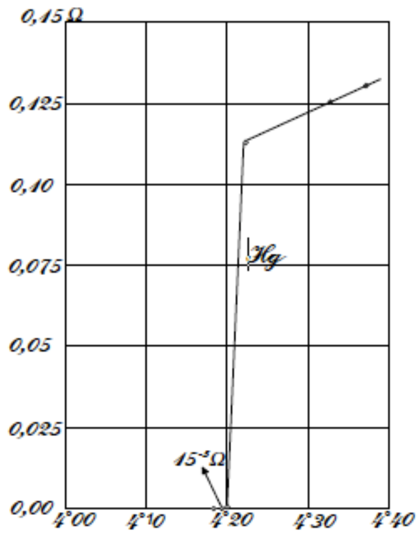
***super conductivity***

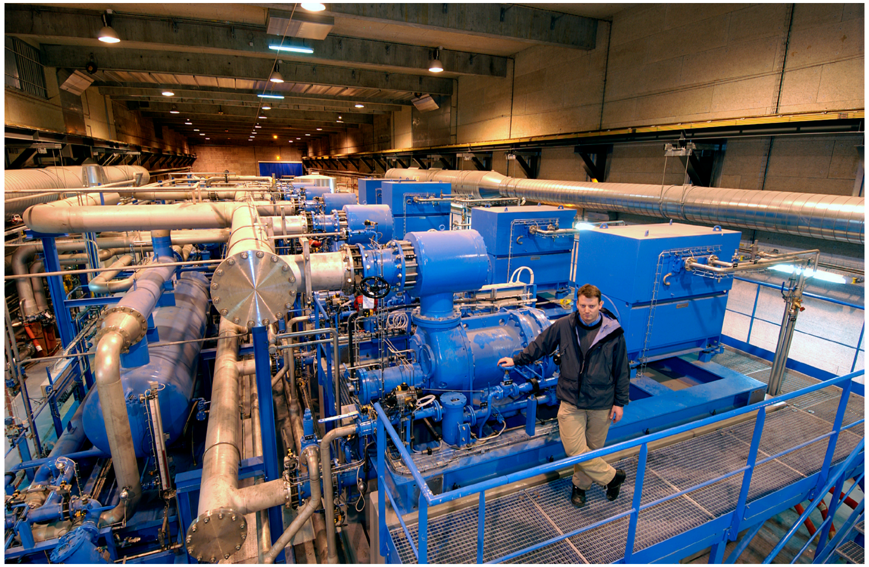


# *Super Conductivity*

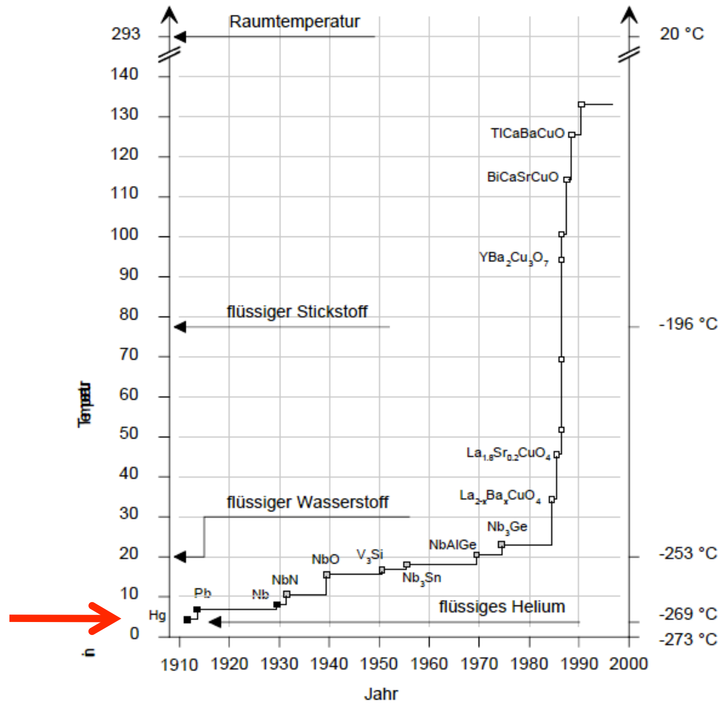


***discovery of sc. by H. Kammerling Onnes, Leiden 1911***



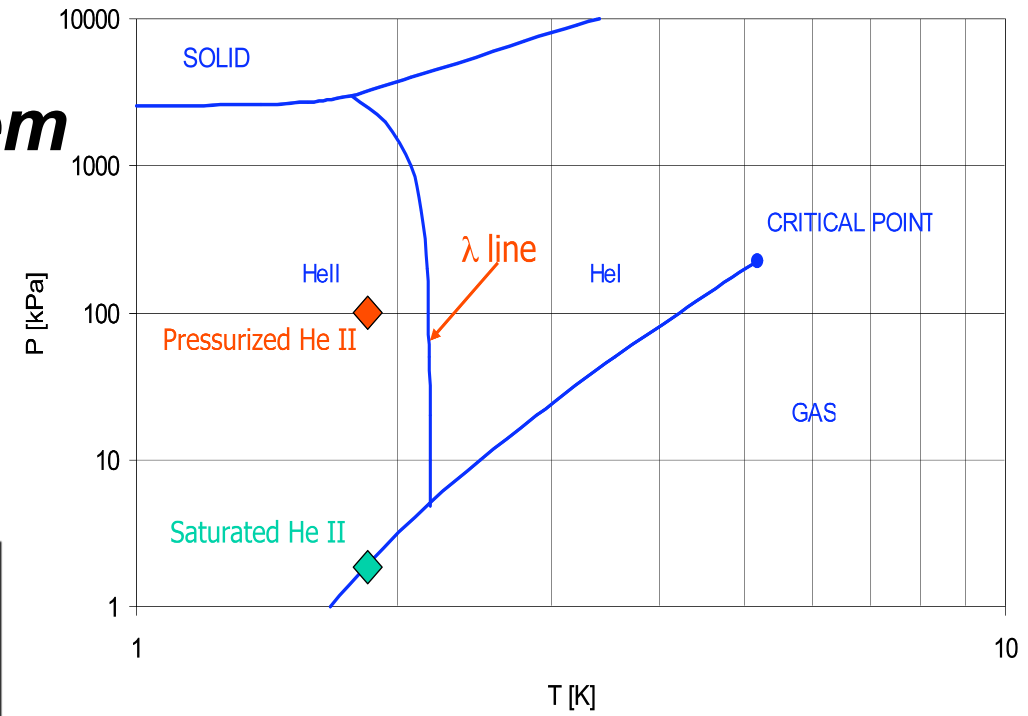



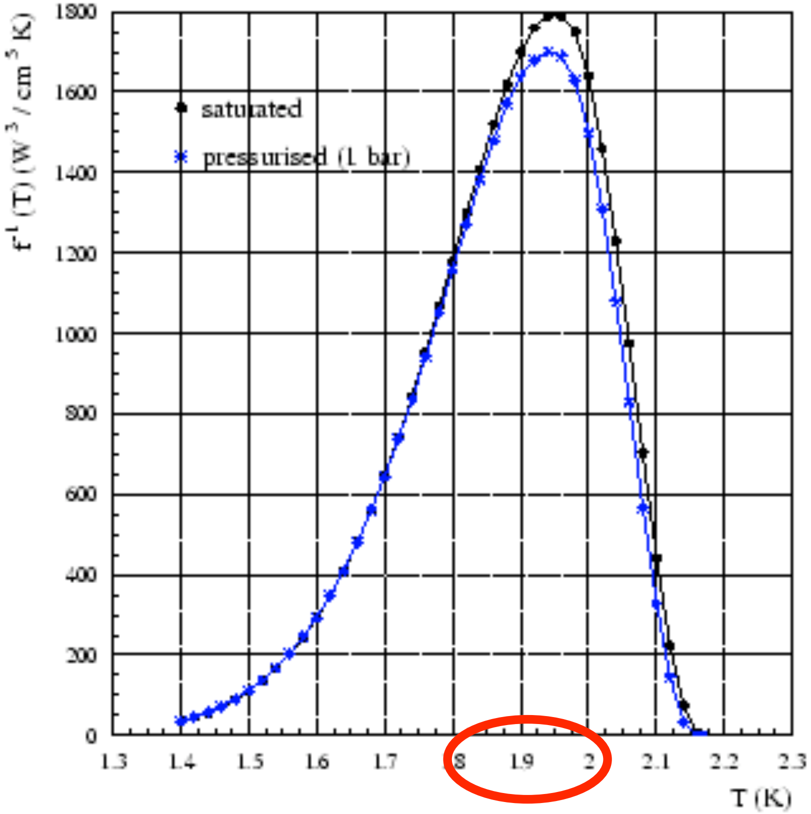
***LHC 1.9 K cryo plant***



# Superfluid helium:
# 1.9 K cryo system

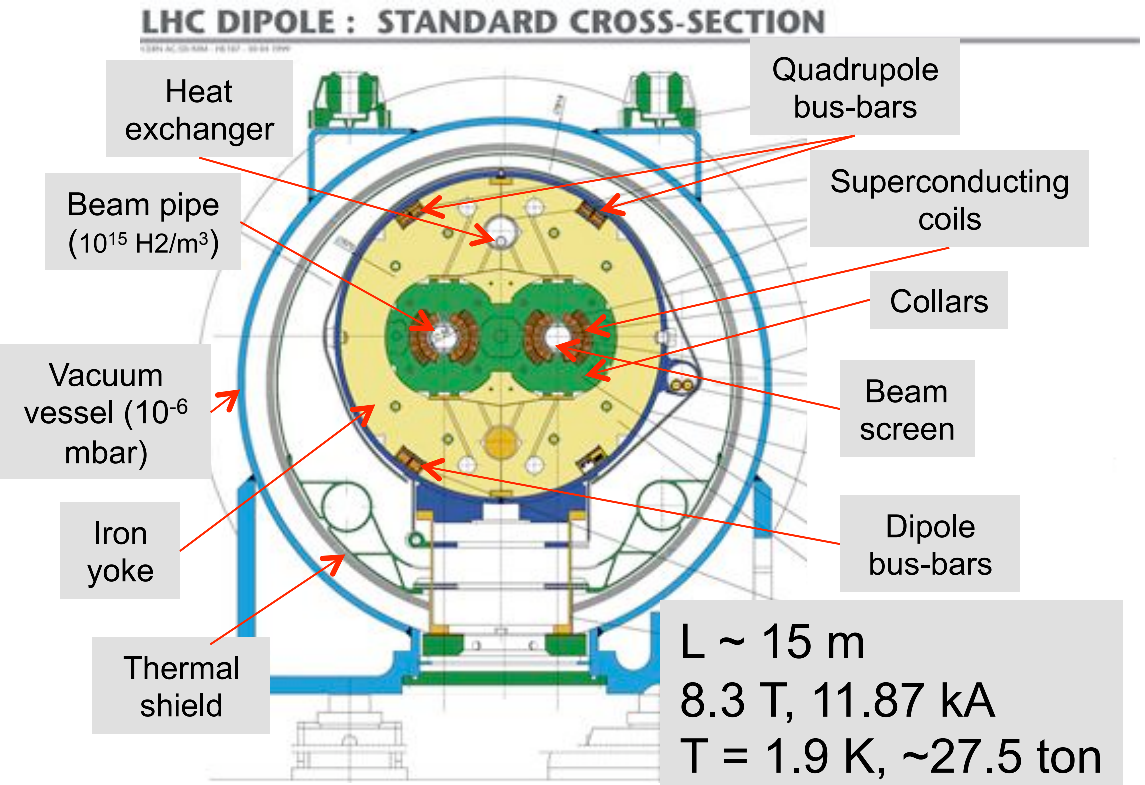
*Phase diagramm of Helium*



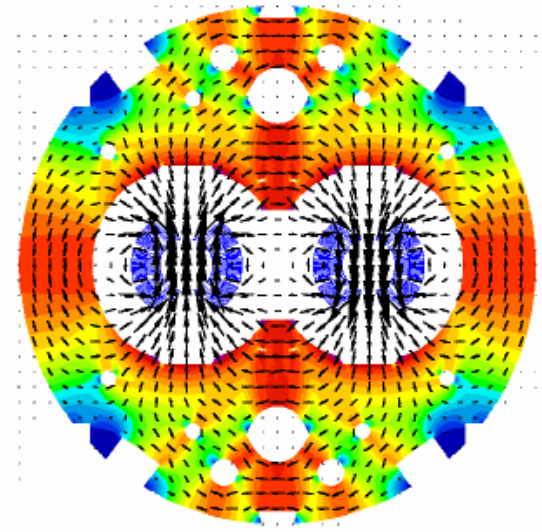




*thermal conductivity of fl. Helium in supra fluid state*

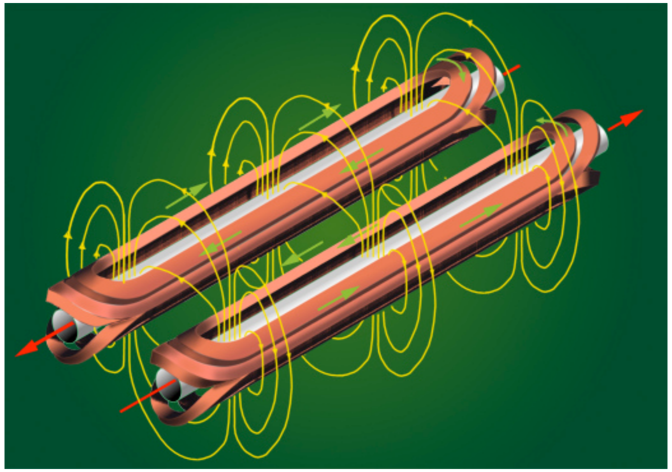
# *LHC: The -1232- Main Dipole Magnets*

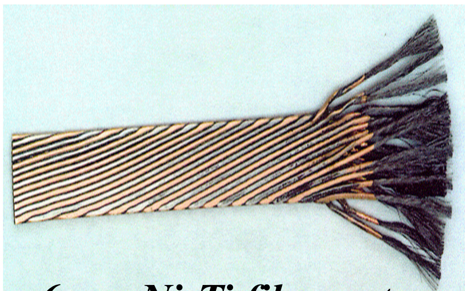



L ~ 15 m
8.3 T, 11.87 kA
T = 1.9 K, ~27.5 ton

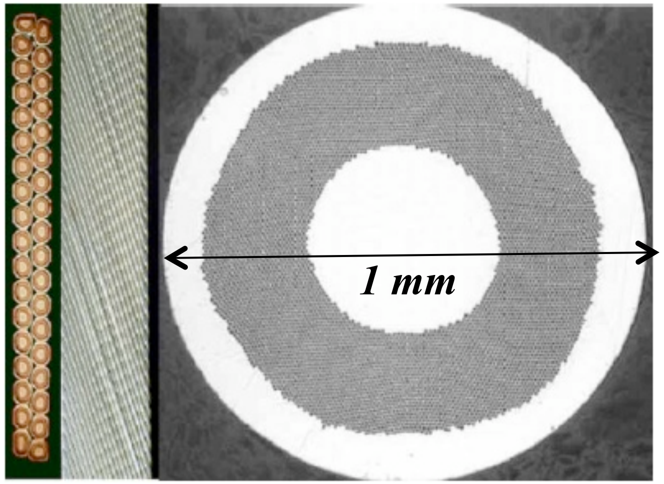


***required field quality:***
***$\Delta B/B=10^{-4}$***



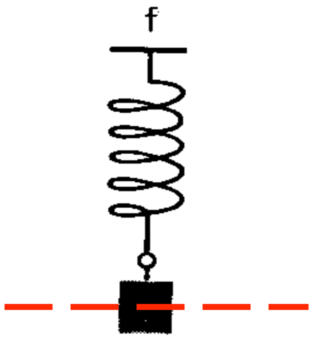


***6 µm Ni-Ti filament***



# 3.) Focusing Properties – Transverse Beam Optics

***classical mechanics: pendulum***




*there is a restoring force, proportional to the elongation x:*

$$m * \frac{d^2 x}{dt^2} = -c * x$$

*general solution: free harmonic oszillation*

$$x(t) = A * \cos(\omega t + \varphi)$$

***Storage Ring:* we need a *Lorentz force* that rises as a function of the distance to ........ ?**

***.................. the design orbit***

$$F(x) = q * v * B(x)$$

# *Quadrupole Magnets:*

*required:* *focusing forces to keep trajectories in vicinity of the ideal orbit*

*linear increasing Lorentz force*

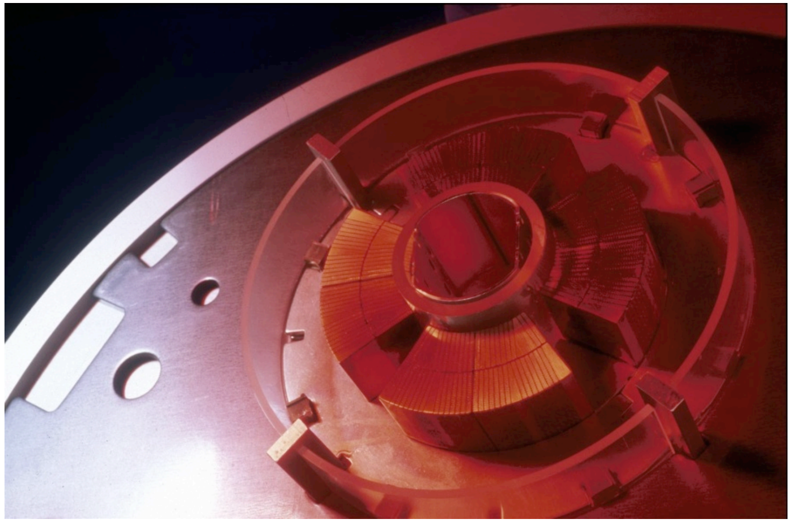
*linear increasing magnetic field* $B_y = g\, x \qquad B_x = g\, y$

*normalised quadrupole field:*

$$\longrightarrow \qquad k = \frac{g}{p/e}$$

*simple rule:*

$$k = 0.3\,\frac{g(T/m)}{p(GeV/c)}$$



*LHC main quadrupole magnet*

$$g \approx 25 \ldots 220 \;\; T/m$$

*what about the vertical plane:*
*... Maxwell*

$$\vec{\nabla} \times \vec{B} = \cancel{\vec{j}} + \cancel{\frac{\partial \vec{E}}{\partial t}} = 0 \qquad \Rightarrow \qquad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = g$$

# *Focusing forces and particle trajectories:*

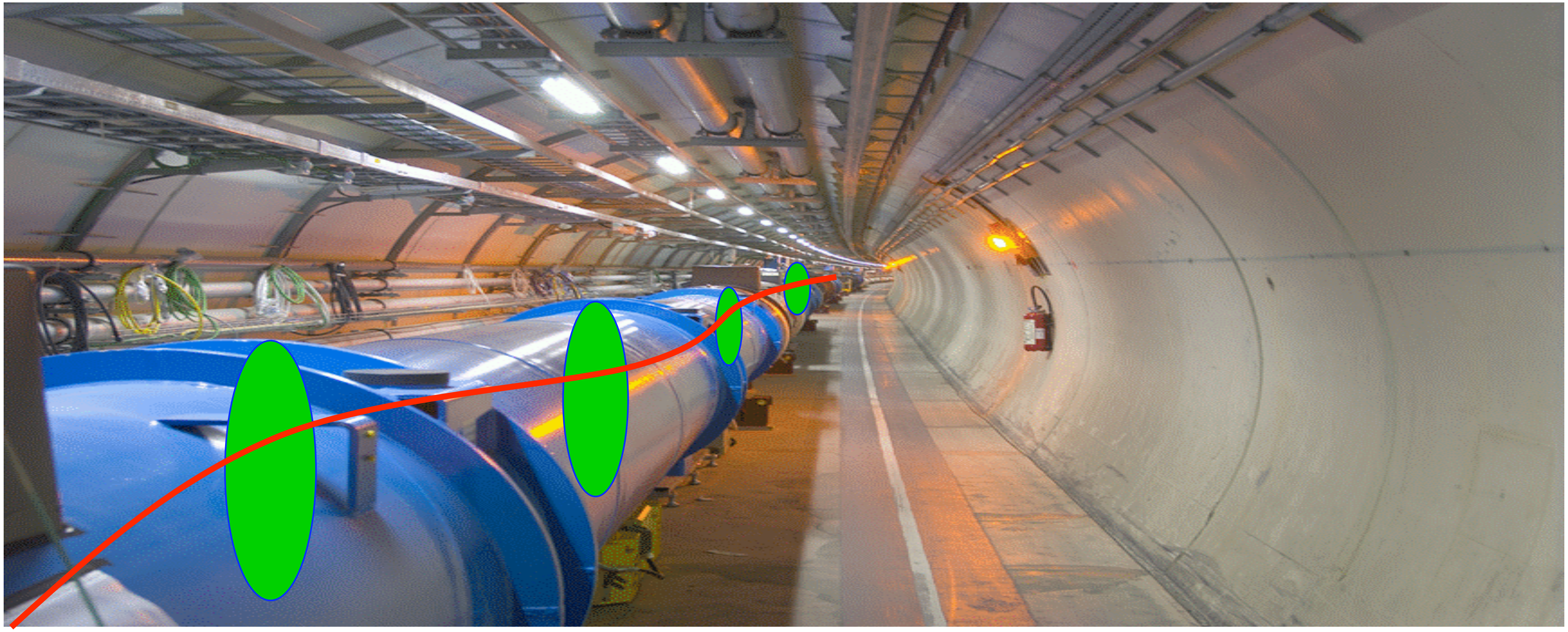
*normalise magnet fields to momentum*
*(remember:* ***B*ρ = p / q*** *)*

***Dipole Magnet***

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

***Quadrupole Magnet***

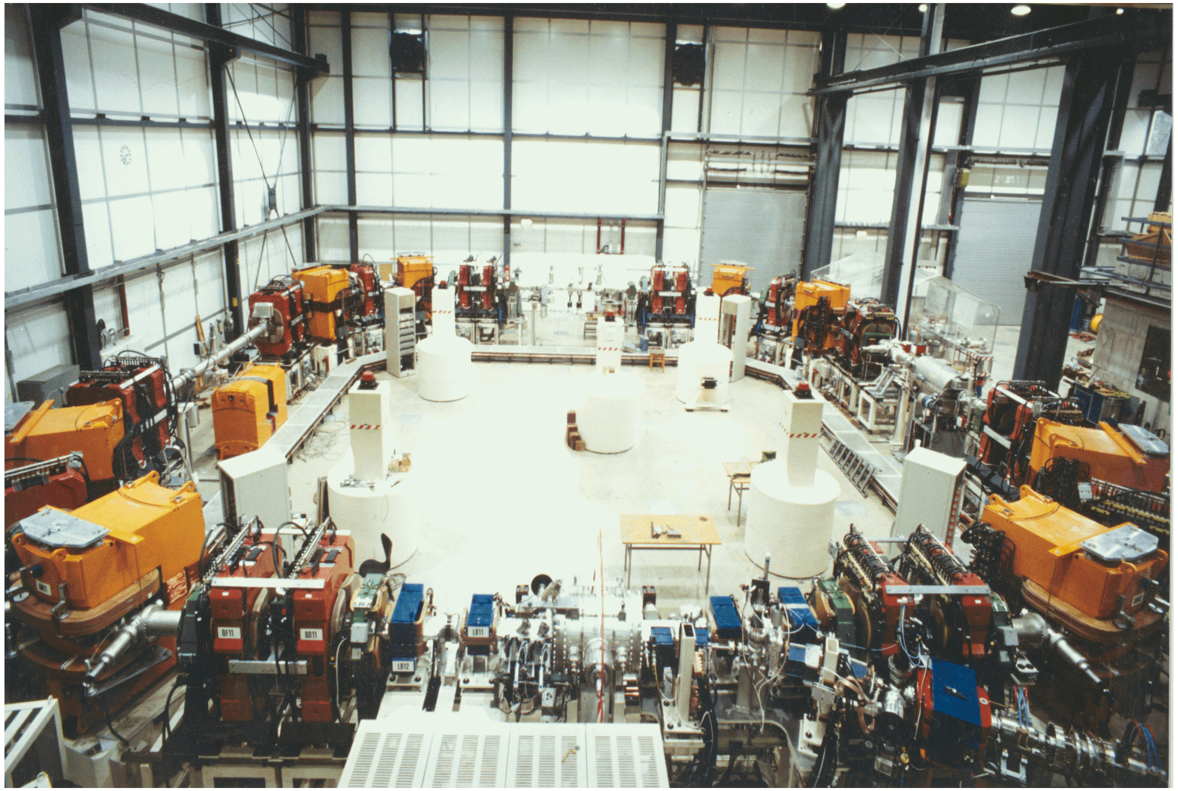
$$k := \frac{g}{p/q}$$



# The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \frac{1}{2!}m\,x^2 + \frac{1}{3!}n\,x^3 + \ldots$$

*only terms linear in x, y taken into account* *dipole fields*
*quadrupole fields*



*Separate Function Machines:*

***Split the magnets and optimise them according to their job:***

***bending, focusing etc***

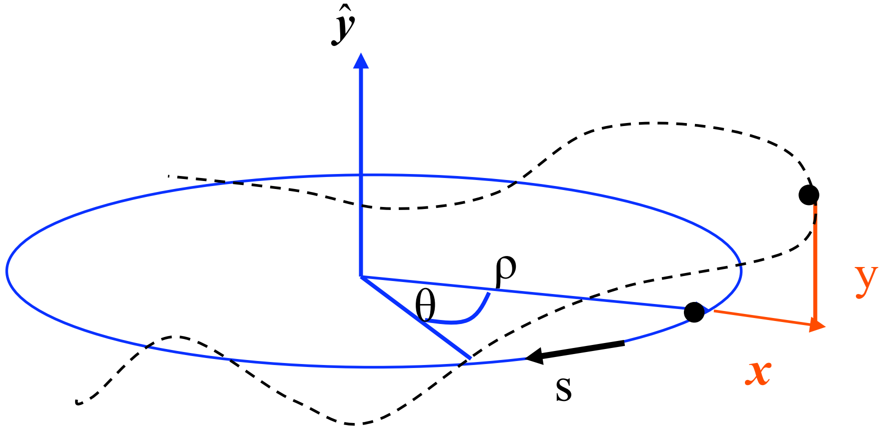
*Example:*
*heavy ion storage ring TSR*

* *man sieht nur dipole und quads → linear*

# The Equation of Motion:

* *Equation for the horizontal motion:*

$$x'' + x\left(\frac{1}{\rho^2} + k\right) = 0$$



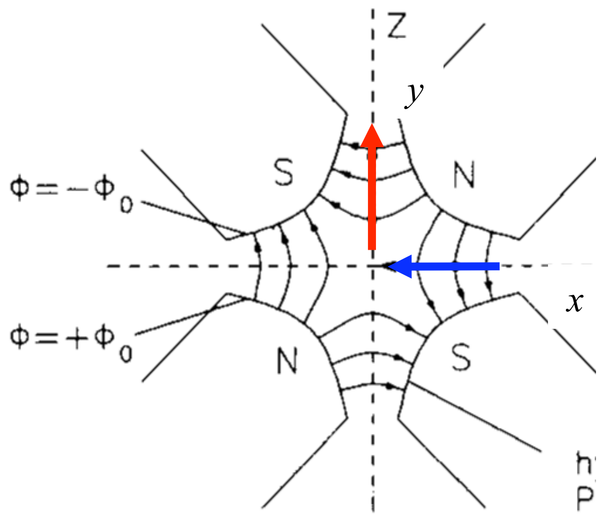

***x = particle amplitude***
***x' = angle of particle trajectory (wrt ideal path line)***

* *Equation for the vertical motion:*

$$\frac{1}{\rho^2} = 0$$ ***no dipoles … in general …***

$$k \leftrightarrow -k$$ ***quadrupole field changes sign***

$$y'' - k\,y = 0$$



# *5.) Solution of Trajectory Equations*

***Define … hor. plane: K= 1/ρ² + k***

***… vert. Plane: K = - k***

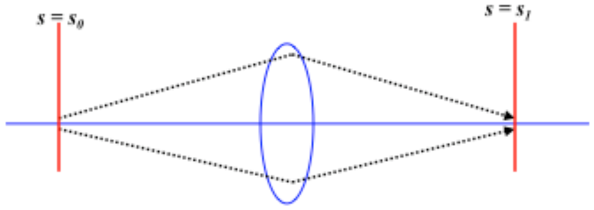
$$\boldsymbol{x'' + K\ x = 0}$$

***Differential Equation of harmonic oscillator … with spring constant K***

***Ansatz:*** ***Hor. Focusing Quadrupole K > 0:***

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x_0' \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x_0' \cdot \cos(\sqrt{|K|}s)$$



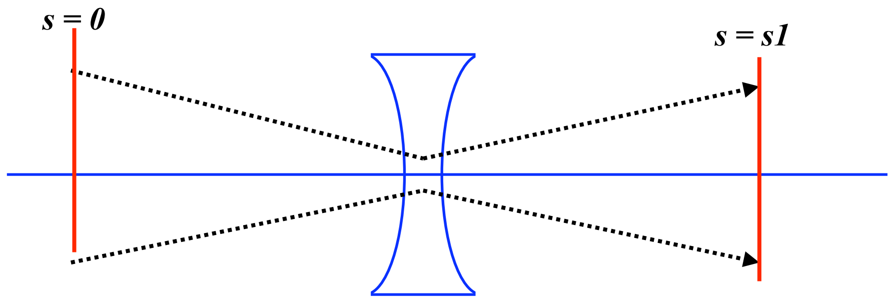

***For convenience expressed in matrix formalism:***

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$\boldsymbol{M_{foc}} = \begin{pmatrix} \cos\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) & \frac{1}{\sqrt{|\boldsymbol{K}|}}\sin\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) \\ -\sqrt{|\boldsymbol{K}|}\sin\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) & \cos\left(\sqrt{|\boldsymbol{K}|}\boldsymbol{l}\right) \end{pmatrix}$$

***hor. defocusing quadrupole:***

$$x'' - K\, x = 0$$



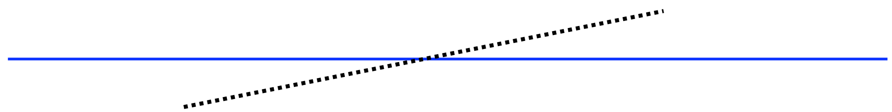

***Ansatz:*** ***Remember from school***

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$M_{defoc} = \begin{pmatrix} \cosh \sqrt{|K|}l & \frac{1}{\sqrt{|K|}} \sinh \sqrt{|K|}l \\ \sqrt{|K|} \sinh \sqrt{|K|}l & \cosh \sqrt{|K|}l \end{pmatrix}$$

***drift space:***

$K = 0$



$$x(s) = x'_0 * s$$

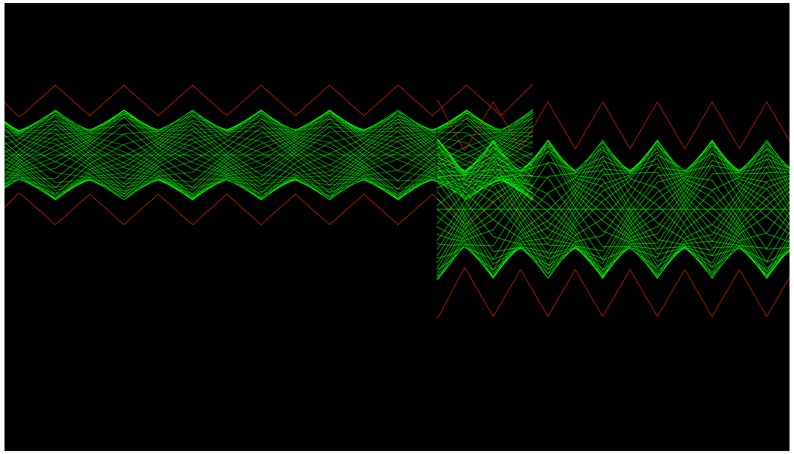
$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

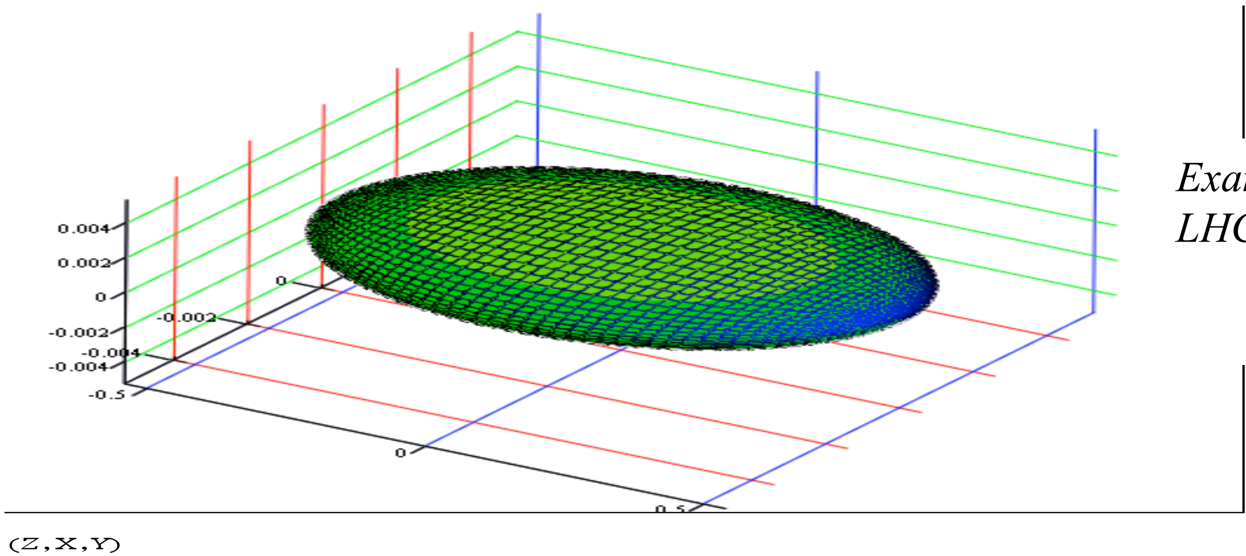
**!** ***with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“***

## *Quadrupole Magnets ...*

***... focus every single particle trajectory towards the centre of the vacuum chamber***
***... define the beam size ... and divergence***
***... „produce" the tune***
***... increase the luminosity***

*Example: Many particle trajectories forming the beam size*






*Example:*
*LHC bunch in the arc of the storage ring*
$l \approx 13$ *cm,*
$x \approx y \approx 0.3mm$

## „veni vidi vici ...“ .... or in english .... „we got it !“

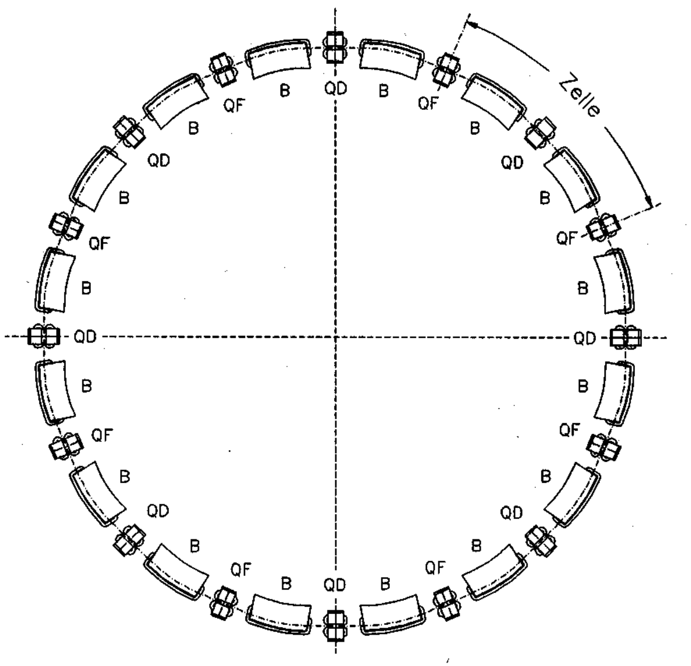
* we can calculate the trajectory of a single particle, inside a storage ring magnet (lattice element)

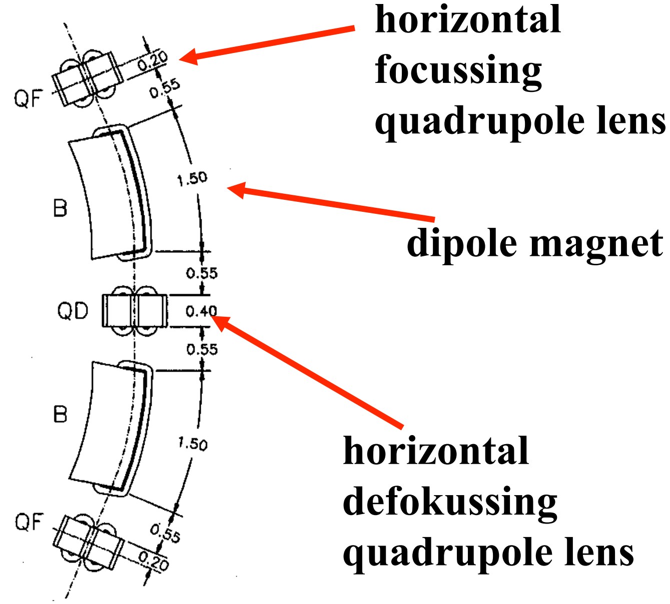
* for arbitrary initial conditions $x_{0,}\ x'_0$

* we can combine these trajectory parts (also mathematically) and so get the complete transverse trajectory around the storage ring

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*}.....$$

*Beispiel: Speichering für Fußgänger (Wille)*



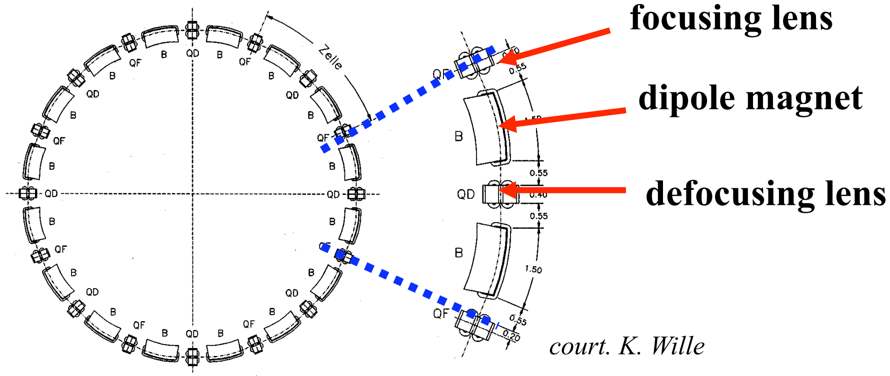



## *Transformation through a system of lattice elements*

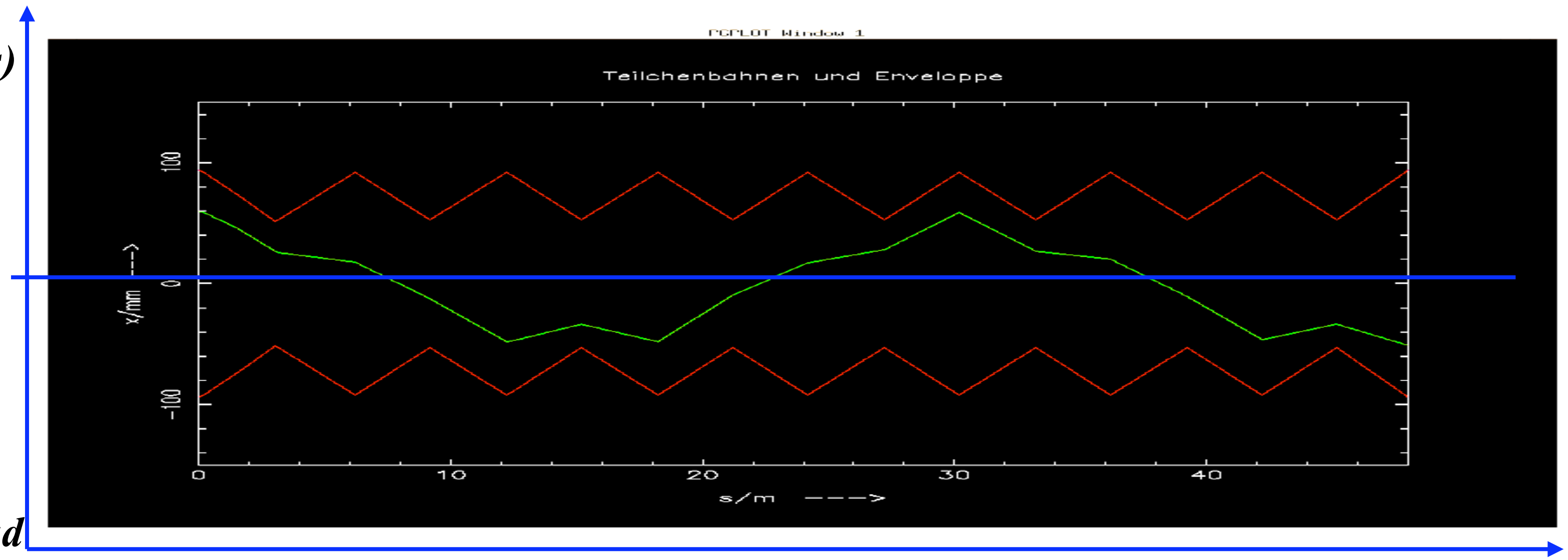
***combine the single element solutions by multiplication of the matrices***

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*.....}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$

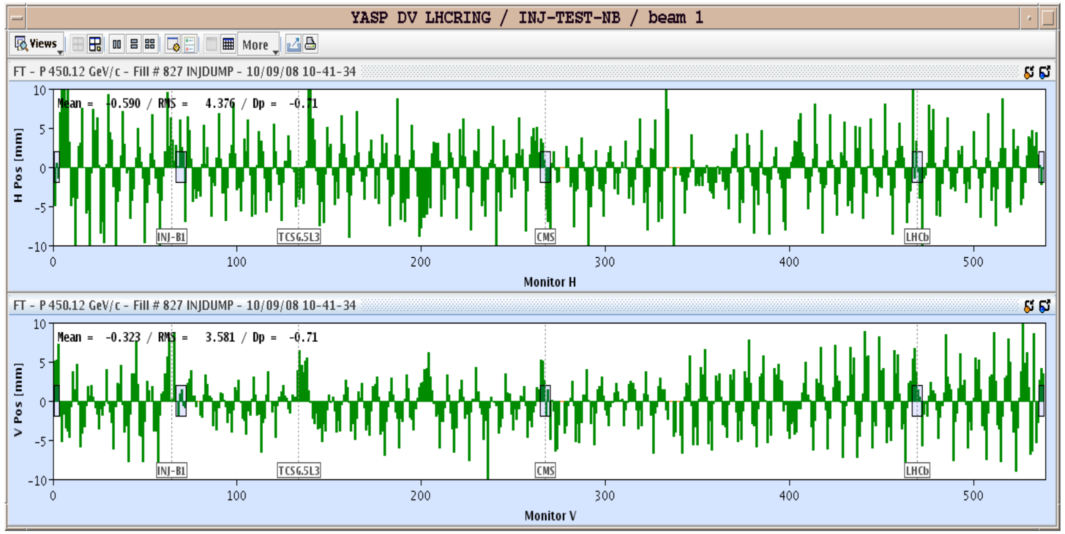



***in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator ! ! !***




***typical values in a strong foc. machine:***
***$x \approx mm$, $x' \leq mrad$***

# 6.) Orbit & Tune:



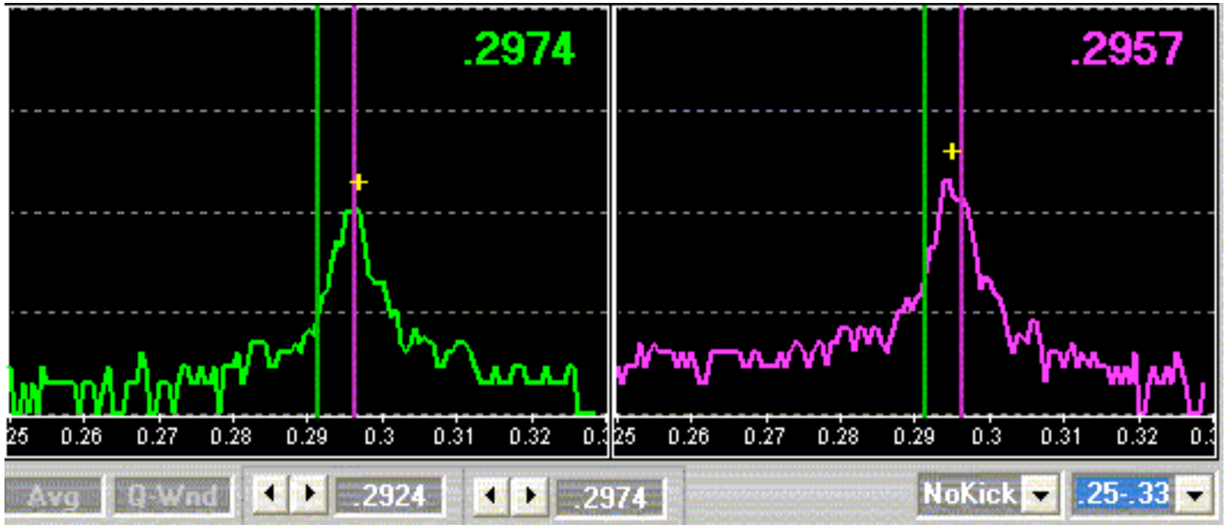

*Tune: number of oscillations per turn*

*64.31*
*59.32*

*Relevant for beam stability:*
*non integer part*

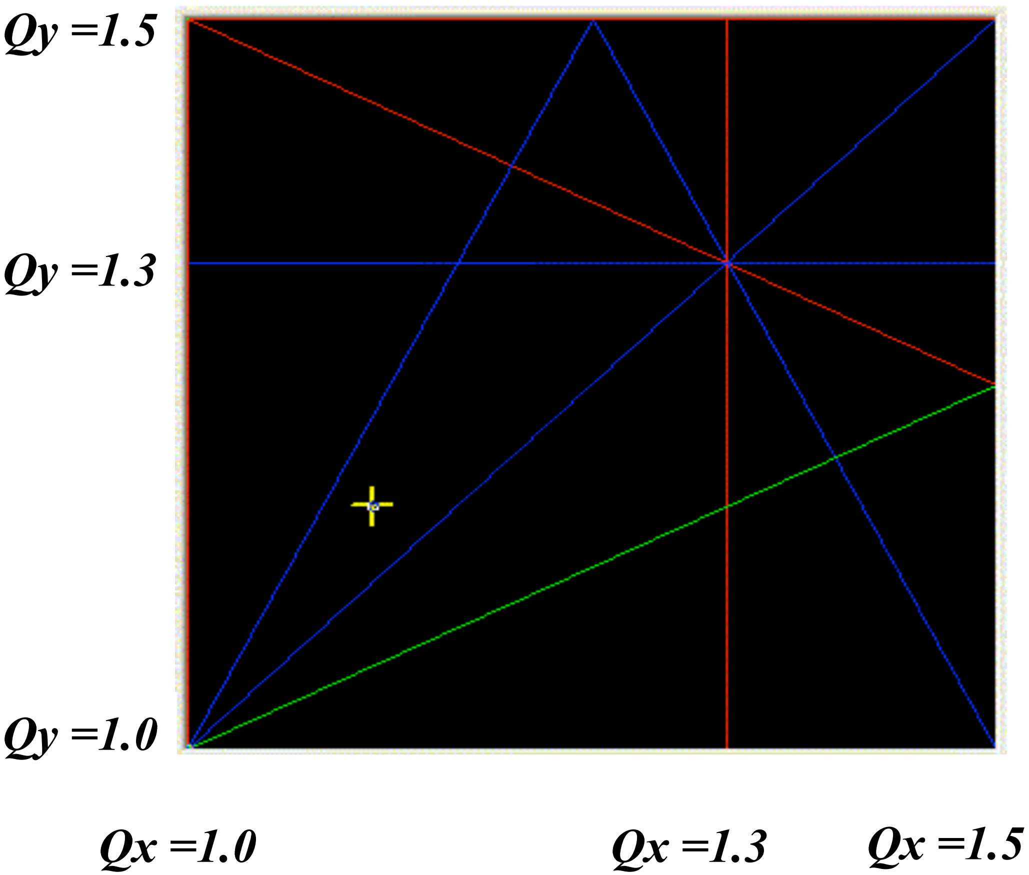
*LHC revolution frequency: 11.3 kHz*

$$0.31 * 11.3 = 3.5kHz$$



# *Tune and Resonances*

***To avoid resonance conditions the frequency of the transverse motion must not be equal (or a integer multiple) of the revolution frequency***




$1*Q_x = 1 \;\; -> \;\; Q_x = 1$

$2*Q_x = 1 \;\; -> \;\; Q_x = 1/2$

***in general:***

$m*Q_x + n*Q_y + l*Q_s = integer$

***Tune diagram up to 3rd order***

# *Resonance Problem:*

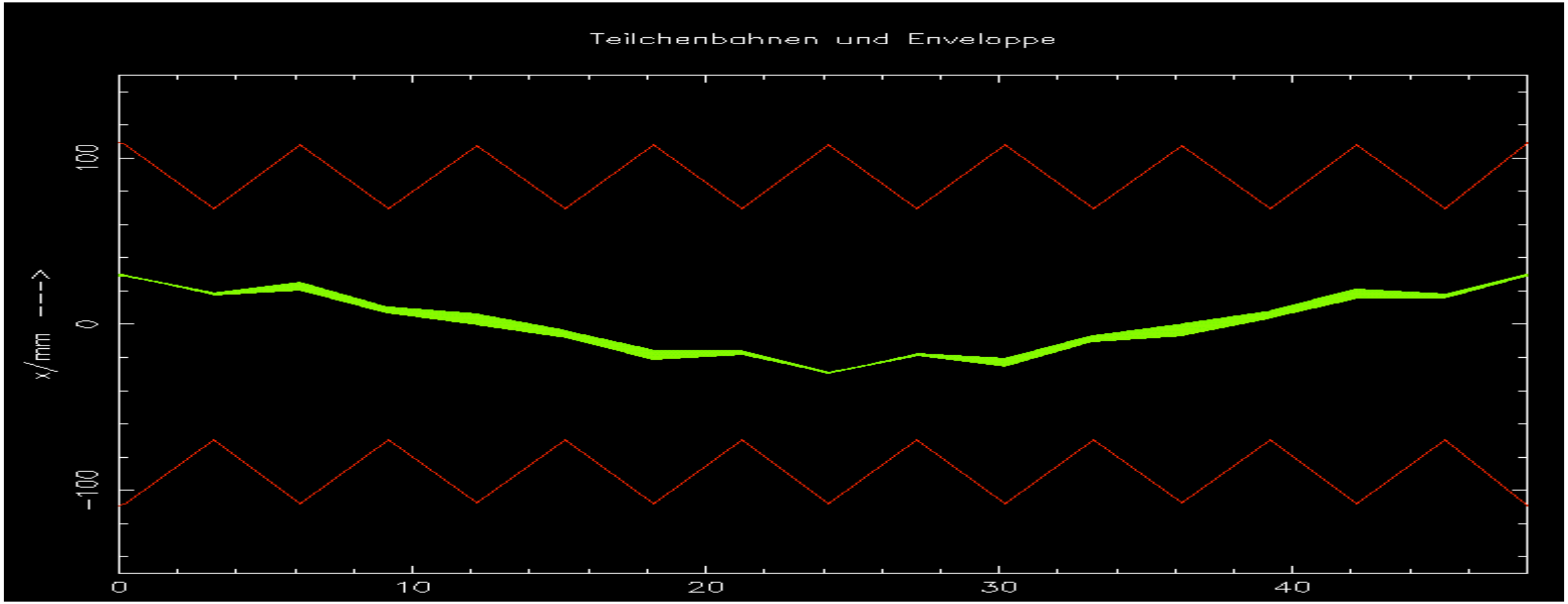
***Orbit in case of a small dipole error:***

$$x_{co}(s) = \frac{\sqrt{\beta(s)} * \int \frac{1}{\rho_{s1}} \sqrt{\beta_{s1}} * \cos(\psi_{s1} - \psi_s - \pi Q)\, ds}{2 \sin \pi Q}$$

***Assume: Tune = integer*** $Q = 1 \quad \rightarrow \quad 0$

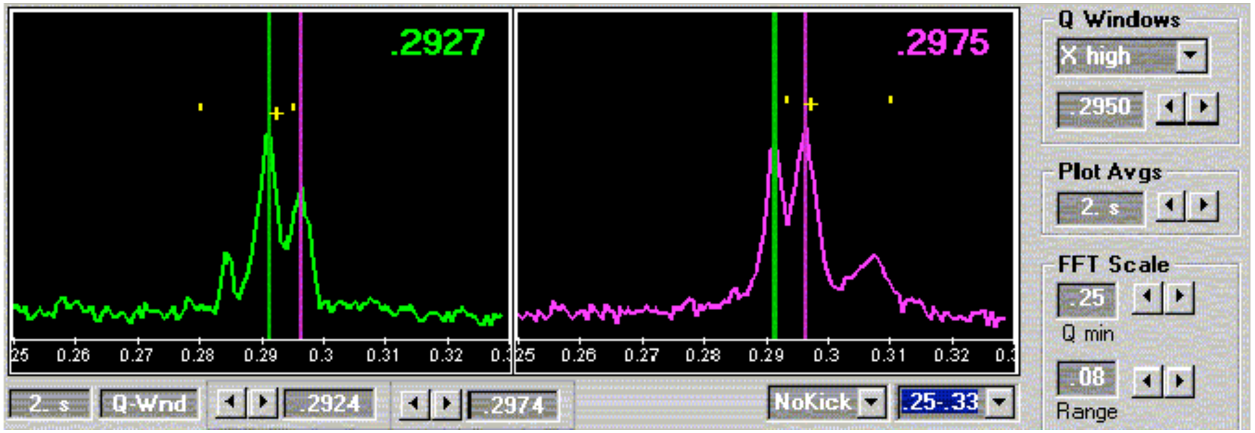
***Qualitatively spoken:***

***Integer tunes lead to a resonant increase of the closed orbit amplitude in presence of the smallest dipole field error.***



# *Coupling*

! *with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“*




*Coupling …*

*1.) hurts but does not kill the beam, however makes it sensitive to additional resonances*

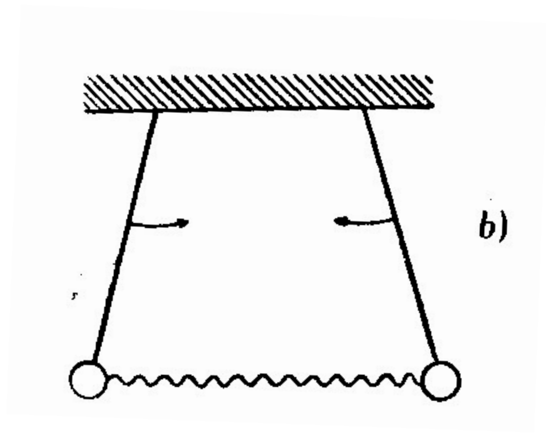
*2.) the amplitude of one oscillation (e.g. hor.) can completely transfer to the other oscillation*
*-> aperture need*

*3.) the tune peaks are split*
*… it is not possible anymore to run with equal tunes in both planes*

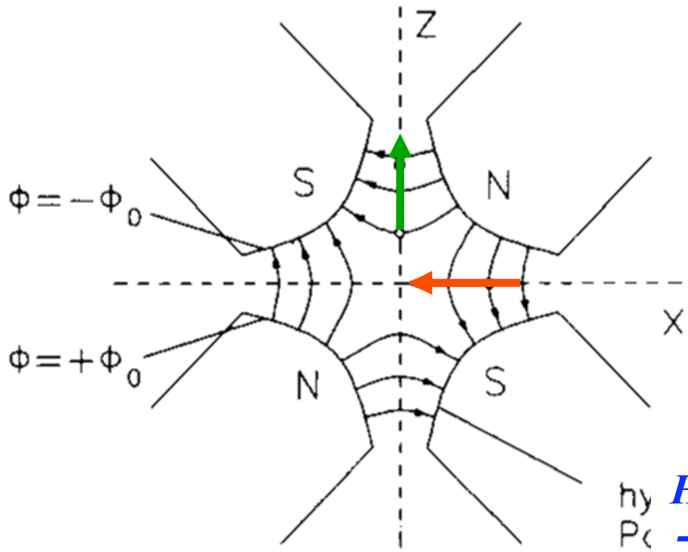
*4.) coupling does not mean that therее are always two peaks visible.*

## *Sources of coupling:*

***We are looking form a force that deflects ...***
***a particle with amplitude in one plane***
***into the other plane***



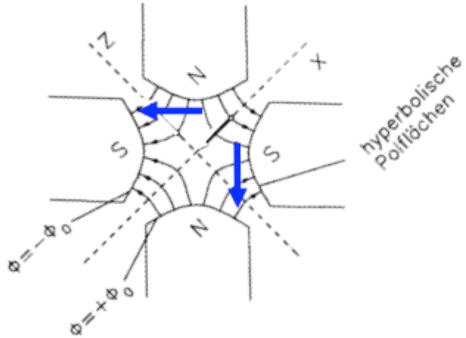

## *Reminder: Field lines of a normal quadrupole*




***Force acts always perpendicular***
***to the field lines***

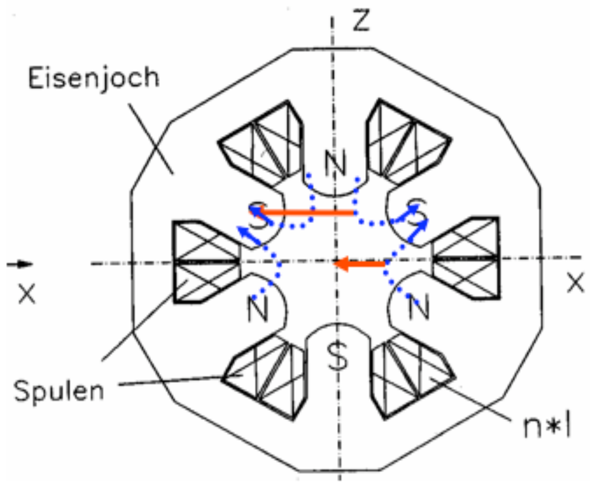
***Horizontal trajectory amplitude***
***→ horizontal focusing***

# Sources of coupling:



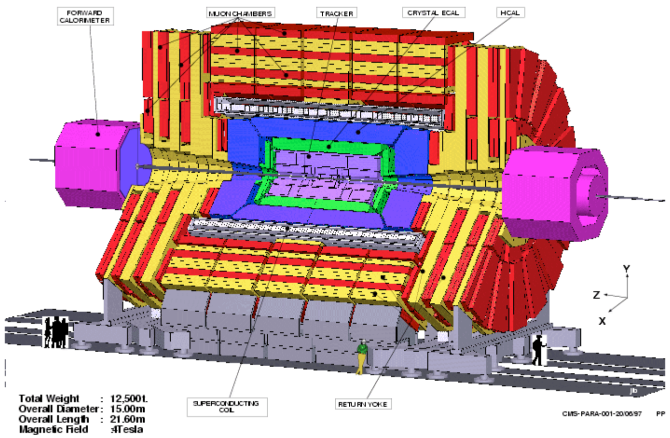

***Skew** (rotated") **Quadrupole:***

*Horizontal amplitude leads to vertical force*
*Vertical amplitude leads to horizontal force*

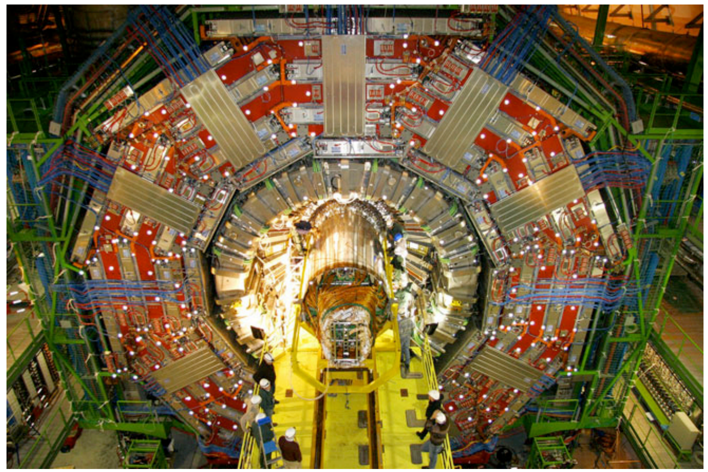



***Sextupole Magnet:***

***horizontal plane:*** *increases in the quadrupole effect*
***vertical plane:*** *creates coupling*




***Detector Solenoids***

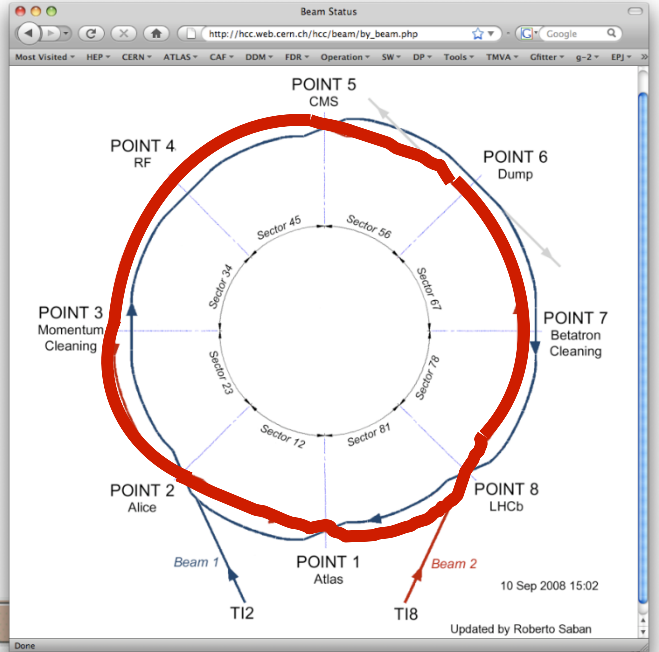


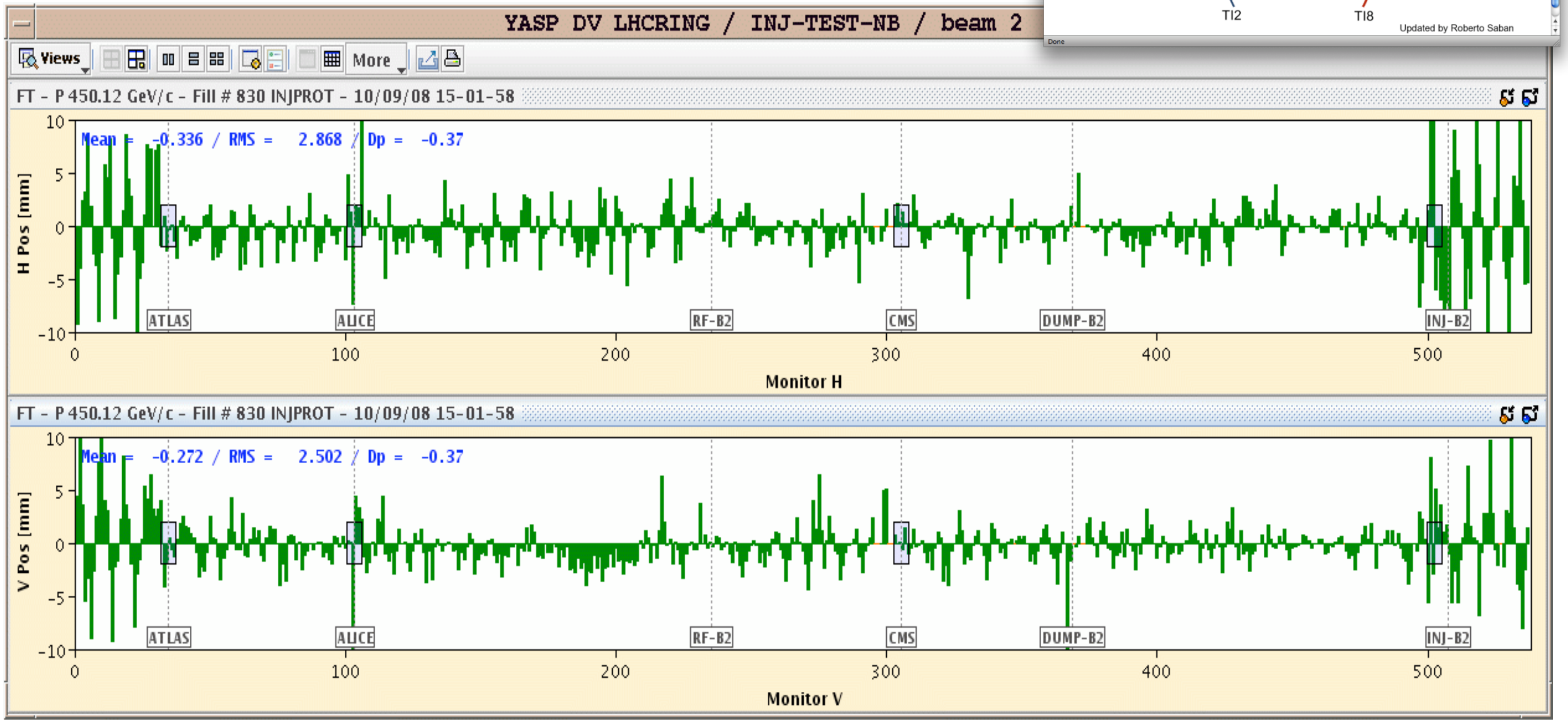
# LHC Operation: Beam Commissioning

***First turn steering "by sector:"***

- One beam at the time
- Beam through 1 sector (1/8 ring),

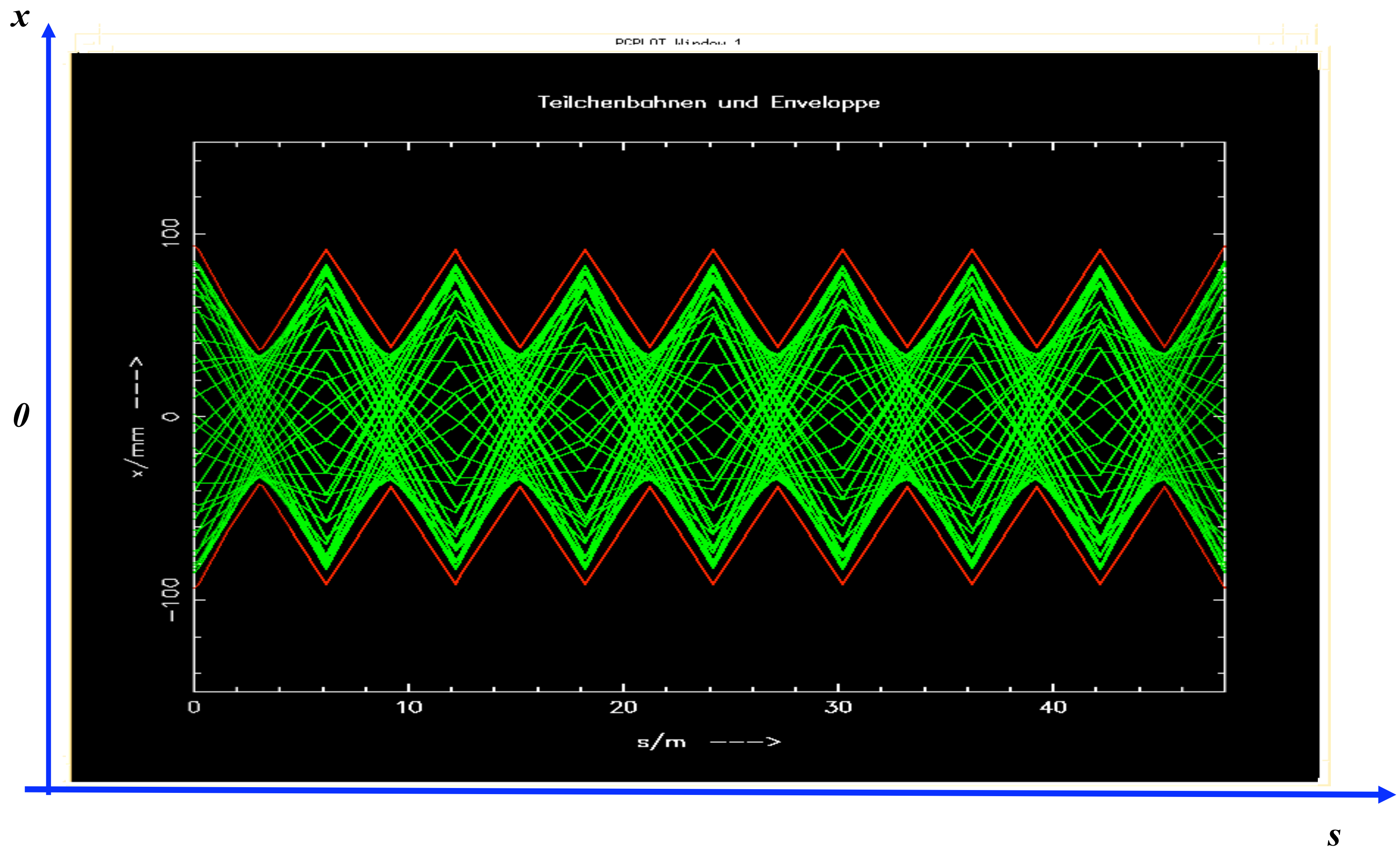
correct trajectory, open collimator and move on.



*Question: what will happen, if the particle performs a second turn ?*

*... or a third one or ... $10^{10}$ turns*



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