Fixed Field Alternating Gradient (FFAG) Accelerator

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26 May, 2011
CERN Accelerator School

Content

Remarks

- Energy in this lecture is kinetic energy, not total energy.

  In the energy range below ~10 GeV for a hadron beam, it is still not ultra-relativistic and “kinetic energy” != “total energy”.

  e.g.
  - 1 GeV kinetic energy of proton
  - Proton mass is 0.938 GeV/c^2
  - Total energy is ~ 2 GeV
  - Momentum is ~1.7 GeV/c

Accelerators and high beam power

Three major frontier of particle accelerators

High energy frontier
LHC, ILC
Energy of each particle
•Always a driving force of accelerator development.
•Small emittance is preferred.

Synchrotron light source
ESRF, DIAMOND
Brightness of light
•Ultimate goal is a beam as a point source. (or zero emittance.)

High power beam source
μ, K, Neutron source, ν factory
Power carried by a beam
•Large emittance to avoid multi particle effects
•Proton, can be ion, but not electron.

Particle energy vs beam power

- High energy frontier machine,
  Particle energy is converted to create new particles.
- Synchrotron light source,
  Particle energy is converted to photon.
- High power beam source,
  Beam power is converted to produce secondary or tertiary beams.

Trend
linear accelerator in all three area

- High energy frontier machine,
  Linear collider machine because synchrotron radiation at bending magnet is too wasteful.
  ILC
- Synchrotron radiation source,
  Linear machine because small emittance cannot be preserved for many turns.
  XFEL
- High power beam source,
  Linear machine because it is believed to be easier to handle beam loss.
  ESS

Beam power of accelerator

Energy of individual particle [GeV]

Higher energy is preferable, but size should be moderate

x

Number of particle per beam [ppp]

Enlarge aperture as much as possible to mitigate space charge

x

Repetition rate [Hz]

Continuous operation is the best, but very high repetition is acceptable

Low loss

Keep accessibility

Reliability

Hardware failures cut integrated beam power

What limits "Energy of individual particle"? (1) *accelerator size*

- Linear accelerator:

  1 GeV kinetic energy with 5 MV/m radiofrequency field gradient

  The machine length becomes <u>200 m.</u>

- Circular accelerator:

  1 GeV kinetic energy proton has momentum of 1.7 GeV/c.

  With 2 T bending magnet, bending radius is 2.8 m.

  When only 20% of the circumference is bending magnet

  The machine radius becomes <u>14 m.</u>

It is OK for 1 GeV, but not for 10 GeV.

What limits "Energy of individual particle"? (2) *target*

- Target size

  Interaction length becomes longer with higher energy.

- Cross section (lower limit)

  Have to be above threshold to produce secondary particles.

What limits “Number of particles per beam”? *space charge*

- Interaction among particles in a beam becomes not negligible when a charge density increases.

- Transverse space charge tune shift.

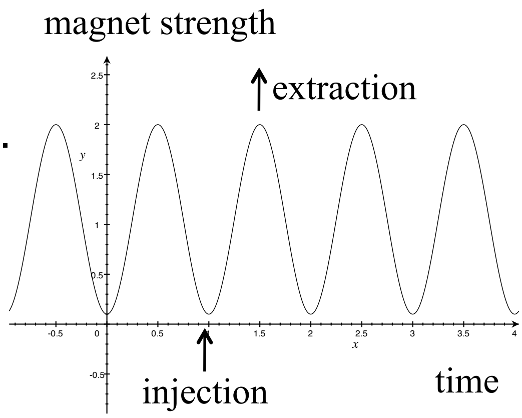
$$\Delta Q = -\frac{r_p n_t}{2\pi\beta^2\gamma^3\varepsilon_t B_f}$$

- Large aperture to decrease charge density $\frac{n_t}{\varepsilon_t}$.

What limits “Repetition rate”? (1) *ramping magnet*

- Synchrotron’s repletion is determined by magnet ramping.

  maximum is 50 Hz at ISIS



- CW (continuous wave) beam like in a cyclotron is the best.
  However, the maximum cyclotron energy is 0.5 GeV at PSI.
  No longitudinal focusing because there is no bucket.

What limits “Repetition rate”? (2)
user demand

- Different user demands different request.
  Neutron user prefer low repletion (10 to 20 Hz).
  ADSR user likes a continuous beam (> 1 kHz).

- Stacking at maximum momentum to reshape pulse structure if necessary.

FFAG’s
(Fixed Field Alternating Gradient accelerator) approach

- Focusing is similar to synchrotron
  Use alternating gradient focusing in transverse plane like a synchrotron.
  Use rf bucket to capture a beam in longitudinal plane like a synchrotron.

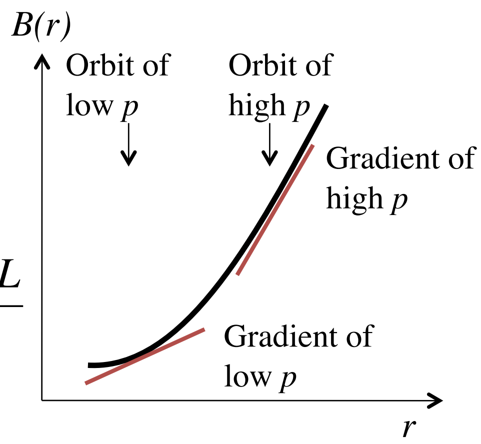
- No ramping of magnets like a cyclotron
  Increase repetition as much as possible by sweeping RF frequency as fast as possible.

How could we eliminate magnet ramping?

FFAG basics (1)

orbit and optics

Conventional strong focusing synchrotron	FFAG
Same orbit shape	Orbit shape can not be the same
$$\theta = \frac{B(t)L}{p(t)/e}$$	$$\theta(t) = \frac{BL}{p(t)/e}$$
Same focal length	Same focal length
$$\frac{1}{f} = \frac{dB(t)/dr \cdot L}{p(t)/e}$$	$$\frac{1}{f} = \frac{dB(r)/dr \cdot L}{p(t)/e}$$
Keep $\frac{B(t)}{p(t)}$ constant.	"cardinal condition" (next slide)



FFAG basics (2)

cardinal conditions of a FFAG

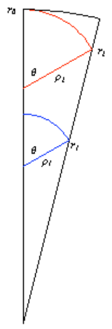
Geometrical similarity

$$\left.\frac{\partial}{\partial p}\left(\frac{\rho}{\rho_0}\right)\right|_{\vartheta = const.} = 0$$

ρ_0 : average curvature

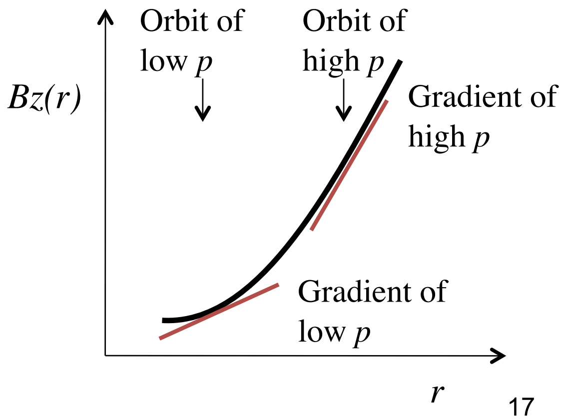
ρ : local curvature

: generalized azimuth



Constancy of k at corresponding orbit points

$$\left.\frac{\partial k}{\partial p}\right|_{\vartheta = const.} = 0 \qquad k = \frac{r}{B}\left(\frac{\partial B}{\partial r}\right)$$



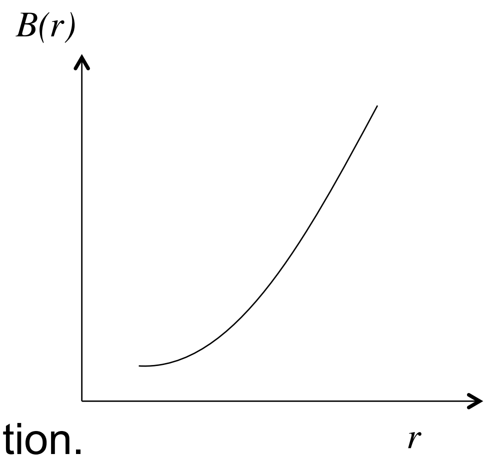
FFAG basics (3)

field profile

If the field profile has the shape of

$$B(r,\theta) = B_0\left(\frac{r}{r_0}\right)^{k} F(\vartheta)$$

Cardinal condition is satisfied.



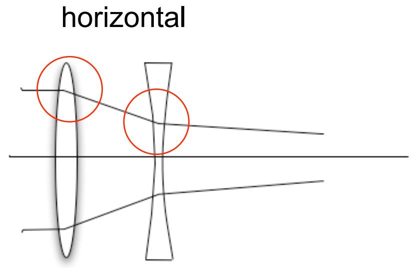
Gradient is determined by focusing condition, not by isochronous condition.

$$\omega = \frac{eB}{m\gamma} \neq \text{constant}$$

FFAG basics (4)

alternating gradient

- Alternating gradient is a focusing scheme with both sign of gradient magnets, focusing and defocusing elements.



FFAG basics (5)

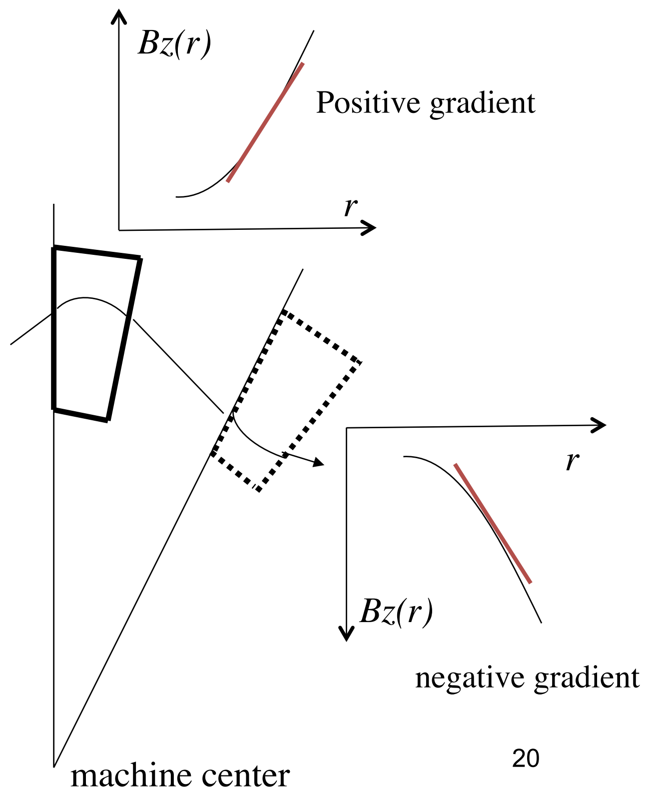
simple case

$$B(r,\theta) = B_0\left(\frac{r}{r_0}\right)^k F(\vartheta)$$

$F(\vartheta) = F(\theta)$

Radial sector type

Alternating magnet has opposite sign of bending.



FFAG basics (6)

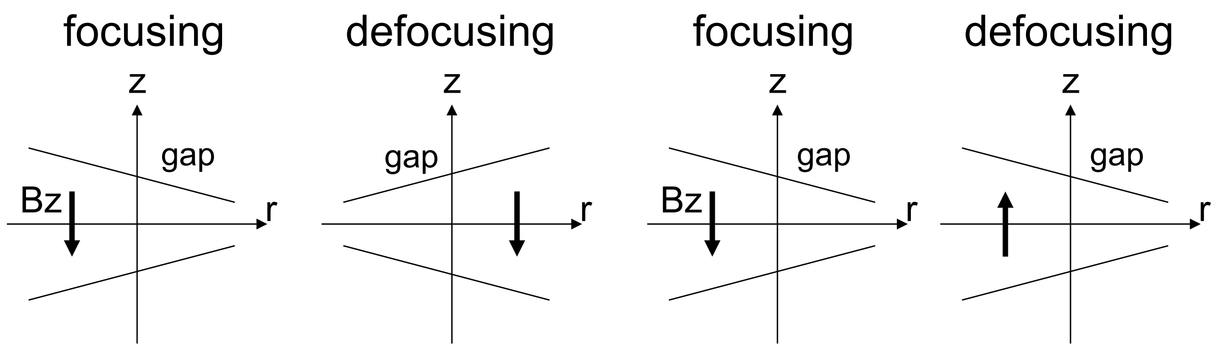
a way to realize AG

Conventional strong focusing synchrotron

Bending a beam in the same direction.

FFAG

Bending a beam in the opposite direction to change the sign.



FFAG basics (7)

another way

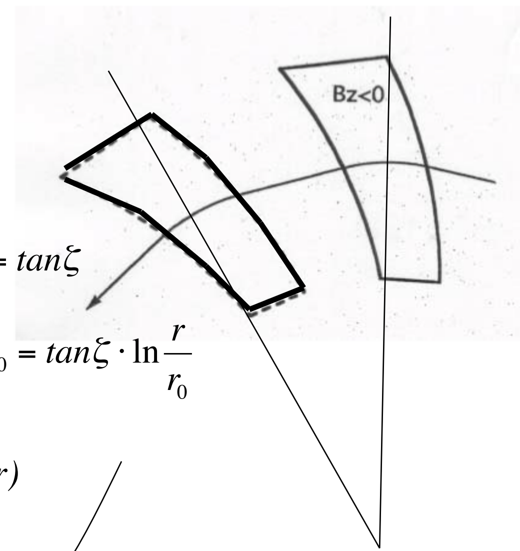
$$B(r,\theta) = B_0\left(\frac{r}{r_0}\right)^k F(\vartheta)$$

$$F(\vartheta) = F\left(\theta - tan\zeta \cdot \ln\frac{r}{r_0}\right)$$

Spiral sector type

$$\frac{rd\theta}{dr} = tan\zeta$$

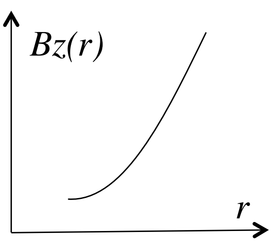
$$\theta - \theta_0 = tan\zeta \cdot \ln\frac{r}{r_0}$$



machine center

Spiral angle gives strong edge focusing.

$$\therefore \Delta p_z = \frac{e}{v_x}\int_{-\infty}^{\infty}\left(-v_y B_x\right)dx = -eB_{z0}\tan\zeta \cdot z$$



FFAG basics (8)

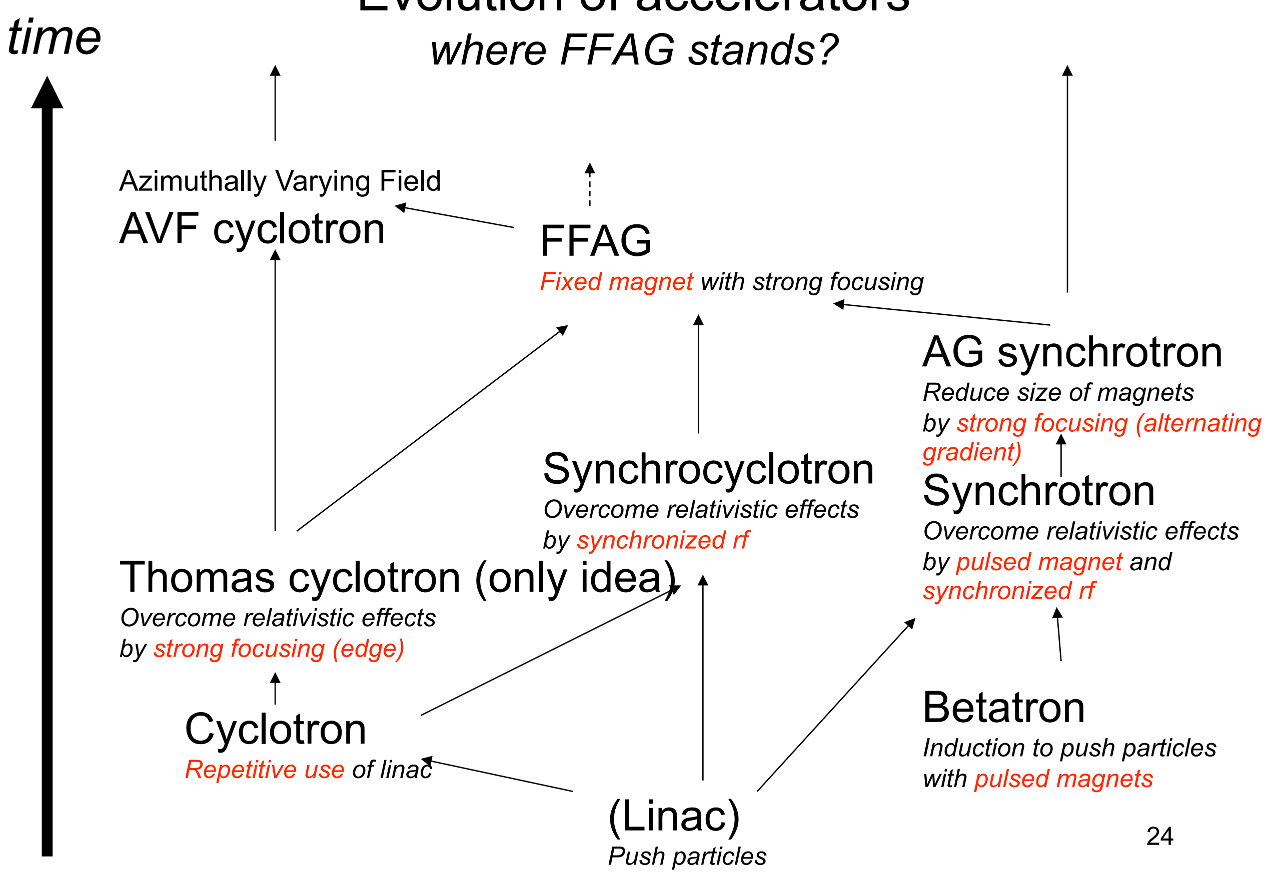
- What is the difference between FFAG and synchrocyclotron?

    very similar
    stronger transverse focusing (alternating gradient)
    transverse tune is constant

- If one tries to make a synchrocyclotron now, it should become a FFAG.

Evolution of accelerators

where FFAG stands?



Why FFAG draw attention recently ?

Neutrino factory and FFAG (1)

how it works?

Produce a well collimated neutrino beam to a detector a few 1000 km away.

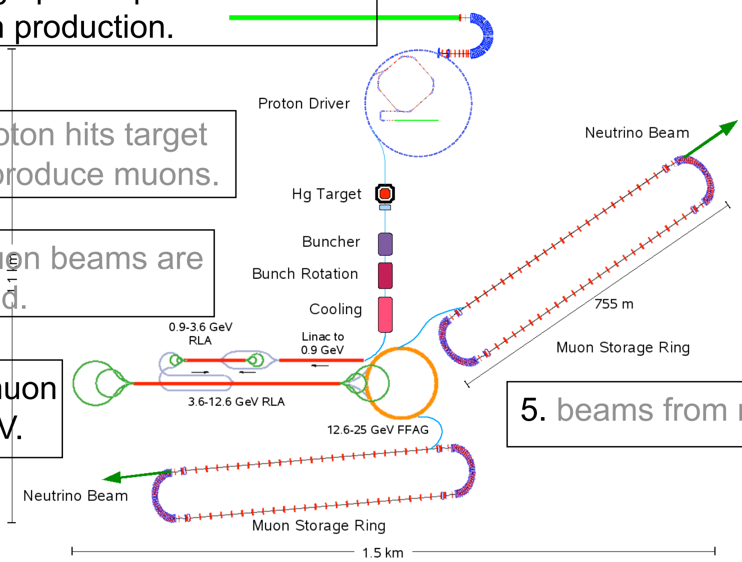
1. High power proton beam for muon production.

2. Proton hits target and produce muons.

3. Muon beams are cooled.

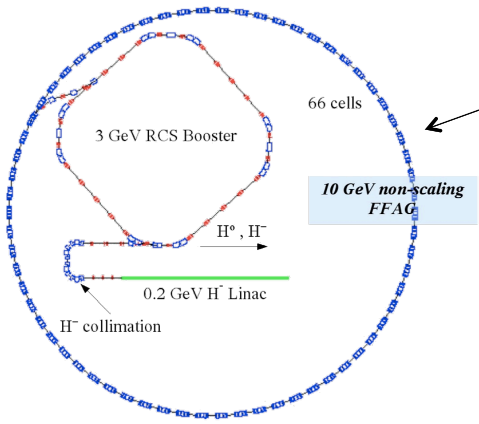
4. Acceleration of muon beams to 20-50 GeV.

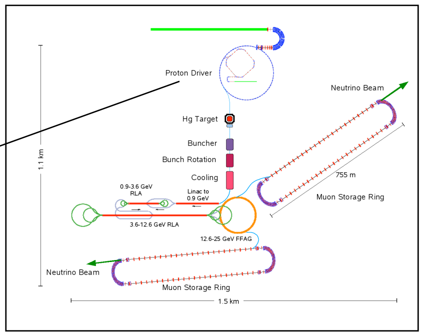
5. beams from muon decay.



Neutrino factory and FFAG (2)

FFAG as high power proton source (proton driver)



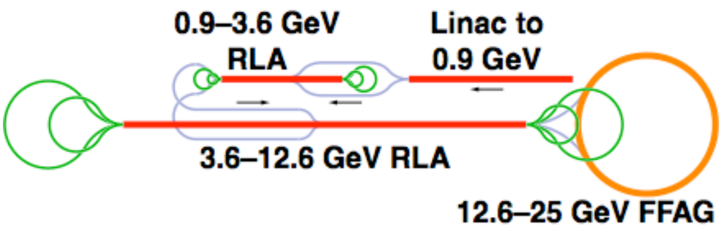


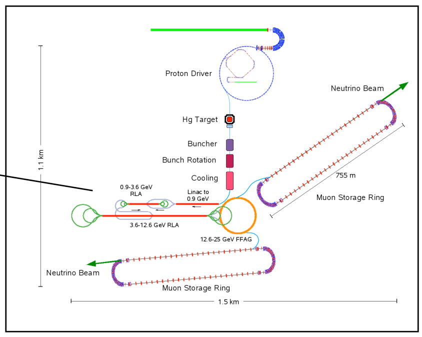
Beam power:	4 MW
Energy:	5 to 10 GeV

c.f. Beam power of SNS is 1.4 MW with 1 GeV.

Neutrino factory and FFAG (3)

requirements for muon acceleration





- Energy up to 20 - 50 GeV
  - It is ultra relativistic.
- Large acceptance
  - Muon emittance is a few tens of thousand π mm mrad even after cooling. (e.g. 30,000 π mm mrad)
- Quick acceleration
  - Muon's lifetime at rest is 2.2 μs.

Neutrino factory and FFAG (4)

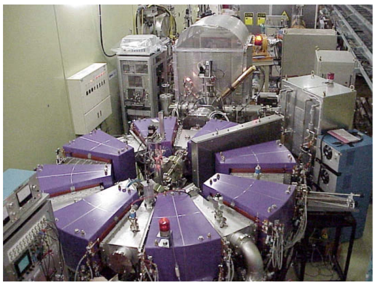
muon and high power beam acceleration

Muon acceleration	High power beam acceleration
Energy up to 20 - 50 [GeV] It is ultra relativistic. x Large acceptance Muon emittance is a few tens of thousand π mm mrad even after cooling. (e.g. 30,000 π mm mrad) x Quick acceleration Muon's lifetime in rest frame is 2.2 μs.	Energy of each particle [GeV] Higher energy is preferable, but size should be moderate x Number of particle per beam [ppp] Enlarge aperture as much as possible x Repetition rate [Hz] Continuous operation is the best, but very high repetition is acceptable

Requirements are similar !

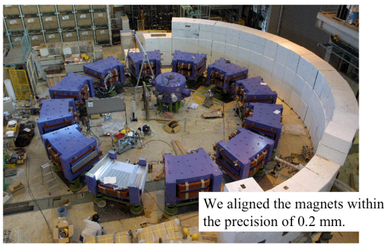
Rebirth of FFAG

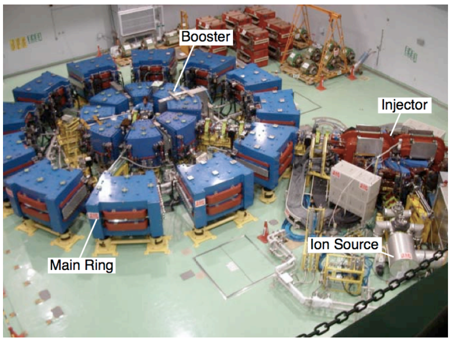
with new technology



Proof of principle (proton acceleration with rf cavity) machine was constructed in 1999 and demonstrate rapid acceleration with 1 kHz.

Scale up version of PoP FFAG was constructed as a prototype of proton therapy machine.





3 stage FFAG for ADSR

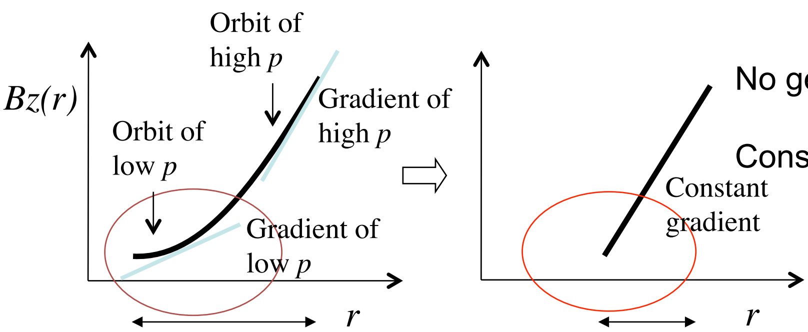
- 2.5 MeV spiral (ion beta) FFAG with induction cores
- 25 MeV radial (booster) FFAG with RF and flat gap
- 150 MeV radial (main) FFAG with RF and tapered gap

Novel FFAG development

FFAG for muon acceleration (1)

making the magnet more compact

- If we could break cardinal conditions (scaling law), FFAG would be much simpler and magnet would be smaller.



No gentle slope at low momentum.
- Orbit excursion is shorter.

Constant gradient.
- Linear magnet.

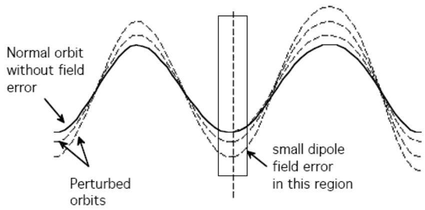
- Why we wanted to have cardinal conditions?

To avoid resonance in accelerator.

FFAG for muon acceleration (2)

resonances

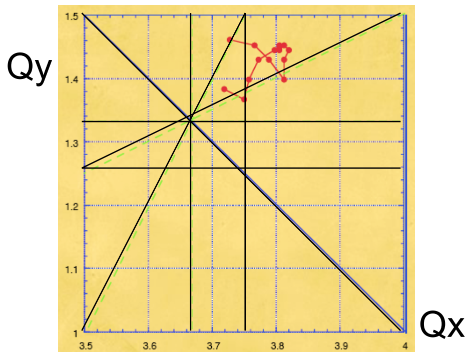
- There are many resonances in tune space. Normally, once a particle hits one of them, it will be lost.



- However, particles can survive after crossing resonances <u>if resonance is weak and crossing is fast.</u>

Furthermore, in reality, tune moves due to imperfections of magnet (red zigzag line).

Tune diagram of 150 MeV FFAG



FFAG for muon acceleration (3)

new idea: nonscaling FFAG

Muons circulate only a few (~10) turns in FFAG.

Is resonance really harmful to a beam?

Maybe not, because

- Resonance has a cumulative effect.
- We can avoid strong resonance such as integer one.

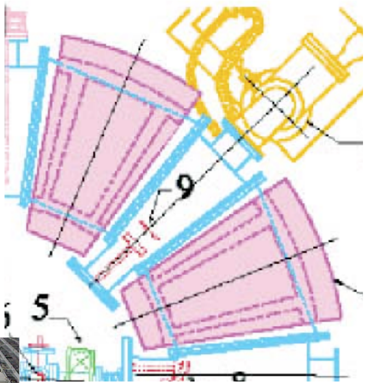
In FFAG for muon acceleration, “cardinal conditions” are not necessary.

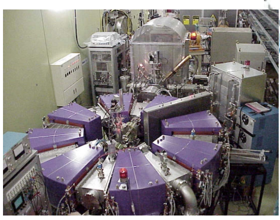
Instead, let us use simple fixed field Bend and Quad magnets to optimize optics and make a compact machine.

Birth of nonscaling FFAG.

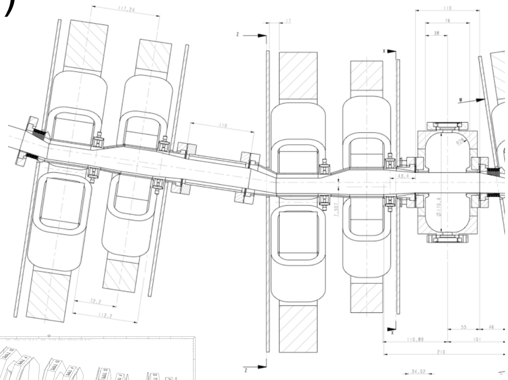
FFAG for muon acceleration (4)
lattice elements

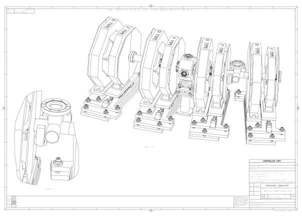
Original FFAG (scaling FFAG) layout which satisfies cardinal conditions.





Nonscaling FFAG uses simple shifted Quad (to produce Bend field.)





FFAG for muon acceleration (5)
conditions

- Lattice with very small momentum compaction α_p

$$\frac{dL}{L} = \alpha_p \frac{dp}{p} \qquad \alpha_p = \frac{1}{C}\oint \frac{D(s)}{\rho} ds$$

  – makes the orbit shift in radial direction small.
  – reduces the magnet size and makes a compact machine.

- Eliminate nonlinear fields.

$$\frac{B(r,\theta)}{B_0} = \left(\frac{r}{r_0}\right)^k = \left[1 + \frac{k}{r_0}(r - r_0) + \frac{k(k-1)}{2r_0^2}(r - r_0)^2 + \frac{k(k-1)(k-2)}{3 \cdot 2r_0^3}(r - r_0)^3 + \cdots\right]$$

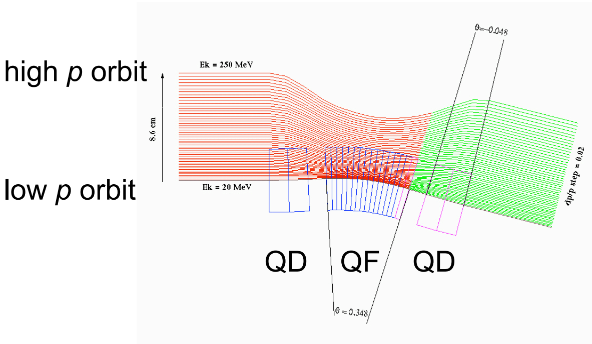
dipole quadrupole sextupole octupole

- This is like a storage ring without chromaticity correction.

FFAG for muon acceleration (6)

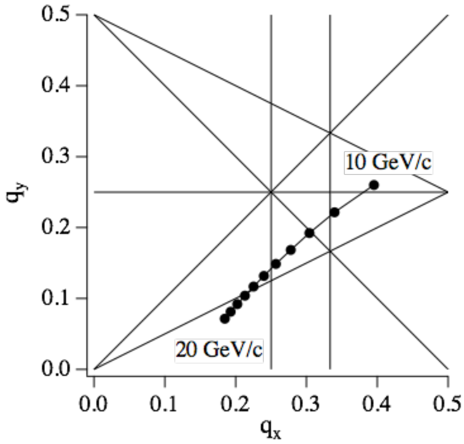
orbit and optics

- Orbits for different momenta are no longer similar.



- Focusing force decreases as momentum increases.

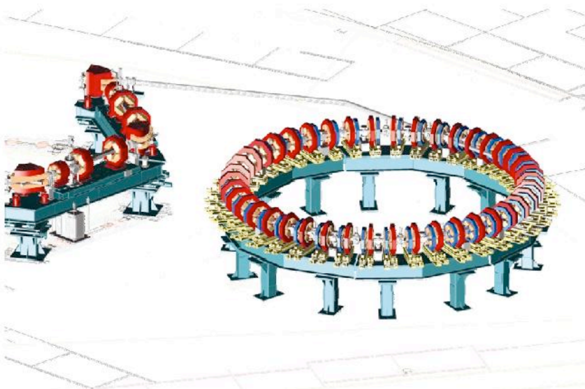
$$\frac{1}{f} = \frac{B'}{p/e}$$



EMMA (1)

Electron Model of Muon Acceleration

- No one ever made a nonscaling FFAG before.



- Demonstrate that nonscaling FFAG works as expected
  - Study resonance crossing in detail.
  - Prove quick acceleration and large acceptance.
  - Study outside bucket acceleration in detail.

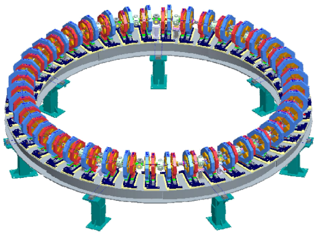
EMMA (2)
periodicity

- Periodicity of 42, that is a half of 10 to 20 GeV muon ring, has been chosen,

  Enough to simulate a muon ring lattice.

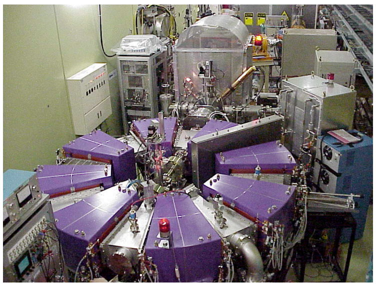
  Not a tabletop size machine. Circumference is 16.57 m

  - Scaling PoP FFAG's circumference is about 3 m.



Nonscaling counterpart.

42 doublet cells



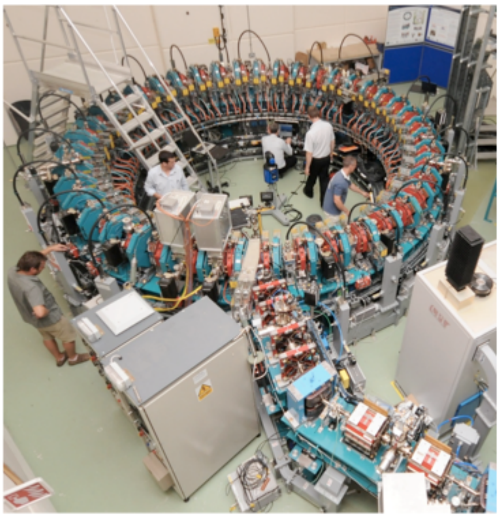
PoP scaling FFAG (2000)

8 triplet cells

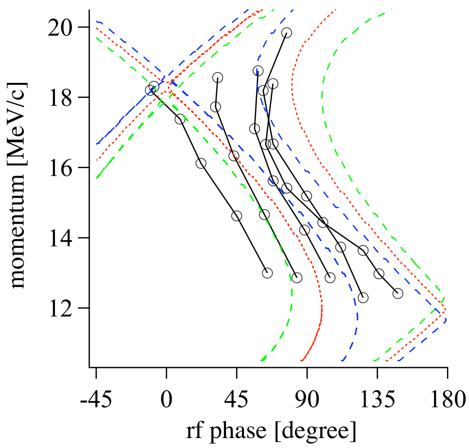
- Space is available next to ERLP.

EMMA (3)
commissioning

- Since last summer, we started commissioning of the world first nonscaling FFAG in Daresbury Laboratory UK.



EMMA construction



Beam is accelerated in 6 turns (March 2011).

Toward higher beam power

Beam power of accelerator

Energy of each particle [GeV]

Higher energy is preferable, but size should be moderate

x

Number of particle per beam [ppp]

Enlarge aperture as much as possible

x

Repetition rate [Hz]

Continuous operation is the best, but very high repetition is acceptable

Low loss

Keep accessibility

Reliability

Hardware failures cut integrated beam power

Is high repetition enough to make high beam power?

- With the same number of particles per bunch, 1 kHz instead of 50 Hz repetition makes 20 times higher beam power.
- In order to achieve 10 MW, we need 20 times higher beam power than the one we have now: 0.5 ~ 1.0 MW.
- It is possible to reduce space charge effects 20 times less if the same beam power is needed.

Space charge effects (1)

same or different as the one in synchrotron?

- Similar

    Single bunch effect.

    Decrease of focusing force: tune shift or depression.

- Specific to FFAG

    In a nonscaling FFAG and isochronous FFAG, transverse zero current tune moves.

    Resonance excited by space charge becomes an issue.

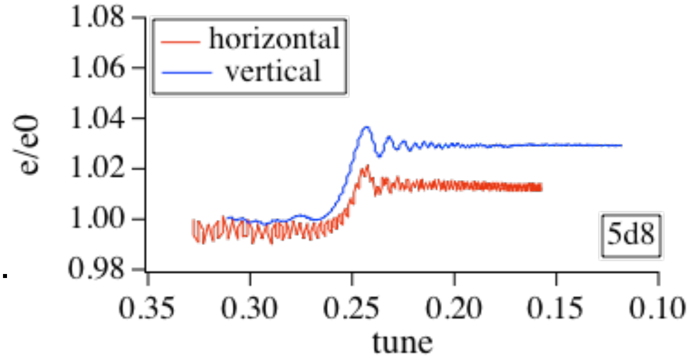
Space charge effects (2)

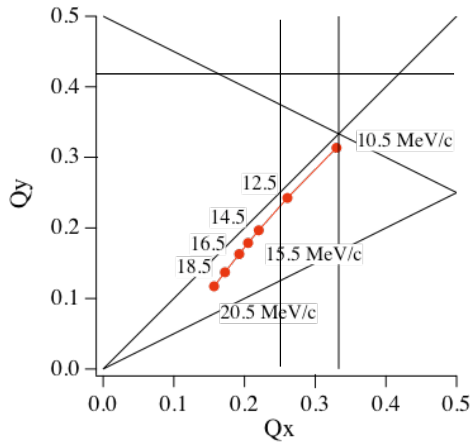
crossing of space charge induced resonances

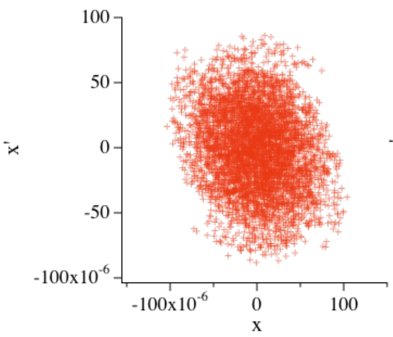
Space charge potential excites nonlinear self fields.

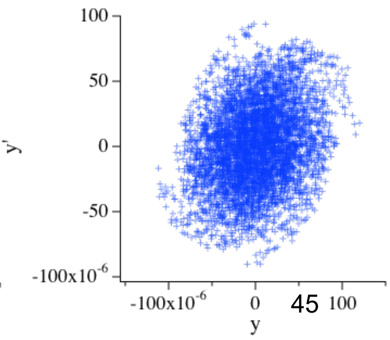
$$\Delta x' = \frac{K_{sc}}{\sigma_x(\sigma_x+\sigma_y)} x \exp\left[-\frac{x^2+y^2}{(\sigma_x+\sigma_y)^2}\right]$$

Particles may be trapped by the self field when it cross Qx,y=0.25.









Space charge effects (3)

space charge due to image charge and current

- Incoherent tune shift electric, DC magnetic, AC magnetic

$$\Delta Q_{inc} - \Delta Q_{inc,self} = -\frac{NRr_0}{\pi Q\beta^2\gamma}\left[\frac{1}{B}\frac{\varepsilon_1}{h^2} + \beta^2\frac{\varepsilon_2}{g^2} - \beta^2\left(\frac{1}{B}-1\right)\frac{\varepsilon_1}{h^2}\right]$$

$$= -\frac{NRr_0}{\pi Q\beta^2\gamma}\left[\frac{1}{\gamma^2 B}\frac{\varepsilon_1}{h^2} + \beta^2\frac{\varepsilon_1}{h^2} + \beta^2\frac{\varepsilon_2}{g^2}\right]$$

- Coherent tune shift electric, DC magnetic, AC mag from trans, AC mag from longi

$$\Delta Q_{coh} = -\frac{NRr_0}{\pi Q\beta^2\gamma}\left[\frac{1}{B}\frac{\xi_1}{h^2} + \beta^2\frac{\varepsilon_2}{g^2} - \beta^2\frac{(\xi_1-\varepsilon_1)}{h^2} - \beta^2\left(\frac{1}{B}-1\right)\frac{\xi_1}{h^2}\right]$$

$$= -\frac{NRr_0}{\pi Q\beta^2\gamma}\left[\frac{1}{\gamma^2 B}\frac{\xi_1}{h^2} + \beta^2\frac{\varepsilon_1}{h^2} + \beta^2\frac{\varepsilon_2}{g^2}\right]$$

Space charge effects (4)

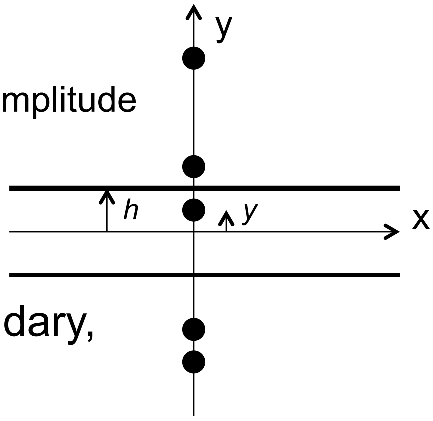
image charge (or current) coefficients

- Coefficients ε_1, ξ_1 are determined by the summation of image charge and current.
- Summation becomes

$$E_y = \frac{\lambda}{2\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2 h^2 - y^2}$$

$$\approx \frac{\lambda}{\pi\varepsilon_0}\frac{\pi^2}{16h^2}y = \frac{\lambda}{\pi\varepsilon_0}\xi_1 y \quad \text{for small amplitude}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\frac{\pi}{4h}\tan\frac{\pi y}{2h}$$



- When a beam goes nearby the boundary, field strength increases sharply.
- In FFAG, a beam usually does not go at the centre of vacuum chamber.

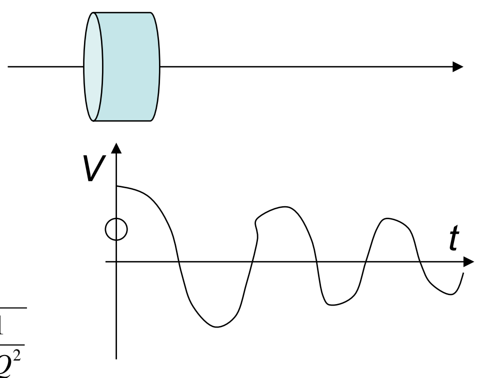
Beam loading (1)

problem

- Beam is decelerated when it passes through longitudinal impedance (rf cavity).
- There are many rf cavities to accelerate muons (or electrons) quickly.
- Beam induced voltage is

$$\begin{cases} 0 & t<0 \\ \alpha R_s q & t=0 \\ 2\alpha R_s e^{-\alpha t}\left(\cos\varpi t - \frac{\alpha}{\varpi}\sin\varpi t\right)q & t>0 \end{cases}$$

$$\text{where } \alpha = \frac{\omega_R}{2Q} \text{ and } \varpi = \sqrt{\omega_R^2 - \alpha^2} = \omega_R\sqrt{1-\frac{1}{4Q^2}}$$

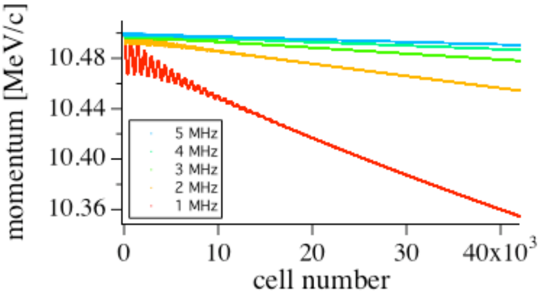


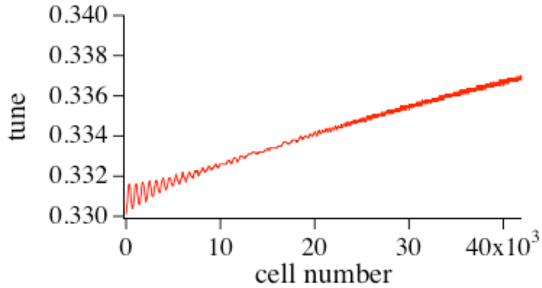
Beam loading (2)

transient effect

When the beam comes back to the same cavity, whether the beam is accelerated or decelerated depends on resonant frequency.

Transient beam loading can be measured as tune oscillation.





Summary

- As a high power beam accelerator, FFAG has advantage of high repetition operation.
- Concept is old (proposed ~50 years ago), but intensive study has been carried out for last ten years.
- FFAG turns out to be attractive choice for muon (or any other short life particle) accelerator as well.
- For last ten years, different type of FFAG (nonscaling FFAG which does not satisfy cardinal condition, isochronous FFAG, etc) has been proposed.
- For 10 MW design, FFAG has similar problems of a present high power synchrotron.
- However a bunch intensity can be always less than that of synchrotron because of high repetition operation.