CERN Accelerator School
Lecture on : Insertion Devices

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Content of the Lecture

- Thursday 3rd July
  - Part I : Radiation from Insertion Devices
- Saturday 5th July Morning
  - Part II: Effect on the electron beam
  - Part III : Technology of Insertion Devices
- Saturday 5th July Afternoon
  - Part IV : Variable Polarisation Insertion Devices
  - Part V : Undulator for FELs

Part I
Radiation from Insertion Devices

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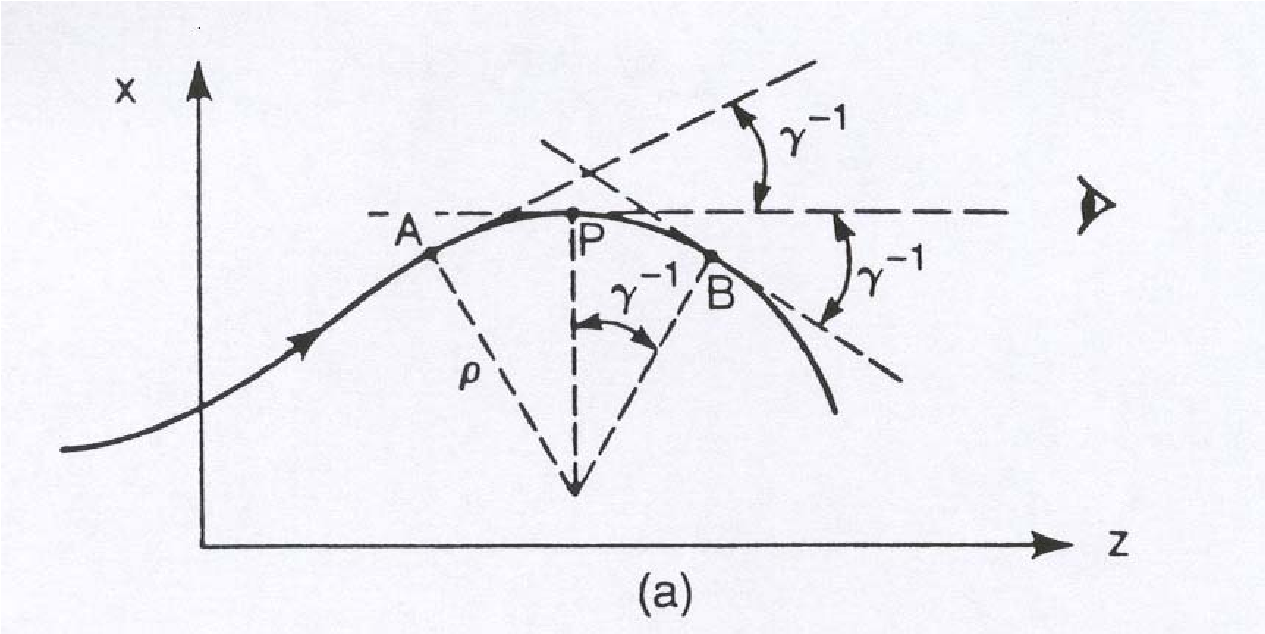
Short Bibliography

- J.D. Jackson, Classical Electrodynamics, Chapter 14, John Wiley
- Kim K.J., Characteristics of Synchrotron Radiation, AIP Conference Proceedings 184, vol. 1 p567 (American Institute of Physics, New York, 1989).
- Walker R.P., CAS - CERN Accelerator School: Synchrotron Radiation and Free Electron Lasers, Grenoble, France, 22 - 27 Apr 1996，CERN 98-04 p 129.
- "Undulators, Wigglers and their Applications", Editors : H. Onuki, P. Elleaume, Publisher : Taylor and Francis, 2003, ISBN 0-415-28040-0.
- And References therein ….

Table of Contents

Bending Magnet Radiation



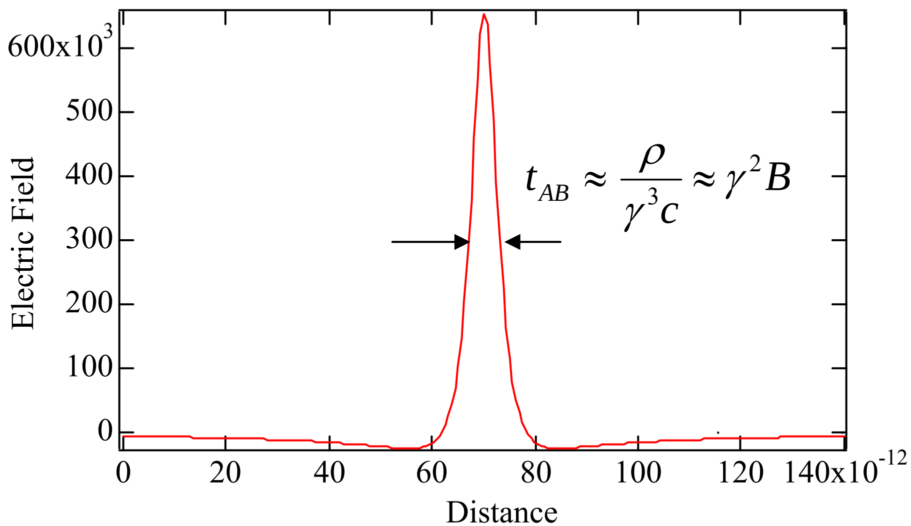
ρ: Radius of Curvature

Lorentz Force in Field B: $\gamma m \frac{d\vec{v}}{dt} = e\vec{v}\times\vec{B} \Rightarrow \frac{1}{\rho} = \frac{eB}{\gamma mc}$

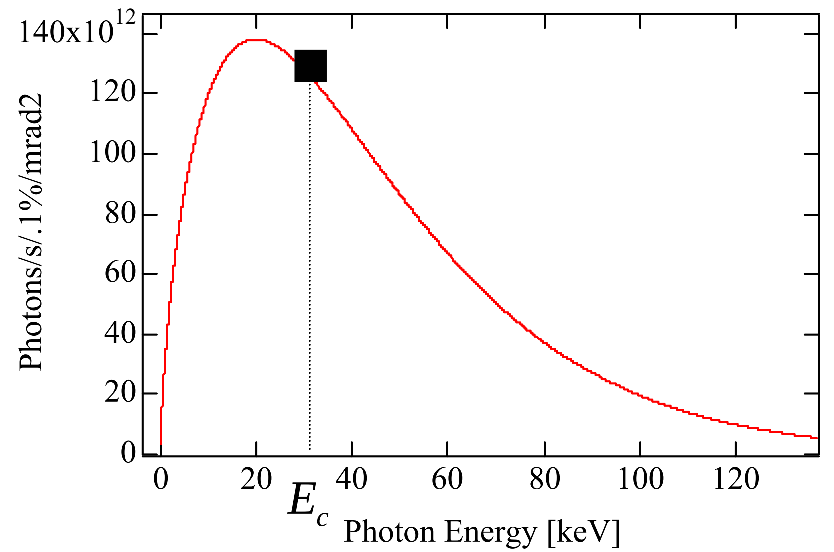
$$\tau_{AB} = \frac{2\rho}{\gamma c}, \quad \frac{t_{AB}}{\tau_{AB}} \approx \frac{1}{\gamma^2} \Rightarrow t_{AB} \approx \frac{\rho}{\gamma^3 c} \Rightarrow h\omega \approx \frac{\gamma^3}{\rho} \approx \gamma^2 B$$

Critical Energy of Bending Magnet Radiation

Electric Field in the Time Domain



Angular Flux in Frequency Domain

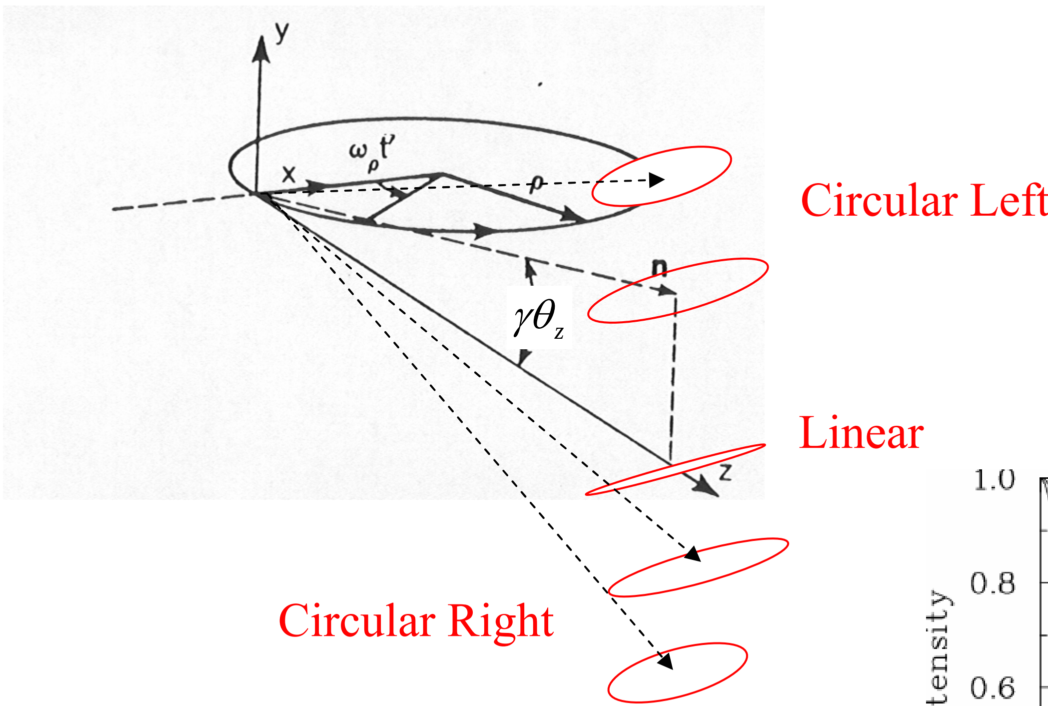


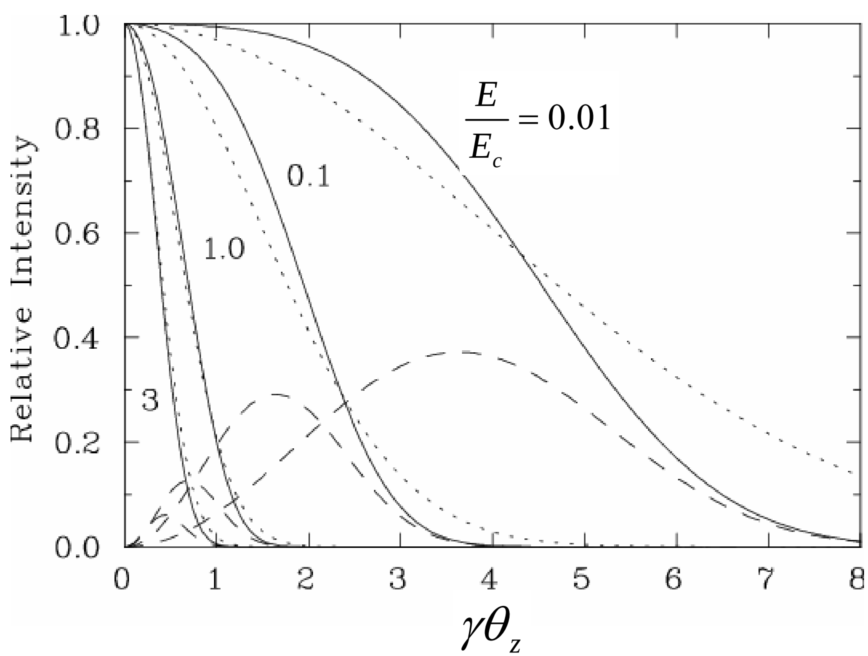
Computed for 6 GeV, I = 200 mA, B =1 tesla

$$E_c = \frac{3hc}{4\pi}\frac{\gamma^3}{\rho} = \frac{3he}{4\pi m}\gamma^2 B$$

$$E_c[keV] = 0.665\ E^2[GeV]\ B[T]$$

Polarization of Bending Magnet Radiation





I, 8/38 , P. Elleaume, CAS, Brunnen July 2-9, 2003.

Electron Trajectory in a General Insertion Device

Consider Ortogonal Frame Oxzs

$Electron\ velocity\ \vec{v} = (v_x, v_z, v_s)$

$Electron\ position\ \vec{R} = (x, z, s)$

$Magnetic\ field\ \vec{B} = (B_x, B_z, B_s)$

$$Lorentz\ Force: \gamma m \frac{d\vec{v}}{dt} = e\vec{v} \times \vec{B}$$

$$=> \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = Cst$$

$$=> \gamma m \frac{dv_x}{dt} = -e(v_s B_z - v_z B_s)$$

$$Assume: v_x, v_z << v_s \approx c$$

$$\frac{v_x(s)}{c} = -\frac{e}{\gamma mc} \int_{-\infty}^{s} B_z(s')ds'$$

$$x(s) = -\frac{e}{\gamma mc} \int_{-\infty}^{s} \int_{-\infty}^{s'} B_z(s'')ds''ds'$$

and similar expression for $v_z(s)$ and $z(s)$

Electron Trajectory in a Planar Sinusoidal Undulator

$$Consider\ \vec{B} = (\,0,\ B_0 \sin(2\pi \frac{s}{\lambda_0}),\ 0\,)$$

$$\frac{v_x}{c} = \frac{K}{\gamma}\cos(2\pi\frac{s}{\lambda_0})$$

$$x = -\frac{\lambda_0}{2\pi}\frac{K}{\gamma}\sin(2\pi\frac{s}{\lambda_0})$$

$$v_z = 0,\ \ z = 0$$

with

$$K = \frac{eB_0\lambda_0}{2\pi mc} = 0.0934\ B_0[T]\,\lambda_0[mm]$$

$$K\ \ is\ \ a\ \ fundamental\ \ parameter\ \ also\ \ called\ \ "Deflection\ \ Parameter"$$

$$Example: ESRF,\ \ Energy = 6GeV,\ \ Undulator\ \ \lambda_0 = 35\ \ mm,\ \ B_0 = 0.7\ \ T$$

$$=> \ \ K = \ 2.3, \quad \frac{K}{\gamma} = 200\ \ \mu rad, \quad \frac{\lambda_0}{2\pi}\frac{K}{\gamma} = \ 1.1\ \ \mu m\,!!$$

Longitudinal Velocity in a Planar Sinusoidal Undulator

$$\frac{1}{\gamma^2} = 1 - \frac{v_x^2}{c^2} - \frac{v_s^2}{c^2} - \frac{v_z^2}{c^2}$$

$$\frac{v_x}{c} = \frac{K}{\gamma}\cos(2\pi\frac{s}{\lambda_0})$$

$$v_z = 0$$

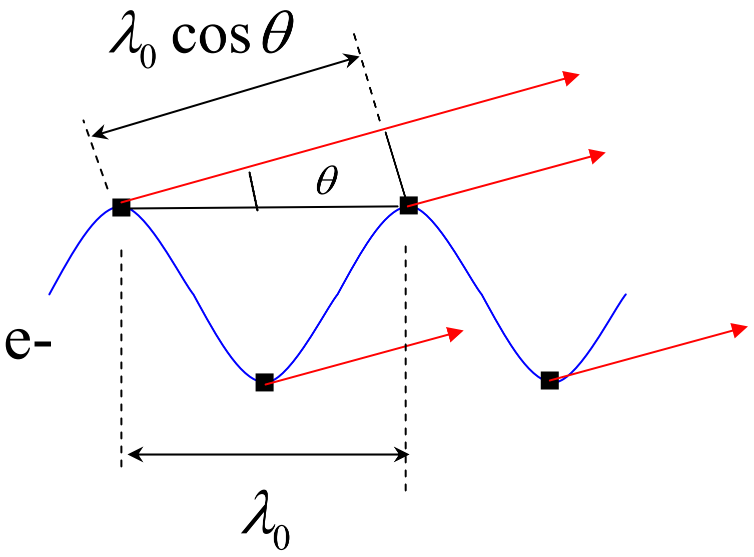
$$\Rightarrow \frac{1}{\gamma^2} + \frac{K^2}{\gamma^2}\cos^2(2\pi\frac{s}{\lambda_0}) = 1 - \frac{v_s^2}{c^2} \cong 2(1 - \frac{v_s}{c})$$

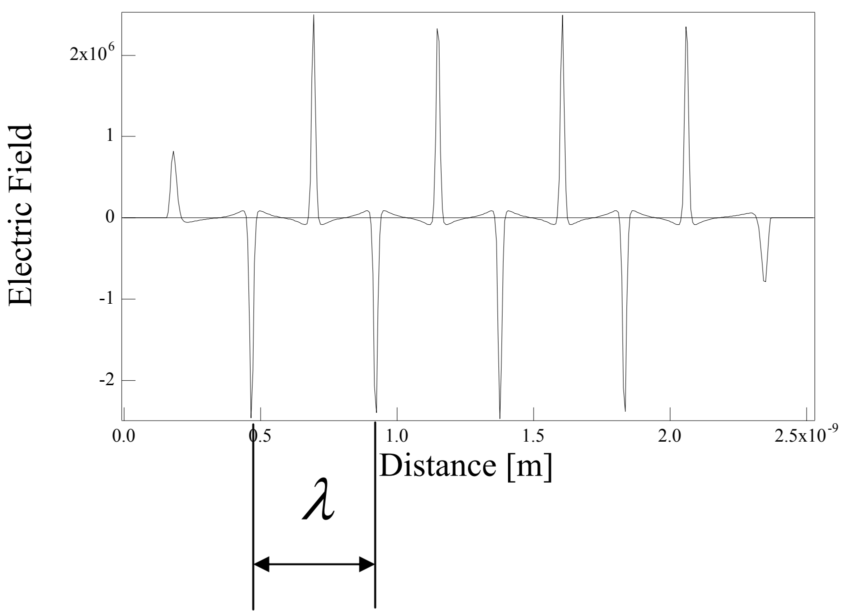
Averaging over one undulator period

$$\frac{v_s}{c} \cong 1 - \frac{1}{2\gamma^2}(1 + \frac{K^2}{2})$$

K can be understood as a measure of how much the longitudinal velocity is slowed down due to the undulator magnetic field

Radiation Field from a Planar Undulator in time Domain





$$\lambda = c\left(\frac{\lambda_0}{v_s} - \frac{\lambda_0}{c}\cos\theta\right) \cong \lambda_0\left(1 - \frac{v_s}{c} + \frac{\theta^2}{2}\right) \cong \frac{\lambda_0}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

Wavelength of the Harmonics

$$\lambda_n = \frac{\lambda_0}{2n\gamma^2}(1+\frac{K^2}{2}+\gamma^2(\theta_x^2+\theta_z^2))$$

In an equivalent manner , the energy E_n of the harmonics are given by

$$E_n[keV] = \frac{9.5\, n\, E^2[GeV]}{\lambda_0[mm](1+\frac{K^2}{2}+\gamma^2(\theta_x^2+\theta_z^2))}$$

λ_n, E_n : *Wavelength, Energy of the* n^{th} *harmonic*

$n = 1, 2, 3, ..$: *Harmonic number*

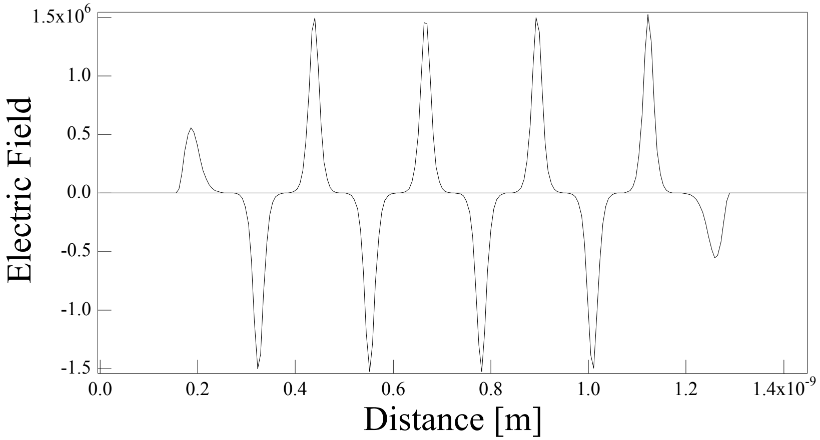
λ_0 : *Undulator period*

$E = \gamma mc^2$: *Electron Energy*

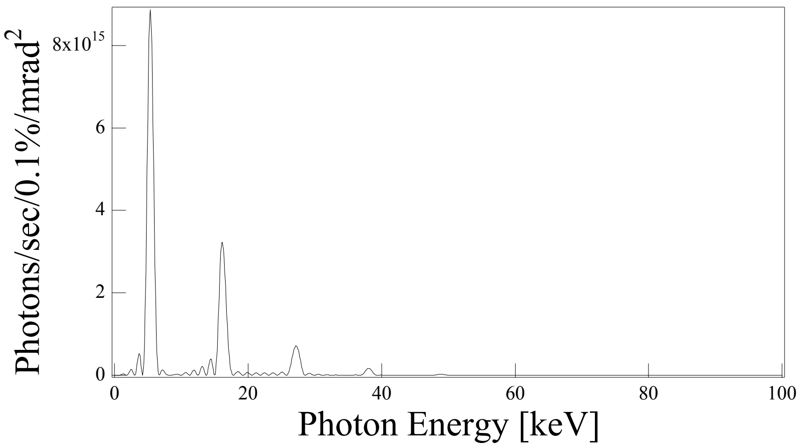
K : *Deflection Parameter* $= 0.0934\, B_0[T]\, \lambda_0[mm]$

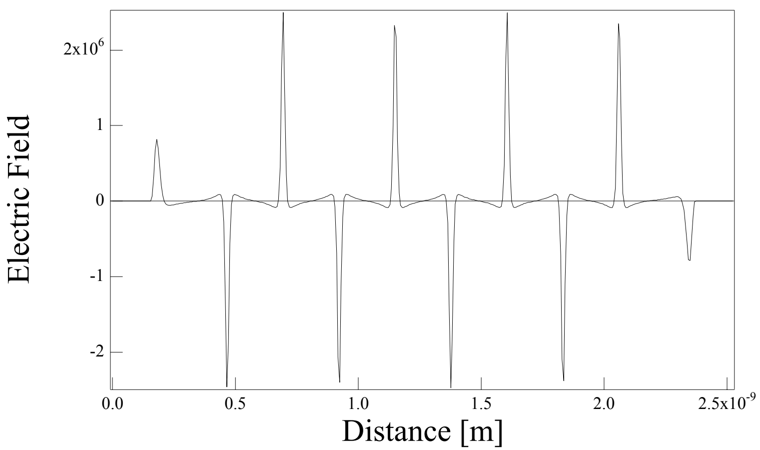
θ_x, θ_z : *Direction of Observation*

Electric Field and Spectrum vs K

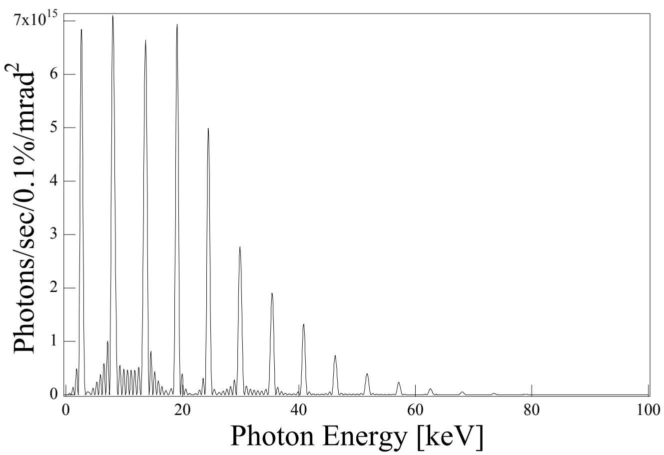


K=1



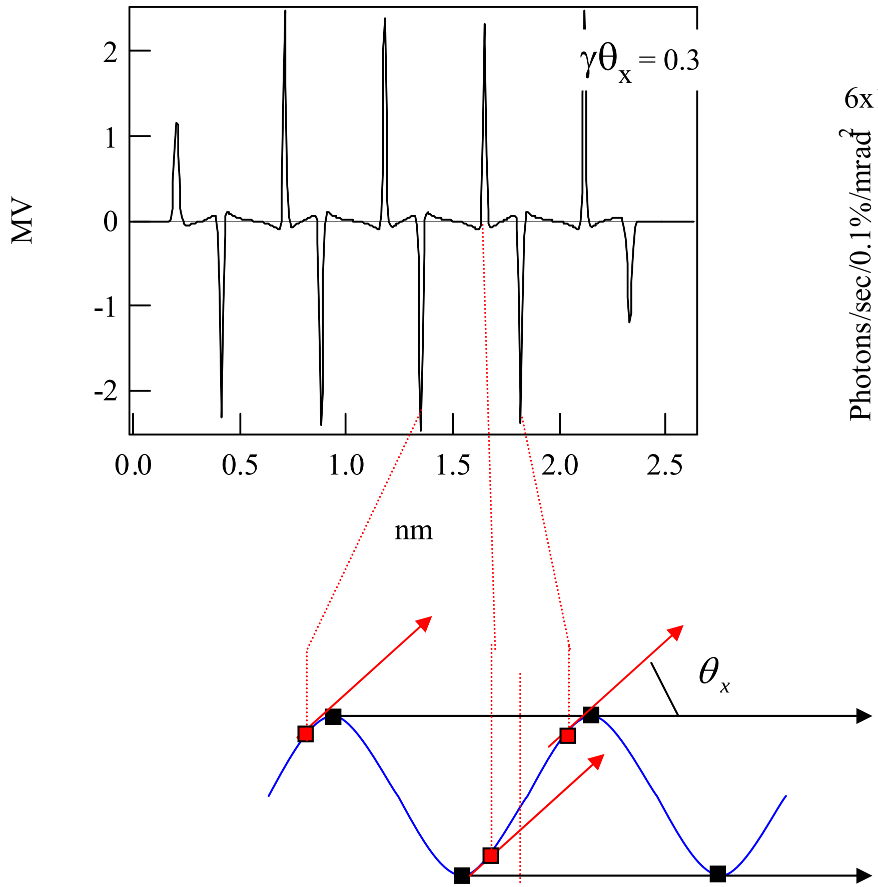


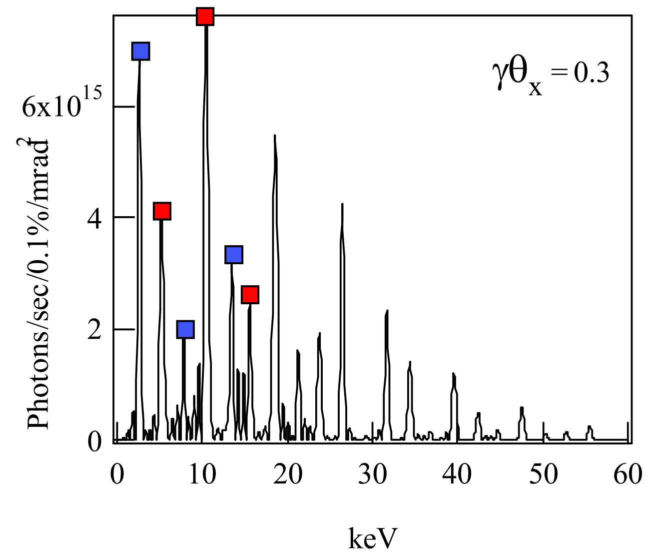
K=2



Computed for 6 GeV, I=200 mA, 35 mm period

Even harmonics are generated off-axis

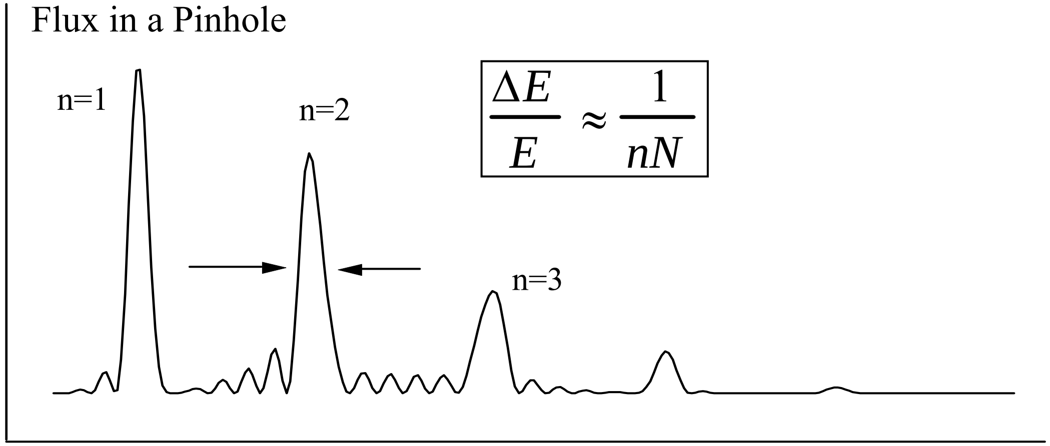




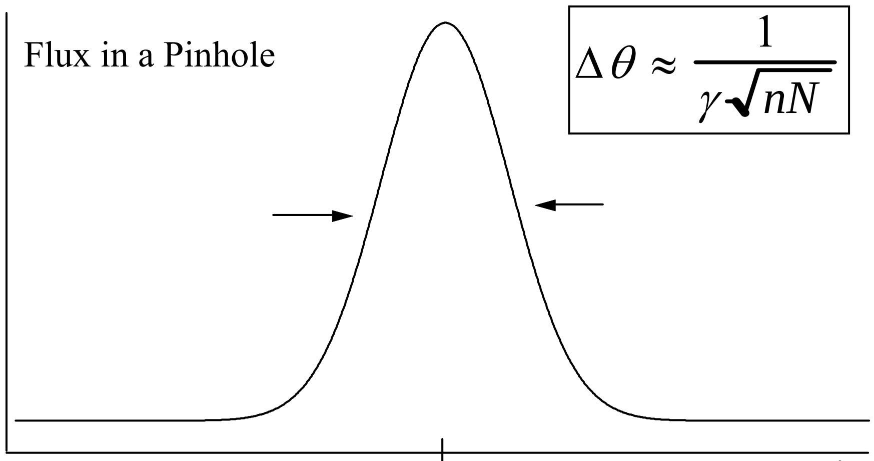
If $\theta_x = \theta_z = 0$, only harmonics n = 1,3,5 are generated

In general all harmonics n = 1,2,3,4,5 are generated

Undulator Emission by a Filament Electron Beam



$$\frac{\Delta E}{E} \approx \frac{1}{nN}$$



$$\Delta\theta \approx \frac{1}{\gamma\sqrt{nN}}$$

n : Harmonic number

N : Number of Periods

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{E}{mc^2}$$

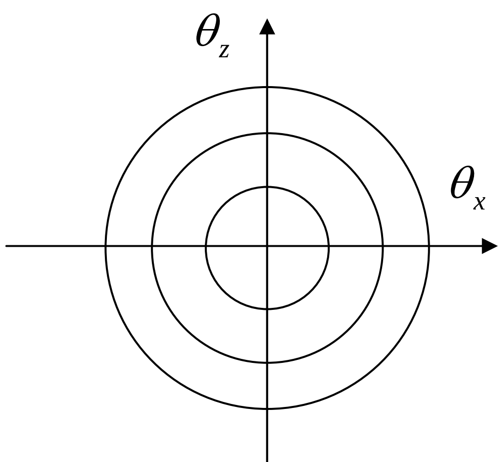
Angular spectral flux

- For a monoenergetic filament electron beam, the angular spectral flux on-axis is given by :

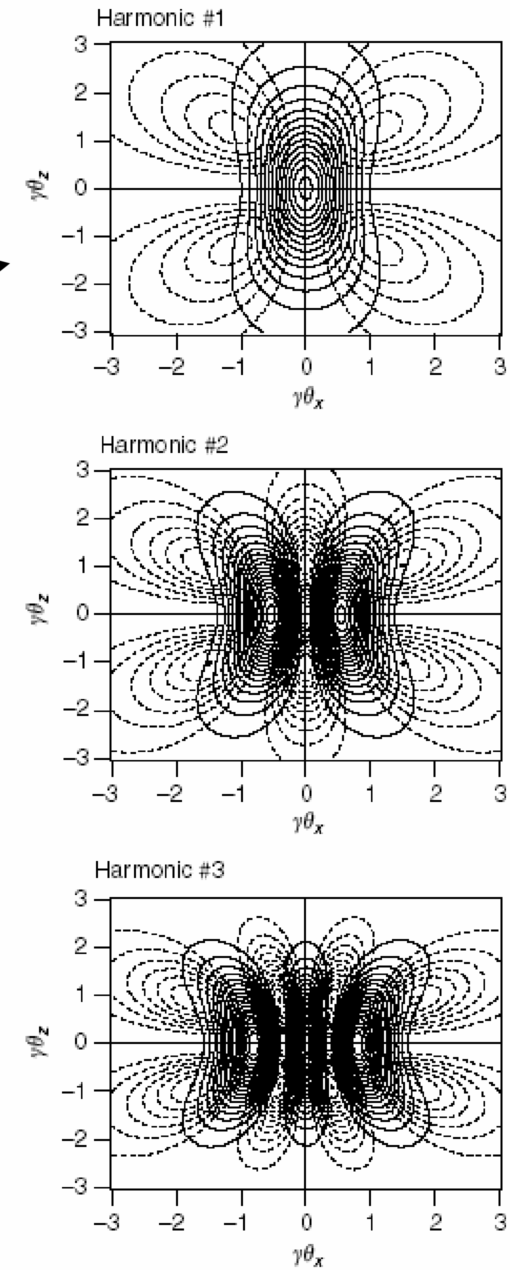
Undulator Field

$$\frac{d\Phi_n}{d\theta_x d\theta_z \frac{d\lambda}{\lambda}}(\theta_x,\theta_z,\lambda) = \alpha \frac{I}{e}\gamma^2 N^2 \overbrace{\left|h_n(\theta_x,\theta_z)\right|^2}\underbrace{\left(\frac{\sin(\pi N n(\frac{\lambda_n}{\lambda}-1))}{\pi N n(\frac{\lambda_n}{\lambda}-1)}\right)^2}$$

Interference



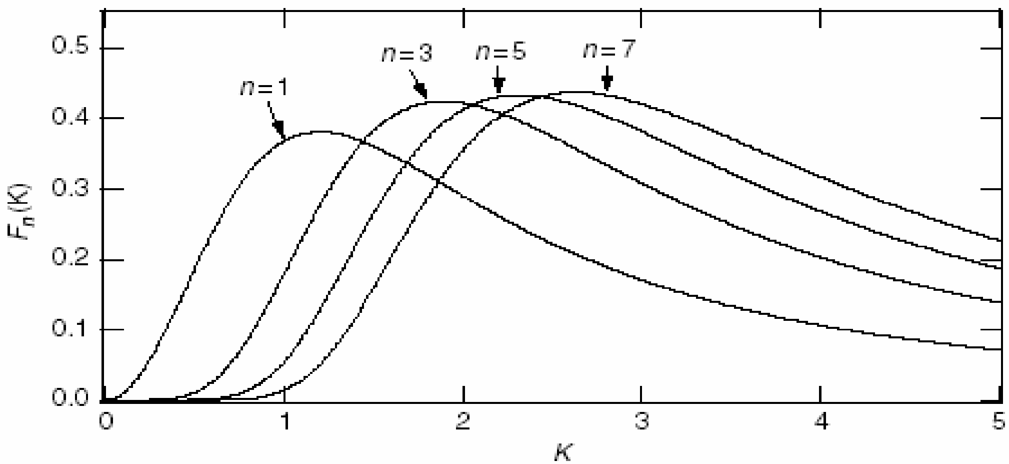
$\left|h_n(\theta_x,\theta_z)\right|^2$ of planar undulator



$$If\ \theta_x = \theta_z = 0 \text{ and } \lambda = \lambda_n \qquad \Rightarrow$$

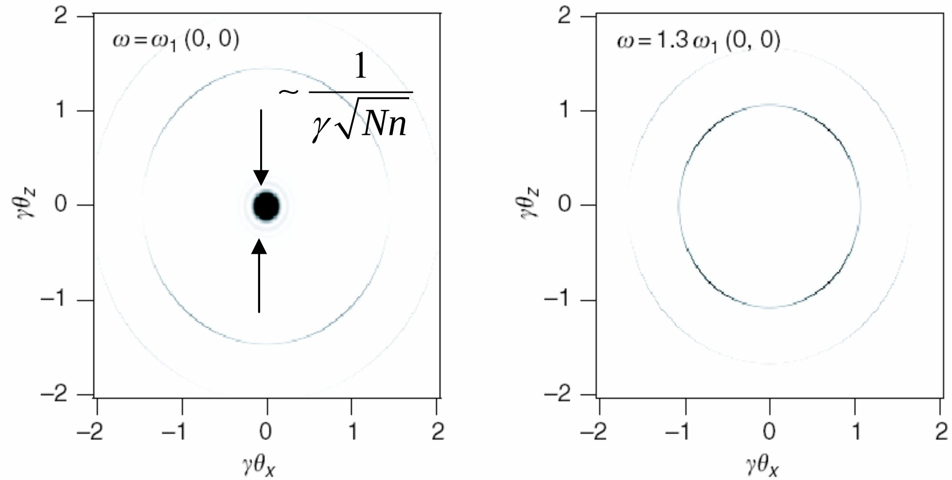
$$\frac{d\Phi_n}{d\theta_x d\theta_z \frac{d\lambda}{\lambda}}(0,0,\lambda) = \alpha \frac{I}{e} N^2 \left| h_n(0,0) \right|^2 = \alpha \frac{I}{e} N^2 \gamma^2 F_n(K)$$

$$\text{with } F_n(K) = \frac{n^2 K^2}{(1+\frac{K^2}{2})^2} \left[J_{\frac{n-1}{2}}\left(\frac{nK^2}{4+2K^2}\right) - J_{\frac{n+1}{2}}\left(\frac{nK^2}{4+2K^2}\right) \right]^2$$



In usefull Units

$$\boxed{\frac{d\Phi_n}{d\theta_x d\theta_z \frac{d\lambda}{\lambda}}(0,0,\lambda)[\text{Phot/s/.1\%/mrad}^2] = 1.744 \times 10^{14} N^2 E^2 [GeV] I[A] F_n(K)}$$



7 Angular spectral flux produced by a filament beam as a function of the horizontal and vertical angles $\gamma\theta_x$ and $\gamma\theta_z$ for two frequencies ω. The undulator is made up of 50 periods with a deflection parameter $K = 1.5$. Each ring is associated to a single harmonic of the spectrum. This ring pattern is typical of the undulator radiation.

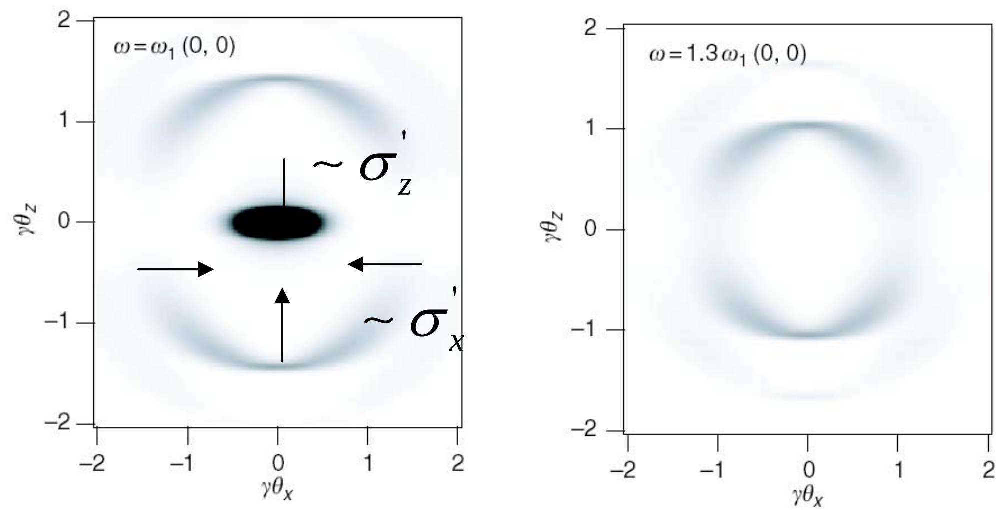
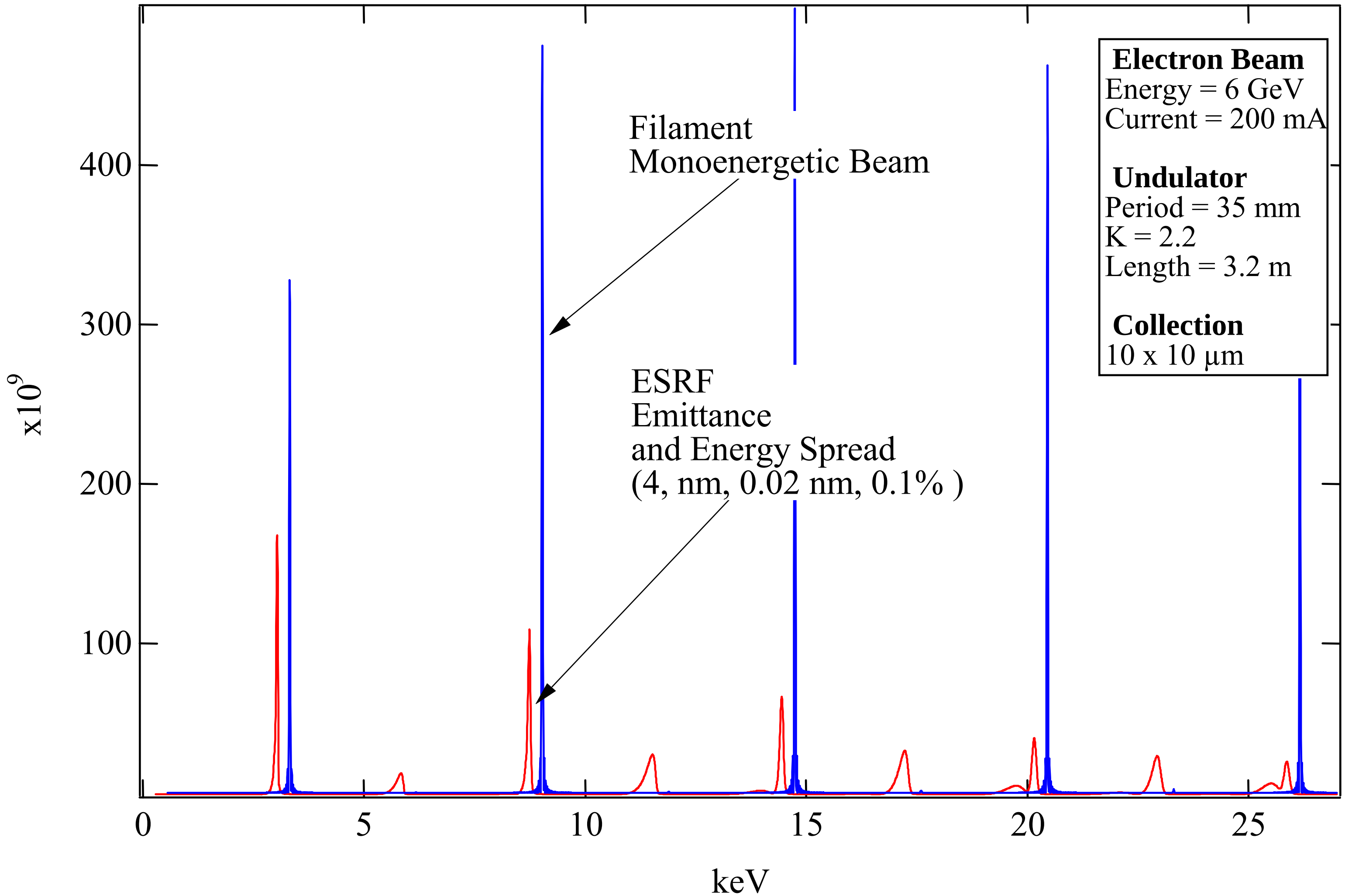
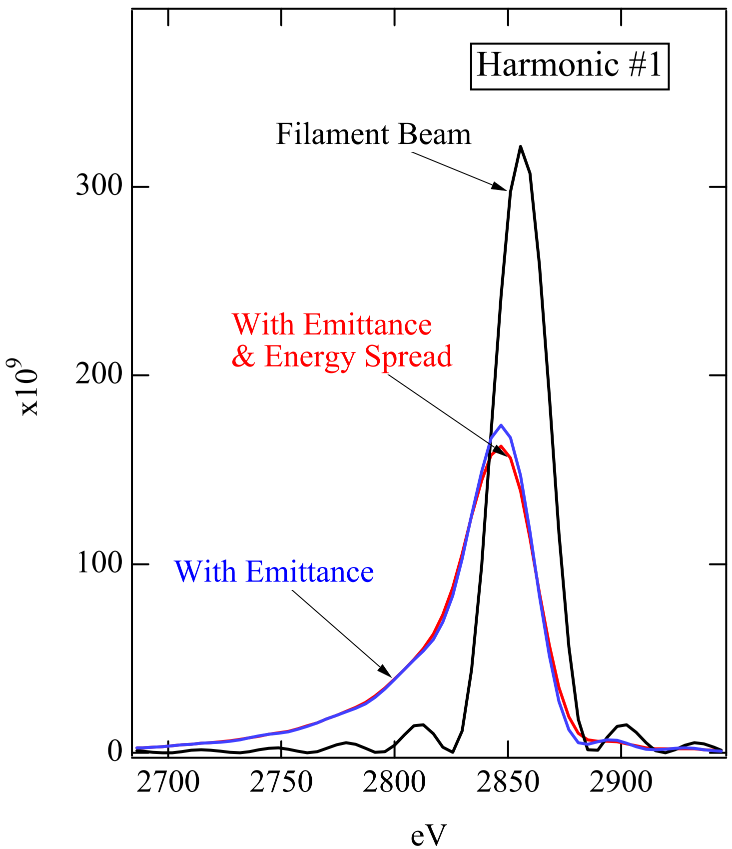


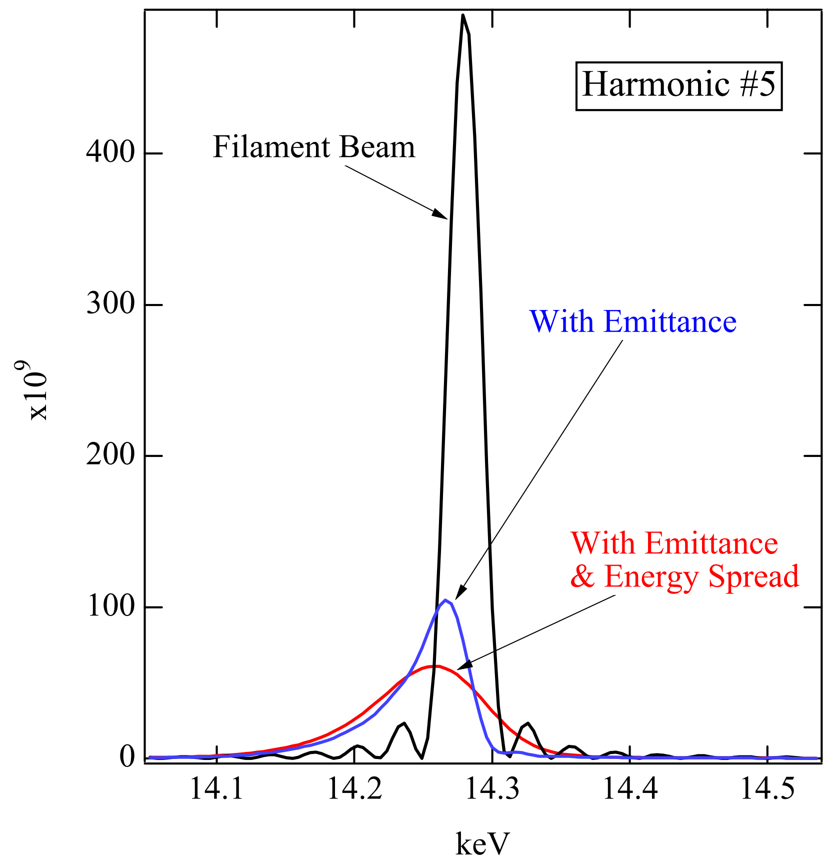
Figure 3.9 Angular spectral flux produced by a 6 GeV electron beam with horizontal (vertical) rms divergence of 10(4) μrad. The frequencies and the undulator parameters are those of Figure 3.7.

Sensitivity to Emittance and Energy Spread of an ESRF Undulator



Sensitivity to Emittance and Energy Spread of an ESRF Undulator



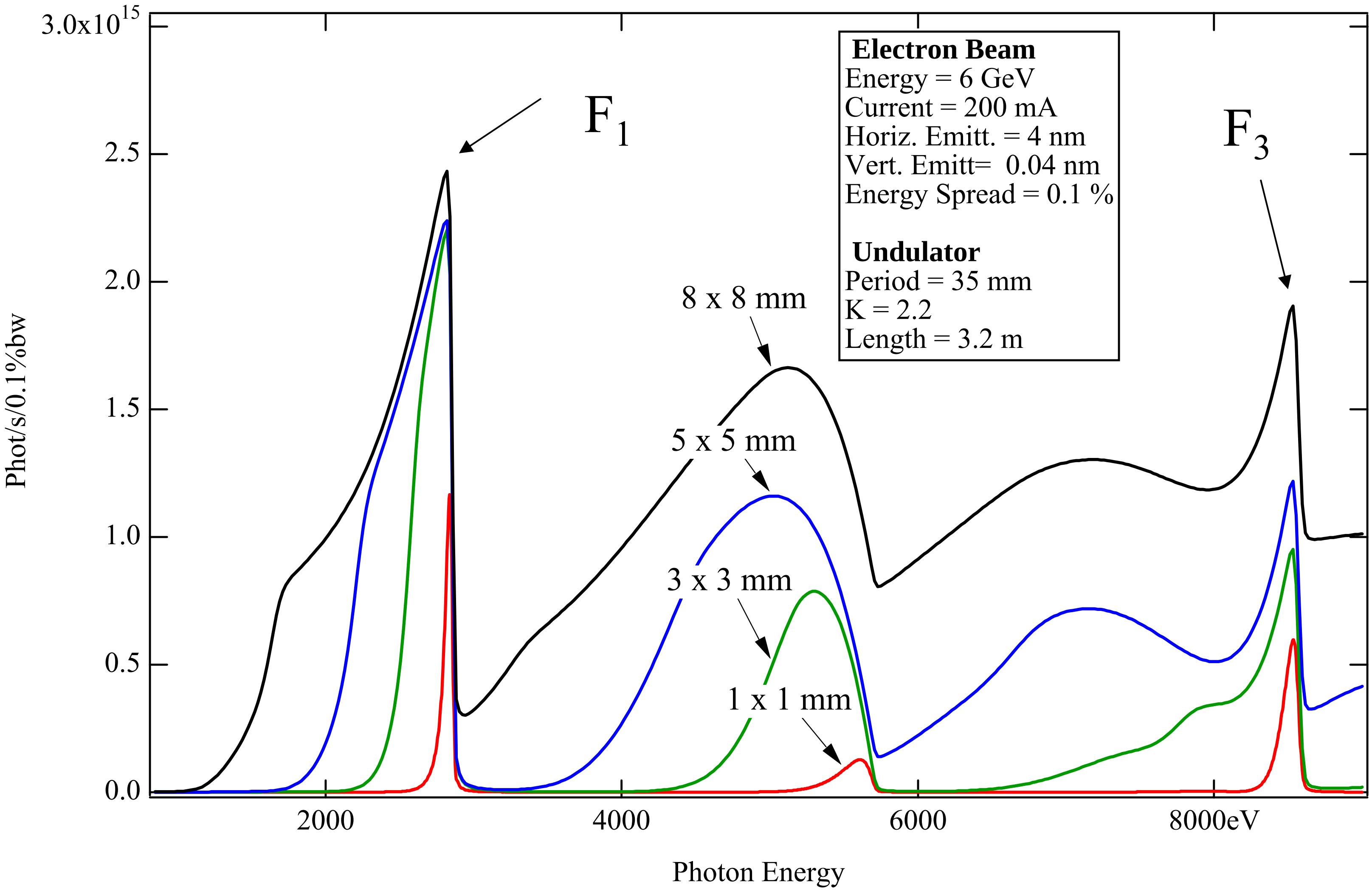


Undulator : K = 2.2, Period = 35 mm, Length = 3.2 m

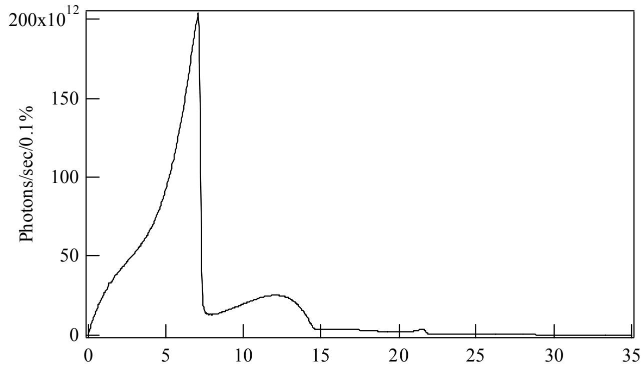
Energy = 6 GeV, Emittance = 4 & 0.04 nm Energy Spread = 0.1 %

Collection Aperture : a 10x10 µm

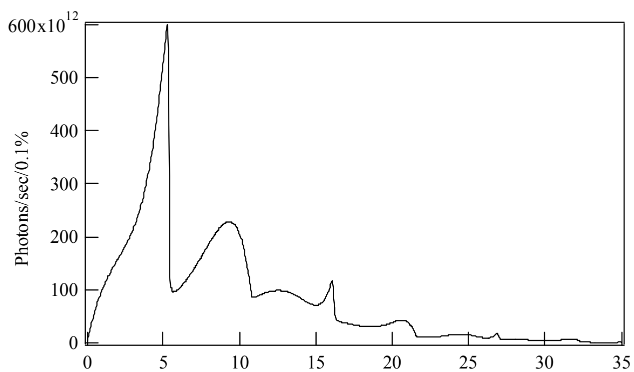
Collecting Undulator Radiation in a Variable Aperture



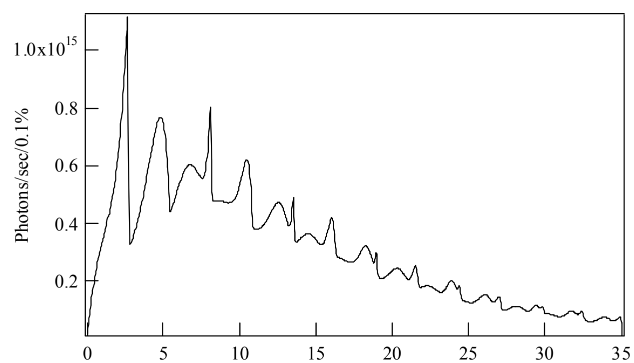
Angle Integrated Flux



K=0.5



K=1



K=2

Maximum Spectral Flux on-axis on odd harmonics

$$\frac{dF_n}{\frac{d\lambda}{\lambda}} = \int \frac{d\Phi_n}{d\theta_x d\theta_z \frac{d\lambda}{\lambda}}(\theta_x,\theta_z,\lambda_n)d\theta_x d\theta_z = \pi\alpha \frac{I}{e} N Q_n(K)$$

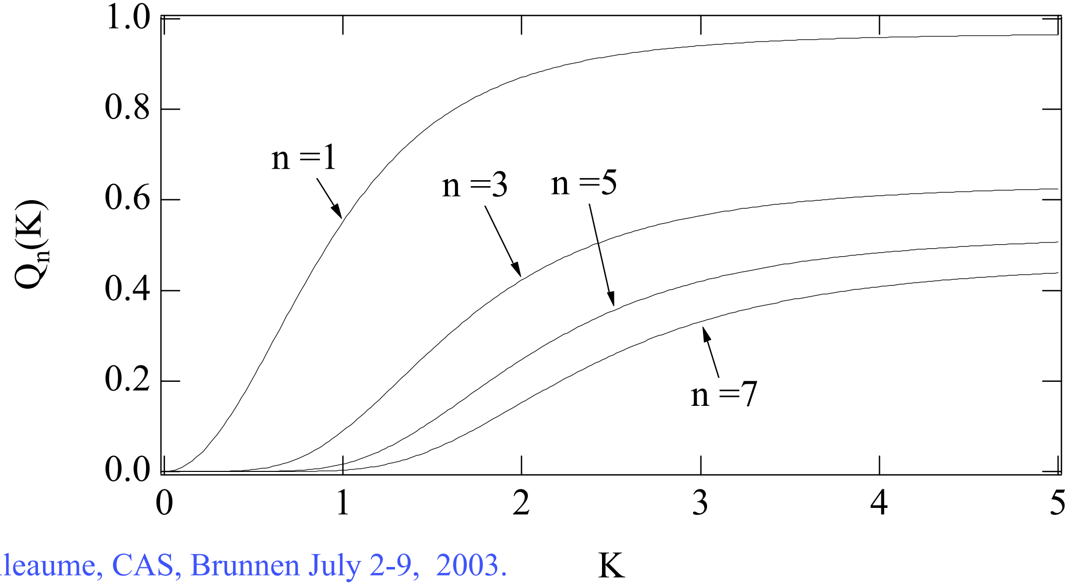
$$F_n\,[Ph/\sec/0.1\%] = 1.431\,10^{14}\; N\; I[A]\, Q_n(K)$$

N : Number of Undulator Periods

I : Ring Current

n : Harmonic Number

$$Q_n(K) = \frac{nK^2}{1+\frac{K^2}{2}}\left(J_{(n+1)/2}\left(\frac{nK^2}{4+2K^2}\right) - J_{(n-1)/2}\left(\frac{nK^2}{4+2K^2}\right)\right)^2$$



(Spectral) Brilliance or Brightness

The (spectral) brilliance or equivalently (spectral) brightness is the equivalent of the photon density in 4D phase space.
It can be rigorously defined using Wigner Distribution Functions.
Its expression is in general complicated but at the energy of an odd harmonic n observed on-axis, it is approximated according to:

$$B_n = \frac{F_n}{(2\pi)^2 \Sigma_x \Sigma'_x \Sigma_z \Sigma'_z}$$

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_R^2} \ , \qquad \sigma_R = \frac{\sqrt{\lambda_n L}}{2\pi}$$

$$\Sigma'_x = \sqrt{\sigma'^2_x + \sigma'^2_R} \ , \qquad \sigma'_R = \frac{\lambda_n}{2L}$$

Σ_x, Σ_z : Photon beam sizes

Σ'_x, Σ'_z: Photon beam divergences

Limiting Cases of Brilliance

$$B_n = \frac{F_n}{(2\pi)^2 \Sigma_x \Sigma'_x \Sigma_z \Sigma'_z}$$

i/ $$\sigma_x, \sigma_z << \frac{\sqrt{\lambda_n L}}{2\pi} \ and \ \sigma'_x, \sigma'_z << \sqrt{\frac{\lambda_n}{2L}} \Rightarrow B_n = \frac{F_n}{(\lambda_n / 2)^2}$$ Diffraction Limit

ii/ $$\sigma_x, \sigma_z >> \frac{\sqrt{\lambda_n L}}{2\pi} \ and \ \sigma'_x, \sigma'_z >> \sqrt{\frac{\lambda_n}{2L}} \Rightarrow B_n = \frac{F_n}{4\pi^2 \varepsilon_x \varepsilon_z}$$

iii/ *Intermediate with Optimum Beta Function* $\Rightarrow$

$$B_n = \frac{F_n}{4\pi^2 (\varepsilon_x + \frac{\lambda_n}{4\pi})(\varepsilon_z + \frac{\lambda_n}{4\pi})}$$

In all cases the brilliance grows like the number N of undulator periods

Some Difficulties with Brilliance

- Brilliance is very often assimilated as photon density in 4D phase space $B(\omega,x,x',z,z')$ => Not a Physical quantity in the sense of Quantum Mechanics. Nevertheless, it is possible to mathematically define such brilliance using Wigner Distribution Functions so that :

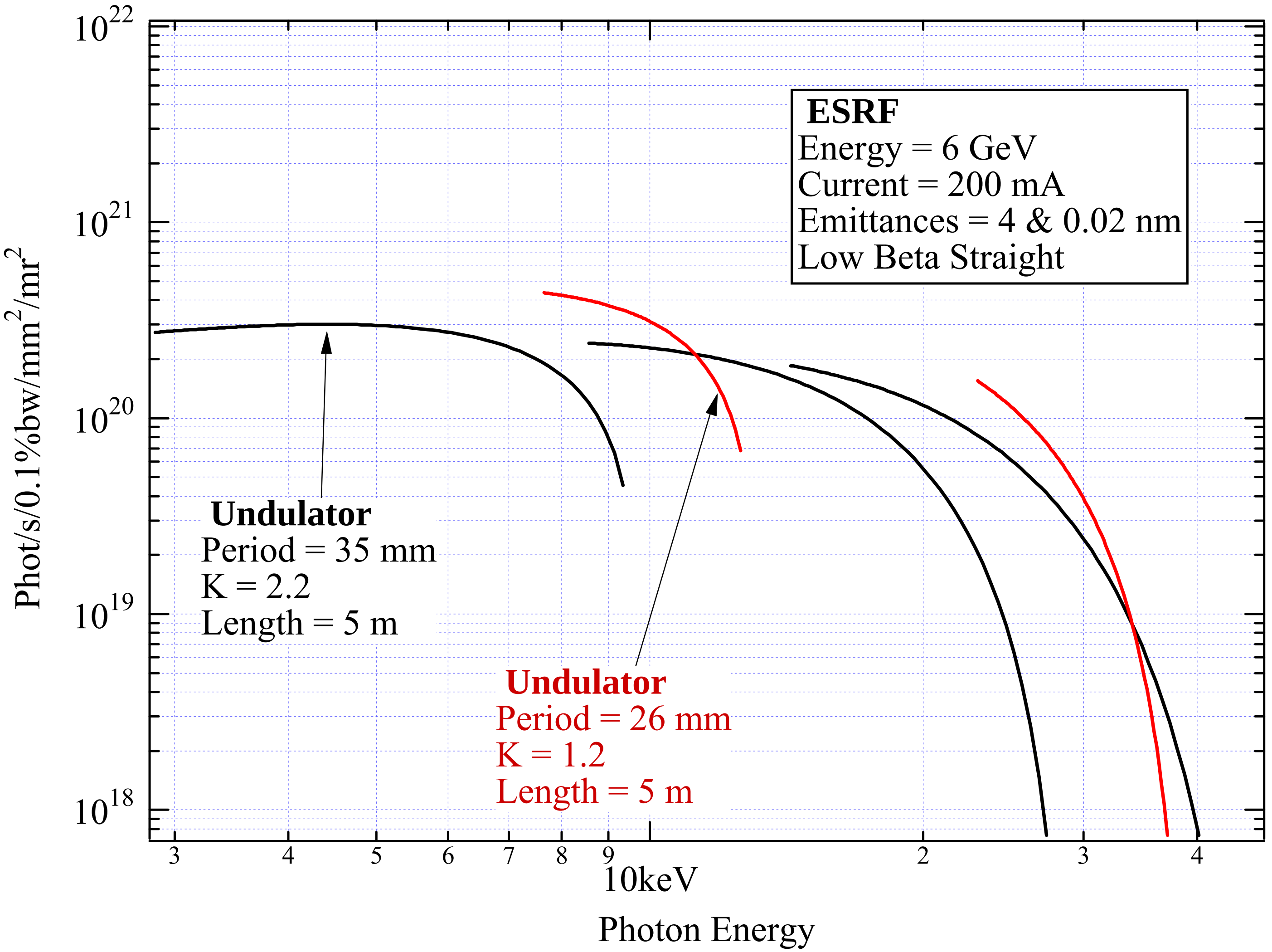
$$\text{Flux} \quad F(\omega) = \int\int\int\int B(\omega, x, x', z, z')\,dx\,dz\,dx'\,dz'$$

$$\text{Angular Flux} \quad \frac{dF}{d\Omega}(\omega) = \int\int\int\int B(\omega, x, x', z, z')\,dx\,dz$$

- Such brilliance can be negative for some x,x',z,z', ω !!
- The spatial and angular distribution of the single electron emission are not Gaussian => There exists different definitions of σ_R and σ'_R
- There is no simple way to include energy spread except performing a complicated convolution that nobody has ever tried to do numerically

The expression of the planar undulator brilliance on an odd harmonic on-axis is nevertheless widely used and is one of the most important figures of merit for a light source. It combines the requirements of: large flux, small divergence and small beam sizes

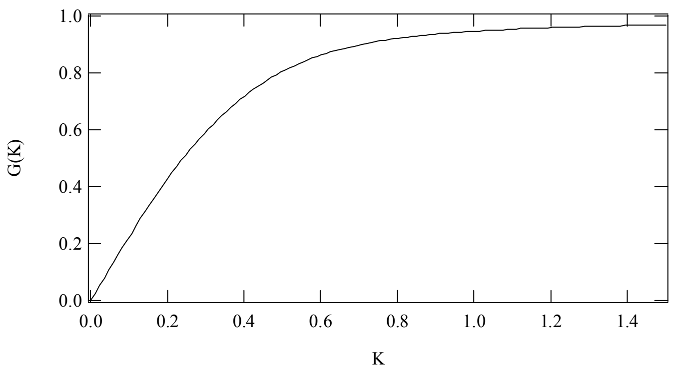
Brilliance vs Photon Energy



Power and Power Density

$$Power[kW] = 0.633\hat{B}^2[T]E^2[GeV]I[A]L[m]$$

$$\frac{Power}{Solid\ Angle}[W/mr^2] = 10.84\hat{B}[T]E^4[GeV]I[A]NG(K)$$

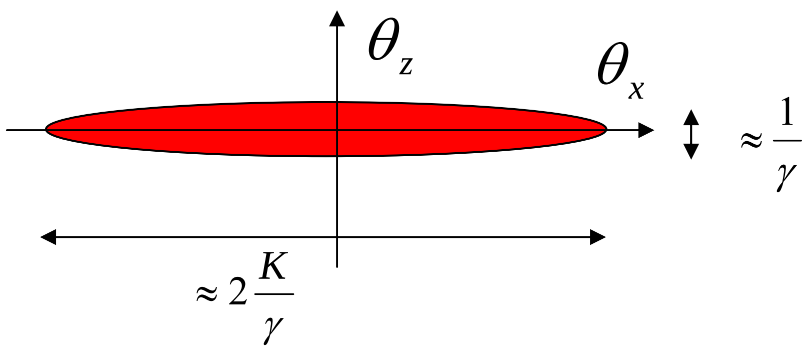


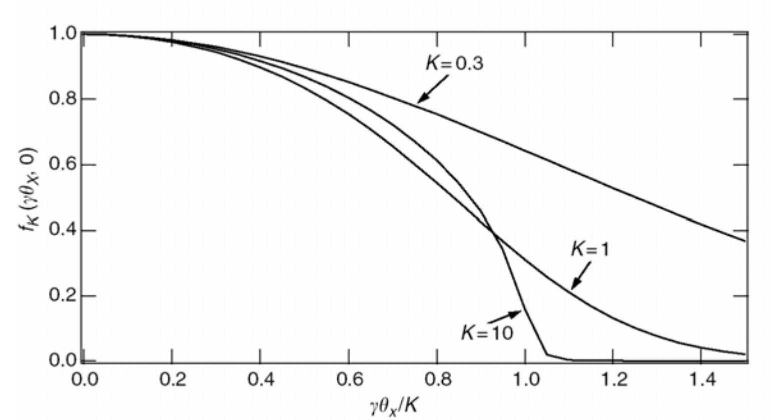
For an undulator : Length = 5 m, Field = 0.75 T, Period =35 mm

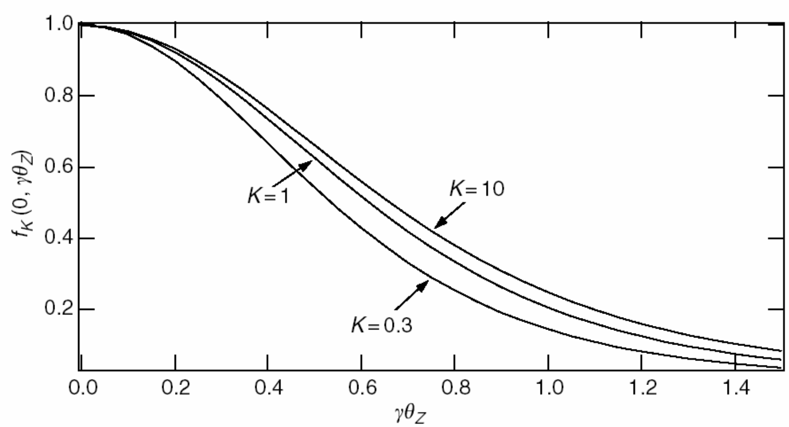
Ring	Energy	Current	Power	Power Density
				@ 10 m on-axis
	[GeV]	[A]	[kW]	[kW/mm2]
SuperACO	0.8	0.5	0.57	0.0023
SLS	2.4	0.3	3.08	0.11
ESRF	6	0.2	12.82	2.96

Power Density Profile

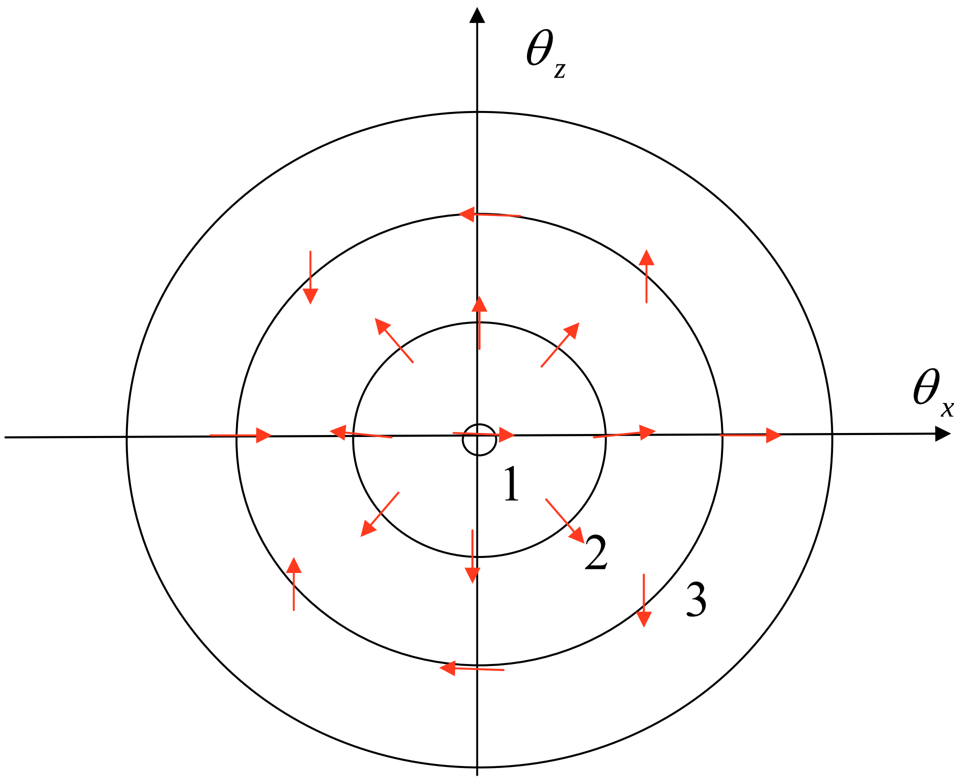
$$\frac{Power}{Solid\ Angle}(\theta_x,\theta_z) = \frac{Power}{Solid\ Angle}(0,0)\, f_K(\gamma\theta_x,\gamma\theta_z)$$







Planar field undulators produce only linearly polarized radiation.



On-axis :

- Single electron emission is 100% linearly polarized
- With emittance (and energy spread) some small depolarisation is generated

Ellipsoidal Undulator

$$Magnetic\ \ field:\ \vec{B} = (\hat{B}_x \sin(2\pi \frac{s}{\lambda_0} + \varphi), \hat{B}_z \sin(2\pi \frac{s}{\lambda_0}), 0)$$

$$Electron\, velocity:\ \vec{v} = (\frac{K_x}{\gamma} \cos(2\pi \frac{s}{\lambda_0}), \frac{K_z}{\gamma} \cos(2\pi \frac{s}{\lambda_0} + \varphi), 1) + o(\frac{1}{\gamma})$$

$$with:\ K_x = \frac{e\hat{B}_z \lambda_0}{2\pi mc^2} \ \ and \ \ K_z = \frac{e\hat{B}_x \lambda_0}{2\pi mc^2}$$

Like the planar undulator, it generates an harmonic spectrum with peaks at :

$$\lambda = \frac{\lambda_0}{2n\gamma^2}(1 + \frac{K_x^2}{2} + \frac{K_y^2}{2} + \left(\gamma\theta_x\right)^2 + \left(\gamma\theta_y\right)^2)$$

The polarisation is in general ellipsoidal depending on direction and wavelength

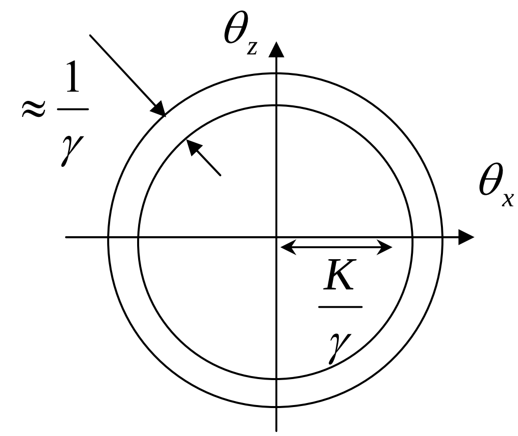
The footprint of the angular power density is an ellipse :

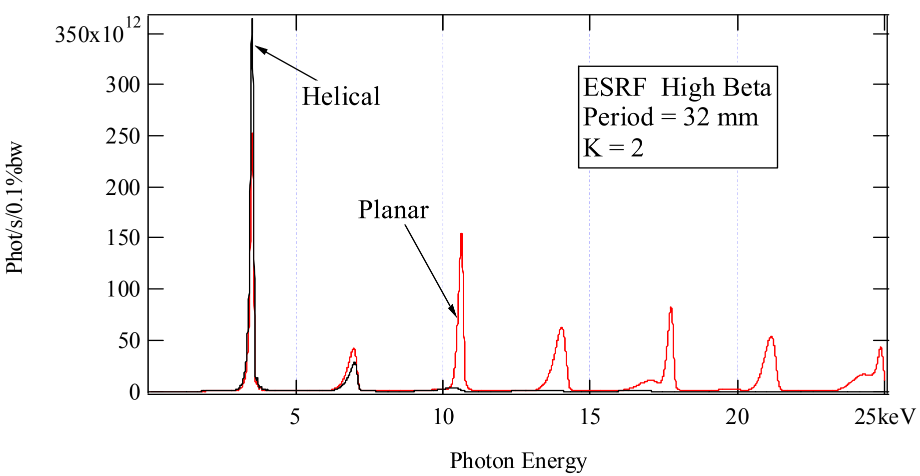
Helical Undulator

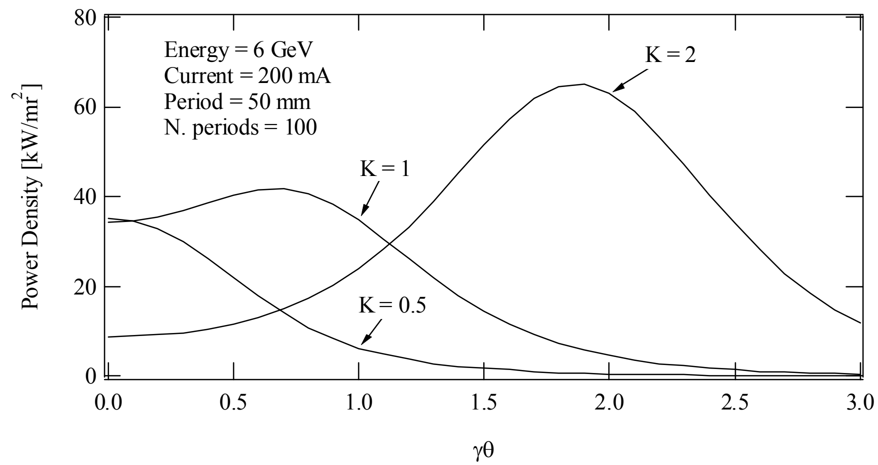
$$Magnetic\ field:\ \vec{B} = (\hat{B}\cos(2\pi\frac{s}{\lambda_0}), \hat{B}\sin(2\pi\frac{s}{\lambda_0}), 0)$$

$$Electron\ velocity:\ \vec{v} = (\frac{K}{\gamma}\cos(2\pi\frac{s}{\lambda_0}), \frac{K}{\gamma}\sin(2\pi\frac{s}{\lambda_0}), 1) + o(\frac{1}{\gamma}) \qquad with:\ K = \frac{e\hat{B}\lambda_0}{2\pi mc^2}$$

The footprint of the radiation observed on a screen makes a ring. The radiation on axis contains essentially the harmonic 1

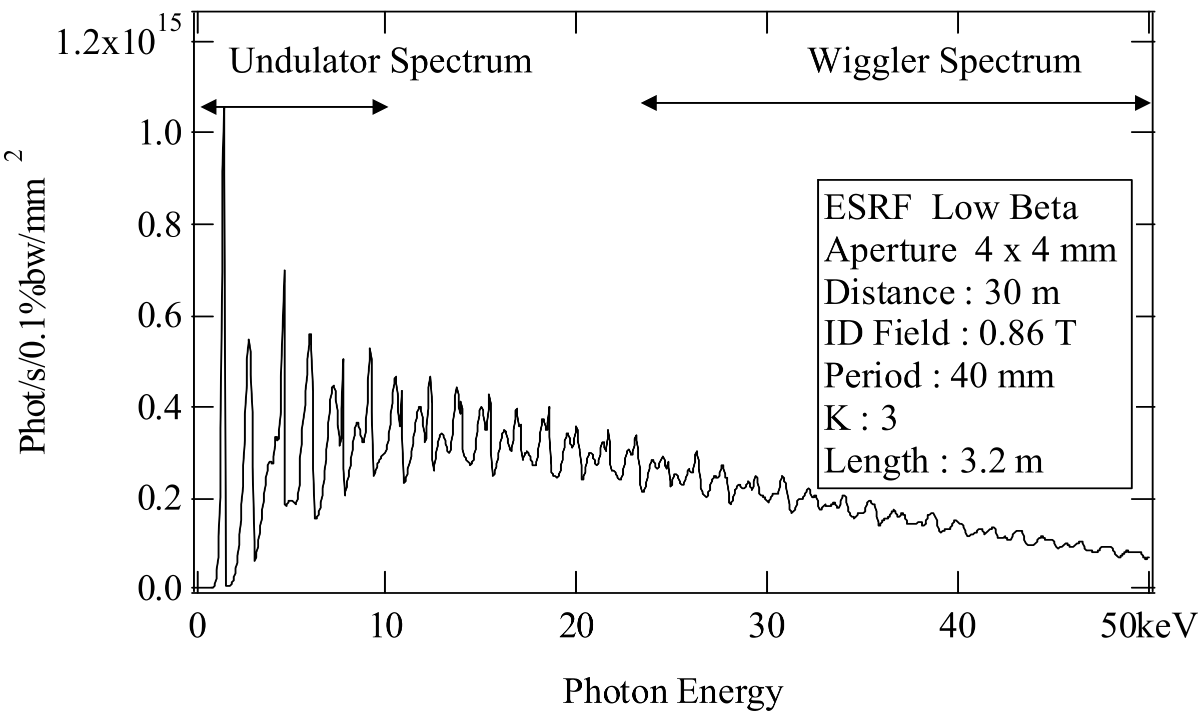




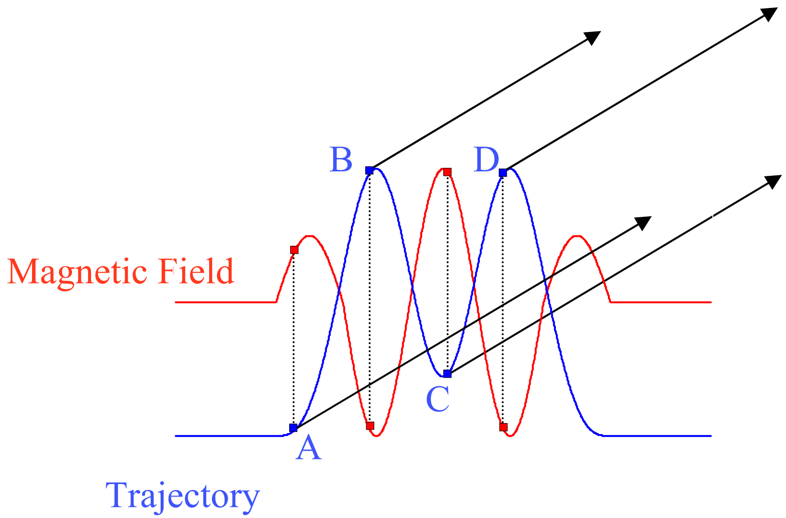


I, 33/38 , P. Elleaume, CAS, Brunnen July 2-9, 2003.

Wiggler Spectrum



Wiggler Radiation



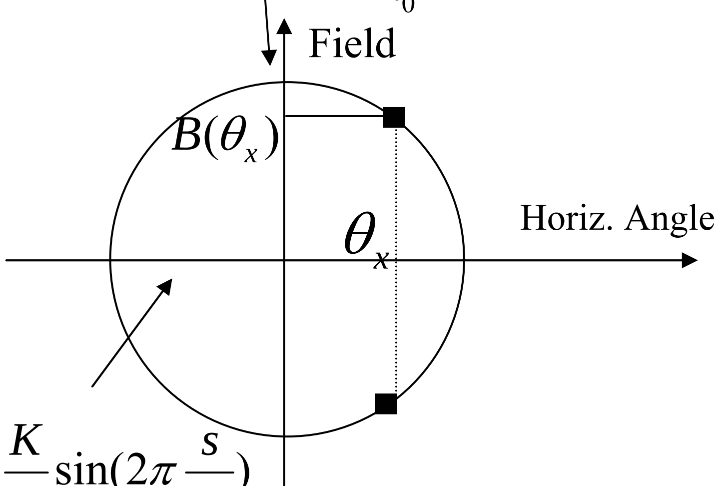
Wiggler Approximation

-Construct source points (tangent to direction of observation)

-Simply add the Spectral Flux from each point treated as a bending magnet (no interference effects)

- Two Source points per period

$$B = \hat{B}\cos(2\pi \frac{s}{\lambda_0})$$



$$\theta_x(s) = \frac{K}{\gamma}\sin(2\pi \frac{s}{\lambda_0})$$

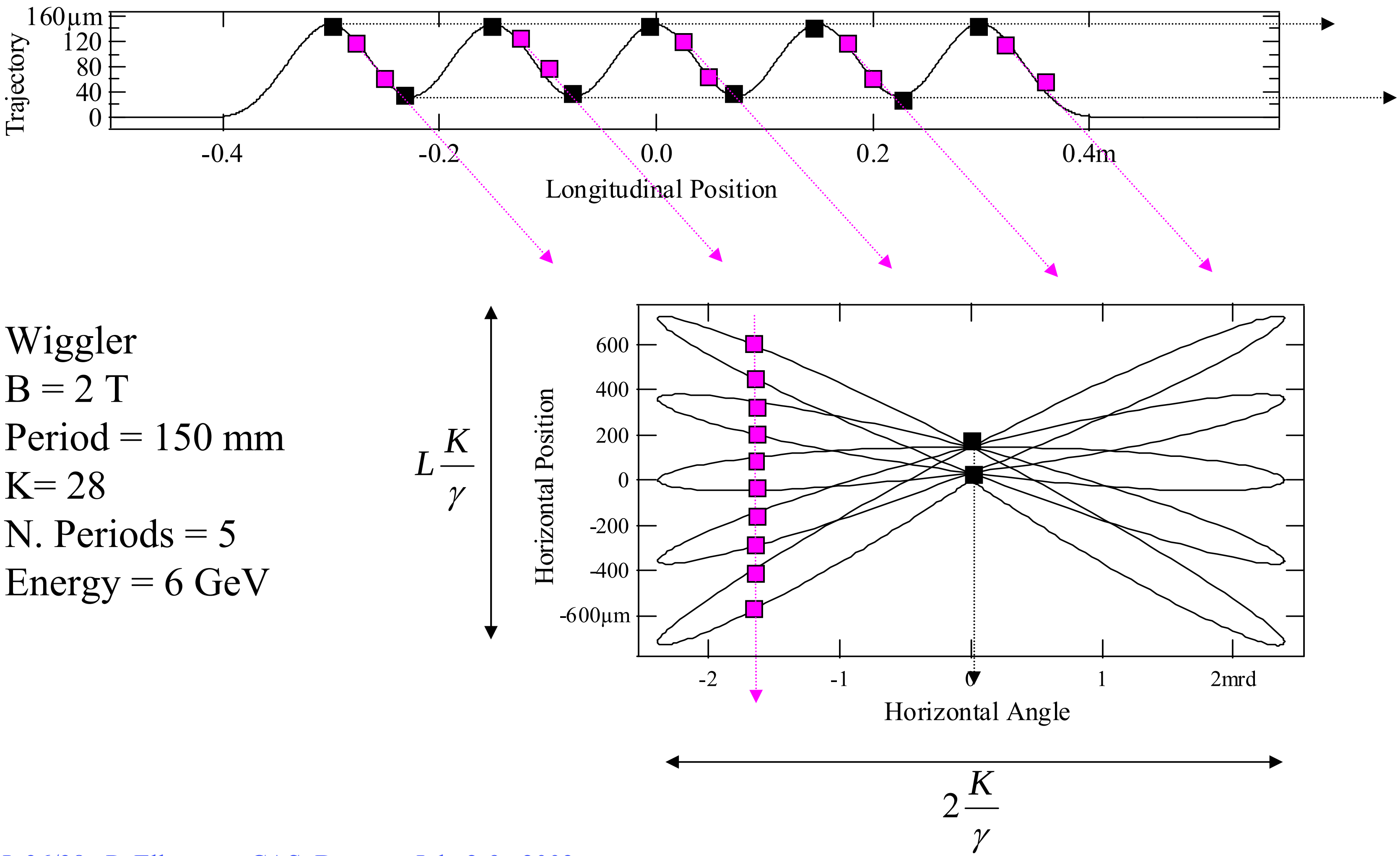
Characteristics

- Numerically simple and inexpensive to compute
- Critical Energy decays off-axis horizontal

$$B(\theta_x) = \hat{B}\sqrt{1-(\frac{\gamma\theta_x}{K})^2}$$

- Continuous spectrum
- Flux, Brilliance,… grows like 2 x N. of Periods
- Polarization :
    - linear in Median Plane
    - depolarized off-axis
- Multiple source points when observing off-axis

Multiple Source Points in Horizontal Plane



Wiggler
B = 2 T
Period = 150 mm
K= 28
N. Periods = 5
Energy = 6 GeV

Remarks on Wigglers and Undulators

- It is customary to call undulators a device with K < 2.5 and wiggler a device with K > 2.5. This is partly justified by the fact that there is little practical interest in making use of an undulator spectrum with K>2.2 except if one wants to reach a very low energy.
- All devices behave partly as an undulator (at low energy) and partly as a wiggler (at high energy)
- In the past 10 years, it has been observed in most synchrotron light sources that undulators are preferred to wigglers :
  - They generate high brilliance
  - They can be used at a higher photon energy than originally expected by reducing the magnetic gap and making use of high harmonic numbers

Computer codes for undulator/wiggler radiation including emittance and energy spread

Name	Authors	Platform	Download
B2E & SRW	O. Chubar, P. Elleaume, ESRF	Mac, Windows	http://www.esrf.fr/machine/groups/insertion_devices/Codes/software.html
URGENT	R.P. Walker, Diamond Light Source	Fortran Source	Contact Author
XOP	M. Sánchez del Río , ESRF & R. J. Dejus, APS	Unix, Windows	http://www.esrf.fr/computing/scientific/xop/
SPECTRA	T. Tanaka, SPring8	Unix, Windows Mac	http://www.spring8.or.jp/ENGLISH/facility/bl/insertion/Softs/index.html