Beam Cooling

Søren Pape Møller, Aarhus University, Denmark

- Introduction to cooling, temperature, phase space and Liouville
- Stochastic cooling
- Electron cooling
- Laser cooling
- Radiation damping
- Ionisation and other cooling

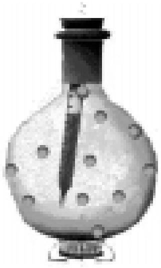
Beam cooling

- Emphasis on physical ideas (description and understanding)
- Will not review existing all facilities and performances
- Will not derive cooling time, but give crude formula and comment on important dependencies

Beam cooling

- Since beamcooling is ”slow”, it is only effective in storage rings;
- however, ionisation and ”stochastic cooling” has been or will be used in beamlines for muons

Introduction



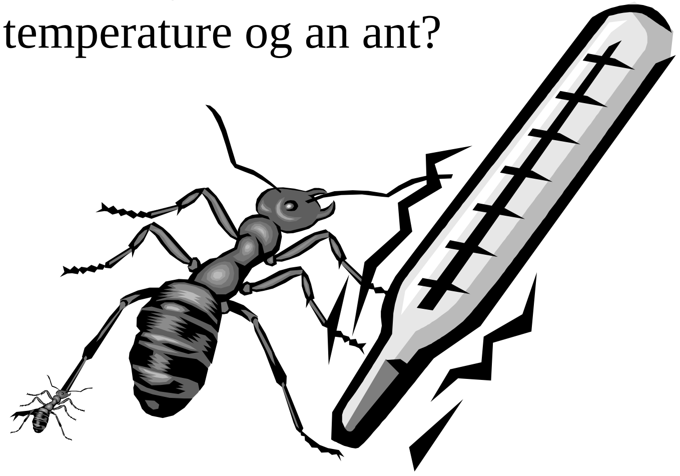
What is cooling? What is Temperature?

$$(\tfrac{3}{2}k)T_{\perp//} = \tfrac{1}{2}m\langle \vec{v}^{\,2}_{\perp//}\rangle$$

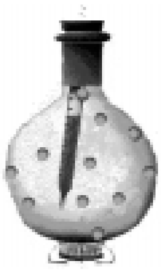
v is the velocity relative to the reference particle moving with the average ion velocity.
Temperature is a measure of the disordered motion.

My old thermodynamics teacher

- How do you measure the temperature og an ant?



Introduction



What is cooling? What is Temperature?

$$(\tfrac{3}{2}\,k)T_{\perp //} = \tfrac{1}{2}\,m\langle \vec{v}^{\,2}_{\perp //}\rangle$$

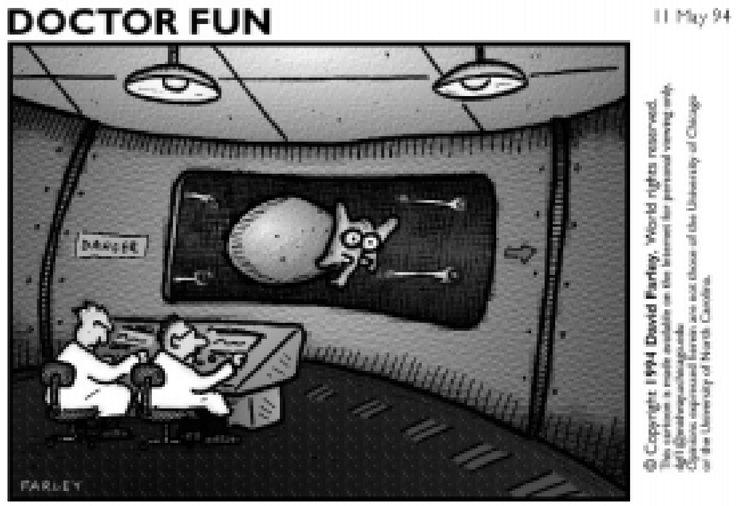
v is the velocity relative to the reference particle moving with the average ion velocity.
Temperature is a measure of the disordered motion.

In an accelerator

$$T_{//} = Mc^2\beta^2\langle \Delta p / p\rangle^2$$

$$T_{\perp} = Mc^2\beta^2\gamma^2\varepsilon\left(\frac{1}{\langle \beta_H\rangle} + \frac{1}{\langle \beta_V\rangle}\right)$$

Why beam cooling?



Deep within the atomic supercollider, the search continues for the elusive elephantino.

Why beam cooling?

Improve beam quality

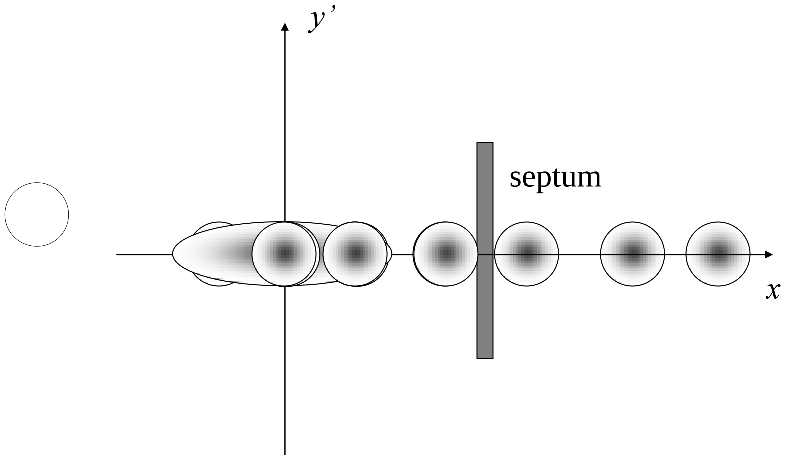
- beam size, emittance
- energy spread
- intensity of beam, accumulation, stacking
- lifetime of beam

Counteract degradation of beam quality

due to interaction of ions with

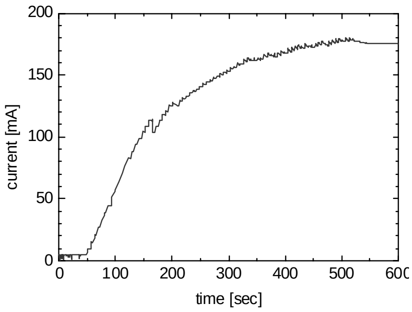
- other ions (intrabeam scattering)
- rest-gas (internal targets)
- non-ideal fields, resonances, instabilities
  injection errors

Stacking by cooling



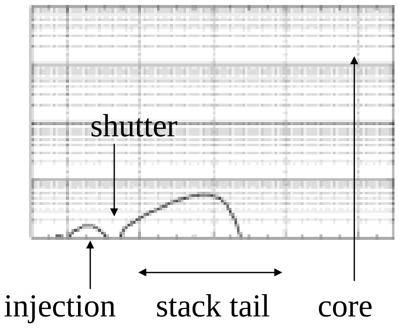
Stacking by cooling 2

ASTRID SR source:
~200 mA accumulated
from many injections
of ~5 mA



Fermilab antiproton
accumulator
stacking for 1 hour

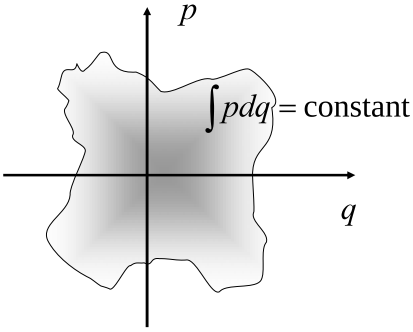
10^8/2 sec pbar at 8 GeV

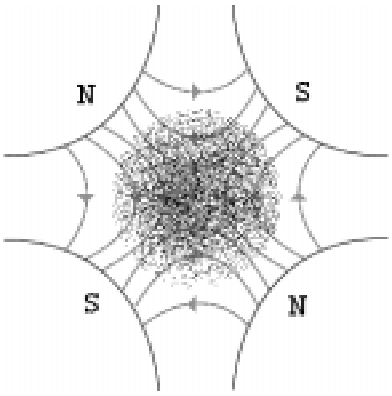


Phase space and Liouville

Liouville: For hamiltonian systems, the phase space density is constant (when measured along a trajectory)
The phase space volume (emittance) is conserved

Often the two transverse and the longitudinal degrees of freedom are decoupled





Phase space, Liouville and cooling

Liouvilles theorem means that cooling is not possible for Hamiltonian systems, that is systems with forces that can be derived from potentials.
In addition particles cannot be injected into already filled areas of Phase space.
All you can do is to change the form of phase space.

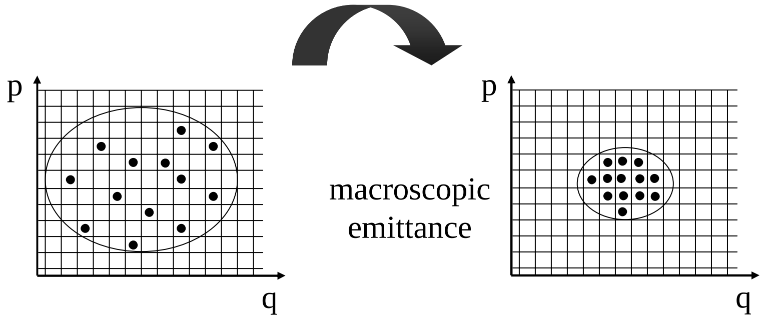
However, with velocity-dependent forces
drag, friction (dissipative) forces

electron, radiation, Laser, ionisation cooling

cooling is indeed possible!!

Coffee, cream, Liouville and Stochastic cooling

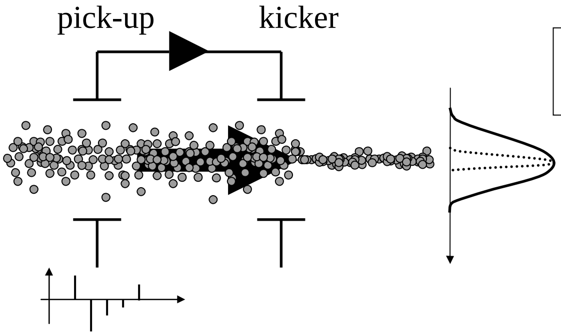
Stochastic cooling principle



macroscopic emittance

Stochastic cooling

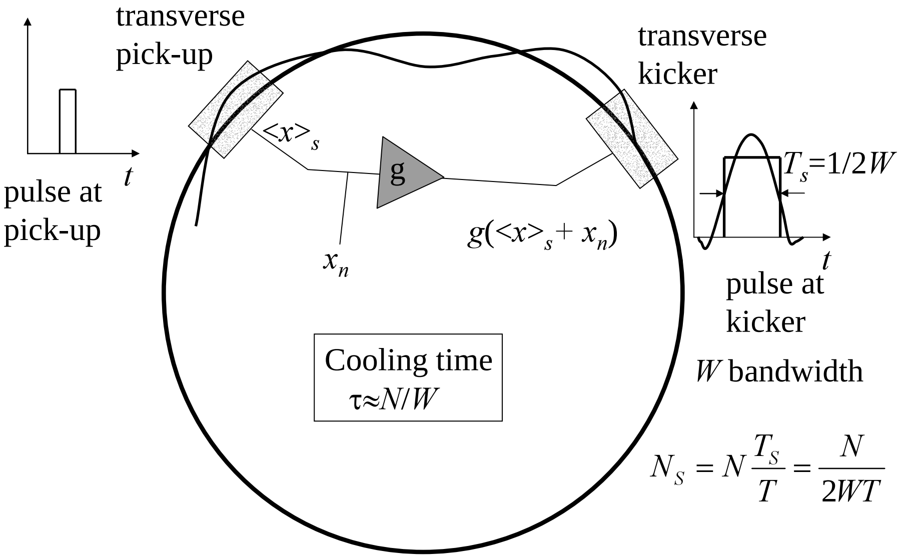
Liouville: Cooling is not possible with electromagnetic forces deflecting the particles (continous fluid, og $N=\infty$).
When single particles can be observed, and a corresponding correction applied, cooling is possible!
This is the secret of stochastic cooling!



$N=10^8$, $\sigma=5\text{mm}$
$\sigma/\sqrt{N}=0.5\mu\text{m}$

In reality $W<\infty$

Stochastic cooling



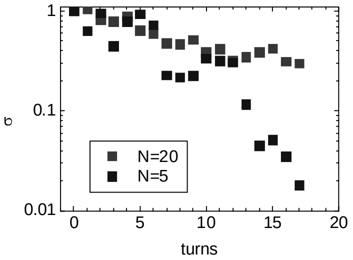
Cooling time
$\tau \approx N/W$

W bandwidth

$$N_S = N\frac{T_S}{T} = \frac{N}{2WT}$$

Stochastic cooling exercise

1) Ask for 5 random numbers with $\langle x\rangle=0$ and $\sigma=1$
2) Find actual $\langle x\rangle$ (in general $\langle x\rangle\neq 0$)
3) Subtract error in mean to restore mean to zero
4) Calculate new σ
5) Goto 1)
6) Watch σ as function of time
7) What is the cooling time?
8) Include electronical noise!



Cooling Time

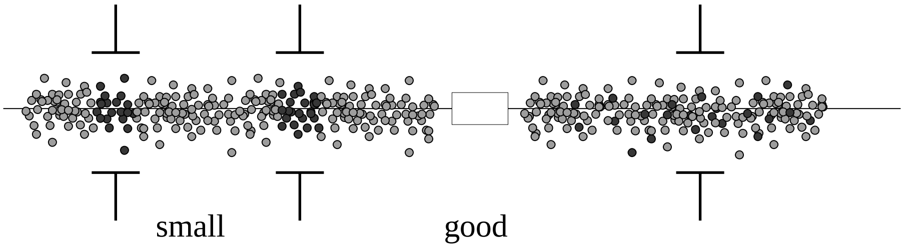
mixing

$$\frac{1}{\tau}=\frac{gW}{N}\left[1-\frac{g}{2}(\Gamma+\upsilon)\right]$$

noise/signal-ratio

optimum gain $g=1/(\Gamma+\upsilon)<1$

optimum cooling time $\dfrac{1}{\tau}=\dfrac{W}{N(\Gamma+\upsilon)}$



Cooling time 2

$$\frac{1}{\tau}=\frac{gW}{N}\left[1-\frac{g}{2}(\Gamma+\upsilon)\right]$$

optimum gain $\quad g=1/(\Gamma+\upsilon)<1$

optimum cooling time $\quad \frac{1}{\tau}=\frac{W}{N(\Gamma+\upsilon)}$

$\tau \propto N$

Decrease gain as cooling proceeds

Good mixing, $\Gamma = 1$, by designing storage ring so

$\eta=\partial(\Delta T/T)/\partial(\Delta p/p)$ is large. However small mixing PU→K

Large bandwidth ($W>$ GHz, $N_s \sim 10^{-3}N$)

Weak dependence on energy

Z dependence in υ

Stochastic cooling

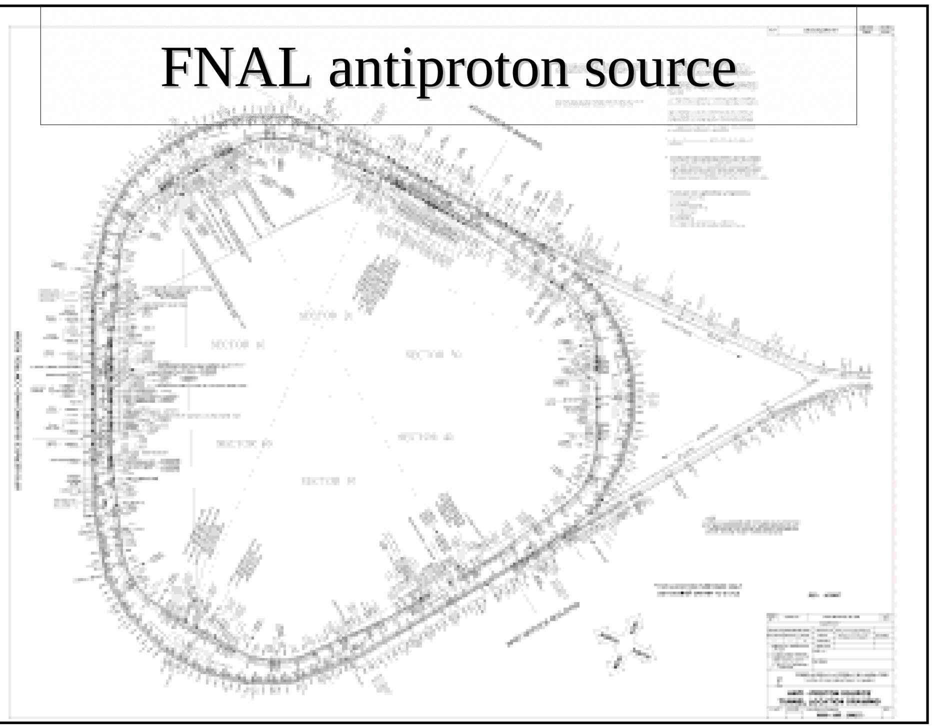
Betatron cooling: 2 systems (hor. and vert.)
  dist. PU → kicker = odd number of $\lambda/4$

Momentum cooling:
  acc. gap instead of transverse kicker

  (*i*) PU in high-dispersion region $\Delta x/x=D\ \Delta p/p$
  (*ii*) detect $\Delta f/f=\eta\ \Delta p/p$ and correct $\Delta p/p$

Stochastic cooling facilities:
ISR (1977), ICE, AA, AC, LEAR, AD @ CERN
Fermilab
TARN
COSY, GSI

FNAL antiproton source

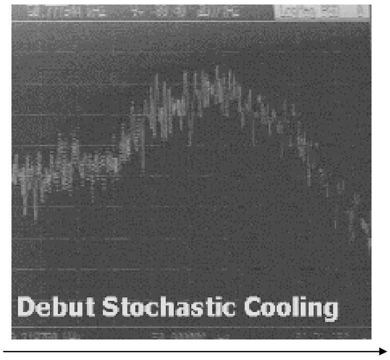


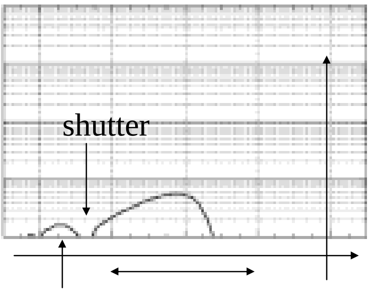
Stochastic Cooling

AD at CERN

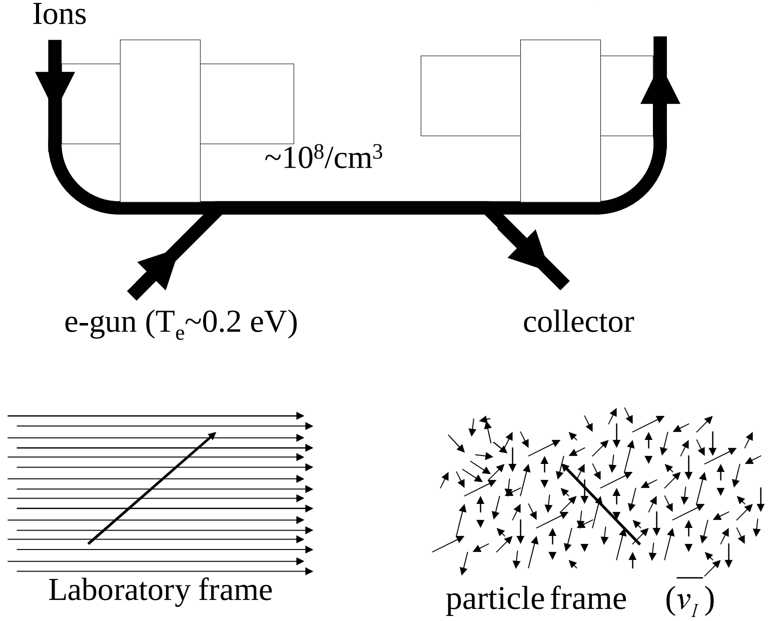
Fermilab antiproton accumulator stacking for 1 hour

10^8/2 sec pbar at 8 GeV

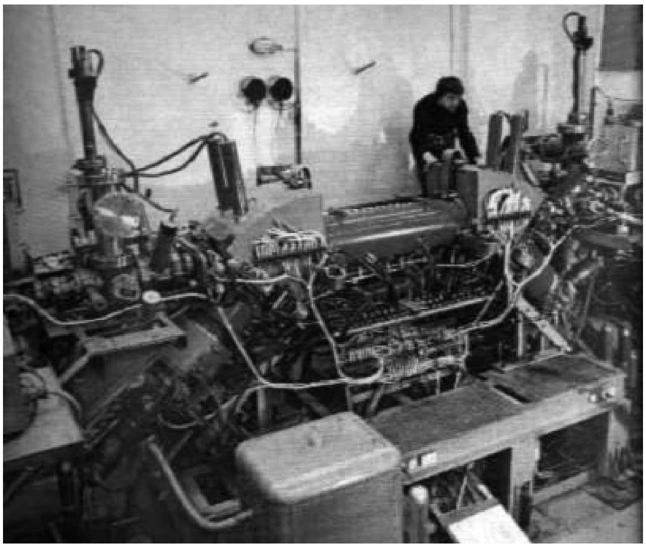




Electron cooling

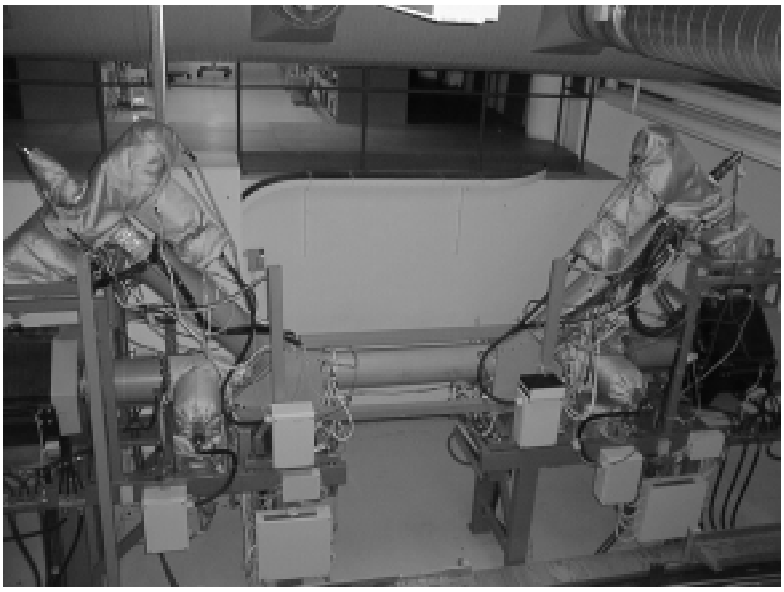


Electron Cooling



NAP-M ring
at INP, 1974,
68 MeV p

ASTRID electron cooler



Electron cooling 2

Initially $\overline{v_I^2} \geq \overline{v_e^2}$

$$T_I^i \equiv \frac{1}{2} M \overline{v_I^2} >> \frac{1}{2} m \overline{v_e^2} \equiv T_e^i$$

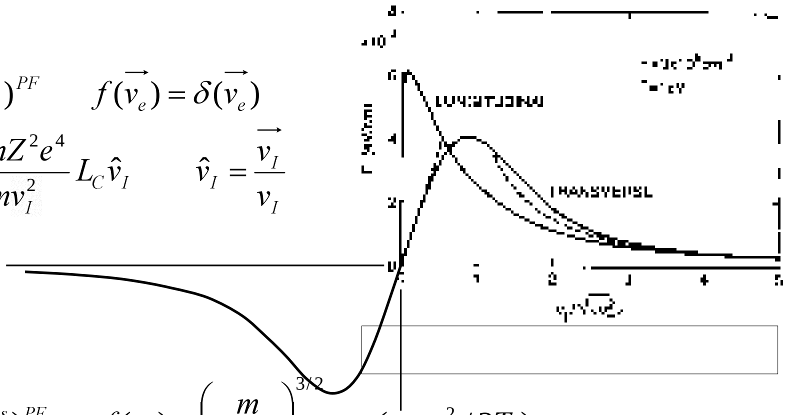
Finally $T_I^f = T_e^f$ no heating

$$v_I^{rms} \equiv \sqrt{\overline{v_I^2}} = \sqrt{\frac{m}{M}} v_e^{rms} \approx \frac{1}{43} \sqrt{\frac{m}{M}} v_e^{rms}$$

Electron cooling drag force

$(v_I > v_e^{rms})^{PF} \qquad f(\overrightarrow{v_e}) = \delta(\overrightarrow{v_e})$

$\overrightarrow{F} = -\frac{4\pi n Z^2 e^4}{m v_I^2} L_C \hat{v}_I \qquad \hat{v}_I = \frac{\overrightarrow{v_I}}{v_I}$



$(v_I < v_e^{rms})^{PF} \qquad f(v_e) = \left(\frac{m}{2\pi T_e}\right)^{3/2} \exp(-m v_e^2 / 2T_e)$

$\overrightarrow{F} = -\frac{4\sqrt{2\pi}}{3} \frac{n e^4 Z^2}{m} L_C \left(\frac{m}{T_e}\right)^{3/2} \overrightarrow{v_I}$

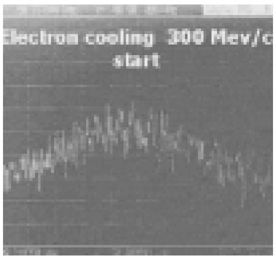
Electron cooling time

$$\tau \equiv \left|\frac{1}{v_I}\frac{dv_I}{dt}\right|^{-1} = \left|\frac{M v_I}{F}\right|$$

$$\tau = \frac{\gamma^2}{\eta}\frac{Mm}{Z^2 e^4}\frac{1}{nL}\begin{cases} \frac{1}{4\pi}(v_I^{PF})^3 & (v_I > v_e^{rms})^{PF} \\ \frac{3}{2\sqrt{2\pi}}\left(\frac{T_e}{m}\right)^{3/2} & (v_I < v_e^{rms})^{PF} \end{cases}$$

Typically τ~tens of seconds (*Z*=1)

Electron cooling



Electron cooling at AD

LEAR, ICE, AD @ CERN

CRYRING, CELSIUS

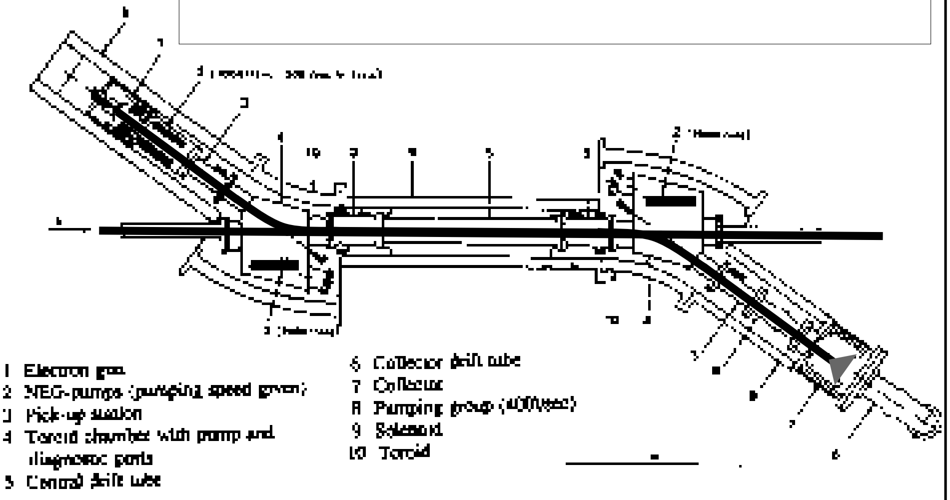
TSR, COSY, SIS, ESR

IUCF, Fermilab

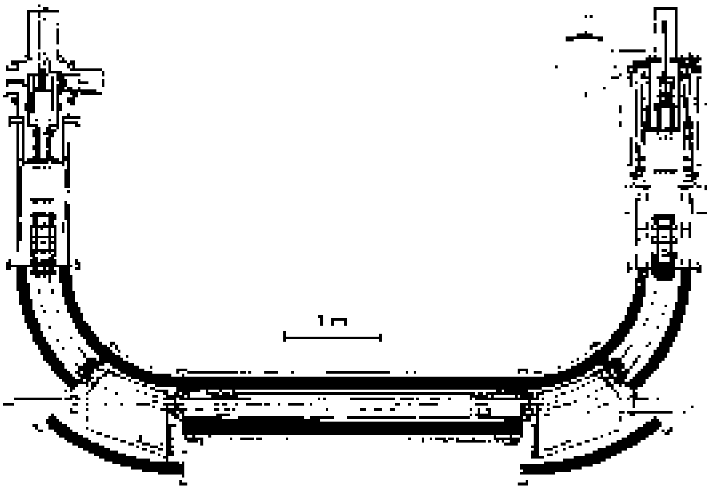
TARN, ..

ASTRID

LEAR/AD electron cooler

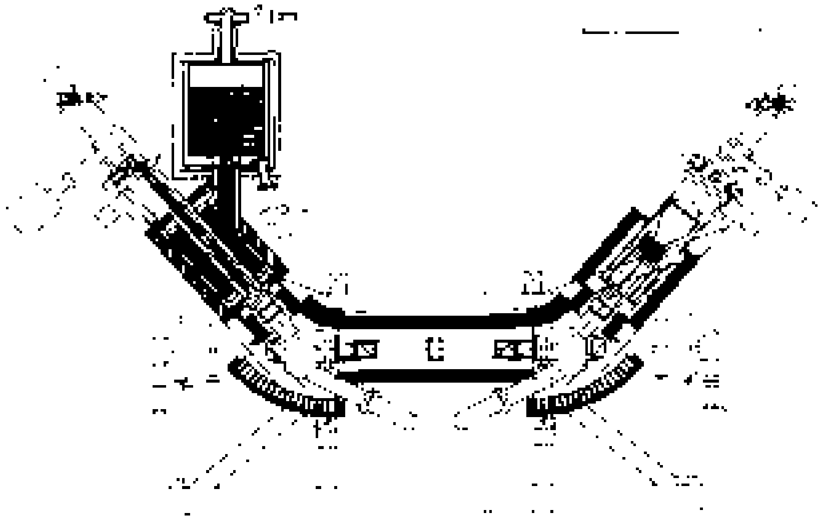


MUSES electron cooler



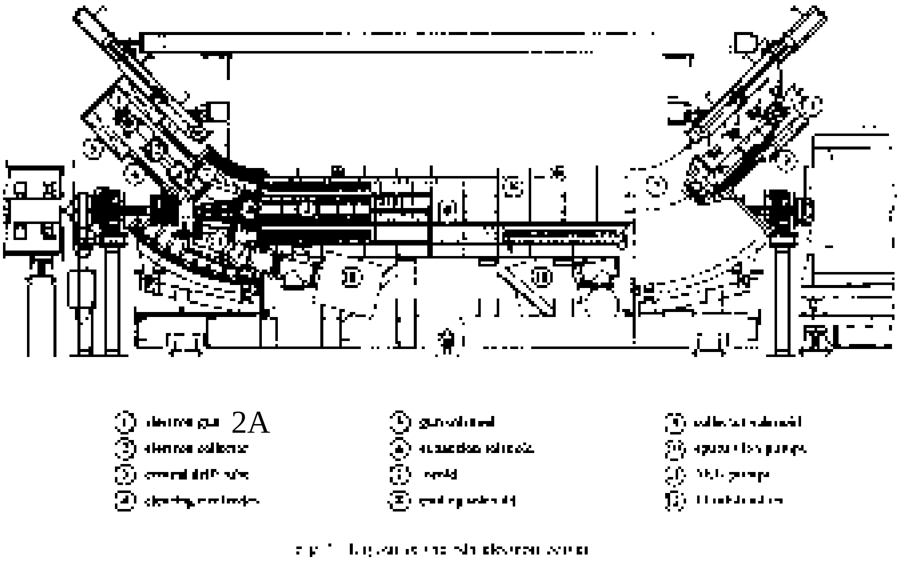
[illegible]

CRYRING electron cooler



[illegible]

SIS electron cooler



Mod. to simple description

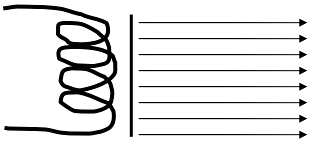
1) Flattened distribution due to acceleration

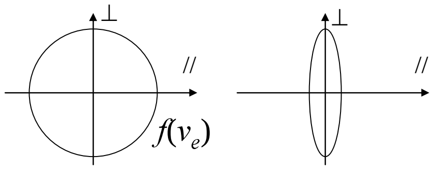
$$E_f = E_i + V$$

$$\Delta E_f = \Delta(\tfrac{1}{2} m v_e^2) = \Delta E_i = T_C$$

$$T_{//}^{PF} = \tfrac{1}{2} m(\Delta v_e)^2 \approx T_C \frac{T_C}{4V} << T_C$$

so $T_{//} \approx 0 \Rightarrow \tau_{//} \downarrow$

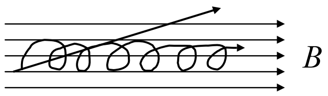




2) $B \neq 0$

$$B = \infty \rightarrow T_\perp \approx 0 \rightarrow \tau_\perp \downarrow (\tau_\perp \propto v_I^{-3})$$

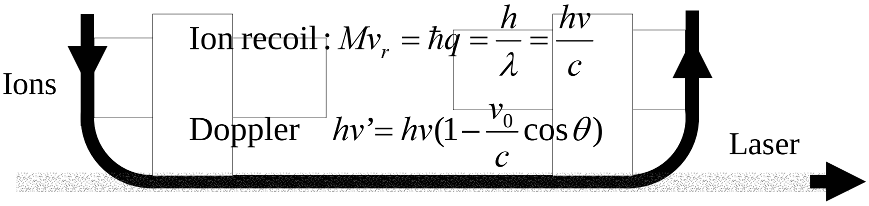
In practice $B=\infty$ only for distant collisions



Virtues of electron cooling

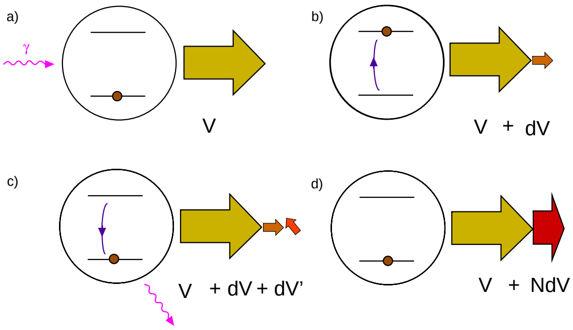
Versatile cooling technique
Longitudinal and transverse cooling
Cooling times $\tau \approx 0.1\text{-}1\,\mathrm{sec}\,A/Q^2$
$T_{//} << 0.1$ eV
$T_{\perp} \approx 0.1$ eV
in addition: adiabatic expansion $T_{//} \propto B$

Laser cooling



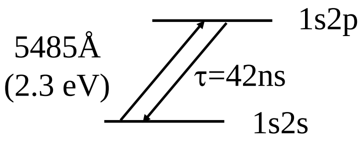
Ion recoil : $Mv_r = \hbar q = \frac{h}{\lambda} = \frac{hv}{c}$

Doppler $hv' = hv(1 - \frac{v_0}{c}\cos\theta)$



Kick from one photon absorption-emission

100 keV Li^+



Change in momentum: $\overline{\Delta p} = h/\lambda$

Change in energy: $\Delta E = p\Delta v = 12\text{meV}$

At saturation (stimulated = spontaneous)(1mW in Ø3mm)

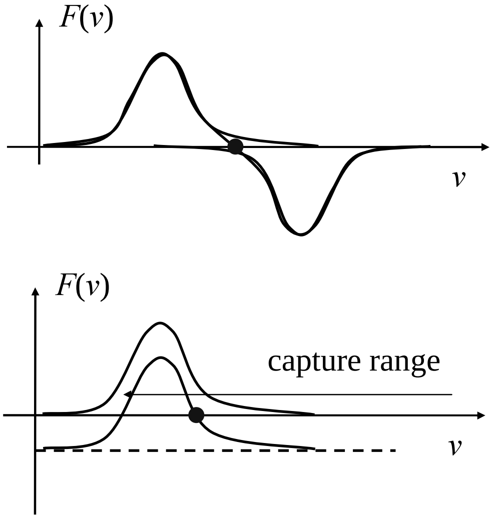
$r = ½\Gamma = 10^7 \text{s}^{-1}$

in 2 m: $r \cdot 2\text{m}/v = 15$

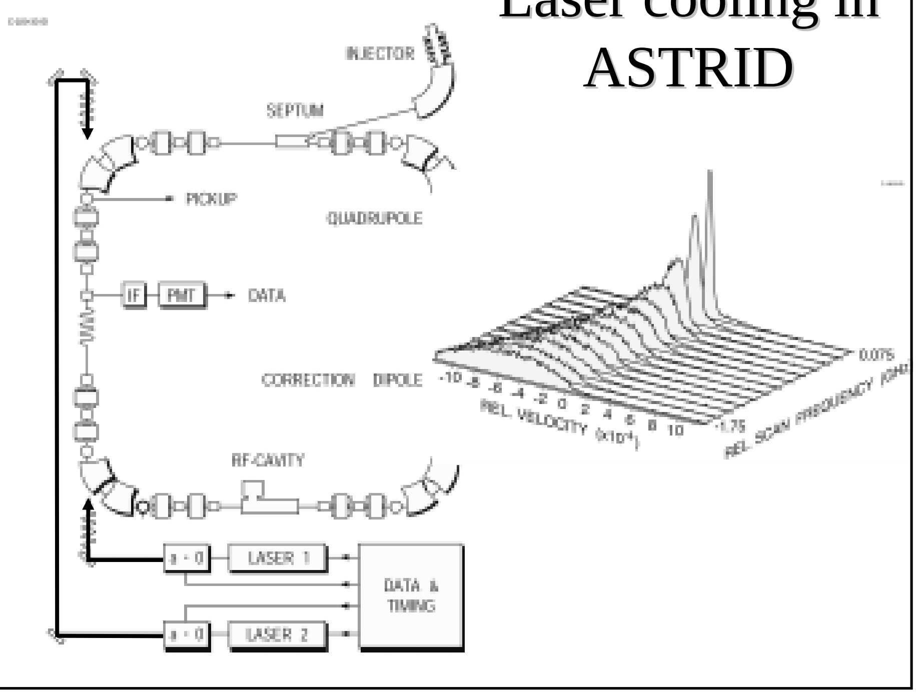
change in 2 m: 0.2 eV

Ultimate limit: single recoil = 12meV

Laser cooling



Laser cooling in ASTRID



Laser cooling

Virtues of laser cooling:

Laser cooling is fast

However:

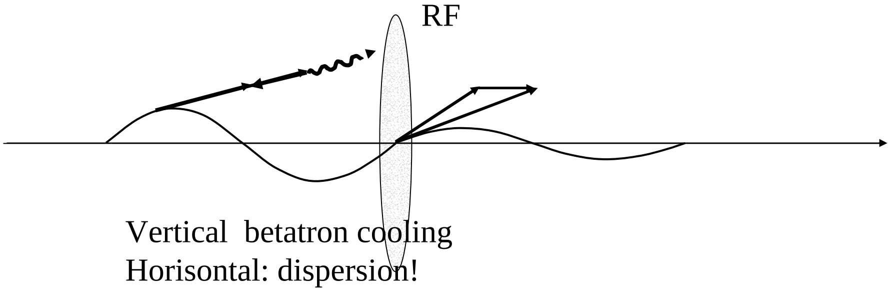
Only effective for longitudinal cooling

Not versatile: Li$^+$, Be$^+$, Mg$^+$, …

Radiation damping

(for details,
see lectures by L. Rivkin)

In principle: any charged particle
in practise: only electrons/positrons
since $\tau \approx E/(U_0/T_0)$



Vertical betatron cooling
Horisontal: dispersion!
Longitudinal: finite energy quanta

Ionisation cooling

Friction force $\vec{F} \propto -\vec{v}$
Slowing down in matter
Not hadrons due to large inelastic cross section
Not electrons due to short radiation length

Can only be used for in μ in μ-collider/ ν-factory

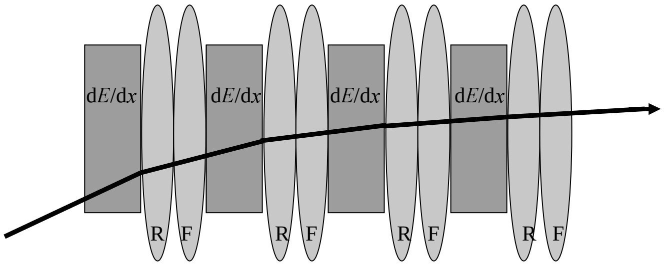
ν's produced by decaying μ's
μ's produced from decaying π's
π's produced by p's on target

Since μ's do not live forever (2.2 μs)
cooling has to be fast.

Also emittances are very large!

Ionisation cooling principle

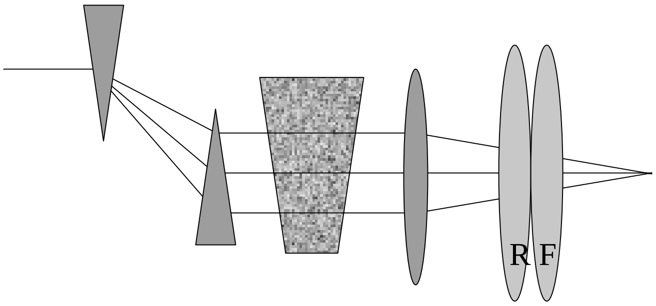
Transverse cooling:
muons lose energy by dE/dx and longitudinal momentum is replaced by RF



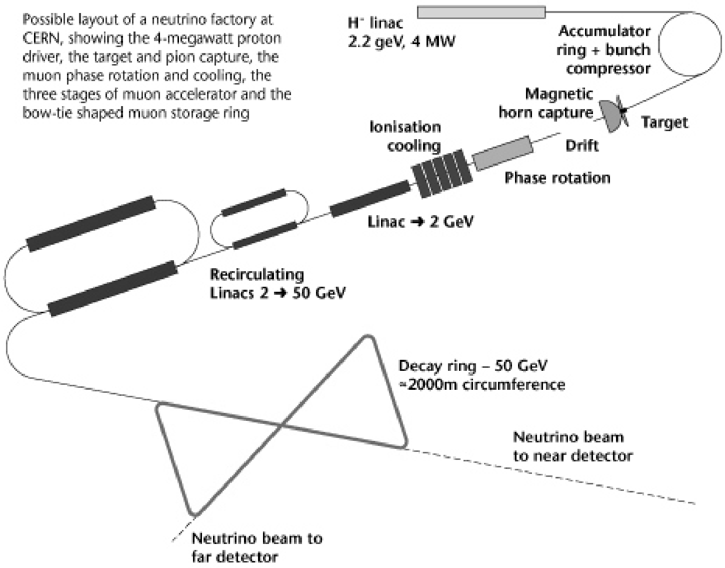
To minimize heating from Coulomb scattering:
- ●Small $\beta_\perp$ (high-field solenoids)
- ●Large L_R (low-*Z* absorber): Liquid H_2

Ionisation energy cooling

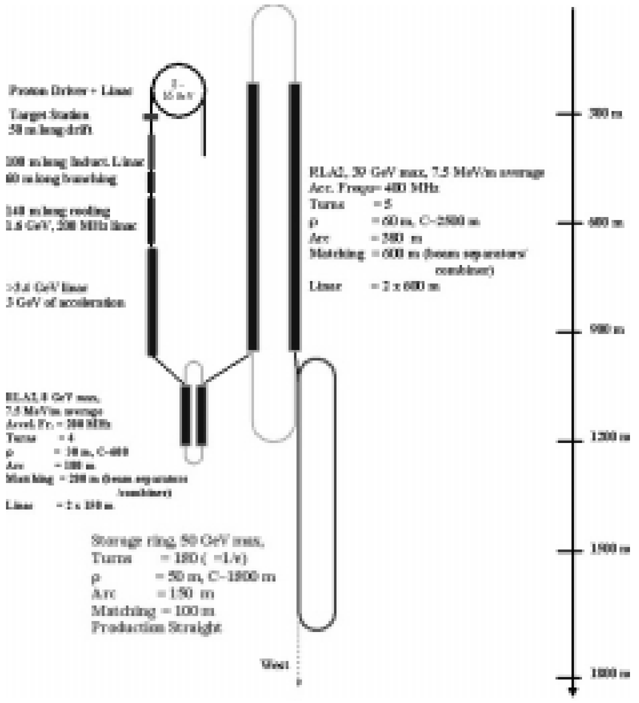
Ionisation energy cooling using a wedge and dispersion



possible ν-factory at CERN

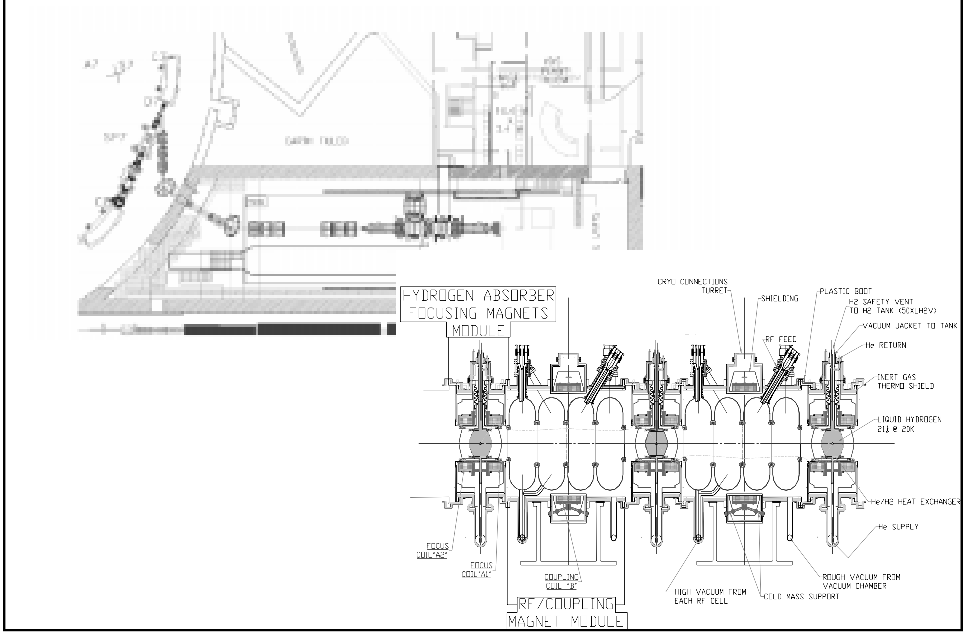


Possible layout of a neutrino factory at CERN, showing the 4-megawatt proton driver, the target and pion capture, the muon phase rotation and cooling, the three stages of muon accelerator and the bow-tie shaped muon storage ring



FNAL ν-factory

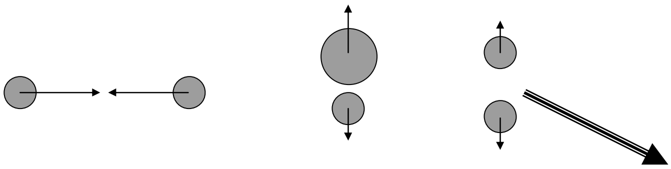
MICE at RAL



Other cooling methods

Stimulated radiation cooling

Radiative cooling



Conclusions

	Stochastic	**Electron**	**Radiation**	**Laser**	**Ionisation**
Species	all	ions	e^-/e^+	some ions	muons
Favoured beam velocity	high	medium $0.01<\beta$ $\beta <0.1$	very high $\gamma>100$	any (but Doppler)	any
Favoured beam intensity	low	any	any	any	any
Cooling time	$N\cdot 10^{-8}$ s	$10\text{-}10^{-2}$ s	$>10^{-3}$ s	$10^{-4}\text{-}10^{-5}$ s	10^{-6} s
Favoured beam temperature	high	low	any	low	any