

CERN Accelerator School  
Specialized Course on Magnets  
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# Basic design and engineering of normal-conducting, iron-dominated electro-magnets

Lecture 2 + 3  
'Basic analytical design'

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# Basic analytical design

Goals in magnet design

Magnet components

Yoke design

Magnetic steel

Coil design

Cooling

Cost optimization



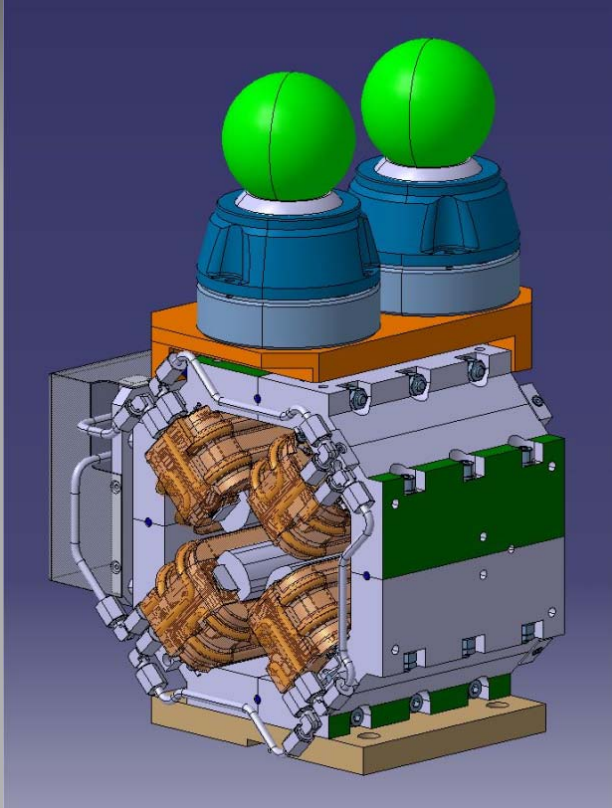
# Goals in magnet design

The goal is to produce a product just **good enough** to perform **reliably** with a sufficient **safety factor** at the **lowest cost** and on **time**.

- Good enough:
  - Obvious parameters clearly specified, but tolerance difficult to define.
  - Tight tolerances lead to increased costs
- Reliability:
  - Get MTBF and MTTR reasonably low
  - Reliability is usually unknown for new design
  - Requires experience to search for a compromise between extreme caution and extreme risk (reviews)
- Safety factor:
  - Allows operating a device under more demanding condition as initially foreseen
  - To be negotiated between the project engineer and the management
  - Avoid inserting safety factors a multiple levels (costs!)



# Magnet Components



Alignment targets

Yoke

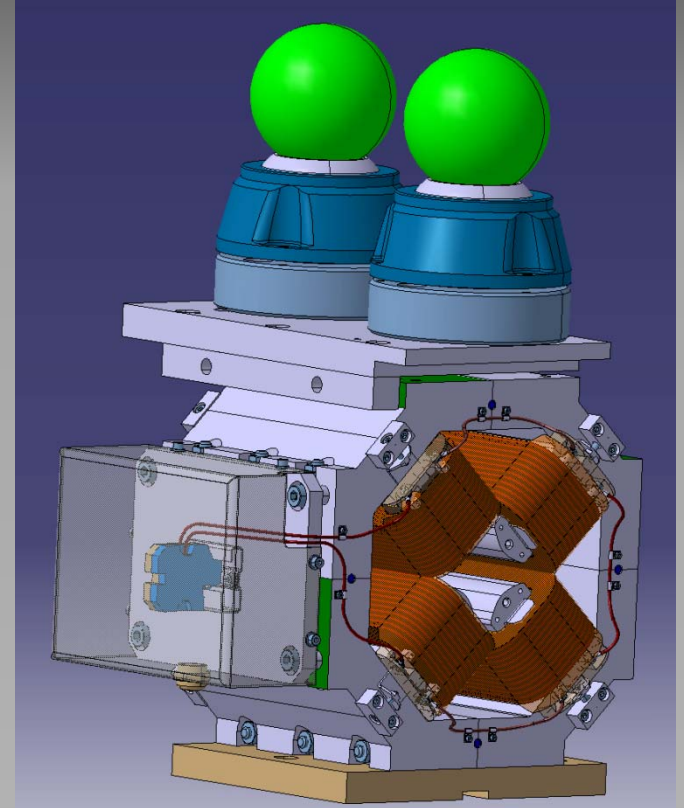
Coils

Sensors

Cooling circuit

Connections

Support





# Yoke design

Translate the beam optic requirements into a magnetic design

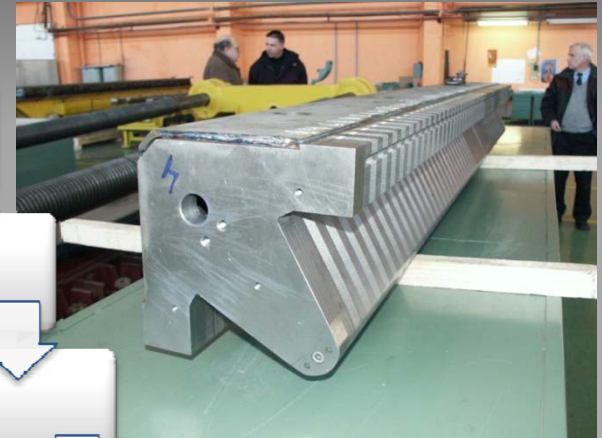
Required magnetic induction

Aperture size

Magnet excitations (Amp-turns)

Yoke cross-section

Yoke material





# Beam rigidity

Beam rigidity  $B\rho$  [Tm]:

$$B\rho = \frac{1}{qc} \sqrt{T^2 + 2T E_0} \quad (1)$$

- $q$ : particle charge number
- $c$ : speed of light [m/s]
- $T$ : kinetic beam energy [eV]
- $E_0$ : particle rest mass energy [eV]  
( 0.51 MeV for electrons, 938 MeV for protons)



# Magnetic induction

Dipole bending field  $B$  [T]:

- $B$ : Flux density or magnetic induction (vector) [T, Gauss]
- $r_M$ : magnet bending radius [m]

$$B = \frac{B\rho}{r_M} \quad (2a)$$

Quadrupole field gradient  $B'$  [T/m]:

- $k$ : quadrupole strength [ $\text{m}^{-2}$ ]

$$B' = B\rho k \quad (2b)$$

Sextupole differential gradient  $B''$  [ $\text{T}/\text{m}^2$ ]:

- $m$ : sextupole strength [ $\text{m}^{-3}$ ]

$$B'' = B\rho m \quad (2c)$$



# Aperture size

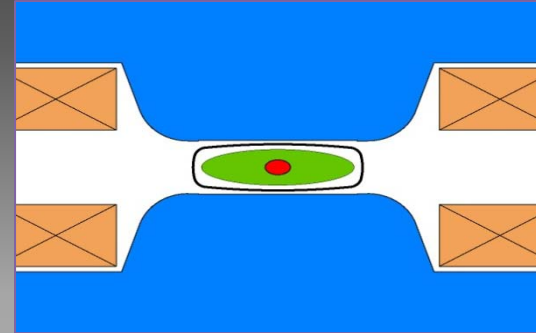
Aperture =

Good field region

- Maximum beam size
  - Lattice functions: beta functions and dispersion
  - Geometrical transverse emittances (energy depended)
  - Momentum spread
  - Envelope (typical several sigma)
  - Largest beam size usually at injection
- Closed orbit distortions (~5 mm)

+ Vacuum chamber thickness (0.3 – 2 mm)

+ Installation and alignment margin (0 – 2 mm)



$$\sigma = \sqrt{\varepsilon \beta + \left( D \frac{\Delta p}{p} \right)^2}$$





# Excitation current in a dipole

Amperes law  $\oint \vec{H} \cdot d\vec{l} = NI$  and  $\vec{B} = \mu \vec{H}$  and  $\mu = \mu_0 \mu_r$

leads to 
$$NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{l} = \int_{\text{gap}} \frac{\vec{B}}{\mu_{\text{air}}} \cdot d\vec{l} + \int_{\text{yoke}} \frac{\vec{B}}{\mu_{\text{iron}}} \cdot d\vec{l} = \frac{Bh}{\mu_{\text{air}}} + \frac{B\lambda}{\mu_{\text{iron}}} \quad (3)$$

assuming, that B is constant around the path.

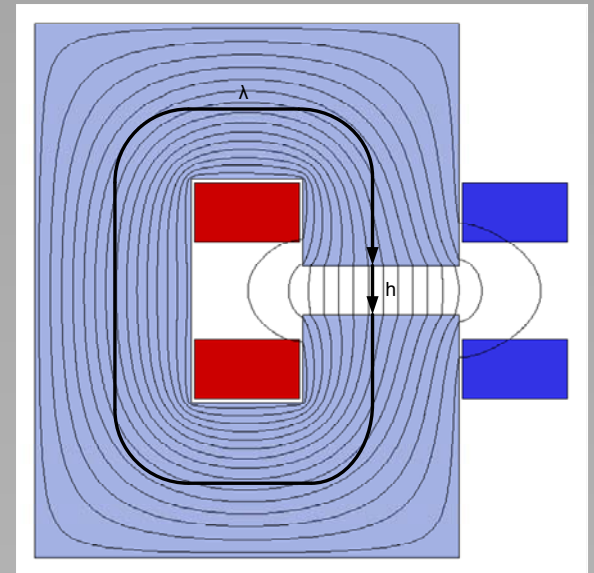
If the iron is not saturated:

$$\frac{h}{\mu_{\text{air}}} \gg \frac{\lambda}{\mu_{\text{iron}}}$$

then:

$$NI_{(\text{per pole})} = \frac{Bh}{2\eta\mu_0} \quad (4^*)$$

- $h$ : gap height [m]
- $H$ : magnetic field (vector) [A/m, Oersted]
- $\eta$ : efficiency (typically 99 %)
- $\mu_0$ : permeability of free space:  $4 \pi 10^{-7}$  [Vs/Am]
- $\mu_r$ : relative permeability:  $\mu_{\text{air}} = 1$ ;  $\mu_{\text{iron}} > 1000$  (not saturated)





# Reluctance and efficiency

Similar to Ohm's law, one can define the 'resistance' of a magnetic circuit, called 'reluctance', as:

$$R_M = \frac{NI}{\Phi} = \frac{l_M}{A_M \mu_r \mu_0} \quad (5)$$

- $\Phi$ : magnetic flux [Wb]
- $l_M$ : flux path length in iron [m]
- $A_M$ : iron cross section perpendicular to flux [m<sup>2</sup>]

Second term ( $\frac{\lambda}{\mu_{iron}}$ ) in (3) is called 'normalized reluctance' of the yoke

Iron saturation (small  $\mu_{iron}$ ) leads to inefficiencies

Keep iron yoke reluctance less than a few % of air reluctance ( $\frac{h}{\mu_0}$ ) by providing sufficient iron cross section ( $B_{iron} < 1.5$  T)

Efficiency: 
$$\eta = \frac{R_{M,gap}}{R_{M,gap} + R_{M,yoke}} \approx 99\%$$



# Magnetic length

Coming from  $\infty$ , B increases towards the magnet center (stray flux)

'Magnetic' length > iron length

$$\text{Magnetic length: } l_{mag} = \frac{\int_{-\infty}^{\infty} B(z) \cdot dz}{B_0}$$

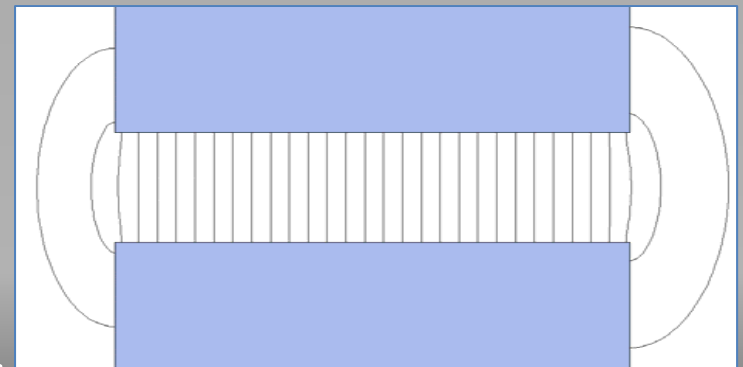
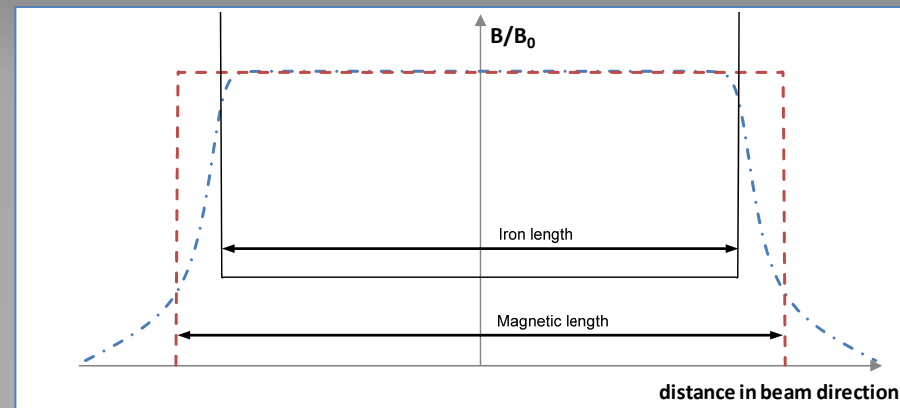
$$\text{For a dipole: } l_{mag} = l_{iron} + 2hk \quad (6^*)$$

- $k$ : geometry specific constant ( $\approx 0.56$ )

$k$  gets smaller in case of:

- Pole width < gap height
- Saturation

Precise determination of  $k$  only by measurements or numerical calculations



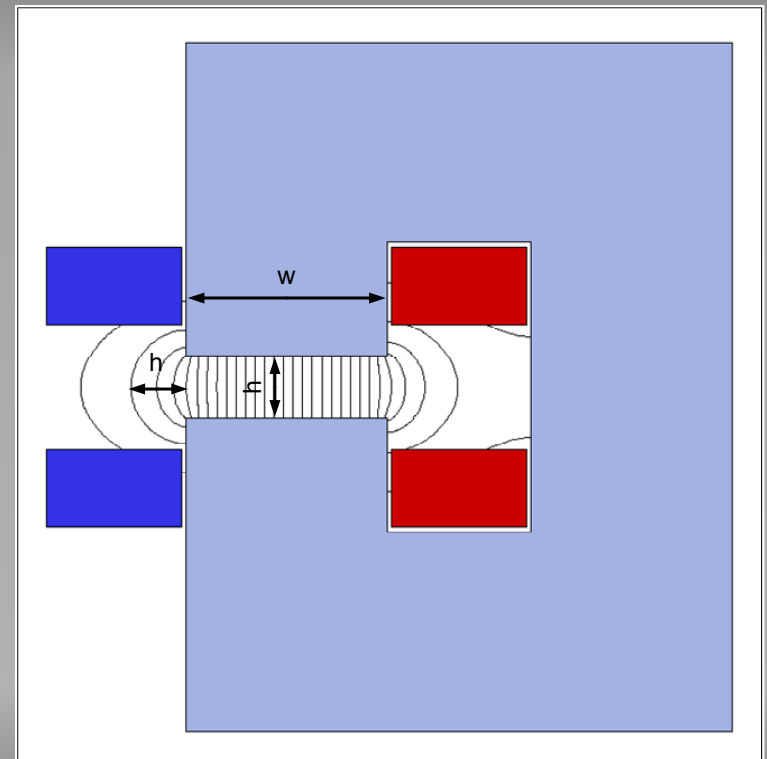


# Magnetic flux

Flux in the yoke includes the gap flux and stray flux

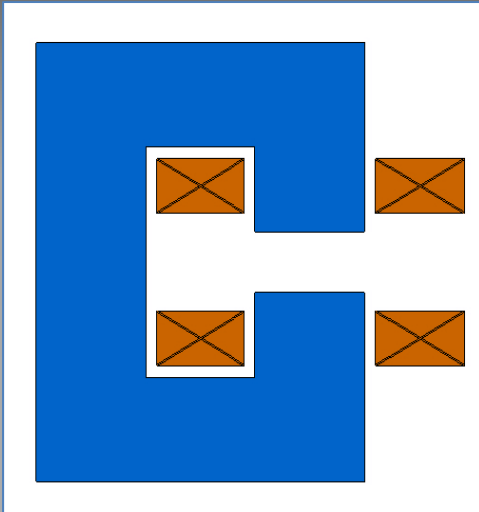
Total flux in the return yoke:

$$\Phi = \int_A B \cdot dA \approx B_{gap} (w + 2h) l_{mag} \quad (7^*)$$



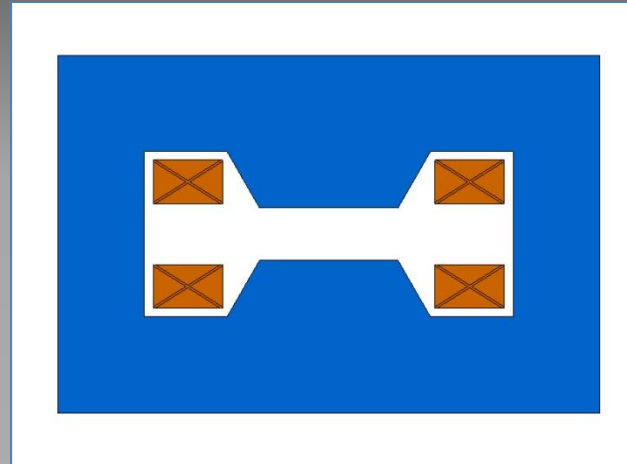
# Dipole types

## C-Shape



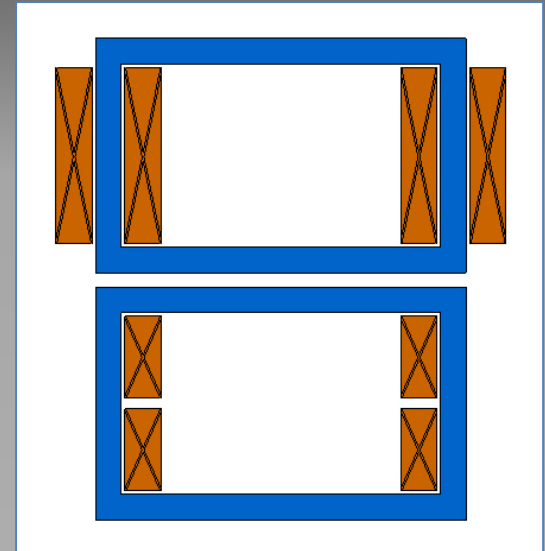
Accessibility: very good  
 Field quality: asymmetric  
 Shims required: yes  
 Mechanical stability: poor  
 Iron (weight): high

## H-Shape



Accessibility: poor  
 Field quality: symmetric  
 Shims required: yes  
 Mechanical stability: good  
 Iron (weight): medium

## O-Shape



Accessibility: very poor  
 Field quality: very good,  
 symmetric  
 Shims required: no  
 Mechanical stability: good  
 Iron (weight): low/medium



# Asymmetry in a C-magnet

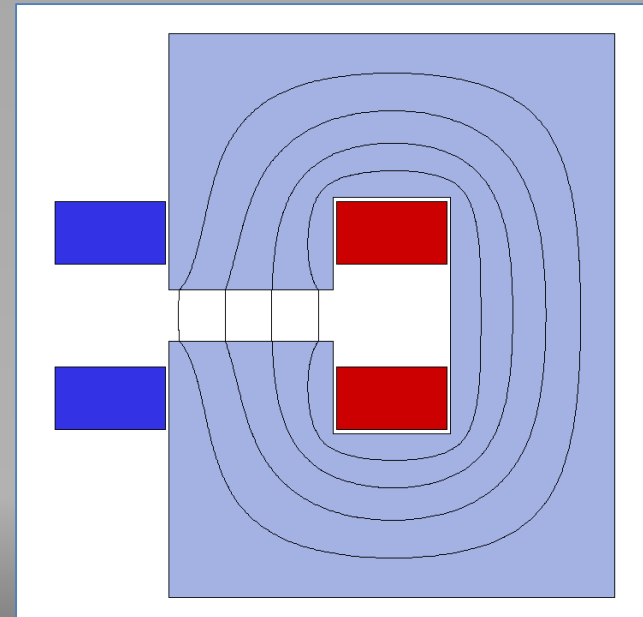
C-magnet: one-fold symmetry

Since  $NI = \oint \vec{H} \cdot d\vec{l} = \text{const.}$  the contribution to the integral in the iron has different path lengths

Finite (low) permeability will create lower B on the outside of the gap than on the inside

Generates 'forbidden' harmonics with  $n = 2, 4, 6, \dots$  changing with saturation

Quadrupole term resulting in a gradient around 0.1% across the pole





# Excitation current in a Quadrupole

Choosing the shown integration path gives:

$$NI = \oint \vec{H} \cdot d\vec{l} = \int_{s1} \vec{H}_1 \cdot d\vec{l} + \int_{s2} \vec{H}_2 \cdot d\vec{l} + \int_{s3} \vec{H}_3 \cdot d\vec{l}$$

For a quadrupole, the gradient  $B' = \frac{dB}{dr}$  is constant and  $B_y = B'x$   $B_x = B'y$

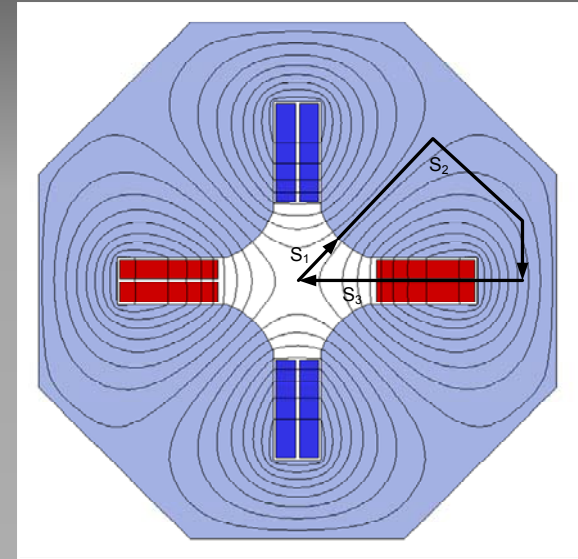
Field modulus along  $s_1$ :  $H(r) = \frac{B'}{\mu_0} \sqrt{x^2 + y^2} = \frac{B'}{\mu_0} r$

Neglecting B in  $s_2$  because:  $R_{M,s2} = \frac{S_2}{\mu_{iron}} \ll$

and along  $s_3$ :  $\int_{s3} \vec{H}_3 \cdot d\vec{l} = 0$

Leads to:  $NI \approx \int_0^R H(r) dr = \frac{B'}{\mu_0} \int_0^R r dr$

$$NI_{(per\ pole)} = \frac{B' r^2}{2\eta\mu_0} \quad (8^*)$$





# Magnetic length

Magnetic length for a quadrupole:

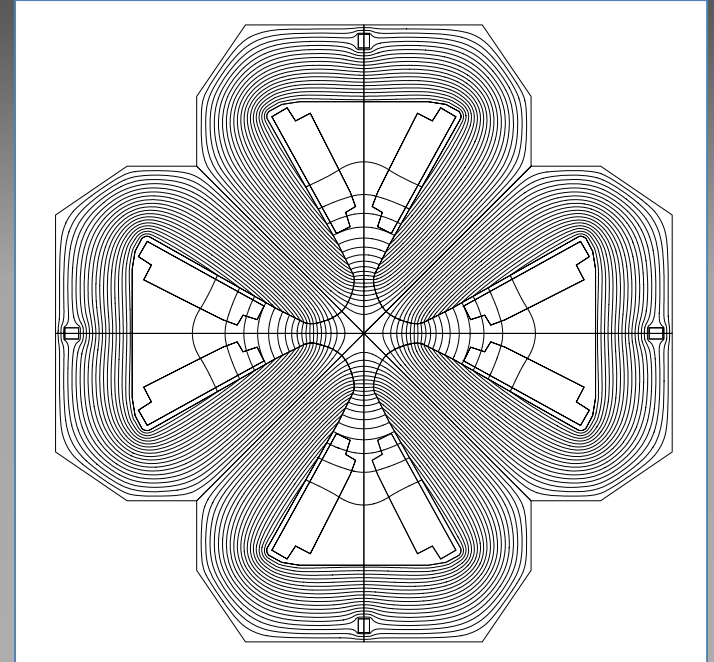
$$l_{mag} = l_{iron} + 2rk \quad (9^*)$$

- $k$ : geometry specific constant ( $\approx 0.45$ )

NI increases with the square of the quadrupole aperture:

$$NI \propto r^2 \quad P \propto r^4$$

More difficult to get the necessary Ampere-turns (= coil cross section) in  
-> termination of the hyperbola

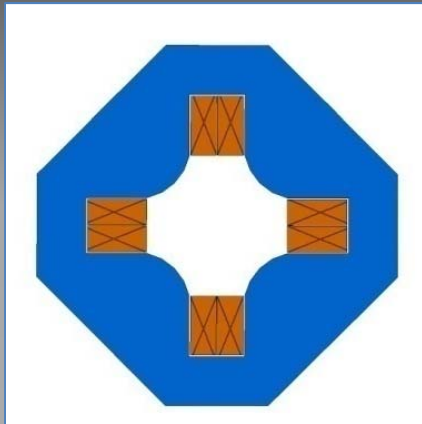




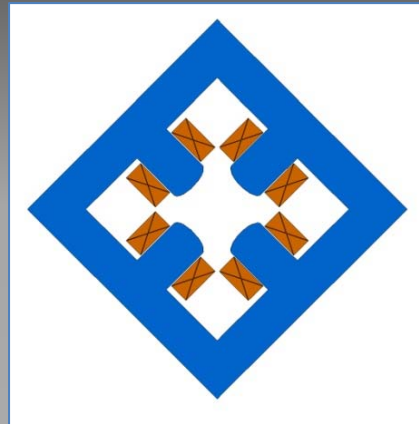


# Quadrupole types

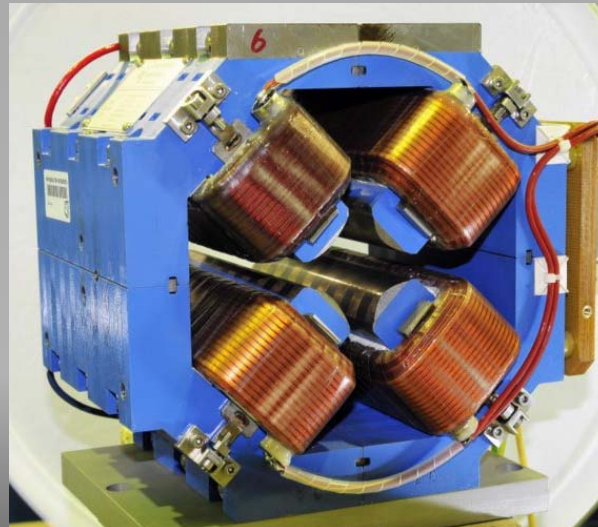
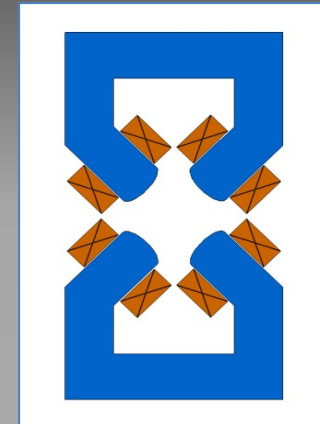
Standard quadrupole 1



Standard quadrupole 2



Collins or Figure-of-Eight





# Excitation current in a Sextupole

Same approach as for quadrupole:

For a sextupole, the field is parabolic and  $B'' = \frac{d^2 B}{dr^2}$  is constant

$$\text{so } H(r) = \frac{B''}{2\mu_0} r^2$$

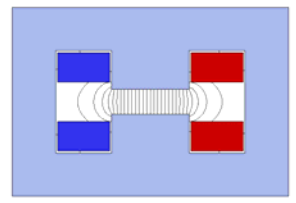
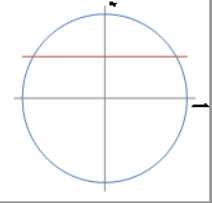
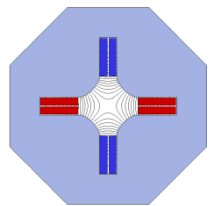
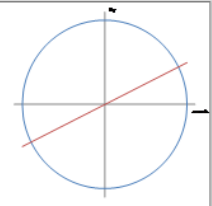
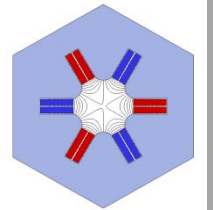
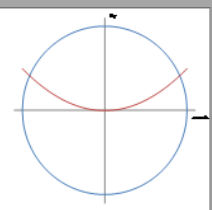
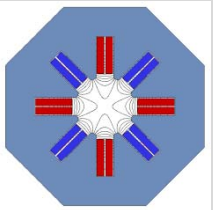
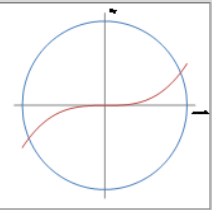
$$\text{leads to: } NI = \oint \vec{H} \cdot d\vec{l} \approx \int_0^R H(r) dr = \frac{B''}{2\mu_0} \int_0^R r^2 dr \quad \boxed{NI_{(per\ pole)} = \frac{B'' r^3}{6\eta\mu_0}} \quad (10^*)$$

NI increases with the 3<sup>rd</sup> power of the aperture:

$$NI \propto r^3 \quad P \propto r^6$$

Fortunately, sextupole fields are usually much smaller than quadrupole fields

# Summary

Pole shape	Field distribution	Pole equation	$B_y$ on x-axis
		$y = \pm r$	$B_y = a_1 = B_0 = \text{const.}$
		$2xy = \pm r^2$	$B_y = a_2 x$
		$3x^2y - y^3 = \pm r^3$	$B_y = a_3(x^2 - y^2)$
		$4(x^3y - xy^3) = \pm r^4$	$B_y = a_4(x^3 - 3xy^2)$



# Yoke materials

Massive (cast ingot) iron only for dc magnets

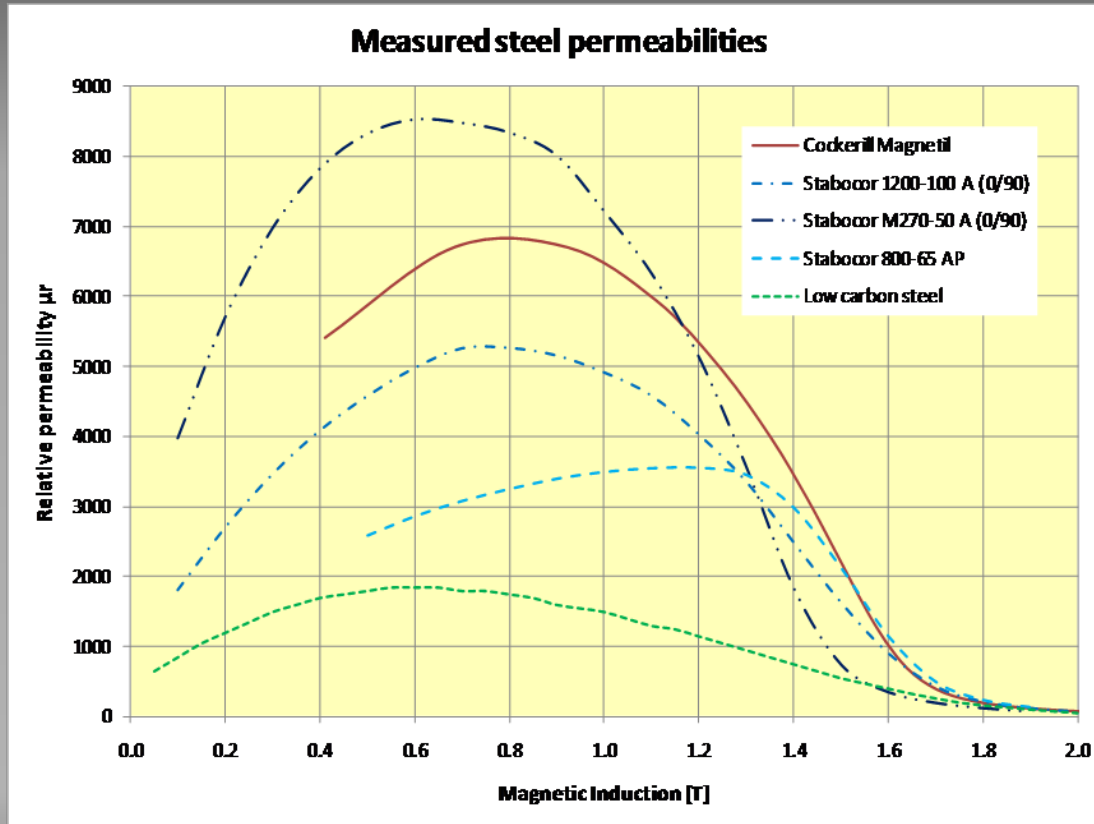
**Today's standard: cold rolled, non-oriented electro-steel sheets (EN 10106)**

- Magnetic and mechanical properties can be adjusted by final annealing
- Reproducible steel quality even over large productions
- Magnetic properties (permeability, coercivity) within small tolerances
- Homogeneity and reproducibility among the magnets of a series can be enhanced by selection, sorting or shuffling
- Organic or inorganic coating for insulation and bonding
- Material is usually cheaper, but laminated yokes are labour intensive and require more expensive toolings (fine blanking, stacking)

For more specific details see lecture of S. Sgobba



# NGO electro-steel properties



Sheet thickness:  
 $0.3 \leq t \leq 1.5$  mm

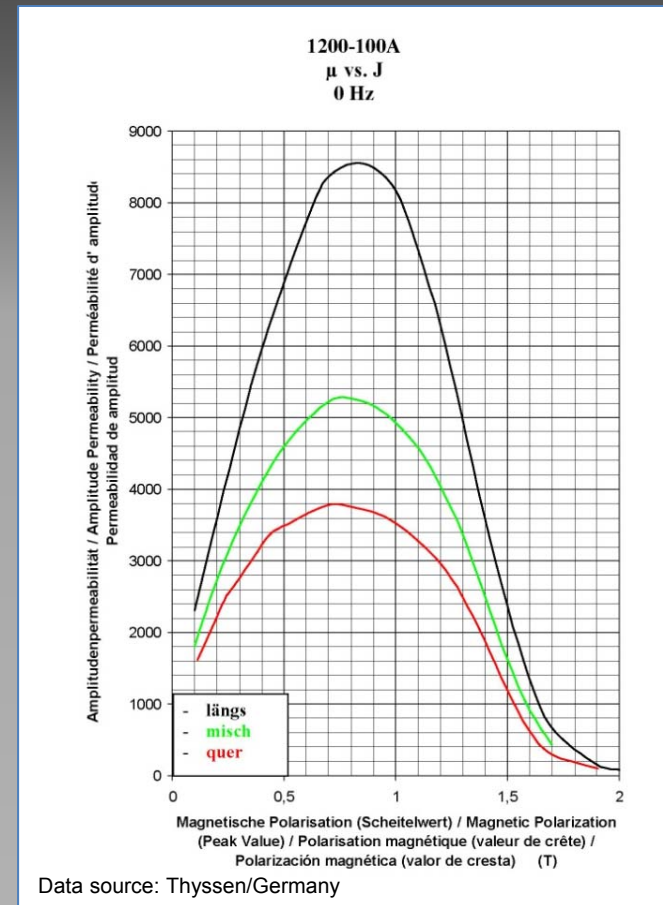
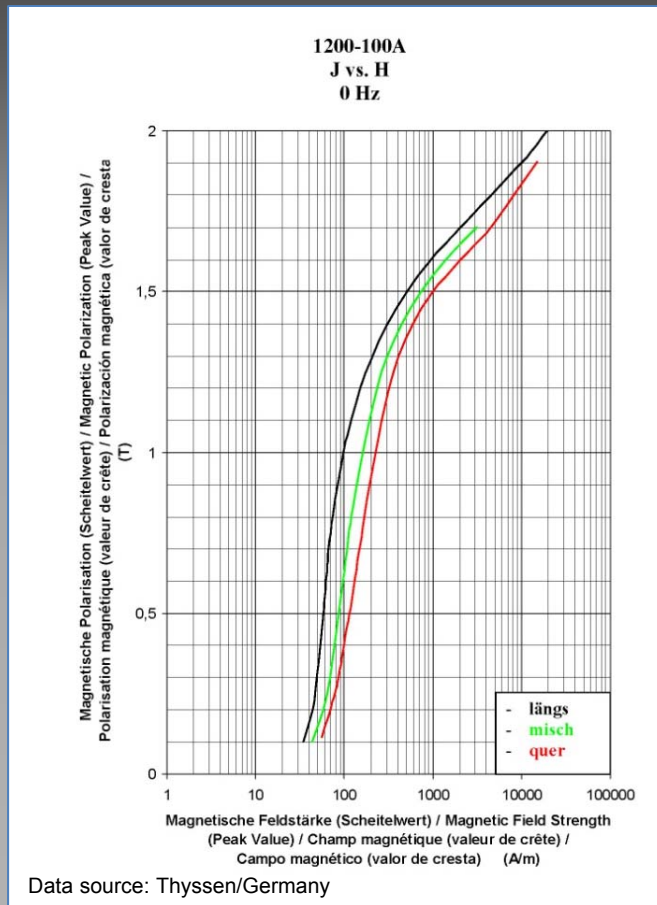
Specific weight:  
 $7.60 \leq \delta \leq 7.85$  g/cm<sup>3</sup>

Coercivity:  
 $H_c < 65$  ( $\pm 10$ ) A/m

Electrical resistivity @20°C:  
 $0.16$  (low Si)  $\leq \rho \leq$   
 $0.61$   $\mu\Omega\text{m}$  (high Si)



# Anisotropy of rolled steel



Anisotropy can be partly cured by final annealing

- $J$ : magnetic polarization [T] according to:  $B = \mu_0 H + J$

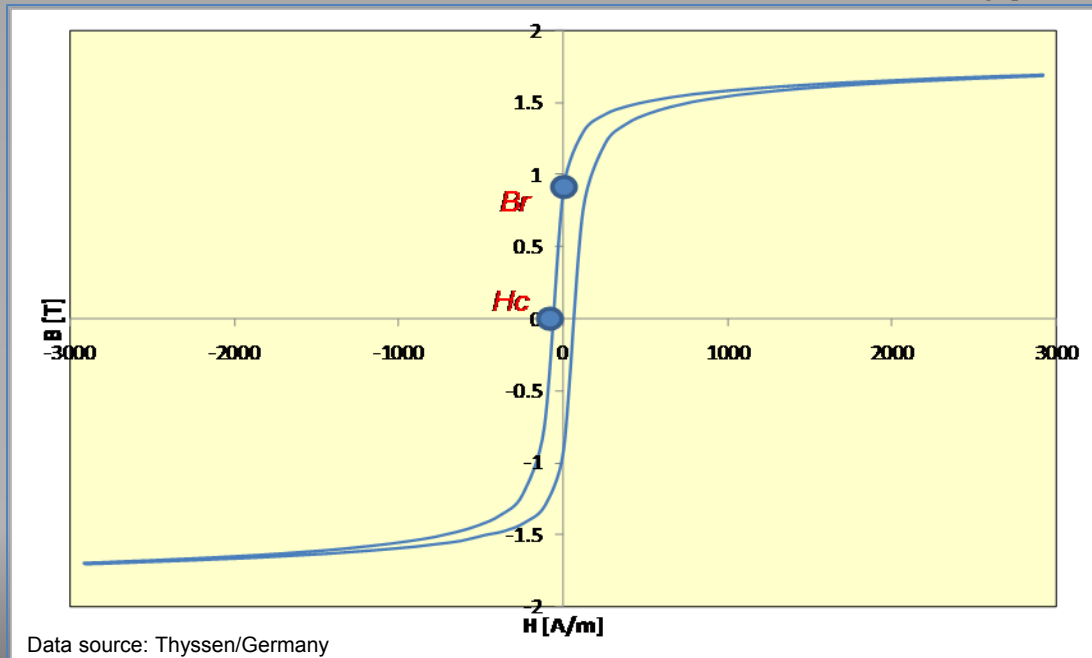


# Remanent field & coercivity

In a continuous ferro-magnetic core (transformer) the residual field is determined by the remanence  $B_r$

In a magnet core with gap, the residual field is determined by  $H_c$

Assuming the coil current  $I=0$ :  $\oint \vec{H} \cdot d\vec{l} = \int_{gap} \vec{H}_{gap} \cdot d\vec{l} + \int_{yoke} \vec{H}_c \cdot d\vec{l} = 0$

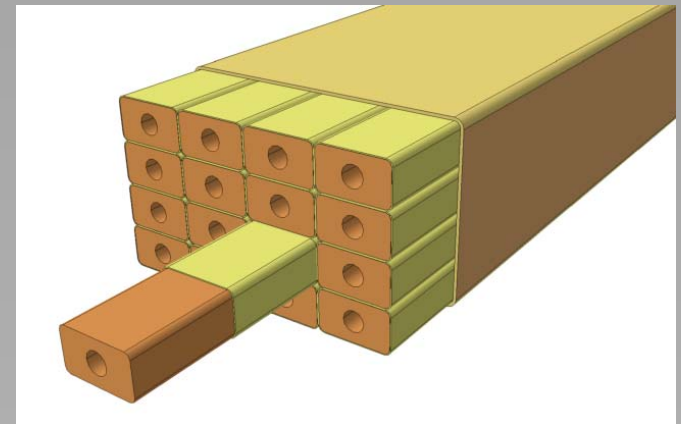
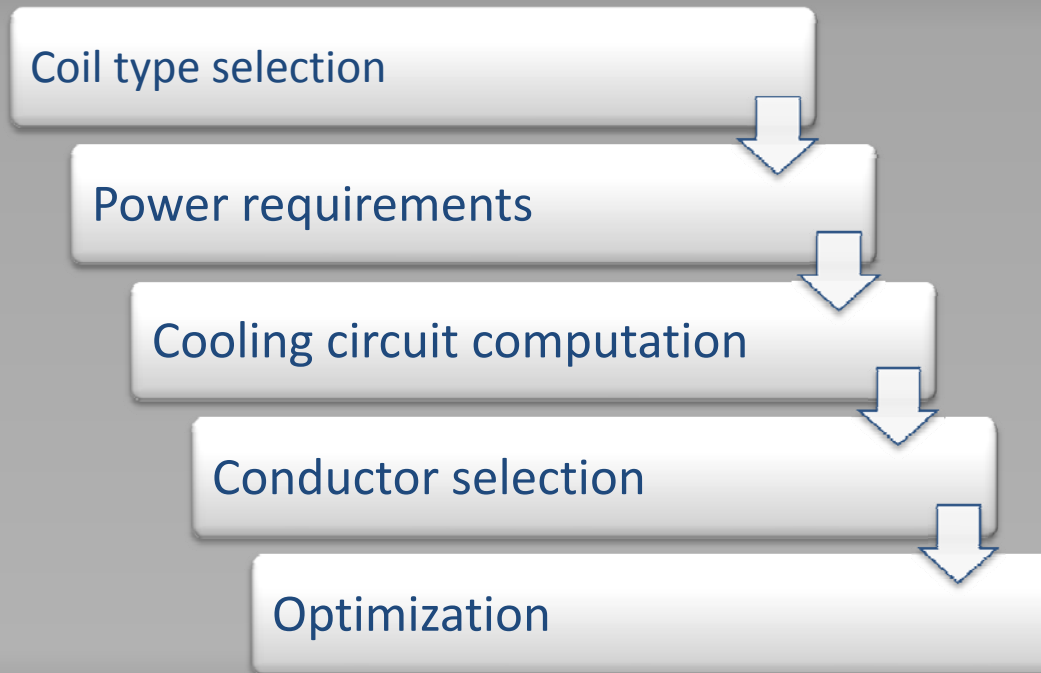


$$B_{residual} = -\mu_0 H_c \frac{\lambda}{g} \quad (11^*)$$



# Coil design

Ampere-turns  $NI$  are determined, but the current density  $j$ , the number of turns  $N$  and the coil cross section need to be decided

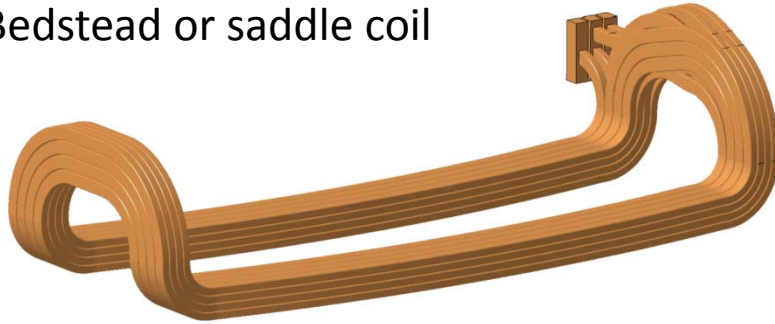




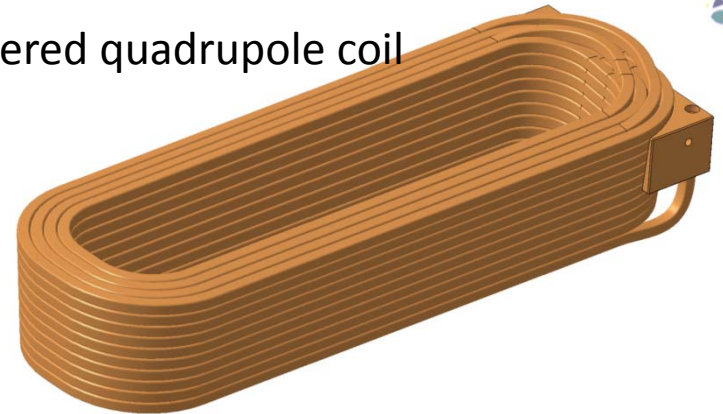


# Standard coil types

Bedstead or saddle coil



Tapered quadrupole coil



Racetrack coil





# Power requirements

Assuming the magnet cross-section and the yoke length are known, one can calculate the total dissipated power per magnet:

$$P_{dipole} = \rho \frac{Bh}{\eta\mu_0} j l_{avg} \quad (12^*)$$

$$P_{quadrupole} = 2\rho \frac{B' r^2}{\eta\mu_0} j l_{avg} \quad (13^*)$$

$$P_{sextupole} = \rho \frac{B'' r^3}{\eta\mu_0} j l_{avg} \quad (14^*)$$

- $j$ : current density [A/m<sup>2</sup>]:  $j = \frac{NI}{f_c A} = \frac{I}{a_{cond}}$  (15)
- $\rho$ : resistivity [ $\Omega\text{m}$ ] (for copper:  $1.86 \cdot 10^{-8} \Omega\text{m}$  @ 40°C)
- $l_{avg}$ : average turn length [m]; approximation:  $2.5 l_{iron} < l_{avg} < 3 l_{iron}$  for racetrack coils
- $a_{cond}$ : conductor cross section [m<sup>2</sup>]
- $A$ : coil cross section [m<sup>2</sup>]
- $f_c$ : filling factor =  $\frac{\text{net conductor area}}{\text{coil cross section}}$  (geometric filling factor, insulation, cooling duct, edge rounding)

**Note:** for a constant geometry, the power loss  $P$  is proportional to the current density  $j$ .



# Number of turns

Basic relations:  $R_{magnet} \propto N^2 j$        $V_{magnet} \propto Nj$        $P_{magnet} \propto j$

The determined power can be divided into voltage and current:  $P = UI$

The number of turns  $N$  are chosen to match the impedances of the power converter and connections:

## Large $N$ = low current = high voltage

- Small terminals
- Small conductor cross-section
- Thick insulation for coils and cables
- Less good filling factor in the coils
- Large coil volume
- Low power transmission loss

## Small $N$ = high current = low voltage

- Large terminals
- Large conductor cross-section
- Thin insulation in coils and cables
- Good filling factor in the coils
- Small coil volume
- High power transmission loss



# Coil examples

## Dipole

- # of turns  $N_{(\text{per pole})}$ : 16
- Current: 3000 A
- Voltage: 100 V

## Quadrupole

- # of turns  $N_{(\text{per pole})}$ : 20
- Current: 650 A
- Voltage: 12 V

## Sextupole

- # of turns  $N_{(\text{per pole})}$ : 14
- Current: 650 A
- Voltage: 16 V

## Corrector

- # of turns  $N_{(\text{per pole})}$ : 240/96
- Current: 15/30 A
- Voltage: 7/3 V



# Air cooling

## Air cooling by natural convection:

- Current density:
  - $j \leq 2 \text{ A/mm}^2$  for small, thin coils
  - $j \leq 1 \text{ A/mm}^2$  for large, captured coils
- Difficult to calculate analytically
- Numerical computations required to get reasonable results
- Round, rectangular or square conductor
  - Filling factor: 0.63 (round) to 0.8 (rectangular)
- Conductor pre-impregnated with varnish ( $0.02 \leq t \leq 0.1 \text{ mm}$ ) or half-lapped polyimide (Kapton®) tape ( $0.1 \leq t \leq 0.2 \text{ mm}$ )
- Outer coil insulation: epoxy impregnated glass fibre tape



## Cooling enhancement:

- Heat sink with enlarged radiation surface
- Forced air flow (cooling fan)

Only for magnets with limited strength (correctors, steering magnets....)



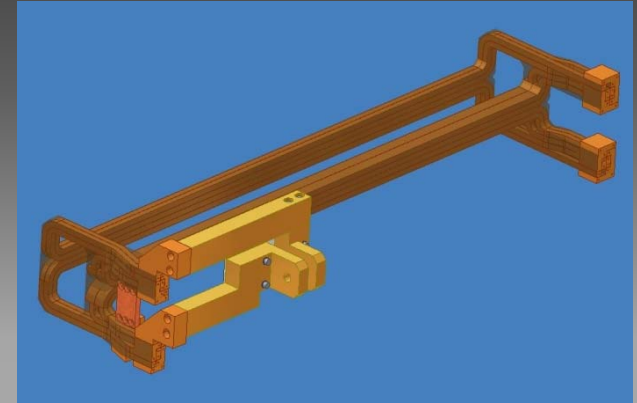
# Water cooling

## Direct water cooling:

- Current density typically up to  $j = 10 \text{ A/mm}^2$
- $j = 80 \text{ A/mm}^2$  have been realized, but difficult and risky (single turn cooling)
- Rectangular or square copper (or aluminium) conductor with central cooling duct for demineralised water
- Inter-turn and ground insulation: one or more layers of half-lapped epoxy impregnated glass fibre tape
- Inter-turn insulation thickness:  $0.3 \leq t \leq 1.0 \text{ mm}$
- Ground insulation thickness:  $0.5 \leq t \leq 3.0 \text{ mm}$

## Indirect water cooling:

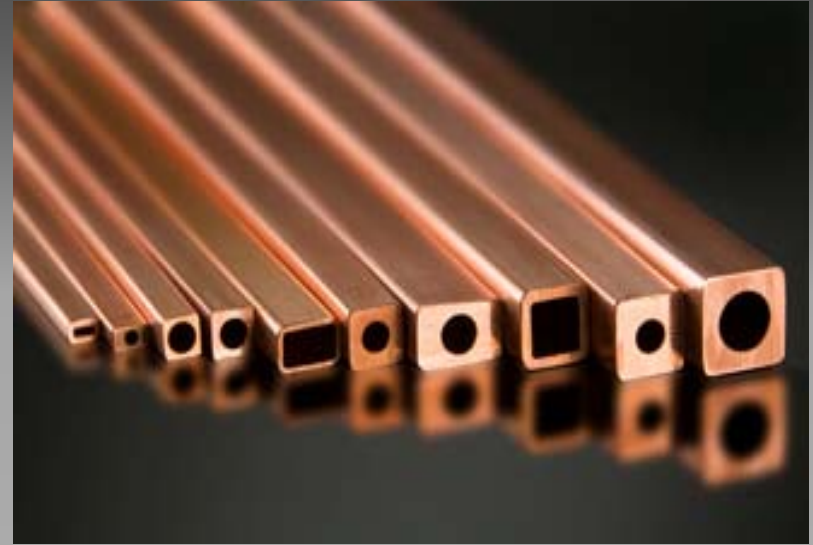
- Current density  $j \leq 2 \text{ A/mm}^2$
- Tap water can be used



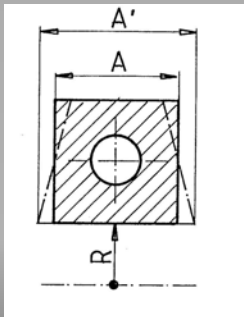


# Conductor materials

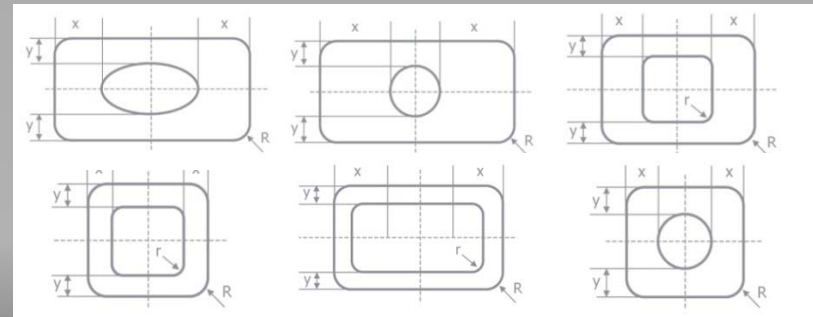
	Al	Cu (OF)
Purity	99.7 %	99.95 %
Resistivity @ 20°C	2.83 $\mu\Omega$ cm	1.72 $\mu\Omega$ cm
Thermal resistivity coeff.	0.004 K <sup>-1</sup>	0.004 K <sup>-1</sup>
Specific weight	2.70 g/cm <sup>3</sup>	8.94 g/cm <sup>3</sup>
Thermal conductivity	2.37 W/cm K	3.91 W/cm K
Price	4.7 Euros/kg	11 Euros/kg



**Keystoning:** risk of insulation damage & decrease of cooling duct cross section



$$R = 3 \cdot A \Rightarrow \frac{\Delta A}{A} = 3.6\%$$



Details on coil insulation materials see lecture of D. Tommasini



# Cooling parameters

## Recommendations and canonical values:

- Water cooling:  $2 \text{ A/mm}^2 \leq j \leq 10 \text{ A/mm}^2$
- Pressure drop:  $0.1 \leq \Delta p \leq 1.0 \text{ MPa}$  (possible up to 2.0 MPa)
- Low pressure drop might lead to more complex and expensive coil design
- Flow velocity should be high enough so flow is turbulent
- Flow velocity  $u_{av} \leq 5 \text{ m/s}$  to avoid erosion and vibrations
- Acceptable temperature rise:  $\Delta T \leq 30^\circ\text{C}$
- For advanced stability:  $\Delta T \leq 15^\circ\text{C}$

## Assuming:

- Long, straight and smooth pipes without perturbations
- Turbulent flow = high Reynolds number
- Good heat transfer from conductor to cooling medium
- Temperature of inner conductor surface equal to coolant temperature
- Isothermal conductor cross section





# Cooling parameters

Pressure drop through a water circuit: 
$$\Delta p = f \frac{l}{d} \frac{\delta u_{av}^2}{2} \quad (16)$$

- $p$ : pressure [Pa, N/m<sup>2</sup>]
- $f$ : friction factor [.]
- $l, d$ : cooling circuit length and diameter [m]
- $\delta$ : coolant mass density [kg/m<sup>3</sup>] (for water: 1000 kg/m<sup>3</sup> = 1 kg/liter)
- $u_{avg}$ : average coolant velocity [m/s]

Friction factor  $f$  depends on the Reynolds number  $Re$  
$$Re = \frac{u_{avg} d}{\nu} \quad (17)$$

Laminar flow:  $Re < 2000$  and  $f = 64/Re$

- $\nu$ : kinematic viscosity of coolant is temperature depending, for simplification it is assumed to be constant ( $9.85 \cdot 10^{-7}$  m<sup>2</sup>/s @ 21°C for water)

Turbulent flow:  $Re > 4000$  and  $f$  is transcendental:

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left( \frac{\varepsilon}{3.7d} + \frac{2.51}{Re \sqrt{f}} \right) \quad (18)$$

- $\varepsilon$ : roughness of cooling channels ( $\sim 1.5 \cdot 10^{-3}$  mm)



# Cooling parameters

Velocity and friction factor using  $Re(u_{avg}) \rightarrow u_{avg}$  to be solved iteratively:

$$u_{avg} = \sqrt{\frac{2\Delta p d}{\delta f l}} \quad (19)$$

$$Re = \frac{d}{\nu} \sqrt{\frac{2\Delta p d}{\delta f l}} \quad (20)$$

Substituting  $Re$  in (18) with (20) leads to:

$$u_{avg} = -2 \sqrt{\frac{2\Delta p d}{\delta l}} \log_{10} \left( \frac{\varepsilon}{3.7d} + \frac{2.51}{\frac{d}{\nu} \sqrt{\frac{2\Delta p d}{\delta l}}} \right) \quad (21)$$

Simplified approach using water as cooling fluid:

$$u_{avg} \approx 0.3926 \cdot d^{0.714} \left( \frac{\Delta p}{l} \right)^{0.571} \quad (22^*)$$



# Cooling parameters

Heat absorbed by coolant medium across a heated surface:

$$P = \dot{m} c_p \Delta T \quad \dot{m} = \delta Q$$

- $c_p$ : heat capacity [W s/kg °C] (4.19 kW s/kg °C for water)
- $Q$ : flow rate [liter/s]
- $P$ : power [W]
- $\Delta T$ : temperature increase [°C]

Flow  $Q$  necessary to remove heat  $P$ :  $Q = \frac{P}{\delta c_p \Delta T} \quad Q_{\text{water}} = 0.2388 \frac{P}{\Delta T} \quad (22)$

Coolant flow inside a round tube with a bore diameter  $d$ :  $Q = u_{\text{av}} \frac{\pi d^2}{4} 10^3 \quad (23)$

Temperature increase using water as cooling fluid:  $\Delta T = 0.304 \frac{P}{u_{\text{avg}} d^2} \cdot 10^{-6} \quad (24)$



# Cooling parameters

Number of cooling circuits per coil:  $\Delta p \propto \frac{1}{K_w^3}$

→ Doubling the number of cooling circuits reduces the pressure drop by a factor of eight for a constant flow

Diameter of cooling channel:  $\Delta p \propto \frac{1}{d^5}$

→ Increasing the cooling channel by a small factor can reduce the required pressure drop significantly



# Cooling circuit design

Already determined: current density  $j$ , power  $P$ , current  $I$ , # of turns  $N$

1. Select # of layers  $m$  and # of turns per layer  $n$
2. Round up  $N$  if necessary to get reasonable  $m$  and  $n$
3. Define coil height  $c$  and coil width  $b$ :  $A=bc=\frac{NI}{jf_c}$  (Aspect ratio  $c : b$  between 1 : 1 and 1 : 1.7 and  $0.6 \leq f_c \leq 0.8$ )
4. Calculate  $l_{avg}$  = pole perimeter + 8 x clearance + 4 x coil width
5. Start with single cooling circuit per coil:  $l = \frac{K_c N l_{avg}}{K_w}$  (25)
6. Select  $\Delta T$ ,  $\Delta p$  and calculate cooling hole diameter  $d$ :  $d = 5.59 \cdot 10^{-3} \left( \frac{P}{\Delta T K_w} \right)^{0.368} \left( \frac{l}{\Delta p} \right)^{0.21}$  (26\*)
7. Change  $\Delta p$  or number of cooling circuits, if necessary
8. Determine conductor area  $a$ :  $a = \frac{I}{j} + \frac{d^2 \pi}{4} + r_{edge} (4 - \pi)$  (27)
9. Select conductor dimensions and insulation thickness
10. Verify if resulting coil dimensions,  $N$ ,  $I$ ,  $V$ ,  $\Delta T$  are still compatible with the initial requirements (if not, start new iteration)
11. Compute coolant velocity and coolant flow using (21) and (22)
12. Verify if Reynolds number is inside turbulent range ( $Re > 4000$ ) using (17)



# Cooling water properties

## Water properties:

- For the cooling of hollow conductor coils demineralised water is used (exception: indirect cooled coils)
- Water quality essential for the performance and the reliability of the coil (corrosion, erosion, short circuits)
- Resistivity  $> 0.1 \times 10^6 \Omega\text{m}$
- pH between 6 and 6.5
- Dissolved oxygen below 0.1 ppm
- Filters to remove particles, loose deposits and grease to avoid cooling duct obstruction



# Stored energy

Stored energy in a magnet depends on (non-uniform) field distribution in the gap, coils, and iron yoke

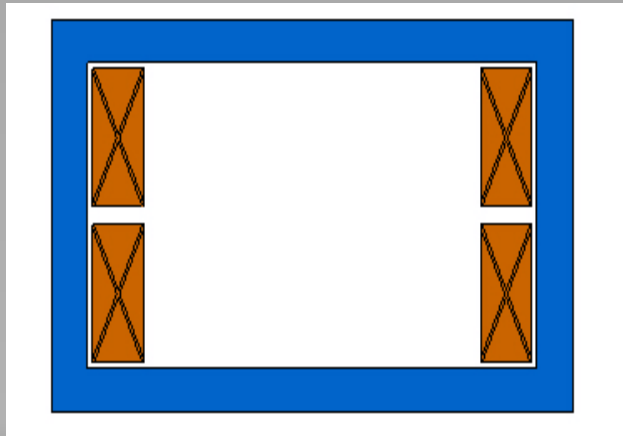
- $U$ : stored energy [J, joules]:

$$U = \frac{1}{2} \int_V BH \cdot dv \quad (28^*)$$

In general, difficult to calculate analytically

- usually done by numerical computations

For a window frame magnet with constant field in the gap:



$$U_{gap} = \frac{B^2}{2\mu_0} V_{gap} \quad U_{coil} = \frac{B^2}{6\mu_0} V_{coil} \quad U_{yoke} = \frac{B^2}{2\mu_r\mu_0} V_{yoke}$$

$$U_{magnet} = U_{gap} + 2U_{coil} + U_{yoke} = \frac{B^2}{2\mu_0} \left( V_{gap} + 2\frac{V_{coil}}{6} + \frac{1}{\mu_r} V_{yoke} \right) \quad (29^*)$$



# Inductance

The inductance of a magnet is given by:  $L = \frac{2U}{I^2}$  (30)

- $L$ : Inductance [H]

- Total voltage on a pulsed magnet:

$$V_{tot} = RI + L \frac{dI}{dt} = RI + \frac{2U}{I^2} \frac{dI}{dt} \quad (31)$$

- Total voltage on a magnet cycled with  $I = I_0 \sin(\omega t)$  :

$$V_{tot} = RI_0 \sin(\omega t + \varphi) \quad (32)$$

$$\varphi = \tan^{-1} \frac{L\omega}{R} \quad (33)$$





# Cost estimate example

## Production specific toolings:

5 to 15 k€/tool

## Material:

Steel sheets: 1.5 € /kg

Copper conductor: 11 to 13 € /kg

## Yoke manufacturing:

Dipoles: 6 to 10 € /kg (> 1000 kg)

Quads/Sextupoles: 50 to 80 € /kg (> 200 kg)

Small magnets: up to 300 € /kg

## Coil manufacturing:

Dipoles: 30 to 50 € /kg (> 200 kg)

Quads/Sextupoles: 65 to 80 € /kg (> 30 kg)

Small magnets: up to 300 € /kg

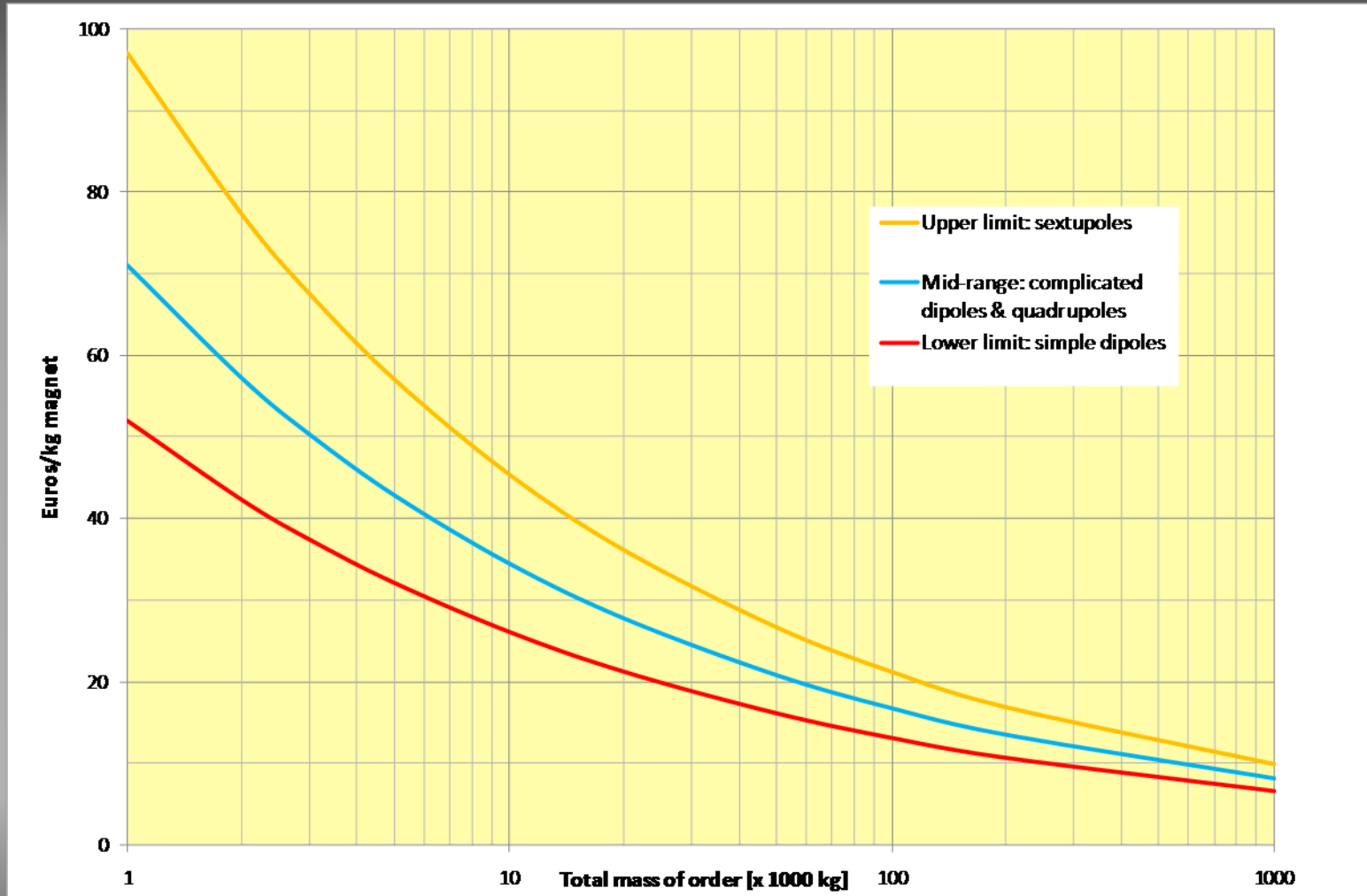
## Contingency: 10 to 20 %

<b>Magnet</b>	<b>Magnet type</b>	<b>Dipole</b>
	Number of magnets (incl. spares)	18
	Total mass/magnet	8330 kg
<b>Fixed costs</b>	Design	14 kEuros
	Punching die	12 kEuros
	Stacking tool	15 kEuros
	Winding/molding tool	30 kEuros
<b>Yoke</b>	Yoke mass/magnet	7600 kg
	Used steel (incl. blends)/magnet	10000 kg
	Yoke manufacturing costs	8 Euros/kg
	Steel costs	1.5 Euros/kg
<b>Coil</b>	Coil mass/magnet	730 kg
	Coil manufacturing costs	50 Euros/kg
	Cooper costs (incl. insulation)	12 Euros/kg
<b>Total costs</b>	Total order mass	150 Tonnes
	Total fixed costs	71 kEuros
	Total Material costs	428 kEuros
	Total manufacturing costs	1751 kEuros
	Total magnet costs	2250 kEuros
	Contingency	20 %
	<b>Total overall costs</b>	<b>2700 kEuros</b>

**NOT included:** magnetic design, supports, cables, water connections, alignment equipment, magnetic measurements, transport, installation  
Prices for 2009



# Cost estimate



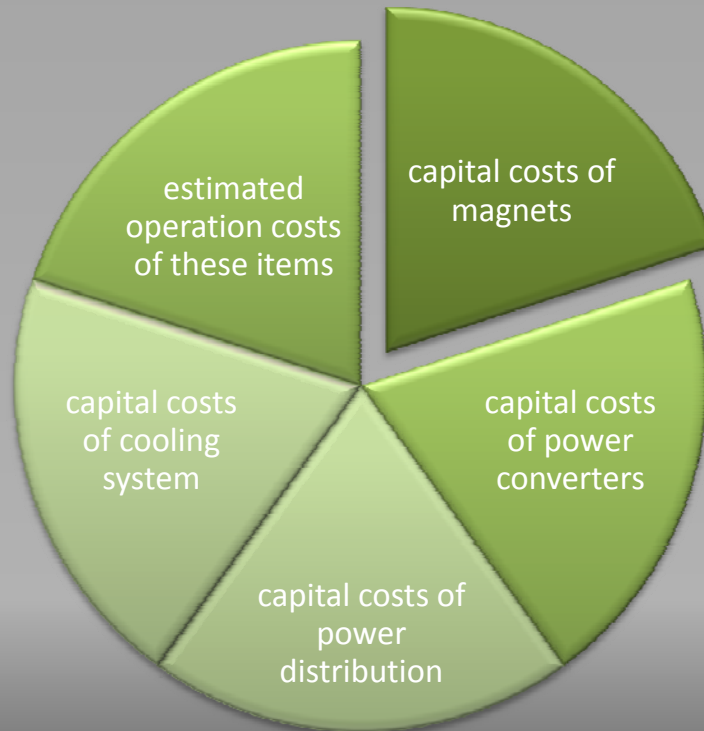


# Cost optimization

*Focus on economic design!*

**Design goal:** Minimum total costs over projected magnet life time by optimization of capital (investment) costs against running costs (power consumption)

Total costs include:





# Cost optimization

Optimization procedure for dc magnets [CERN/SI/Int. DL/70-10]

For the example case:

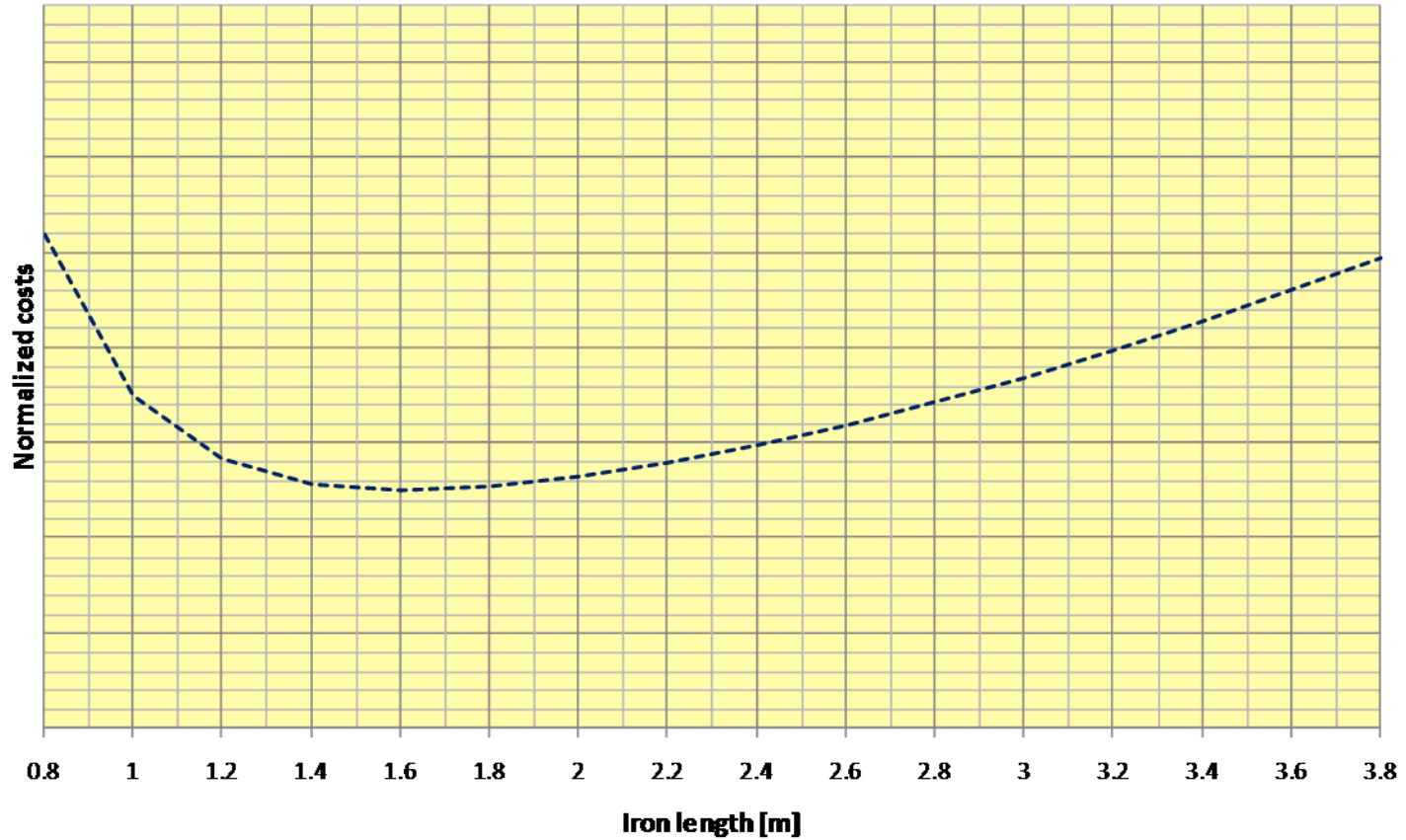
- Scaling parameters: magnet length and current density
- Power is important - it enters into capital costs and running costs
- Magnetic fluxes should be between 1.2 and 1.7 T
- Minima of  $j$  between 3 and 5 A/mm<sup>2</sup>
- Optimum yoke length around 1.6 m
- Power converter cost can be minimized by running several magnets in series
- Costs are presented in **relative numbers!**

**Note:** for other projects, the optima might be different



# Cost optimization

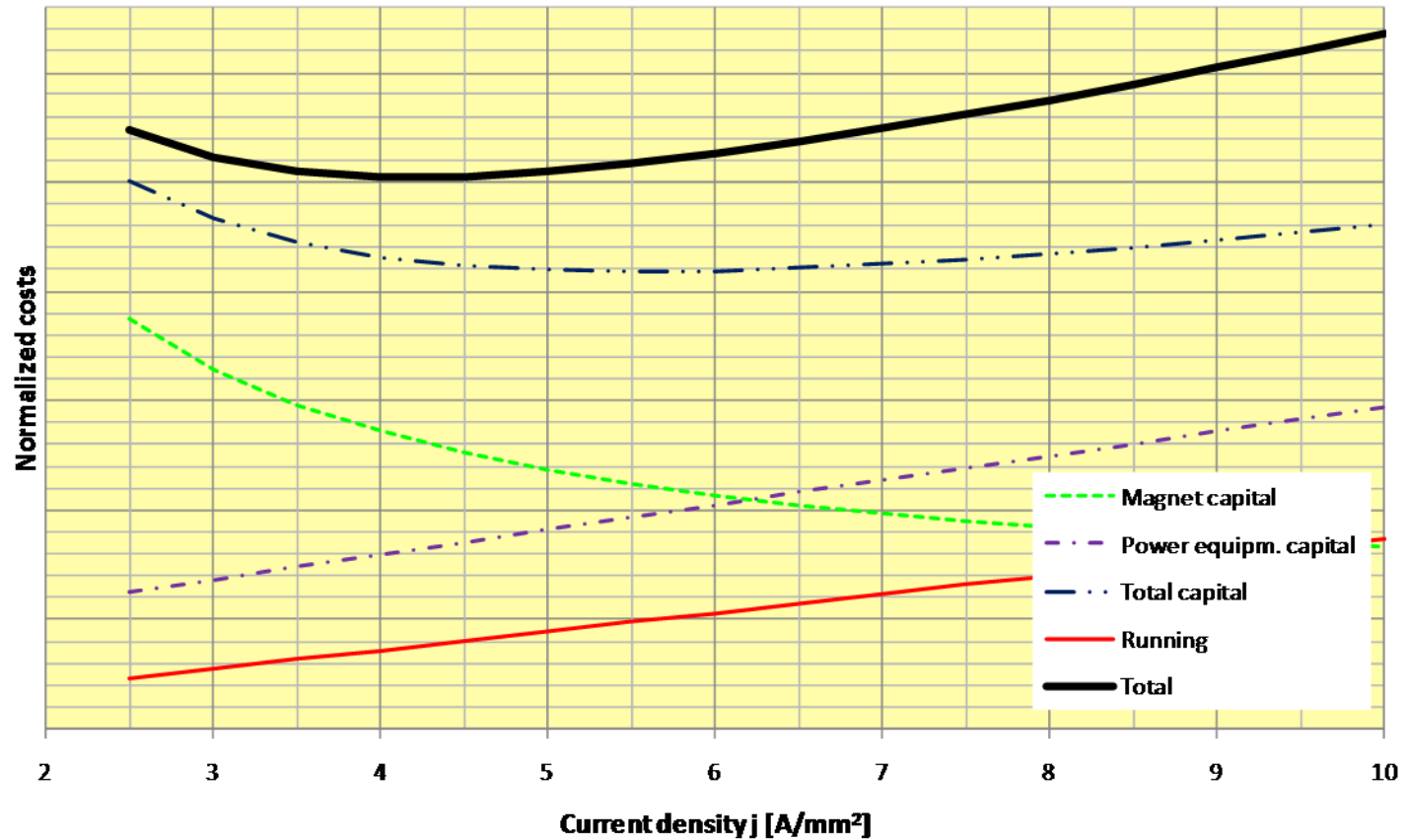
## Magnet costs (iron length)





# Cost optimization

## Investment vs running costs





# Cost optimization

