Transverse Beam Dynamics III

I) Linear Beam Optics

Single Particle Trajectories
Magnets and Focusing Fields
Tune & Orbit

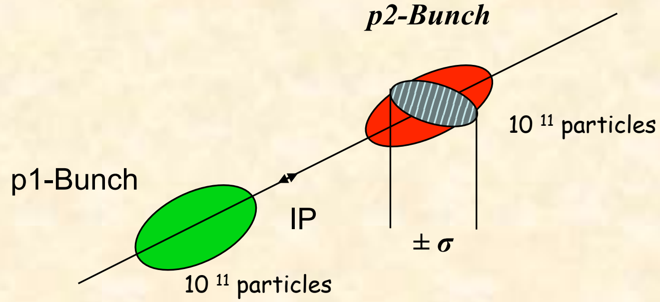
II) The State of the Art in High Energy Machines:

The Beam as Particle Ensemble
Emittance and Beta-Function
Colliding Beams & Luminosity

III) Errors in Field and Gradient:

Liouville during Acceleration
The $\Delta p/p \neq 0$ problem
Dispersion
Chromaticity

Luminosity



$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

Example: Luminosity run at LHC

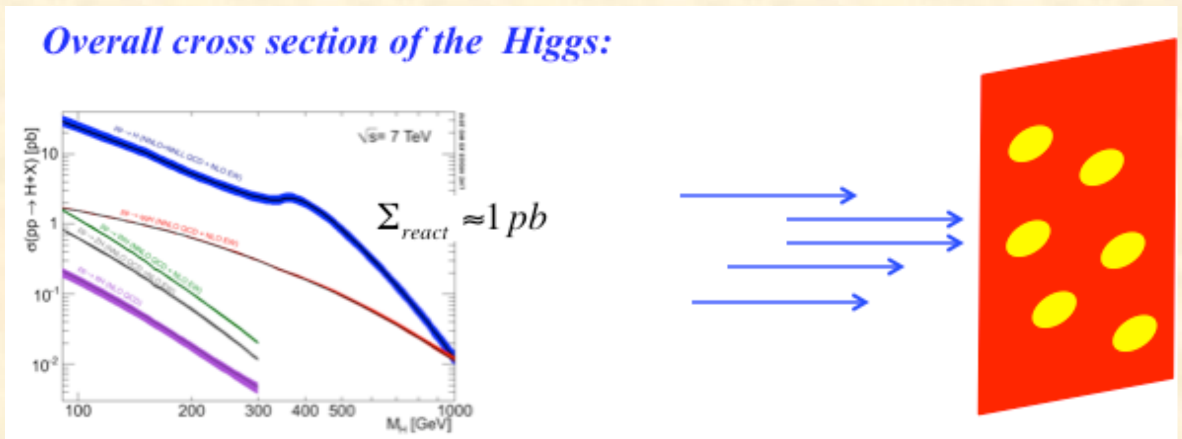
$$\beta^*_{x,y} = 0.55\, m \qquad f_0 = 11.245\, kHz$$

$$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m \qquad n_b = 2808$$

$$\sigma_{x,y} = 17\ \mu m$$

$$I_p = 584\ mA$$

$$L = 1.0 * 10^{34}\ \frac{1}{cm^2 s}$$

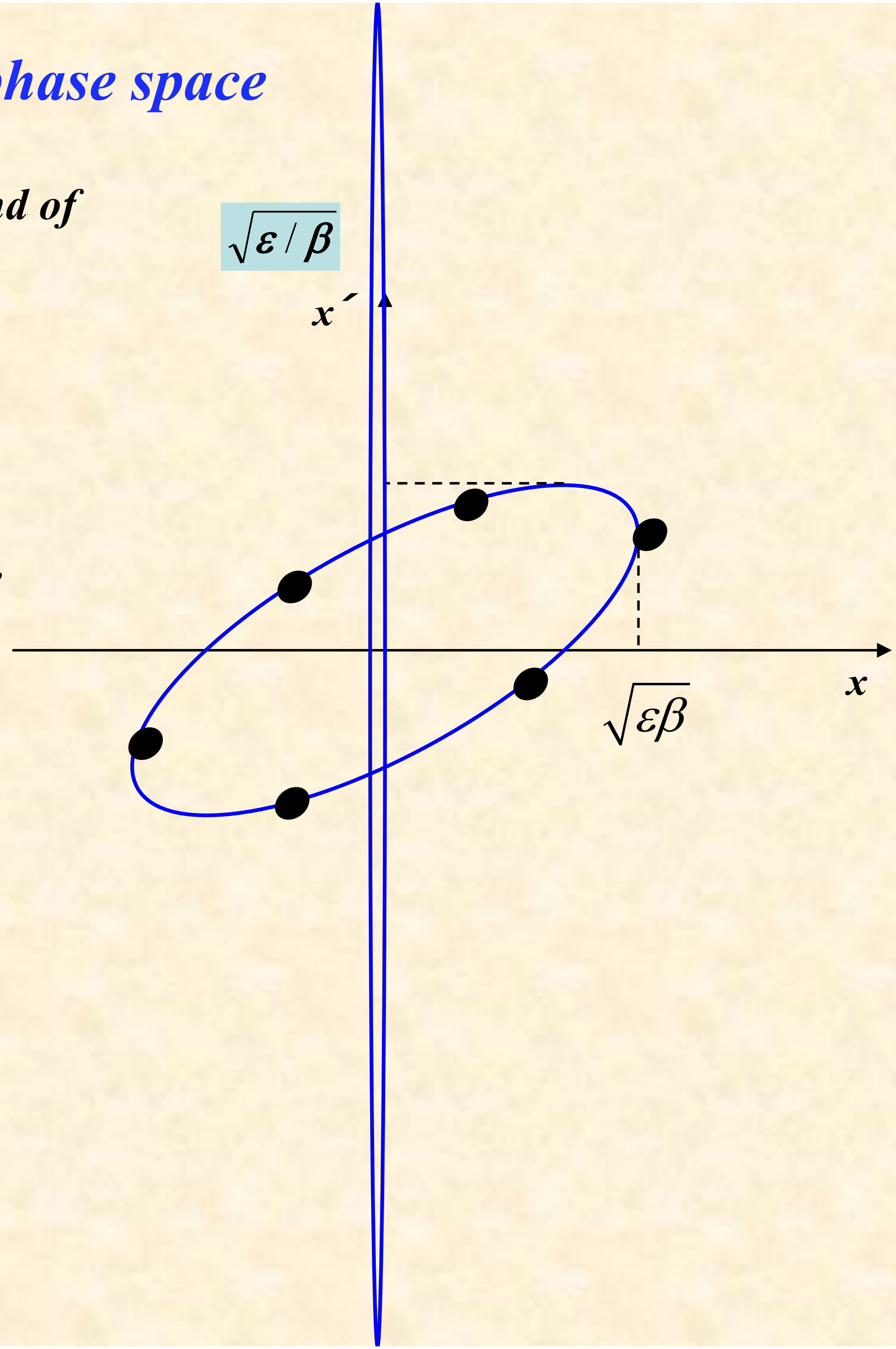


Make β* as small as possible !!!

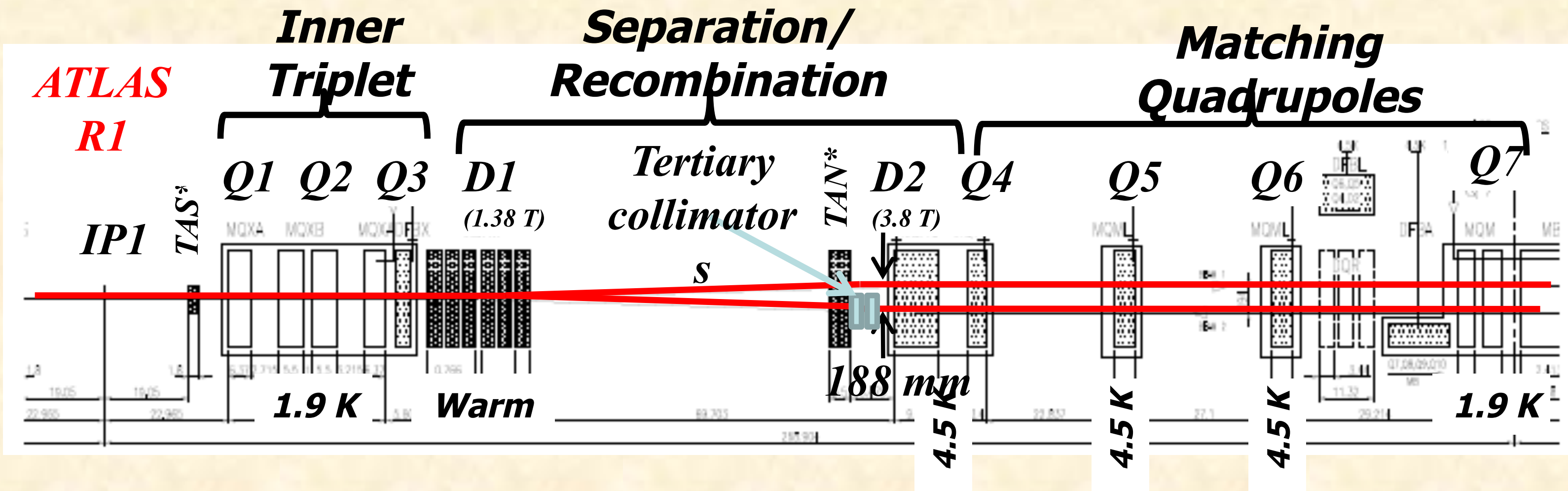
Mini-Beta-Insertions in phase space

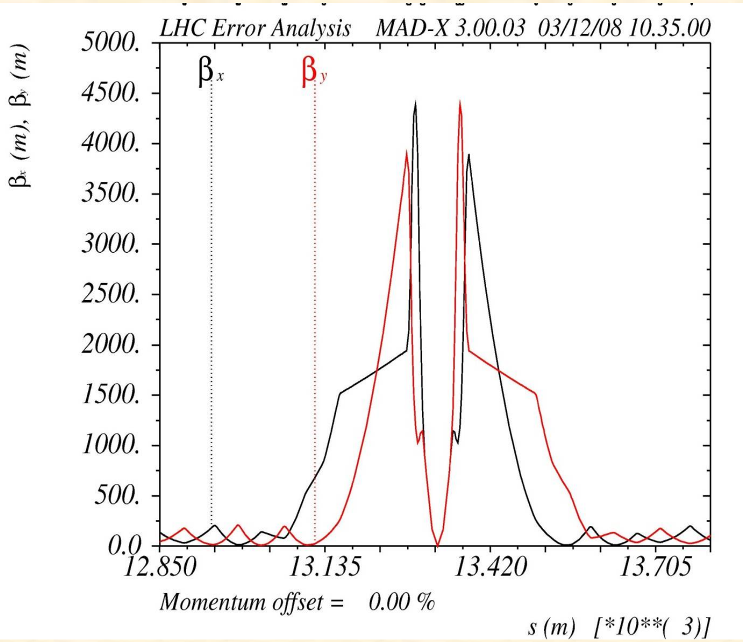
A mini-β insertion is always a kind of
special symmetric drift space.
→ greetings from Liouville

the smaller the beam size
the larger the bam divergence

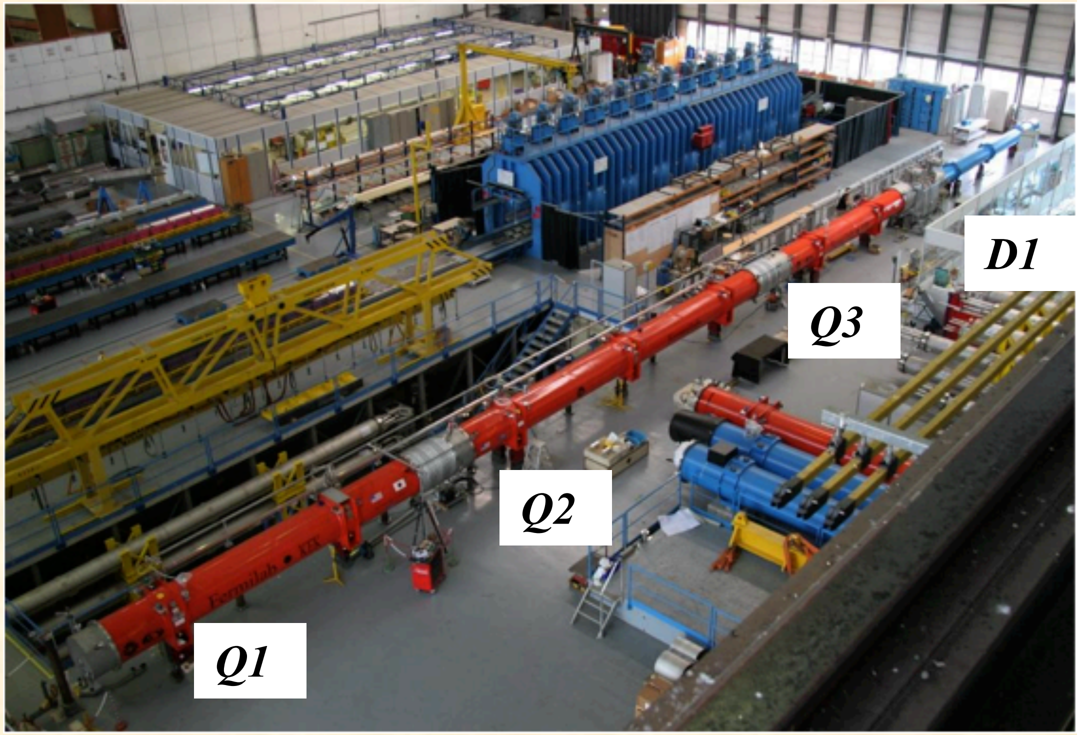


The LHC Insertions



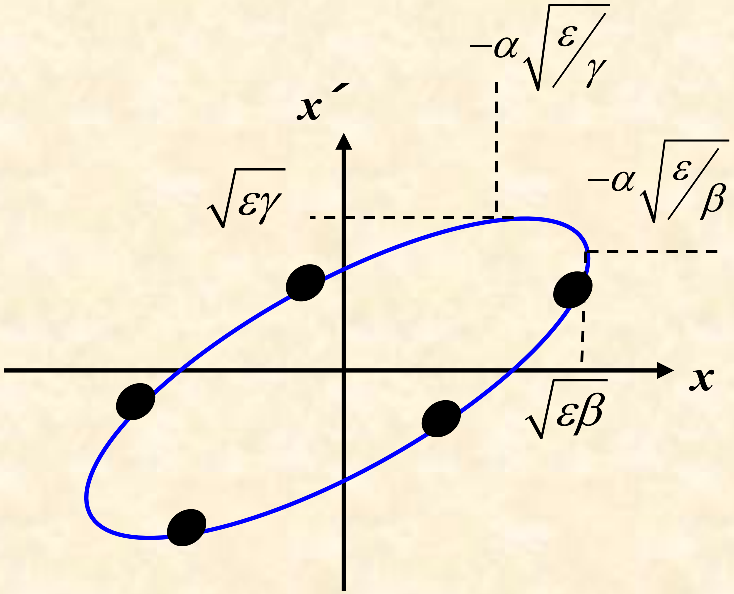


mini β optics



14.) Liouville during Acceleration

$$\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s) x(s) x'(s) + \beta(s)\, x'^2(s)$$



***Beam Emittance** corresponds to the area covered in the*
x, x´ Phase Space Ellipse

***Liouville:** Area in phase space is constant.*

But so sorry ... $\varepsilon \neq const$!

Classical Mechanics:

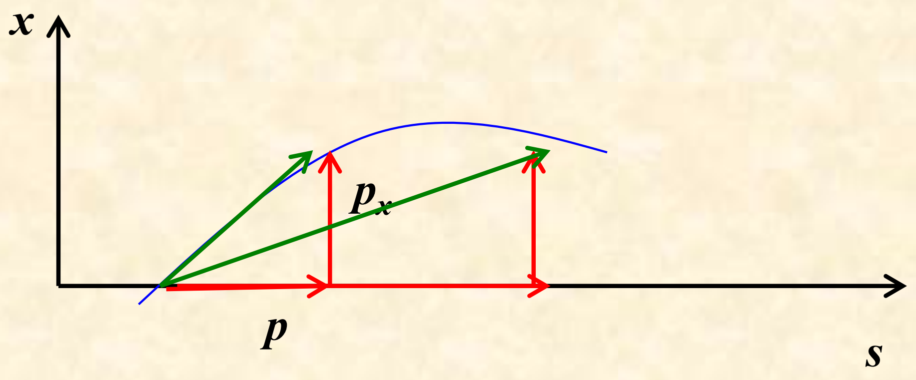
phase space = diagram of the two canonical variables
position & momentum
x p_x

According to Hamiltonian mechanics:
phase space diagram relates the variables q and p

Liouvilles Theorem: $$\int p\,dq = const$$

$$\int p_x\,dx = const$$

for convenience (i.e. because we are lazy bones) we use in accelerator theory:



$$x' = \frac{dx}{ds} = \frac{dx}{dt}\frac{dt}{ds} = \frac{\beta_x}{\beta} = \frac{p_x}{p}$$

$$\underbrace{\int x'\,dx}_{\varepsilon} = \frac{\int p_x\,dx}{p} \propto \frac{const}{m_0 c\cdot\gamma\beta}$$

$$\Rightarrow \quad \varepsilon = \int x'dx \propto \frac{1}{\beta\gamma}$$

the beam emittance shrinks during acceleration $\varepsilon \sim 1/\gamma$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\beta_x = \frac{v_x}{c}$$

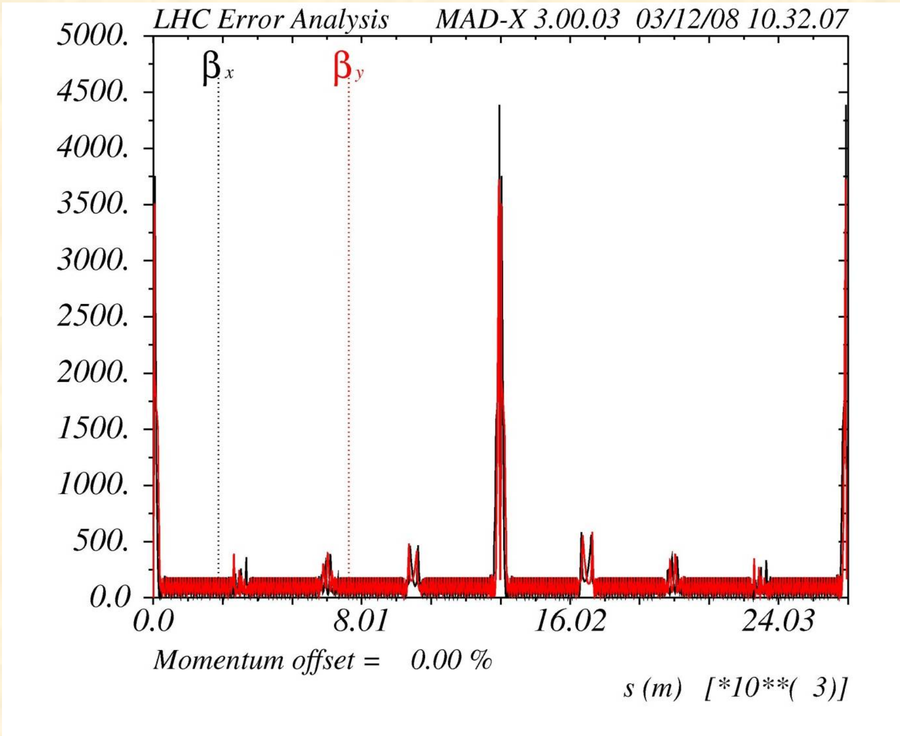
Nota bene:

1.) A proton machine … or an electron linac … needs the highest aperture at injection energy !!!
as soon as we start to accelerate the beam size shrinks as $\gamma^{-1/2}$ in both planes.

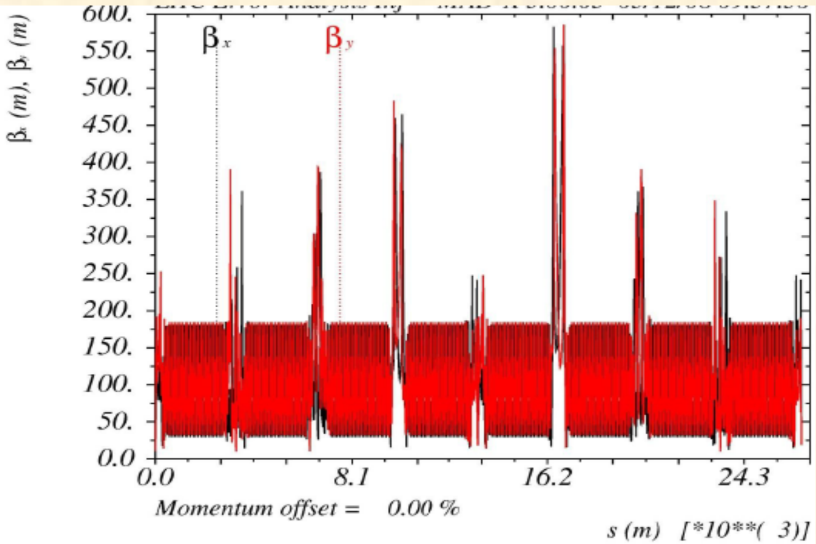
$$\sigma = \sqrt{\varepsilon\beta}$$

2.) At lowest energy the machine will have the major aperture problems,
→ here we have to minimise $\hat{\beta}$

3.) we need different beam optics adopted to the energy:
A Mini Beta concept will only be adequate at flat top.



LHC mini beta optics at 7000 GeV

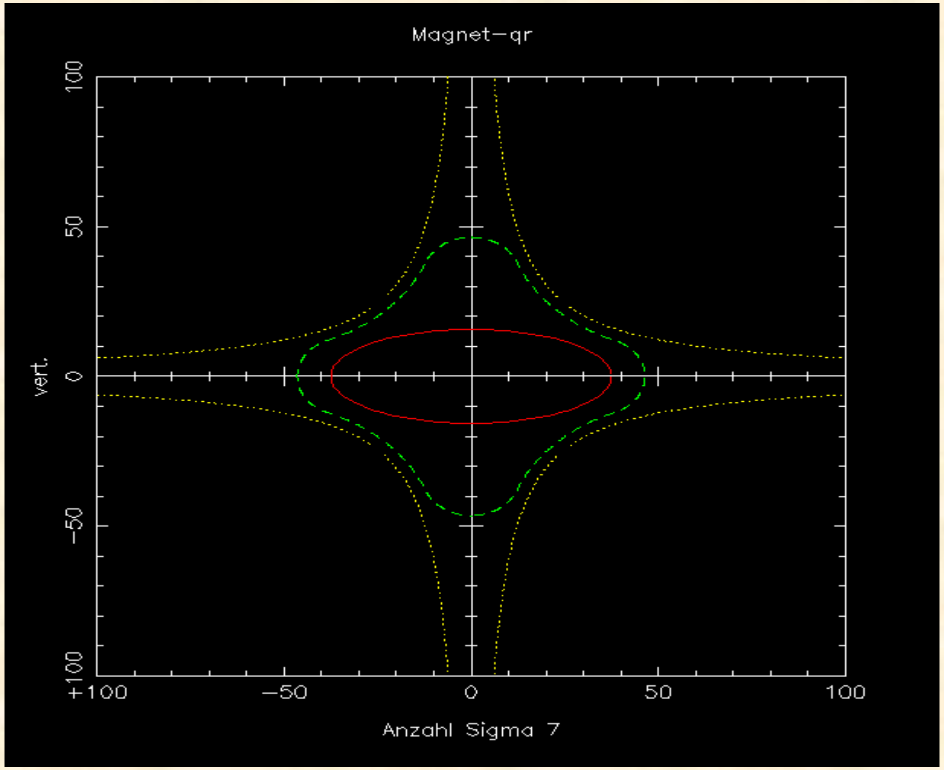


LHC injection optics at 450 GeV

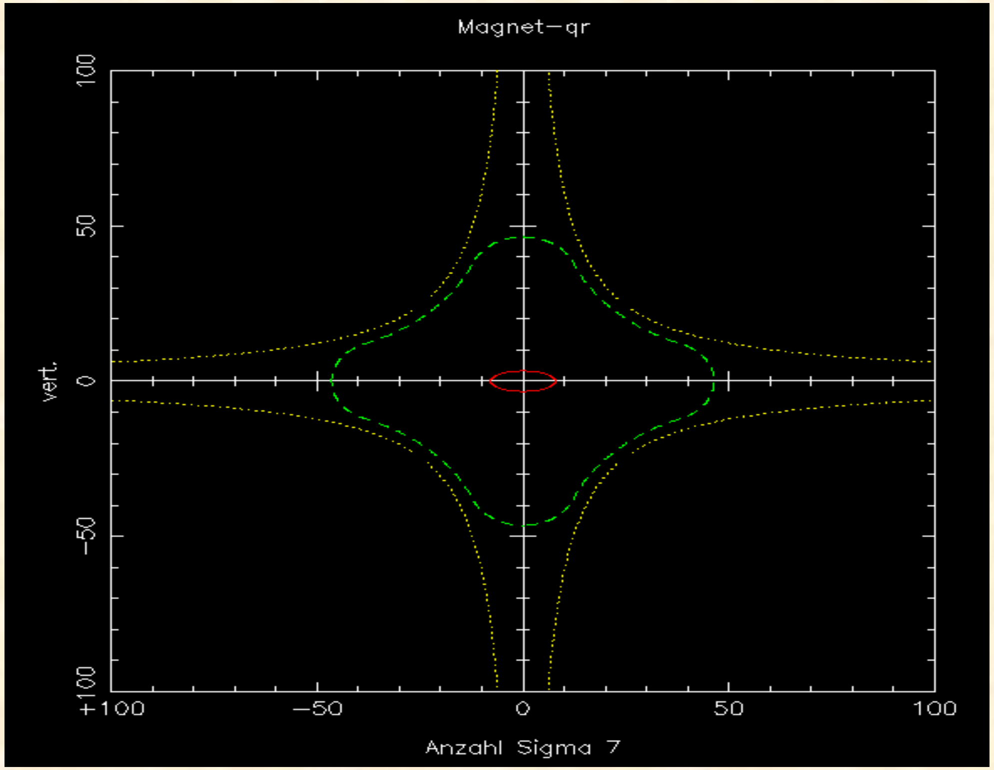
Example: HERA proton ring

injection energy: 40 GeV $\gamma = 43$
flat top energy: 920 GeV $\gamma = 980$

emittance $\varepsilon\,(40GeV) = 1.2 * 10^{-7}$
$\varepsilon\,(920GeV) = 5.1 * 10^{-9}$



7 σ beam envelope at E = 40 GeV



… and at E = 920 GeV

The „ not so ideal world “

15.) The „ $\Delta p / p \neq 0$“ Problem

ideal accelerator: *all particles will see the same accelerating voltage.*

→ $\Delta p / p = 0$

„nearly ideal“ accelerator: *Cockroft Walton or van de Graaf*

$\Delta p / p \approx 10^{-5}$

Vivitron, Straßbourg, inner structure of the acc. section

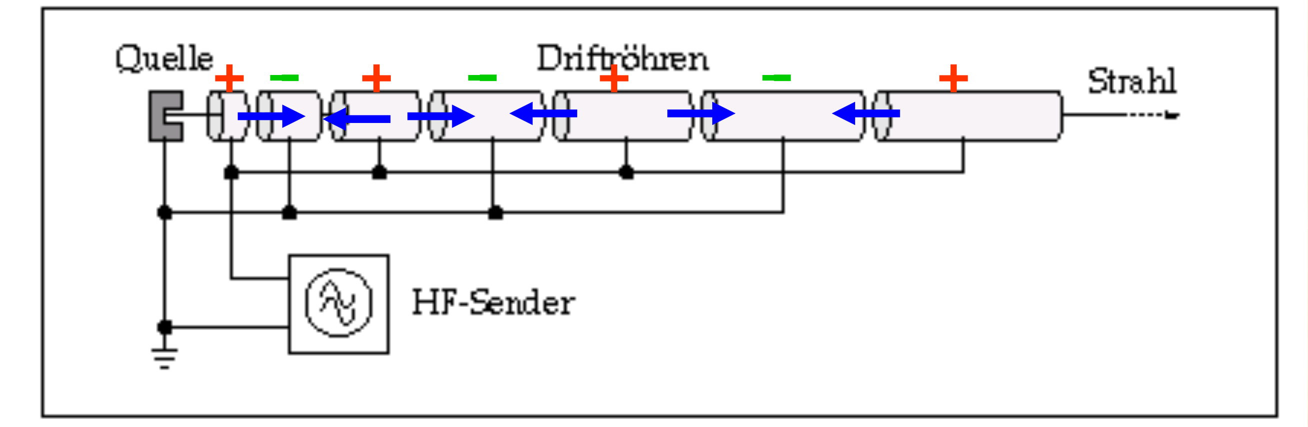
MP Tandem van de Graaf Accelerator at MPI for Nucl. Phys. Heidelberg

RF Acceleration

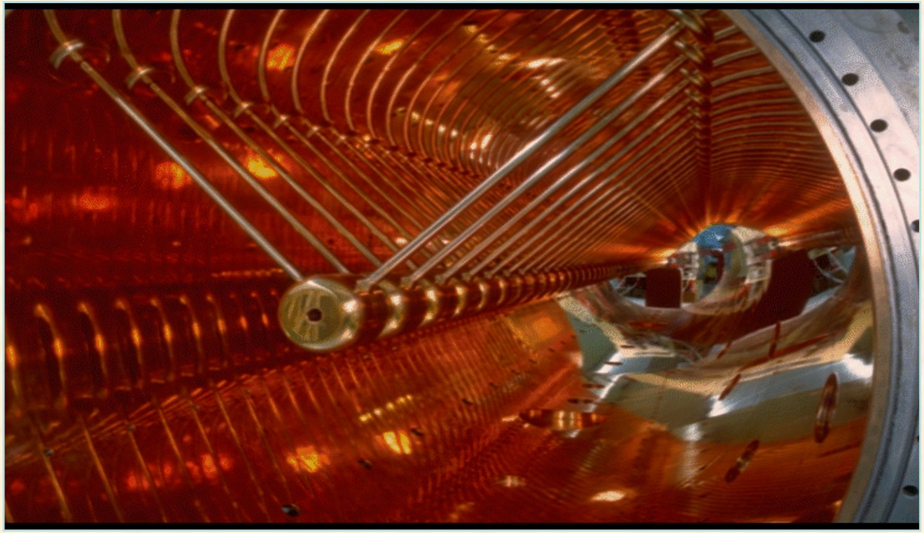
Energy Gain per „Gap“:

$$W = n * q\, U_0 \sin\omega_{RF} t$$

1928, Wideroe

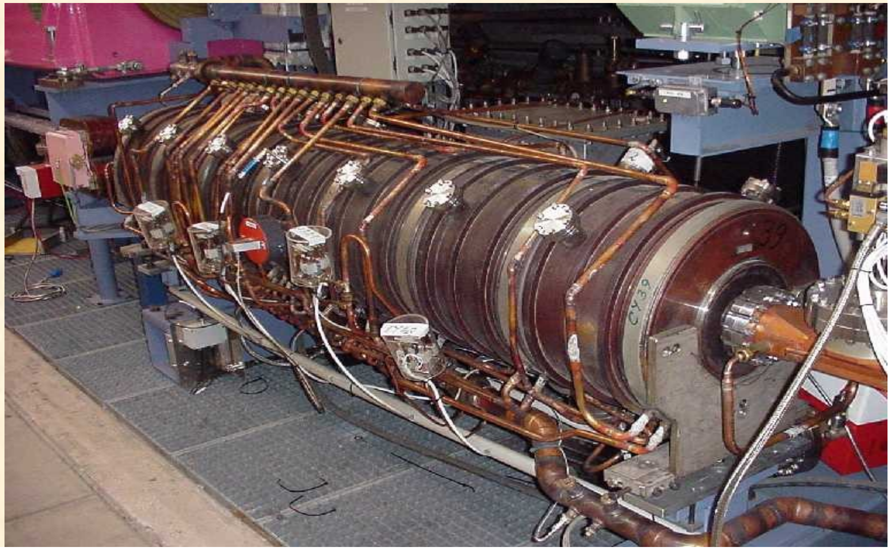


drift tube structure at a proton linac (GSI Unilac)



n *number of gaps between the drift tubes*
q *charge of the particle*
$\boldsymbol{U_0}$ *Peak voltage of the RF System*
$\boldsymbol{\Psi_S}$ *synchronous phase of the particle*

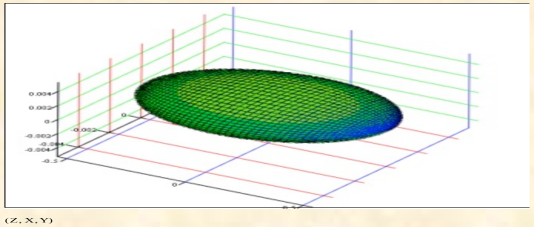
500 MHz cavities in an electron storage ring



* *RF Acceleration:* ***multiple application of the same acceleration voltage; brillant idea to gain higher energies***

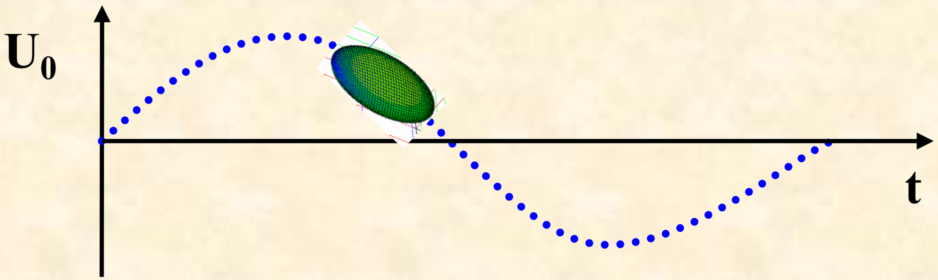
RF Acceleration-Problem: panta rhei !!!

(Heraklit: 540-480 v. Chr.)



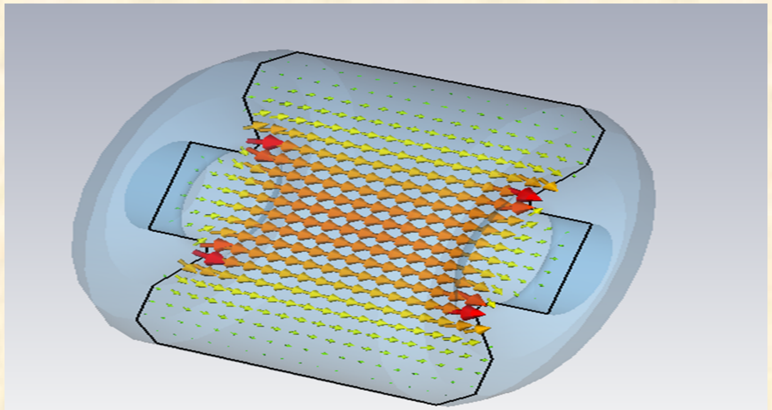
Bunch length of Electrons ≈ 1cm

just a stupid (and nearly wrong) example)



$$\lambda = 75\ cm$$

$$\nu = 400\,MHz$$
$$c = \lambda\,\nu$$
$$\lambda = 75\ cm$$



$$\sin(90^o) = 1$$
$$\sin(84^o) = 0.994$$

$$\frac{\Delta U}{U} = 6.0\ \ 10^{-3}$$

typical momentum spread of an electron bunch:

$$\frac{\Delta p}{p} \approx 1.0\ \ 10^{-3}$$

Dispersive and Chromatic Effects: $\Delta p/p \neq 0$

Are there any Problems ???

Sure there are !!!

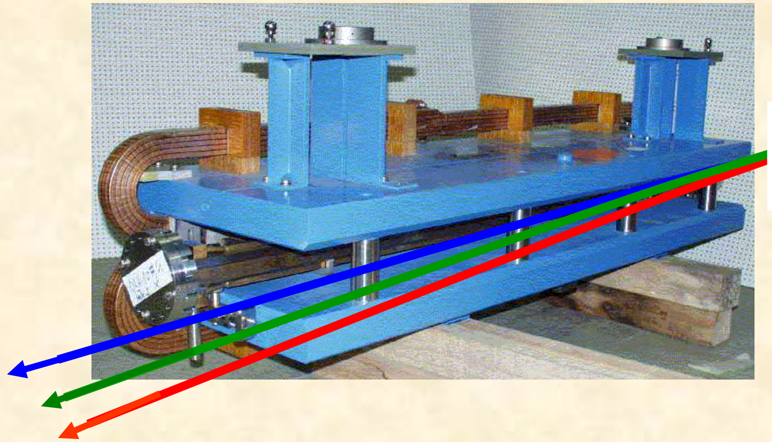
font colors due to pedagogical reasons

16.) Dispersion and Chromaticity: Magnet Errors for $\Delta p/p \neq 0$

Influence of external fields on the beam: *prop. to magn. field & prop. zu 1/p*

dipole magnet

$$\alpha = \frac{\int B\,dl}{p/e}$$



$$x_D(s) = D(s)\,\frac{\Delta p}{p}$$

focusing lens

$$k = \frac{g}{p/e}$$

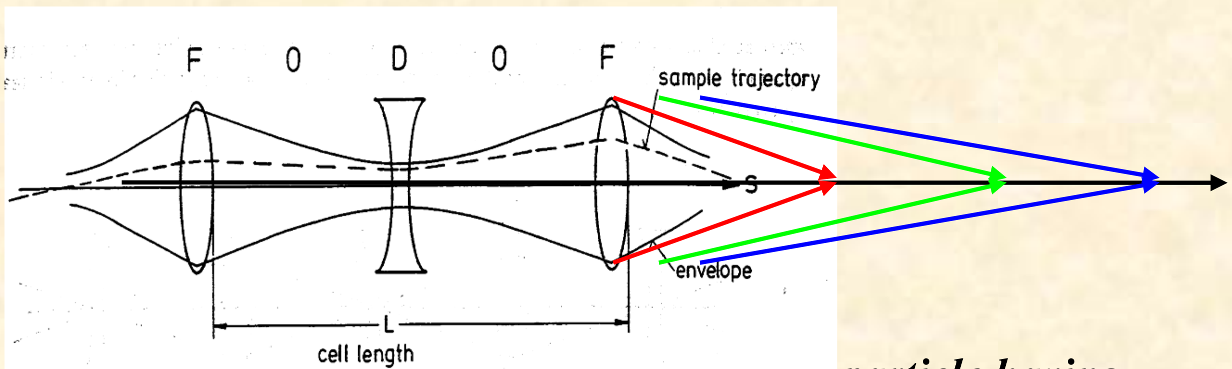


Figure 29: FODO cell

particle having ...

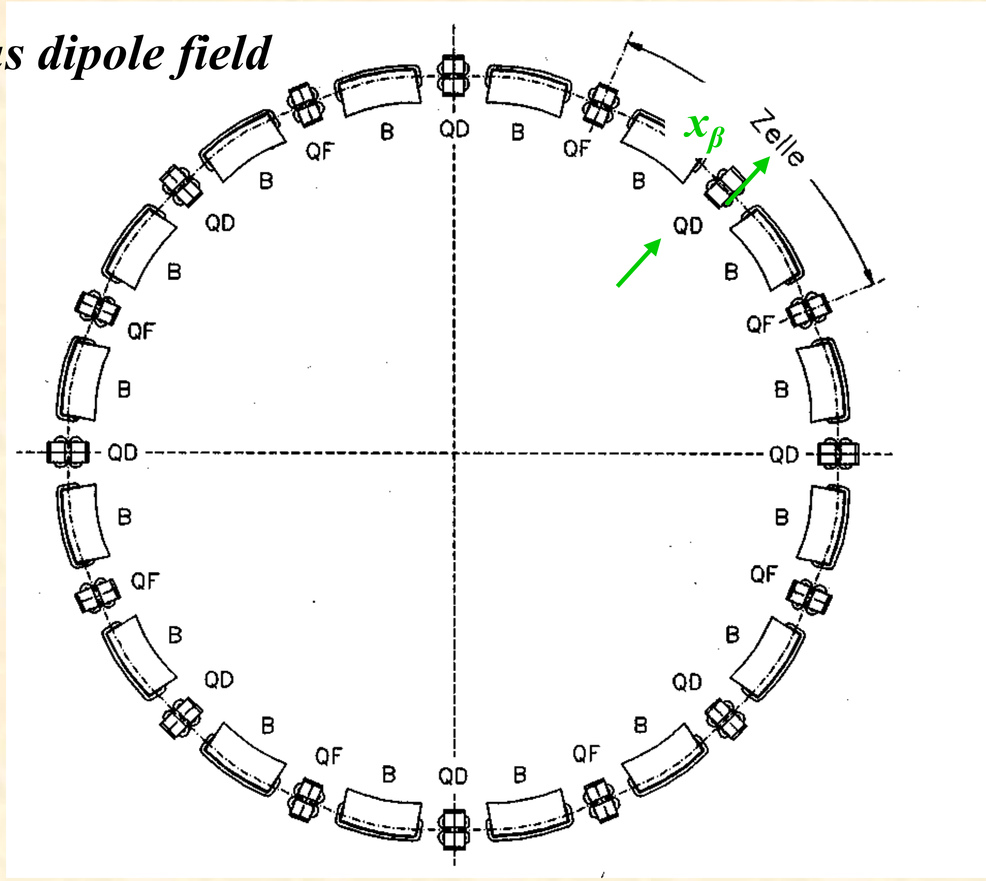
to high energy

to low energy

ideal energy

Dispersion

Example: homogeneous dipole field



…it for Δp/p > 0

$$\ldots = D(s) \cdot \frac{\Delta p}{p}$$

Matrix formalism:

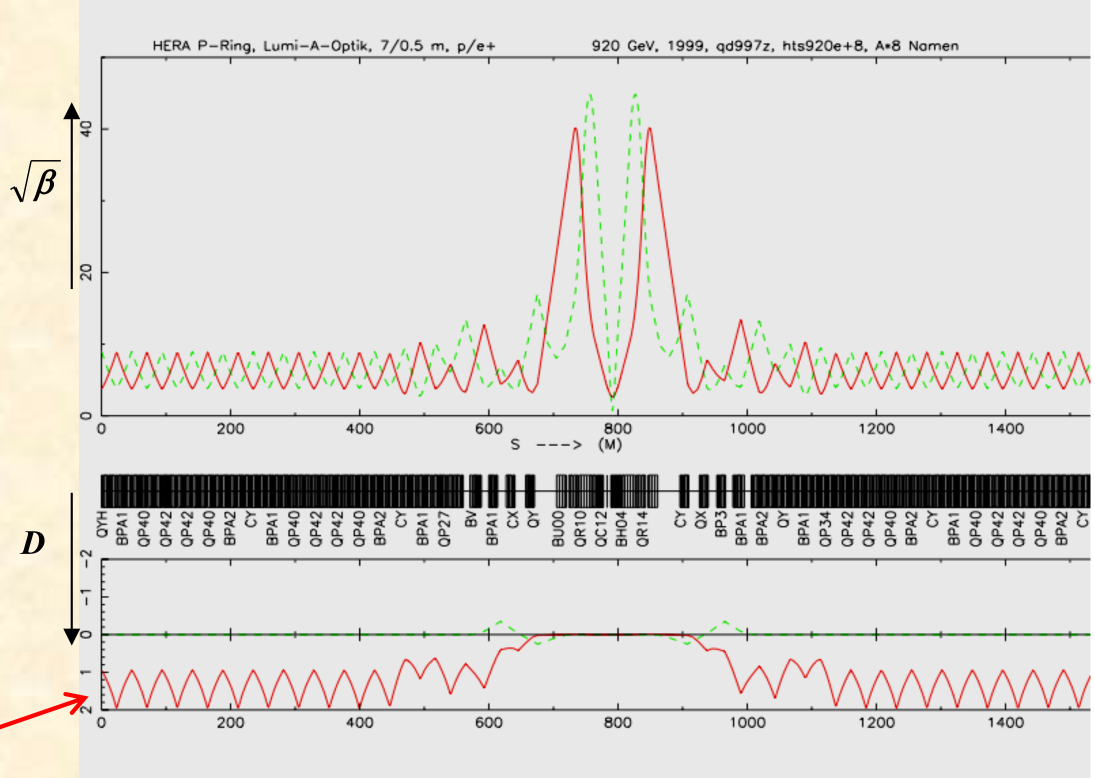
$$x(s) = x_\beta(s) + D(s) \cdot \frac{\Delta p}{p}$$

$$x(s) = C(s) \cdot x_0 + S(s) \cdot x'_0 + D(s) \cdot \frac{\Delta p}{p}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0 + \frac{\Delta p}{p} \begin{pmatrix} D \\ D' \end{pmatrix}_0$$

or expressed as 3x3 matrix

$$\begin{pmatrix} x \\ x' \\ \Delta p/p \end{pmatrix}_s = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ \Delta p/p \end{pmatrix}_0$$



Example

$$x_\beta = 1 \ldots 2\, mm$$
$$D(s) \approx 1 \ldots 2\, m$$
$$\Delta p/p \approx 1 \cdot 10^{-3}$$

Amplitude of Orbit oscillation contribution due to Dispersion ≈ beam size
→ Dispersion must vanish at the collision point

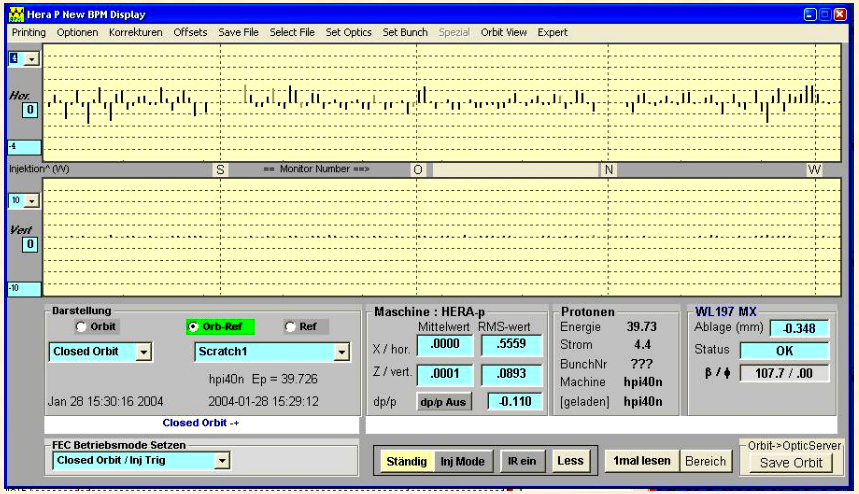
!

Calculate D, D´: ... takes a couple of sunny Sunday evenings !

$$D(s) = S(s) \int_{s_0}^{s_1} \frac{1}{\rho} C(\tilde{s}) d\tilde{s} - C(s) \int_{s_0}^{s_1} \frac{1}{\rho} S(\tilde{s}) d\tilde{s}$$

(proof see CAS proc.)

Dispersion is visible



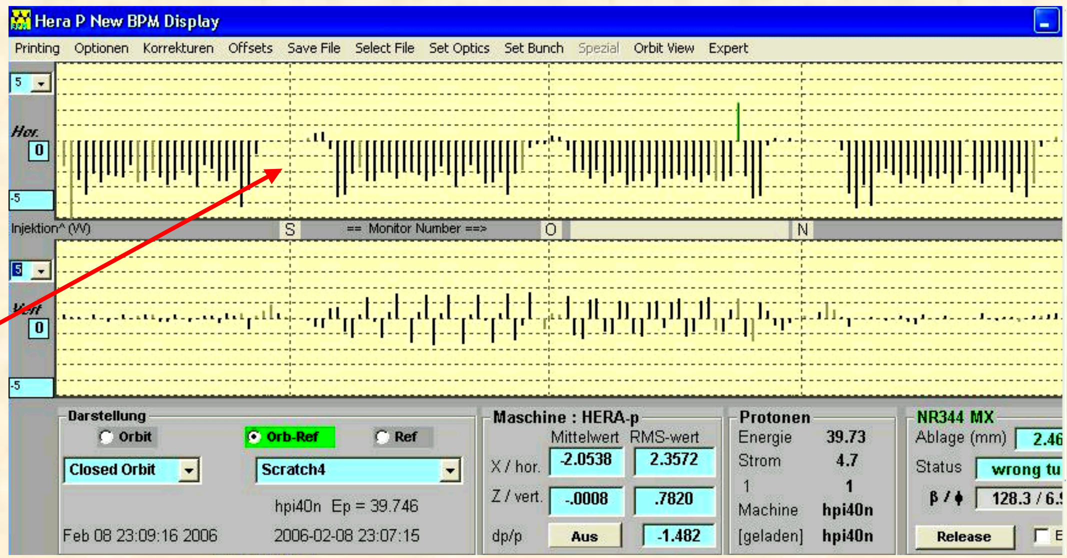
HERA Standard Orbit

HERA Dispersion Orbit

dedicated energy change of the stored beam

→ closed orbit is moved to a dispersions trajectory

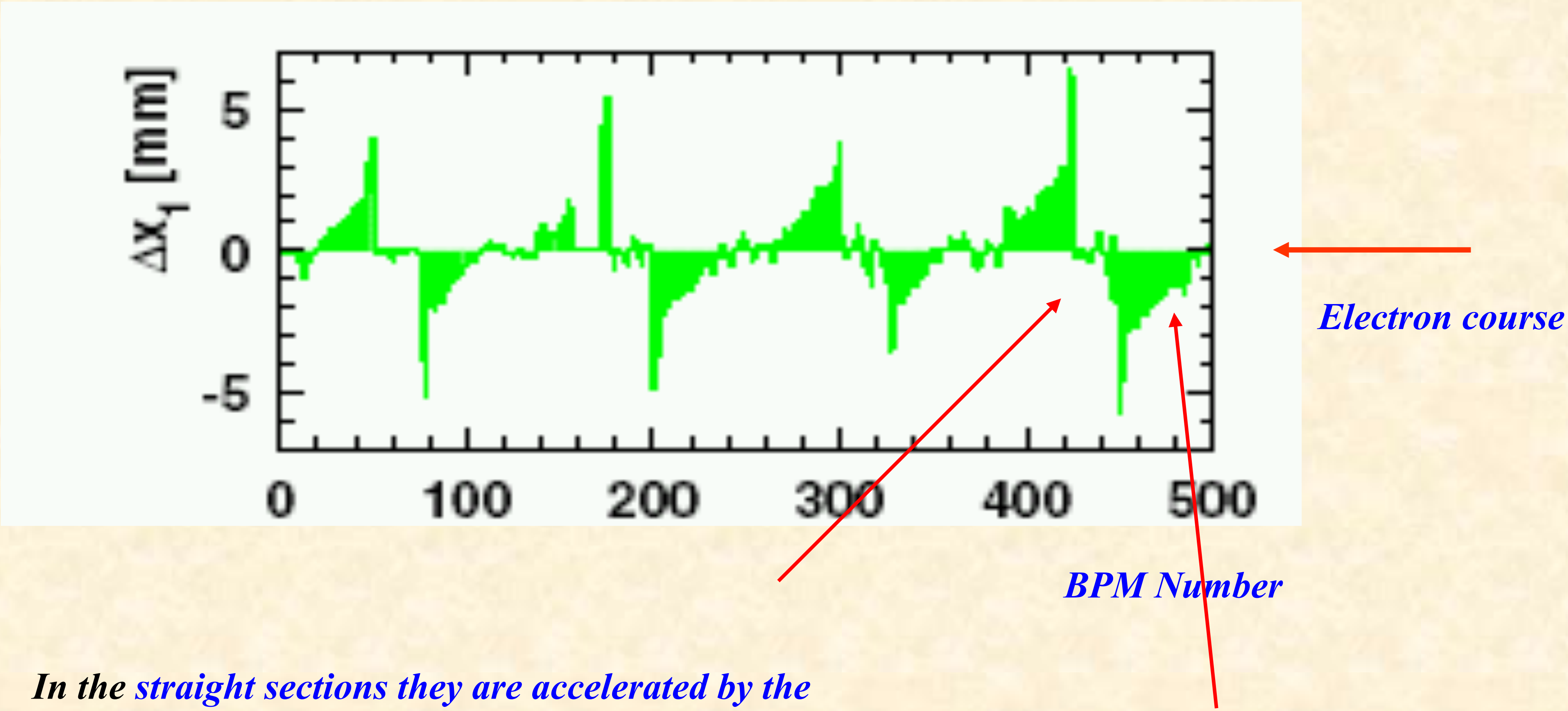
$$x_D = D(s) * \frac{\Delta p}{p}$$



Attention: at the Interaction Points we require D=D´= 0

Periodic Dispersion:

„Sawtooth Effect“ at LEP (CERN)



In the straight sections they are accelerated by the rf cavities so much that they „overshoot“ and reach nearly the outer side of the vacuum chamber.

In the arc the electron beam loses so much energy in each octant that the particle are running more and more on a dispersion trajectory.

17.) Chromaticity:
A Quadrupole Error for Δp/p ≠ 0

Influence of external fields on the beam: ***prop. to magn. field & prop. zu 1/p***

Remember the normalisation of the external fields:

focusing lens

$$k = \frac{g}{p/e}$$

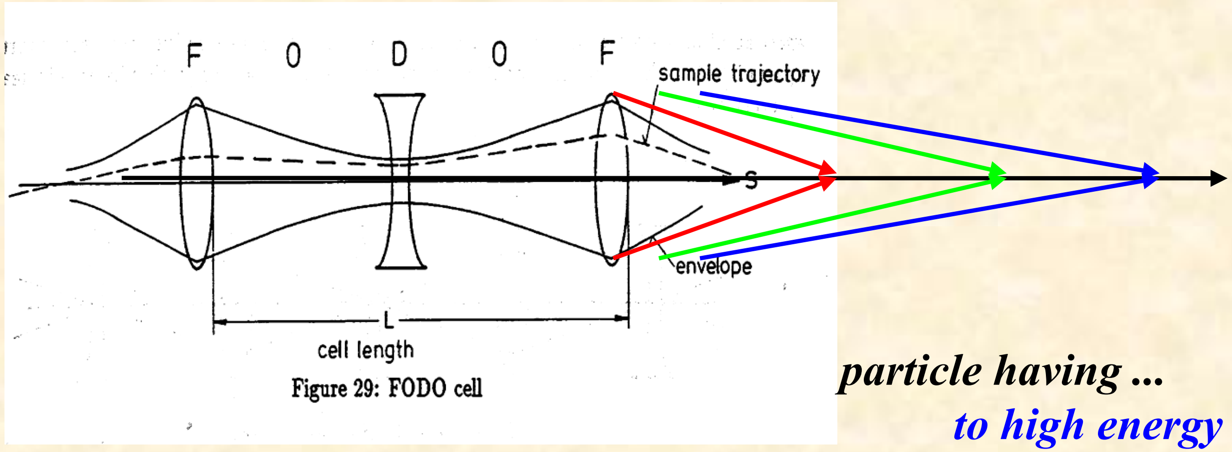


Figure 29: FODO cell

particle having ...
to high energy
to low energy
ideal energy

a particle that has a higher momentum feels a weaker quadrupole gradient and has a lower tune.

$$\Delta Q = -\frac{1}{4\pi}\frac{\Delta p}{p_0} k_0 \beta(s) ds$$

definition of chromaticity:

$$\Delta Q = Q' \frac{\Delta p}{p}$$

… what is wrong about Chromaticity:

Problem: chromaticity is generated by the lattice itself !!

***Q'* is a *number* indicating the *size of the tune spot* in the working diagram,**
***Q'* is always created if the beam is focussed**
→ it is determined by the focusing strength *k* of all quadrupoles

$$\Delta Q = -\frac{1}{4\pi}\frac{\Delta p}{p_0} k_0 \beta(s) ds$$

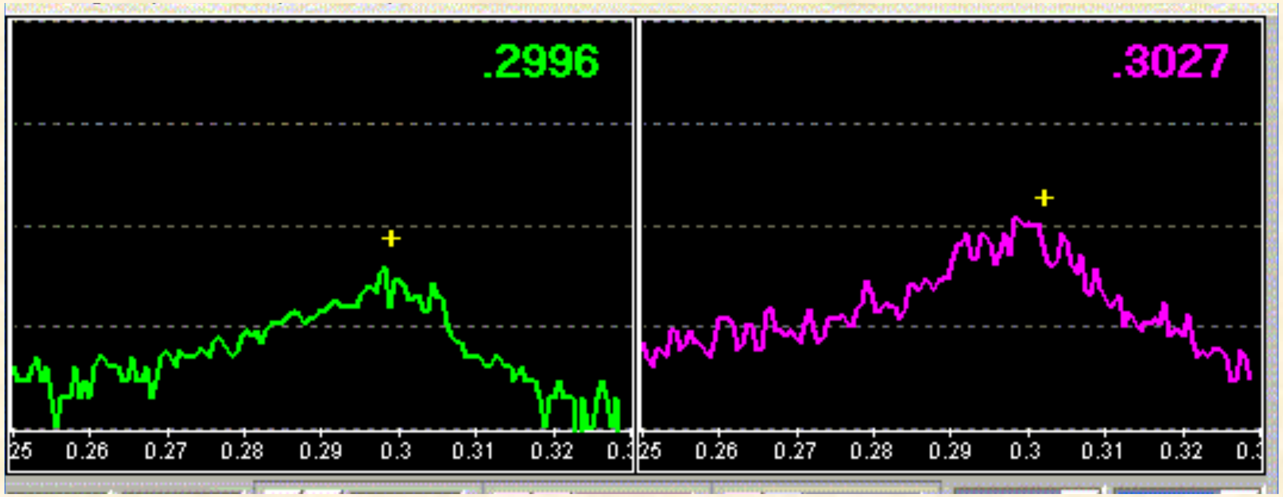
***k* = quadrupole strength**
***β* = betafunction indicates the beam size … and even more the sensitivity of the beam to external fields**

Example: LHC

$Q' = 250$
$\Delta p/p = +/- 0.2 \cdot 10^{-3}$
$\Delta Q = 0.256 \ldots 0.36$

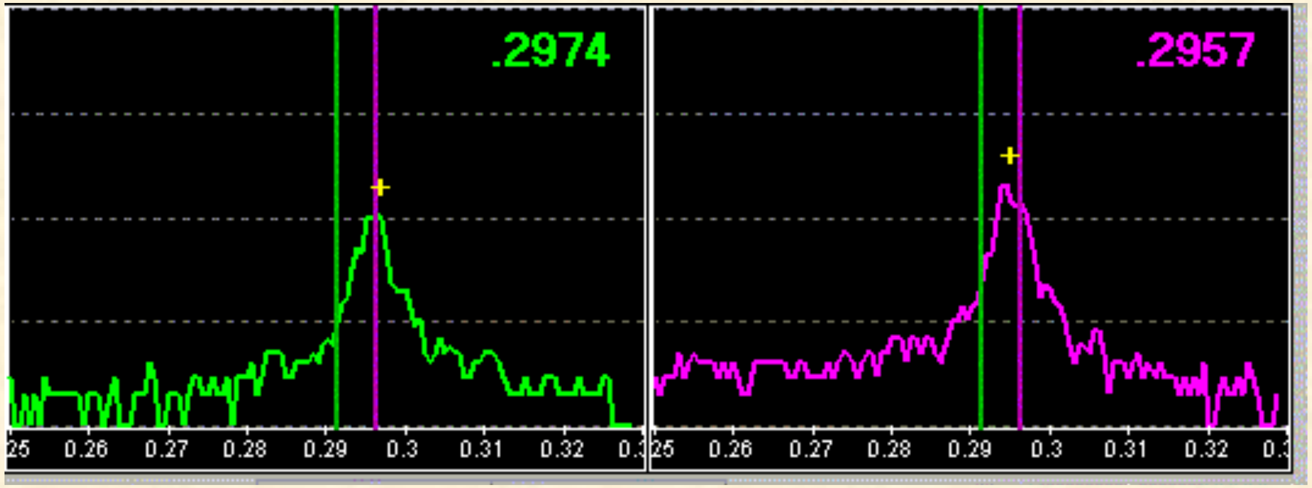
→ Some particles get very close to resonances and are lost

in other words: the tune is not a point it is a pancake



Tune signal for a nearly uncompensated cromaticity (Q' ≈ 20)

Ideal situation: cromaticity well corrected, (Q' ≈ 1)

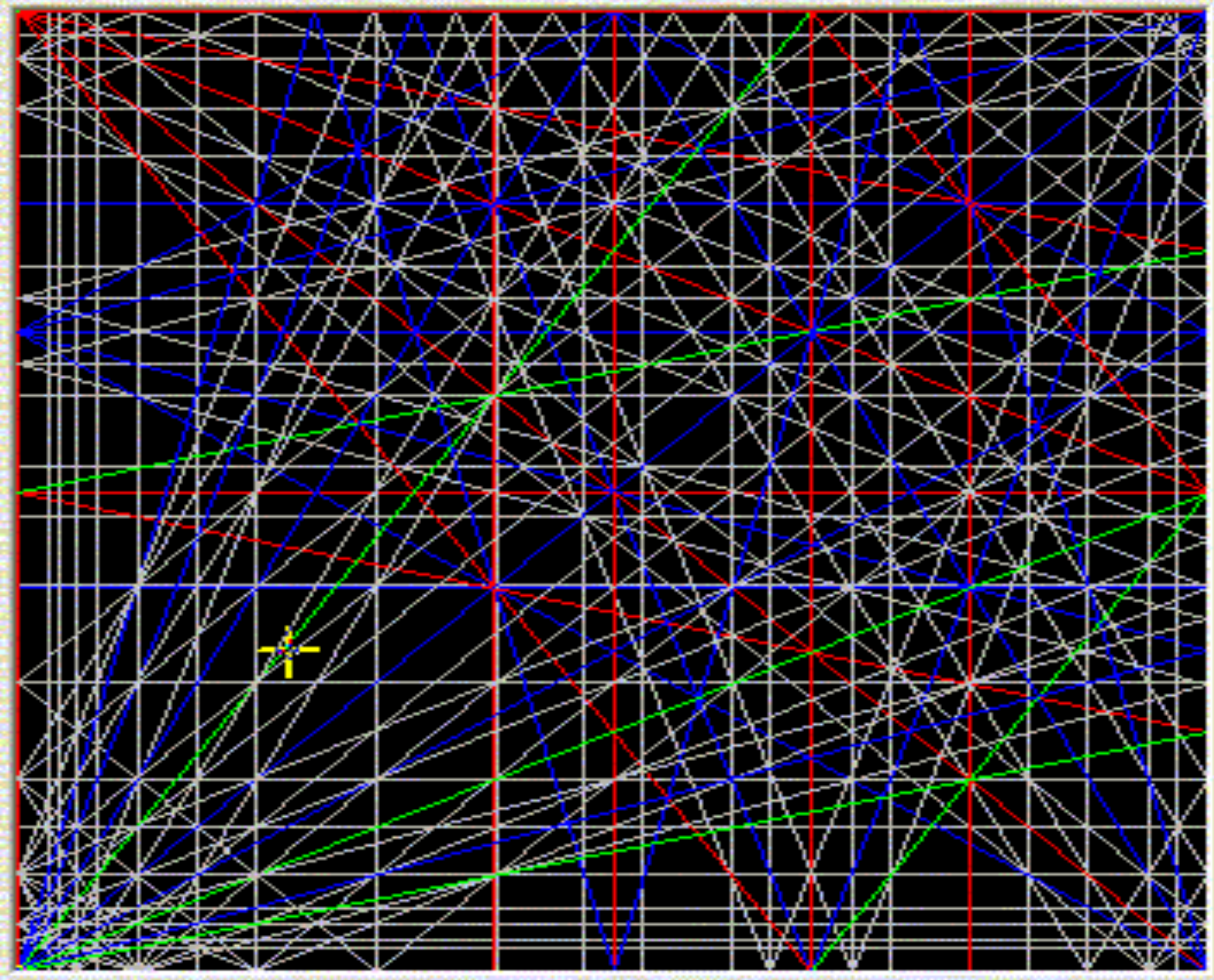


Tune and Resonances

$$m*Q_x+n*Q_y+l*Q_s = integer$$

Tune diagram up to 3rd order

… and up to 7th order



Homework for the operateurs: find a nice place for the tune where against all probability the beam will survive

Chromaticity Correction:

We need a magnetic field that focuses stronger those individual particles that have larger momentum and focuses weaker those with lower momentum.

... but that does not exist.

The way the trick goes:

1.) sort the particle trajectories according to their energy
we use the dispersion to do the job

2.) introduce magnetic fields that increase stronger than linear with the distance Δx to the centre

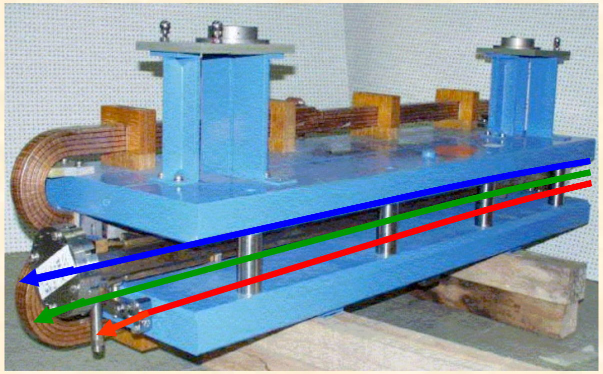
3.) calculate these fields (sextupoles) in a way that the lack of focusing strength is exactly compensated.

Correction of Q':

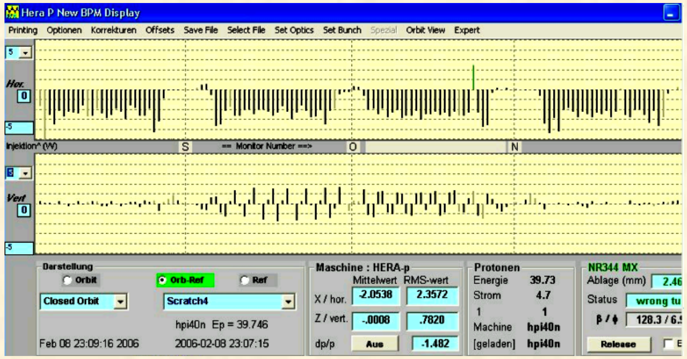
Need: additional quadrupole strength for each momentum deviation Δp/p

1.) sort the particles acording to their momentum

$$x_D(s) = D(s)\frac{\Delta p}{p}$$



… using the dispersion function



2.) apply a magnetic field that rises quadratically with x (sextupole field)

$$\left.\begin{aligned} B_x &= \tilde{g}xy \\ B_y &= \frac{1}{2}\tilde{g}(x^2 - y^2) \end{aligned}\right\} \qquad \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = \tilde{g}x$$

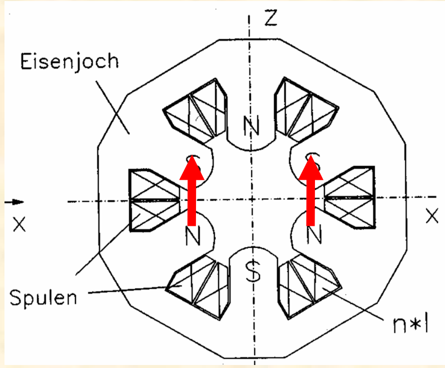
linear amplitude dependent „gradient“:

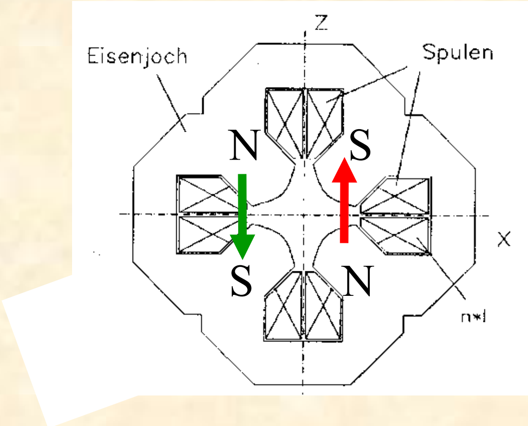
Correction of Q':

k_1 normalised quadrupole strength

k_2 normalised sextupole strength

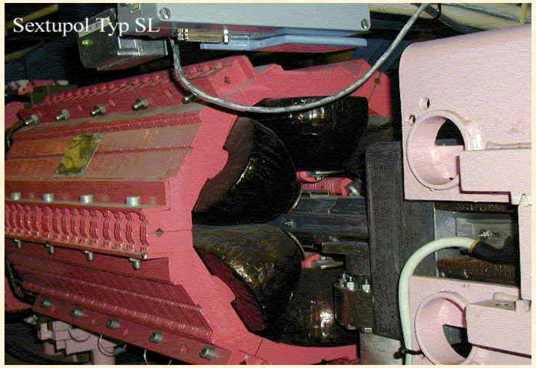
Sextupole Magnets:





$$k_1(sext) = \frac{\tilde{g}x}{p/e} = k_2 * x$$

$$= k_2 * D\frac{\Delta p}{p}$$

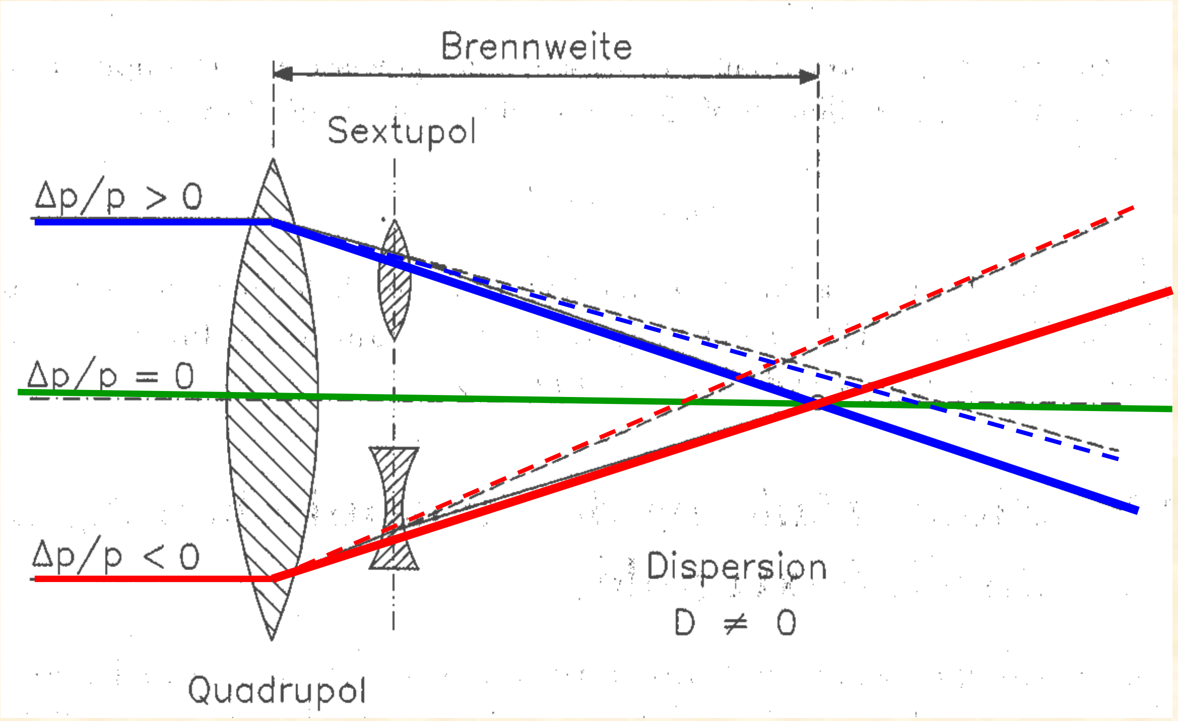


Combined effect of „natural chromaticity“ and Sextupole Magnets:

$$Q' = -\frac{1}{4\pi}\left\{\int k_1(s)\beta(s)ds + \int k_2 * D(s)\beta(s)ds\right\}$$

You only should not forget to correct Q ‘ in both planes ...
and take into account the contribution from quadrupoles of both polarities.

Chromatizitätskorrektur:

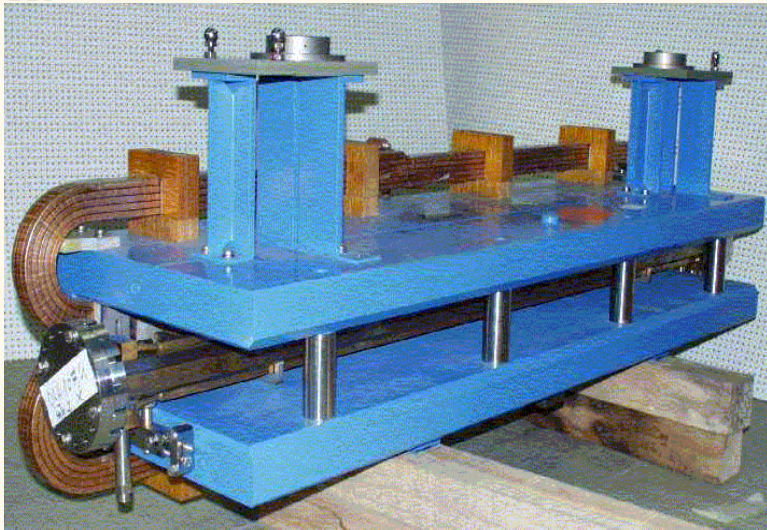


Einstellung am Speicherring:

Sextupolströme so variieren, dass $\xi \approx +1 \ldots +2$

A word of caution: keep non-linear terms in your storage ring low.

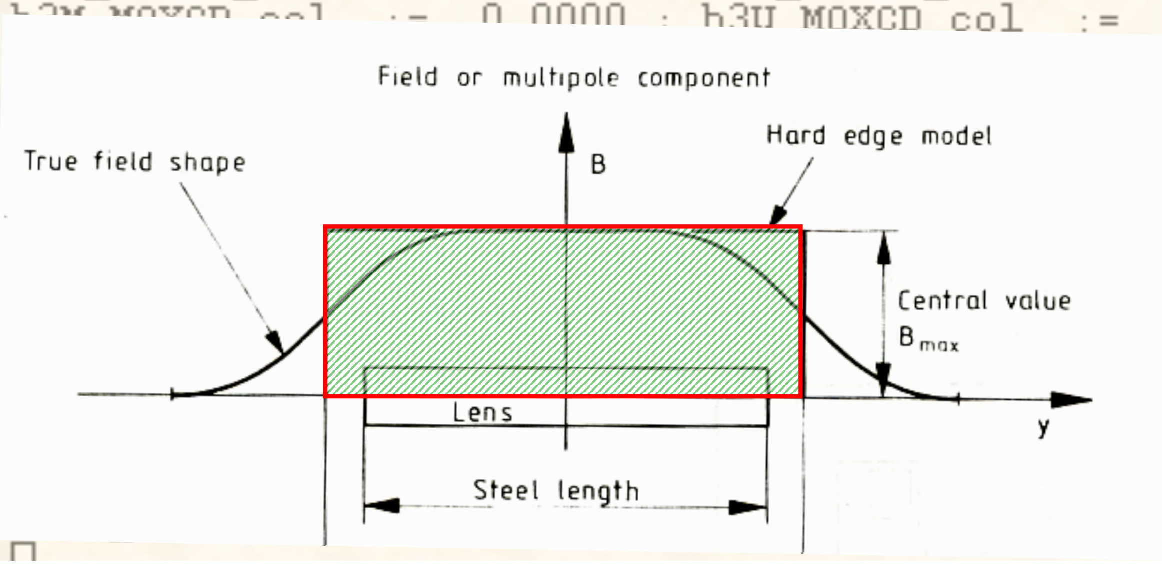
$$B_y + iB_x = B_{ref} * \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{r_0} \right)^{n-1}$$

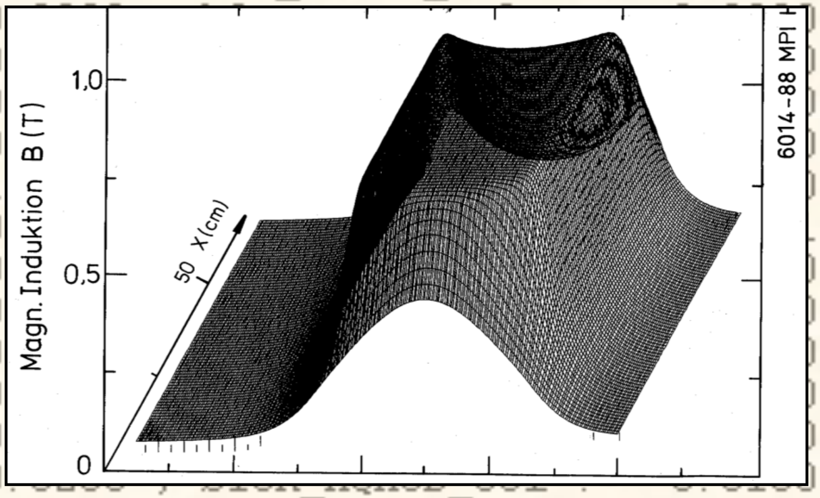


"effective magnetic length"

$$B * l_{eff} = \int_0^{l_{mag}} B ds$$

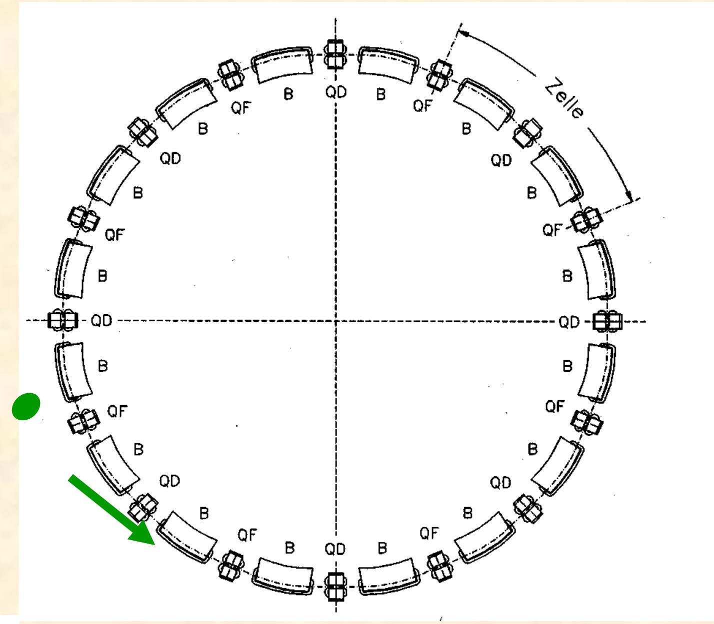
```
bn at injection
b1M_MQXCD_inj  :=  0.0000 ; b1U_MQXCD_inj  :=  ... 0000
b2M_MQXCD_inj  :=  0.0000 ; b2U_MQXCD_inj  :=  ... 0000
                                            ... 8900
                                            ... 6400
                                            ... 4600
                                            ... 2800
                                            ... 2100
                                            ... 1600
b9M_MQXCD_inj  :=  0.0000 ; b9U_MQXCD_inj  :=  ... 0800
b10M_MQXCD_inj :=  0.5000 ; b10U_MQXCD_inj :=  ... 0600
b11M_MQXCD_inj :=  0.0000 ; b11U_MQXCD_inj :=  ... 0300
b12... 12U_MQXCD_inj :=  ... 0200
b13... 13U_MQXCD_inj :=  ... 0100
b14M_MQXCD_inj := -0.2700 ; b14U_MQXCD_inj :=  0.0300 ; b14R_MQXCD_inj :=  0.0100
b15M_MQXCD ...            ; b15U_MQXCD_inj :=  0.0000 ; b15R_MQXCD_inj :=  0.0000

bn in collision
b1M_MQXCD_c...            ; b1U_MQXCD_col  :=  0.0000 ; b1R_MQXCD_col  :=  0.0000
b2M_MQXCD_col  :=  0.0000 ; b2U_MQXCD_col  :=  0...
                                            0.0400 ; b14R_MQXCD_col :=  0.0100
                                            0.0000 ; b15R_MQXCD_col :=  0.0000
```





Clearly there is another problem ...

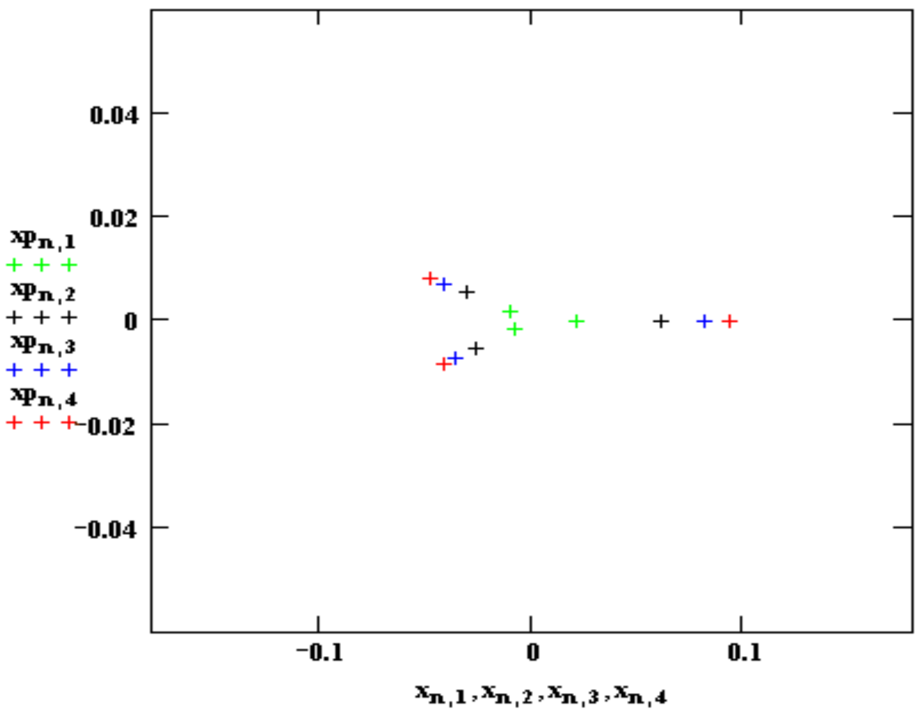
... if it were easy everybody could do it



Again: the phase space ellipse

for each turn write down – at a given position „s“ in the ring – the single partilce amplitude x and the angle x´... and plot it.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

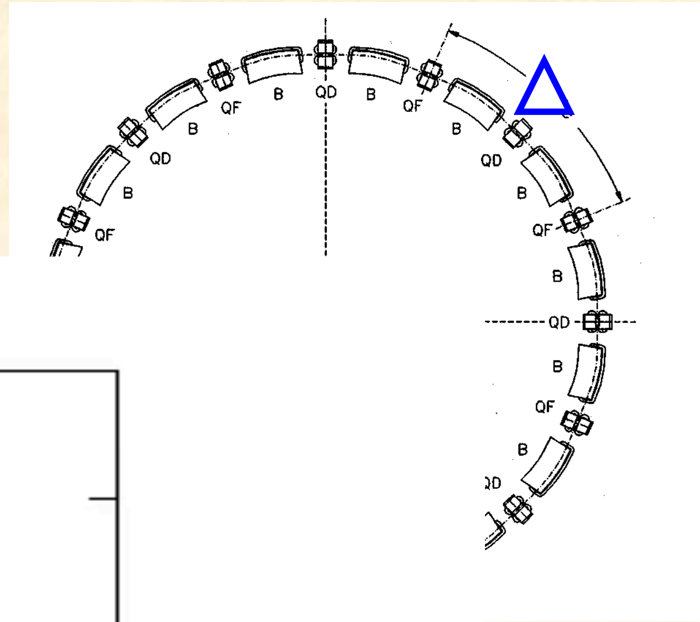


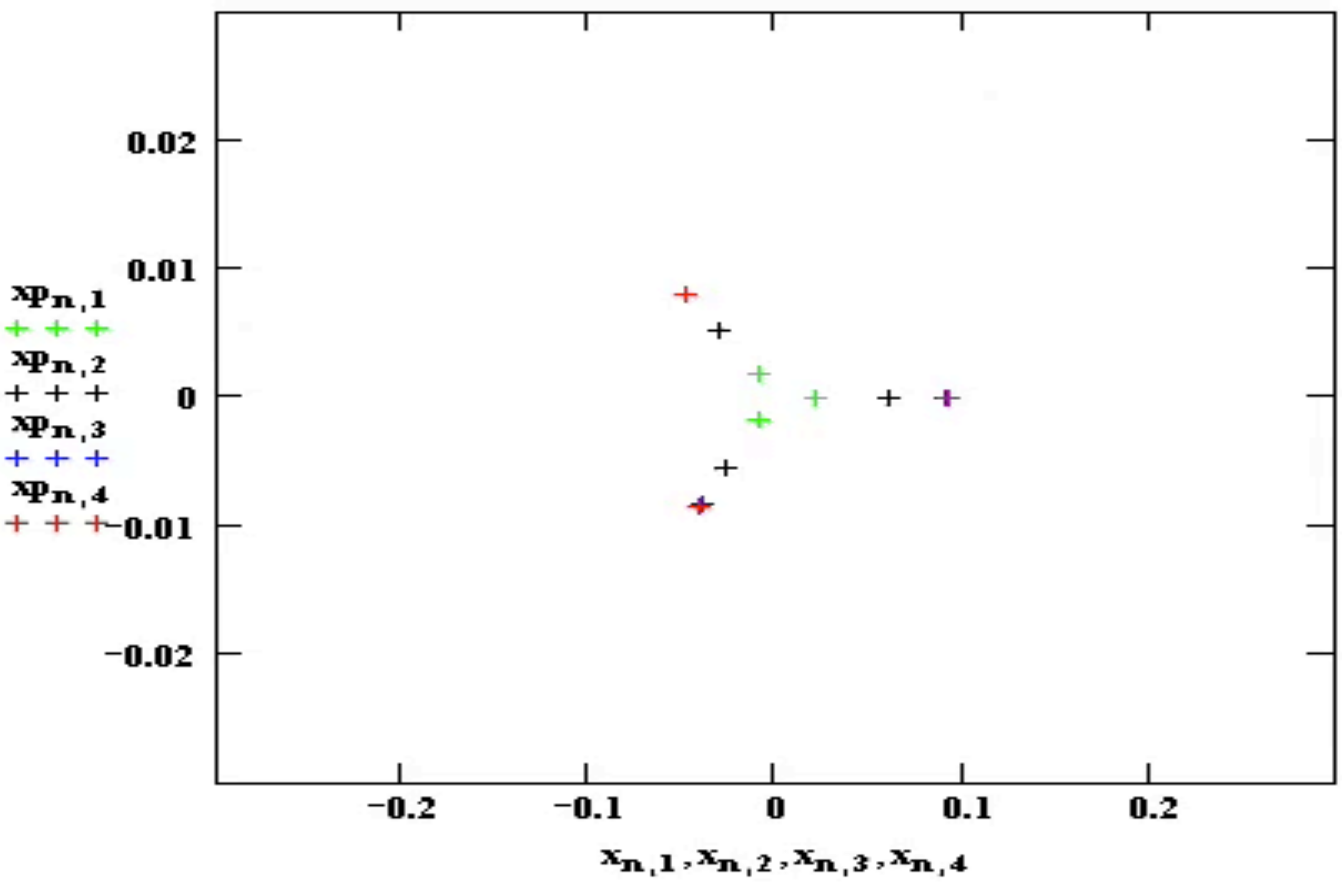
A beam of 4 particles

– each having a slightly different emittance:

Installation of a weak (!!!) sextupole magnet

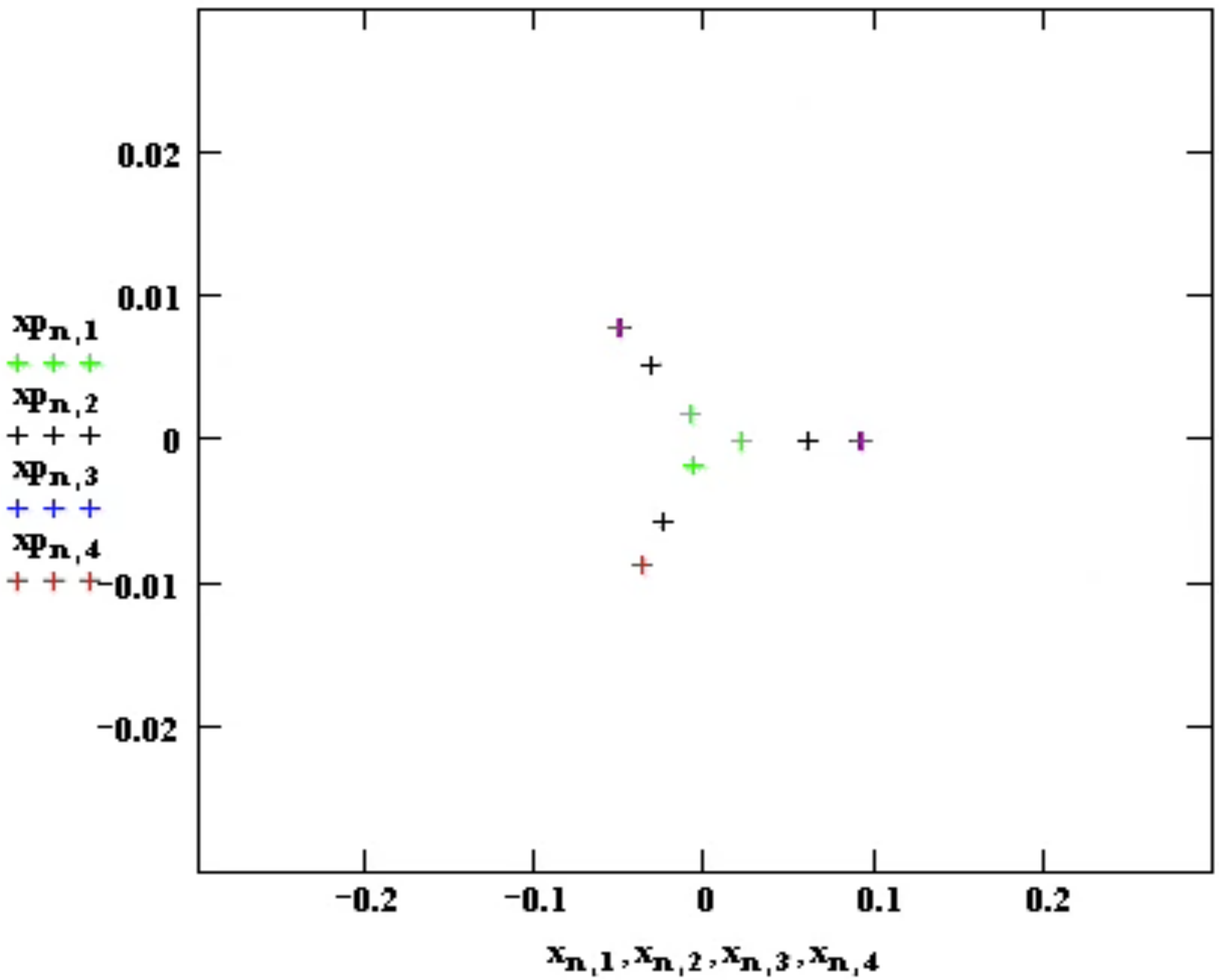
The good news: sextupole fields in accelerators cannot be treated analytically anymore.
→ no equatiuons; instead: Computer simulation

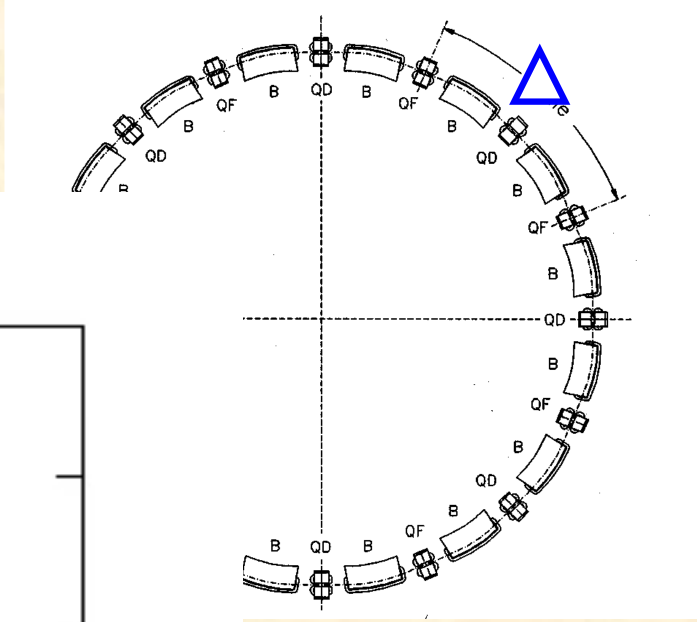




Effect of a strong (!!!) Sextupole …

→ *Catastrophy !*





„dynamic aperture"

Luminosity...
...describes the performance of a collider to hit the „target" (i.e. the other particles) and so to produce „hits".

The Mini-Beta scheme ...
... focusses striongly the beams to achieve smallest possible beam sizes at the IP. The obtained small beta function at the IP is called β.*
Don't forget the cat.

A proton beam shrinks during acceleration, we call it unfortunately „adiabatic shrinking".
Nota bene: An electron beam in a ring is growing with energy !!

Dispersion ...
... is the particle orbit for a given momentum difference.

Chromaticity ...
... is a focusing problem. Different momenta lead to different tunes → attention ... resonances !!

Sextupoles ...
have non-linear fields and are used to compensate chromaticity

Strong non-linear fields can lead to particle losses (dynamic aperture)

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