Synchrotron Radiation
An Introduction

L. Rivkin, PSI

Introduction to Accelerator Physics Course
CERN Accelerator School, Baden bei Wien, September 2004

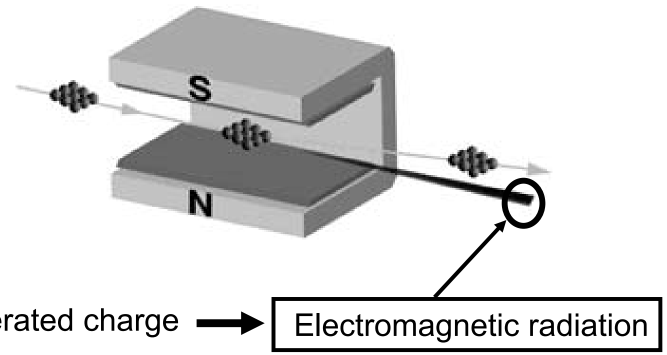
Books

Helmut Wiedemann
- Synchrotron Radiation
  Springer-Verlag Berlin Heidelberg New York 2003
- Particle Accelerator Physics I
  Springer-Verlag Berlin Heidelberg New York 2003

A. W. Chao, M. Tigner
- Handbook of Accelerator Physics and Engineering
  World Scientific 1999

CERN Accelerator School Proceedings
- http://cas.web.cern.ch/

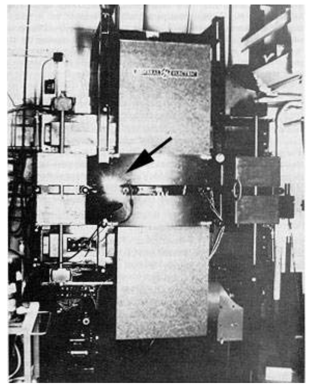
Curved orbit of electrons in magnet field



Crab Nebula
6000 light years away

First light observed
1054 AD

GE Synchrotron
New York State



First light observed
1947

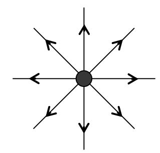
Maxwell equations (poetry)

War es ein Gott, der diese Zeichen schrieb
Die mit geheimnisvoll verborg'nem Trieb
Die Kräfte der Natur um mich enthüllen
Und mir das Herz mit stiller Freude füllen.
Ludwig Boltzman

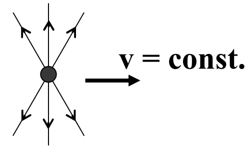
Was it a God whose inspiration
Led him to write these fine equations
Nature's fields to me he shows
And so my heart with pleasure glows.
translated by John P. Blewett

Why do they radiate?

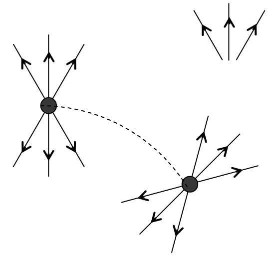
Charge at rest: Coulomb field



Uniformly moving charge



Accelerated charge

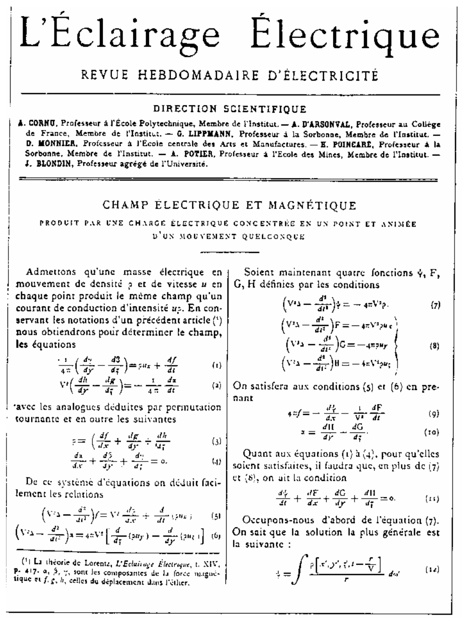


Bremstrahlung

1898 Liénard:

ELECTRIC AND MAGNETIC FIELDS PRODUCED BY A POINT CHARGE MOVING ON AN ARBITRARY PATH

(by means of retarded potentials)



L'Éclairage Électrique

REVUE HEBDOMADAIRE D'ÉLECTRICITÉ

DIRECTION SCIENTIFIQUE

A. CORNU, Professeur à l'École Polytechnique, Membre de l'Institut. — **A. D'ARSONVAL**, Professeur au Collège de France, Membre de l'Institut. — **G. LIPPMANN**, Professeur à la Sorbonne, Membre de l'Institut. — **D. MONNIER**, Professeur à l'École centrale des Arts et Manufactures. — **H. POINCARÉ**, Professeur à la Sorbonne, Membre de l'Institut. — **A. POTIER**, Professeur à l'Ecole des Mines, Membre de l'Institut. — **J. BLONDIN**, Professeur agrégé de l'Université.

CHAMP ÉLECTRIQUE ET MAGNÉTIQUE

PRODUIT PAR UNE CHARGE ÉLECTRIQUE CONCENTRÉE EN UN POINT ET ANIMÉE D'UN MOUVEMENT QUELCONQUE

Admettons qu'une masse électrique en mouvement de densité ρ et de vitesse u en chaque point produit le même champ qu'un courant de conduction d'intensité $u\rho$. En conservant les notations d'un précédent article (1) nous obtiendrons pour déterminer le champ, les équations

$$\frac{1}{4\pi}\left(\frac{d\gamma}{dy}-\frac{d\beta}{dz}\right)=\rho u_x+\frac{df}{dt} \quad (1)$$

$$V^2\left(\frac{dh}{dy}-\frac{dg}{dz}\right)=-\frac{1}{4\pi}\frac{d\alpha}{dt} \quad (2)$$

avec les analogues déduites par permutation tournante et en outre les suivantes

$$\rho=\left(\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}\right) \quad (3)$$

$$\frac{d\alpha}{dx}+\frac{d\beta}{dy}+\frac{d\gamma}{dz}=0. \quad (4)$$

De ce système d'équations on déduit facilement les relations

$$\left(V^2\Delta-\frac{d^2}{dt^2}\right)f=V^2\frac{d\rho}{dx}+\frac{d}{dt}(\rho u_x) \quad (5)$$

$$\left(V^2\Delta-\frac{d^2}{dt^2}\right)\alpha=4\pi V^2\left[\frac{d}{dz}(\rho u_y)-\frac{d}{dy}(\rho u_z)\right] \quad (6)$$

(1) La théorie de Lorentz, *L'Éclairage Électrique*, t. XIV, p. 417. α, β, γ, sont les composantes de la force magnétique et f, g, h, celles du déplacement dans l'éther.

Soient maintenant quatre fonctions ψ, F, G, H définies par les conditions

$$\left(V^2\Delta-\frac{d^2}{dt^2}\right)\psi=-4\pi V^2\rho. \quad (7)$$

$$\left.\begin{aligned}\left(V^2\Delta-\frac{d^2}{dt^2}\right)F&=-4\pi V^2\rho u_x\\ \left(V^2\Delta-\frac{d^2}{dt^2}\right)G&=-4\pi\rho u_y\\ \left(V^2\Delta-\frac{d^2}{dt^2}\right)H&=-4\pi V^2\rho u_z\end{aligned}\right\} \quad (8)$$

On satisfera aux conditions (5) et (6) en prenant

$$4\pi f=-\frac{d\psi}{dx}-\frac{1}{V^2}\frac{dF}{dt} \quad (9)$$

$$\alpha=\frac{dH}{dy}-\frac{dG}{dz}. \quad (10)$$

Quant aux équations (1) à (4), pour qu'elles soient satisfaites, il faudra que, en plus de (7) et (8), on ait la condition

$$\frac{d\psi}{dt}+\frac{dF}{dx}+\frac{dG}{dy}+\frac{dH}{dz}=0. \quad (11)$$

Occupons-nous d'abord de l'équation (7). On sait que la solution la plus générale est la suivante :

$$\psi=\int\frac{\rho\left[x',y',z',t-\frac{r}{V}\right]}{r}\,d\omega' \quad (12)$$

Fig. 1. First page of Liénard's 1898 paper.

Liénard-Wiechert potentials

$$\varphi(t) = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left[r\left(1-\vec{\mathbf{n}}\cdot\vec{\beta}\right)\right]_{ret}} \qquad \vec{\mathbf{A}}(t) = \frac{q}{4\pi\varepsilon_0 c^2}\left[\frac{\vec{\mathbf{v}}}{r\left(1-\vec{\mathbf{n}}\cdot\vec{\beta}\right)}\right]_{ret}$$

and the electromagnetic fields:

$$\nabla\cdot\vec{\mathbf{A}} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0 \qquad \text{(Lorentz gauge)}$$

$$\beta \equiv v/c$$

$$\vec{\mathbf{B}} = \nabla\times\vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = -\nabla\varphi - \frac{\partial\vec{\mathbf{A}}}{\partial t}$$

$$\gamma \equiv \frac{1}{\sqrt{1-v^2/c^2}}$$

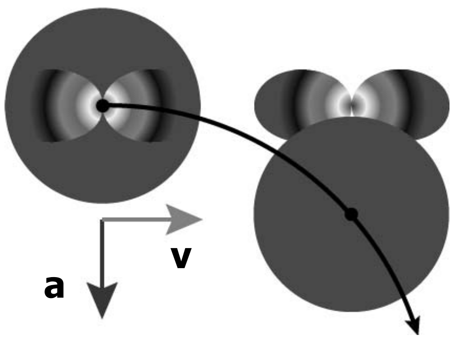
Fields of a moving charge

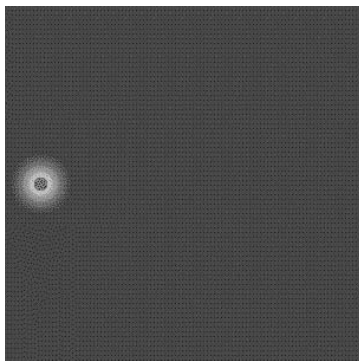
$$\vec{\mathbf{E}}(t) = \frac{q}{4\pi\varepsilon_0}\left[\frac{\vec{\mathbf{n}}-\vec{\beta}}{\left(1-\vec{\mathbf{n}}\cdot\vec{\beta}\right)^3\gamma^2}\cdot\frac{1}{\mathbf{r}^2}\right]_{ret} +$$

$$\frac{q}{4\pi\varepsilon_0 c}\left[\frac{\vec{\mathbf{n}}\times\left[\left(\vec{\mathbf{n}}-\vec{\beta}\right)\times\dot{\vec{\beta}}\right]}{\left(1-\vec{\mathbf{n}}\cdot\vec{\beta}\right)^3\gamma^2}\cdot\frac{1}{\mathbf{r}}\right]_{ret}$$

$$\vec{\mathbf{B}}(t) = \frac{1}{c}\left[\vec{\mathbf{n}}\times\vec{\mathbf{E}}\right]$$

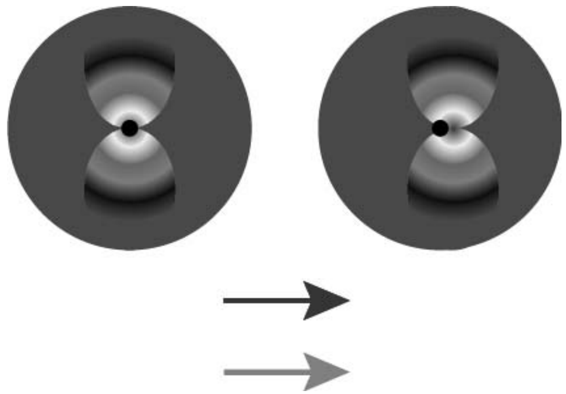
Transverse acceleration





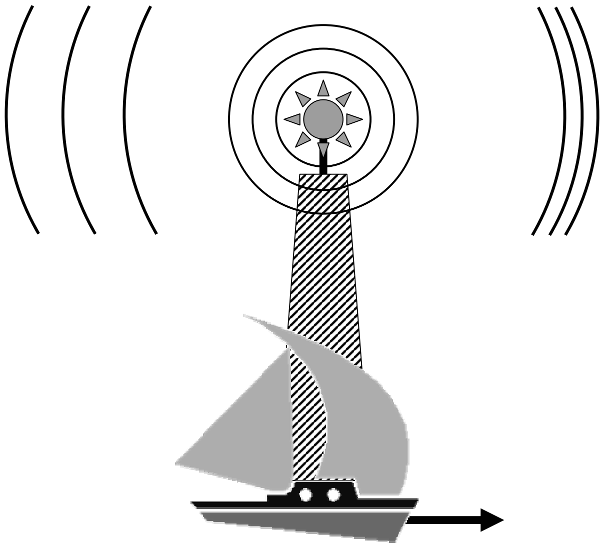
Radiation field quickly separates itself from the Coulomb field

Longitudinal acceleration



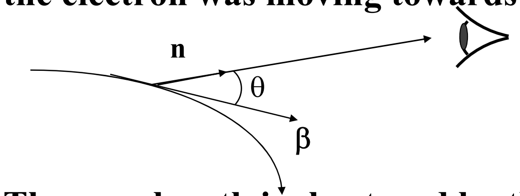
Radiation field cannot separate itself from the Coulomb field

Moving Source of Waves



Time compression

Electron with velocity β emits a wave with period T_{emit} while the observer sees a different period T_{obs} because the electron was moving towards the observer



$$T_{obs} = (1 - \mathbf{n} \cdot \boldsymbol{\beta})\, T_{emit}$$

The wavelength is shortened by the same factor

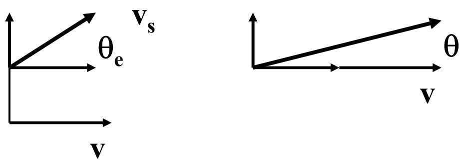
$$\lambda_{obs} = (1 - \beta \cos\theta)\, \lambda_{emit}$$

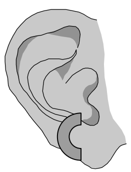
in ultra-relativistic case, looking along a tangent to the trajectory

$$\lambda_{obs} = \frac{1}{2\gamma^2} \lambda_{emit}$$ **since** $$1 - \beta = \frac{1 - \beta^2}{1 + \beta} \cong \frac{1}{2\gamma^2}$$

Angular Collimation

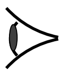
Galileo: sound waves v_s = 331 m/s





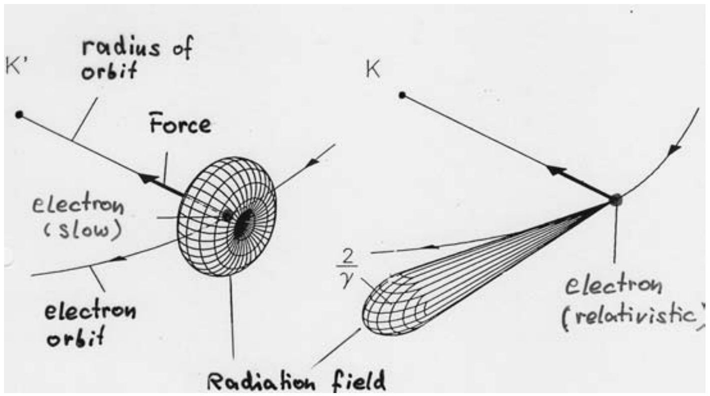
$$\theta = \frac{v_{s\perp}}{v_{s\|} + \mathbf{v}} = \frac{v_{s\perp}}{v_{s\|}} \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}} \approx \theta_e \cdot \frac{1}{1 + \frac{\mathbf{v}}{v_s}}$$

Lorentz: speed of light c = 3·10^8 m/s

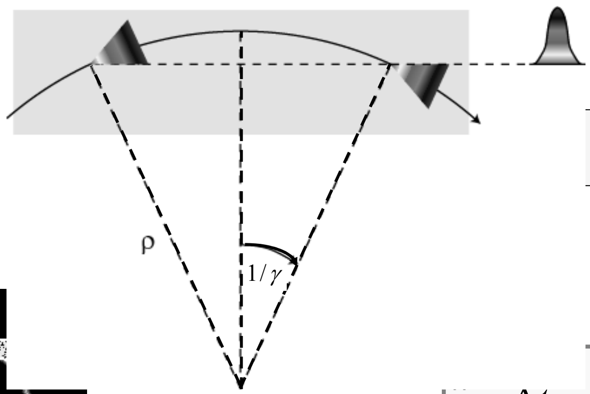


$$\theta = \frac{1}{\gamma} \cdot \theta_e$$

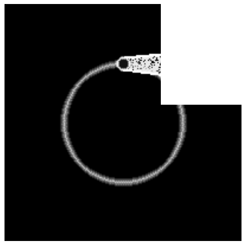
Radiation is emitted into a narrow cone



v << c **v ≈ c**



$$\frac{2\rho}{\gamma c} \cdot \frac{1}{2\gamma^2}$$



$$\Delta t \quad \gamma^3 \omega_0$$

Typical frequency of synchrotron light

Due to extreme collimation of light

• observer sees only a small portion of electron trajectory **(a few mm)**

$$l \sim \frac{2\rho}{\gamma}$$

• Pulse length: difference in times it takes an electron and a photon to cover this distance

$$\Delta t \sim \frac{l}{\beta c} - \frac{l}{c} = \frac{l}{\beta c}(1-\beta)$$

Synchrotron radiation power

Power emitted is proportional to:

$$\boldsymbol{P \propto E^2 B^2}$$

$$P_{\mathrm{SR}} = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$P_{\mathrm{SR}} = \frac{2}{3}\alpha\hbar c^2 \frac{\gamma^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3}\frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5}\left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

$$\alpha = \frac{1}{137}$$

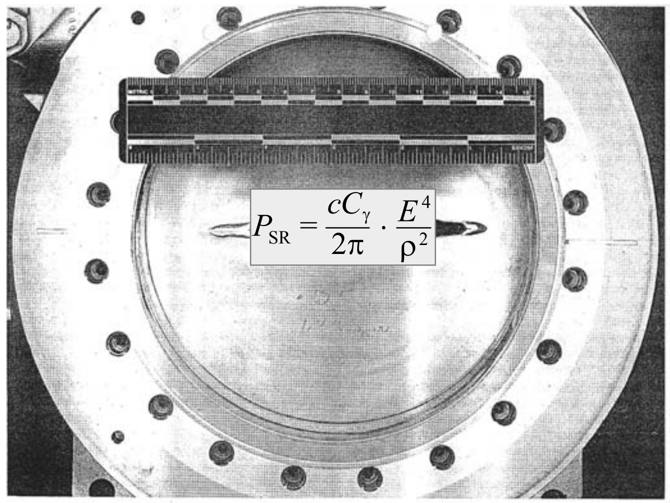
$$\hbar c = 197\ \mathrm{Mev} \cdot \mathrm{fm}$$

Energy loss per turn:

$$U_0 = C_\gamma \cdot \frac{E^4}{\rho}$$

$$U_0 = \frac{4\pi}{3}\alpha\hbar c \frac{\gamma^4}{\rho}$$

The power is all too real!

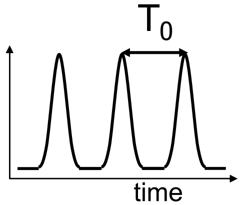


$$P_{\mathrm{SR}} = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

ig. 12. Damaged X-ray ring front end gate valve. The power incident on the valve was approximately 1 kW for a duration estimated to 2–10 min and drilled a hole through the valve plate.

Spectrum of synchrotron radiation

- Synchrotron light comes in a series of flashes every T_0 (revolution period)



- the spectrum consists of harmonics of $$\omega_0 = \frac{1}{T_0}$$

- flashes are extremely short: harmonics reach up to very high frequencies $$\omega_{typ} \cong \gamma^3 \omega_0$$

$$\omega_0 \sim 1\ \text{MHz}$$
$$\gamma \sim 4000$$
$$\omega_{typ} \sim 10^{16}\ \text{Hz!}$$

- At high frequencies the individual harmonics overlap

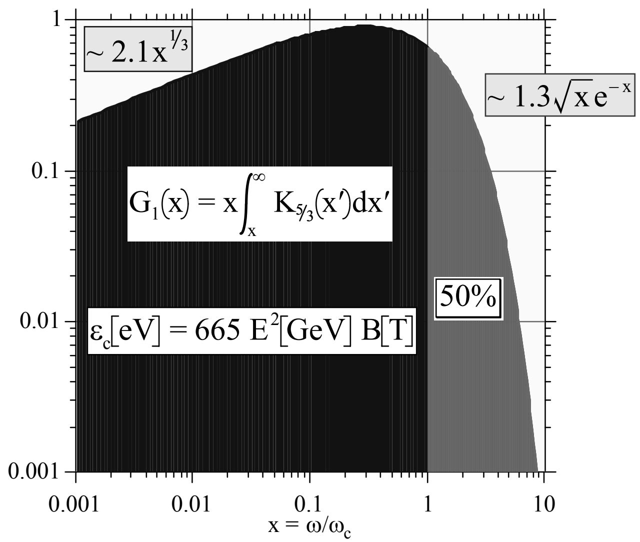
continuous spectrum !

$$\frac{dP}{d\omega} = \frac{P_{tot}}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)$$

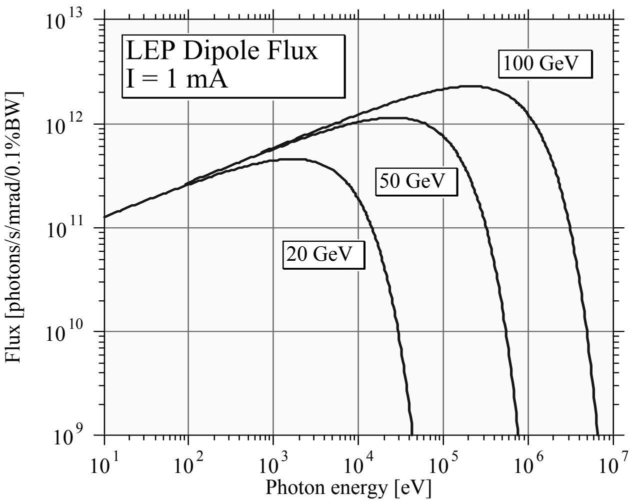
$$S(x) = \frac{9\sqrt{3}}{8\pi}\, x \int_x^\infty K_{5/3}(x')\,dx' \qquad \int_0^\infty S(x')\,dx' = 1$$

$$P_{tot} = \frac{2}{3} \hbar c^2 \alpha \frac{\gamma^4}{\rho^2}$$

$$\omega_c = \frac{3}{2} \frac{c\gamma^3}{\rho}$$



Synchrotron radiation flux for different LEP energies



$$\text{Flux}\left[\frac{\text{photons}}{\text{s} \cdot \text{mrad} \cdot 0.1\%\text{BW}}\right] = 2.46 \cdot 10^{13} E[\text{GeV}]\ I[\text{A}]\ G_1(x)$$

Angular divergence of radiation

The rms opening angle R'

- **at the critical frequency:** $\omega = \omega_c \qquad R' \approx \frac{0.54}{\gamma}$

- **well below** $\omega \ll \omega_c \qquad R' \approx \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3} \approx 0.4 \left(\frac{\lambda}{\rho}\right)^{1/3}$

independent of γ !

- **well above** $\omega \gg \omega_c \qquad R' \approx \frac{0.6}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/2}$