

# FFAG Accelerators

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Many thanks to Dr. S. Machida for his advice and previous lecture materials

# 'Fixed Field Alternating Gradient' Accelerators

- Are FFAGs like a synchrotron or cyclotron?
  - EMMA non-scaling FFAG
- Fixed field magnets
- Beam dynamics
- Scaling FFAGs
- Advanced FFAG types and optics

# Motivation

- Many challenges for future accelerators:

High power

Neutrons, muons, ADS

Reliable

Medical, ADS

Flexible

Is industry limited by existing technology?

Rapid acceleration

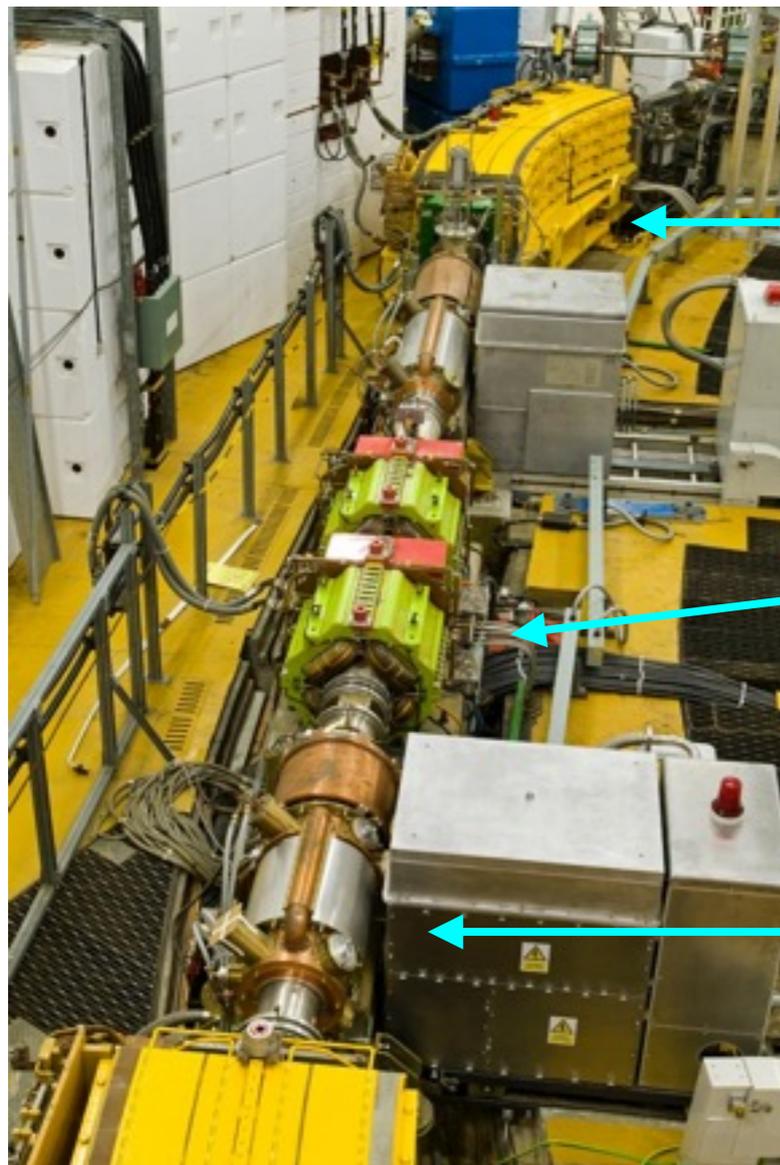
Muon beams, Unstable nuclei

Cheap

Hadron accelerators aren't known for being cheap

# Is an FFAG like a synchrotron? (1)

*“Particles should be constrained to move in a circle of constant radius thus enabling the use of an annular ring of magnetic field ... which would be varied in such a way that the radius of curvature remains constant as the particles gain energy through successive accelerations” - Marcus Oliphant, 1943*

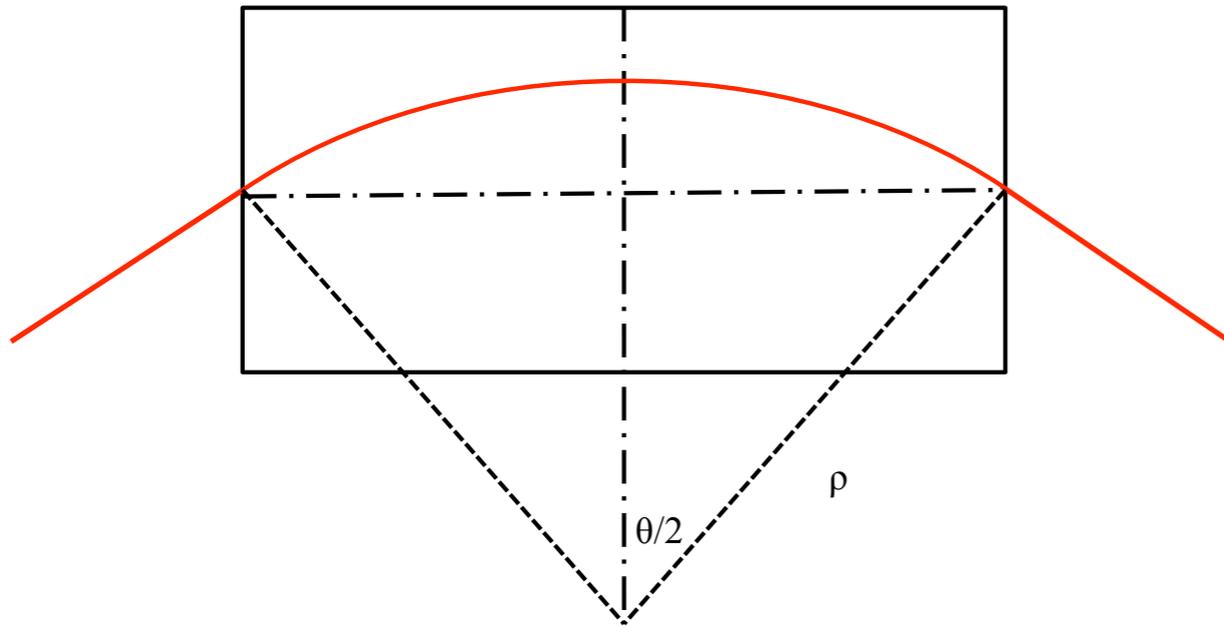


dipole magnets

quadrupole magnets

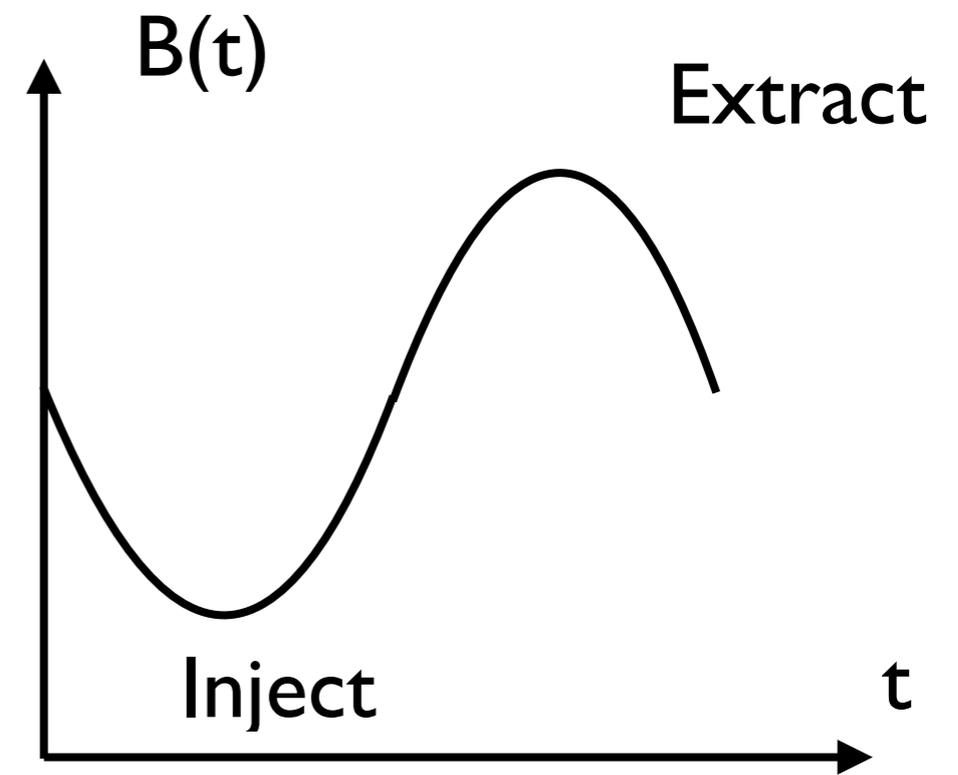
rf cavity

# Is an FFAG like a synchrotron? (2)



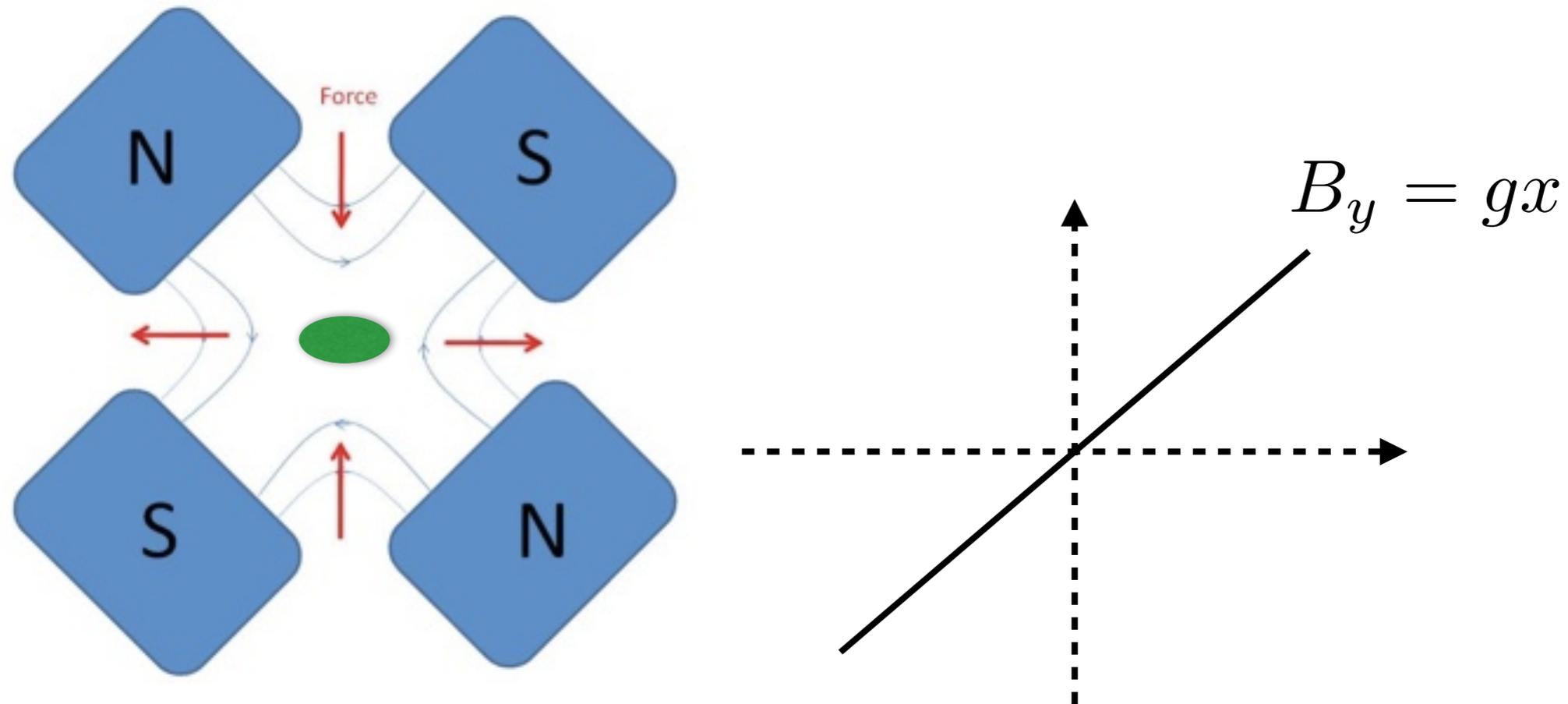
$$\sin(\theta / 2) = \frac{B(t)L}{2(B(t)\rho)}$$

$$\theta \approx \frac{B(t)L}{p(t)/q}$$



What happens if I don't ramp the B field with E?

# Is an FFAG like a synchrotron? (3)



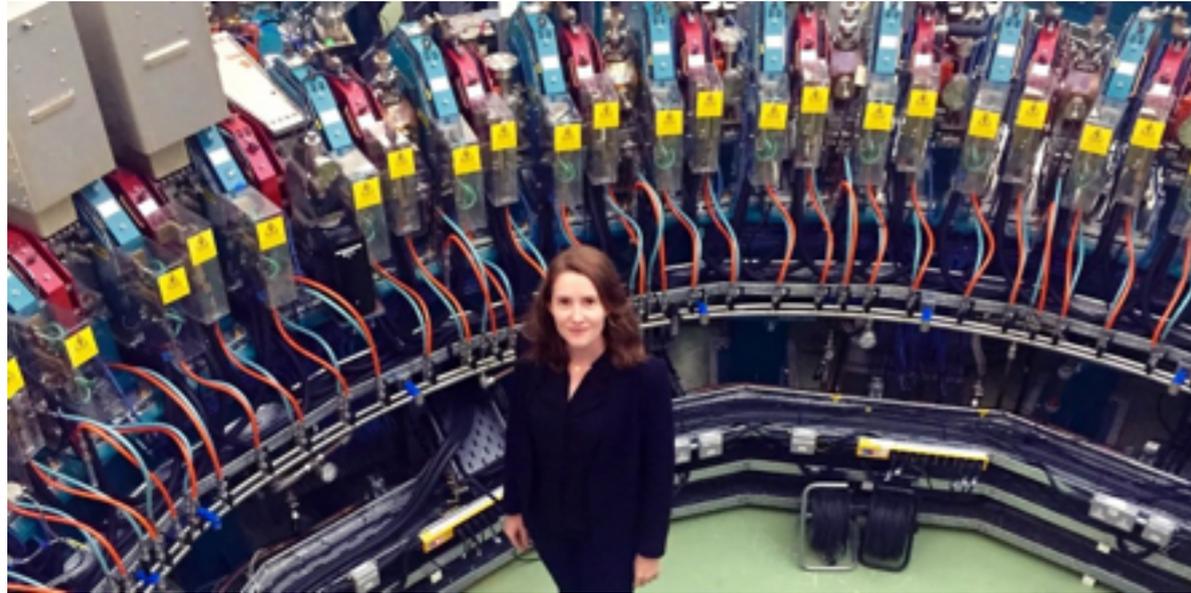
**Do we also ramp the quadrupoles in a synchrotron?**

$$k = \frac{g}{p/q}$$

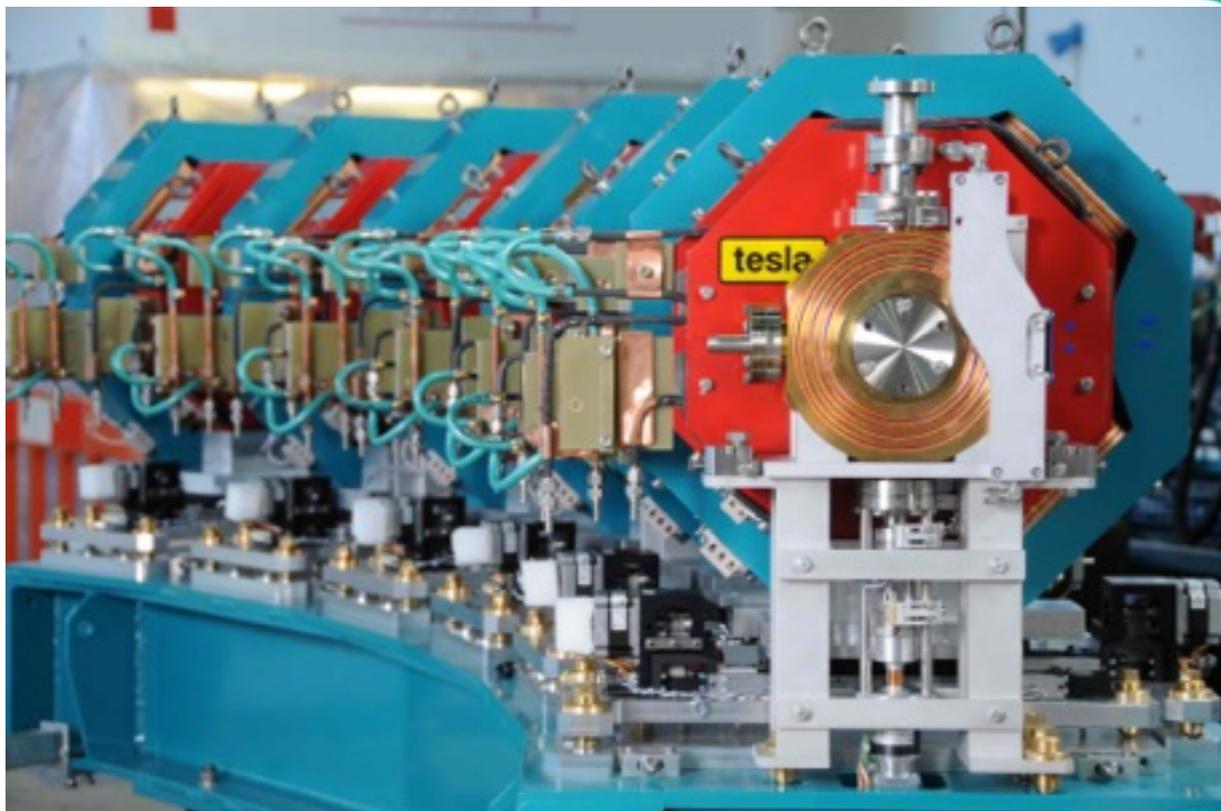
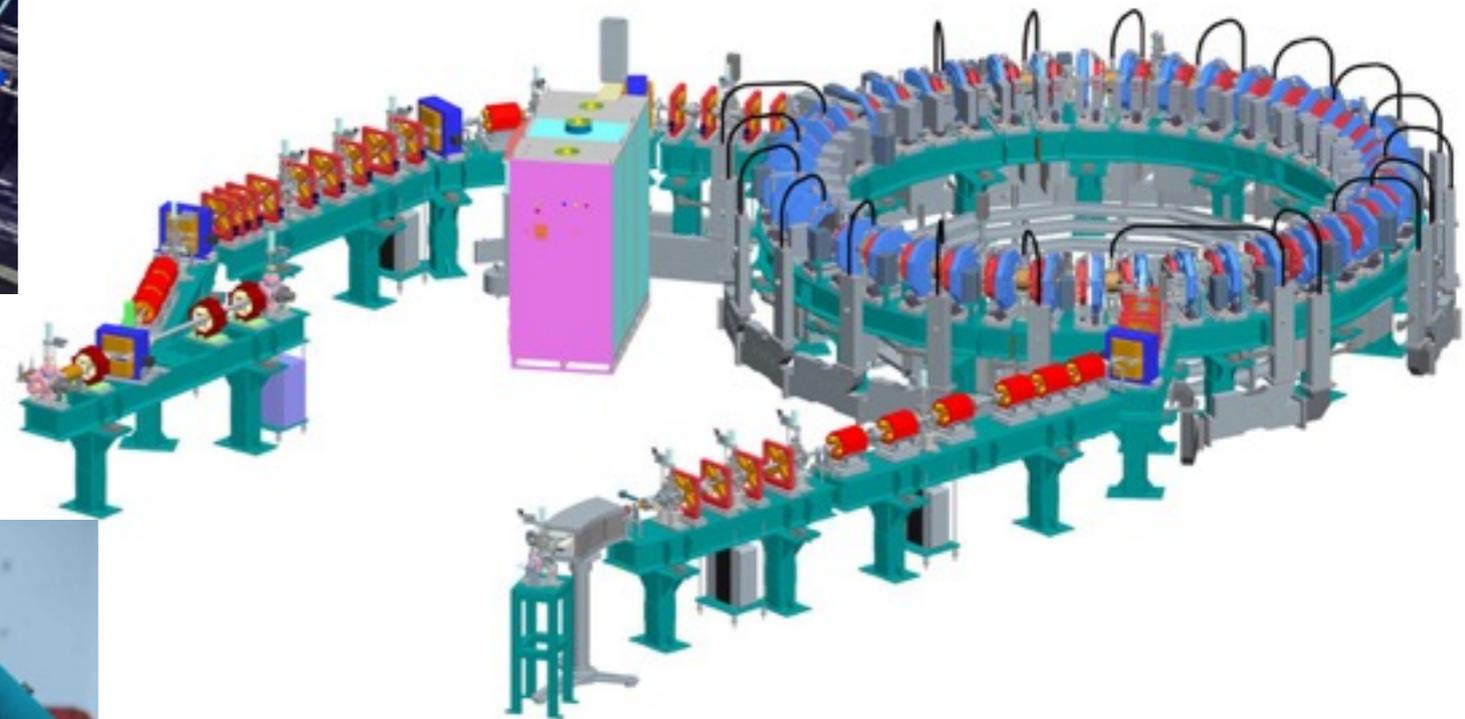
'normalised gradient' of quad

$$\frac{1}{f} = \frac{L(dB(t)/dx)}{p(t)/q}$$

# The 'EMMA' accelerator



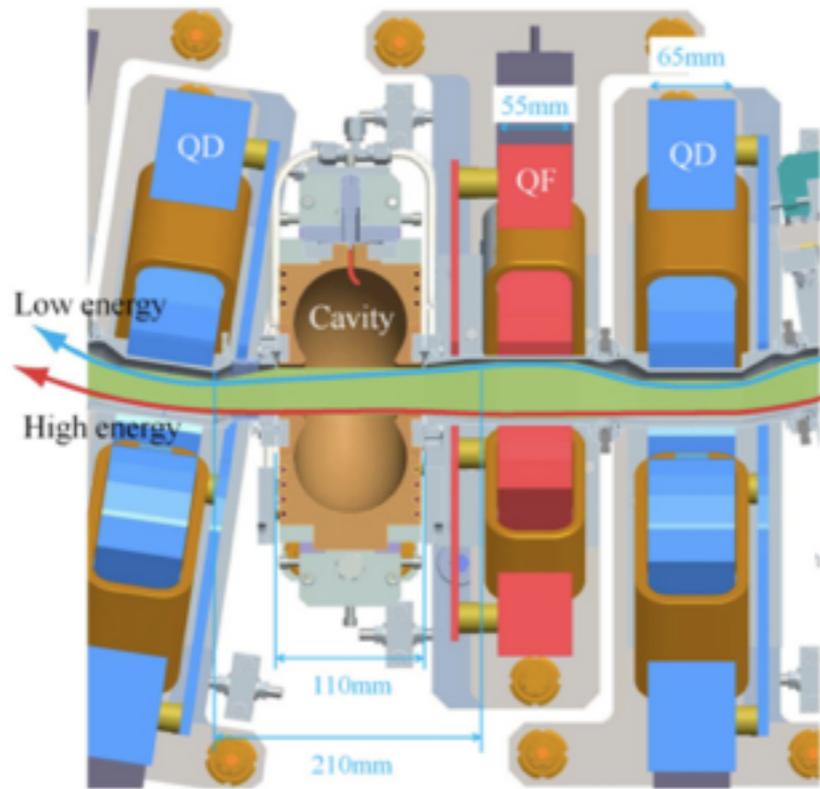
42 Quadrupole doublets  
10-20 MeV e-  
Demonstrates 'non-scaling' FFAG



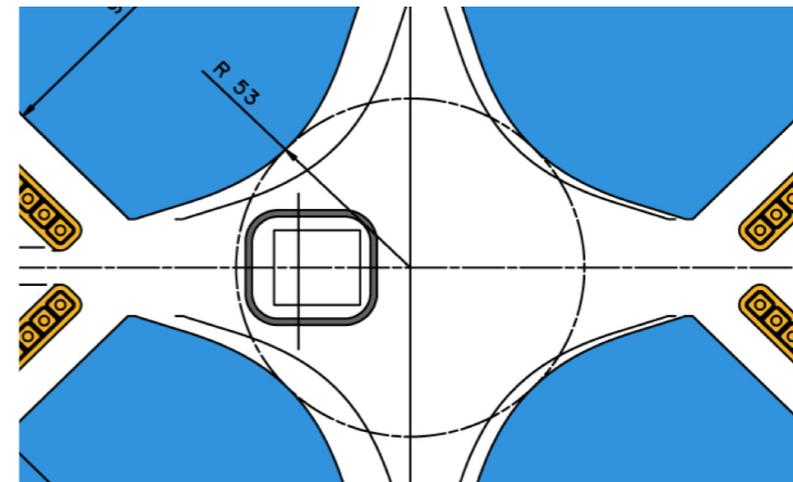
'Electron Model for Many Applications' = EMMA  
Built and commissioned at  
STFC Daresbury Laboratory, UK

# EMMA doesn't ramp the B field with time

'Fixed Field Alternating Gradient' = FFAG



Quadrupole with radial offset creates bending component



Note: this is just like a 'combined function' magnet

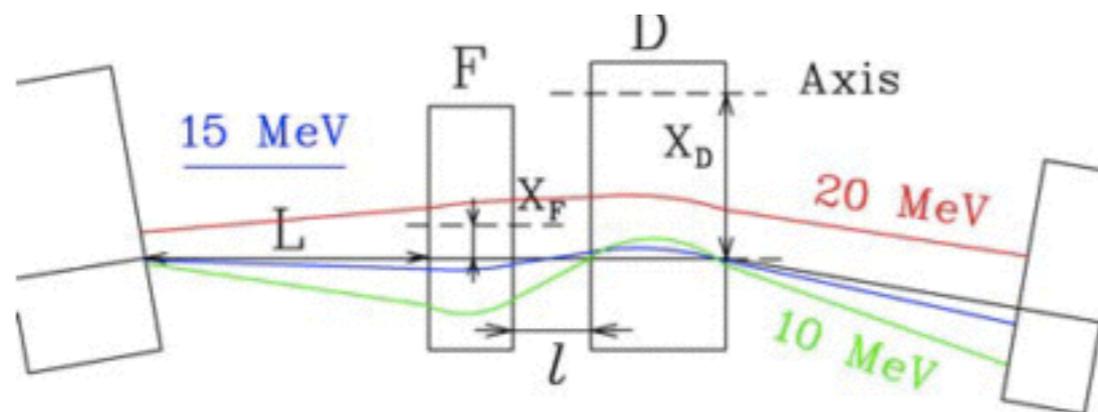


Figure 2: Orbits in a quadrupole doublet cell.

M. Craddock, PAC'07

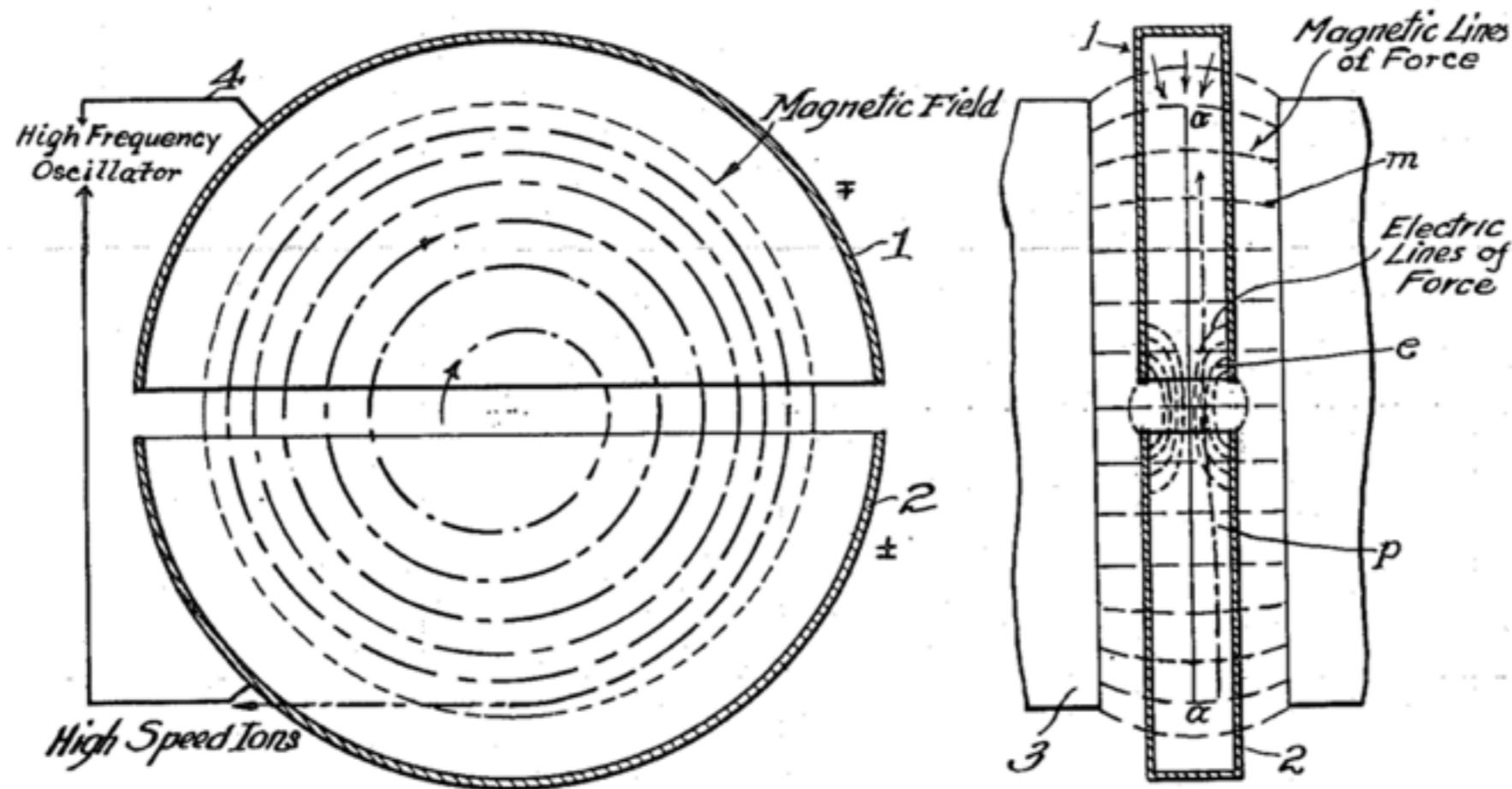
*It doesn't ramp up the magnetic field with energy*

# Fixed-field magnets have advantages

- Simple power supplies and no synchronisation issues
- You can accelerate very quickly (as fast as your RF allows...)
  - in EMMA and in muon FFAGs this is  $\sim 10$  turns
- Higher repetition rate, so higher average current.

# Is an FFAG like a cyclotron? (1)

It has fixed field magnets too



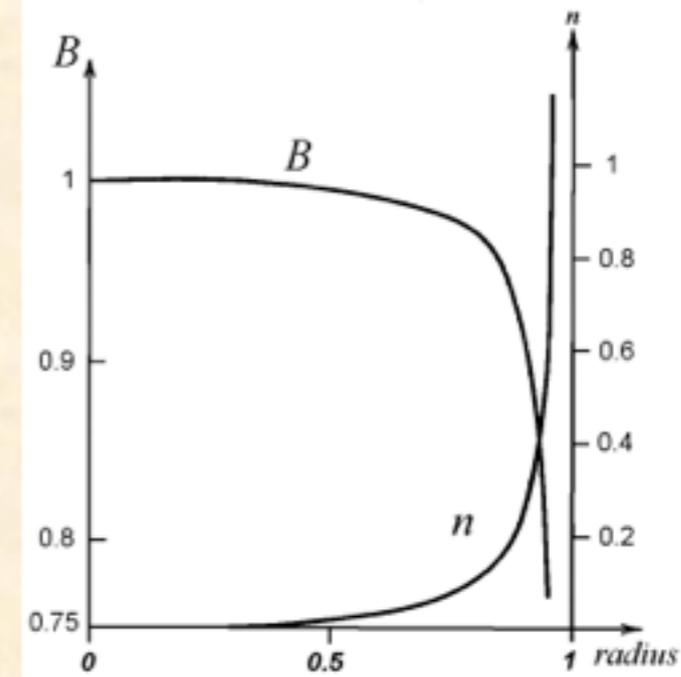
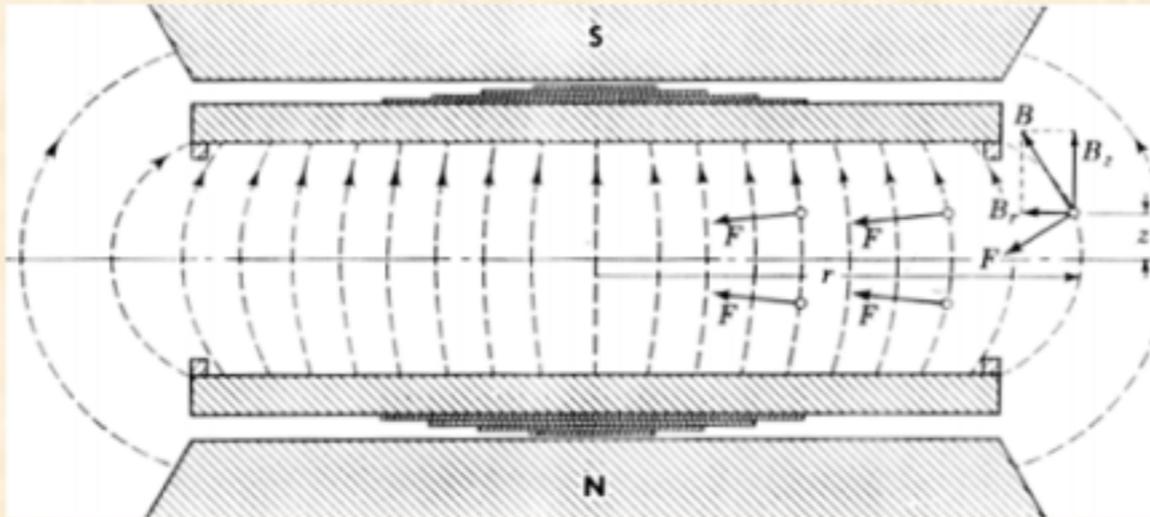
The particles spiral outward as they gain energy

# Is an FFAG like a cyclotron? (2)

## Weak focusing

Simultaneous radial and axial focusing : **Weak focusing**

$$0 \leq n \approx -\frac{\partial B_z}{\partial x} \leq 1 \quad \text{slightly decreasing field}$$



Horizontal focusing  $n < 1$  means :

- $0 < n < 1$   $B_z$  can slightly decrease
- $n < 0$   $B_z$  can increase as much as wanted

Vertical focusing  $n > 0$  means :

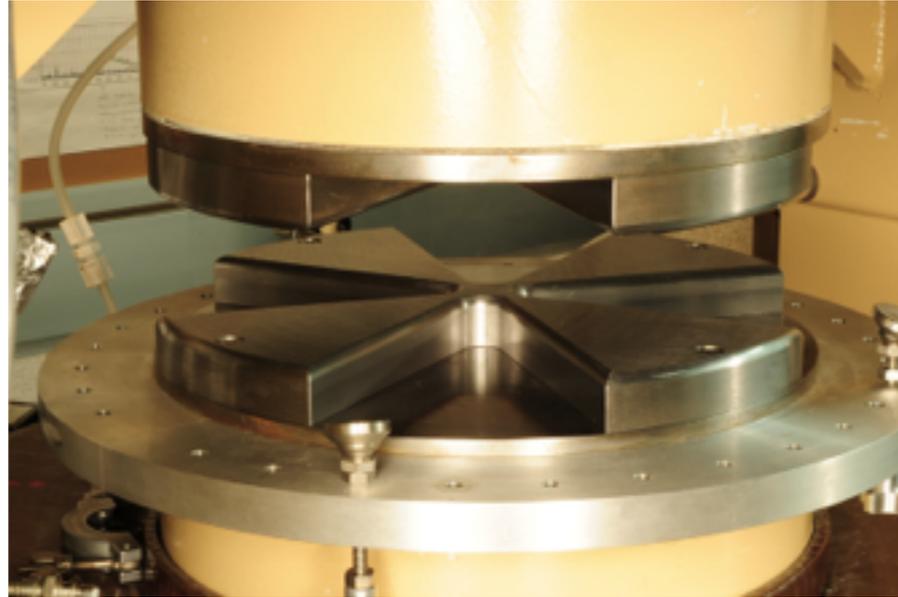
- $B_z$  should decrease with the radius

F. Chautard

18

Slide source: F. Chautard, 2012 CAS

# Is an FFAG like a cyclotron? (3)



## What about the AVF cyclotron?

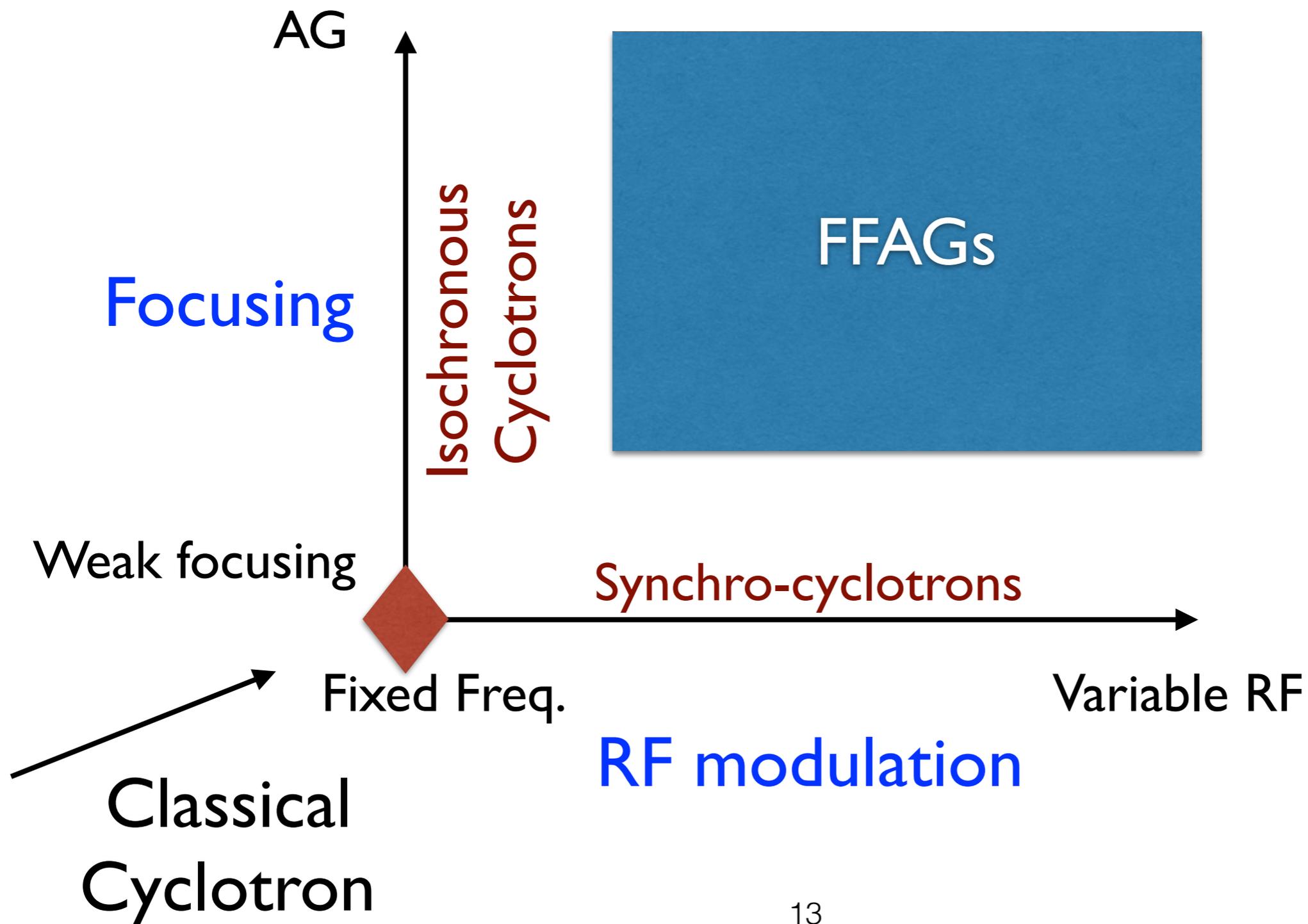
You may have heard of 'flutter' in an AVF cyclotron

An FFAG has:

- Flutter so large that the field reverses sign between 'hills' and 'valleys'.
- Also: FFAG has a field gradient with radius

In the AVF cyclotron the weak focusing is still important, but in the FFAG the dynamics is controlled by the strong focusing

# The circular fixed-field accelerator family



# But that's not the whole story...

- So an FFAG is like a synchrotron but with fixed-field magnets
- OR like a cyclotron with a field gradient and strong focusing, (and variable RF frequency<sup>\*\*</sup>)
- But that's not all there is to it...

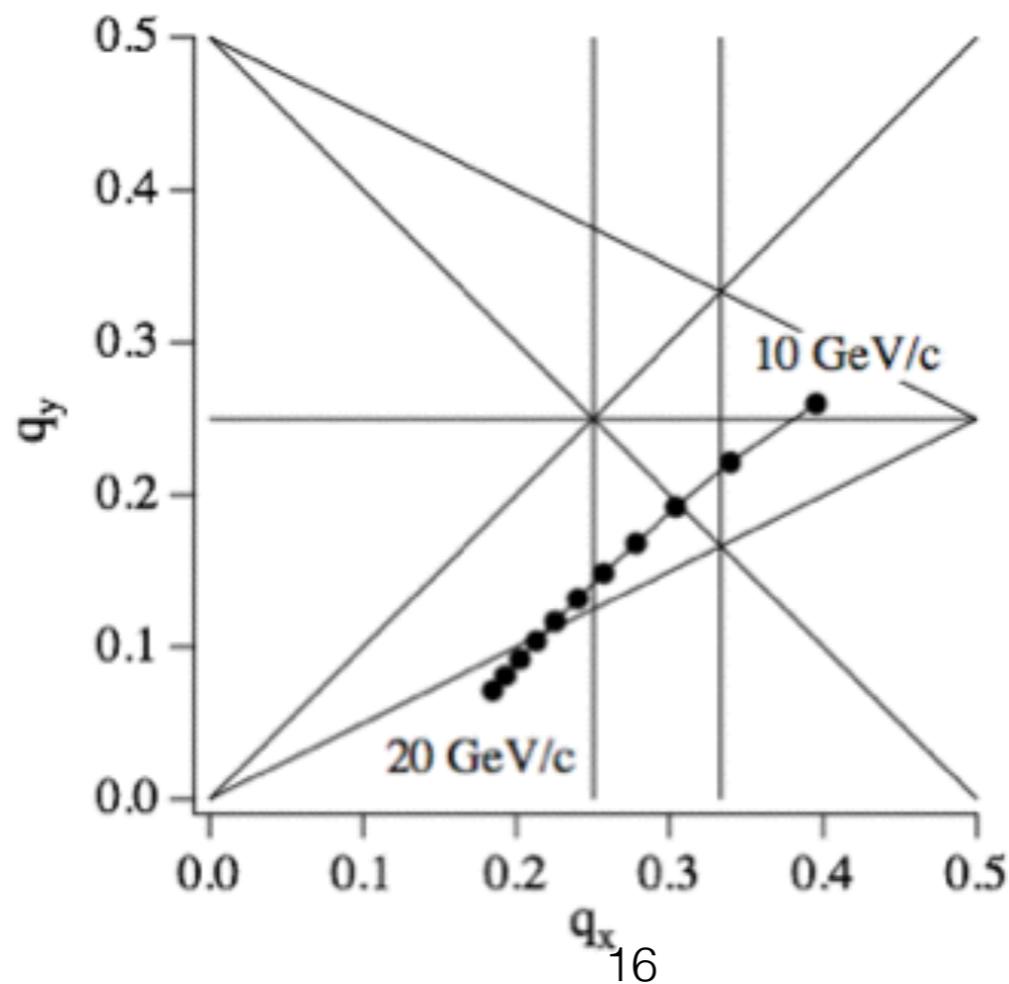
<sup>\*\*</sup>FFAGs do not always have variable RF frequency...

# Circular Accelerators

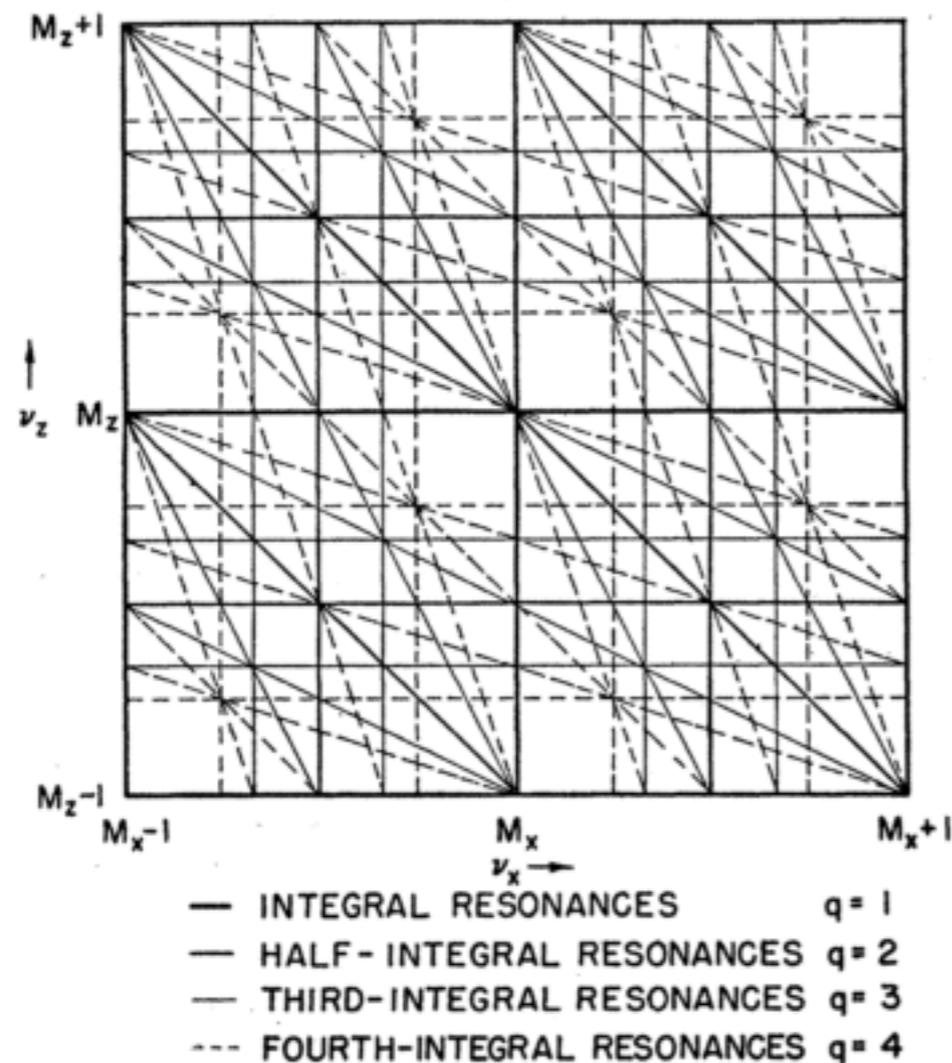
	Cyclotron	Synchrotron	FFAG
Revolution time	Constant	Variable (except relativistic)	Variable
Orbit radius	Variable	Constant	Variable
Transverse focusing	Variable	Constant	Variable

# What does variable focusing mean?

- In a synchrotron the tune is fixed away from resonance lines
- But in an FFAG, the betatron tunes can vary...



# Resonance crossing



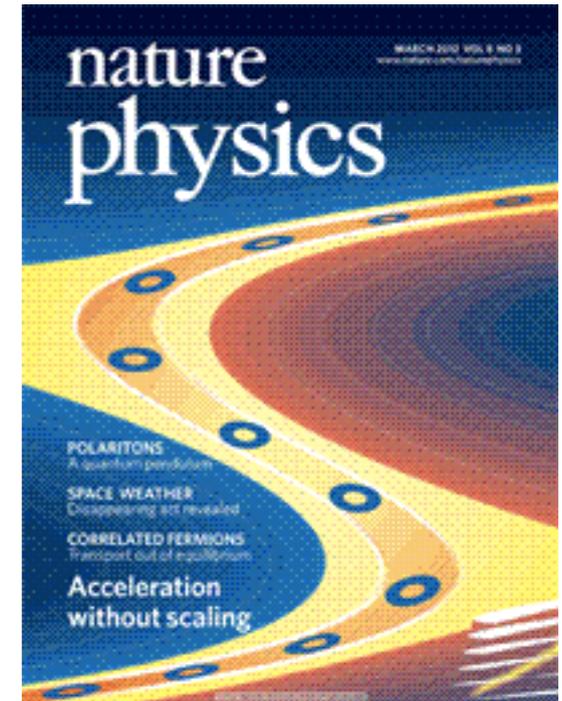
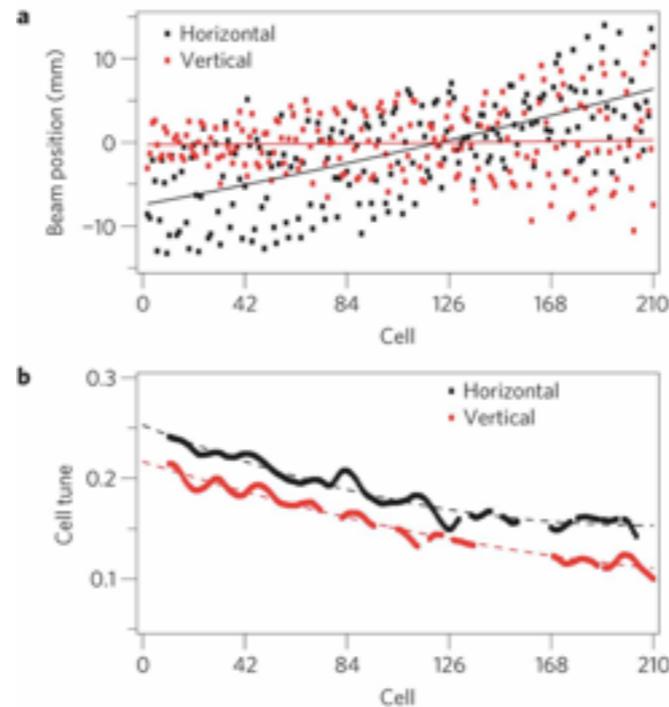
$$n\nu_x + m\nu_y = 0, 1, 2, \dots$$

There are many resonance lines in tune space

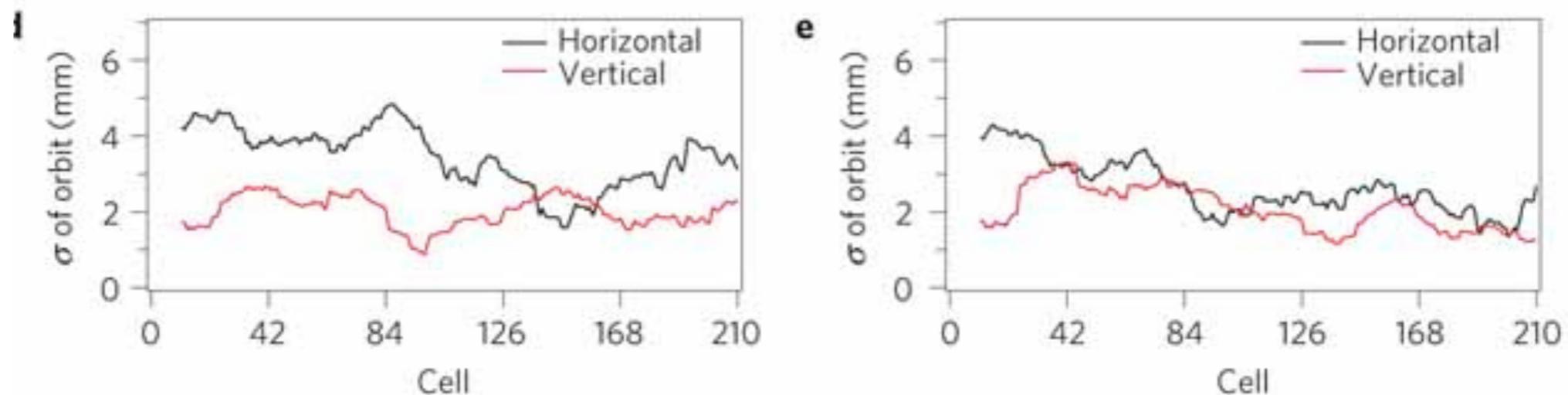
Normally, particles would be lost on resonance, but if the resonance is weak and the crossing is fast the beam can survive.

# Results from EMMA

Orbit and  
tune shift  
with  
momentum



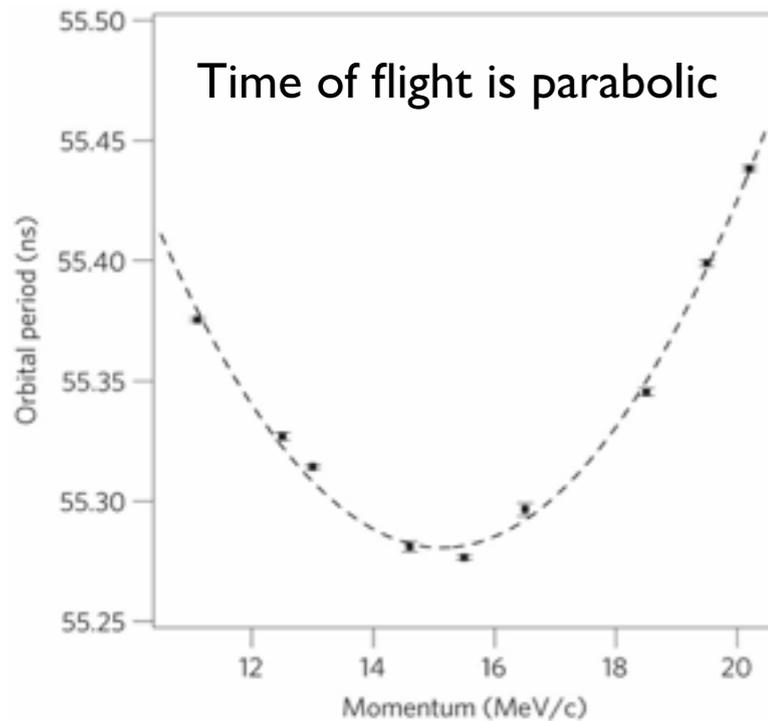
No beam 'blowup' despite resonance crossing



S. Machida et. al., Nature Physics 8, 243–247 (2012)

# EMMA - longitudinal

**Can you have an FFAG with fixed RF frequency?**

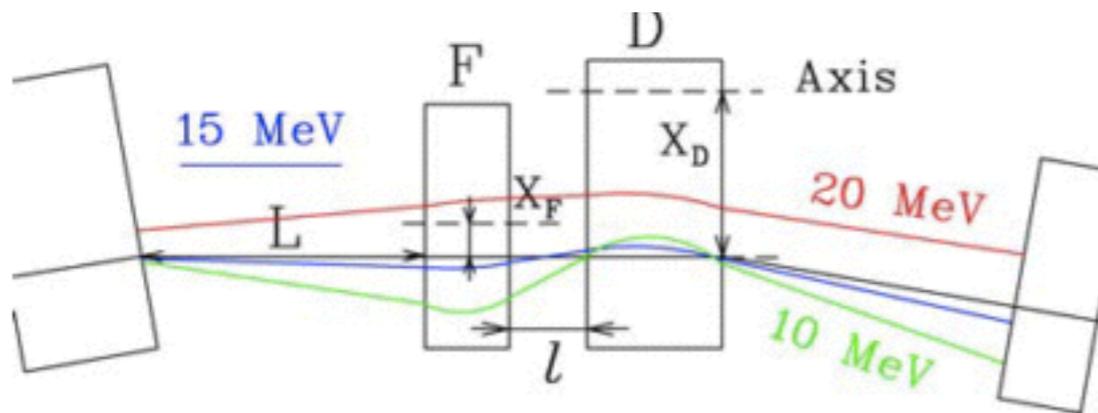
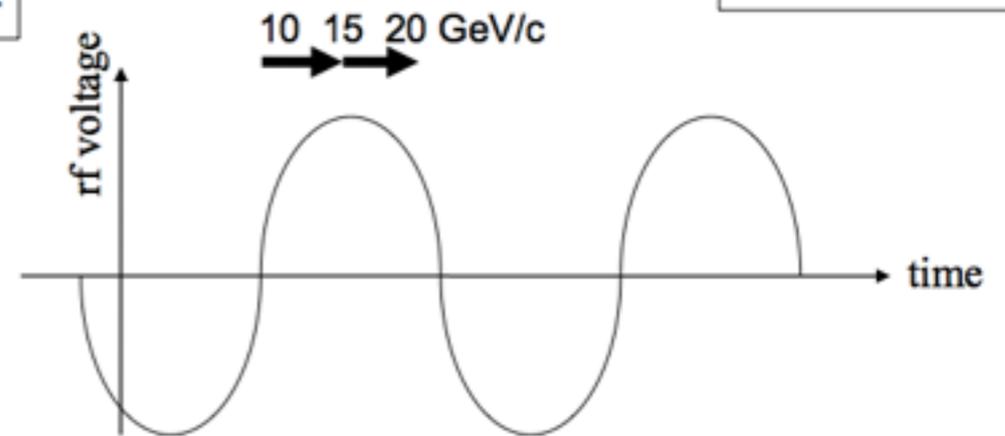


- Suppose we choose rf frequency that is synchronized with revolution frequency at the center.

In the first half of a cycle, a particle lags behind the rf.

At the center momentum, a particle is synchronized with rf.

In the second half, a particle lags again.



If the total time lag is less than a half of rf cycle, a beam has net energy gain.

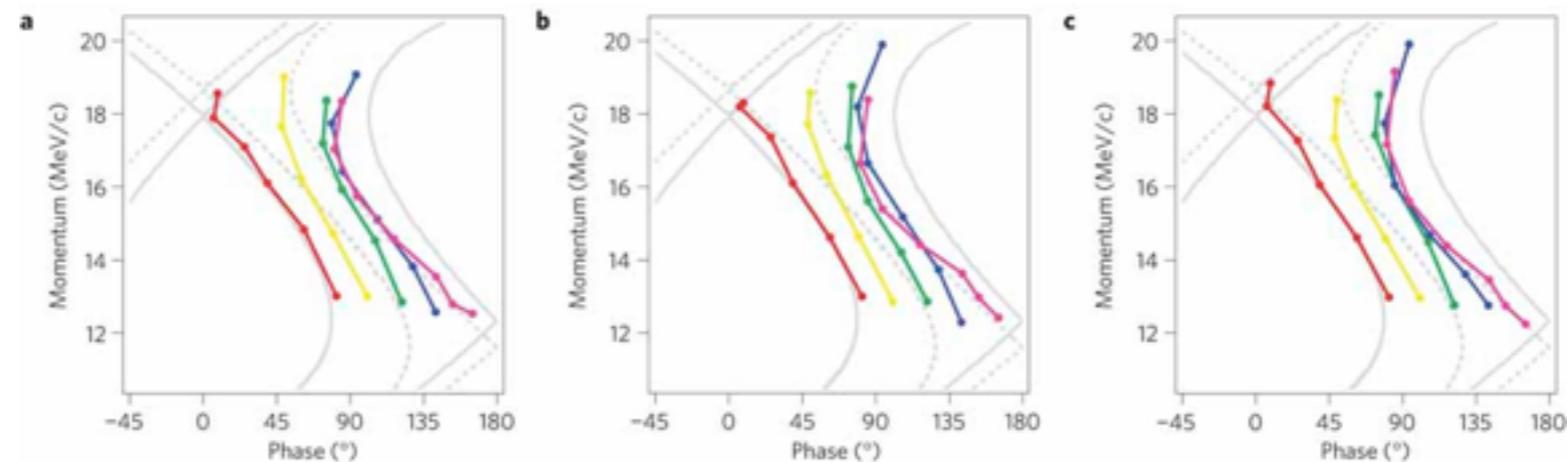
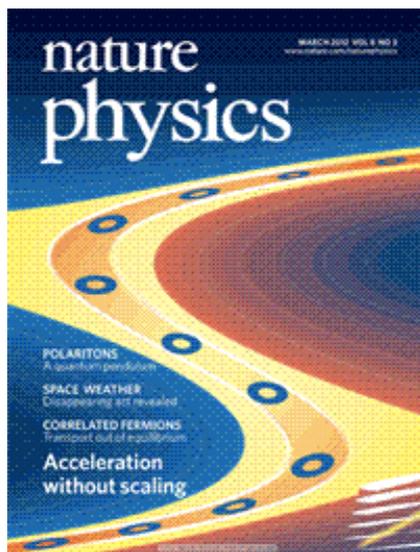
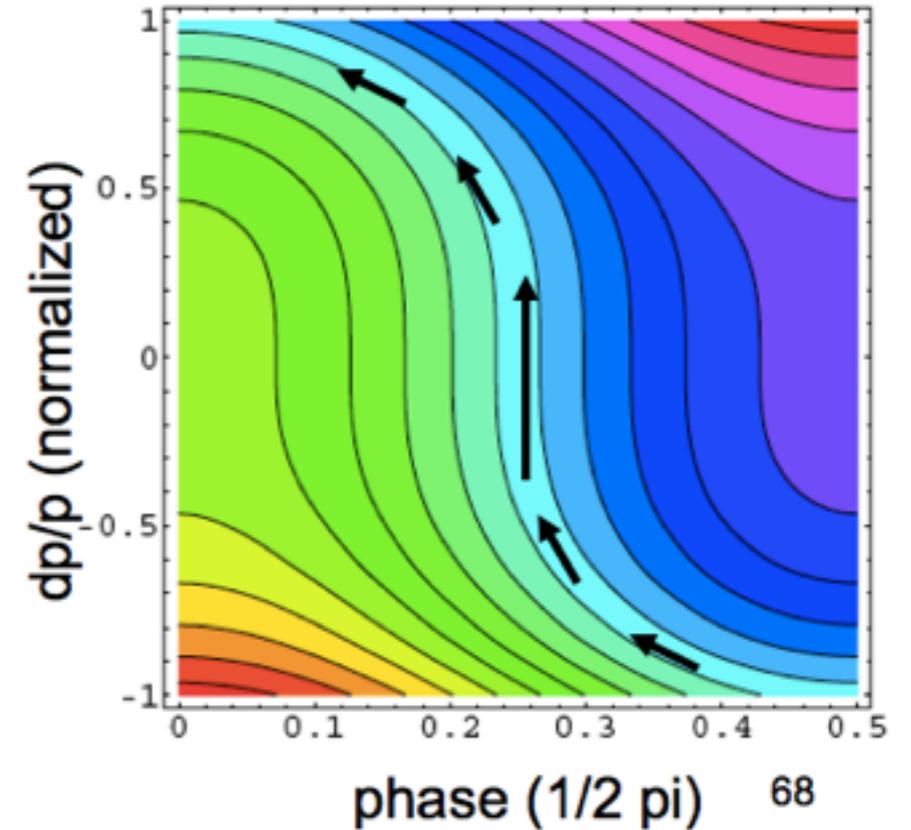
Figure 2: Orbits in a quadrupole doublet cell.

# EMMA - longitudinal

If the RF voltage is sufficient, we can accelerate over the whole energy range

Similar to acceleration in a cyclotron but with imperfect isochronicity

This is called 'serpentine' acceleration and was demonstrated in EMMA



S. Machida et. al., Nature Physics 8, 243–247 (2012)

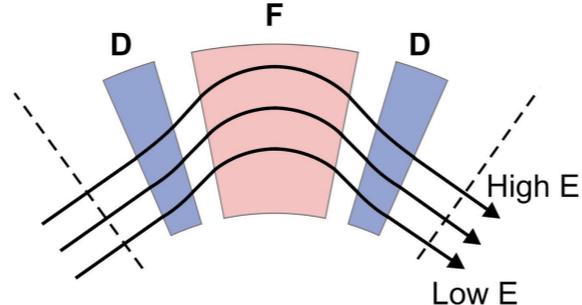
# But that's not the whole story...

- Electrons & muons are easy to accelerate quickly, but for hadrons it's harder...
- If resonance crossing could be harmful for hadron FFAGs, what can we do to fix it?
- In a synchrotron, off-momentum tune variations = chromaticity
- Can we have stable tunes in an FFAG?

# Scaling FFAG

- In fact, the first FFAGs had constant tunes and were designed not to cross resonances, we call them 'scaling' FFAGs

The orbits are made 'similar'

$$\left. \frac{\partial}{\partial p} \left( \frac{\rho}{\rho_0} \right) \right|_{\theta=const.} = 0$$


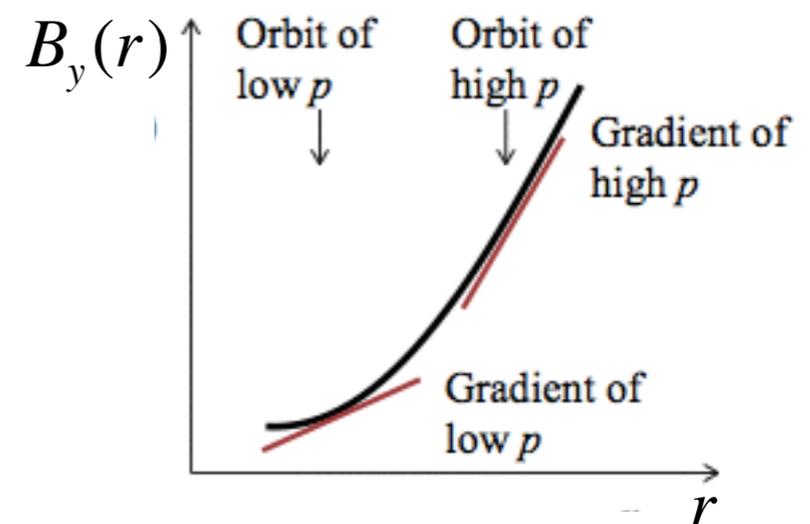
$\rho_0$  Average bending radius

$\rho$  Local bending radius

$\theta$  Generalised azimuth

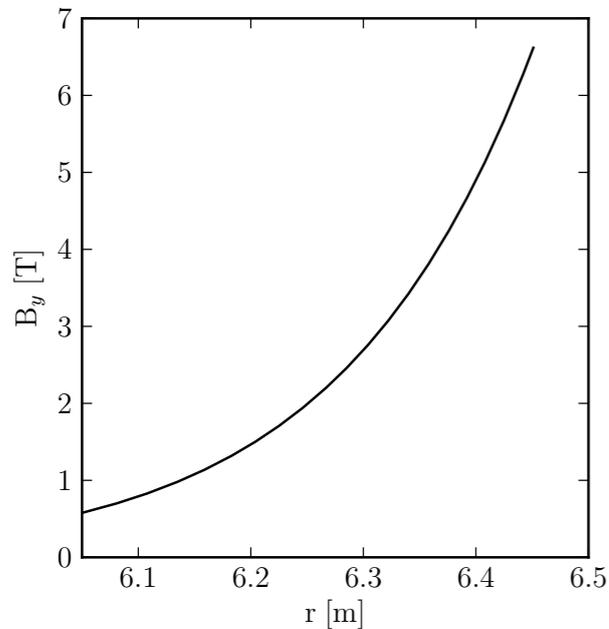
The 'field index' is constant

$$\left. \frac{\partial k}{\partial p} \right|_{\theta=const.} = 0 \quad k = \frac{r}{B} \left( \frac{\partial B}{\partial r} \right)$$

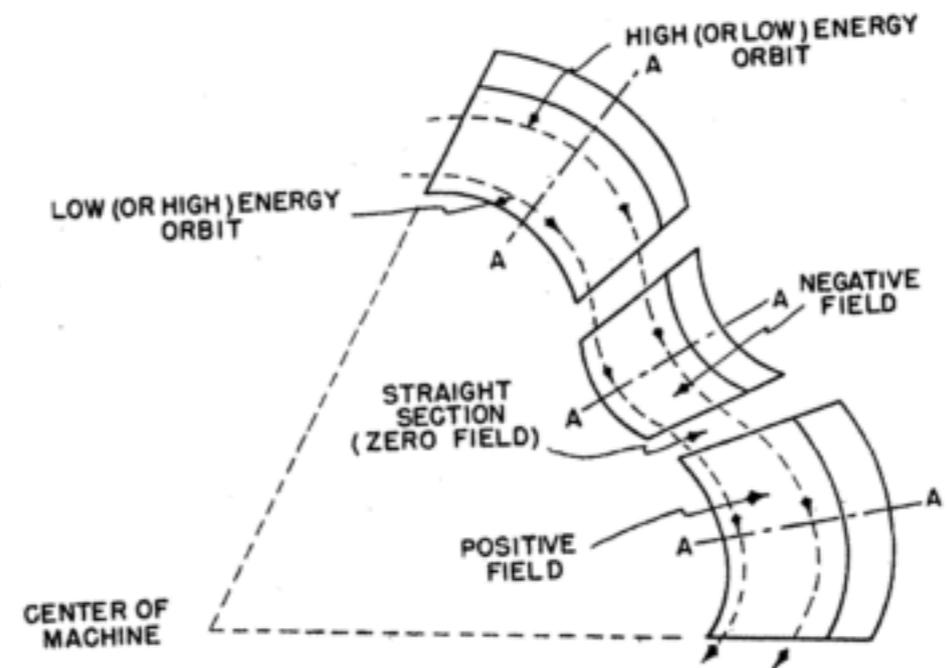


# Scaling FFAG

- If the field profile is of this form, the ‘cardinal conditions’ are satisfied.
- We call this type of FFAG a ‘Scaling’ type.
- Alternating magnets have opposite bending fields



$$B_y = B_0 \left( \frac{r}{r_0} \right)^k F(\theta)$$

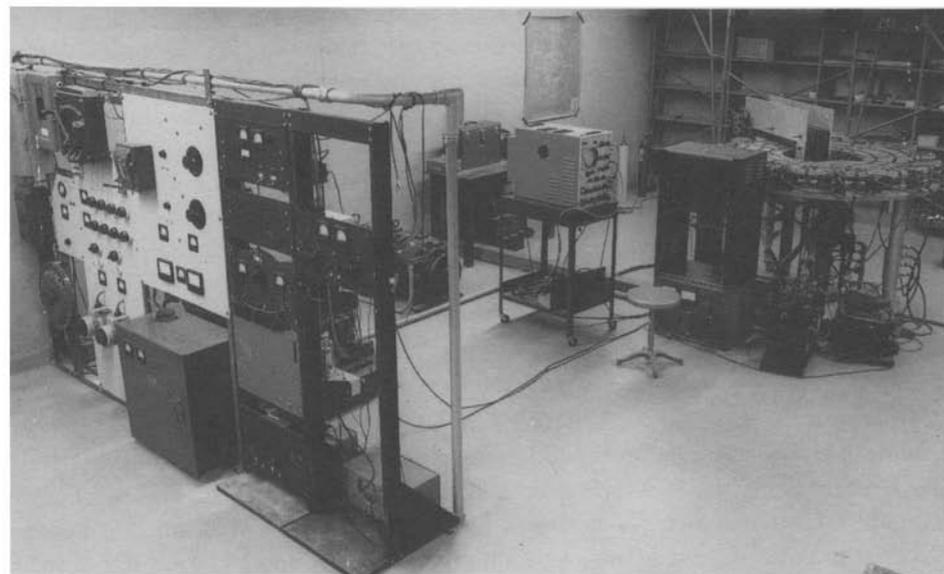
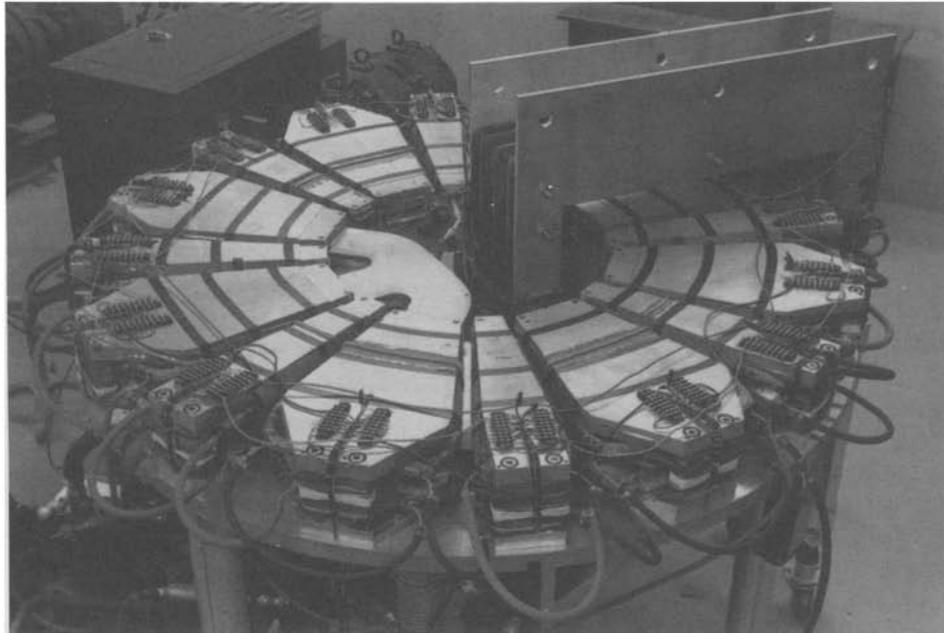


*Note that this field profile does NOT satisfy isochronicity (see M. Seidel's cyclotron lecture)*

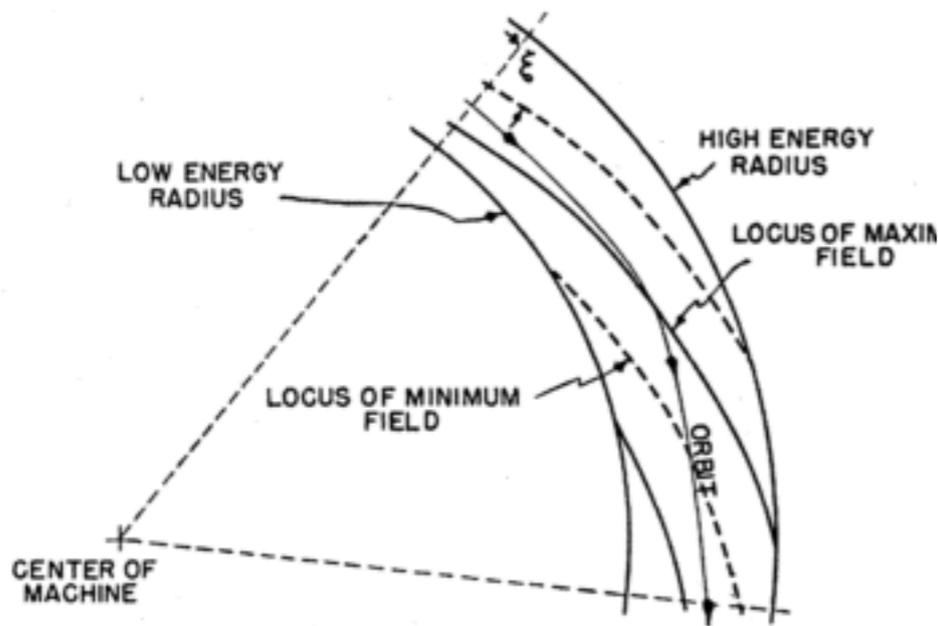
$$\omega = \frac{eB}{m\gamma} \neq \text{const.}$$

# The FFAG is not so new...

1956



# Scaling FFAG types



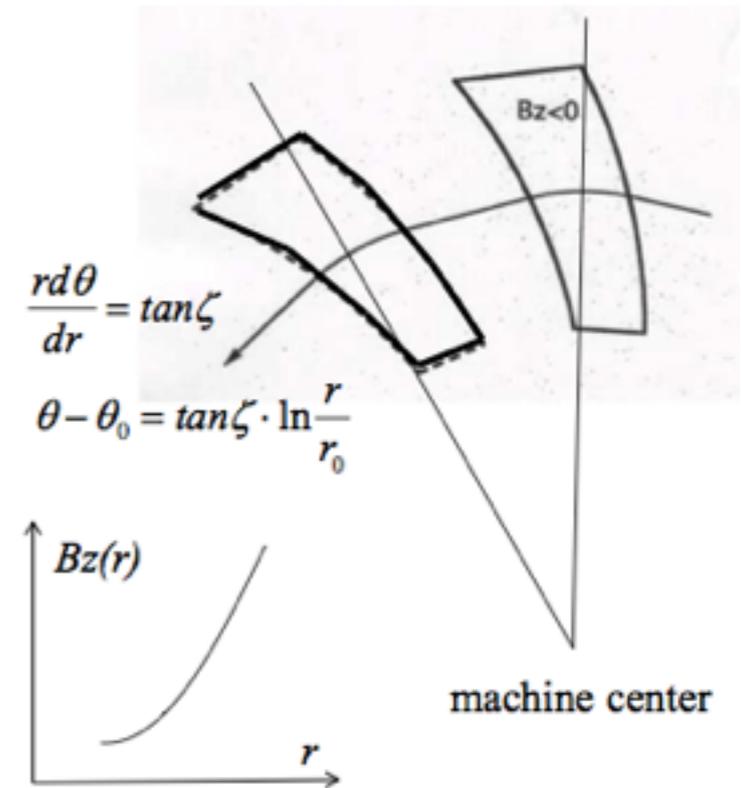
$$B(r, \theta) = B_0 \left( \frac{r}{r_0} \right)^k F(\mathcal{G})$$

$$F(\mathcal{G}) = F\left(\theta - \tan\zeta \cdot \ln \frac{r}{r_0}\right)$$

Spiral sector type

Spiral angle gives strong edge focusing.

$$\therefore \Delta p_z = \frac{e}{v_x} \int_{-\infty}^{\infty} (-v_y B_x) dx = -e B_{z0} \tan\zeta \cdot z$$



S. Machida, CAS 2012

Image source: K. Symon, D. Kerst, L. Jones, L. Laslett, and K. Terwilliger, "Fixed-Field Alternating-Gradient Particle Accelerators," Phys. Rev., vol. 103, no. 6, pp. 1837–1859, Sep. 1956.

# Recent Scaling FFAGs

- In the late 90's and in 2000's, the FFAG idea was re-awakened in Japan,
- Particular focus on hadron FFAGs of scaling type



Proof of Principle machine finished in 1999 at KEK, demonstrated 1kHz rep. rate

3-stage FFAG for ADSR studies:  
2.5 MeV spiral (ion beta) FFAG with induction cores  
25 MeV radial (booster) FFAG with RF  
150 MeV radial (main) FFAG with RF



# Technology for scaling FFAGs



Image credit:A. Takagi



Image credit:Y. Mori,

Magnetic Alloy (MA) Cavity

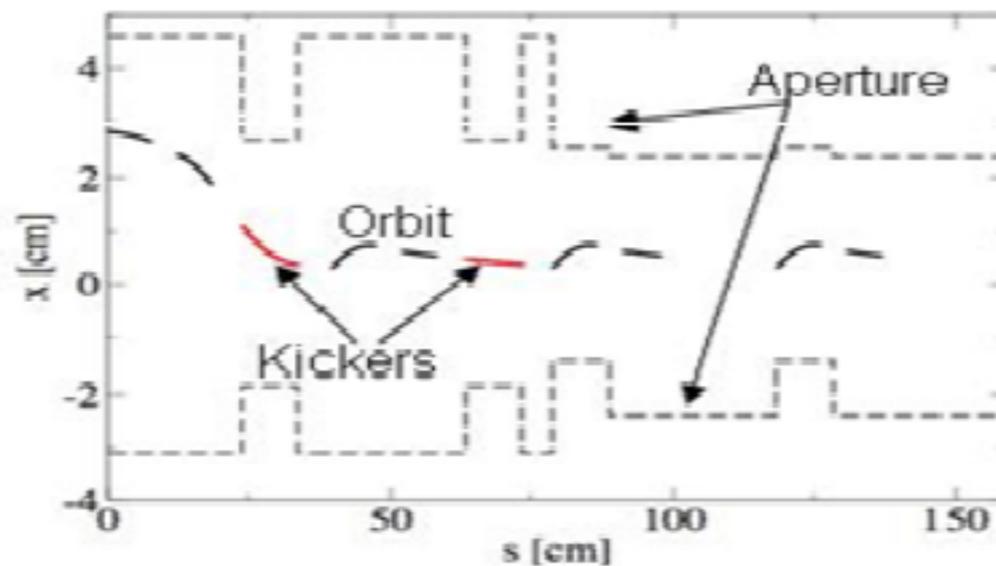
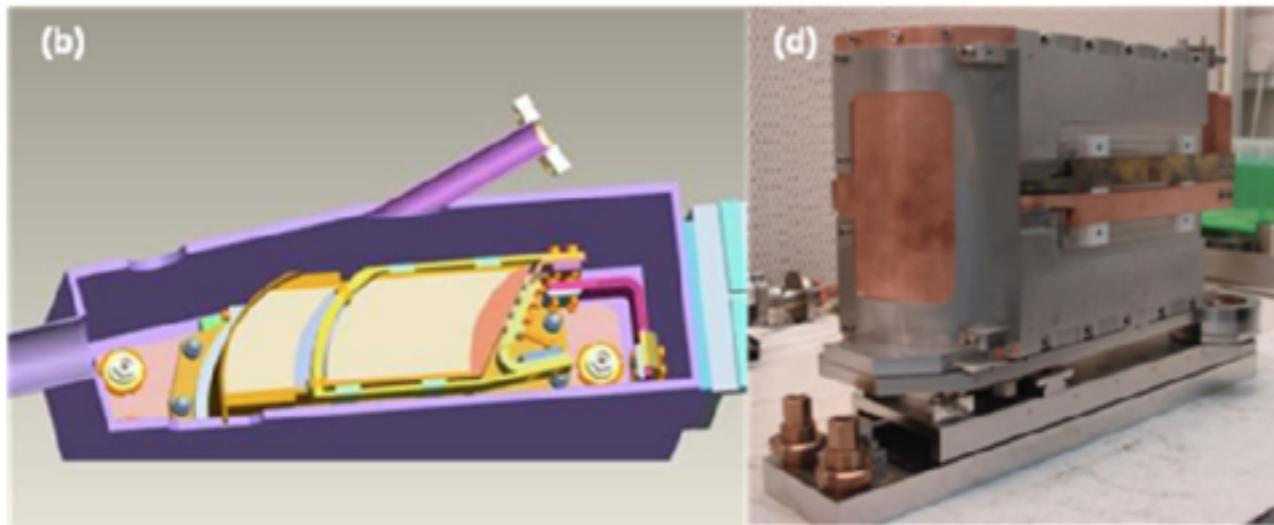
Large aperture

High shunt impedance

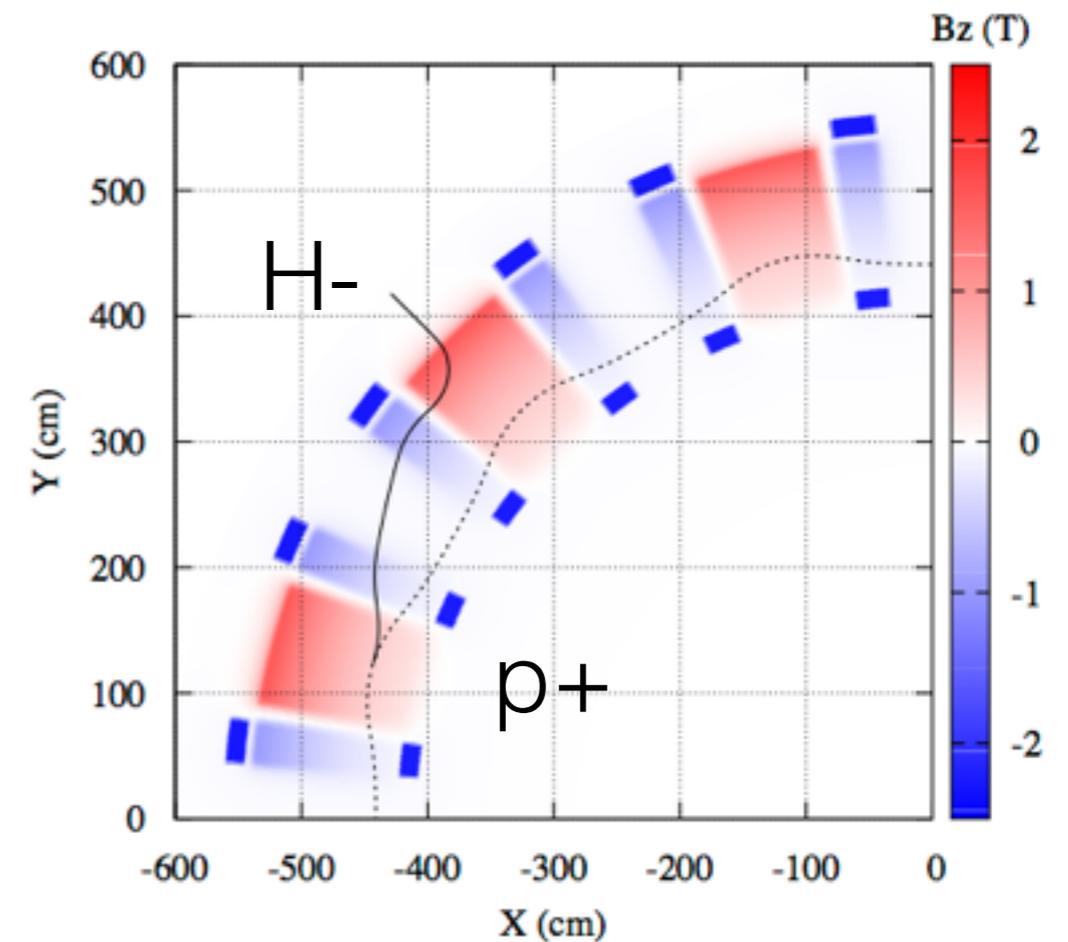
Low Q - can cover large range of frequencies.

# Aside: Injection/extraction

- How do we inject/extract beams without a time dependent field?
- Well, pulsed kickers/septum can still be used.
- Can also exploit the orbit movement with acceleration



Injection/extraction in EMMA



Septum-free injection in KURRI FFAG

# Circular Accelerators

	Cyclotron	Synchrotron	Non-scaling FFAG	Scaling FFAG
Revolution time	Constant	Variable (except relativistic)	Variable (small)	Variable
Orbit radius	Variable	Constant	Variable (small)	Variable
Transverse focusing	Variable	Constant	Variable	Constant

# A quick summary...

- 'Scaling' type is a very specific type of FFAG. Anything else is the 'non-scaling' type.
- EMMA is a linear non-scaling FFAG, which again is quite specific.
- ...Are there any other possibilities?

# Advanced FFAG optics (1)

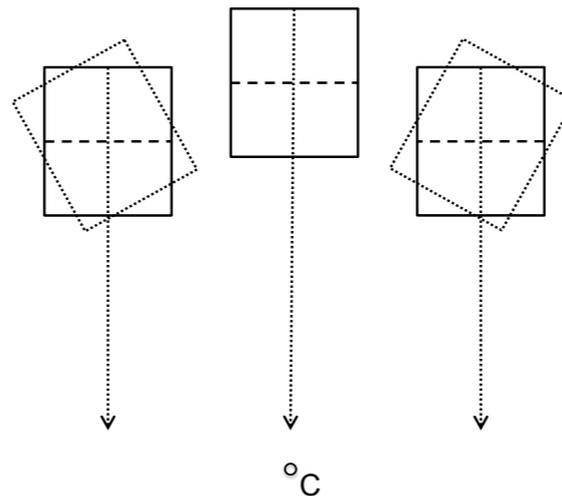
*“There are other variations of these designs which preserve betatron oscillation stability, hold  $v_x$  and  $v_y$  constant, but do not retain the property of similar or equilibrium orbits.”*

*“The magnet edges of focusing and defocusing sectors can be made non-radial, and the fields in positive- and negative- field magnets made different functions of radius”*

- K. Symon, D. Kerst, L. Jones, L. Laslett, and K. Terwilliger, “Fixed-Field Alternating-Gradient Particle Accelerators,” Phys. Rev., vol. 103, no. 6, pp. 1837–1859, Sep. 1956.

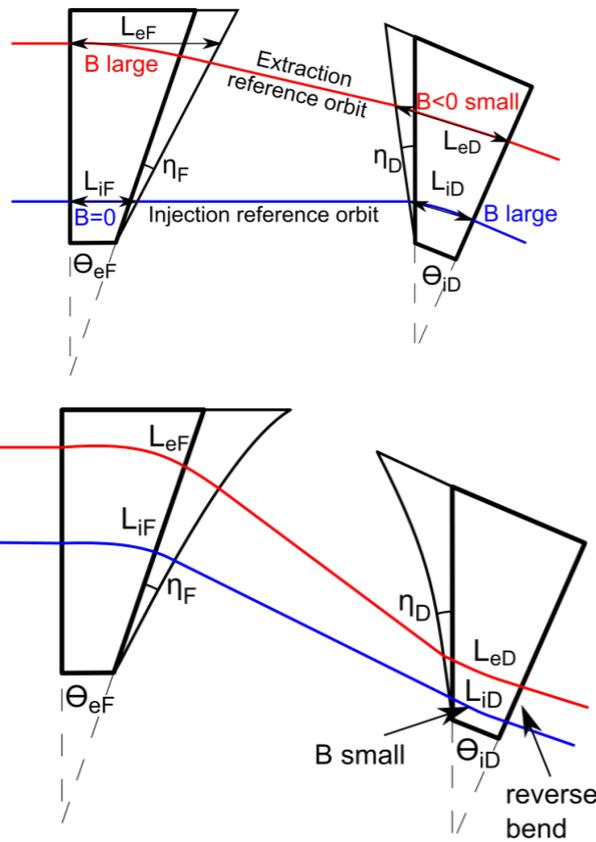
**Tune-stable non-scaling FFAG designs have been developed**

$$B_z = B_{z0} \left( \frac{r_0 + r}{r_0} \right)^k = B_{z0} \left( 1 + \sum_{n=1} \frac{1}{n!} \frac{k(k-1)\cdots(k-n+1)}{r_0^n} r^n \right)$$



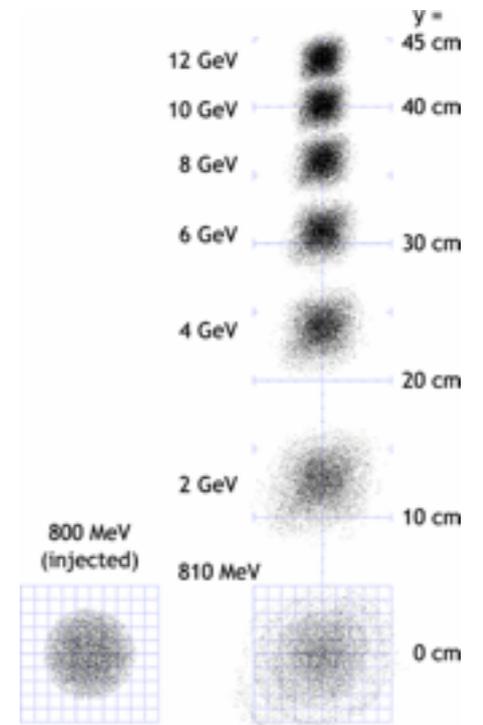
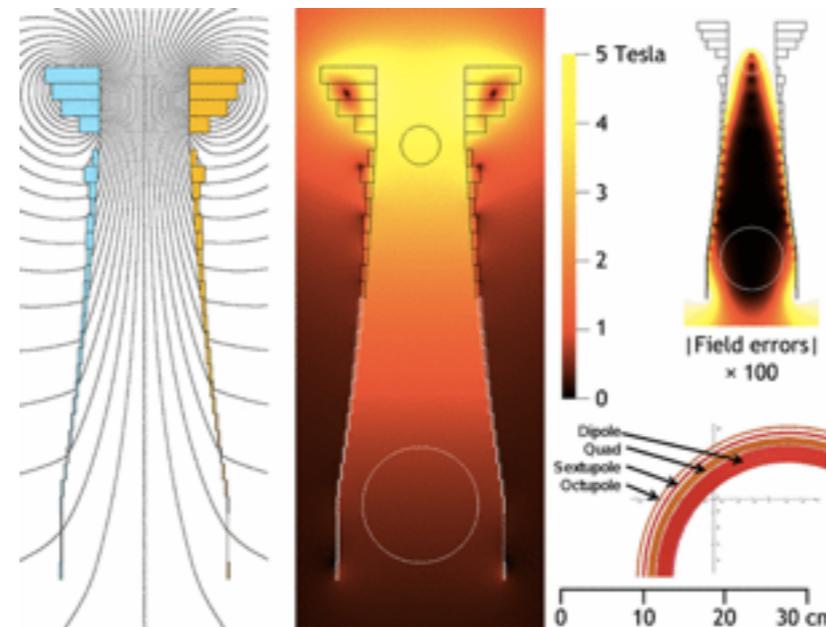
**Rectangular magnets,  
Simplified field profile  
Higher stability region  
(S. Machida, S. Sheehy)**

# Advanced FFAG optics (2)



Radial designs with edge profiles  
(C. Johnstone)

Vertical orbit excursion FFAG  
(S. Brooks)



# Current status of FFAG designs

- A whole spectrum of designs have emerged in the last 5-10 years

Potential applications include:

- Accelerator Driven Subcritical Reactor
- Boron Neutron Capture Therapy
- Proton/carbon therapy
- Accelerator-based Neutron Source
- Emittance/Energy Recovery with Internal Target (ERIT)
  - e-RHIC injector
  - Muon or neutrino factory source
  - + many more...



# Summary

- FFAGs are just a generalisation of synchrotrons or cyclotrons
- Two main types 'scaling' and 'non-scaling'
  - Scaling: specific optics and orbit requirements put a strict requirement on the field profile (zero-chromaticity)
  - Non-scaling: removes these restrictions, very general type (chromatic)
- FFAGs may be suitable for many future applications
- In my view, the next big challenge is demonstrating high power operation

# Reading List

- CERN Courier, “Rebirth of the FFAG”, 2004. <http://cerncourier.com/cws/article/cern/29119>
- K. Symon, D. Kerst, L. Jones, L. Laslett, and K. Terwilliger, “Fixed-Field Alternating-Gradient Particle Accelerators,” Phys. Rev., vol. 103, no. 6, pp. 1837–1859, Sep. 1956.
- S. Machida, “Acceleration in the linear non-scaling fixed-field alternating-gradient accelerator EMMA,” Nat. Phys., vol. 8, no. 3, pp. 243–247, Jan. 2012.
- Proceedings of the FFAG workshops

Notes on FFAGs from CAS schools:

- S. Machida, FFAGs, CAS Bulgaria 2010, <https://cas.web.cern.ch/cas/Bulgaria-2010/Talks-web/Machida-web.pdf>
- S. L. Sheehy, Fixed Field Alternating Gradient Accelerators, <https://arxiv.org/abs/1604.05221> In proceedings of CAS Specialised School on Medical Accelerators, 2015.