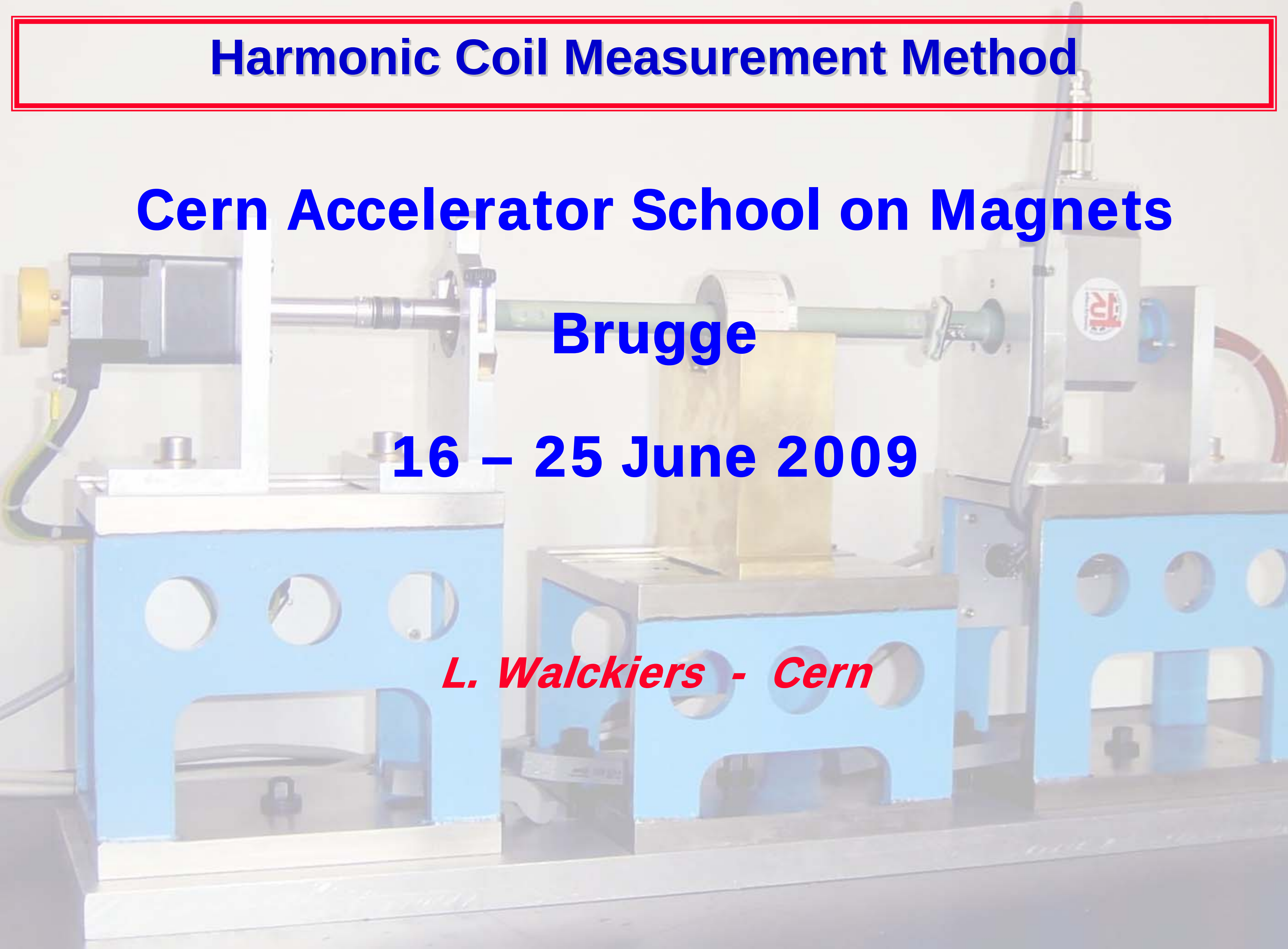
Harmonic Coil Measurement Method



Cern Accelerator School on Magnets

Brugge

16 – 25 June 2009

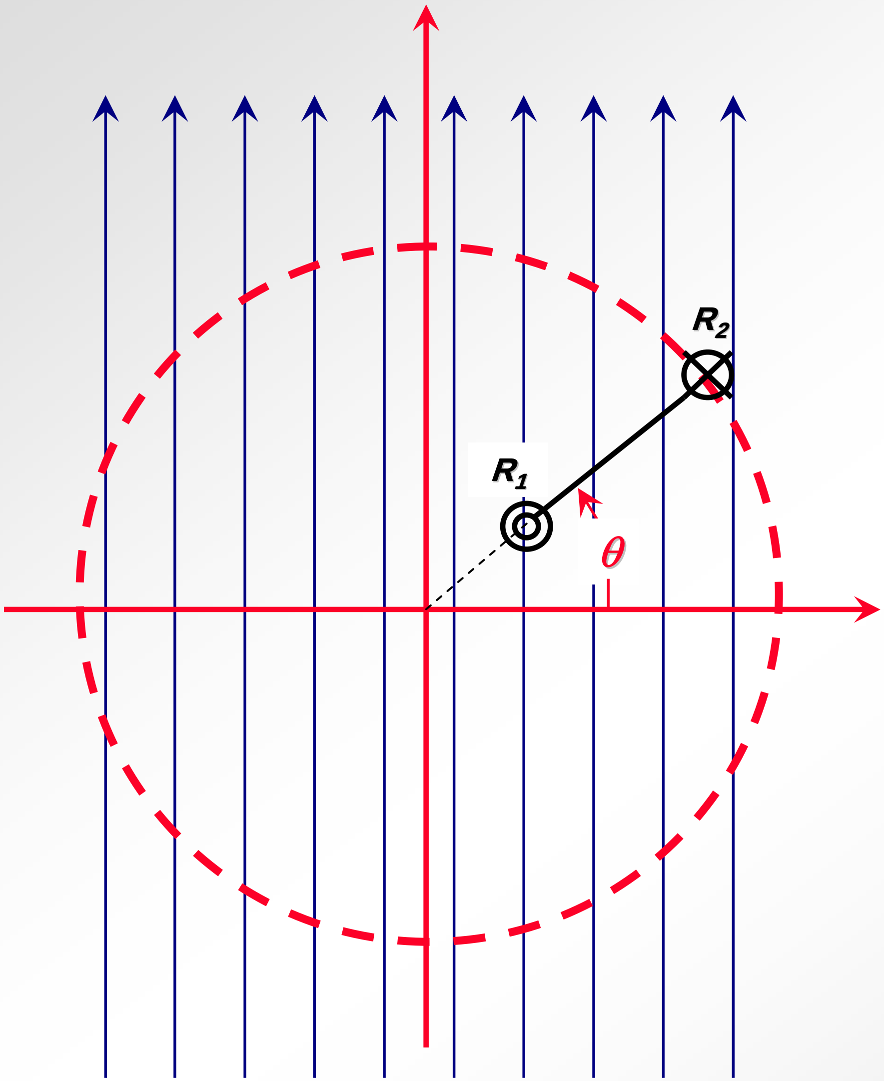
L. Walckiers - Cern

Harmonic Coil Measurement

Outline

- Basic Equations for 2D Field & Multipoles
- Harmonic Coil Measurement
- Measure the Field Direction & Quadrupole Axis
- Accuracy issues & possible improvements
    - Voltage Integrator
    - Angle Encoder & torsional stiffness
    - Imperfection in rotation & shaft rigidity
- Calculate the Coil factors
- Measure multipoles in pulsed magnets
- Pro's & Con's of rotating coil measurement

Flux seen by a (simple) rotating coil

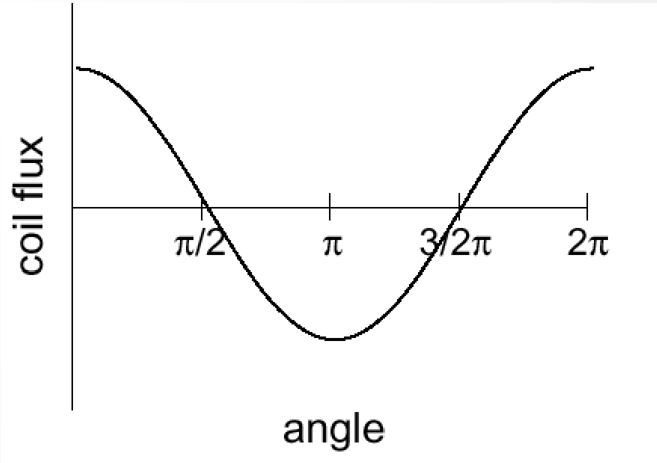


Flux picked by a measuring coil rotating in a dipole field

$$\Psi(\theta) = N_t \cdot L \cdot \int_{R_1}^{R_2} B_1 \cos(\theta) \cdot d\theta$$

With Nt = number of turns

L = Length of the measuring coil



Basic Equations for Field & Multipoles Description

$$B(z) = \sum_{1}^{N(=\infty)} C_n \cdot \left(\frac{z}{R_r} \right)^{n-1} \quad \text{with} \quad C_n = B_n + iA_n$$

$$z = x + i \cdot y$$

C_n are in Tesla at reference radius R_r

Often in use to describe high order multipoles :
units = errors relative to the main harmonic B_N
at reference radius $\mathrm{R_r}$

$$c_n = b_n + ia_n = 10^4 \frac{C_n}{B_N}$$

Why a Reference Radius ?

$$B(z) = \sum_{1}^{N(=\infty)} C_n \cdot \left(\frac{z}{R_r} \right)^{n-1} \quad \text{with} \quad C_n = B_n + iA_n$$

The reference radius R_r in practice corresponds to :

- 2/3 of the yoke aperture in resistive magnets
- " coil " superconducting magnets
- Useful aperture for the beam
- Radius when the multipoles relative to main field have same order of magnitude

- Choose carefully your reference radius
- Measure with $R_{meas} > R_{ref}$

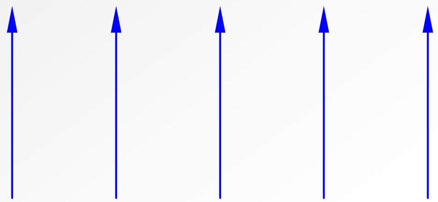
$$c_n = b_n + ia_n = 10^4 \frac{C_n}{B_n}$$

Main Field Components

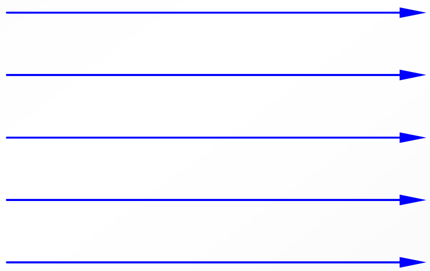
$$\mathbf{C}_n = B_n + iA_n$$

n=1

$B_1 \neq 0$, normal dipole

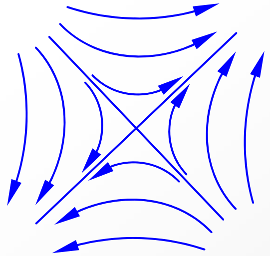


$A_1 \neq 0$, skew dipole

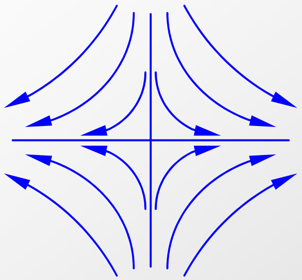


n=2

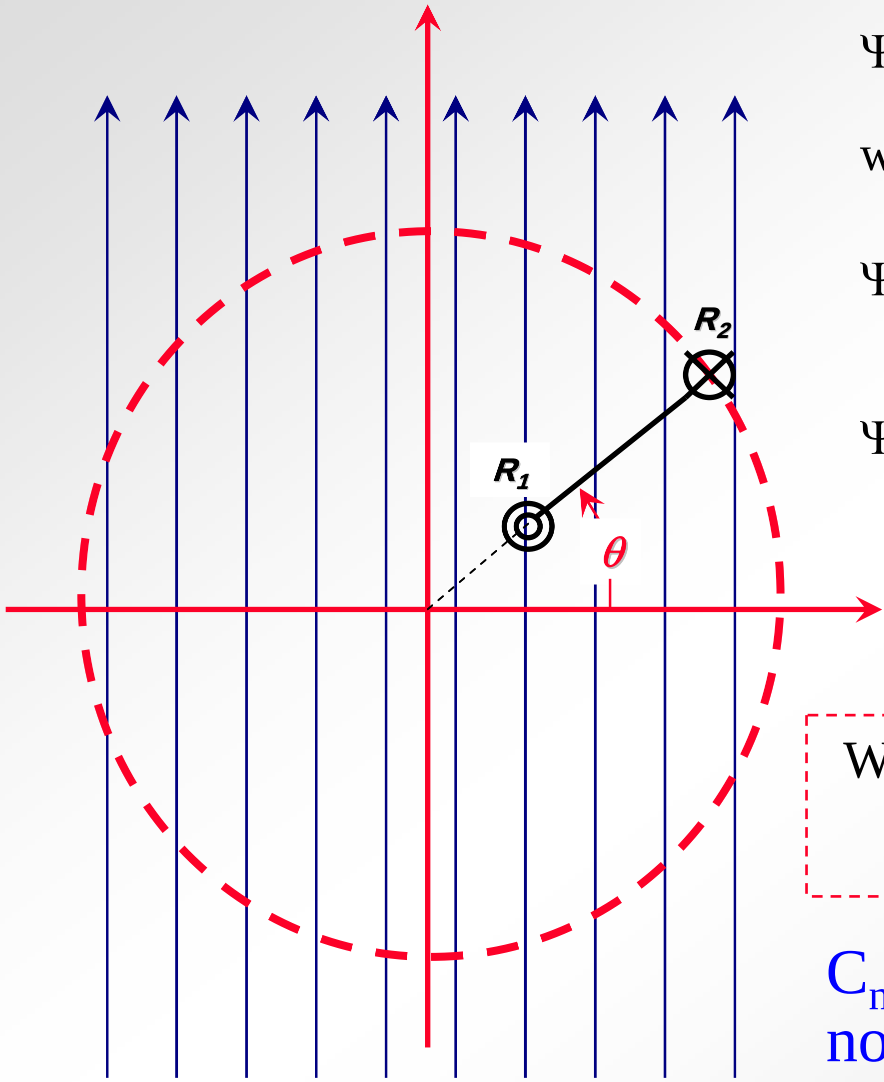
$B_2 \neq 0$, normal quadrupole



$A_2 \neq 0$, skew quadrupole



Simple rotating coil – Any Field



$$\Psi(z) = N_t \cdot L \cdot \mathrm{Re}\int_{R_1}^{R_2} B(z)\cdot dz$$

with $z = x + i\cdot y = R\cdot e^{i\theta}$

$$\Psi(z) = N_t \cdot L \cdot \mathrm{Re}\int_{R_1}^{R_2} \sum_{1}^{N(=\infty)} C_n \cdot \left(\frac{z}{R_r}\right)^{n-1} \cdot dz$$

$$\Psi(\theta = \omega\cdot t) = \mathrm{Re}\left(\sum_{1}^{N(=\infty)} \boxed{N_t \cdot L \cdot \frac{(R_2^n - R_1^n)}{n\cdot R_r^{n-1}}} \cdot C_n \cdot e^{in\theta}\right)$$

Coil $[K_n]$ Field Time

Dependence

With a Fourier Analysis of Ψ(θ):

$$\Psi_n = K_n \cdot C_n = K_n \cdot (B_n + i\cdot A_n)$$

C_n , K_n , Ψ_n , are 2D complex numbers

normal $[B_y(x)]$ & skew $[B_x(x)]$ terms

A real bench with a permanent magnet

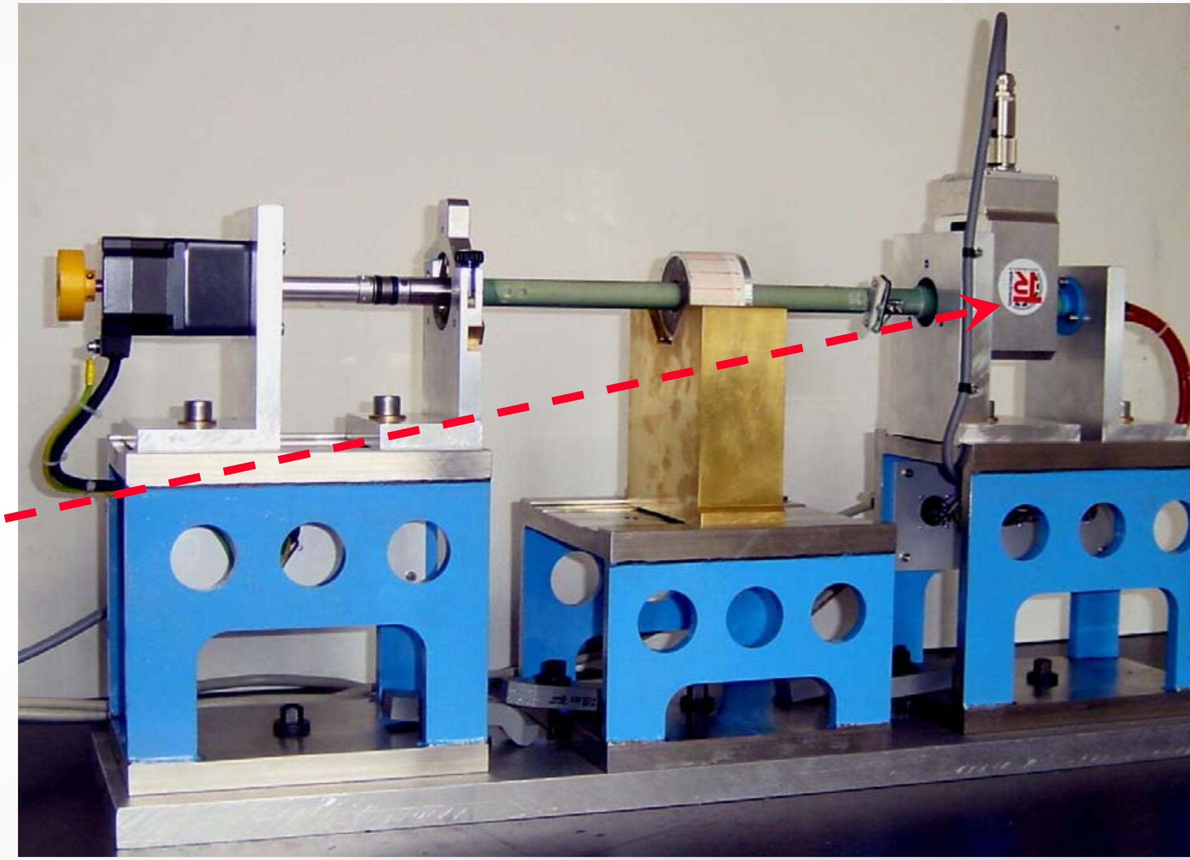
$$\Psi(\theta) = \mathrm{Re}\left(\sum_{1}^{N(=\infty)} N_t \cdot L \cdot \frac{(R_2^n - R_1^n)}{n \cdot R_r^{n-1}} \cdot C_n \cdot e^{in\theta(t)} \right)$$

Measured :

$\Psi(\theta_{i+1}) - \Psi(\theta_i)$

Rotating coil connected to a voltage integrator (time disappears)

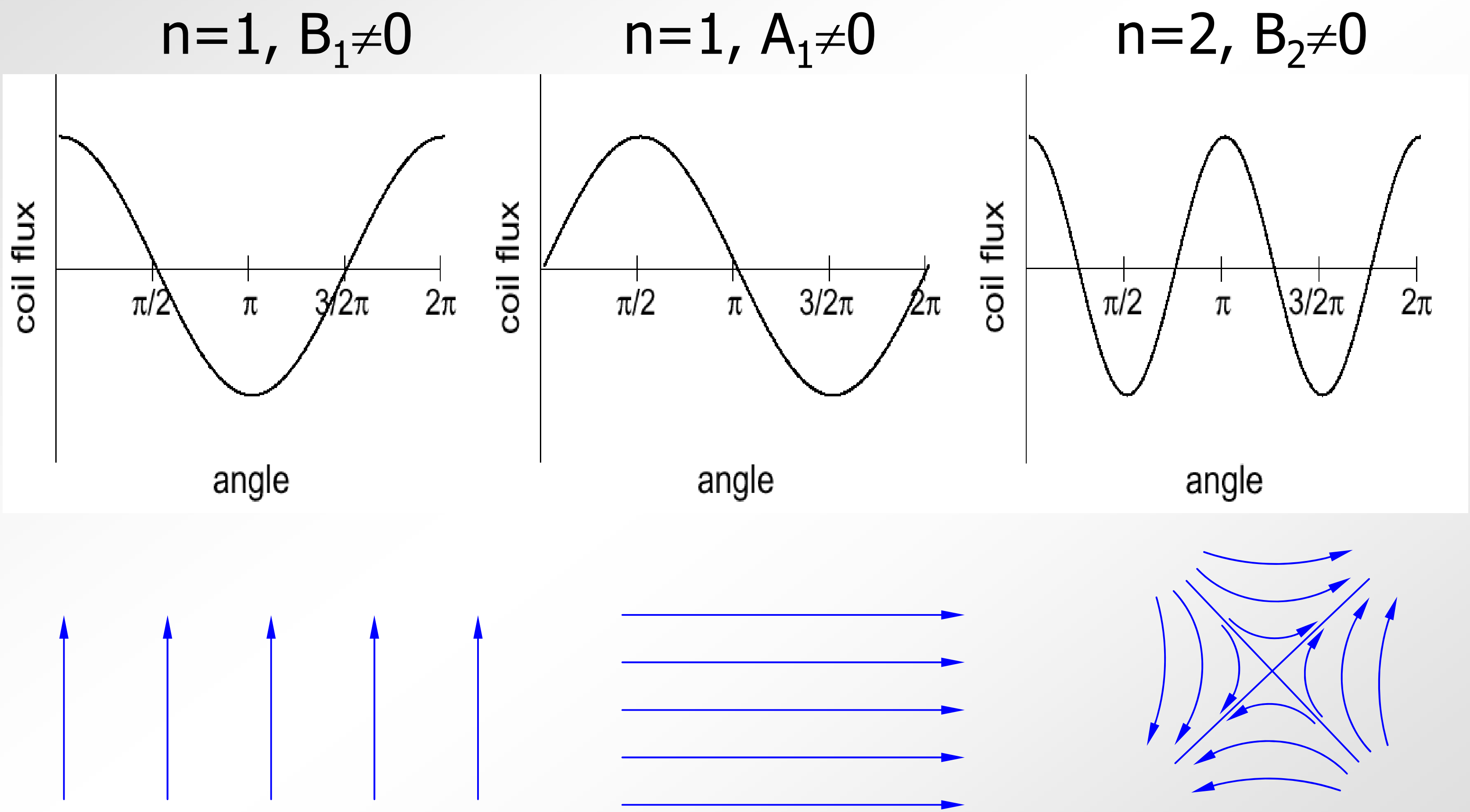
Triggered by an angular encoder



At angles i = 1 .. 2^M

(2^M = 256 or 512 in most cases)

Fourier Analysis of the Flux Ψ seen by a rotating coil



Harmonic Coil Measurement

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- **Measure the Field Direction & Quadrupole Axis**
- Accuracy issues &possible improvements
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Reference Angle misaligned - Measure the Field Direction

Need to go from one reference to the other ?

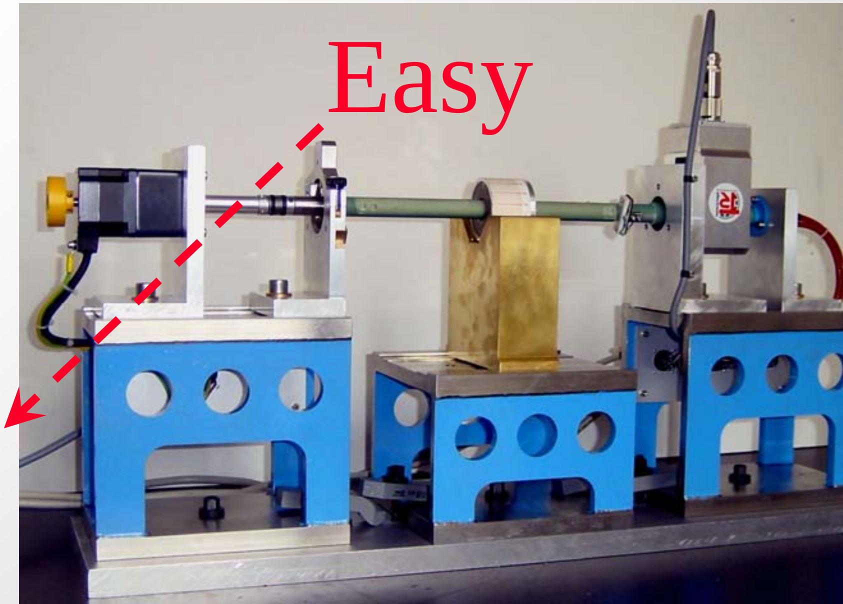
- θ_m reference for the Fourier analysis , zero of the encoder
- θ_g Gravity , magnet fiducials when aligned
- θ_f Field defines vertical A1 = 0 in dipole (A2 in quadrupole)

$$C_n^m = C_n^f \cdot \exp(in(\theta_f - \theta_m))$$

Measure the field Direction :

- Resolution (for A_1/B_1) < 0.1 mrad
- Issue : refer θ_m (encoder) to θ_g (magnet fiducials)
- Issue : calibrate K_1 (coil direction) with θ_m (encoder)

when possible : turn the full system (or the magnet) end to end



Axis misaligned – measure Quadrupole Axis

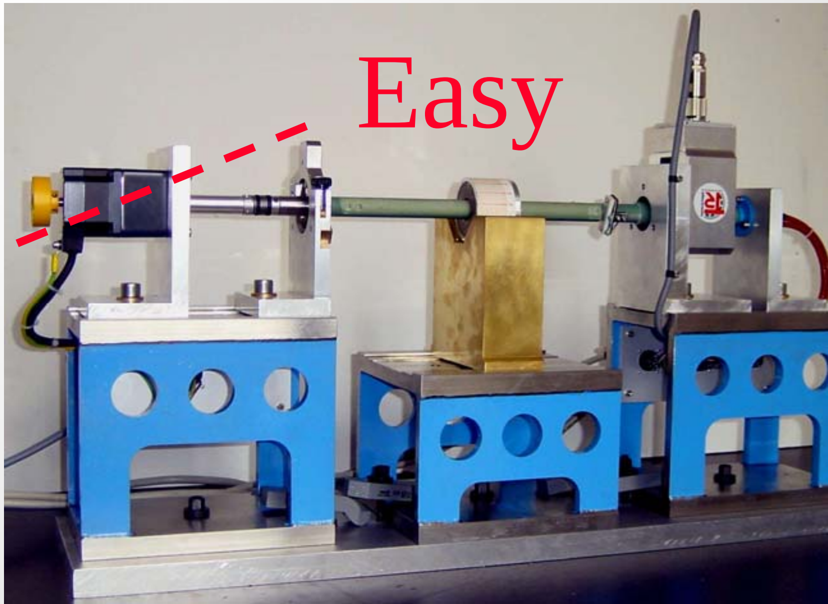
Need to go from one reference (z_c = center of quadrupole) to the other (z_m = rotation axis of the measuring coil) ?

$$\text{with}\quad z_m = z_c - d \cdot R_{ref} \qquad C_n^m = \sum_{k=n}^{\infty} \frac{(k-1)!}{(n-1)!\,(k-n)} C_k^c \cdot d^{k-n}$$

Find quadrupole axis ? $d = -\dfrac{C_1^m}{C_2^m}$

- Resolution (for d) ≈ 0.01 mm
- Issue : refer rotation axis to magnet fiducials

when possible : turn the magnet top to bottom once centred




Sometimes
Impossible

And Useless

Need to center the system ?

For LHC dipoles, measurement axis sometimes 1 to 2 mm from mechanical dipole axis

So need to correct for the feed down from high "allowed" multipoles (b_3, b_5, b_7) to lower ones where we wanted c_n at 10 ppm (0.1 unit) resolution

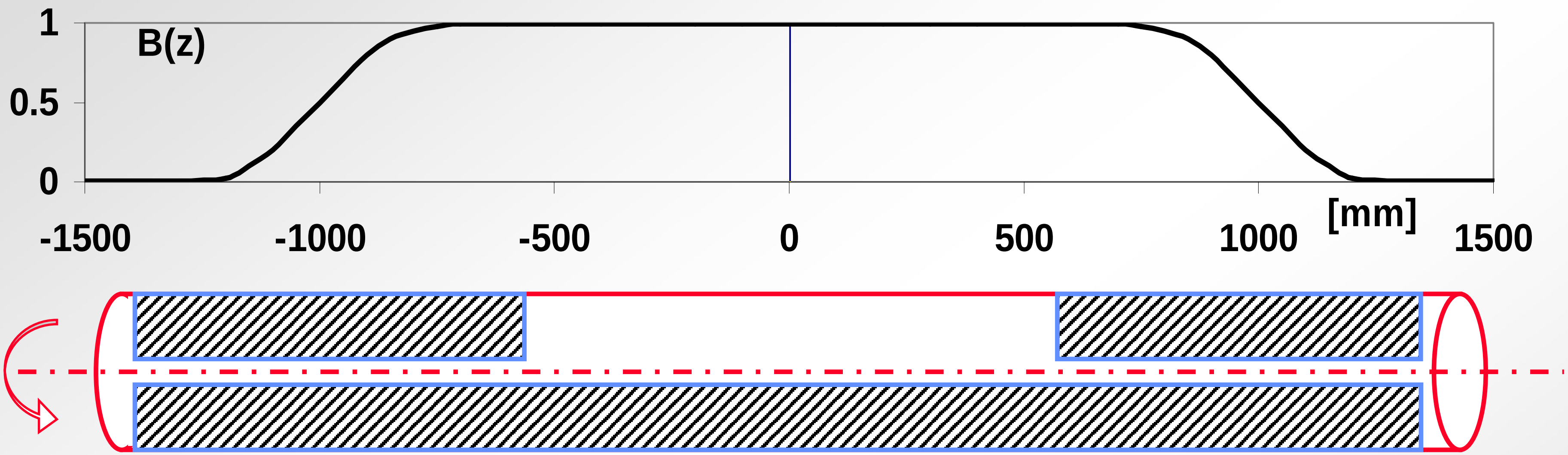
$b_3 \approx 5$ to 10 unit & changing with time

$$with \;\; z_m = z_c - d \cdot R_{ref} \quad C_n^m = \sum_{k=n}^{\infty} \frac{(k-1)!}{(n-1)!\cdot(k-n)} C_k^c \cdot d^{k-n}$$

$$\text{At 1st order } (\mathrm{d} < \mathrm{R}_{\mathrm{ref}}) \;\; C_n^m = n \cdot C_{n+1}^c \cdot d$$

Choice done: $d = C_{10}/10 \cdot C_{11}$ defines axis for the results of the LHC dipoles

Align measurement system with Quad axis



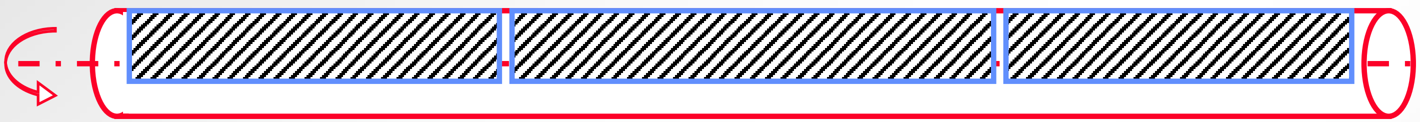
LEP measuring shaft : aperture 120 mm , maximum length 3.5 m

Align with the 2 end coils
Fine tune of alignement with integral coil
Gives full multipoles for ends & central part

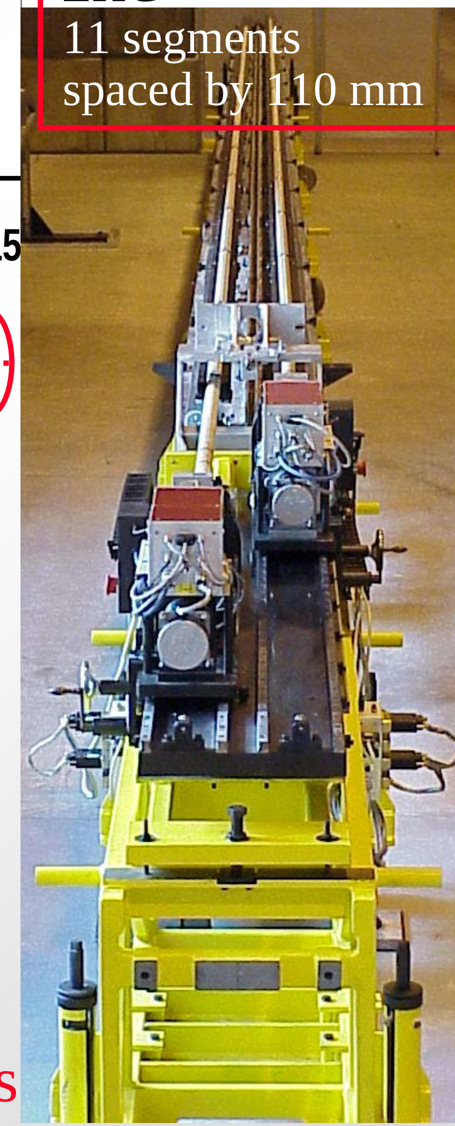
- central field (by difference) then effective length
- measurement of end fields to compare to 3D calculation

Impossible for smaller apertures and/or longer magnets

All coils on the same size



LHC
11 segments
spaced by 110 mm



Allows more radial room for the coil segments
(and compensation scheme)

Need to make sum of 3 (or more) measurement
and take into account the holes between coils
to get ∫Bdl (& ∫Gdl)

Does not work for axis finding if intermediate bearings

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Using a Voltage Integrator

Magnetic Fluxes are $[T \cdot m^2] = [V \cdot s]$ (Maxwell)
=> Integrating the voltage between 2 angular positions eliminates "time"

What about the Amplifier offset ? Can be eliminated

- over a turn $$\Psi(2\pi) = \Psi(0) + \oint Offset \cdot dt$$

Not true if excitation current changes with time

- By "washing machine" : average between go and return

If ω (rotation rate) non constant ?

Can be eliminated if δt measured with each angular interval

If both ω (rotation rate) & Offset(t) non constant ?

- creates coupling between harmonics,
- generally the limit for the measurement accuracy

! Have a good motor and smooth mechanics !

Angular Encoder not perfect

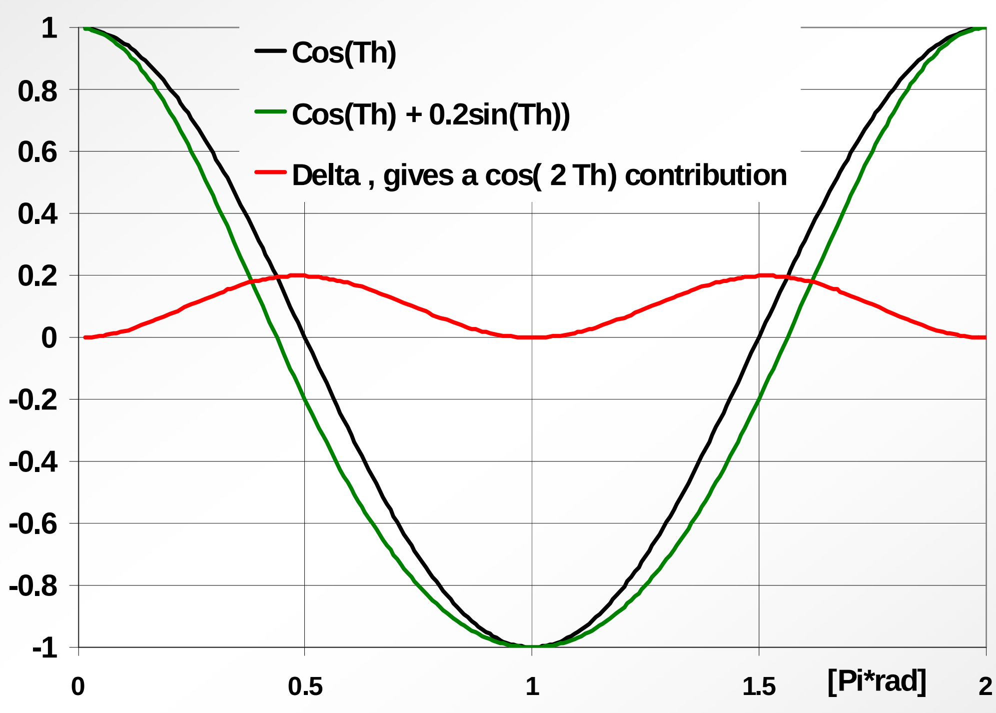
Encoder axis non parallel or non co-axial with shaft axis gives :

$\theta_{meas.} = \theta + \varepsilon \cdot \sin(\theta)$

in pure dipole (B1≠0)

$\Psi(\theta) \propto \cos(\theta_{meas}) \approx \cos(\theta) - \varepsilon/2 \cdot (1-\cos(2\theta))$

Non existing B_2 term induced : $\partial B_2 / B_1 = \partial b_2 = \varepsilon / 2 \cdot K_1 / K_2$



General case

$$\theta_{meas} = \theta + \sum_k (\gamma_k \cos k\theta + \varepsilon_k \sin k\theta)$$

in pure dipole

$$\partial b_n = \frac{nK_n}{2K_1}(\varepsilon_{n-1} + \varepsilon_{n+1})$$

$$\partial a_n = \frac{nK_n}{2K_1}(\gamma_{n-1} + \gamma_{n+1})$$

Torsional vibrations give similar random errors

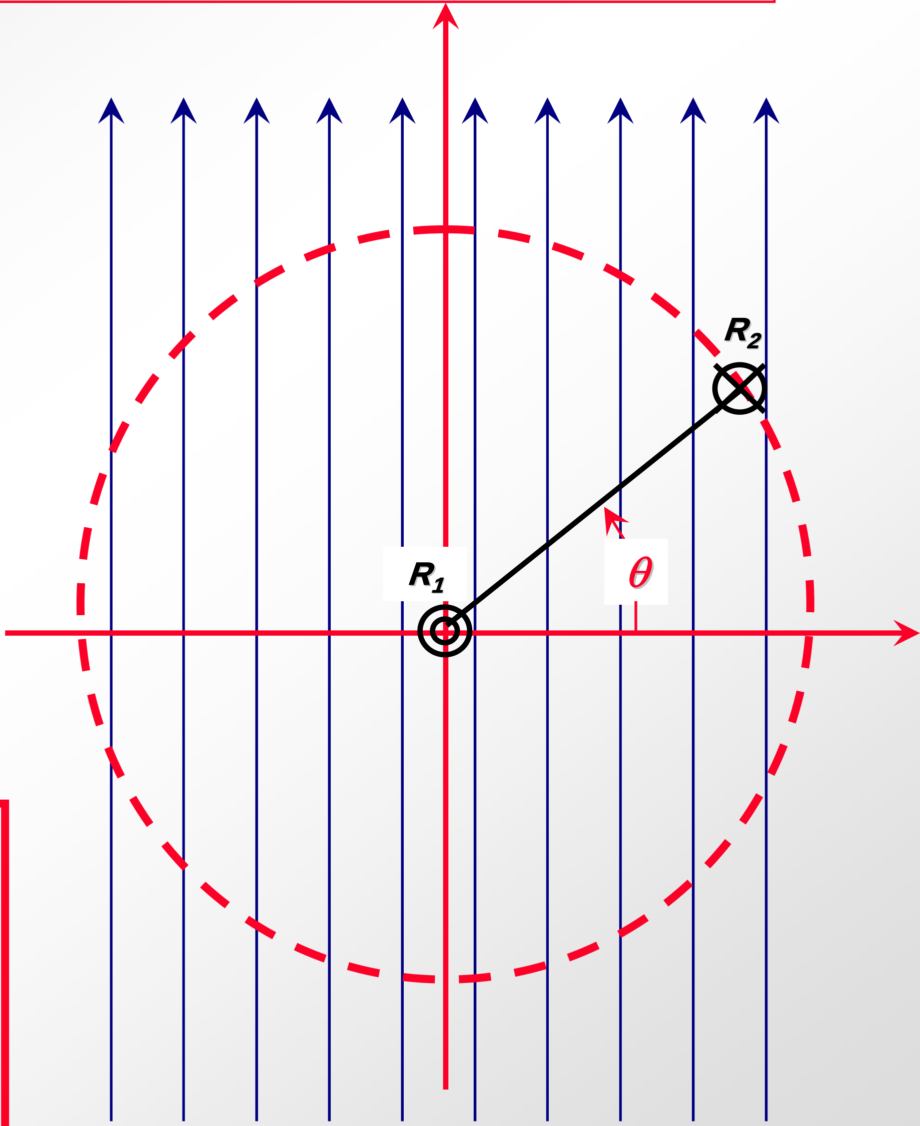
Angular Encoder not perfect (2)

Order of Magnitude

$$\text{Hyp}: R_2 = R_{ref} \;\; ; \;\; R_1 = 0$$

$$\frac{K_n}{K_1} = \frac{1}{n} \cdot \left(\frac{R_2}{R_{ref}} \right)^{n-1} = \frac{1}{n}$$

$$\varepsilon = 1\,\text{mrad gives } \partial b_2 = 10^{-3} \;\; (10\text{ unit})$$



Encoder with 2^{12} points (4096) is

- Specified to be just good enough
- is in fact better (if incremental encoder)

! Take Care !

- Torsional stiffness of coil
- Encoder mechanical mounting (add special bellow)

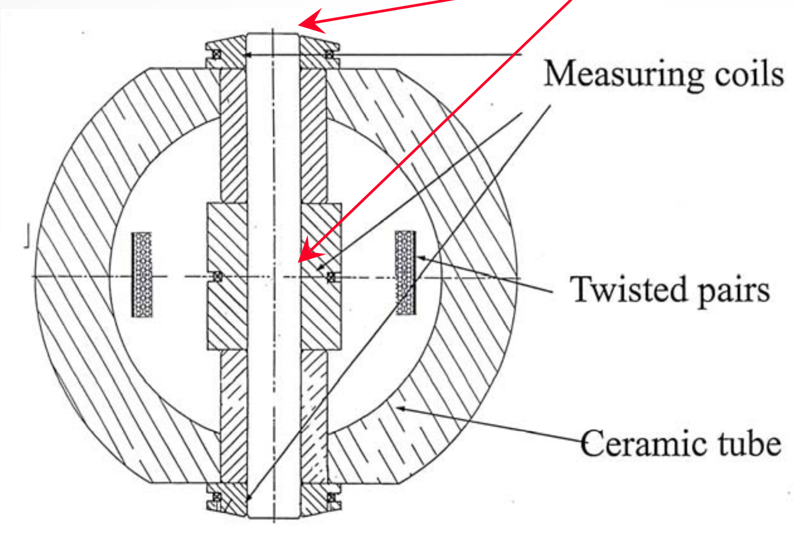
Eliminate Angle imperfection

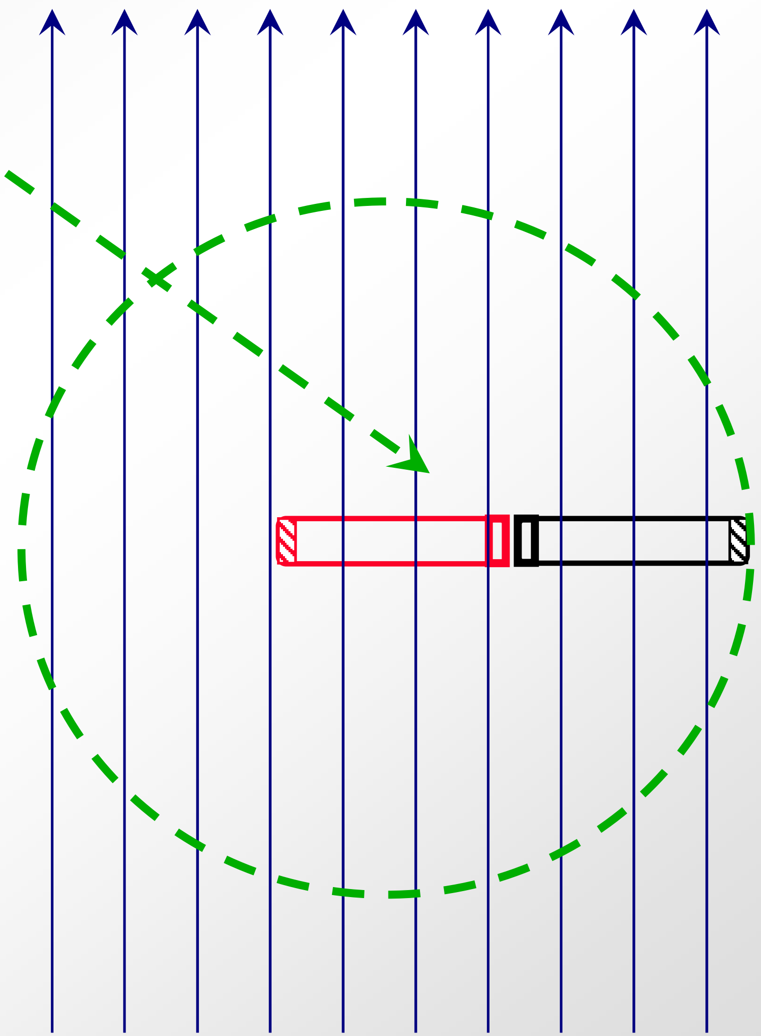
2 identical coils in electrical opposition are unsensitive to a dipole field : $K_1 = 0$ (true whether you displace or rotate them)

Rejection ratio ≈ 300 to 2000 if the two coils are

- sorted according to effective area
- (adjusted to be) parallel

Example with tangential coils : [a] – [c]





Coil with lateral displacement when rotating in Quadrupole

Exemple : coil shaft bends due to gravity

Hyp : $R_2 = R_r$; $R_1 = 0$; Displ. = $i \cdot d \cdot R_r \cdot \cos(2\theta)$

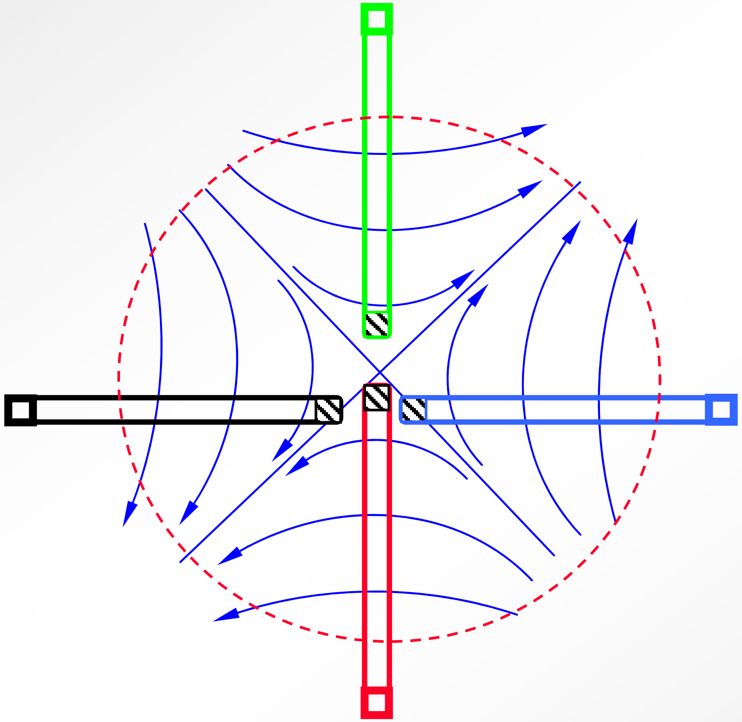
In $\Psi(z = e^{i\theta}) = N_t \cdot L \cdot \mathrm{Re} \int_{R_1}^{R_2} \sum_{1}^{N(=\infty)} C_n \cdot \left(\frac{z}{R_r} \right)^{n-1} \cdot dz$

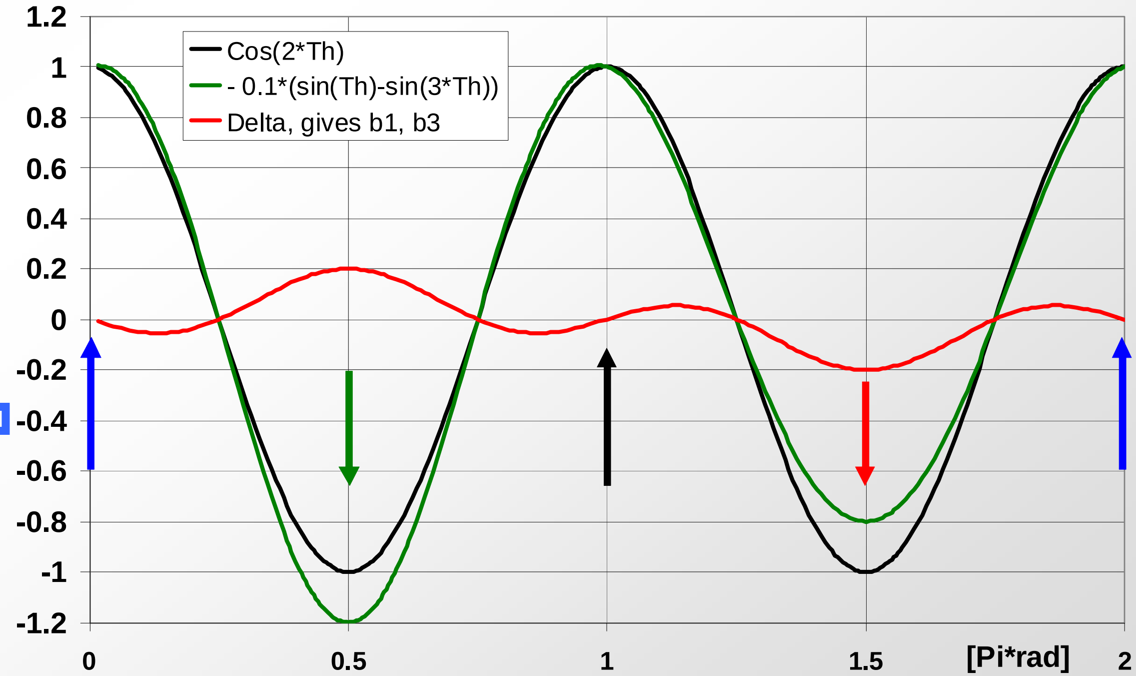
$R_2 = R_r \cdot (i \cdot d \cdot \cos(2\theta) + e^{i\theta})$; $R_1 = R_r \cdot i \cdot d \cdot \cos(2\theta)$

Erroneous dipole & sextupole

$$\partial B_1 / B_2 = \partial b_1 = d$$

$$\partial b_3 = -3 \cdot d$$





Imperfection in rotation (in Quadrupole)

Order of Magnitude : Hyp: $(R_2 =)\ R_{ref} = 20\ \text{mm}$

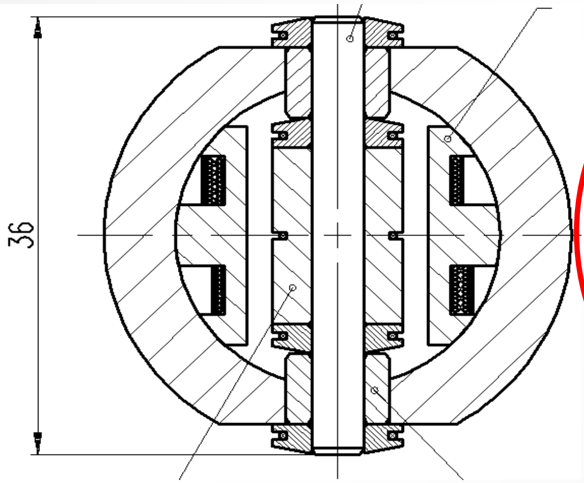
$$d = 0.02\ \ \text{mm}$$

! Take Care !

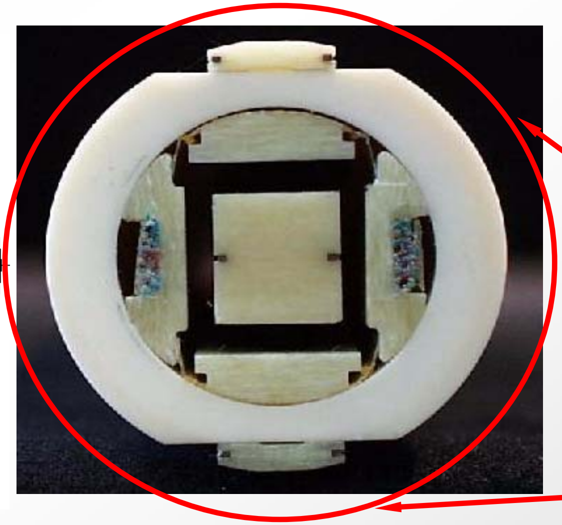
- Stiffness of coil (shaft)
- Quality of the bearings
- Compensate main harmonic

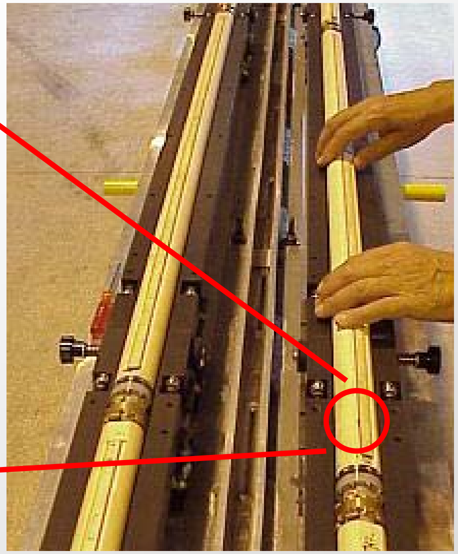
$$\partial \frac{B_1}{B_2} \cdot R_{ref} = 0.02\ \ \ \text{mm}$$

$$\partial \frac{B_3}{B_2} = \partial b_3 = 3 \cdot 10^{-3}\ \ (30\ \text{unit})$$



Quadrupole coil





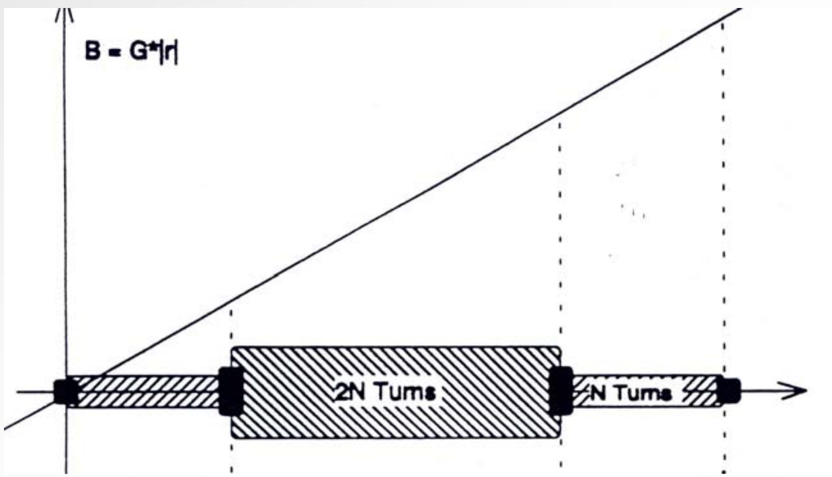
Eliminate imperfection in rotation

Compensating coil picks same flux as main

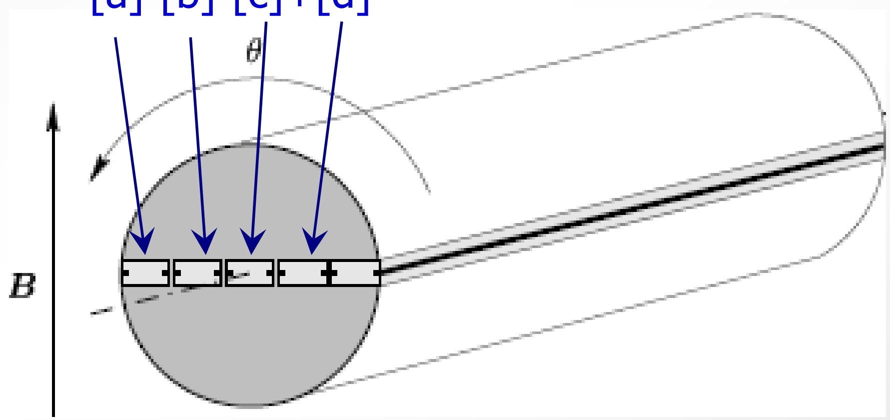
- ➢half width & twice number of turn
- ➢centrally located

in Gradient (B(x) = G·x)

When rotated or displaced : $K_1 = K_2 = 0$



More symmetric construction



Rejection ratio ≈ 50 to 500

if the 4 (5) coils are

- ➢sorted according to effective area
- ➢(adjusted to be) parallel

Compensation Coil Schemes
Improves signal to noise ratio

Only multipoles > main harmonic are measured with compensation coils

Since C_n $(n \neq 1) << B_1$ in dipole ($(n \neq 2)$ in quadrupole)

- Voltage Ripple & slow current change by current supply

  disappear (1st order)

- Coupling between main and higher harmonic disappears

  (cf. varying rotation rate & offset)

- Voltage on integrator smaller (by rejection ratio)

  => can be amplified

  higher resolution

  [signal / offset] better ratio

Compensation Coil Schemes
Compare different implementations

Type of Coil Bucking		Common Mode Rejection	Rotation imperfection correction	Flexibility	Notes
Analog Bucking	Equal coils ± Series Connection	Yes	Yes	Average	Requires large array for higher orders
	Different Nturns ± Series Connection	Yes	Yes	Poor	Array optimized for one specific order
	Equal coils Variable-gain preampli	Yes	Yes	High	Highly stable and linear ampli required, otherwise unacceptable errors
Digital Bucking	Equal coils Numerical treatment	No	Yes	Best	Gains may be fine tuned *a posteriori* Multiple DAQ channels required

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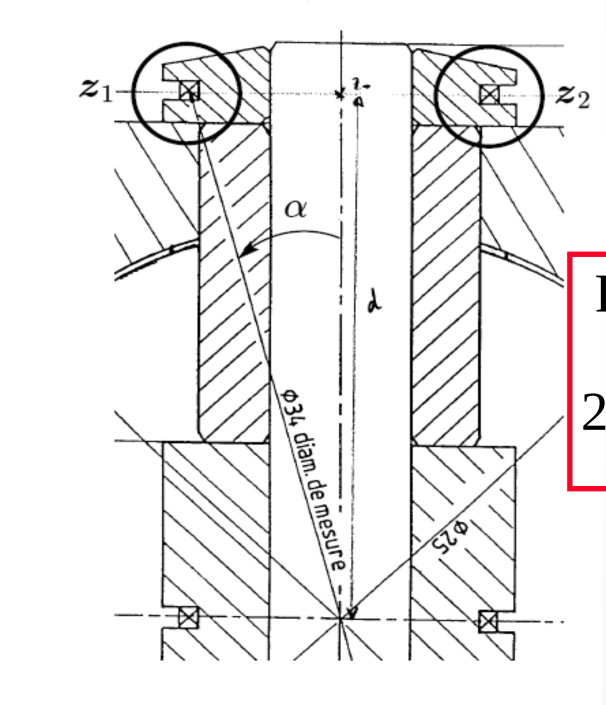
Tangential vs. Radial coils

Allows more rigid coil frame

Unsensitive for n·Angle=2π

Gives imaginary K_n

$\Psi_n = K_n \cdot C_n$ holds



$$R_c = R_r = 17\ \text{mm}$$

$$2 \cdot \alpha = \frac{2\pi}{12.5}$$

General Expression

$$K_n = N_t \cdot L \cdot \frac{(z_2^n - z_1^n)}{n \cdot R_r^{n-1}}$$

Radial Coil : R_2 & R_1 are real

$$K_n = N_t \cdot L \cdot \frac{(R_2^n - R_1^n)}{n \cdot R_r^{n-1}}$$

Tangential Coil :

$$z_2 = R_c \cdot e^{-i\alpha} \quad ; \quad z_1 = R_c \cdot e^{i\alpha}$$

$$K_n = -2 \cdot i \cdot N_t \cdot L \cdot \frac{R_c^n \sin(n\alpha)}{n \cdot R_r^{n-1}}$$

Calculate K_n with finite windings

So far, pointlike coil winding

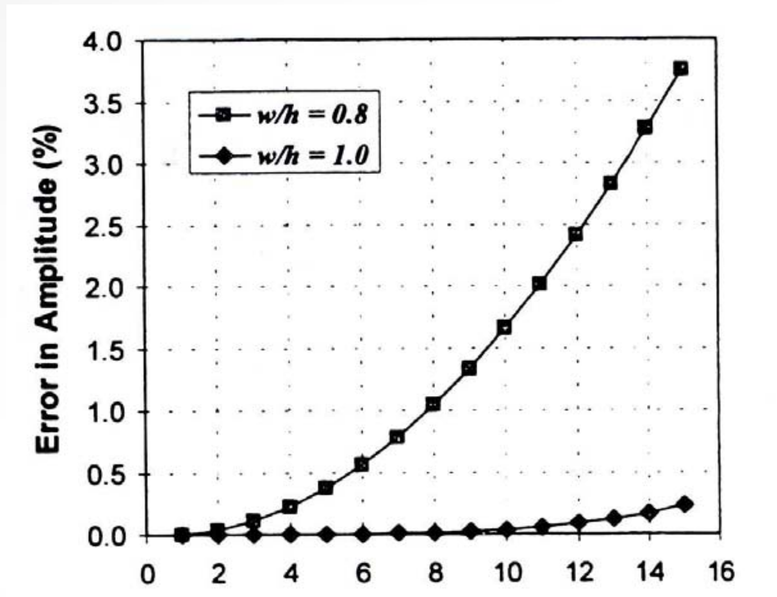
If finite dimension of coil winding

Replace z_2

by $\langle z_2^n \rangle = \frac{1}{S} \int z^n \cdot dz$

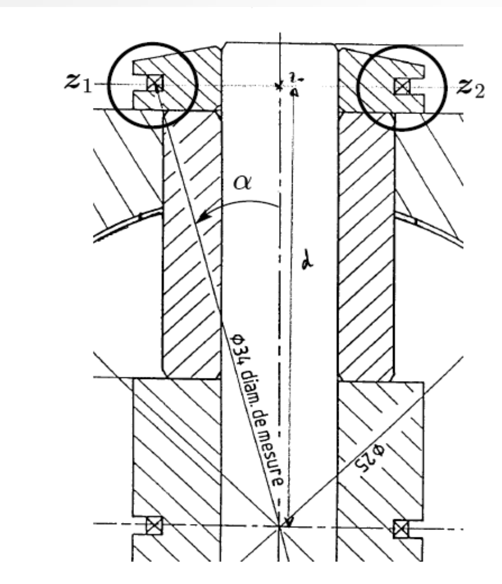
in $K_n = N_t \cdot L \cdot \frac{(z_2^n - z_1^n)}{n \cdot R_r^{n-1}}$

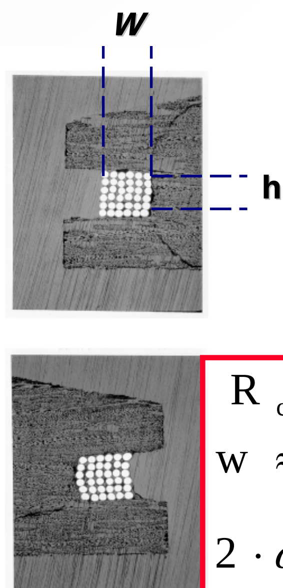
Not much different if w=h<<R_c



$R_c = 10\ \text{mm} \quad w \cdot h = 1\ \text{mm}^2$

Cf. A. Jain ,Anacapri, CAS 97





$R_c = R_r = 17\ \text{mm}$

$w \approx h \approx 0.5\ \text{mm}$

$2 \cdot \alpha = \frac{2\pi}{12.5}$

Calculate K_n with finite windings

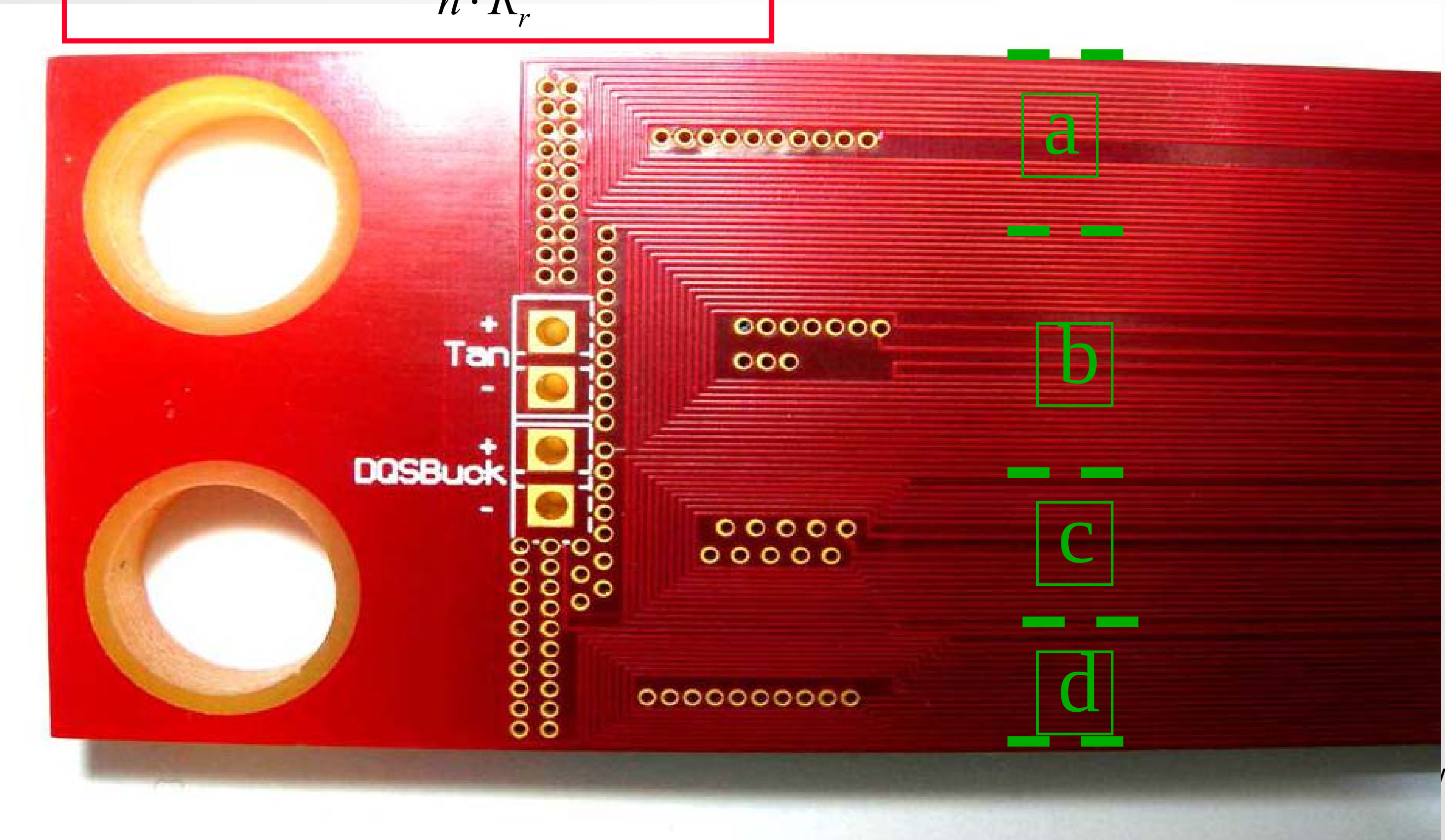
If finite dimension of coil winding

Replace z_2

by $\langle z_2^n \rangle = \frac{1}{S}\int z^n \cdot dz$

in $K_n = N_t \cdot L \cdot \frac{(z_2^n - z_1^n)}{n \cdot R_r^{n-1}}$

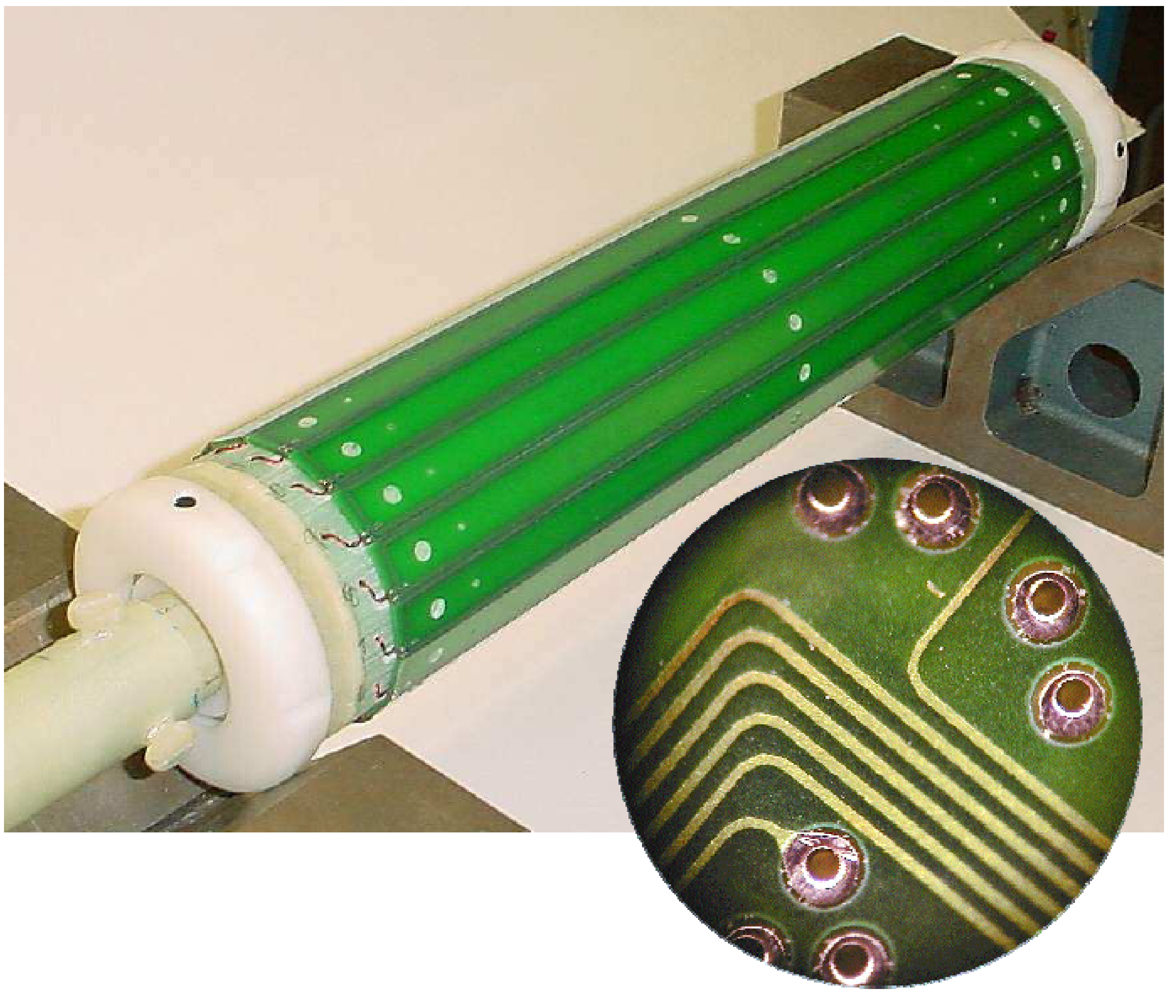
Correct calculation needed in this case



Developped @ FNAL

Coils Array to measure harmonics during ramps

BNL Harmonic Coil Array



16 Printed Circuit coils, 10 layers

6 turns/layer

300 mm long

0.1 mm lines with 0.1 mm gaps

Matching coils selected from a production batch

Radius =
35.7 mm (BioMed)
26.8 mm (GSI)

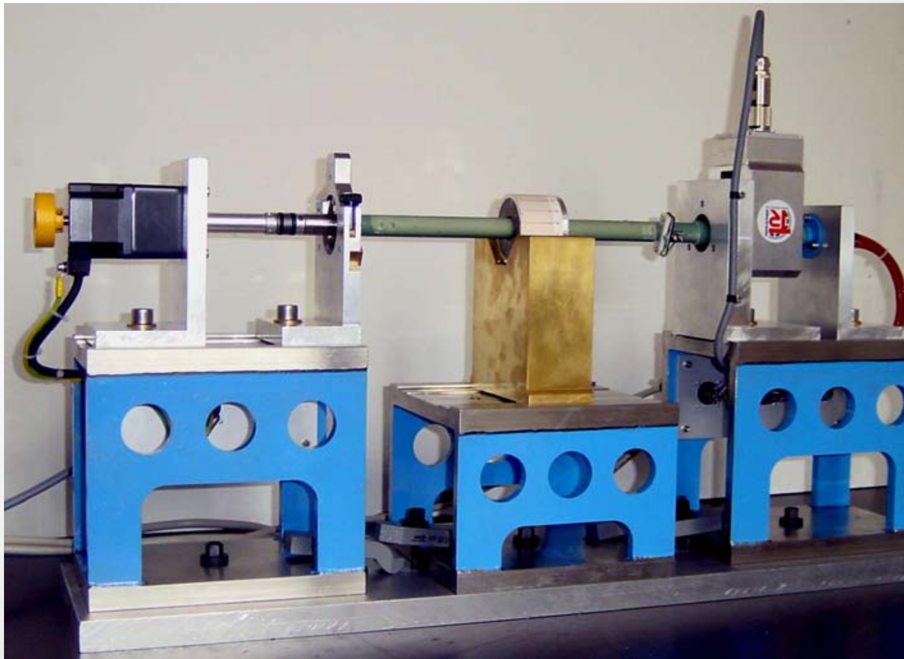
How to measure multipoles in pulsed magnet

$\Delta B/\Delta t$ cannot be neglected over one coil revolution period

- Increase rotation rate (and bandwidth of acquisition) (cf. P. Arpaia & M. Buzio)
- Coil static at given θ_i & pulse the current. then go to next angle (32 or 64 points per turn)

Linac 4 pulsed quadrupoles have 2 ms flat duration

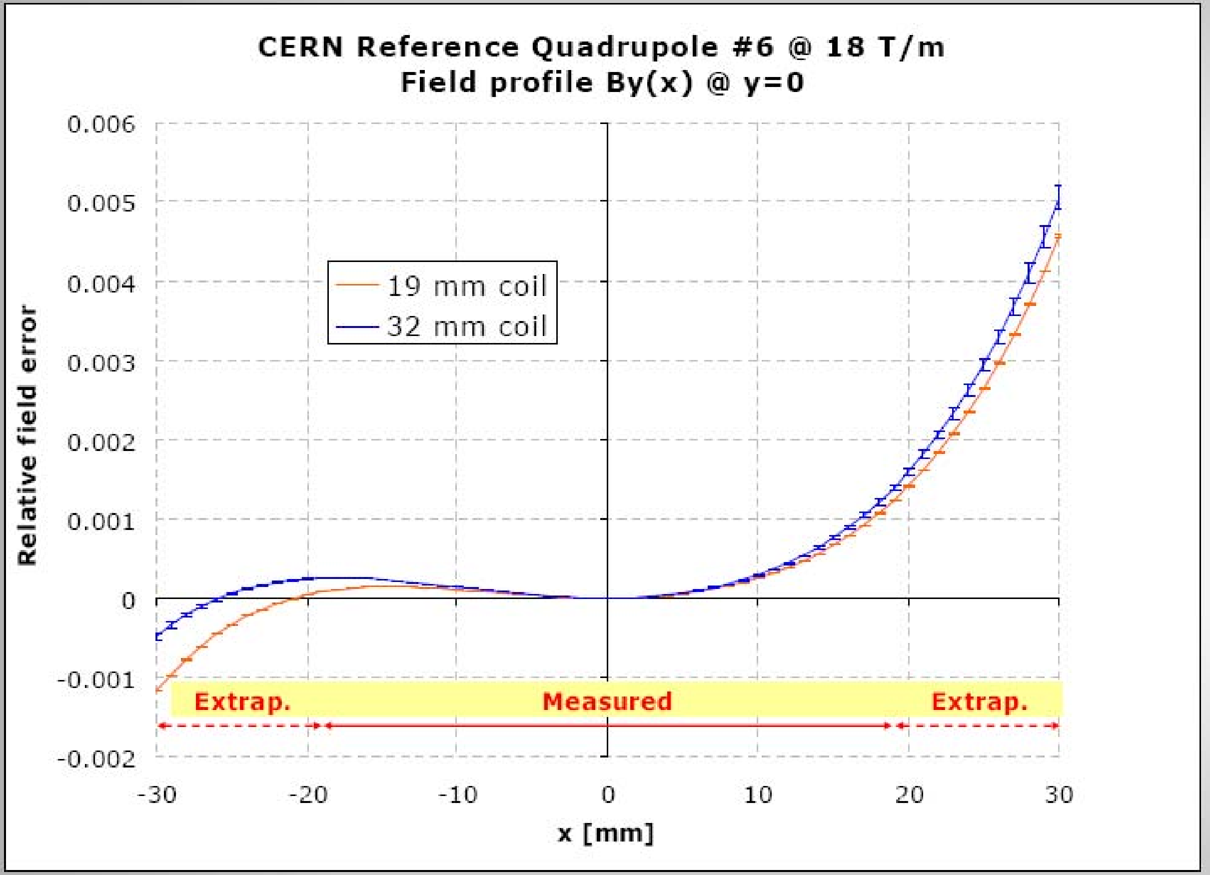
Experimental work going on



Can we extrapolate outside Rmes ?

Example of results: harmonics in large aperture by extrapolation at $r>r_{coil}$

Reference quadrupole measured in the same conditions with two moles having different coil ∅



$$\varepsilon(z)=\frac{\Delta B}{B_{\max}}=\frac{r_{ref}}{\|C_2\|R_2}\sum_{n=3}^{15}C_n\frac{z^{n-1}}{r_{ref}^{n-1}}$$

Max. relative error in when extrapolating results @ R=19 mm $\Delta B/B \approx 6 \cdot 10^{-4}$

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The Harmonic Coil Measurement Pro's & Con's

Pro's

- full 2 D measurement (normal and skew terms)
  corresponding to beam simulation needs
  gives axis and field direction
- High accuracy of multipole measurement
  with help of Coil Compensation Schemes
- Analysis and results with general formalism
  (in particular for "Coil Factors")

Con's

- More complex mechanics (encoder, motor)
- Not suited to high angle bending magnets
- Not suited to wide horizontal aperture magnets