Eddy currents
(in accelerator magnets)

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Introduction

- **Definition**

  According to Faraday‘s law a voltage is induced in a conductor loop, if it is subjected to a time-varying flux.As a result current flows in the conductor, if there exist a closed path.

  ‚Eddy currents‘ appear, if extended conducting media are subjected to time varying fields. They are now distributed in the conducting media.

- **Effects**
  - Field delay (Lenz ´s Law), field distortion
  - Power loss
  - Lorentz-forces

  – **Beneficial** in some applications (brakes, dampers, shielding, induction heating, levitated train etc.)

  – **Mostly unwanted** in accelerator magnets: ⟹ appropriate design to avoid them / to minimize the unwanted effects.

Outline

- Introduction
  - Definition, Effects (desired, undesired)
- Basics
  - Maxwell-equations
  - Diffusion approach
    - Analytical solutions: Examples
    - Numerical solutions: Introduction of numerical codes
  - Direct application of Maxwell-equations (small perturbation)
- Eddy currents in accelerator magnets
  - Yoke, mechanical structure, resistive coil, beam pipe
- Design principles / Summary
- Appendix (references)

Basics

Maxwell-equations

- Quasistationary approach
- No excess charge

Ampere's Law

$$\nabla \times H = j \qquad \oint H \cdot ds = \int_A j \cdot dA$$

Faraday's Law

$$\nabla \times E = -\frac{\delta B}{\delta t} \qquad \oint E \cdot ds = -\frac{\delta}{\delta t}\int_A B \cdot dA$$

$$\nabla \cdot E = 0 \qquad \oint_A E \cdot dA = 0$$

$$\nabla \cdot B = 0 \qquad \oint_A B \cdot dA = 0$$

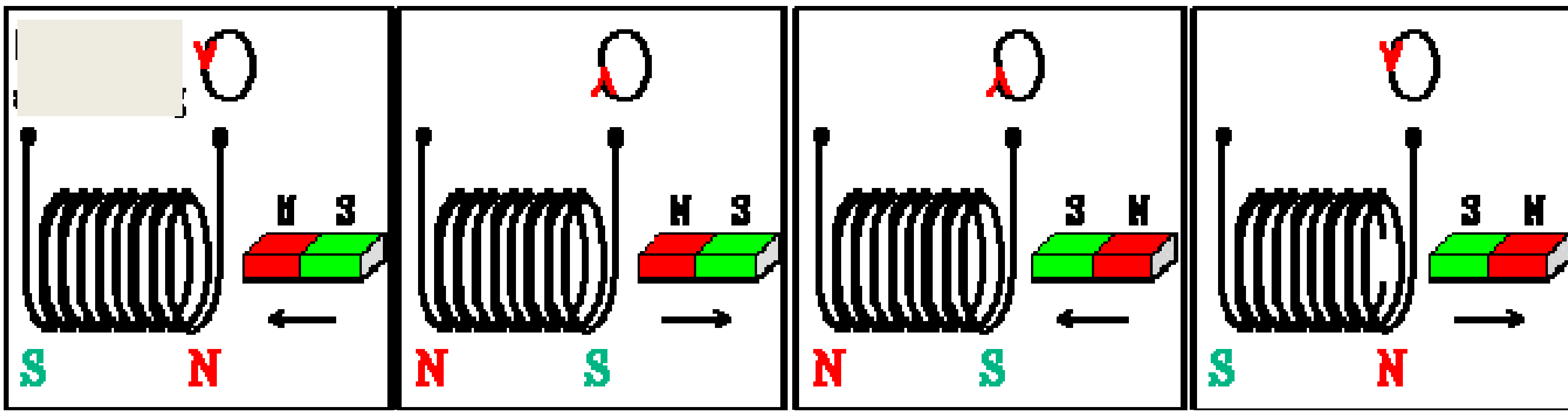
Material properties

$$B = \mu_0 \underline{\underline{\mu_r}} H \qquad j = \underline{\underline{\sigma}} E$$

Lenz‘s law

"the emf induced in an electric circuit always acts in such a direction that the current it drives around a closed circuit produces a magnetic field which opposes the change in magnetic flux."

$$\oint_{\partial A} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s} = -\frac{\partial}{\partial t} \int_A \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{A}$$



Lenz‘s Law: Reason for field delay → slow diffusion process

Field diffusion equation

one way of eddy current calculation

$$\vec{j} = \sigma \vec{E}$$

vanish

$$\nabla \times \vec{H} = \vec{j} \quad \rightarrow \quad \nabla \times \nabla \times \vec{H} = \nabla \times \vec{j} = \sigma\,(\nabla \times \mathrm{E})$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla^2 \vec{H} = \sigma\mu \frac{\partial \vec{H}}{\partial t}$$

$$\kappa = \frac{1}{\sigma \cdot \mu}$$

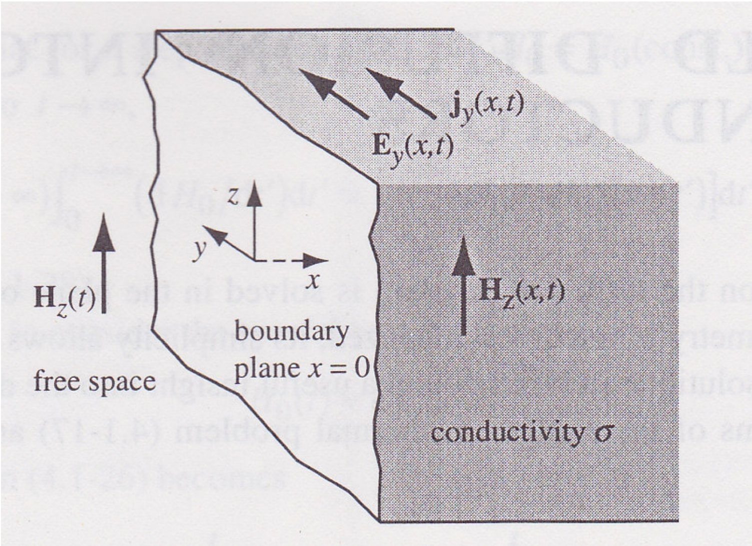
Magnetic diffusivity

- Assumption: σ uniform in space
- Diffusion equation does also exist also for current density j, magnetic Induction B and magnet Vector Potential A !
- Having solved the differential equation for H, the eddy current density j can be calculated by Ampere's Law and consequently the power loss P.

Analytical solutions: Half-space conductor (1D-approach)(1)

(following closely H.E. Knoepfel ‚Magnetic Fields‘)

$$\frac{\partial^2 H_z}{\partial x^2} = \sigma\mu \frac{\partial H_z}{\partial t}$$



Application of external field: $H_z(t)$
Solution $H_z(x,t)$ = ?

<u>Boundary conditions</u>:

$H_z(0,t)=0$ $t<0$

$H_z(0,t)= H_z(t)$ $t\geq 0$

$H_z(x,0)= 0$ $0<x<\infty$

1. Step-function field $H_z(t) = H_0 = \text{constant}$
2. Transient linear field $H_z(t) = H_0/t_0 * t$
3. Transient sinusoidal field $H_z(t) = H_0 * \sin(\omega t)$

Analytical solutions: Half-space conductor (1D-approach) (2)

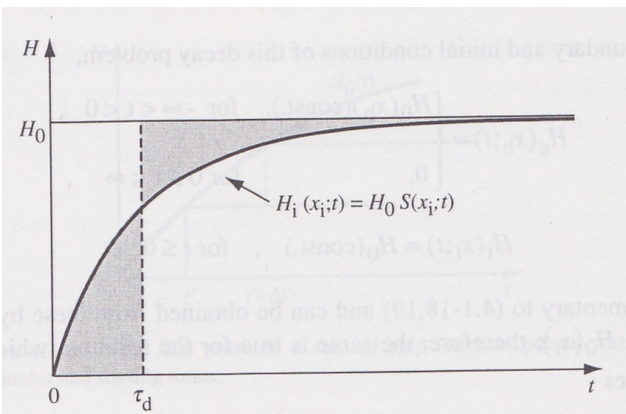
1. Step-function field $H_z(t) = H_0 = \text{constant}$

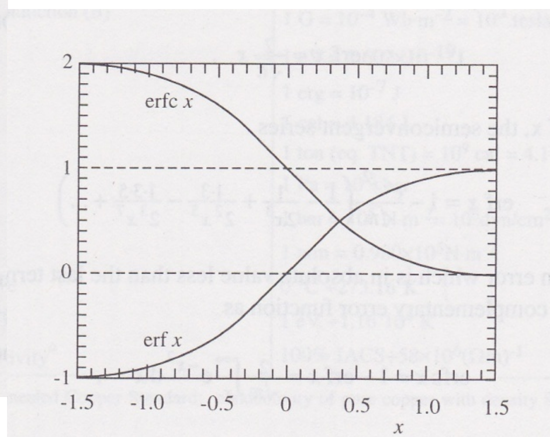
$Hz(x,t) = H_0 * S(x,t)$

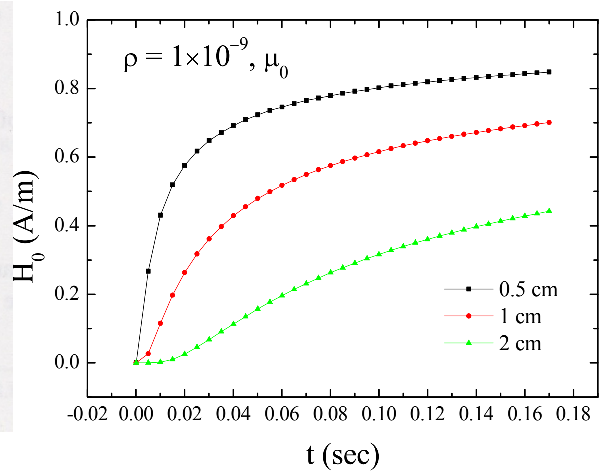
With response function S(x,t)
and $S(x,0)=0$ and $S(x,t \rightarrow \infty)=1$

Diffusion time constant

$$\tau_d = \int_0^\infty (1 - S(x,t))\,dt$$







$$\xi = \frac{x}{2\sqrt{\kappa t}}$$

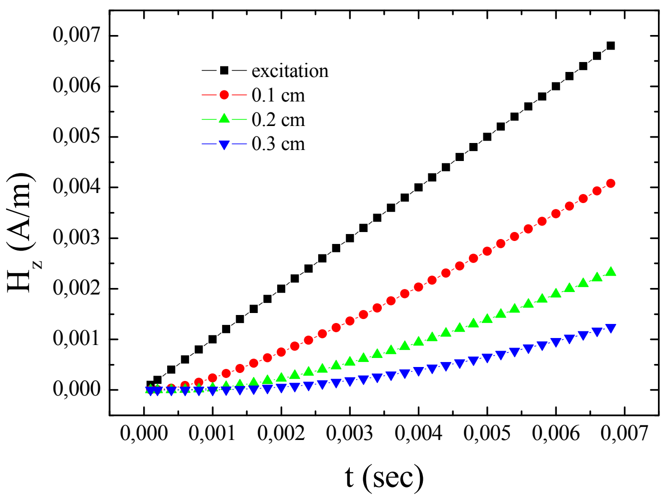
Similarity variable

$$H_{z(x,t)} = H_0(1 - erf\,\xi)$$

Special response function S(x,t)

Analytical solutions: Half-space conductor (1D-approach) (3)

2. Transient linear field $H_z(t) = H_0/t_0 * t$

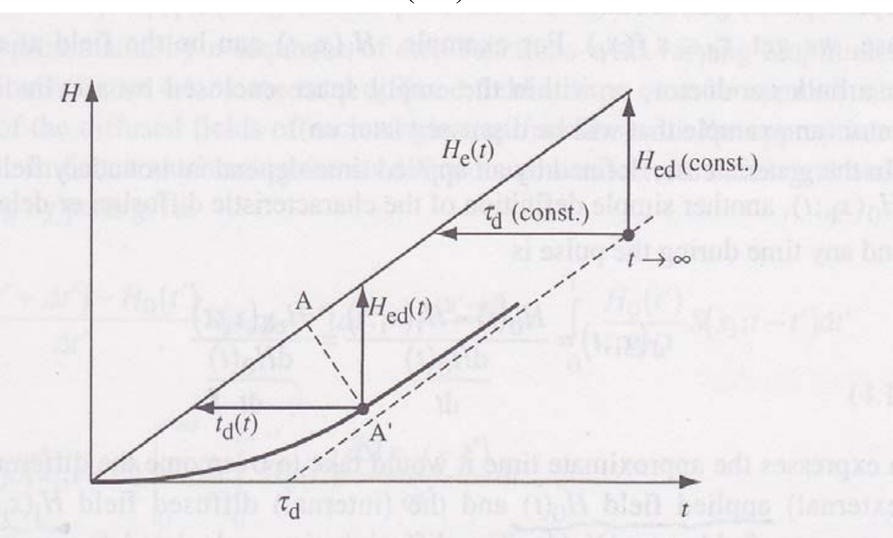


$$H_z(x,t) = \frac{H_0}{t_0} t \left[\left(1 + 2\xi^2\right) \operatorname{erfc} \xi - \frac{2}{\sqrt{\pi}} \xi e^{-\xi^2} \right]$$

$$= H_z^s(x,t) + H_z^t(x,t)$$

$$= \frac{H_0}{t_0} \cdot t - \tau_d(x) \frac{H_0}{t_0} + H_z^t(x,t)$$

$\xrightarrow{}$ 0, if t >> τ_d



Field lag $H_{ed}(x)$

Analytical solutions: Half space conductor (1D-approach) (4)

3. Transient sinusoidal field $H_z(t) = H_0 * \sin(\omega t)$

$$H_z(x,t) = H_z^s(x,t) + H_z^t(x,t)$$

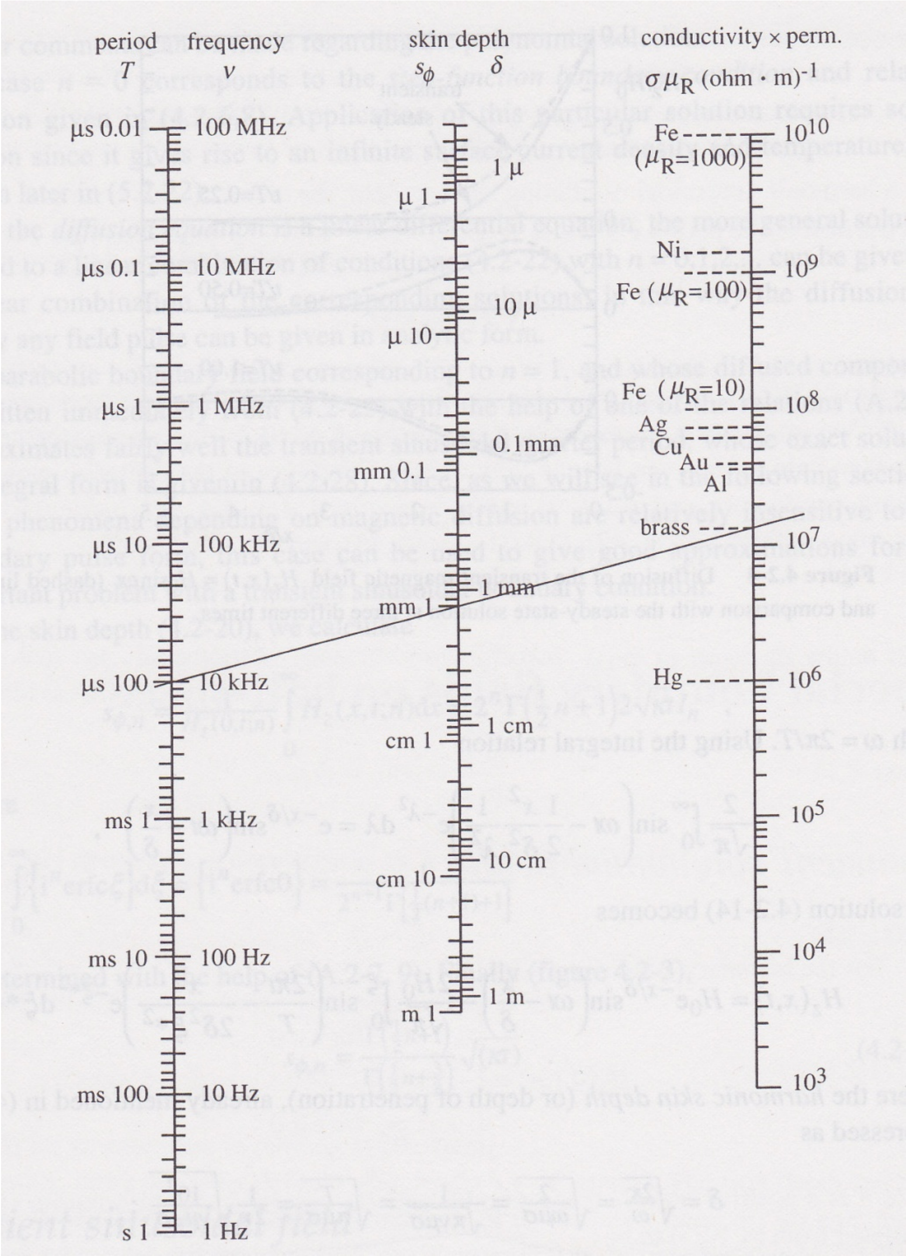
stationary transient

$$H_z^s(x,t) = H_0 \cdot e^{-\frac{x}{\delta}} \cdot \sin(\omega t - \frac{x}{\delta})$$

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

Harmonic skin depth

Skin depth as function of frequency and conductivity



Refer: Knoepfel, fig. 4.2-5

Analytical solutions: Slab conductor (lamination)

Boundary conditions:for step function field

$H_z(\pm d,t)=0$ $t<0$

$H_z(\pm d,t)= H_0$ $t\geq 0$

$H_z(x,0)= 0$ $-d<x<+d$

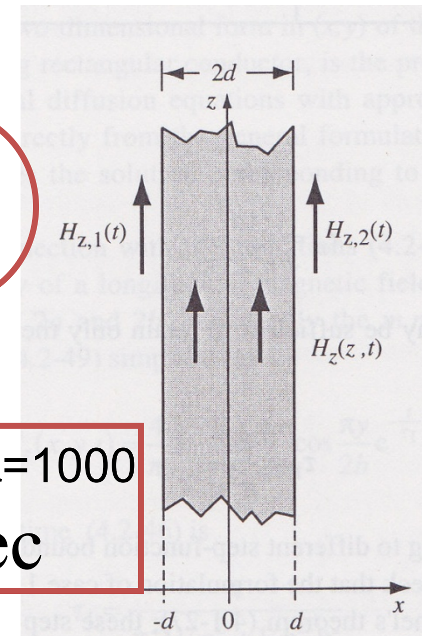
Refer to Knoepfel 4.2, Table 4.2-I

Step-function field (1D)

$$H_z(x,t) = H_0\left[1-4\sum_{n=1}^{\infty}\frac{\cos\frac{n\pi x}{2d}}{n\pi(-1)^{\frac{n-1}{2}}}e^{-\frac{t}{\tau_n}}\right]$$

$$\tau_n = \frac{4}{n^2\pi^2}\cdot\frac{d^2}{\kappa}$$

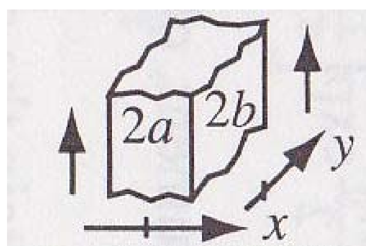
n odd



LC iron, 1mm , μ_r=1000

$$\tau_1 < 1m\sec$$

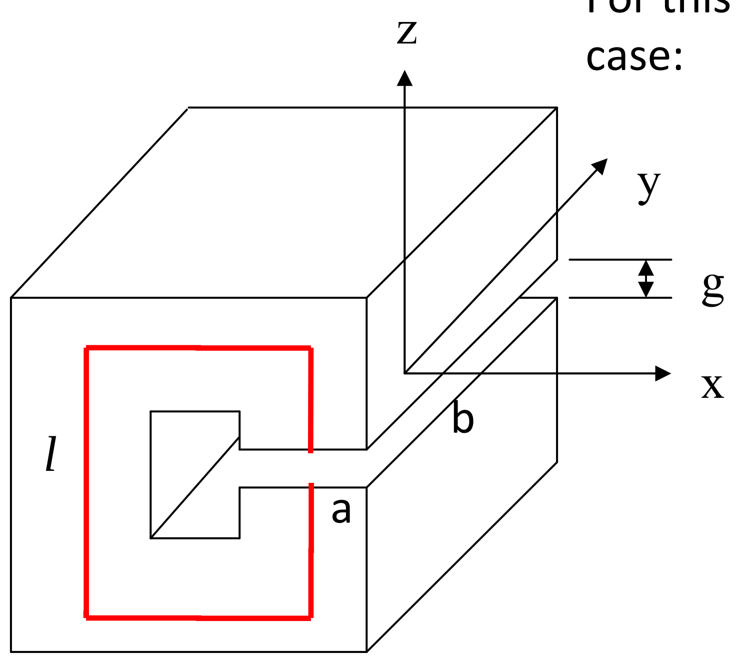
Step-function field (2D)



$$H_z(x,y,t) = H_0\left[1-4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\cos\frac{n\pi x}{2a}\cos\frac{m\pi y}{2b}}{\pi^2\cdot f(n.m)}e^{-\frac{t}{\tau_{n,m}}}\right]$$

$$\tau_{n,m} = 4\Big/\left[\pi^2\kappa\left(\frac{n^2}{a^2}+\frac{m^2}{b^2}\right)\right]$$ n,m odd

Analytical solutions: Field in the gap of an iron-dominated C-dipole



For this special case:

$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}=\sigma\mu_0\frac{l}{g+\left(l/\mu_r\right)}\frac{\partial H}{\partial t}$$

G. Brianti et al., CERN SI/Int. DL/71-3 (1971)

Case1 : g=0 Standard diffusion equation

Case 2: $l/\mu_r << g$ Special diffusion equation

$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}=\sigma\mu_0\frac{l_i}{g}\frac{\partial H}{\partial t}=\frac{1}{\kappa_1}\frac{\partial H}{\partial t}$$

$$\kappa_1=\frac{1}{\mu_0\sigma}\cdot\frac{g}{l}$$

With this diffusivity κ_1 we can use the slab solutions!

Analytical vs. numerical methods

	pros	cons
Analytical Methods	physical understanding	•simple geometry (mainly1D/ 2D) •Homogeneous, isotropic and linear materials •simple excitation
Numerical Methods	•complex geometry (3D) •inhomogeneous, anisotropic and nonlinear materials •complex excitation	long computing times

Vector Potential A - the most common way of numerical eddy current calculation

$$B = \nabla \times A$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}\left(\nabla \times \vec{A}\right) = \nabla \times \left(-\frac{\partial \vec{A}}{\partial t}\right)$$

$$\vec{J} = \sigma \vec{E} = -\sigma \frac{\partial \vec{A}}{\partial t}$$

Field calculation

$$\nabla \times \vec{B} = -\nabla^2 \vec{A} = \mu \vec{J}$$

$$\nabla^2 \vec{A} = \sigma \mu \frac{\partial \vec{A}}{\partial t}$$

Find vector potential

Diffusion equation for Vector potential A

Sometimes the current vector potential T is used: $j = \sigma \cdot \nabla \times T$

Refer: MULTIMAG - program for calculating and optimizing magnetic 2D and 3D fields in accelerator magnets (Alexander Kalimov [kalimov@sptu.spb.su])

Widely used numerical codes for the calculation of eddy current in magnets

- Opera (Vector Fields Software, Cobham Techn. Services, Oxford) *www.**vectorfields**.com*
  - FEM
  - Opera 2d,AC and TR, Opera 3d, ELEKTRA , (TEMPO-thermal and stress-analysis)
- ROXIE (Routine for the Optimization of Magnet X-Sections, Inverse Field Calculation and Coil End Design) (S. Russenschuck, CERN) https://espace.cern.ch/roxie/default.aspx
  - BEM/FEM
  - Optimization of cosnθ-magnets, coil coupling currents only
- ANSYS (ANSYS Inc.) *www.**ansys**.com/*
  - Finite Element Method
  - Direct and in-direct coupled analysis (Multiphysics)
    - eddy current → heat → rising temperature → change resistivity → change eddy current
  - "This feature is important especially in the region of cryogenic temperature. Because most of physical parameters depend highly on temperature in that region" .

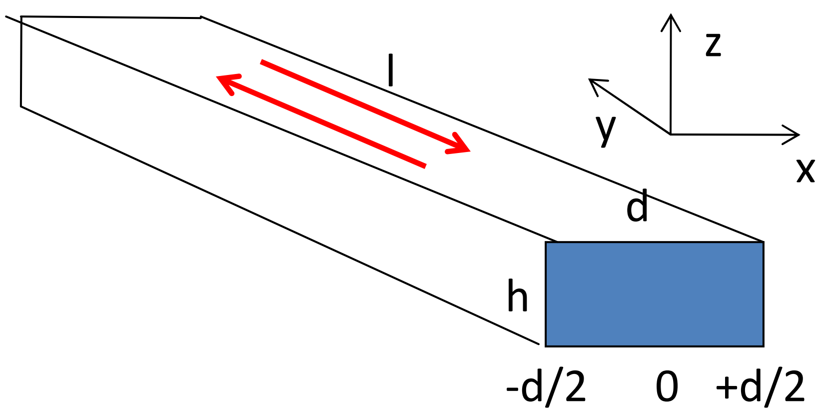
Direct application of maxwell equations -another way of eddy current calculation

Eddy Current effect handled as small perturbation:

- geometrical dimension << skin depth (high resistivity!)
- low field ramp rate

$B=(0,0,B_z)$

1. Eddy currents in a rectangular thin plate



- assumptions
  - Field B_z only, uniform
  - d,h<<l
  - d,h<<penetration depth s (magnetically thin!)
  - Steady state: $t \gg \tau_d$

From Faraday´s law (integral form)

$$j_y(x) = -\frac{\dot{B}_z}{\rho} \cdot x$$

neglecting the resistance contribution of the ends, since 2d<<l

From Ampere's law:

$$H_z^{eddy}(x) = \frac{\dot{B}_z^2}{2\rho}\left(\boxed{x^2 - \frac{d^2}{4}}\right)$$

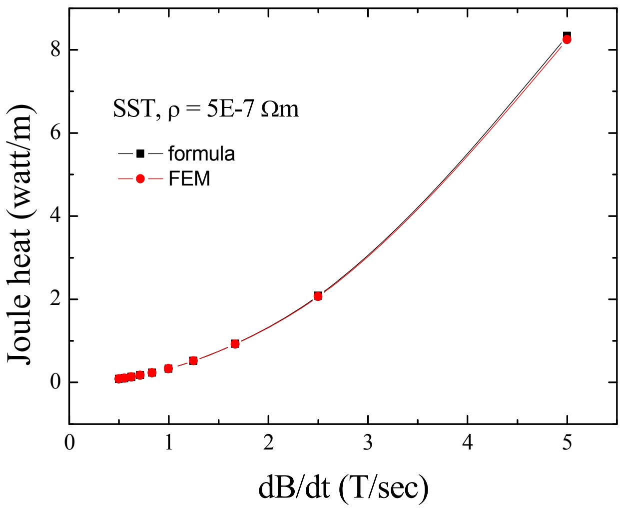
Top/bottom of a rectangular beam pipe!

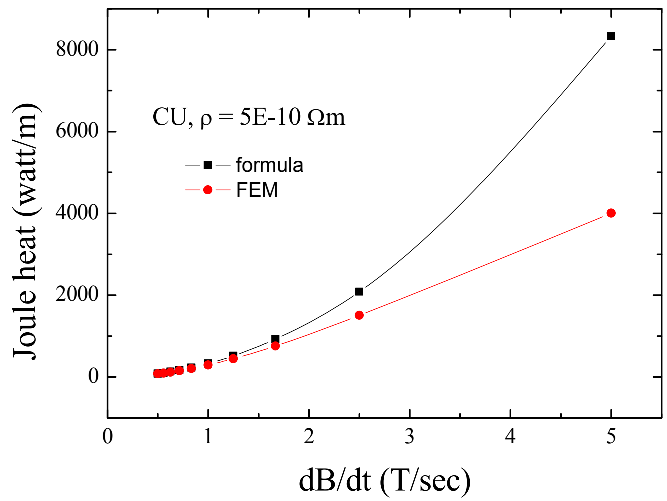
And then for the loss dP in an area A=hdx:

$$dP = \rho \frac{l}{A} \cdot \left(j_y(x) \cdot A \right)^2 = \rho \cdot l \cdot j_y{}^2(x) \cdot h \cdot dx$$

After integration

$$P = 2 \int_0^{d/2} dP = \frac{lh}{12} \frac{d^3}{\rho} \dot{B}_z{}^2 \quad or \quad P/volume = \frac{1}{12} \frac{d^2}{\rho} \dot{B}_z{}^2$$





Same formula as for a thin slab!

For Copper: No small perturbation anymore!

2. Eddy loss in a long,thin cylinder (radius r, length l, thickness d (r>>d))

(round thin beam pipe)

$$j = \frac{r \cos\theta}{\rho} \dot{B} \qquad P = \frac{r^3}{\rho} \dot{B}^2 \pi d l \quad or \quad P/V = \frac{r^2}{2\rho} \dot{B}^2$$

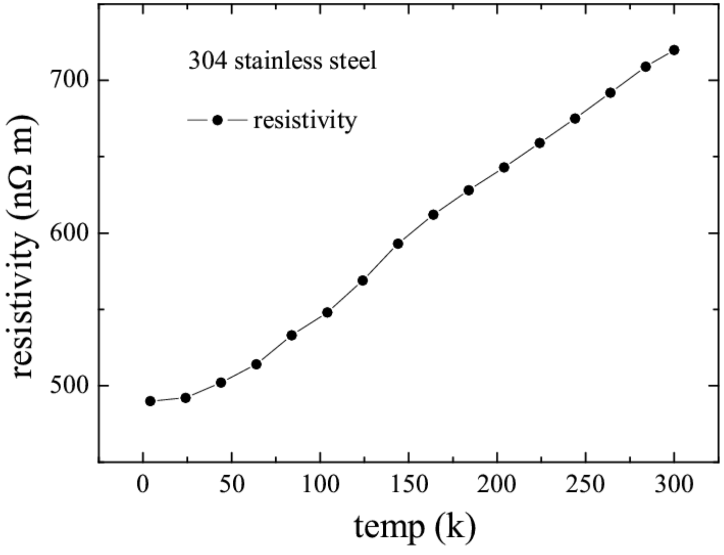
3. Eddy loss of round plate/disk (radius r, thickness d , r>>d)

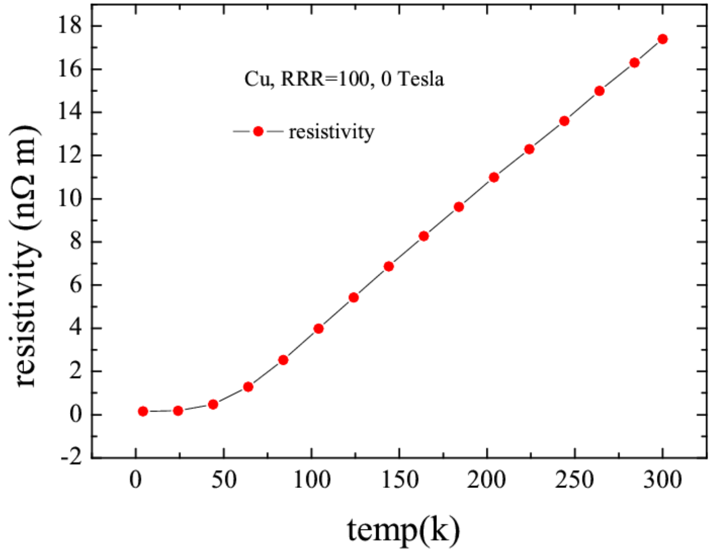
$$P/V = \frac{r^2}{8\rho} \dot{B}^2$$

Resistivity ρ (Ohm*m) @300K/4K (typical)

	300K	4K
LC steel (3% Silicon)	590×10^{-9}	$440 \text{ x } 10^{-9}$
Stainless steel	720×10^{-9}	490×10^{-9}
Copper	17.4×10^{-9}	0.156×10^{-9}

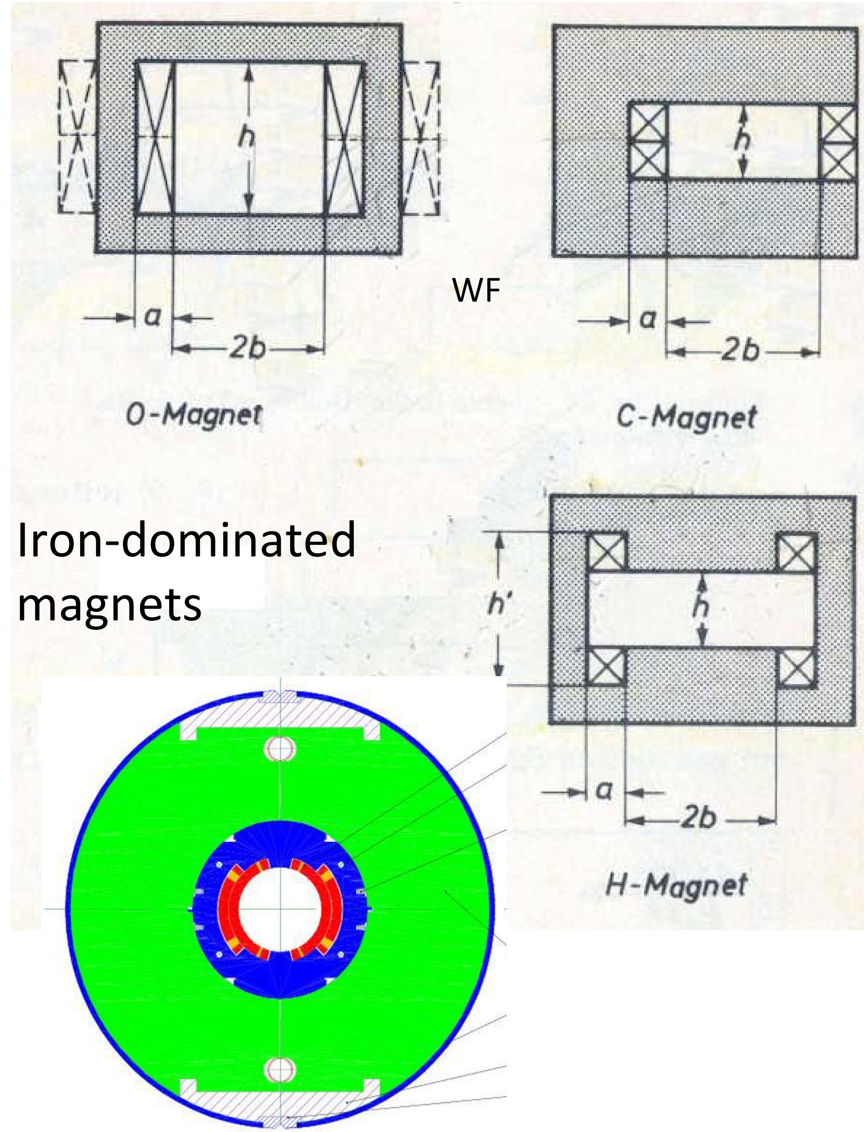
Avoid copper!





Eddy currents in magnets

Different Magnet types



Coil dominated cos nθ- magnet

Eddy currents in all conductive elements, especially in:

- •iron yoke (low carbon iron)
- •mechanical structure (low carbon iron
- • or stainless steel)
- •Coil (Copper, superconductor)
- • beam pipe (stainless steel

Laminated yoke (no isotropy!!)

Recap: thin slab $P \sim d^2 * \sigma$ ⇒ Laminated magnets with insulated laminations and low conductivity!!

Def.:

Packing factor fp = Wi / (Wi + Wa)

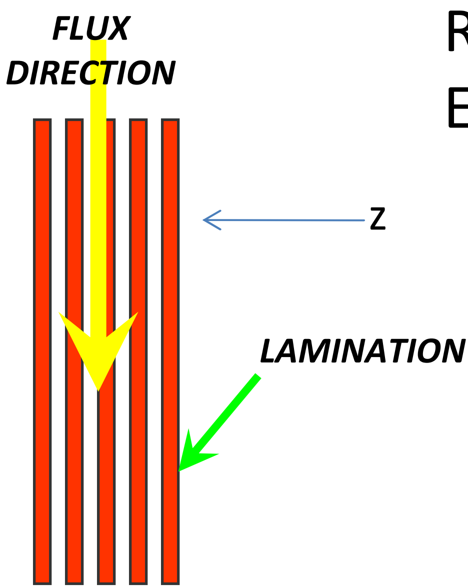
W_i - thickness single lamination

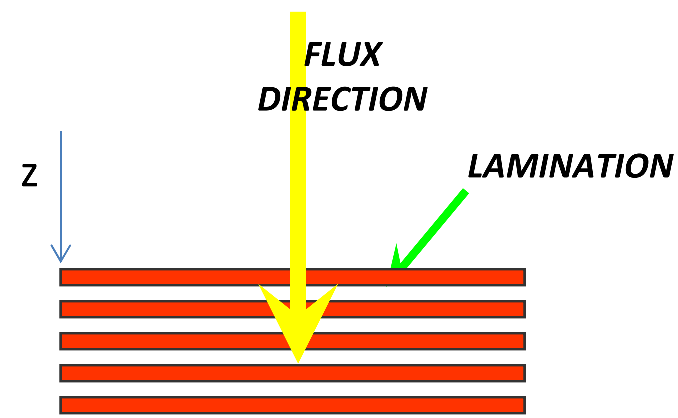
W_a - thickness of insulation

Conductivity: $\sigma_z=0$ $\sigma_{xy}\neq 0$

Rel. permeability of laminations μ_r

Effective permeability: μ_z, μ_{xy}





For laminations tangential to the flux:

$$\mu_{eff} = f_p \cdot (\mu_r - 1) + 1$$

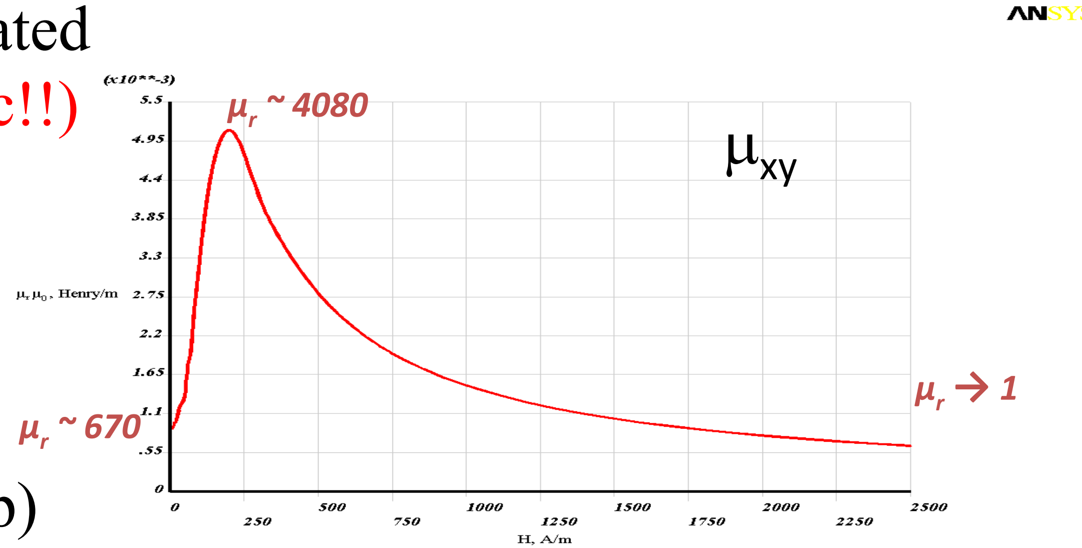
For laminations normal to the flux:

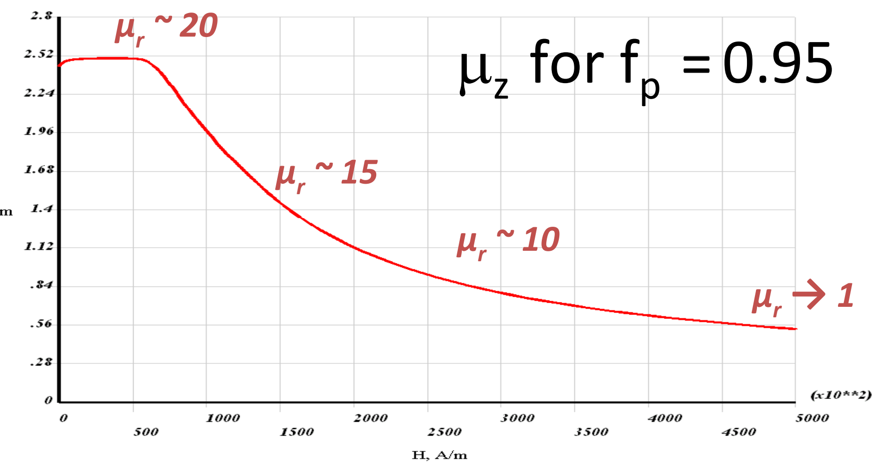
$$\mu_{eff} = \mu_r / (\mu_r - f_p \cdot (\mu_r - 1))$$

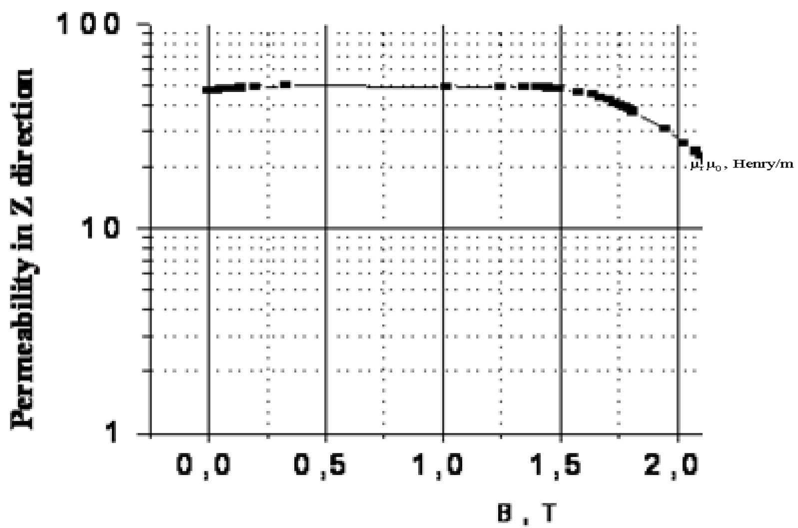
Laminated yoke : μ_{xy}, μ_z

- μ_z , μ_{xy} different for laminated magnets (highly anisotropic!!)
  - $\mu_{xy} = \mu_r(H)$
  - μ_z
    - $f_p = 1 : \mu_z = \mu_r(H)$
    - $f_p < 1: \mu_z = 1 / (1 - fp)$

      $\mu_z = 15 – 50$





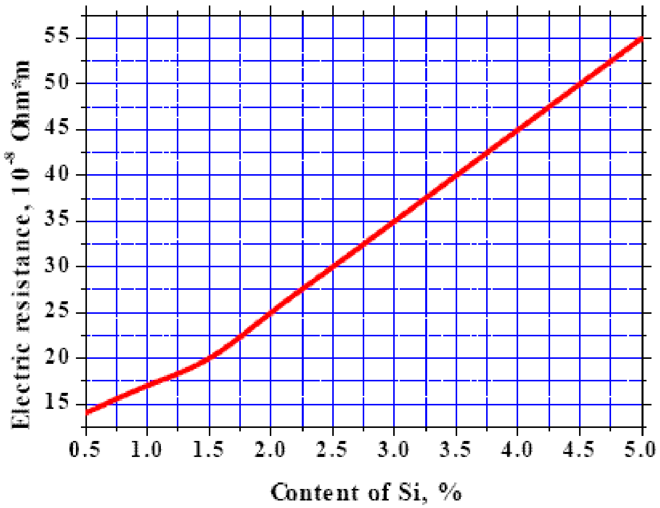


(Courtesy of E. Fischer)

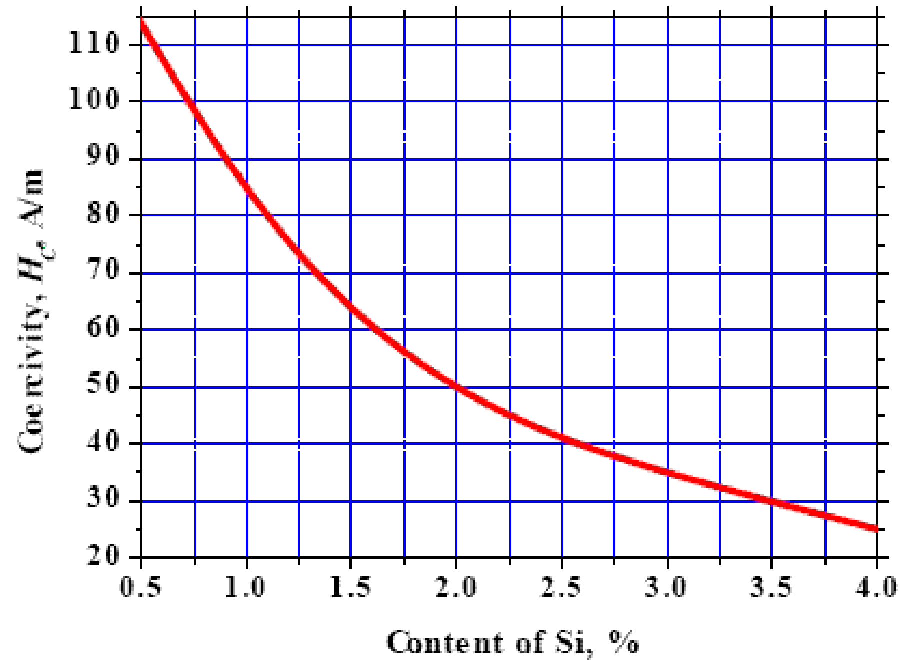
Laminated yoke : choice of iron

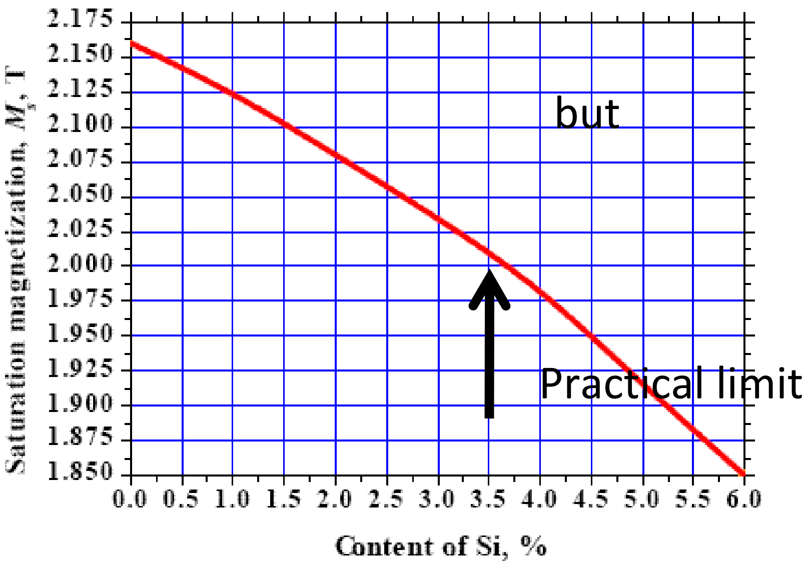
Low carbon silicon steel reduces

Eddy current losses due to higher resistivity



Hysteresis losses due to lower coercivity





P. Shcherbakov et al., Design Report SIS 300 6T dipole (2004)

Iron losses (steel supplier)

Note: Steel suppliers give typically total losses at 50 Hz

Total losses = eddy losses ($\sim \nu^2$)
+ hysteresis losses ($\sim \nu$)
+ anomalous losses ($\sim \nu^{1.5}$)

Eddy losses:

$$P\left[{W}/{m^3}\right] = \frac{\pi^2 \nu^2}{6\rho} B_p^2 d^2$$

ν-frequency (Hz)
d -lamination thickness (m)
ρ-resistivity (Ohm*m)
B_p- Induction amplitude (T)

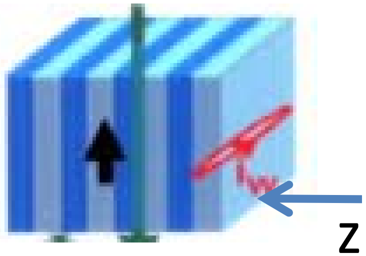
Hysteresis losses: from measurements wit a permeameter

Anomalous losses = rest

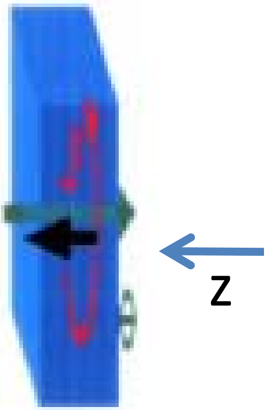
P. Fabbriccatore, et al.
Technical design Report SIS 300
4.5T model dipole

Yoke design of pulsed magnets

- 2D-design (ideal) → No B_z, only B_x, B_y
  - Appropriate lamination thickness d (practical limit 0.3 mm)
  - Low steel conductivity
  - Low coercivity (to reduce the hysteresis losses)



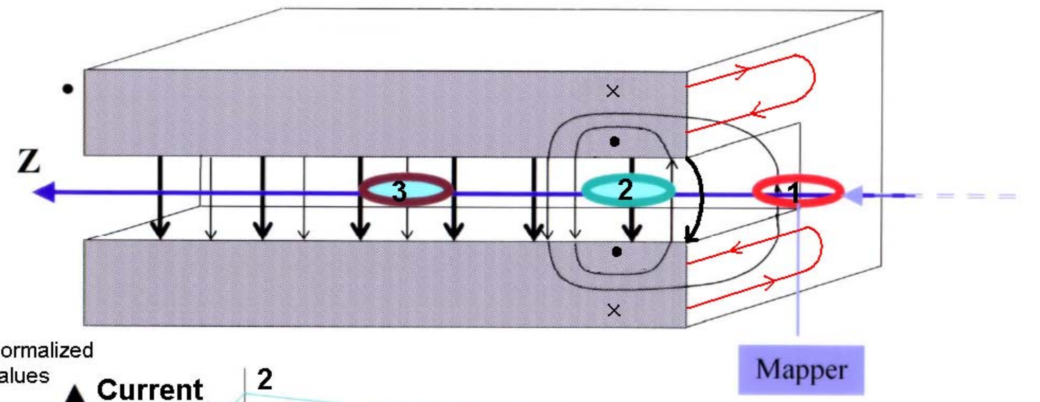
- 3D-design → exist B_z → eddy currents in the lamination sheet surface
  - Yoke end region
  - Areas with low packing factor

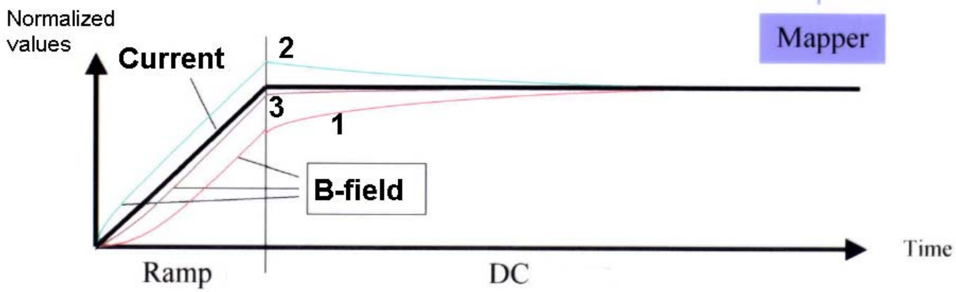


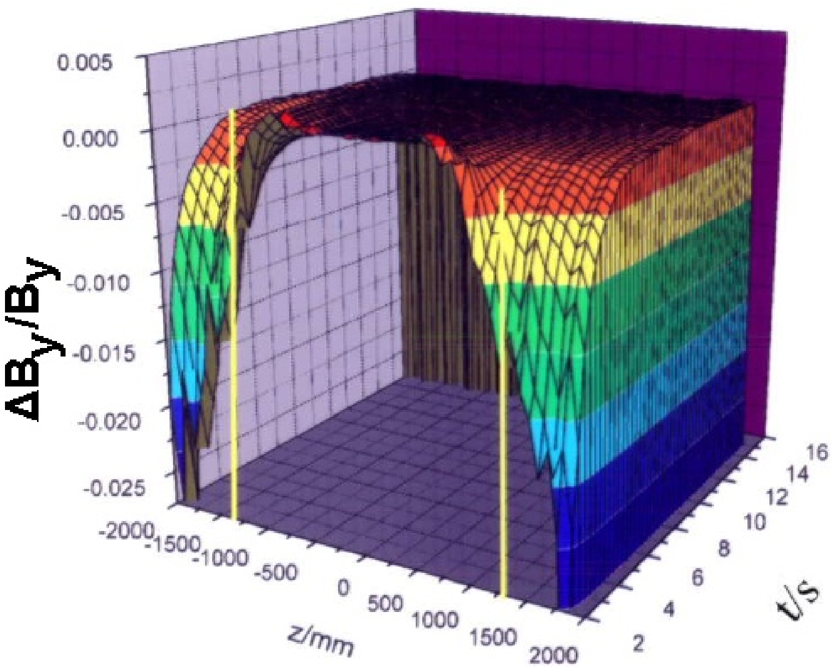
Yoke end region

Large B_z components (up to 2.5T depending on the magnet)

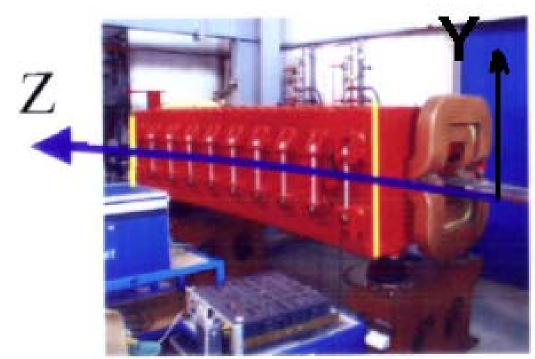
→ Source of eddy currents near the magnet end







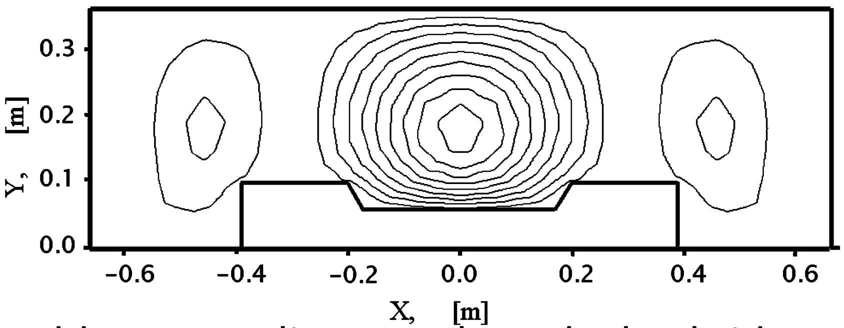
GSI: SIS18 dipole



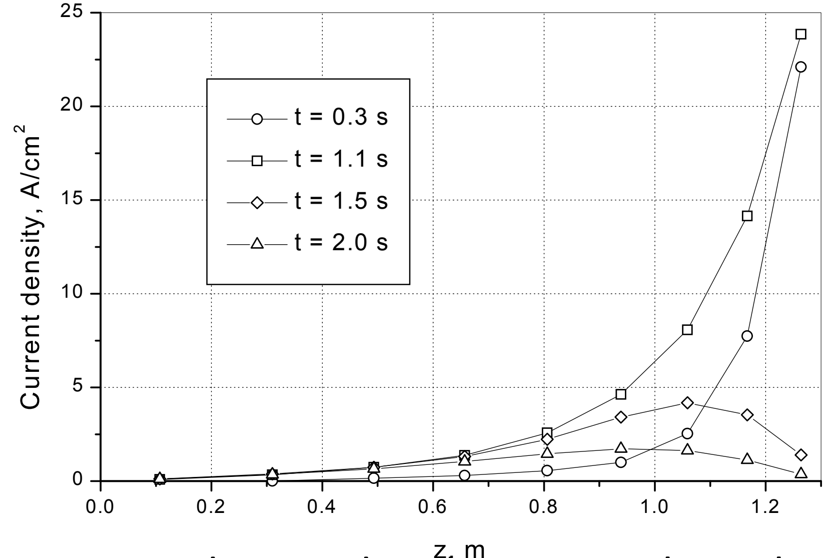
Courtesy of F. Klos

Results

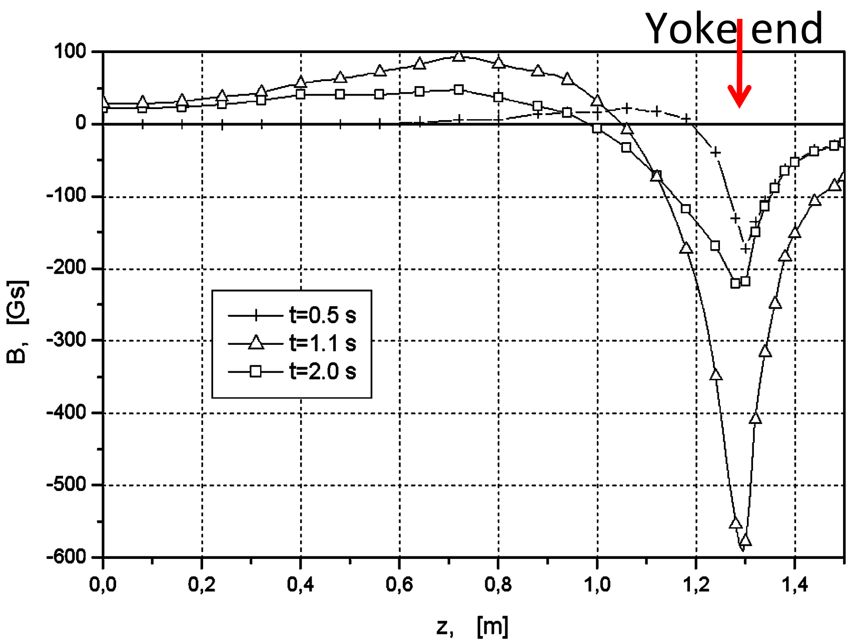
A.Kalimov, et al., IEEE Transactions on Applied Superconductivity., vol.12, No.1 pp. 98-101, 2002 (MT17)



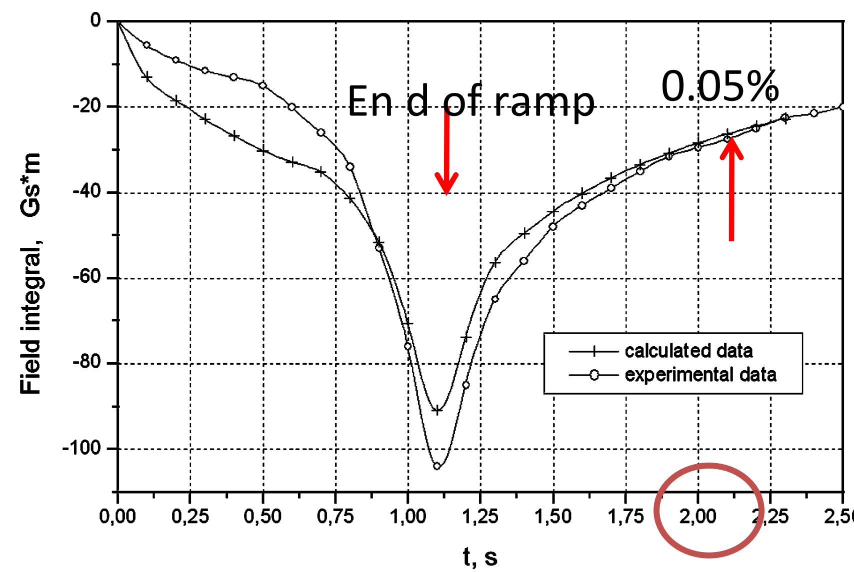
Eddy current lines on the yoke backside surface.



. Current density distribution on the pole surface along the central line of the magnet

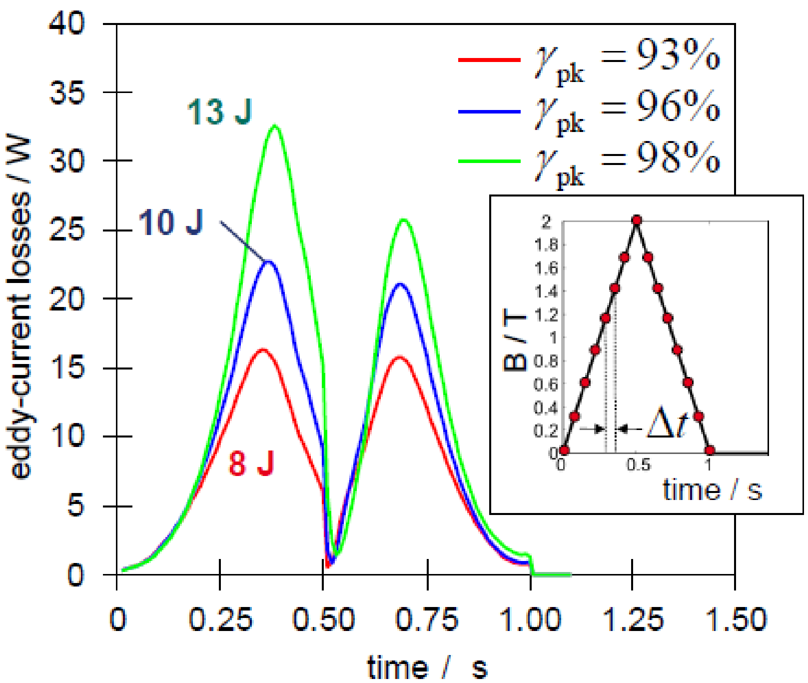


Aperture field induced by the eddy currents

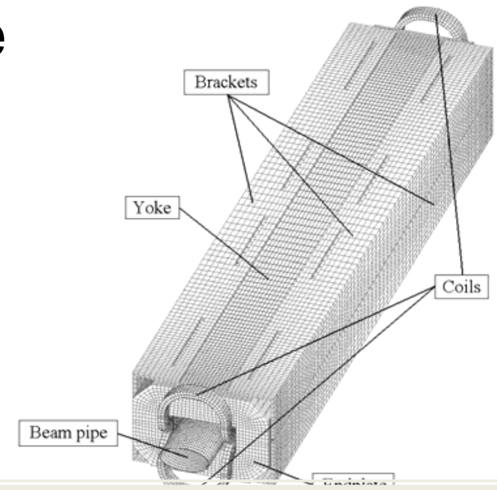


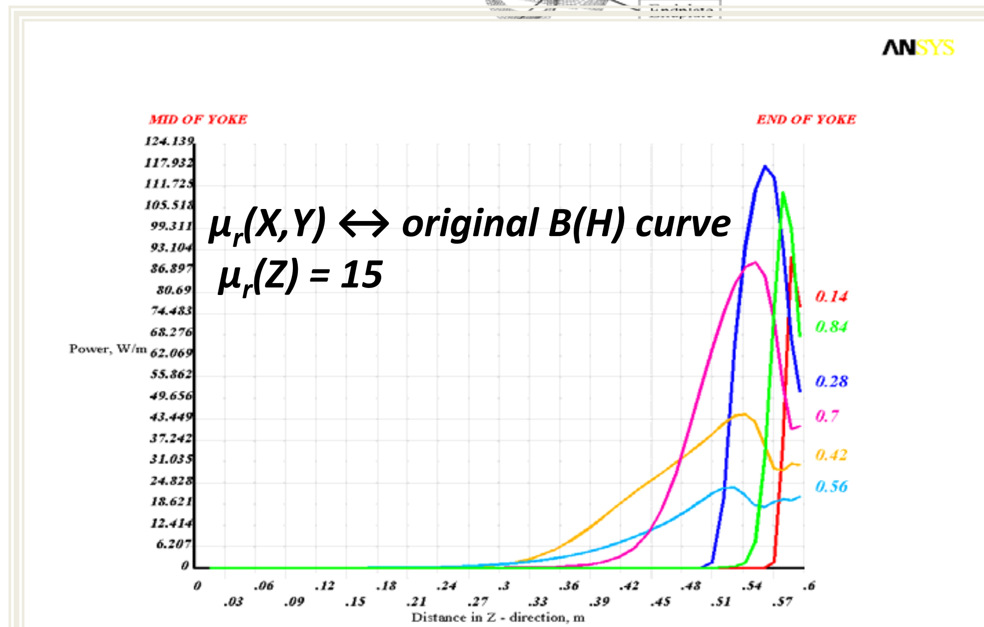
Field integral: difference between static and dynamic value. field integral max: 48000 Gs.m

SIS100 sc dipole model – laminated yoke



eddy current power in laminated yoke (S. Koch, H. de Gersem, T. Weiland (TU Darmstadt))

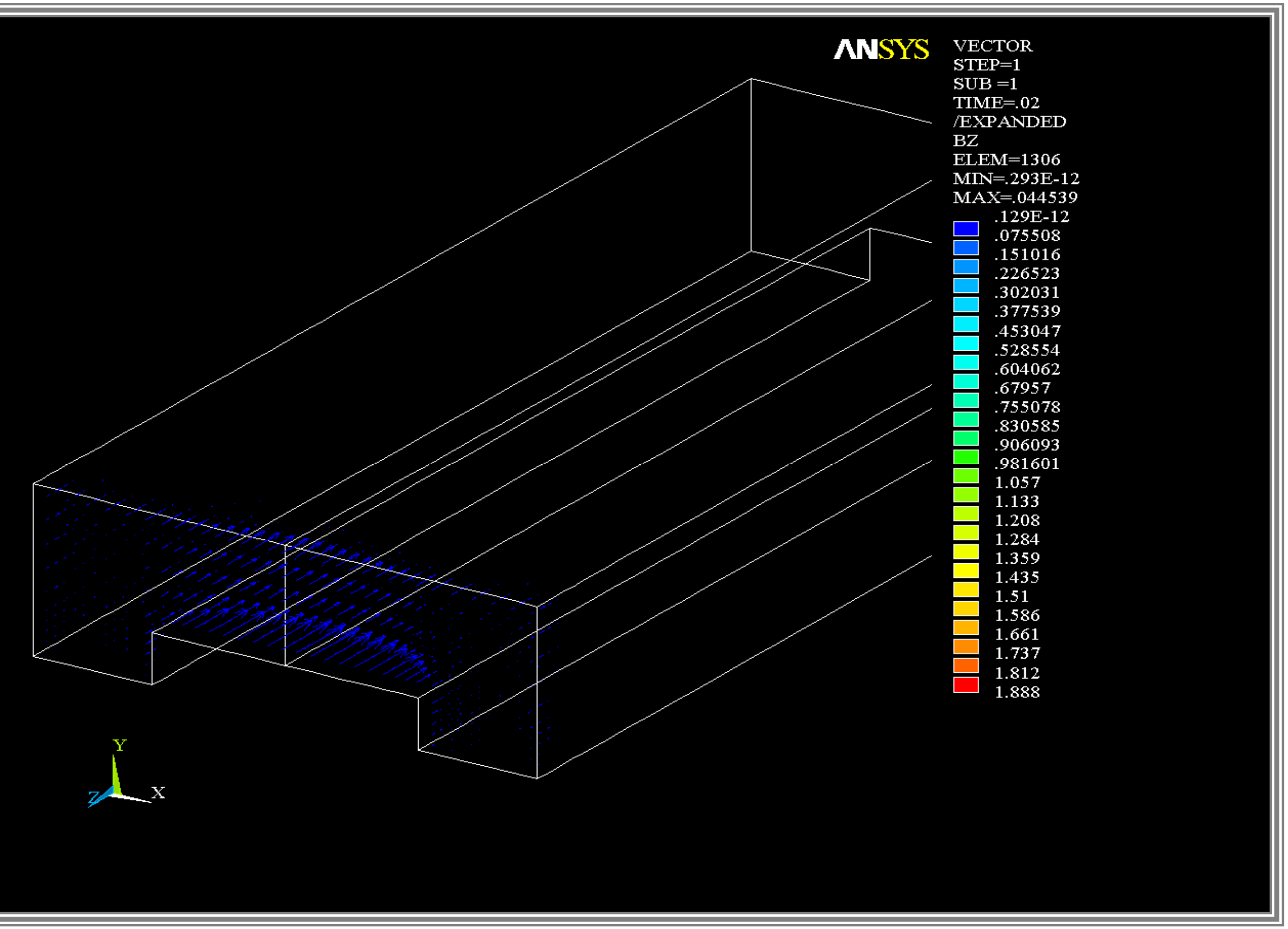




Eddy current power in the yoke along the yoke

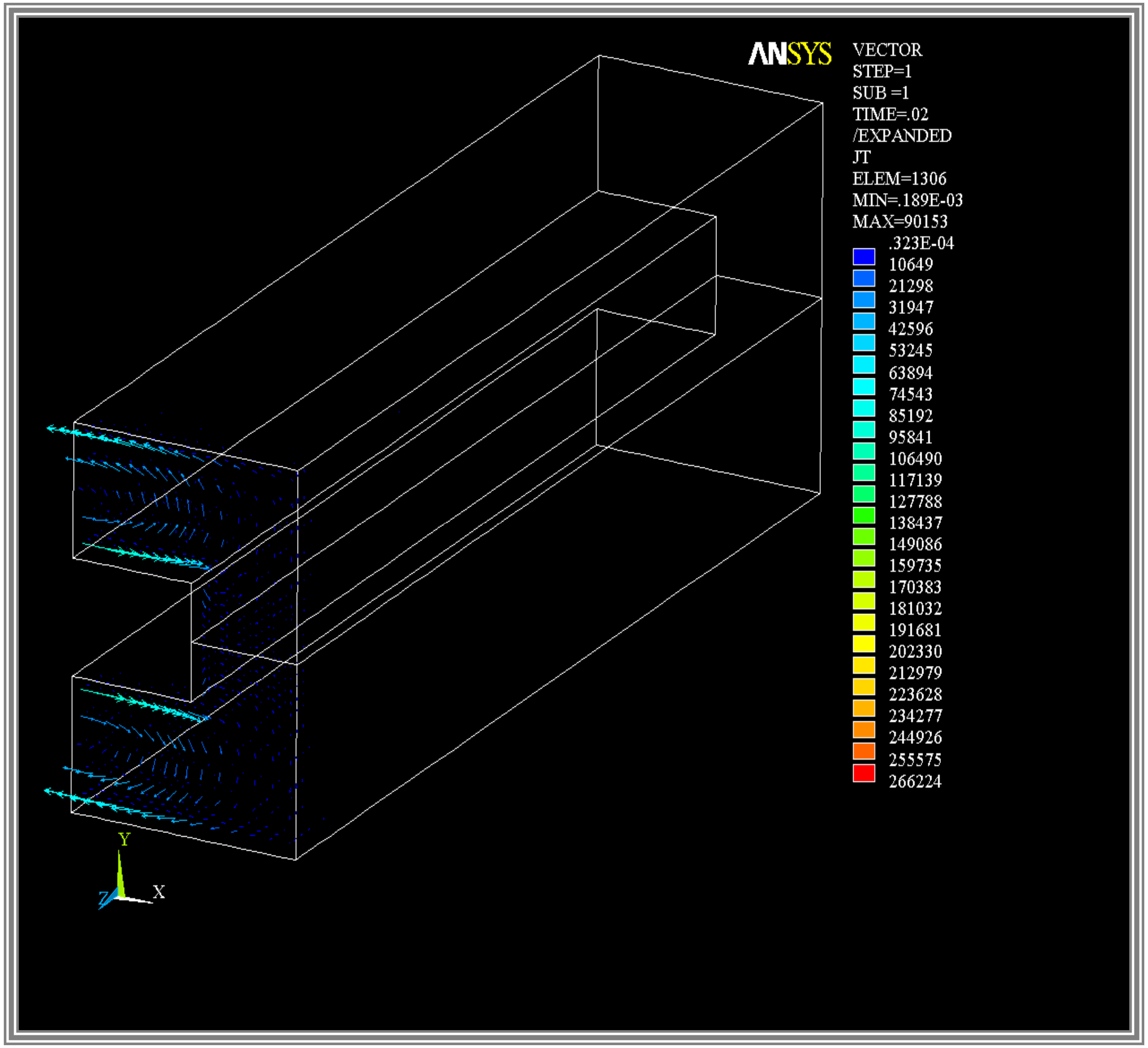
E. Fischer et al. / Cryogenics 47 (2007) 583–594

Vectors of B_Z in yoke vs time (μ_z=25)



R. Kurnyshov et al., Report on FE-R&D, Contract No. 5, GSI, October 2008

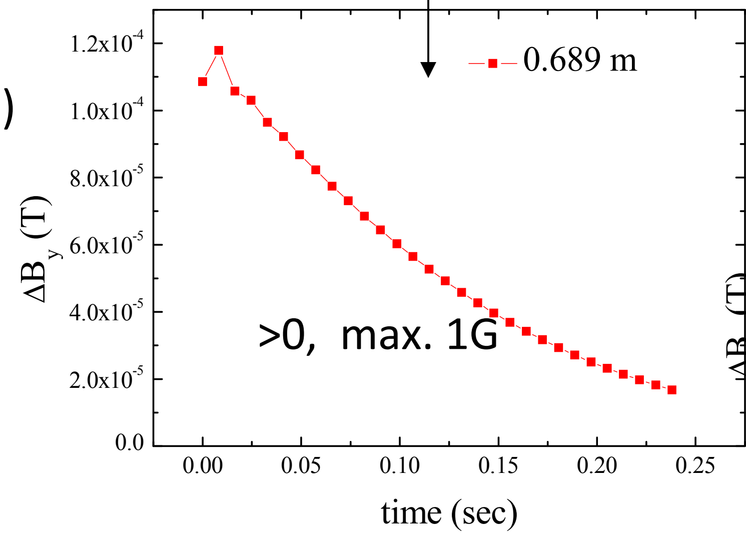
Vectors of eddy current density in yoke vs time (μ_z=25)

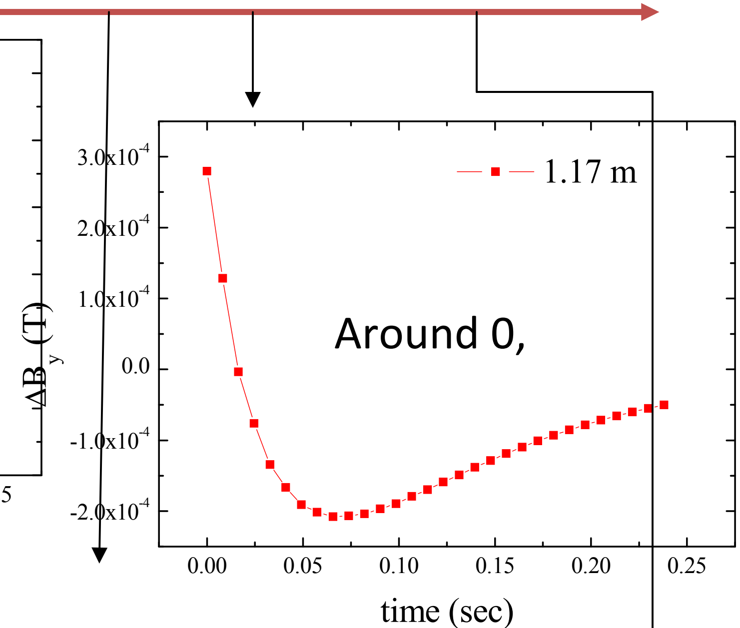


R. Kurnyshov et al.,
Report on FE-R&D,
Contract No. 5, GSI,

Field relaxation at different longitudinal positions

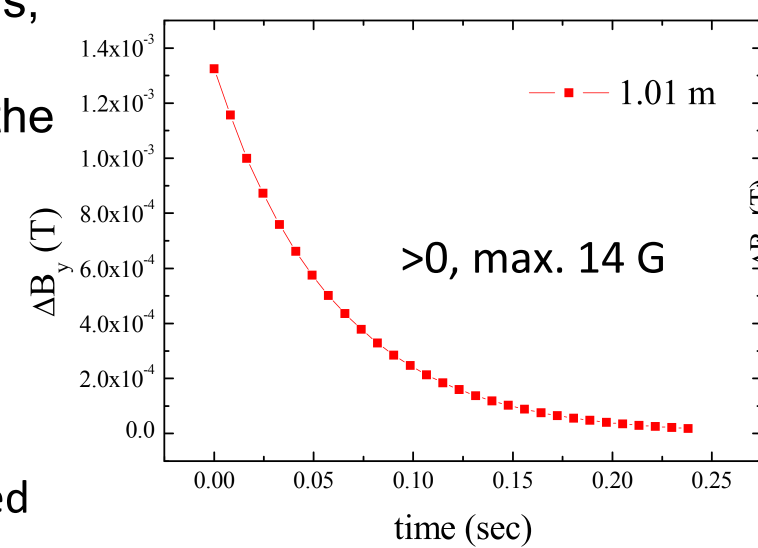
Z=0
(Center of the magnet)

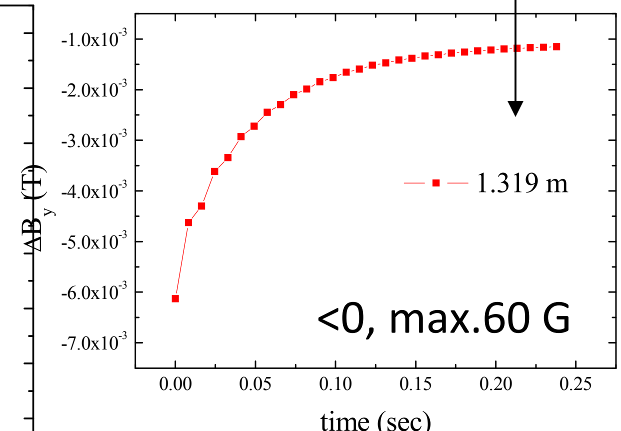




SIS100 dipole

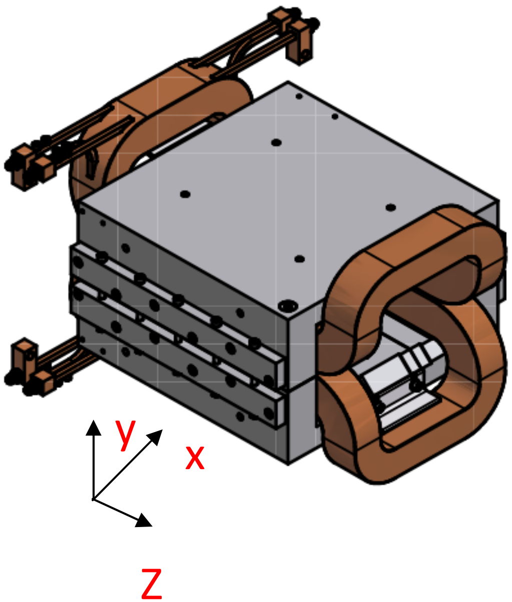
- Linear ramp, up with 4 T/s,
- 1.9 T
- All curves start at t=0 at the end of the ramp





S. Y. Shim, to be published

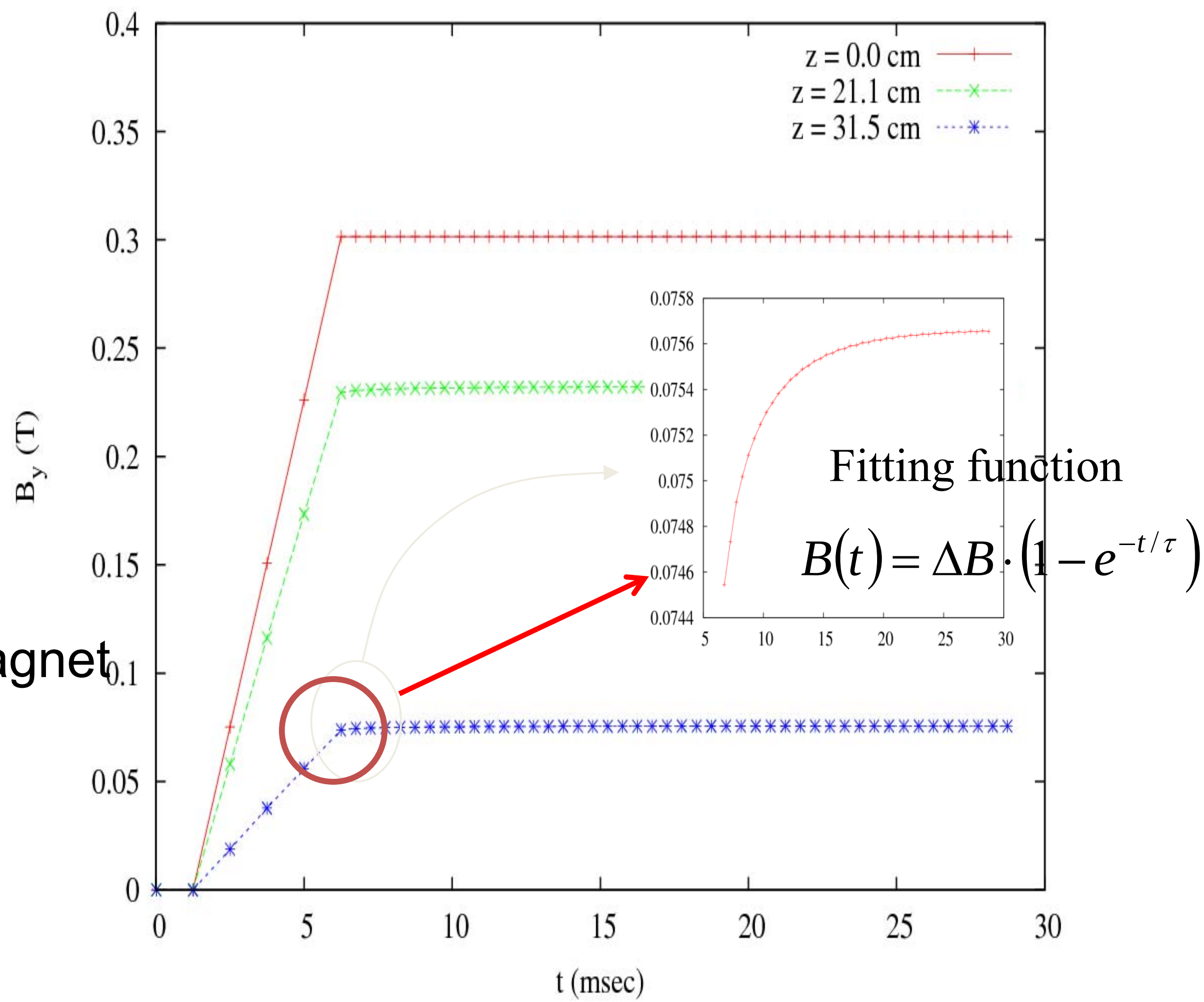
Transient field behaviour after a linear ramp



CNAO Scanning Dipole Magnet

max. center field: 0.3 T
500 T/s
Lamination thickness: 0.3 mm

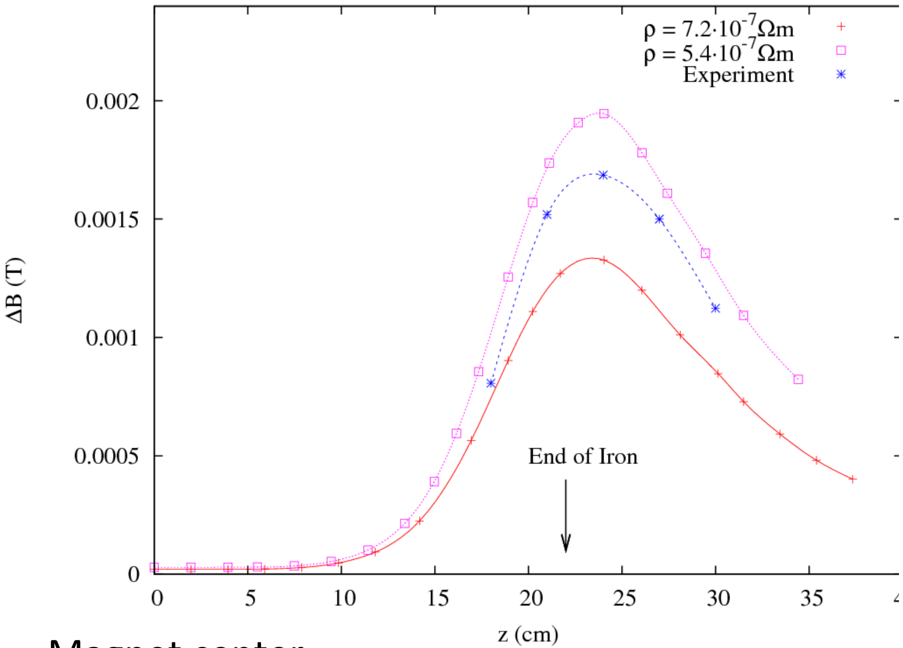
z= 0 magnet center,
z=22.0 cm: end of yoke



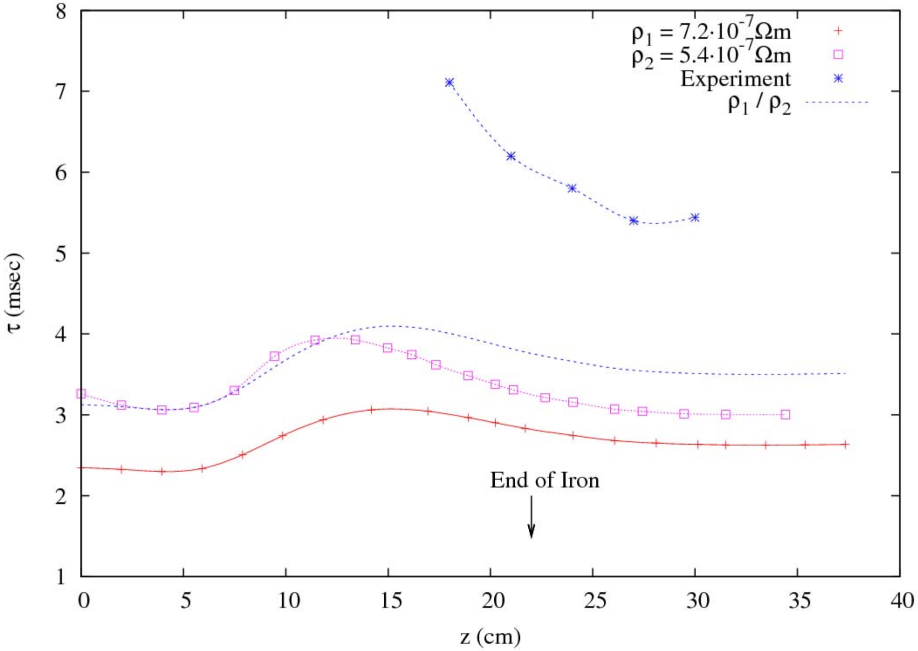
Fitting function

$$B(t) = \Delta B \cdot \left(1 - e^{-t/\tau}\right)$$

Field delay



Diffusion time constant



Magnet center

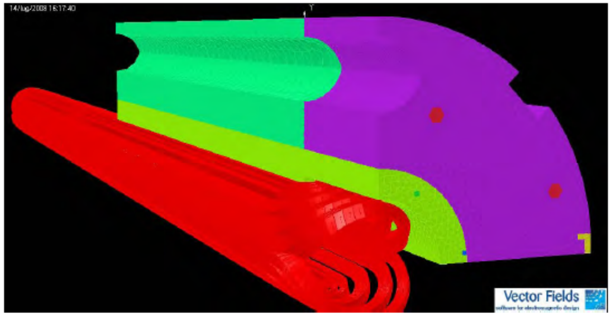
ΔB_{y}_max ≅ 0.7 % of maximum operation field

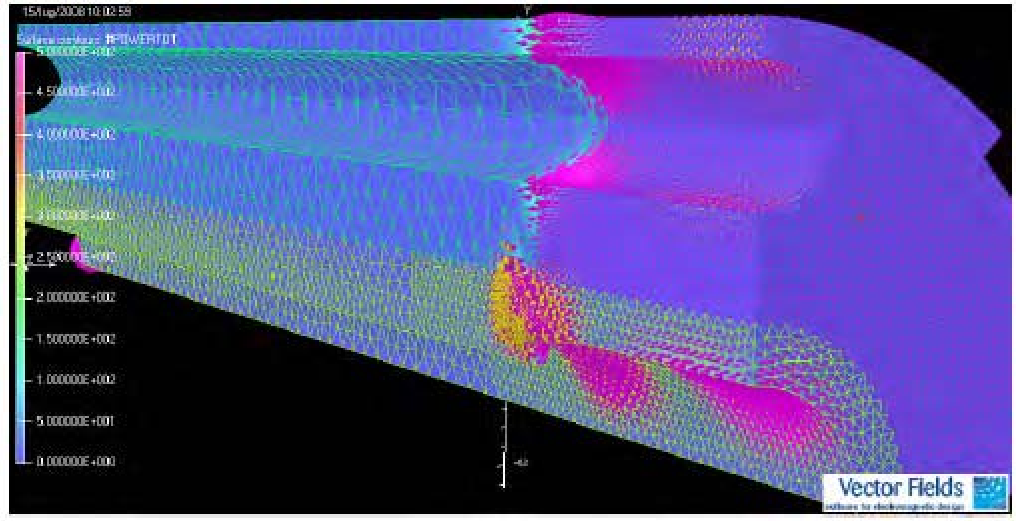
Time constant ≅ some milliseconds

Remark: In a medical accelerator like CNAO the beam energy is varied in small time steps less then the diffusion time constant!

Courtesy of S. Y. Shim

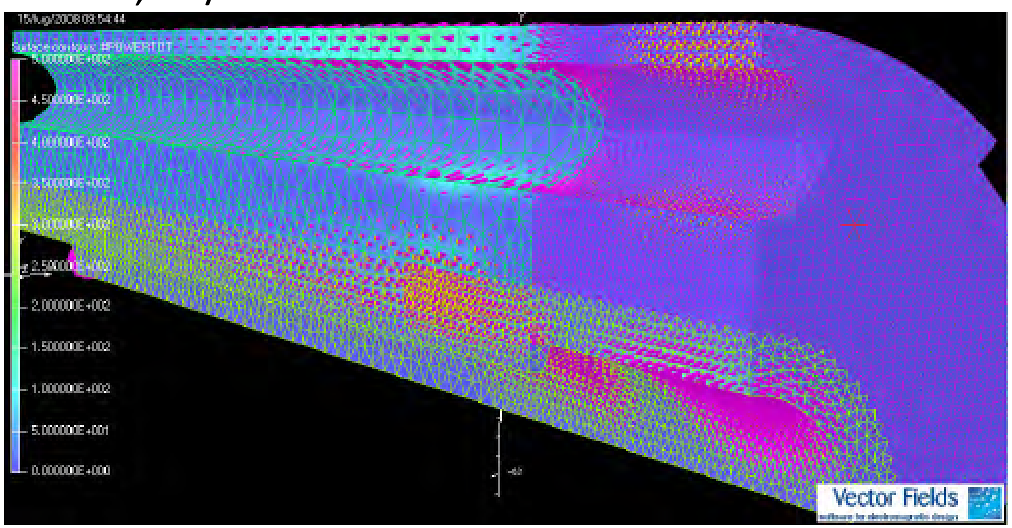
SIS 300 Dipole- eddy currents(direction and current density in the magnet ends





3.0 T, 1T/s

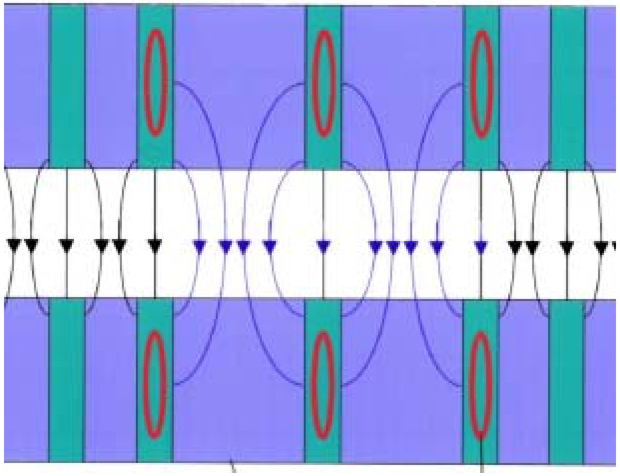
4.5 T, 1T/s



M. Sorbi, et al., “Electromagnetic Design of the Coil-Ends for the FAIR SIS300 Model Dipole,” in Proc. ASC’08, Chicago, 2008

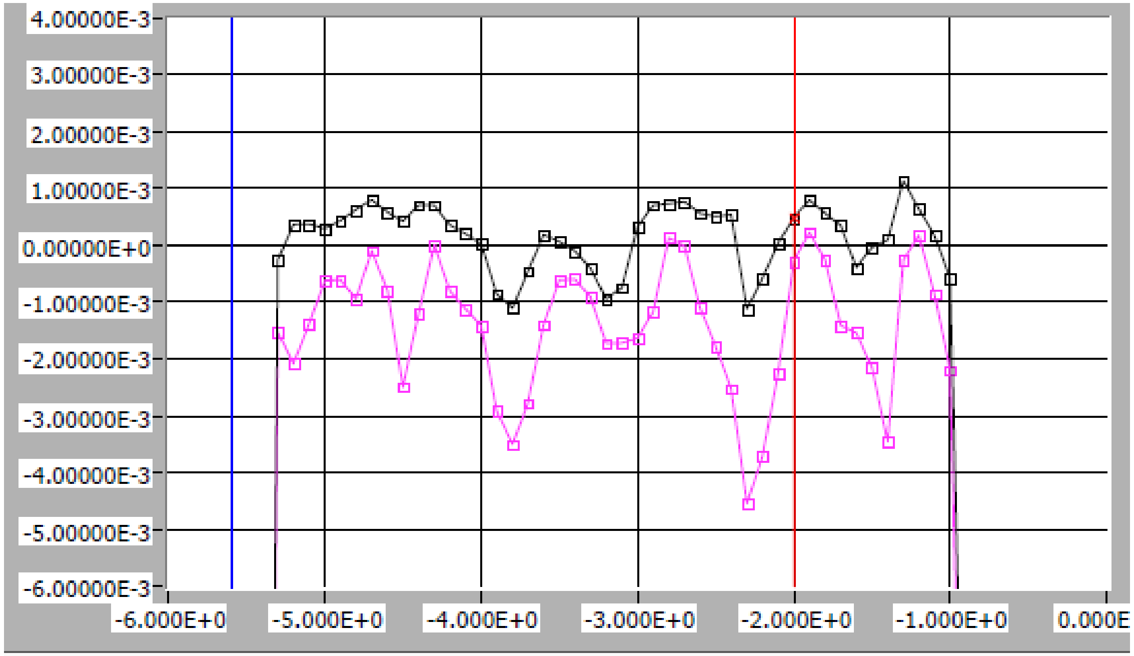
Variation of the packing factor along the magnet

(Synchrotron dipole of the HIT-facility Heidelberg)



Six block structure leads to

- Variation of the DC-field
- Field reduction by eddy currents (induced by local Bz components) in the AC-case

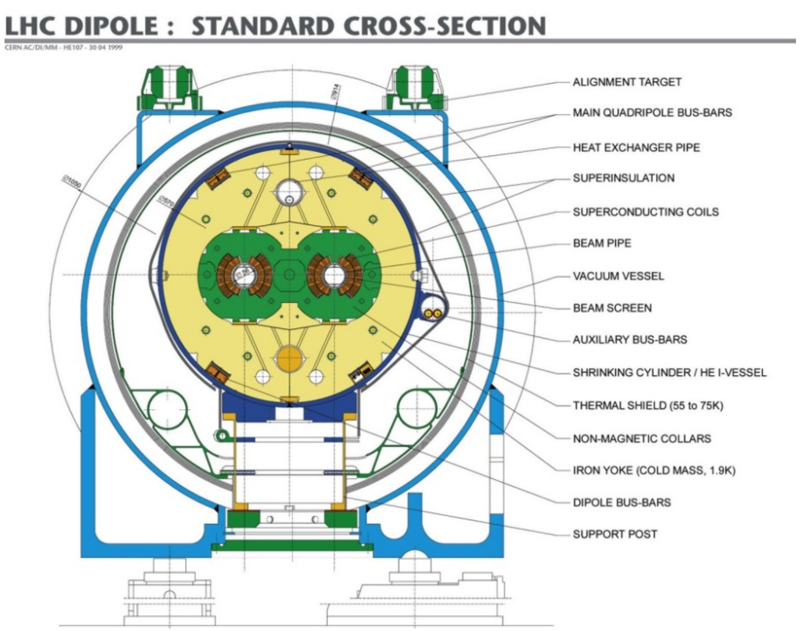


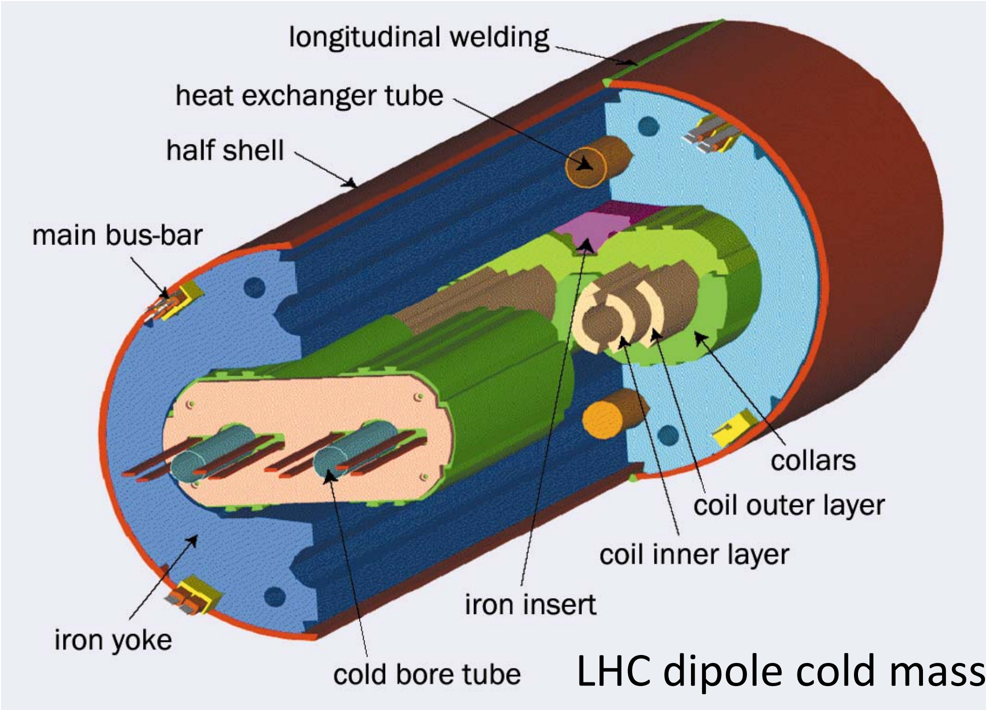
Courtesy of F. Klos

Eddy currents in mechanical structure

- Brackets
- Endplates
- Collar pins, Collar keys, Rods
- Shield, shell,

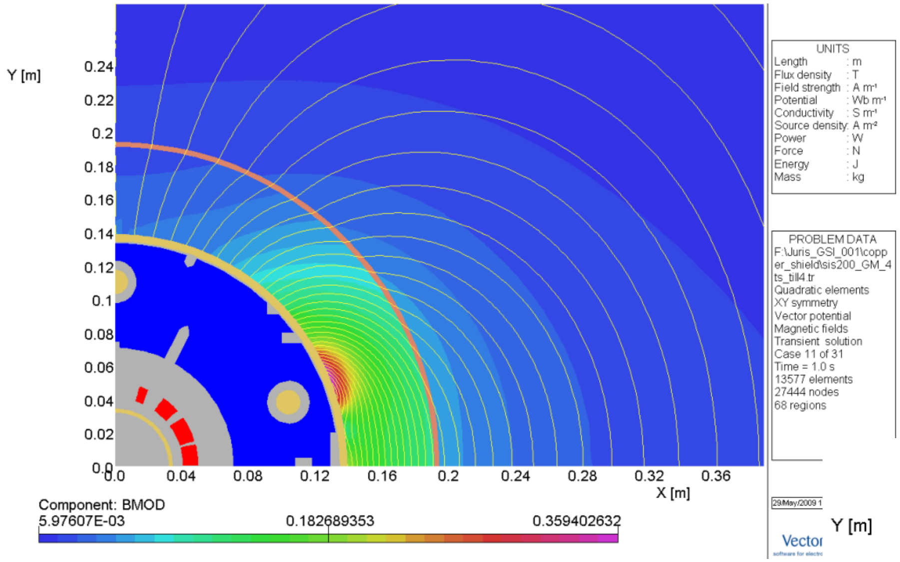
Try to avoid closed flux loops ! (for example by welding seams at the pole)





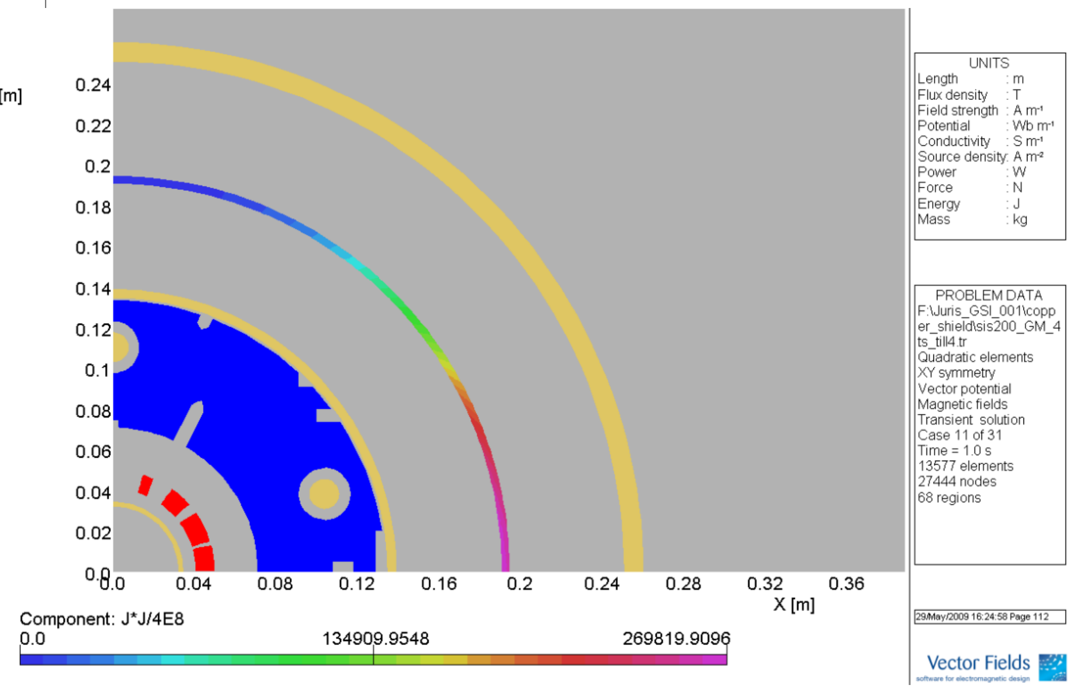
LHC dipole cold mass

Example: eddy curents in the copper shield of a sc dipole



R&D magnet GSI001

Bmod at 4T nominal field

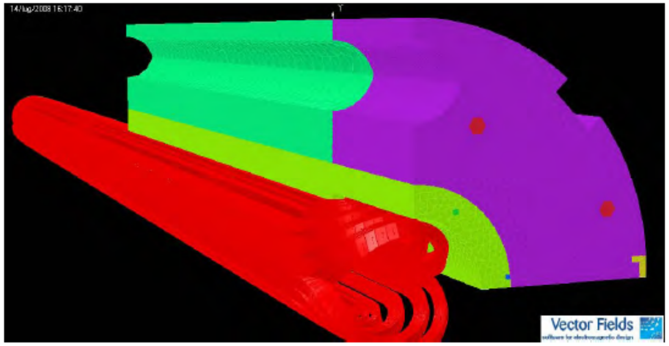


Power density (W/m^3) in the copper shield (resistivity: 2.5 nΩm) at 4T nominal field and a ramp rate of 4T/s.

Maximum power density: 270 kW/m^3

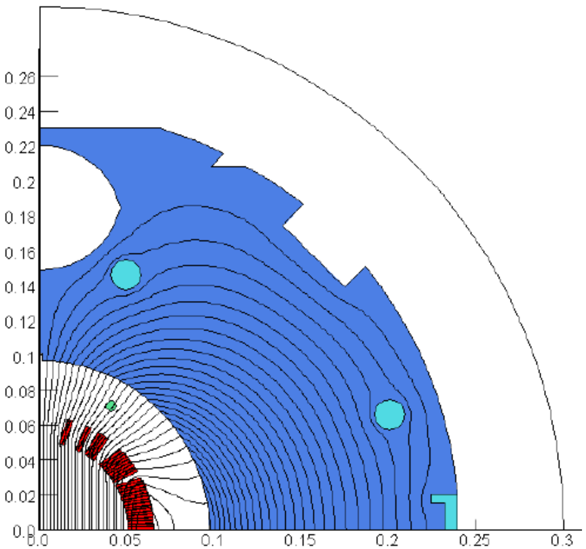
Courtesy of H. Leibrock

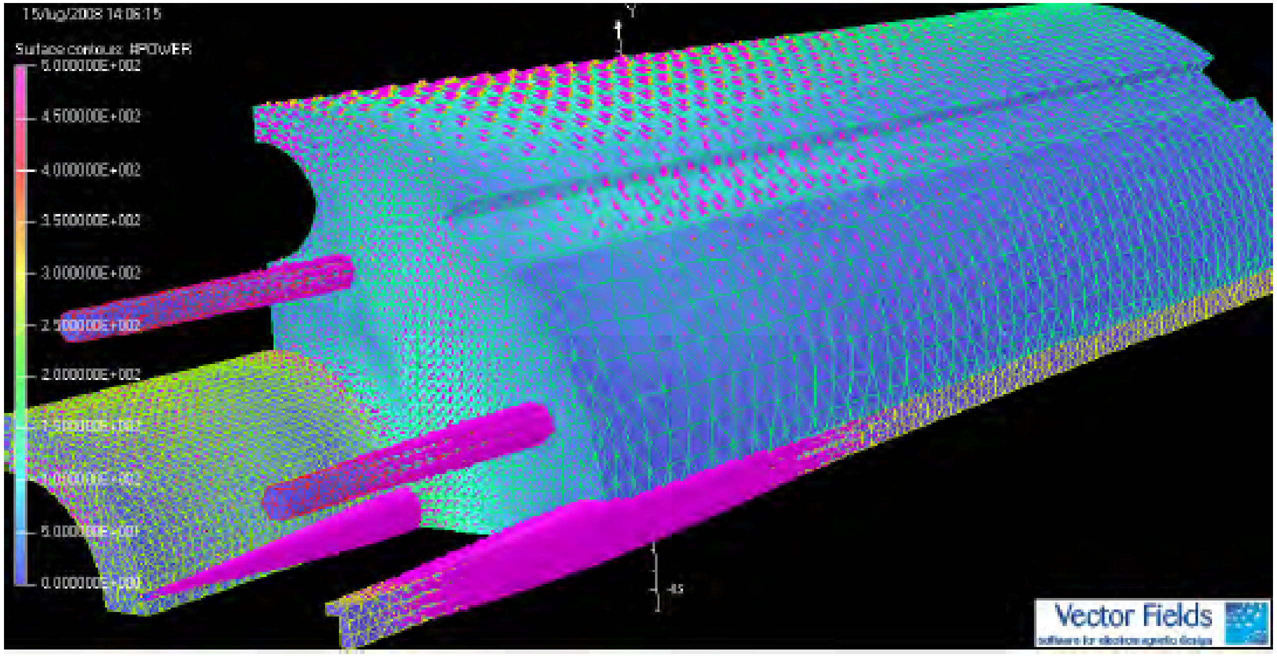
Eddy currents in Pins, Rods and Keys (SIS 300 dipole)



If possible: insulate them
to avoid closed flux looops!

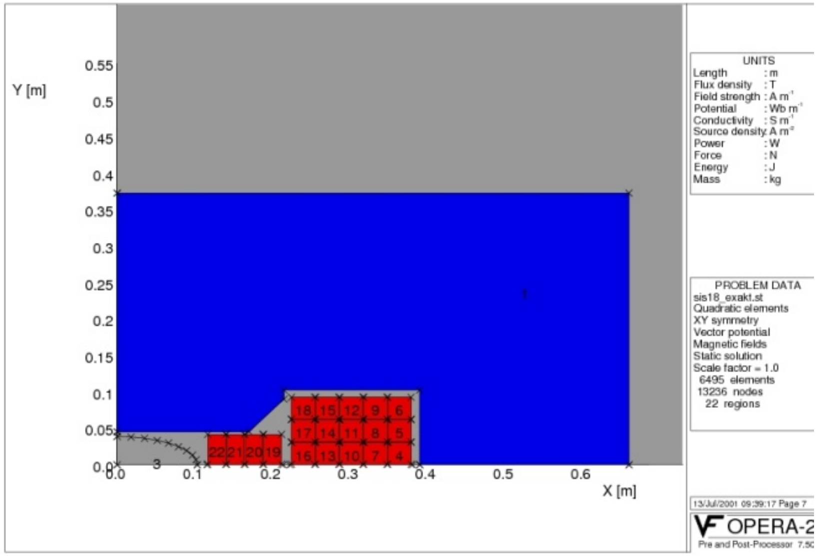
M. Sorbi et al.
Technical Design Report
SIS 300 4.5T model
dipole



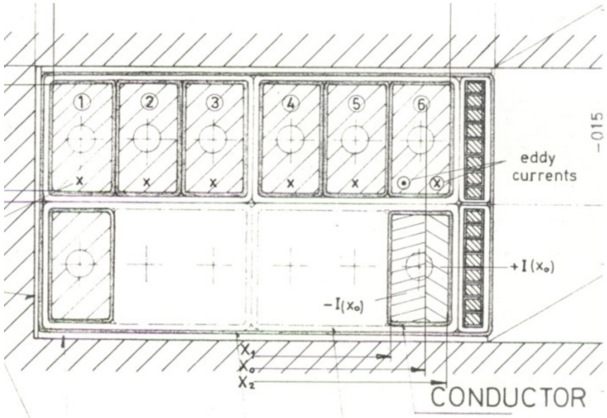


Flux-loop
minimized!!

Eddy currents in resistive coil

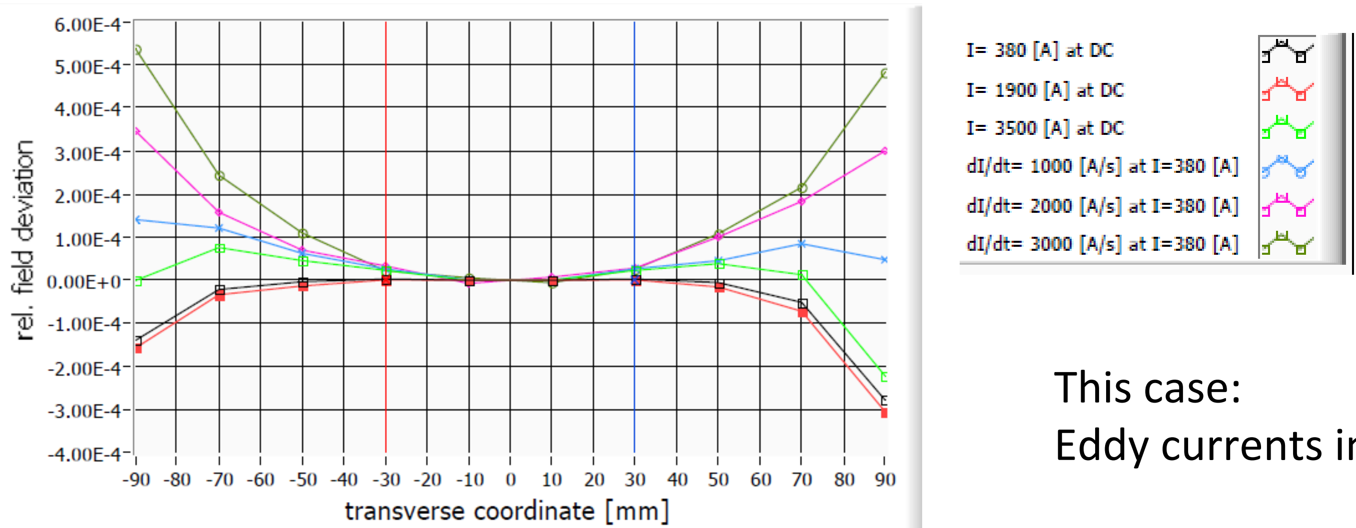


SIS 18 dipole



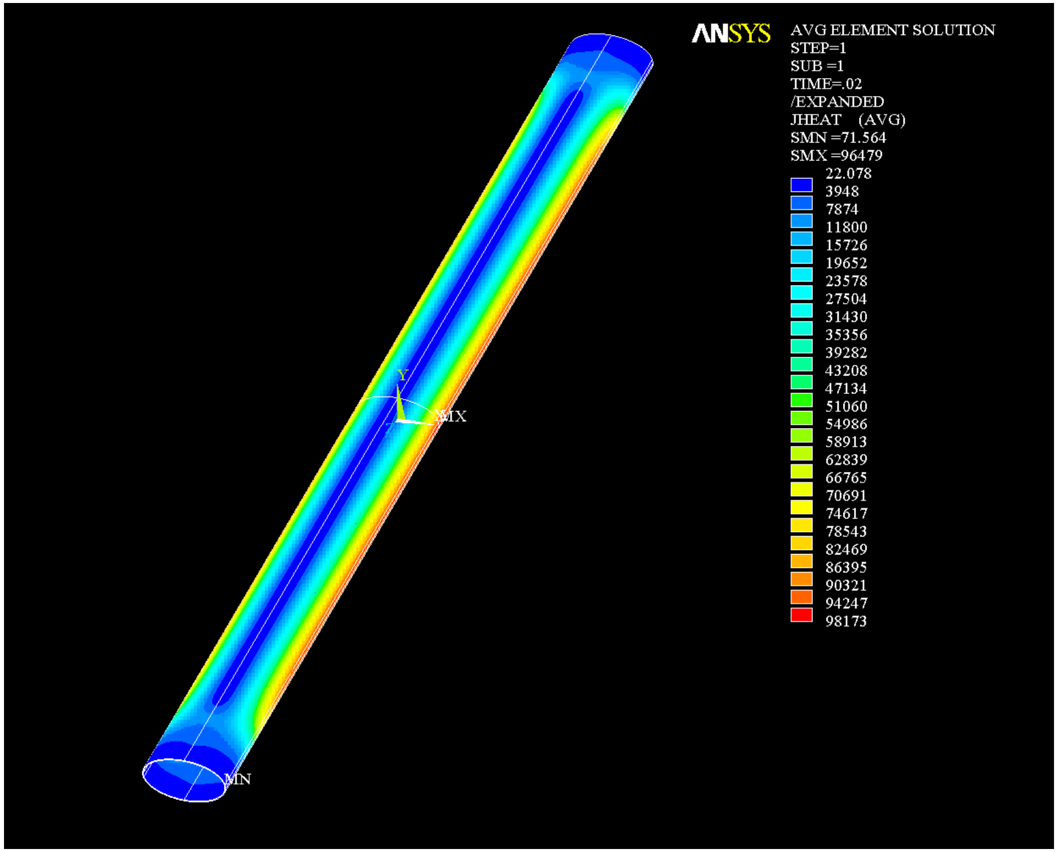
A. Asner et al., SI/Int.DL/69-2 9.6.1969 (Booster Bending Magnet)

Application of Biot-Savart gives the eddy current contribution ito the field n the magnet gap



This case:
Eddy currents improve field quality!!

Elliptical beam pipe (SIS 100 dipole)



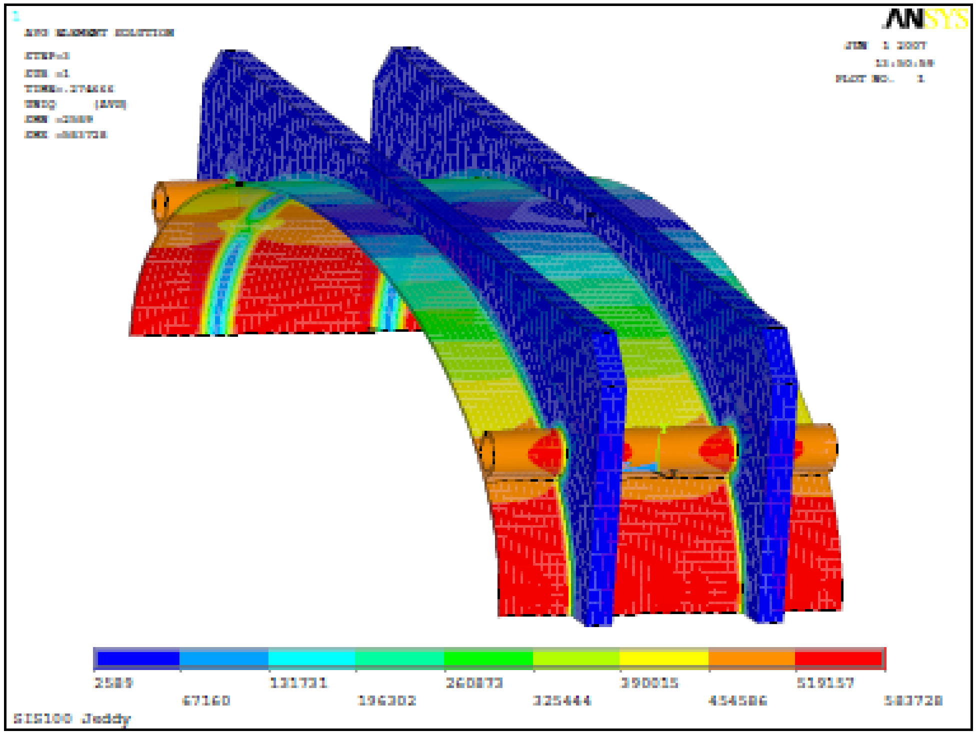
simulation of the eddy current loss density in the beam pipe
Thickness 0.3 mm
Average loss: 4.9 W/m (0-2T, 4T/s)

For the use as cryopump: need to be cooled!)

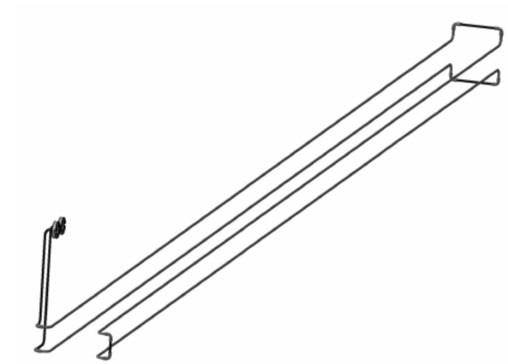
R. Kurnyshov et al., Report on FE-R&D, Contract No. 5, GSI, October 2008

Elliptical beam pipe (SIS 100 dipole)

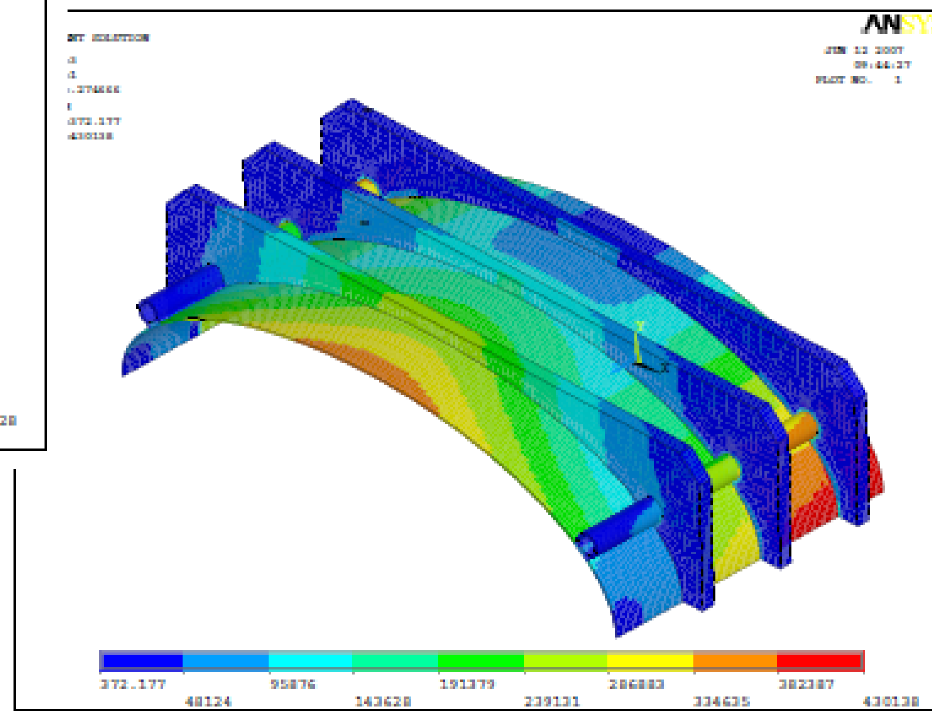
- complete 3D model with ribs and cooling pipe



Current density (A/m²), central part



Designed to avoid closed flux loops



Current density (A/m²), end part

Cycle:0 -2T, 4 T/s

Average loss (W/m):

- 4.9 pipe only
- 8.1 with tubes
- 8.7 with tubes and ribs

Courtesy of S. Y. Shim

Summary: Pulsed magnet design principles

(to minimize eddy current effects)

- Insulated laminations
- Choice of iron
- Appropriate magnet design
  - Avoid saturation (μ_r>>1)
  - Rogowski-profile of the magnet pole ends
  - Slits in the end laminations
  - Non-conductive material at the magnet ends
  - ‚long' magnets (also from the eddy current aspect!)
- Appropriate design of the mechanical structure
  - Choice of materials (non-conductive wherever possible)
  - Avoid ‚bulky' components
  - Avoid magnetic ‚flux loops'
- Field Control (‚B-Train')

References

- Books
  - Heinz E. Knoepfel, ‘Magnetic Fields’, John Wiley and Sons, INC. , New York…., 2000
  - Jack T. Tanabe, Iron dominated electromagnets: design, fabrication, assembly and measurements, WORLD Scientific 2005
  - Y. Iwasa, ‘Case studies in superconducting magnets’, Plenum Press, New York and London, 1994
- Reviews
  - K. Halbach, ‘Some eddy current effects in solid core magnets’, Nuclear Instruments and Methods, 107 (1973), 529-540
  - E.E. Kriezis et al., 'Eddy Currents: Theory and Applications', Proceedings of the IEEE, Vol. 80, NO. 10. October 1992, p.1599-1589

Acknowledgement:
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