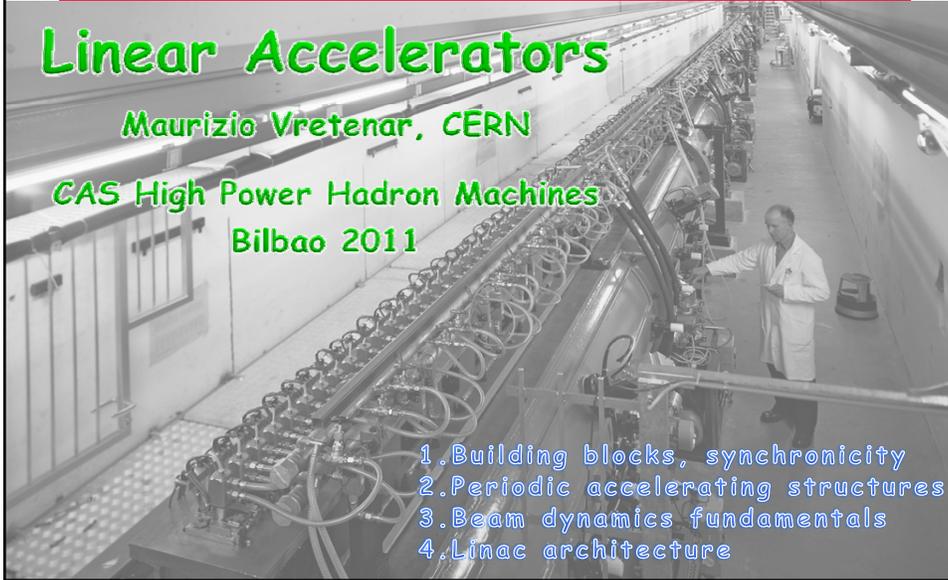# CAS

Linear Accelerators

Maurizio Vretenar, CERN

CAS High Power Hadron Machines
Bilbao 2011

1. Building blocks, synchronicity
2. Periodic accelerating structures
3. Beam dynamics fundamentals
4. Linac architecture

CAS

1. Introduction:
building blocks, synchronicity
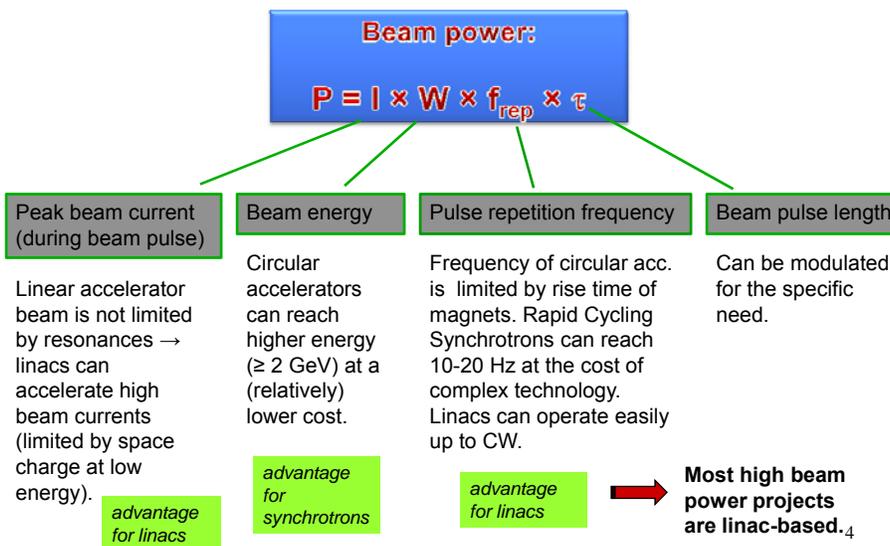
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CAS Why Linear Accelerators

Linear Accelerators are used for:

1. <u>Low-Energy acceleration</u> (injectors to synchrotrons or stand-alone): for protons and ions, linacs can be synchronous with the RF fields in the range where velocity increases with energy. When velocity is ~constant, synchrotrons are more efficient (multiple crossings of the RF gaps instead of single crossing).
   Protons : β = v/c =0.51 at 150 MeV, 0.95 at 2 GeV.

2. <u>High-Energy acceleration</u> in the case of:

   ➤ Production of <u>high-intensity proton beams</u>
      in comparison with synchrotrons, linacs can go to higher repetition rate, are less affected by resonances and have more distributed beam losses. Higher injection energy from linacs to synchrotrons leads to lower space charge effects in the synchrotron and allows increasing the beam intensity.

   ➤ <u>High energy linear colliders for leptons</u>, where the main advantage is the absence of synchrotron radiation.

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CAS Beam power: linear vs. circular

Beam power:

$$P = I \times W \times f_{rep} \times \tau$$

Peak beam current (during beam pulse)	Beam energy	Pulse repetition frequency	Beam pulse length
Linear accelerator beam is not limited by resonances → linacs can accelerate high beam currents (limited by space charge at low energy).	Circular accelerators can reach higher energy (≥ 2 GeV) at a (relatively) lower cost.	Frequency of circular acc. is limited by rise time of magnets. Rapid Cycling Synchrotrons can reach 10-20 Hz at the cost of complex technology. Linacs can operate easily up to CW.	Can be modulated for the specific need.
advantage for linacs	*advantage for synchrotrons*	*advantage for linacs*	**Most high beam power projects are linac-based.**

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CAS — Challenges

Generating the proton beams required for high-intensity applications is a challenge for the linear accelerator builder. Problems are:

1. Generation, acceleration and preservation of **high-brightness particle beams**. (brightness ~ density in phase space). Means defeating space charge (Coulomb repulsion), which is stronger at low energy.

2. Minimization of **beam loss**, for maintainability of the machine. Means a careful design of the beam optics, a compromise between efficiency and beam aperture and a careful collimation.

3. Designing machines with high **power efficiency**. Means mastering and optimizing the process of transferring energy to the beam.

4. Providing high **reliability and availability** of the accelerator (in particular for energy applications).

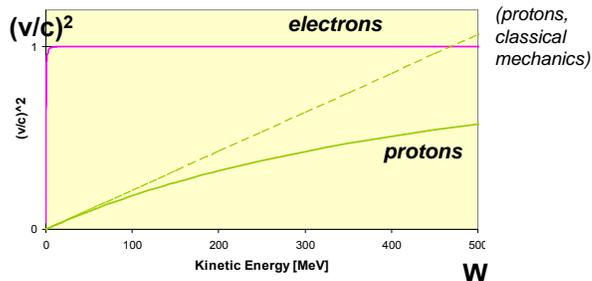5. Last but not least, at a **low cost** !

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CAS — Definitions

In <u>linear accelerators</u> the beam crosses only once the accelerating structures. They are used in the region where the <u>velocity of the particle beam increases</u> with energy

The periodicity of the RF structure must match the increasing particle velocity → development of a complete panoply of structures (NC and SC) with different features.
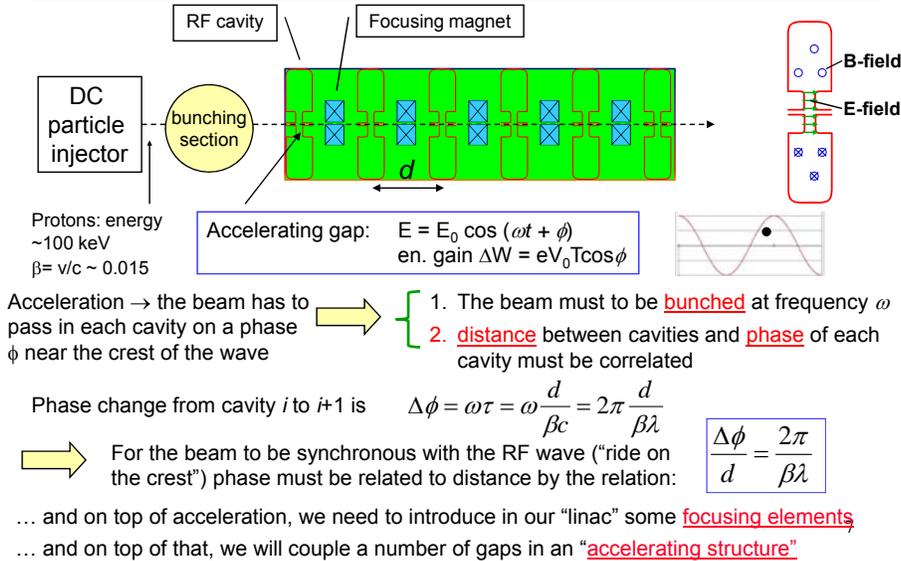
Protons and ions:
at the beginning of the acceleration, beta (=v/c) is rapidly increasing, but after few hundred MeV's (protons) relativity prevails over classical mechanics (β~1) and particle velocity tends to saturate at the speed of light.

The region of increasing β is the region of linear accelerators.

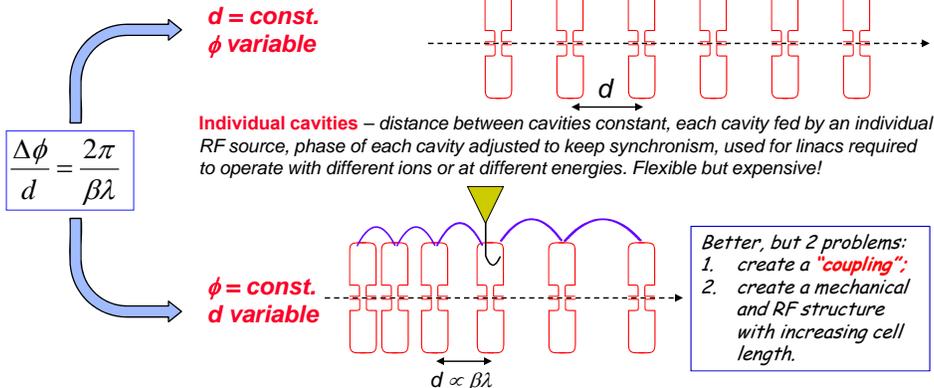The region of nearly constant β is the region of synchrotrons.

(protons, classical mechanics)

$(v/c)^2$

electrons

protons

Kinetic Energy [MeV]

W

Basic linear accelerator structure

CAS

RF cavity | Focusing magnet

B-field
E-field

DC particle injector

bunching section

d

Protons: energy ~100 keV
$\beta = v/c \sim 0.015$

Accelerating gap: $E = E_0 \cos(\omega t + \phi)$
en. gain $\Delta W = eV_0 T\cos\phi$

Acceleration $\rightarrow$ the beam has to pass in each cavity on a phase ϕ near the crest of the wave

1. The beam must to be bunched at frequency ω
2. distance between cavities and phase of each cavity must be correlated

Phase change from cavity i to i+1 is $\quad \Delta\phi = \omega\tau = \omega\dfrac{d}{\beta c} = 2\pi\dfrac{d}{\beta\lambda}$

For the beam to be synchronous with the RF wave ("ride on the crest") phase must be related to distance by the relation:

$$\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

… and on top of acceleration, we need to introduce in our "linac" some focusing elements

… and on top of that, we will couple a number of gaps in an "accelerating structure"

Accelerating structure architecture

CAS

When β increases during acceleration, either the phase difference between cavities $\Delta\phi$ must decrease or their distance d must increase.

d = const.
ϕ variable

d

$$\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

Individual cavities – *distance between cavities constant, each cavity fed by an individual RF source, phase of each cavity adjusted to keep synchronism, used for linacs required to operate with different ions or at different energies. Flexible but expensive!*

ϕ = const.
d variable

$d \propto \beta\lambda$

Better, but 2 problems:
1. create a *"coupling"*;
2. create a mechanical and RF structure with increasing cell length.

Coupled cell cavities - *a single RF source feeds a large number of cells (up to ~100!) - the phase between adjacent cells is defined by the coupling and the distance between cells increases to keep synchronism . Once the geometry is defined, it can accelerate only one type of ion for a given energy range. Effective but not flexible.*
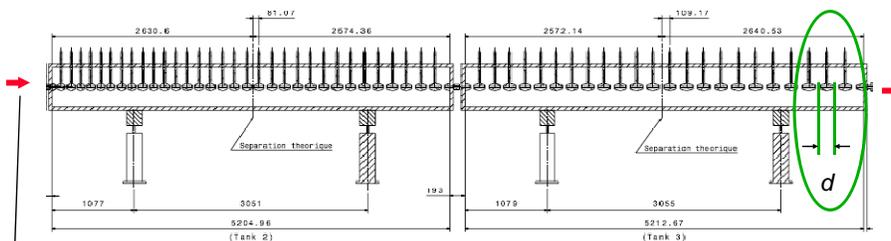
CAS Case 1: a single-cavity linac

The goal is flexibility: acceleration of different ions (e/m) at different energies
$\rightarrow$ *need to change phase relation for each ion-energy*

Cryomodule High Beta (5 Cavités Ø320)

RF cavity

focusing solenoid

REX-HIE linac
(to be built at CERN - <u>superconducting</u>)
Post-accelerator of radioactive ions
2 sections of identical equally spaced cavities
Quarter-wave RF cavities, 2 gaps
12 + 20 cavities with individual RF amplifiers, 8 focusing solenoids
Energy 1.2 $\rightarrow$ 10 MeV/u, accelerates different A/m

beam

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CAS Case 2 : a Drift Tube Linac

10 MeV, β = 0.145

50 MeV, β = 0.31

Tank 2 and 3 of the new Linac4 at CERN:
57 coupled accelerating gaps
Frequency 352.2 MHz, λ = 85 cm
Cell length (d=$\beta\lambda$) from 12.3 cm to 26.4 cm (factor 2 !).

Slide 1

But:

Between these 2 "extremes" there are many "intermediate" cases, because:

    a. Single-gap cavities are expensive (both cavity and RF source!).

    b. Structures with each cell matched to the beta profile are mechanically complicated and expensive.

→ as soon as the increase of beta with energy becomes small ($\Delta\beta/\Delta W$) we can accept a small error and:

1. Use multi-gap cavities with constant distance between gaps.

2. Use series of identical cavities (standardised design and construction).

d

$d \propto \beta\lambda$

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Slide 2

The distance between accelerating gaps is proportional to particle velocity

Beam

Electric field (at time t_0)

d=*distance between centres of consecutive cells*

Example: a linac superconducting 4-cell accelerating structure

Synchronism condition bw. particle and wave
t (travel between centers of cells) = $T/2$

$$\frac{d}{\beta c} = \frac{1}{2f} \quad \Rightarrow \quad d = \frac{\beta c}{2f} = \frac{\beta\lambda}{2}$$

1. In an ion linac cell length has to increase (up to a factor 200 !), the linac will be made of a sequence of different accelerating structures (changing cell length, frequency, operating mode, etc.) matched to the ion velocity.

2. In sequences of (few) identical cells where β increases (acceleration) only the central cell will be synchronous, and in the other cells the beam will have a phase error.

"phase slippage"

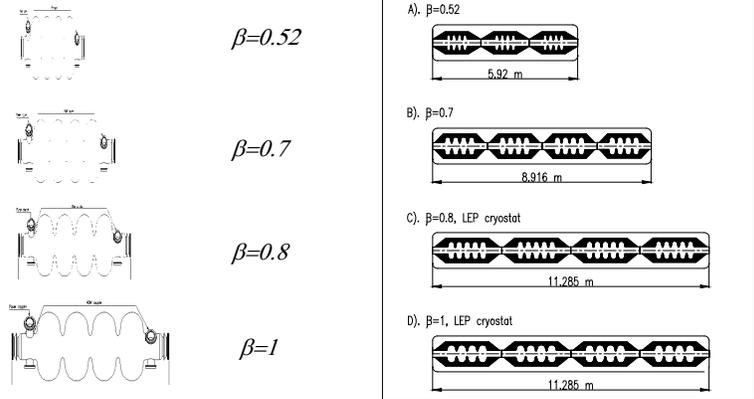$$\Delta\phi = \omega\Delta t = \pi\frac{\Delta\beta}{\beta}$$

= phase error on a gap for a particle with $\beta+\Delta\beta$ crossing a cell designed for β

High and unacceptable for low energy, becomes lower and acceptable for high β

6

Sections of identical cavities: a superconducting linac (medium β)

The same superconducting cavity design can be used for different proton velocities. The linac has different sections, each made of cavities with cell length matched to the average beta in that section. At "medium energy" (>150 MeV) we are not obliged to dimension every cell or every cavity for the particular particle beta at that position, and we can accept a slight "asynchronicity" → phase slippage + reduction in acceleration efficiency from the optimum one.

$\beta=0.52$

$\beta=0.7$

$\beta=0.8$

$\beta=1$

A). β=0.52
5.92 m

B). β=0.7
8.916 m

C). β=0.8, LEP cryostat
11.285 m

D). β=1, LEP cryostat
11.285 m

CERN (old) SPL design, SC linac 120 - 2200 MeV, 680 m length, 230 cavities

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Effects of phase slippage

When sequences of cells are not matched to the particle beta → phase slippage

$$\Delta\phi = \omega\Delta t = \pi\frac{\Delta\beta}{\beta}$$

1. The effective gradient seen by the particle is lower.
2. The phase of the bunch centre moves away from the synchronous phase → possible longitudinal emittance growth and beam loss.

$$\Delta\phi = \pi\frac{\Delta\beta}{\beta} = \pi\frac{1}{\gamma(\gamma-1)}\frac{\Delta W}{W}$$

Very large at small energy ($\gamma\sim1$) becomes negligible at high energy (~2.5 °/m for $\gamma\sim1.5$, W=500 MeV).
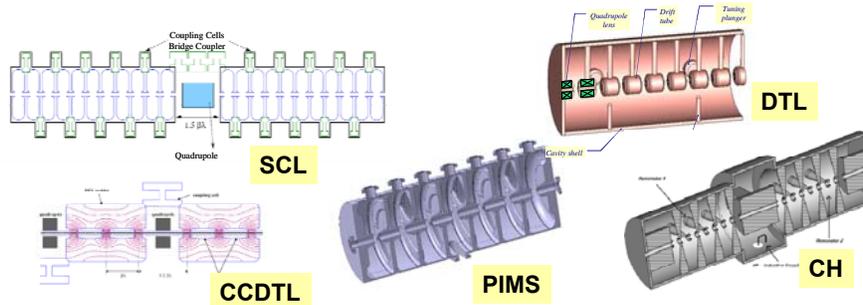
Curves of effective gradient (gradient seen by the beam for a constant gradient in the cavity) for the previous case (4 sections of beta 0.52, 0.7, 0.8 and 1.0).

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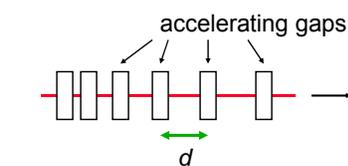
CAS Multi-gap coupled-cell cavities

Between these 2 extreme case (array of independently phased single-gap cavities / single long chain of coupled cells with lengths matching the particle beta) there can be a <u>large number of variations</u> (number of gaps per cavity, length of the cavity, type of coupling) each optimized for a certain <u>range of energy and type of particle</u>.

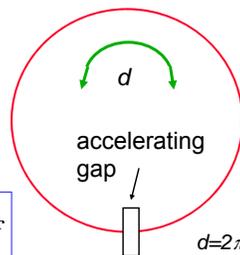The goal of this lecture is to provide the background to understand the main features of these different structures...

CAS Linear and circular accelerators

accelerating gaps

d

d=βλ/2=variable

$$d = \frac{\beta c}{2f} = \frac{\beta \lambda}{2} \; , \; \beta c = 2df$$

d

accelerating gap

d=2πR=constant

Linear accelerator:

Particles accelerated by a sequence of gaps (all at the same RF phase).

Distance between gaps increases proportionally to the particle velocity, to keep synchronicity.
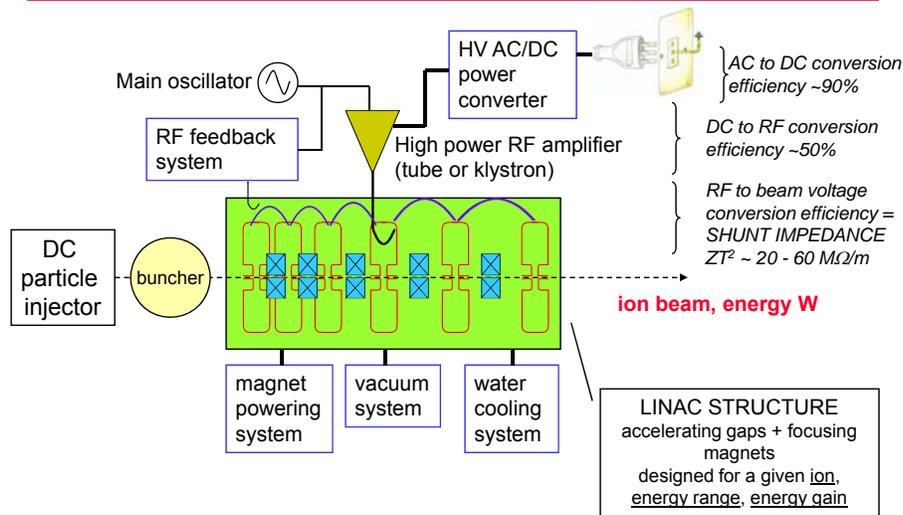
Used in the range where β increases. "Newton" machine

Circular accelerator:

Particles accelerated by one (or more) gaps at given positions in the ring.

Distance between gaps is fixed. Synchronicity only for β~const, or varying (in a limited range!) the RF frequency.

Used in the range where β is nearly constant. "Einstein" machine

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Linac building blocks

HV AC/DC power converter

Main oscillator

RF feedback system

High power RF amplifier (tube or klystron)

AC to DC conversion efficiency ~90%

DC to RF conversion efficiency ~50%

RF to beam voltage conversion efficiency = SHUNT IMPEDANCE $ZT^2 \sim 20 - 60\ M\Omega/m$

DC particle injector

buncher

ion beam, energy W

magnet powering system

vacuum system

water cooling system

LINAC STRUCTURE
accelerating gaps + focusing magnets
designed for a given ion,
energy range, energy gain

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Conclusions – part 1

What did we learn?

1. A linac is composed of an array of accelerating gaps, interlaced with focusing magnets (quadrupoles or solenoids), following an ion source with a DC extraction and a bunching section.

2. When beam velocity is increasing with energy ("Newton" regime), we have to match to the velocity (or to the relativistic β) either the phase difference or the distance between two subsequent gaps.

3. We have to compromise between synchronicity (distance between gaps matched to the increasing particle velocity) and simplicity (number of gaps on the same RF source, sequences of identical cavities). At low energies we have to follow closely the synchronicity law, whereas at high energies we have a certain freedom in the number of identical cells/identical cavities.
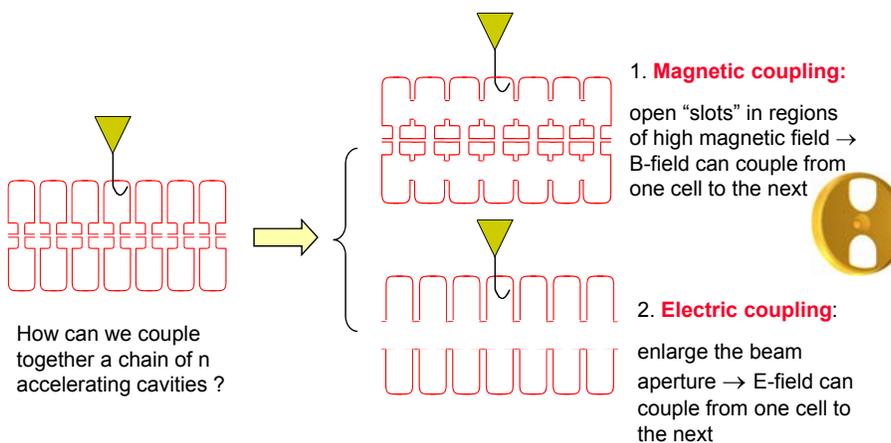
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CAS

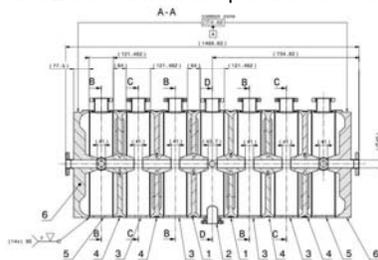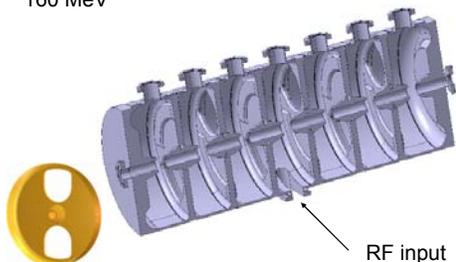# 2 –Accelerating Structures

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CAS Coupling accelerating cells

How can we couple together a chain of n accelerating cavities ?

1. **Magnetic coupling:**

open "slots" in regions of high magnetic field $\rightarrow$ B-field can couple from one cell to the next

2. **Electric coupling**:

enlarge the beam aperture $\rightarrow$ E-field can couple from one cell to the next

The effect of the coupling is that the cells no longer resonate independently, but will have common resonances with well defined field patterns.

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A 7-cell magnetically-coupled structure: the PIMS

CAS

PIMS = Pi-Mode Structure, will be used in Linac4 at CERN to accelerate protons from 100 to 160 MeV

RF input

This structure is composed of 7 accelerating cells, magnetically coupled.
The cells in a cavity have the same length, but they are longer from one cavity to the next, to follow the increase in beam velocity.
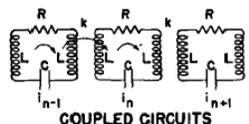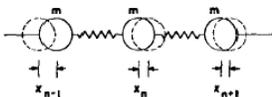
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Linac cavities as chains of coupled resonators

CAS

What is the relative phase and amplitude between cells in a chain of coupled cavities?

A linear chain of accelerating cells can be represented as a chain of resonant circuits magnetically coupled.

COUPLED CAVITIES

Individual cavity resonating at $\omega_0 \rightarrow$ frequenci(es) of the coupled system ?

COUPLED CIRCUITS

Resonant circuit equation for circuit i (R$\cong$0):

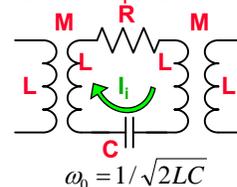$$I_i(2j\omega L + \frac{1}{j\omega C}) + j\omega kL(I_{i-1} + I_{i+1}) = 0$$

LINEAR LATTICE

Dividing both terms by $2j\omega L$:

$$X_i(1 - \frac{\omega_0^2}{\omega^2}) + \frac{k}{2}(X_{i-1} + X_{i+1}) = 0$$

$$\omega_0 = 1/\sqrt{2LC}$$

General response term,
$\propto$ *(stored energy)$^{1/2}$,*
can be voltage, E-field,
B-field, etc.
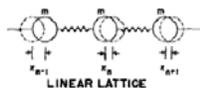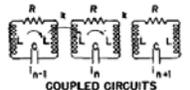
General resonance term
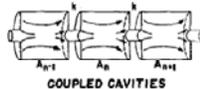
Contribution from adjacent oscillators

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CAS — The Coupled-system Matrix

$$X_i(1-\frac{\omega_0^2}{\omega^2})+\frac{k}{2}(X_{i-1}+X_{i+1})=0$$

$$i=0,..,N$$

A chain of N+1 resonators is described by a (N+1)x(N+1) matrix:

$$\begin{vmatrix} 1-\frac{\omega_0^2}{\omega^2} & \frac{k}{2} & 0 & ... \\ \frac{k}{2} & 1-\frac{\omega_0^2}{\omega^2} & \frac{k}{2} & ... \\ ... & ... & ... & ... \\ 0 & 0 & \frac{k}{2} & 1-\frac{\omega_0^2}{\omega^2} \end{vmatrix} \begin{vmatrix} X_0 \\ X_2 \\ ... \\ X_N \end{vmatrix} = 0 \qquad \text{or} \quad M\,X = 0$$

COUPLED CAVITIES

COUPLED CIRCUITS

LINEAR LATTICE

This matrix equation has solutions only if $\det M = 0$

Eigenvalue problem!

1. System of order (N+1) in $\omega \to$ only N+1 frequencies will be solution of the problem ("eigenvalues", corresponding to the resonances) $\to$ a system of N coupled oscillators has N resonance frequencies $\to$ an *individual resonance opens up into a band of frequencies*.

2. At each frequency ω_i will correspond a set of relative amplitudes in the different cells (X_0, X_2, …, X_N): the "eigenmodes" or "modes".

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CAS — Modes in a linear chain of oscillators

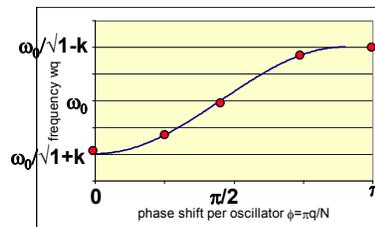We can find an analytical expression for eigenvalues (frequencies) and eigenvectors (modes):

Frequencies of the coupled system :

$$\omega_q^2 = \frac{\omega_0^2}{1+k\cos\frac{\pi q}{N}}, \quad q=0,..,N$$

the index q defines the number of the solution $\to$ is the "mode index"

$\to$ Each mode is characterized by a phase **πq/N**. Frequency vs. phase of each mode can be plotted as a "dispersion curve" ω=f(ϕ):
1. each mode is a point on a sinusoidal curve.
2. modes are equally spaced in phase.
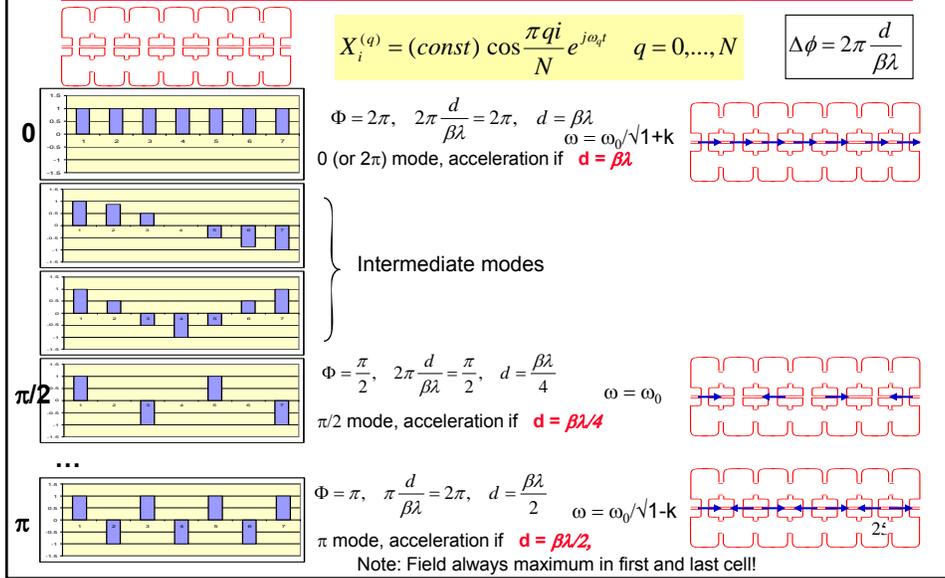

The "eigenvectors = relative amplitude of the field in the cells are:

$$X_i^{(q)} = (const)\cos\frac{\pi q i}{N}e^{j\omega_q t} \quad q=0,...,N$$

STANDING WAVE MODES, defined by a phase πq/N corresponding to the phase shift between an oscillator and the next one $\to$ πq/N=Φ is the phase difference between adjacent cells that we have introduces in the 1st part of the lecture.

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Example: Acceleration on the normal modes of a 7-cell structure

$$X_i^{(q)} = (const)\cos\frac{\pi q i}{N} e^{j\omega_q t} \qquad q = 0,...,N$$

$$\Delta\phi = 2\pi\frac{d}{\beta\lambda}$$

0

$$\Phi = 2\pi, \quad 2\pi\frac{d}{\beta\lambda} = 2\pi, \quad d = \beta\lambda$$
$$\omega = \omega_0/\sqrt{1+k}$$

0 (or 2π) mode, acceleration if **$d = \beta\lambda$**

Intermediate modes

$\pi/2$

$$\Phi = \frac{\pi}{2}, \quad 2\pi\frac{d}{\beta\lambda} = \frac{\pi}{2}, \quad d = \frac{\beta\lambda}{4}$$
$$\omega = \omega_0$$

$\pi/2$ mode, acceleration if **$d = \beta\lambda/4$**

...

π

$$\Phi = \pi, \quad \pi\frac{d}{\beta\lambda} = 2\pi, \quad d = \frac{\beta\lambda}{2}$$
$$\omega = \omega_0/\sqrt{1-k}$$

π mode, acceleration if **$d = \beta\lambda/2$,**

Note: Field always maximum in first and last cell!

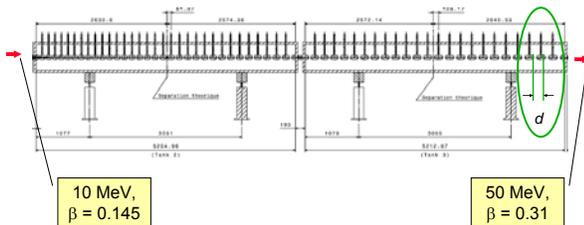## Practical linac accelerating structures

Note: our relations depend only on the cell frequency ω, not on the cell length d !!!

$$\omega_q^2 = \frac{\omega_0^2}{1+k\cos\frac{\pi q}{N}}, \qquad q = 0,...,N$$

$$X_n^{(q)} = (const)\cos\frac{\pi q n}{N} e^{j\omega_q t} \qquad q = 0,...,N$$

$\rightarrow$ As soon as we keep the frequency of each cell constant, we can change the cell length following any acceleration (β) profile!

Example:
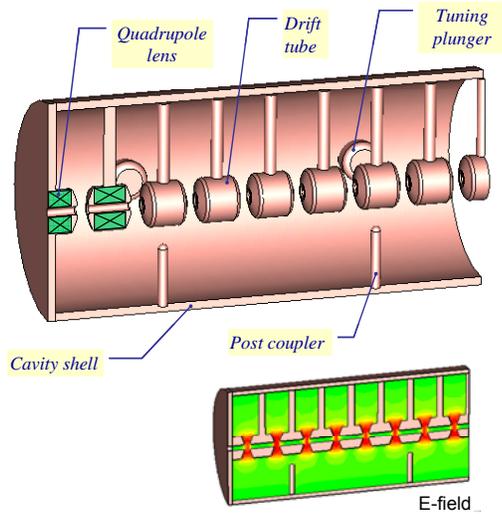The Drift Tube Linac (DTL)

Chain of many (up to 100!) accelerating cells operating in the 0 mode. The ultimate coupling slot: no wall between the cells!

10 MeV, $\beta = 0.145$

50 MeV, $\beta = 0.31$

Each cell has a different length, but the cell frequency remains constant $\rightarrow$ *"the EM fields don't see that the cell length is changing!"*

$d\uparrow \rightarrow (L\uparrow, C\downarrow) \rightarrow LC \sim const \rightarrow \omega \sim const$

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The DTL

Standing wave linac structure for protons and ions, β=0.1-0.5, f=20-400 MHz

Drift tubes are suspended by stems (no net RF current on stem)

Coupling between cells is maximum (no slot, fully open !)

The 0-mode allows a long enough cell (d=$\beta\lambda$) to house focusing quadrupoles inside the drift tubes!
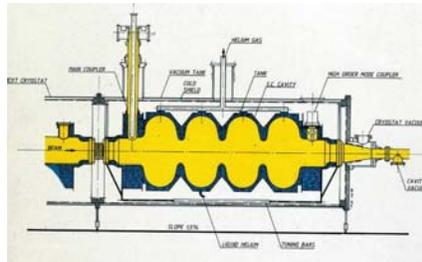
Quadrupole lens

Drift tube

Tuning plunger

Cavity shell

Post coupler

E-field

B-field

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Examples of DTL

Top; CERN Linac2 Drift Tube Linac accelerating tank 1 (200 MHz). The tank is 7m long (diameter 1m) and provides an energy gain of 10 MeV.
Left: DTL prototype for CERN Linac4 (352 MHz). Focusing is provided by (small) quadrupoles inside drift tubes. Length of drift tubes (cell length) increases with proton velocity.
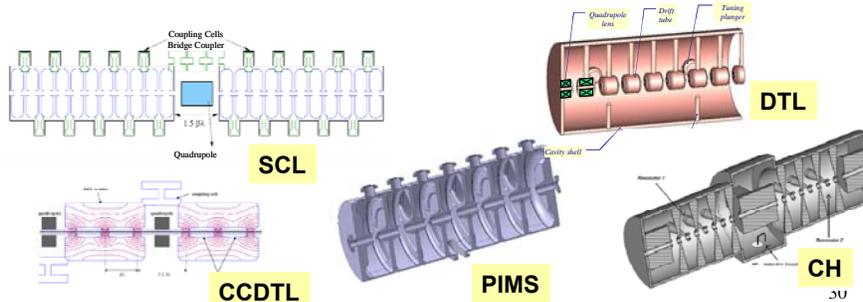
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Multi-gap Superconducting linac structures (elliptical)

CAS

Standing wave structures for particles at β>0.5-0.7, widely used for protons (SNS, etc.) and electrons (ILC, etc.)
f=350-700 MHz (protons),
f=350 MHz – 3 GHz (electrons)

Chain of cells electrically coupled, large apertures (ZT^2 not a concern).

Operating in π-mode, cell length $\beta\lambda/2$

Input coupler placed at one end.

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The normal conducting zoo

CAS

The creativity of cavity designers is endless…

For normal-conducting, the goal is designing high-efficiency structures with a large number of cells (higher power RF sources are less expensive).
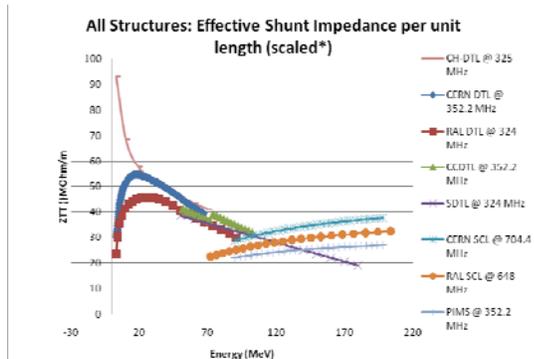
Two important trends:

1. Use $\pi/2$ modes for stability of long chains of resonators → CCDTL (Cell-Coupled Drift Tube Linac), SCL (Side Coupled Linac), ACS (Annular Coupled Structure),....

2. Use alternative modes: H-mode structures (TE band) → Interdigital IH, CH

SCL

DTL

CCDTL

PIMS

CH

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Comparison of structures – Shunt impedance

CAS

Main figure of merit is the shunt impedance
Ratio between energy gain (square) and power dissipation, is a measure of the energy efficiency of a structure.
Depends on the beta, on the energy and on the mode of operation.

However, the choice of the best accelerating structure for a certain energy range depends also on beam dynamics and on construction cost.

All Structures: Effective Shunt Impedance per unit length (scaled*)

- CH-DTL @ 325 MHz
- CCRN DTL @ 352.2 MHz
- RAL DTL @ 324 MHz
- CCDTL @ 352.2 MHz
- SDTL @ 324 MHz
- CCRN SCL @ 704.4 MHz
- RAL SCL @ 648 MHz
- PIMS @ 352.2 MHz

A comparison of shunt-impedances for different low-beta structures done in 2005-08 by the "HIPPI" EU-funded Activity.

In general terms, a DTL-like structure is preferred at low-energy, and π-mode structures at high-energy.
CH is excellent at low energies .

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The superconducting zoo

CAS

Spoke (low beta) [FZJ, Orsay]

4 gaps

CH (low/medium beta) [IAP-FU]

10 gaps

QWR (low beta) [LNL, etc.]

2 gaps

HWR (low beta) [FZJ, LNL, Orsay]

2 gaps

Re-entrant [LNL]

1 gap

4 to 7 gaps

Superconducting structure for linacs can have a small number of gaps → used for low and medium beta.

Elliptical structures with more gaps (4 to 7) are used for medium and high beta.

Elliptical cavities [CEA, INFN-MI, CERN, …]

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CAS   Conclusions – part 2

What did we learn?

1. Coupling together accelerating cells (via the magnetic or electric field) is a way to fix their phase relation.

2. A chain of N coupled resonators will always have N modes of oscillation. Each mode will have a resonance frequency and a field pattern with a corresponding phase shift from cell to cell.

3. Choosing the excitation frequency, we can decide in which mode to operate the structure, and we can select a mode with a phase advance between cells suitable for acceleration. If we change the length of a cell without changing its frequency, we can follow the increase the particle velocity.

4. Practical linac structures operate either on mode 0 (DTL), less efficient but leaving space for internal focusing elements, or on mode π, standard for multi-cell cavities. More exotic modes ($\pi/2$, TE) are used in special cases.
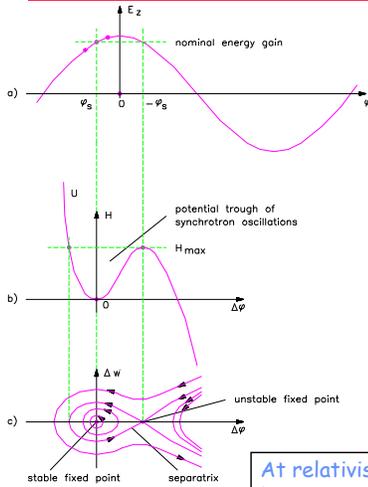
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CAS

3 – Fundamentals of linac beam dynamics

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Slide 1

CAS Longitudinal dynamics

→ Ions are accelerated around a (negative = linac definition) synchronous phase.

→ Particles around the synchronous one perform oscillations in the longitudinal phase space.

→ Frequency of small oscillations:

$$\omega_l^{\,2} = \omega_0^{\,2}\,\frac{qE_0T\sin(-\varphi)\lambda}{2\pi\,mc^2\beta\gamma^3}$$

→ Tends to zero for relativistic particles $\gamma \gg 1$.
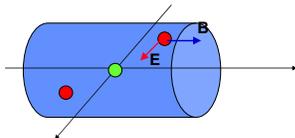
→ Note phase damping of oscillations:

$$\Delta\varphi = \frac{const}{(\beta\gamma)^{3/4}} \qquad \Delta W = const \times (\beta\gamma)^{3/4}$$

At relativistic velocities phase oscillations stop, and the beam is compressed in phase around the initial phase. The crest of the wave can be used for acceleration.

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Slide 2

CAS Transverse dynamics - Space charge

→ Large numbers of particles per bunch (~10^10).

→ Coulomb repulsion between particles (space charge) plays an important role.

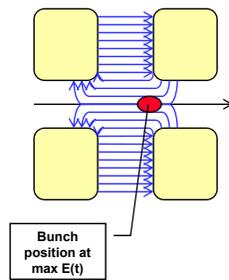→ But space charge forces ~ $1/\gamma^2$ disappear at relativistic velocity

Force on a particle inside a long bunch with density n(r) traveling at velocity v:

$$E_r = \frac{e}{2\pi\varepsilon\,r}\int_0^r n(r)\,r\,dr \qquad B_\varphi = \frac{\mu}{2\pi}\frac{e\,v}{r}\int_0^r n(r)\,r\,dr$$

$$F = e(E_r - vB_\varphi) = eE_r\left(1 - \frac{v^2}{c^2}\right) = eE_r(1 - \beta^2) = \frac{eE_r}{\gamma^2}$$

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Transverse dynamics - RF defocusing

→ RF defocusing experienced by particles crossing a gap on a longitudinally stable phase.

→ In the rest frame of the particle, only electrostatic forces → no stable points (maximum or minimum) → radial defocusing.

→ Lorentz transformation and calculation of radial momentum impulse per period (from electric and magnetic field contribution in the laboratory frame):

$$\Delta p_r = -\frac{\pi e E_0 T L r \sin\varphi}{c \beta^2 \gamma^2 \lambda}$$

→ Transverse defocusing ~ $1/\gamma^2$ disappears at relativistic velocity (transverse magnetic force cancels the transverse RF electric force).

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Transverse beam equilibrium in linacs

The equilibrium between external focusing force and internal defocusing forces defines the frequency of beam oscillations.

Oscillations are characterized in terms of *phase advance per focusing period* σ_t or *phase advance per unit length* k_t.

Ph. advance = Ext. quad focusing - RF defocusing - space charge

$$k_t^2 = \left(\frac{\sigma_t}{N\beta\lambda}\right)^2 = \left(\frac{qGl}{2mc\beta\gamma}\right)^2 - \frac{\pi q E_0 T \sin(-\varphi)}{mc^2\lambda\beta^3\gamma^3} - \frac{3q I \lambda (1-f)}{8\pi\varepsilon_0 r_0^3 mc^3\beta^2\gamma^3}$$

q=charge
G=quad gradient
l=length foc. element
f=bunch form factor
r₀=bunch radius
λ=wavelength
…

Approximate expression valid for:
FODO lattice, smooth focusing approximation, space charge of a uniform 3D ellipsoidal bunch.

A "low-energy" linac is dominated by space charge and RF defocusing forces !!

Phase advance per period must stay in reasonable limits (30-80 deg), phase advance per unit length must be continuous (smooth variations) → at low β, we need a strong focusing term to compensate for the defocusing, but the limited space limits the achievable G and l → needs to use short focusing periods N βλ.

Note that the RF defocusing term ∝f sets a higher limit to the basic linac frequency (whereas for shunt impedance considerations we should aim to the highest possible frequency, Z ∝√f) .

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CAS — Focusing periods

Focusing usually provided by quadrupoles.

Need to keep the phase advance in the good range, with an approximately constant phase advance per unit length → The length of the focusing periods has to change along the linac, going gradually from short periods in the initial part (to compensate for high space charge and RF defocusing) to longer periods at high energy.

For Protons (high beam current and high space charge), distance between two quadrupoles (=1/2 of a FODO focusing period):
- $\beta\lambda$ in the DTL, from ~70mm (3 MeV, 352 MHz) to ~250mm (40 MeV),
- can be increased to 4-10$\beta\lambda$ at higher energy (>40 MeV).
- longer focusing periods require special dynamics (example: the IH linac).

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CAS — Conclusions – part 3

What did we learn?

1. Transverse beam dynamics in linacs is dominated by space charge and RF defocusing forces.

2. In order to keep the transverse phase advance within reasonable limits, focusing has to be strong (large focusing gradients, short focusing periods) at low energy, and can then be relaxed at higher energy.

3. A usual linac is made of a sequence of structures, matched to the beam velocity, and where the length of the focusing period increases with energy.

4. The very low energy section remains a special problem → next lecture

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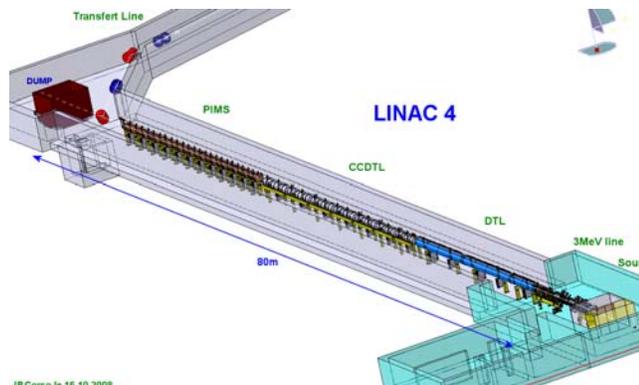**CAS**

4. Proton linac architecture

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CAS

Architecture: cell length, focusing period

EXAMPLE: the **Linac4 project at CERN**. H-, 160 MeV energy, 352 MHz.
A 3 MeV injector + 22 multi-cell standing wave accelerating structures of 3 types

DTL: every cell is different, focusing quadrupoles in each drift tube
CCDTL: sequences of 2 identical cells, quadrupoles every 3 cells
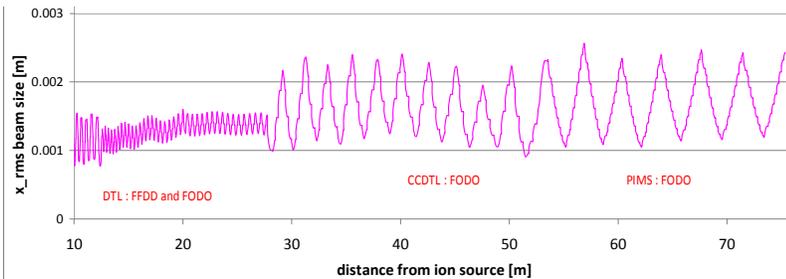**PIMS:** sequences of 7 identical cells, quadrupoles every 7 cells

Two basic principles to remember:

1. As beta increases, phase error between cells of identical length becomes small → we can have short sequences of identical cells (lower construction costs).

2. As beta increases, the distance between focusing elements can increase.

High-intensity protons – the case of Linac4

Transverse (x) r.m.s. beam envelope along Linac4

Example: beam dynamics design for Linac4@CERN.

High intensity protons (60 mA bunch current, duty cycle could go up to 5%), 3 - 160 MeV

Beam dynamics design minimising emittance growth and halo development in order to:
1. avoid uncontrolled beam loss (activation of machine parts)
2. preserve small emittance (high luminosity in the following accelerators)

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Linac architecture: the frequency

approximate scaling laws for linear accelerators:

⮞ RF defocusing (ion linacs)	~ frequency	
⮞ Cell length (=$\beta\lambda/2$)	~ (frequency)$^{-1}$	
⮞ Peak electric field	~ (frequency)$^{1/2}$	
⮞ Shunt impedance (power efficiency)	~ (frequency)$^{1/2}$	
⮞ Accelerating structure dimensions	~ (frequency)$^{-1}$	
⮞ Machining tolerances	~ (frequency)$^{-1}$	

→ Higher frequencies are economically convenient (shorter, less RF power, higher gradients possible) but the limitation comes from mechanical precision required in construction (tight tolerances are expensive!) and beam dynamics for ion linacs.

→ The main limitation to the initial frequency (RFQ) comes from RF defocusing (~ $1/(\lambda\beta^2\gamma^2)$ – 402 MHz is the maximum achievable so far for currents in the range of tens of mA's.

→ High-energy linacs have one or more frequency jumps (start 200-400 MHz, first jump to 400-800 MHz, possible a 3rd jump to 600-1200 MHz): compromise between focusing, cost and size.

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Linac architecture: optimum gradient (NC)

CAS

Note that the optimum design gradient (E_0T) in a normal-conducting linac is not necessarily the highest achievable (limited by sparking).

The cost of a linear accelerator is made of <u>2 terms</u>:
- a "structure" cost proportional to linac length
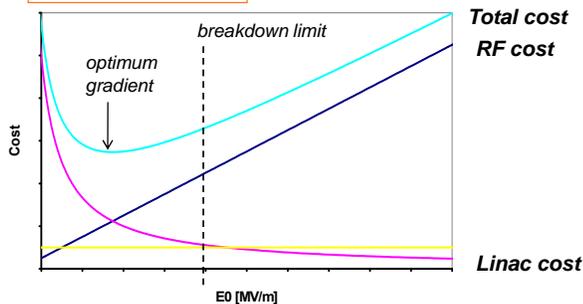- an "RF" cost proportional to total RF power

$$C = C_s l + C_{RF} P$$

C_s, C_{RF} unit costs (€/m, €/W)

$$l \propto 1/E_0T$$

$$P \propto (E_0T)^2 l \propto E_0T$$

$\Rightarrow$

$$C \propto C_s \frac{1}{E_0T} + C_{RF} E_0T$$

Overall cost is the sum of a structure term decreasing with the gradient and of an RF term increasing with the gradient → there is an optimum gradient minimizing cost.

breakdown limit

optimum gradient

Total cost

RF cost

Cost

Linac cost

E0 [MV/m]

Example: for Linac4
C_s … ~200 kCHF/m
C_{RF}…~0.6 CHF/W (recuperating LEP equipment)
E_0T ~ 3 – 4 MV/m

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Linac architecture: superconductivity

CAS

Advantages:

- ➢- Much smaller RF system (only beam power) →
          prefer low current/high duty

- ➢ Large aperture (lower beam loss in the SC section).

- ➢ Lower operating costs (electricity consumption).

Disadvantages:

➢ Need cryogenic system (in pulsed machines, size dominated by static loss → prefer low repetition frequency or CW to minimize filling time/beam time).

➢ Need cold/warm transitions to accommodate quadrupoles → becomes more expensive at low energy (short focusing periods).

➢ Individual gradients difficult to predict (large spread) → need large safety margin in gradient at low energy.

Conclusions:

1. Superconductivity gives a large advantage in cost at high energy / high duty cycle.

2. At low energy / low duty cycle superconducting sections become expensive.

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CAS   Transition warm/cold

The RFQ must be normal conducting (construction problems / inherent beam loss).
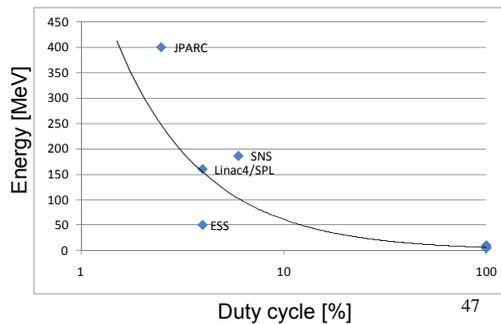
The high-energy (>200 MeV) sections should be superconducting.

But where is the optimum transition energy between normal and superconducting?

The answer is in the cost → the economics has to be worked out correctly !

Overview of warm/cold transition energies for linacs (operating and in design)

Project	Duty Cycle [%]	Transition Energy [MeV]
SNS	6	186
JPARC	2.5	400
Linac4/SPL	4	160
ESS	4	50
Project-X	100	10
EUROTRANS	100	5
EURISOL	100	5
IFMIF/EVEDA	100	5

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CAS   Conclusions – part 4

What did we learn?

1. A linac is always made of different sections optimized for a particular range of beta. Each section is characterized by its RF frequency, by the periodicity of the RF structure and by the focusing distance.

2. The frequency of the first section (RFQ) is limited by beam dynamics (RF defocusing). In the following sections, the frequency can be increased (by multiples of the basic frequency), to gain in efficiency.

3. The first section must be normal conducting, whereas the sections at higher energy can be superconducting. The optimum transition energy between normal and superconducting depends on the duty cycle, on the beam current and on other "environmental" conditions.

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