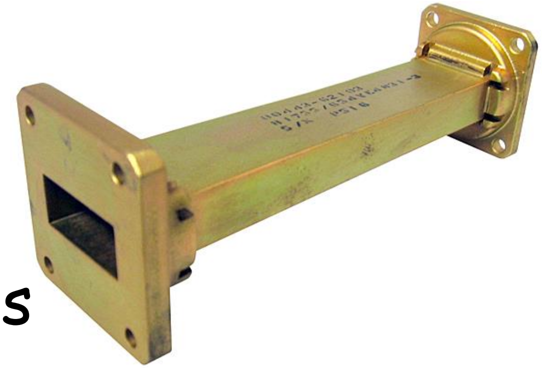
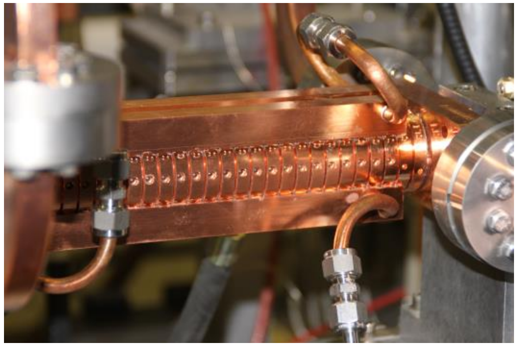
RF Systems

Frank Tecker, CERN, BE-OP



- Waveguides and components
- Cavities
- RF sources
- RF systems



Basics of Accelerator Physics and Technology
ESI, Archamps, 7-11 October 2019

RF Systems

Frank Tecker
CERN, BE-OP

Many thanks to Erk Jensen
from whom I inherited the course
for using much of his material

- Waves in waveguides and modes in cavities
- Types of cavities
  - Standing wave and travelling wave structures
- Cavity parameters:
  - Shunt impedance, transit time factor, quality factor, filling time
- Higher Order Modes and Wakefields
- Power and coupling to cavities
- RF systems and feedback loops

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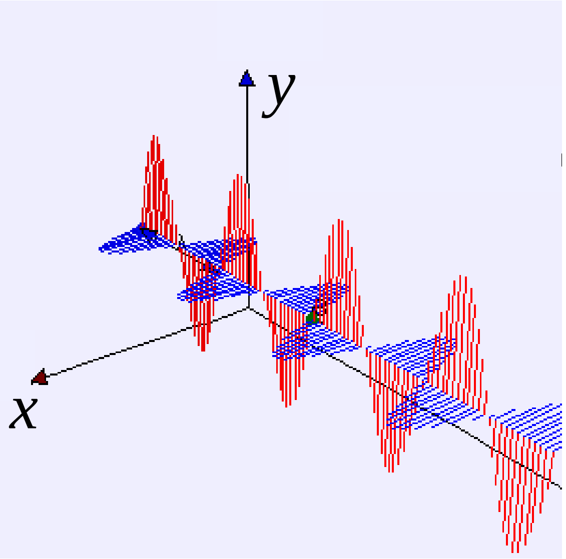
Electromagnetic Homogeneous Plane Wave



Courtesy: http://weelookang.blogspot.ch/2011/10/ejs-open-source-propagation-of.html

In free space:
Electric and magnetic fields are perpendicular to each other and to the direction of the wave

$$\vec{E} \propto \vec{u}_y \cos(\omega t - \vec{k} \cdot \vec{r})$$
$$\vec{B} \propto \vec{u}_x \cos(\omega t - \vec{k} \cdot \vec{r})$$

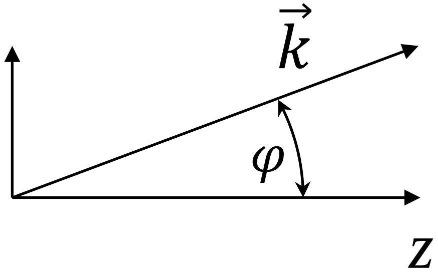
$$\boxed{\omega = 2\pi f}$$

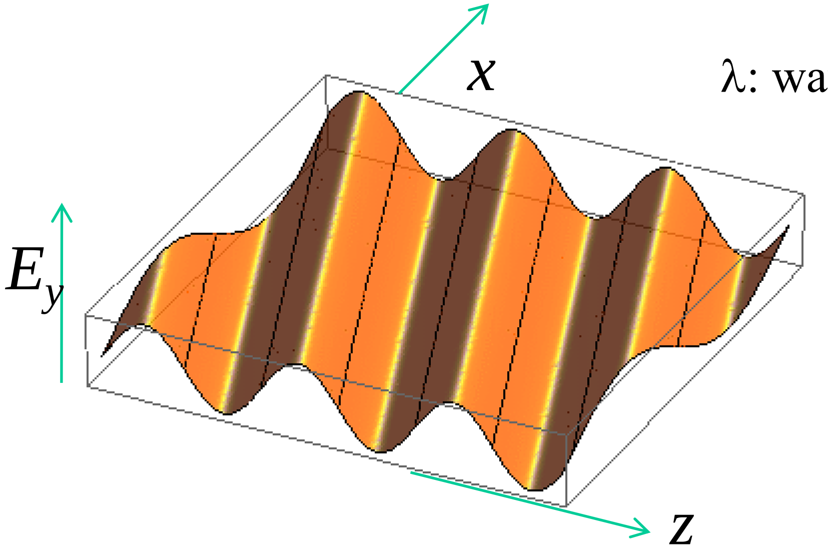
t: time

Wave number k:

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = |\vec{k}|$$

λ: wavelength, c: light speed





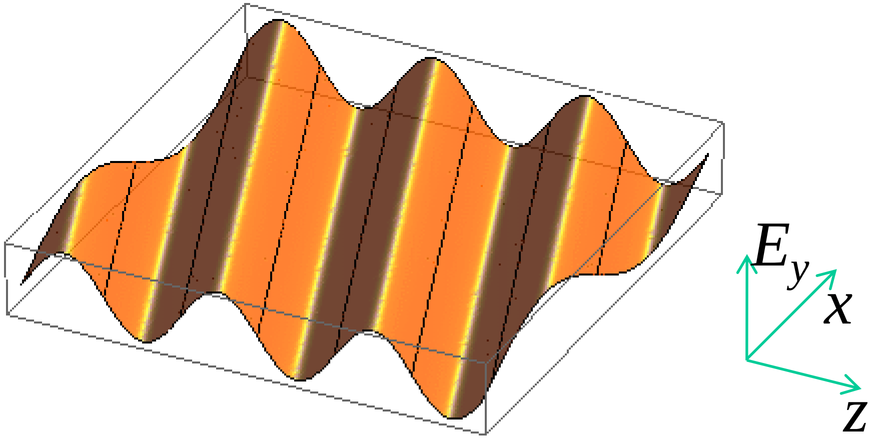
Wave vector $\vec{k}$:

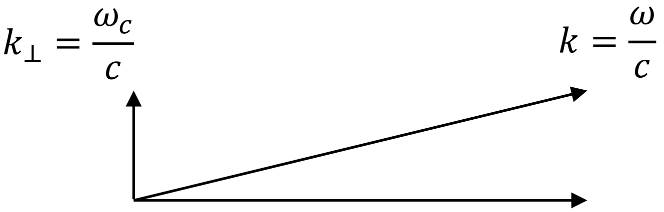
- orthogonal to phase front
- the direction of $\vec{k}$ is (usually) the direction of propagation
- the length of $\vec{k}$ is the phase shift per unit length

Wave length, phase velocity

The components of $\vec{k}$ are related to

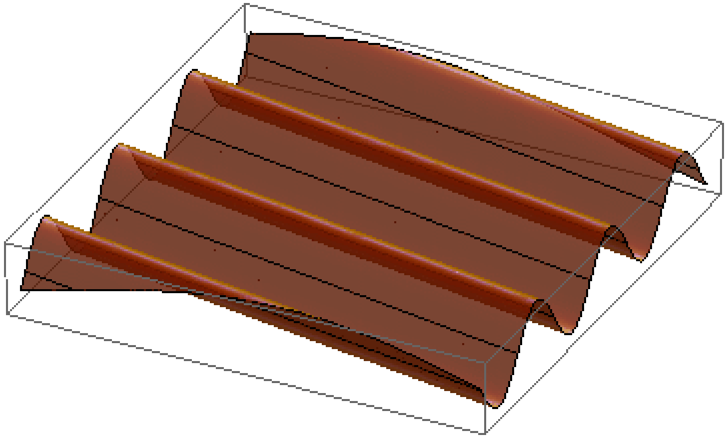
- the wavelength in the direction of that component as $\lambda_z = \frac{2\pi}{k_z}$ etc.
- to the phase velocity as $v_{\varphi,z} = \frac{\omega}{k_z} = f\lambda_z$.

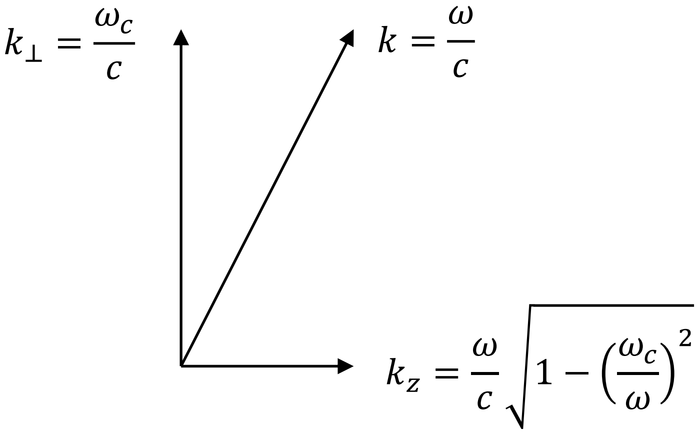




$k_\perp = \frac{\omega_c}{c}$

$k = \frac{\omega}{c}$



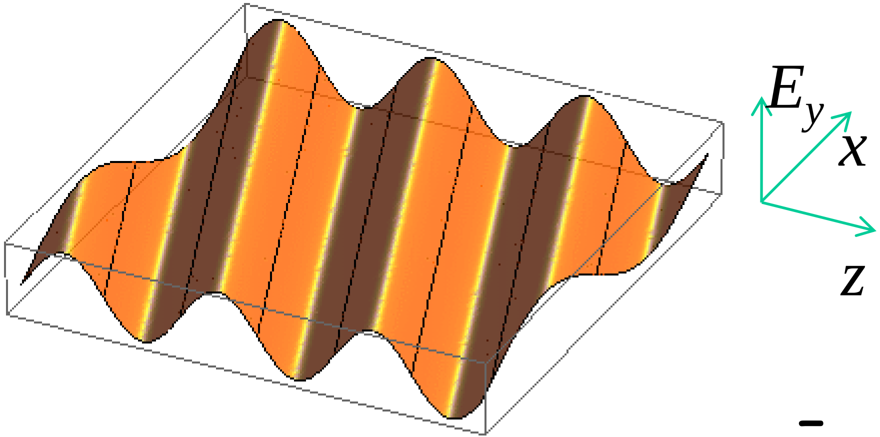


$k_\perp = \frac{\omega_c}{c}$

$k = \frac{\omega}{c}$

$k_z = \frac{\omega}{c}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$

Superposition of 2 homogeneous plane waves



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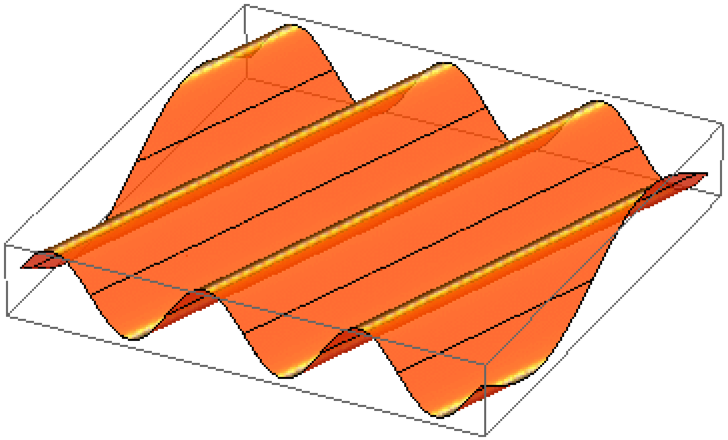

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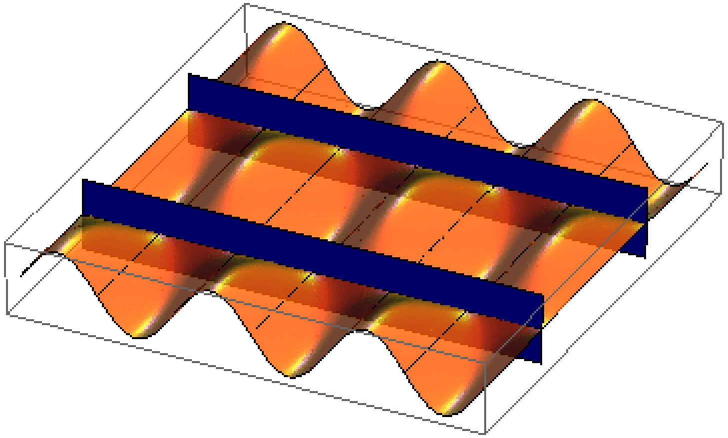


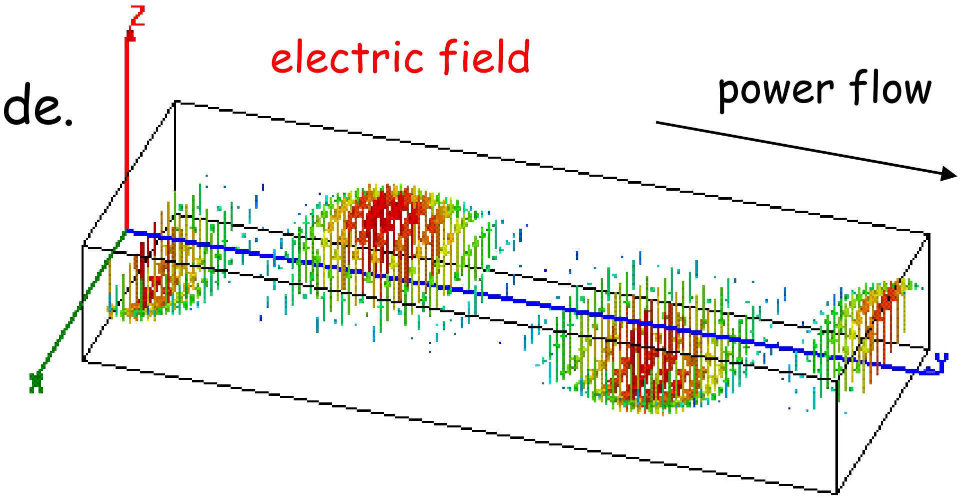
Metallic walls may be inserted where $E_y=0$ without perturbing the fields. Note the standing wave in x-direction!

This way one gets a hollow rectangular waveguide!

Rectangular waveguide

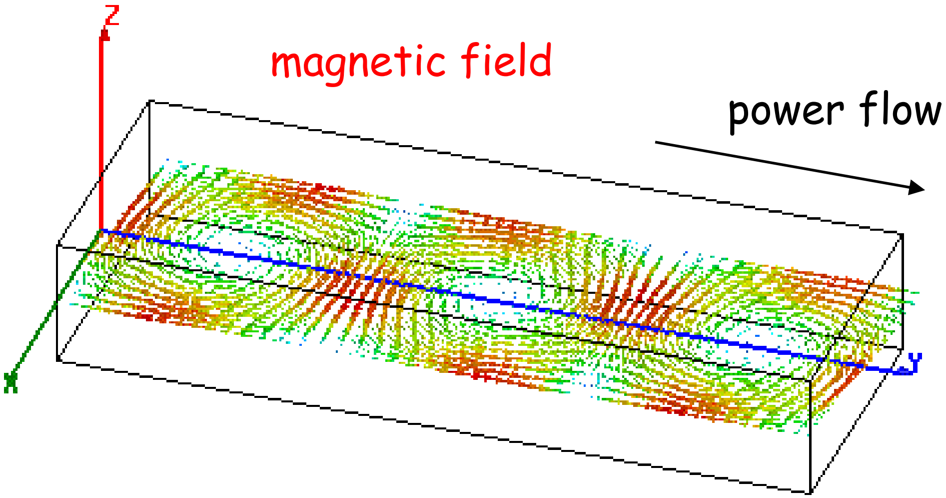
Fundamental (TE$_{10}$ or H$_{10}$) mode
in a standard rectangular waveguide.

E.g. forward wave



Electric and magnetic field

travel in phase in the waveguide



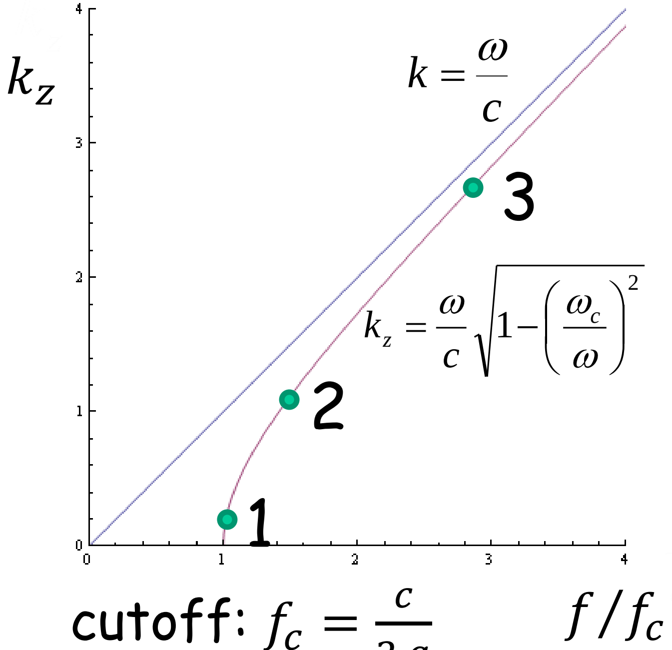
power flow: $\frac{1}{2}\mathrm{Re}\{\iint \vec{E}\times\vec{H}^{*}dA\}$

Waveguide dispersion

Different waveguide width a:
Waves with $\lambda > 2a$ don't propagate. Only frequencies higher **Cutoff** $f_c = \frac{c}{2\,a}$ enter.
The "guided wavelength" λ_g varies from ∞ at f_c to λ at very high frequencies.

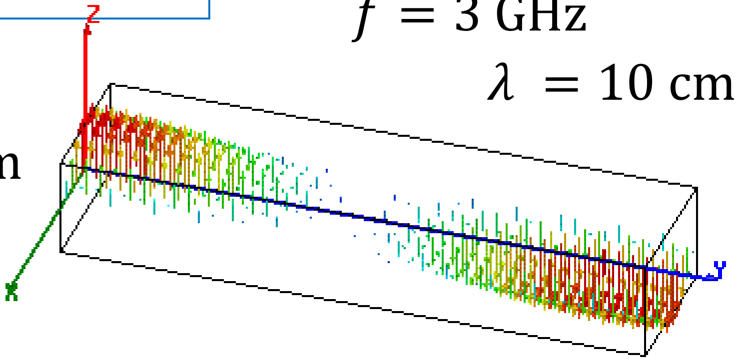
$f = 3\ \mathrm{GHz}$

$\lambda = 10\ \mathrm{cm}$

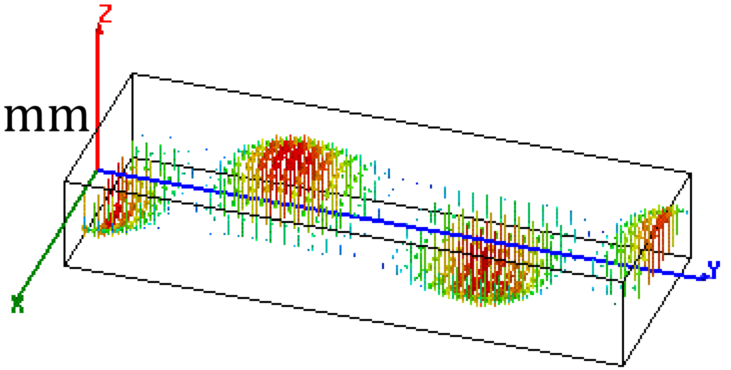


cutoff: $f_c = \frac{c}{2\,a}$

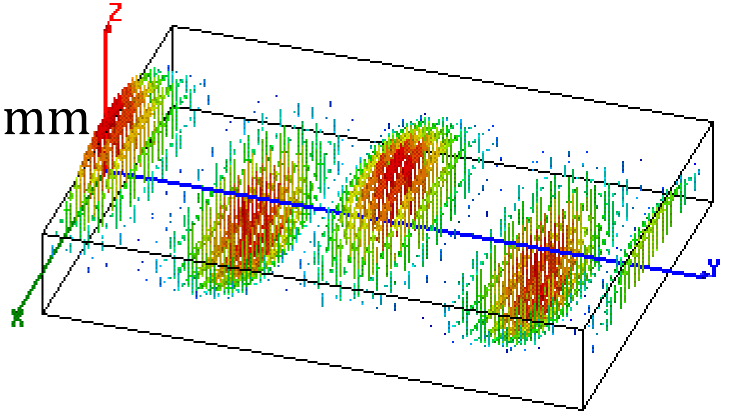
1:
$a = 52\ \mathrm{mm}$
$\frac{f}{f_c} = 1.04$



2:
$a = 72.14\ \mathrm{mm}$
$\frac{f}{f_c} = 1.44$



3:
$a = 144.3\ \mathrm{mm}$
$\frac{f}{f_c} = 2.88$



Phase velocity v_φ

The phase velocity is the speed with which the crest or a zero-crossing travels.
Note in the animations that, at constant f, it is $v_\varphi \propto \lambda_g$.
Note that at $f = f_c$, $v_\varphi = \infty$!
With $f \to \infty$, $v_\varphi \to c$!

Energy travels with group velocity

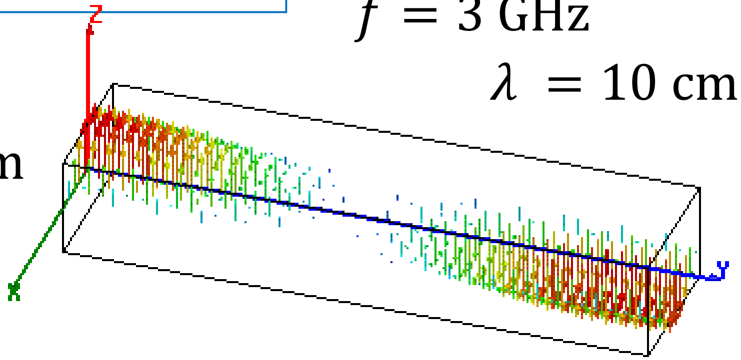
In a hollow waveguide:
- phase velocity $v_\varphi > c$
- group velocity $v_{gr} < c$

$$v_{gr} \cdot v_\varphi = c^2$$

In fixed dimension waveguide
$\Rightarrow$ Different frequencies travel with different speed.

$f = 3\ \mathrm{GHz}$
$\lambda = 10\ \mathrm{cm}$

1:
$a = 52\ \mathrm{mm}$
$\frac{f}{f_c} = 1.04$



2:
$a = 72.14\ \mathrm{mm}$
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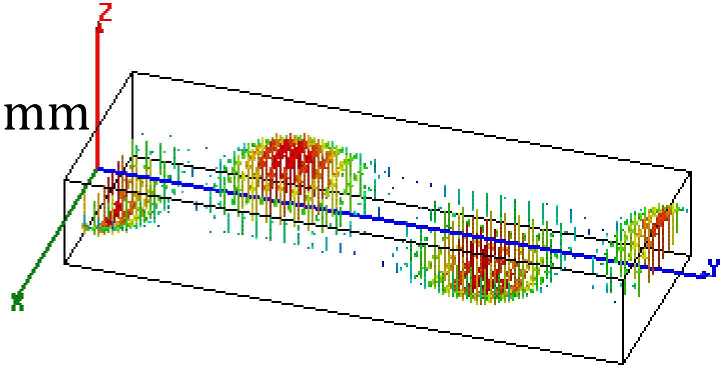
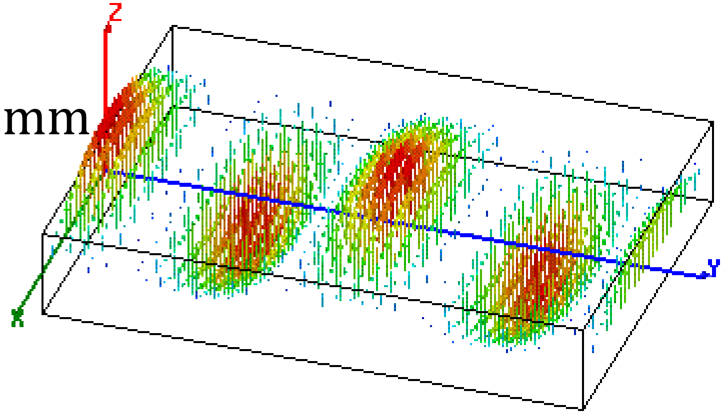


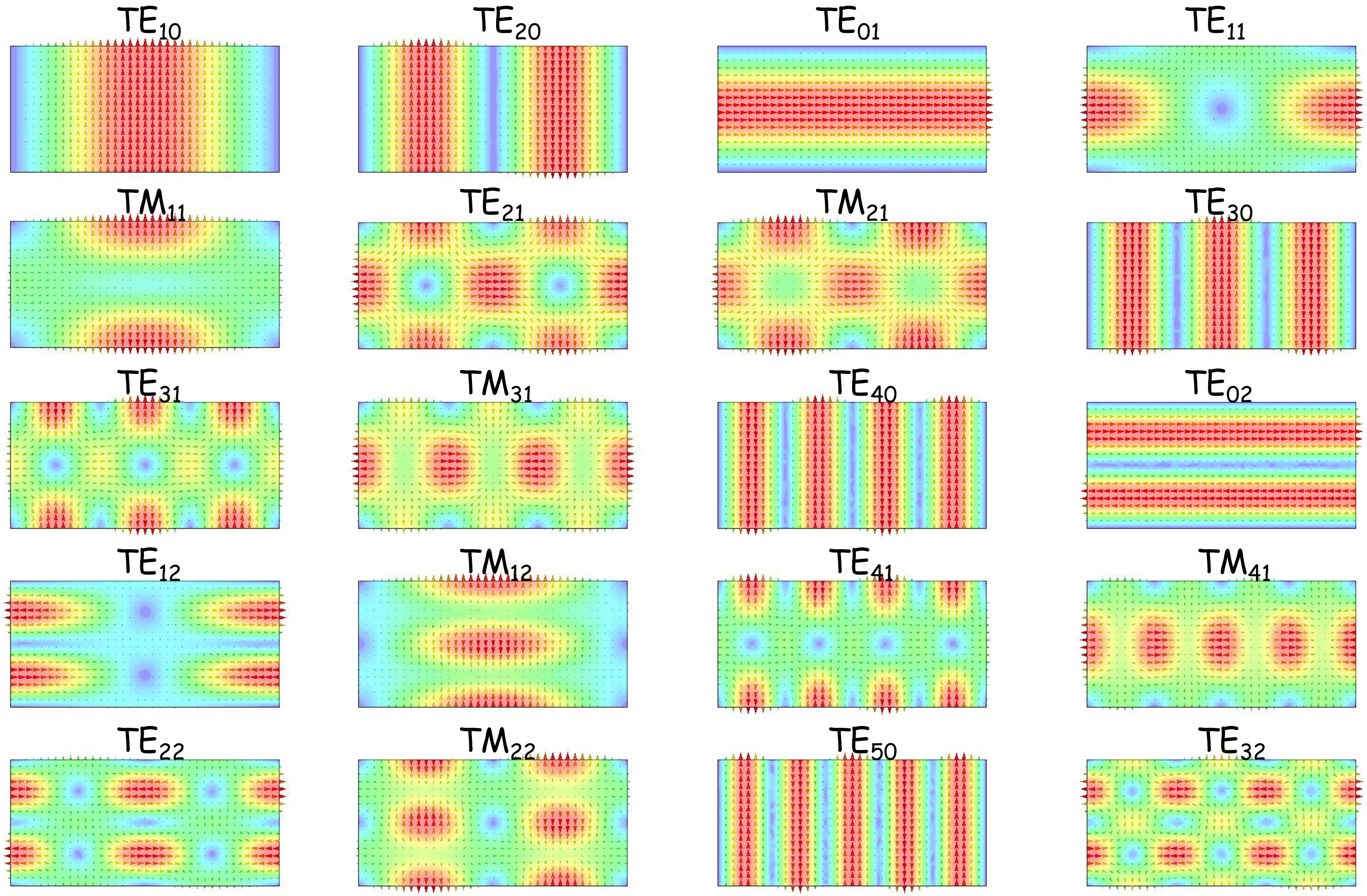
3:
$a = 144.3\ \mathrm{mm}$
$\frac{f}{f_c} = 2.88$



Rectangular waveguide modes

Types: TM_{xy} (transverse magnetic) or TE_{xy} (transverse electric)

Indices indicate number of half-waves in transverse directions.

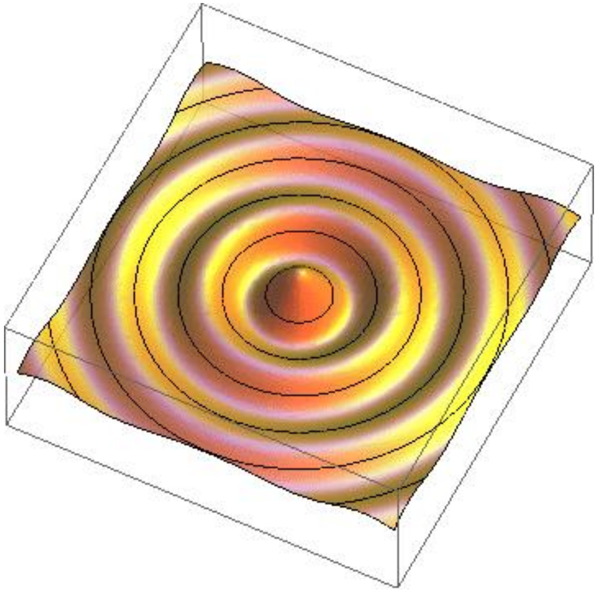


plotted: *E*-field

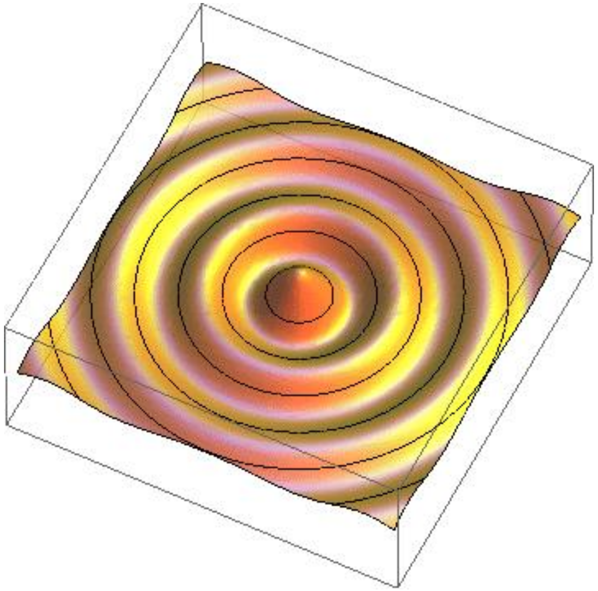
Radial waves

Also radial waves may be interpreted as superposition of plane waves.

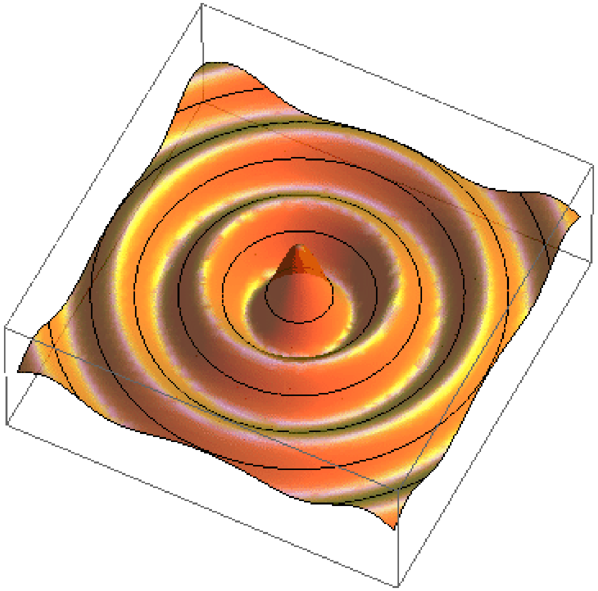
The superposition of an outward and an inward radial wave can result in the field of a round hollow waveguide.



$E_z \propto H_n^{(2)}(k_\rho \rho)\cos(n\varphi)$



$E_z \propto H_n^{(1)}(k_\rho \rho)\cos(n\varphi)$



$E_z \propto J_n(k_\rho \rho)\cos(n\varphi)$

Round waveguide modes

TE_{11} - fundamental

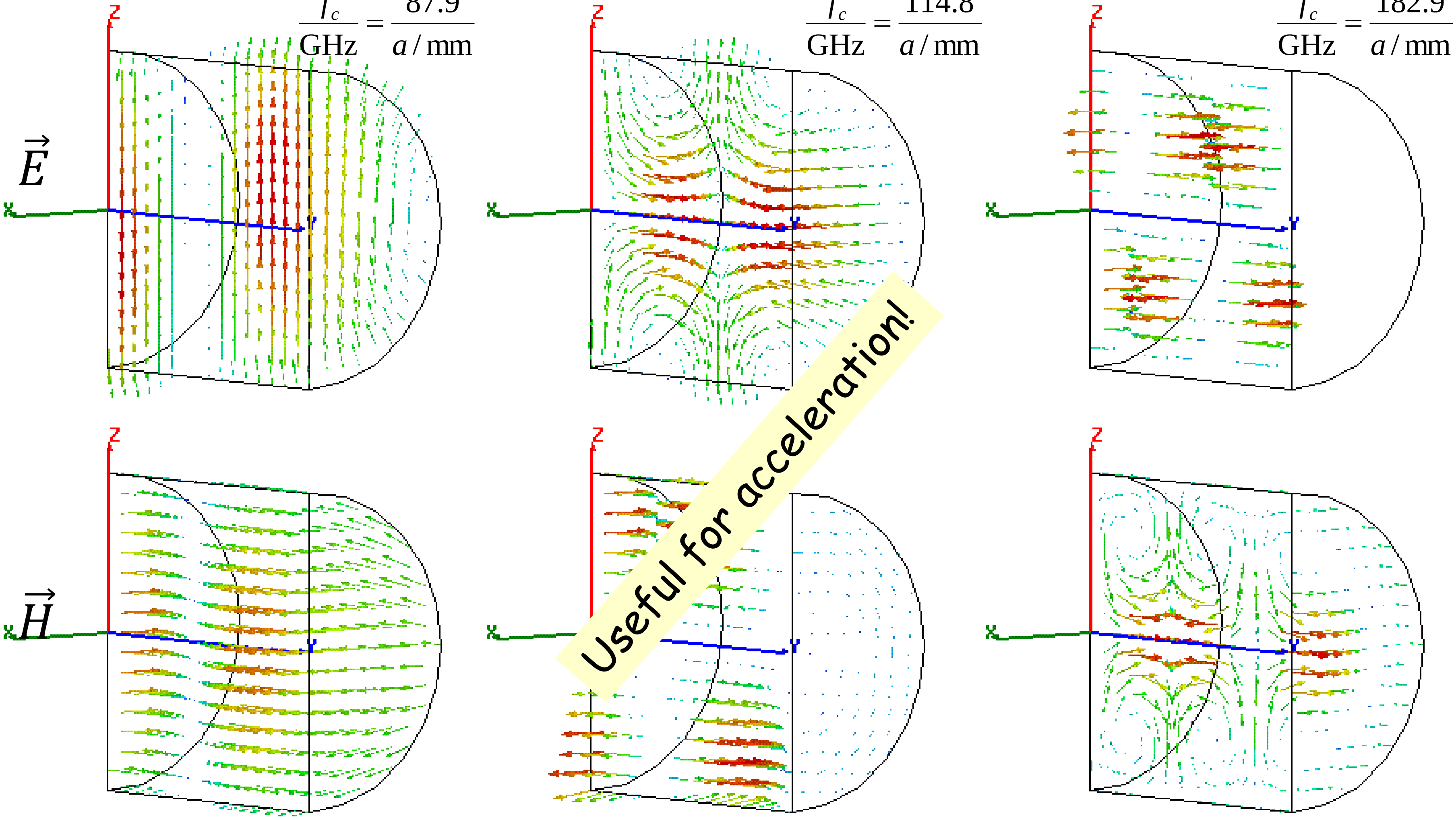
$$\frac{f_c}{\text{GHz}} = \frac{87.9}{a/\text{mm}}$$

TM_{01} - axial field

$$\frac{f_c}{\text{GHz}} = \frac{114.8}{a/\text{mm}}$$

TE_{01} - low loss

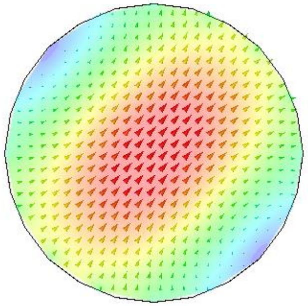
$$\frac{f_c}{\text{GHz}} = \frac{182.9}{a/\text{mm}}$$



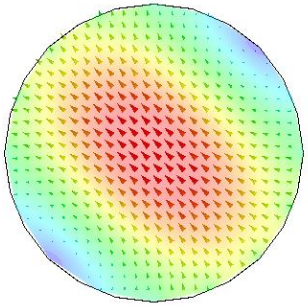
Useful for acceleration!

Circular waveguide modes

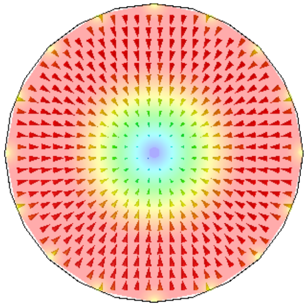
Indices linked to the number of field knots in polar co-ordinates φ, r



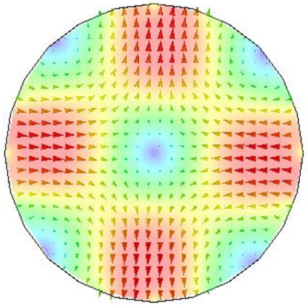
TE_{11}



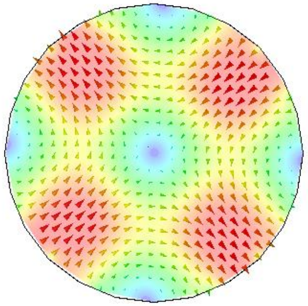
TE_{11}



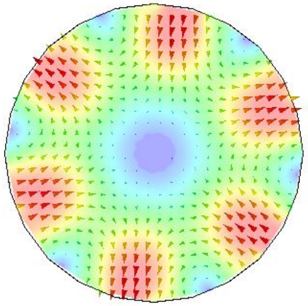
TM_{01}



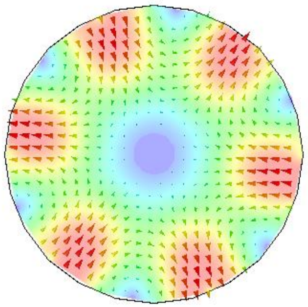
TE_{21}



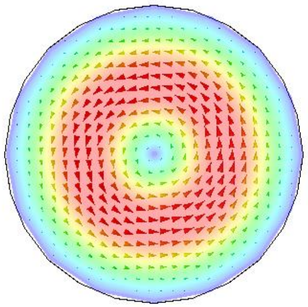
TE_{21}



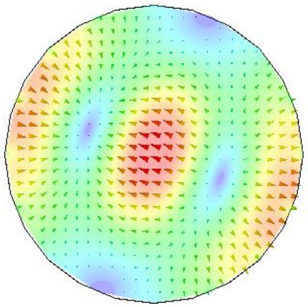
TE_{31}



TE_{31}



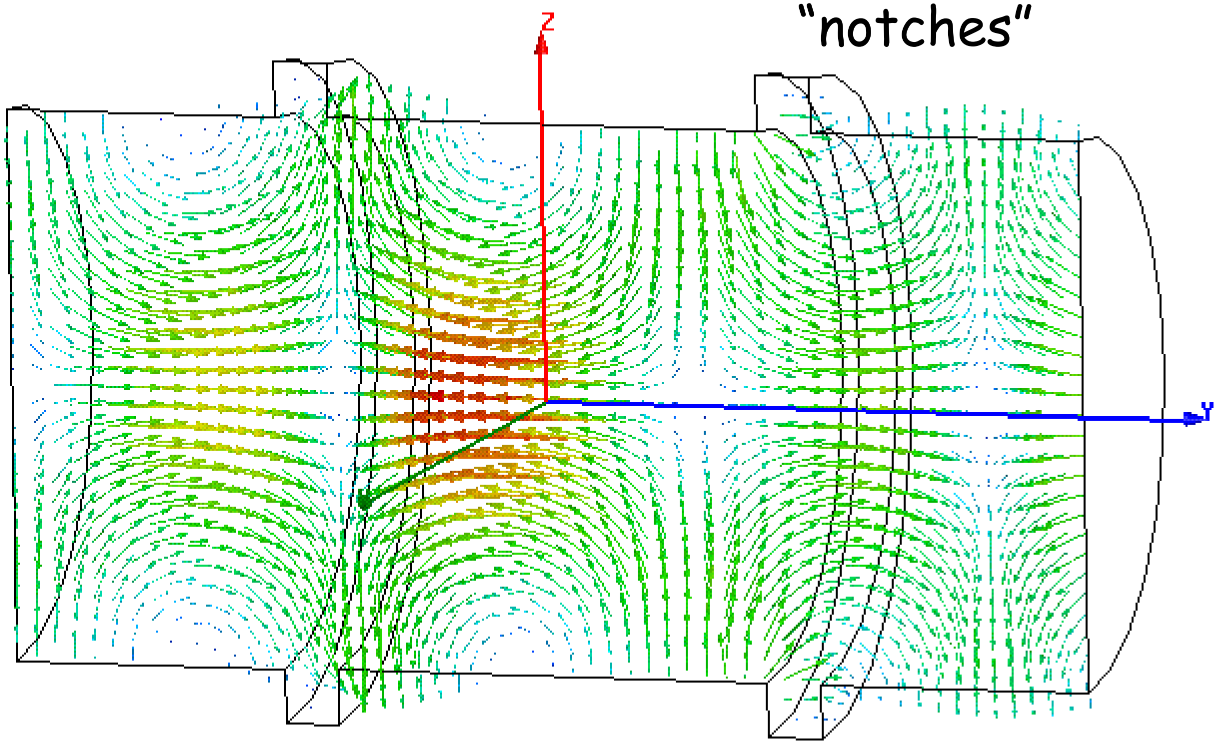
TE_{01}

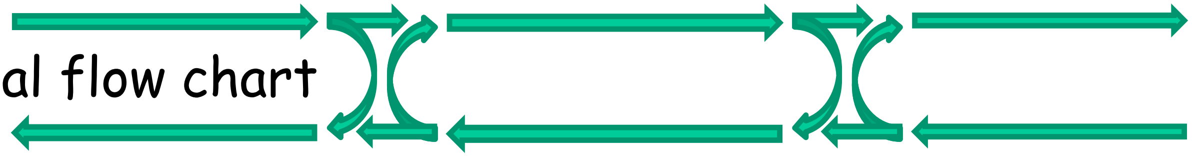


TM_{11}

plotted: *E*-field

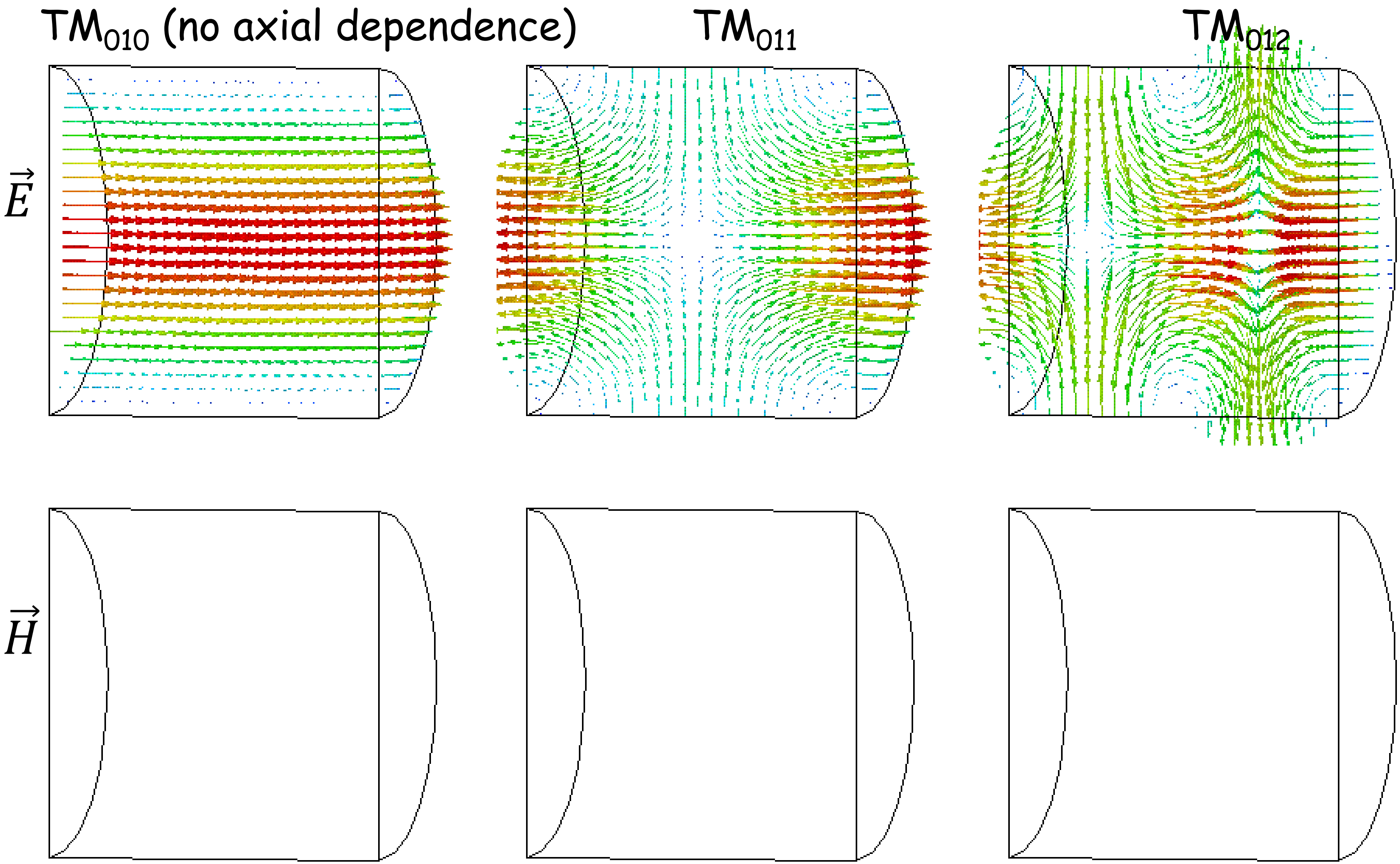
Waveguide perturbed by discontinuities (notches)



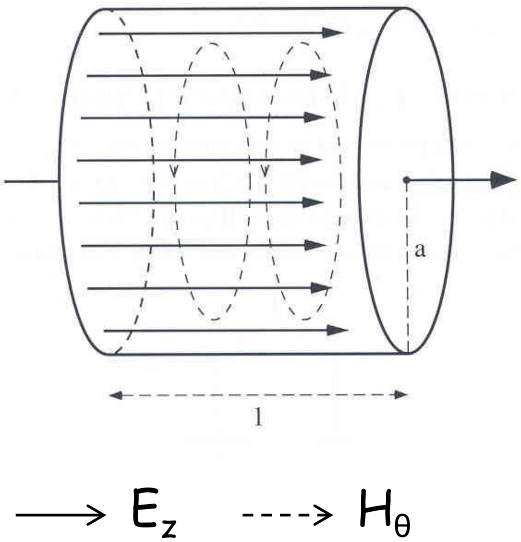

Signal flow chart

Reflections from notches lead to a superimposed standing wave pattern.
"Trapped mode"

Short-circuited waveguide -> Cavity



The 'Pill Box' Cavity

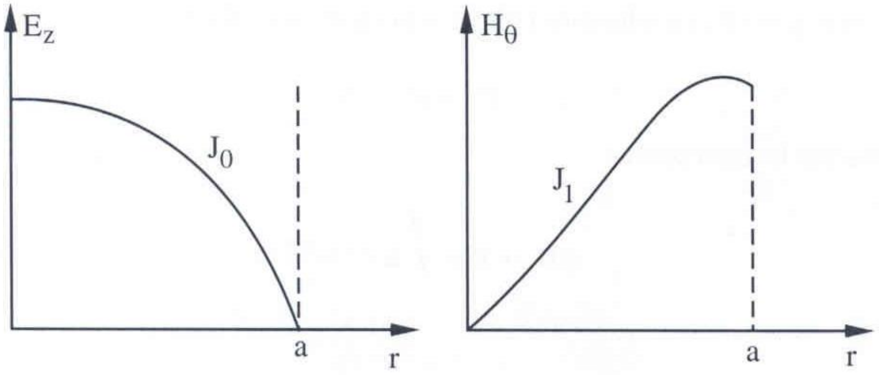


The wave solutions for E and H are oscillating modes, at **discrete frequencies**.

Modes can be type TM_{xyz} (transverse magnetic) or TE_{xyz} (transverse electric).

Indices linked to the number of field knots in polar co-ordinates φ, r and z.

For $l<2a$ the most simple mode, TM_{010}, has the lowest frequency, and has only two field components:



$$E_z = J_0(kr)\, e^{i\omega t}$$

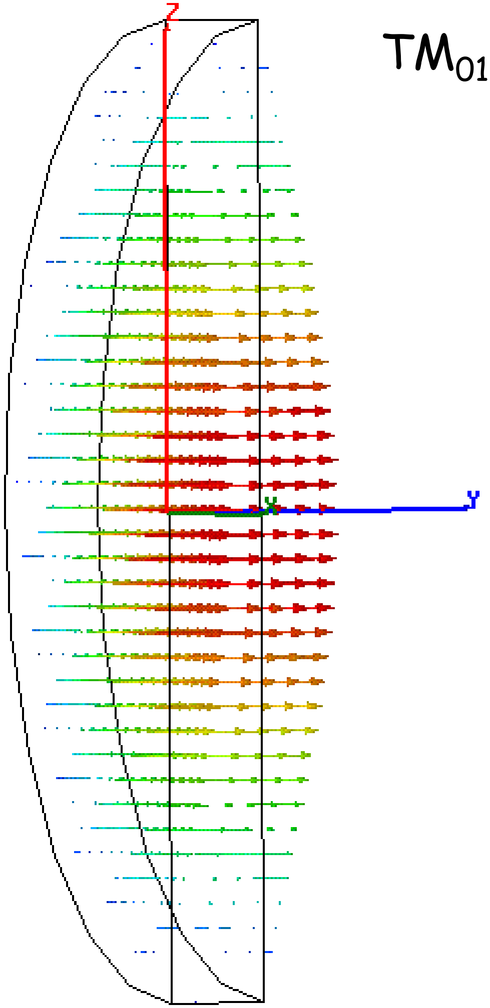
$$H_\theta = -\frac{i}{Z_0} J_1(kr)\, e^{i\omega t}$$

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \qquad \lambda = 2.62a \qquad Z_0 = 377\Omega$$

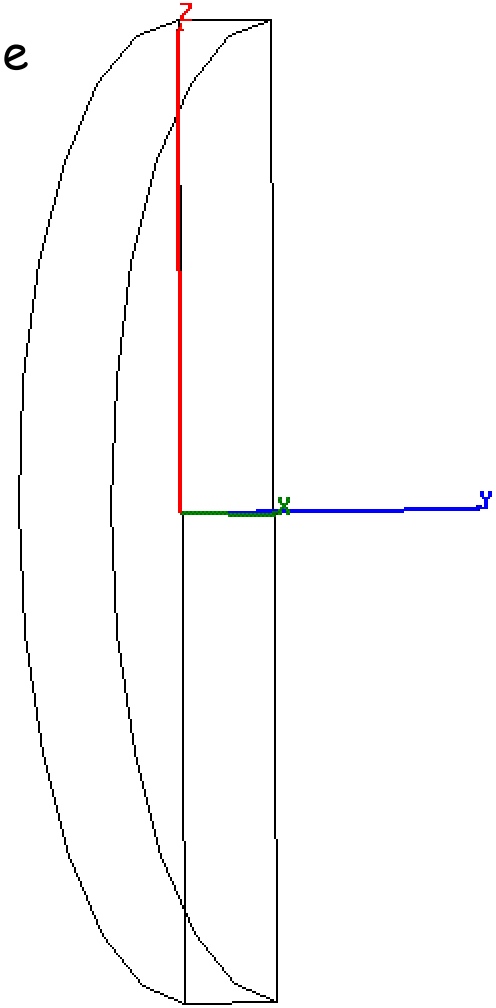
Simple pillbox cavity

(only 1/2 shown)

TM$_{010}$-mode



electric field (purely axial)

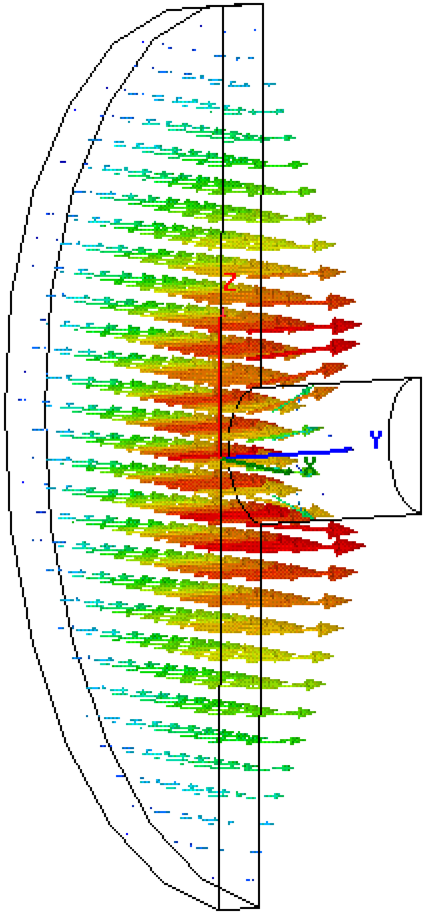


magnetic field (purely azimuthal)

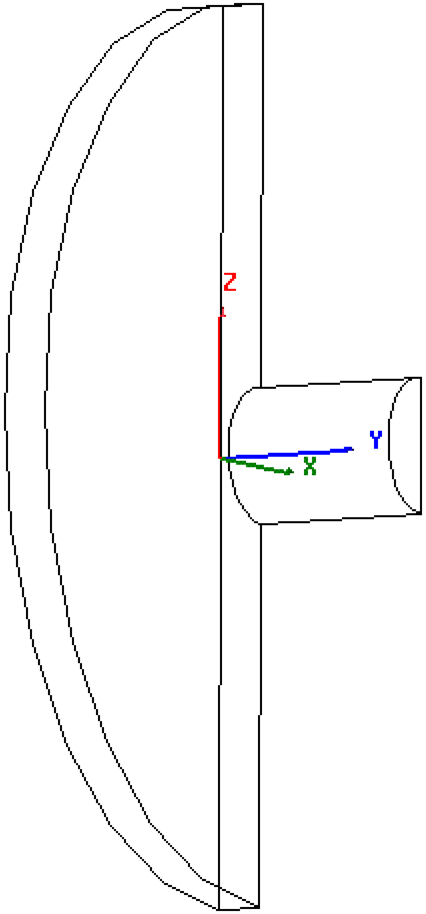
Pillbox with beam pipe

TM_{010}-mode (only 1/4 shown)

One needs a hole for the beam pipe – circular waveguide below cutoff



electric field

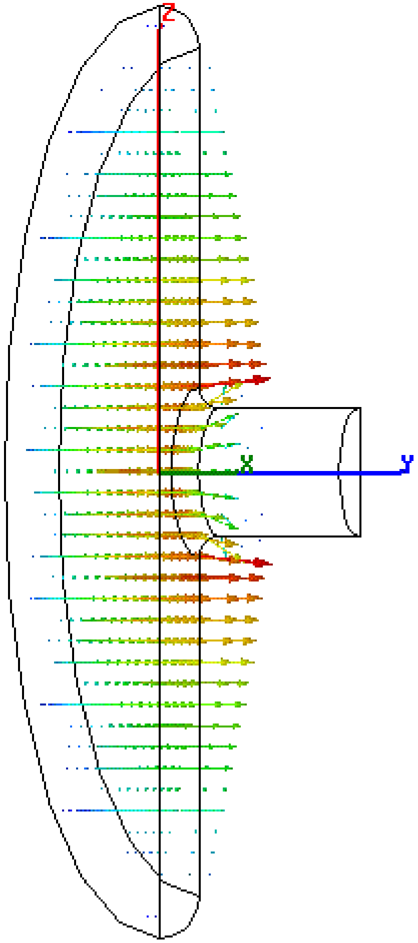


magnetic field

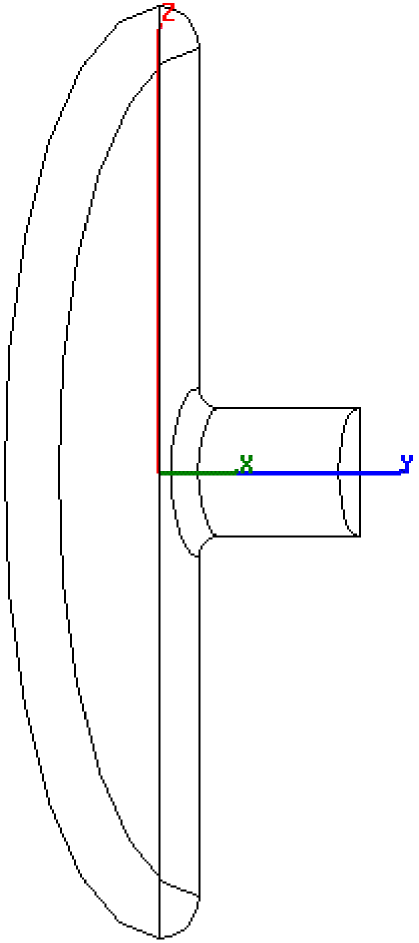
A more practical pillbox cavity

TM_{010}-mode (only 1/4 shown)

Round off sharp edges (field enhancement!)



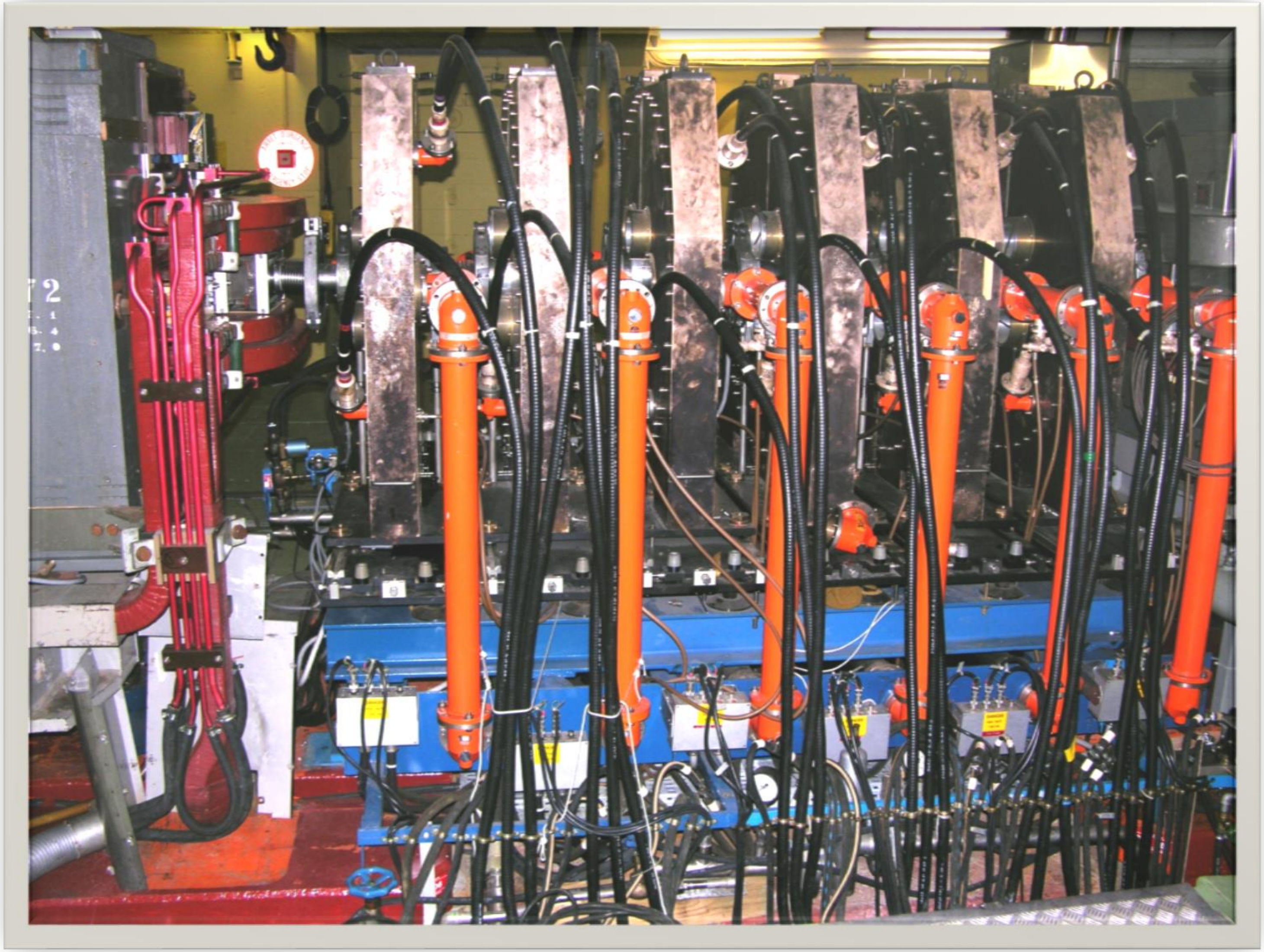
electric field



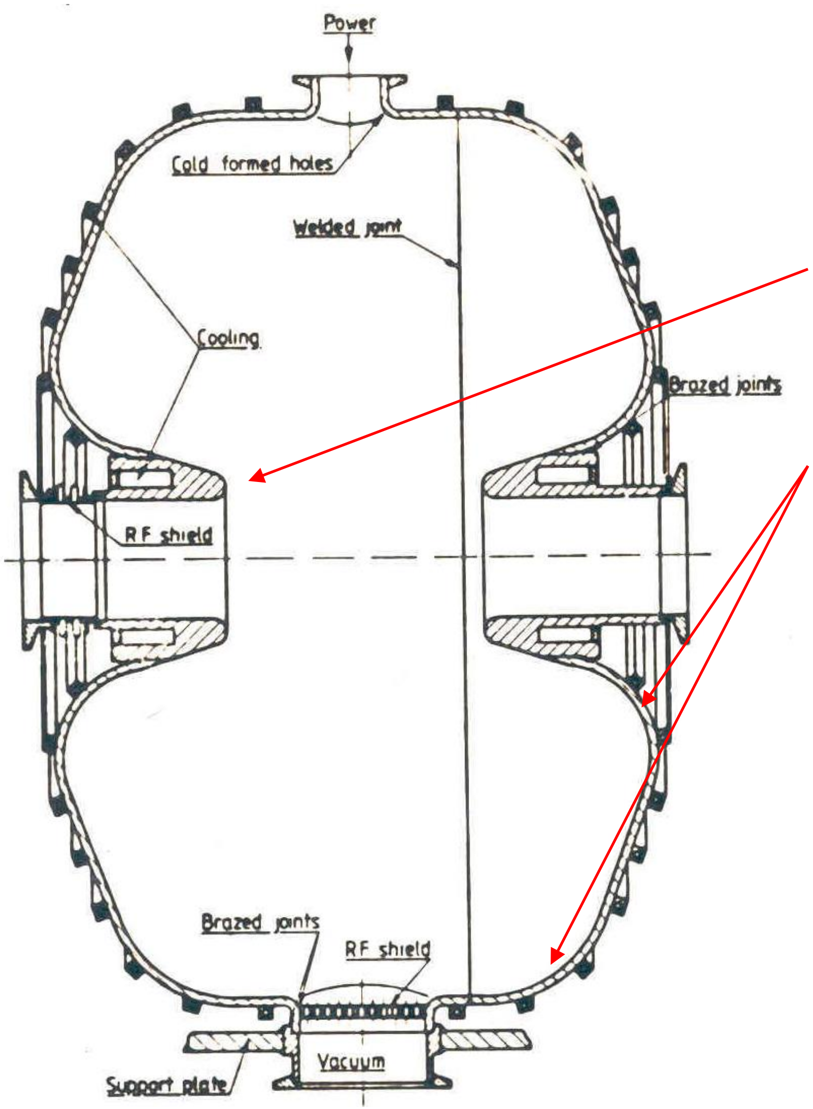
magnetic field

Some real "pillbox" cavities

CERN PS 200 MHz cavities



The Pill Box Cavity -> Real Cavity



The design of a cavity can be sophisticated in order to improve its performances:

- A nose cone can be introduced in order to concentrate the electric field around the axis

- Round shaping of the corners allows a better distribution of the magnetic field on the surface and a reduction of the Joule losses.
It also prevents from multipactoring effects (e- emission and acceleration).

A good cavity efficiently transforms the RF power into accelerating voltage.

Simulation codes allow precise calculation of the properties.

Transit time factor

The accelerating field varies during the passage of the particle
=> particle does not always see maximum field => effective acceleration smaller

Transit time factor defined as:

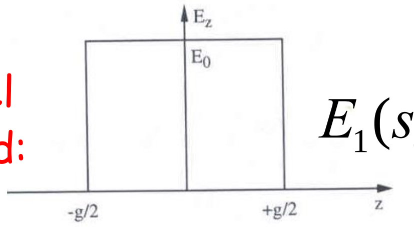
$$T_a = \frac{\text{energy gain of particle with } v = \beta c}{\text{maximum energy gain (particle with } v \to \infty)}$$

In the general case, the transit time factor is:

for $E(s,r,t) = E_1(s,r) \cdot E_2(t)$

$$T_a = \frac{\left| \int_{-\infty}^{+\infty} E_1(s,r) \cos\left(\omega_{RF} \frac{s}{v}\right) \mathrm{d}s \right|}{\int_{-\infty}^{+\infty} E_1(s,r)\, \mathrm{d}s}$$

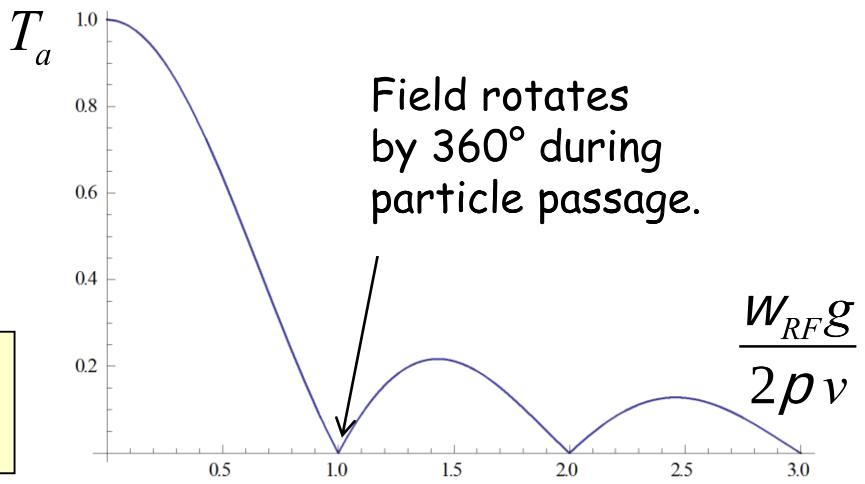
Simple model uniform field:



$$E_1(s,r) = \frac{V_{RF}}{g}$$

follows:

$$T_a = \left| \sin \frac{\omega_{RF} g}{2v} \Big/ \frac{\omega_{RF} g}{2v} \right|$$



Field rotates by 360° during particle passage.

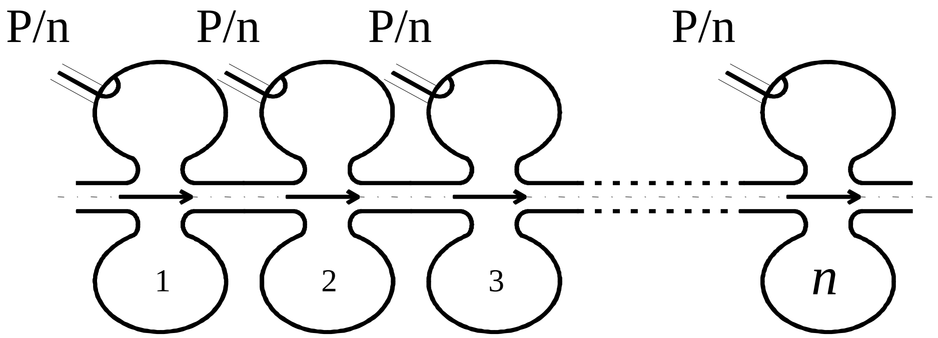
$0 < T_a < 1$, $T_a \to 1$ for $g \to 0$, smaller ω_{RF}
Important for low velocities (ions)

Multi-Cell Cavities

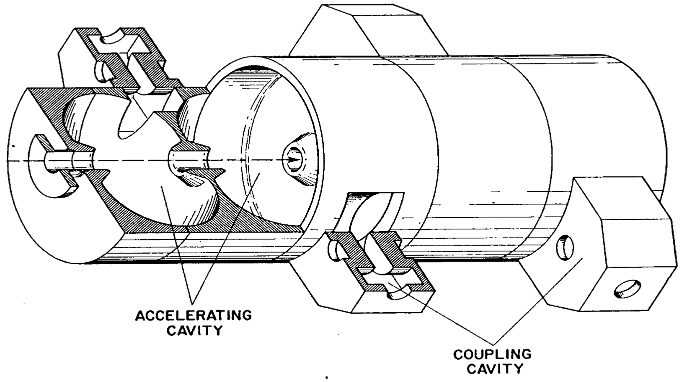
Acceleration of one cavity limited => distribute power over several cells

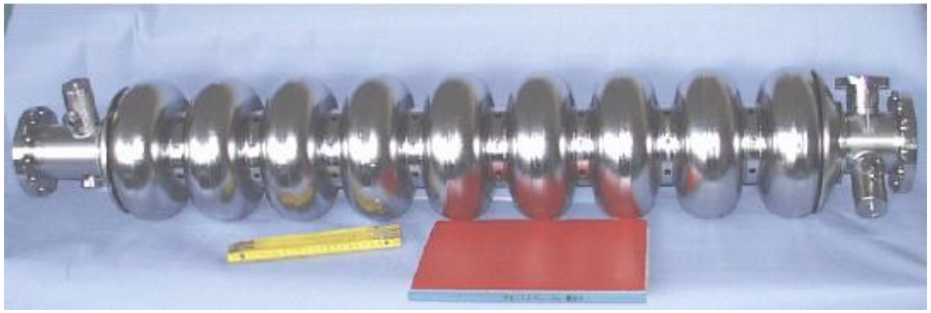
Each cavity receives P/n

Since the field is proportional √P, you get $\sum E_i \propto n\sqrt{P/n} = \boxed{\sqrt{n}}E_0$



Instead of distributing the power from the amplifier, one might as well couple the cavities, such that the power automatically distributes, or have a cavity with many gaps (e.g. drift tube linac).





Multi-Cell Cavities - Modes

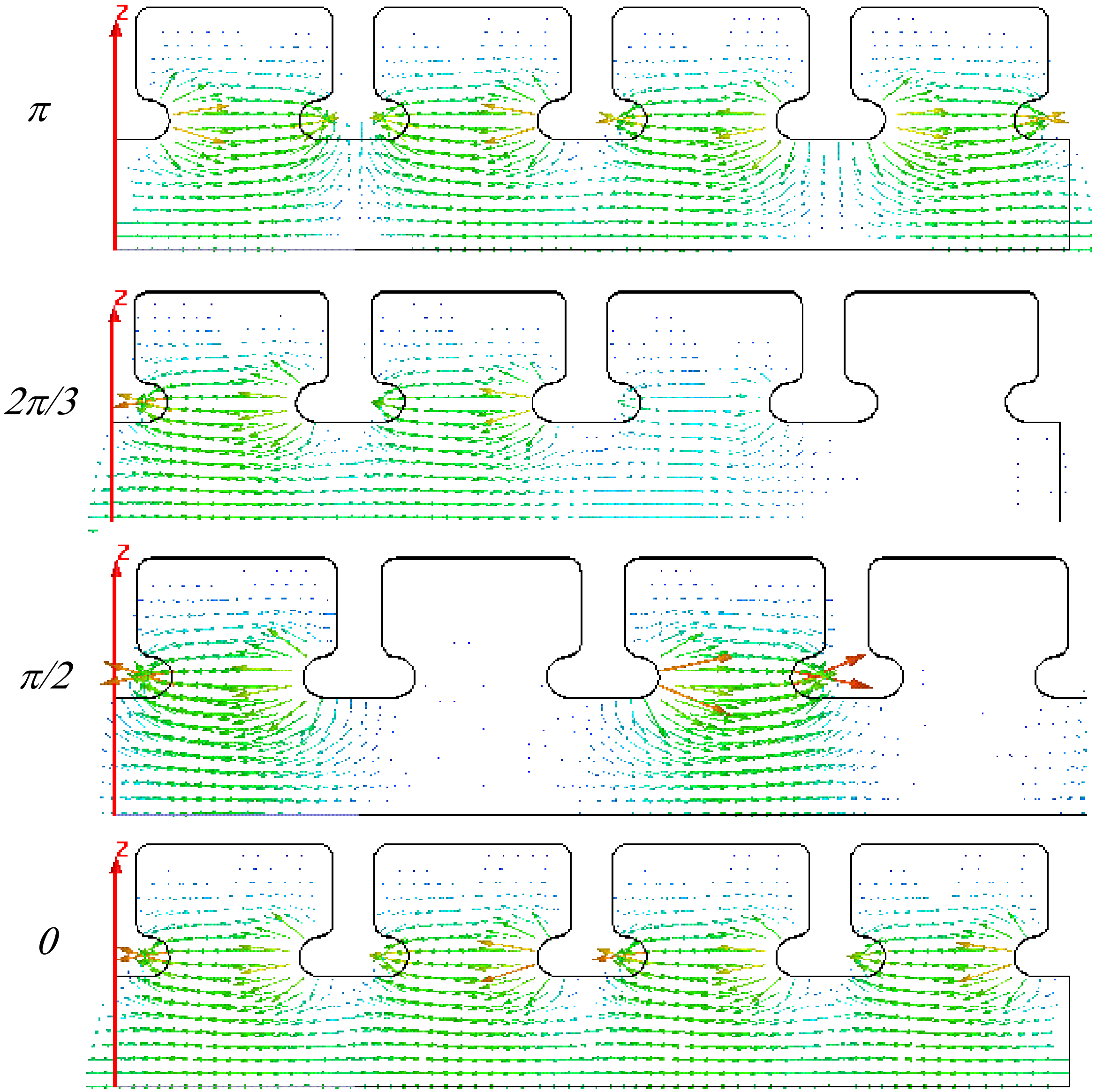
The phase relation between gaps is important!

Coupled harmonic oscillator

=> Modes, named after the phase difference between adjacent cells.

Relates to different synchronism conditions for the cell length L

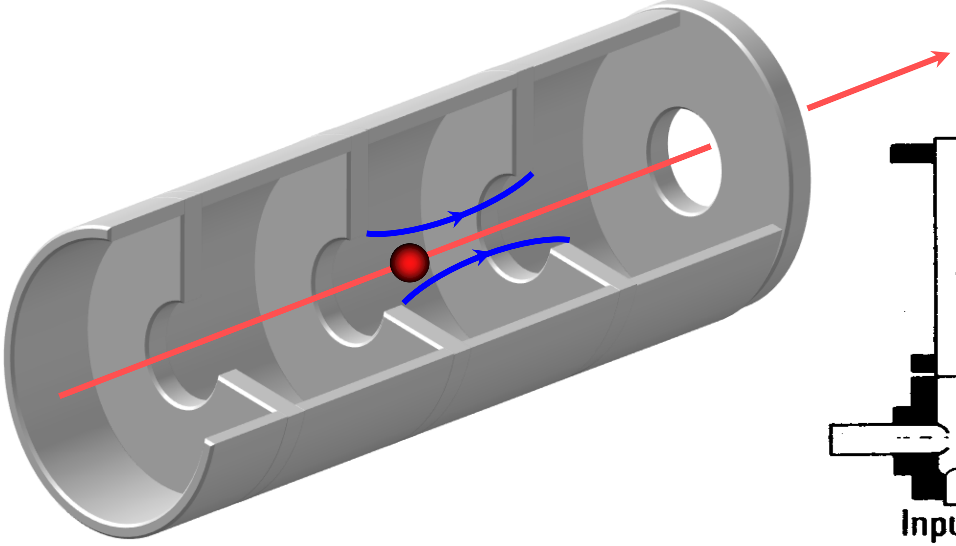
Mode	L
0 (2π)	βλ
π/2	βλ/4
2π/3	βλ/3
π	βλ/2

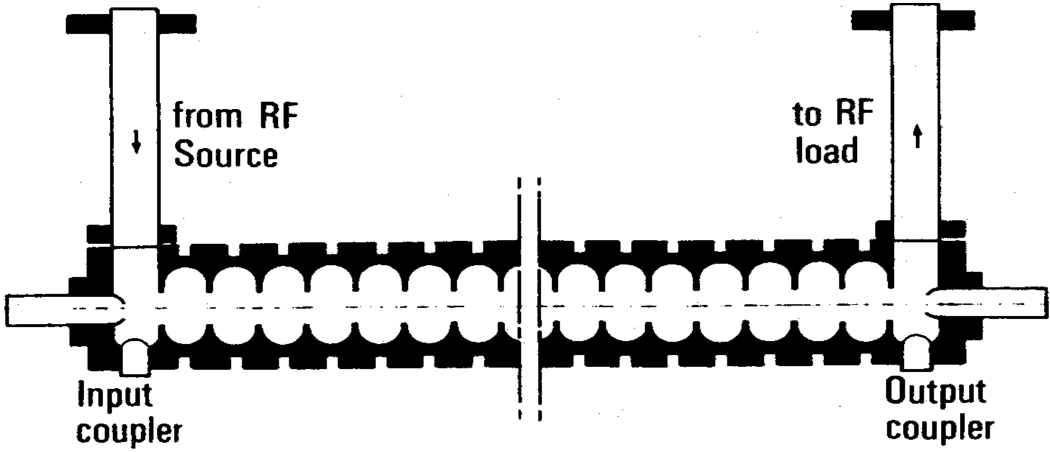


Disc-Loaded Traveling-Wave Structures

When particles get ultra-relativistic (v~c) the drift tubes become very long unless the operating frequency is increased. Late 40's the development of radar led to high power transmitters (klystrons) at very high frequencies (3 GHz).

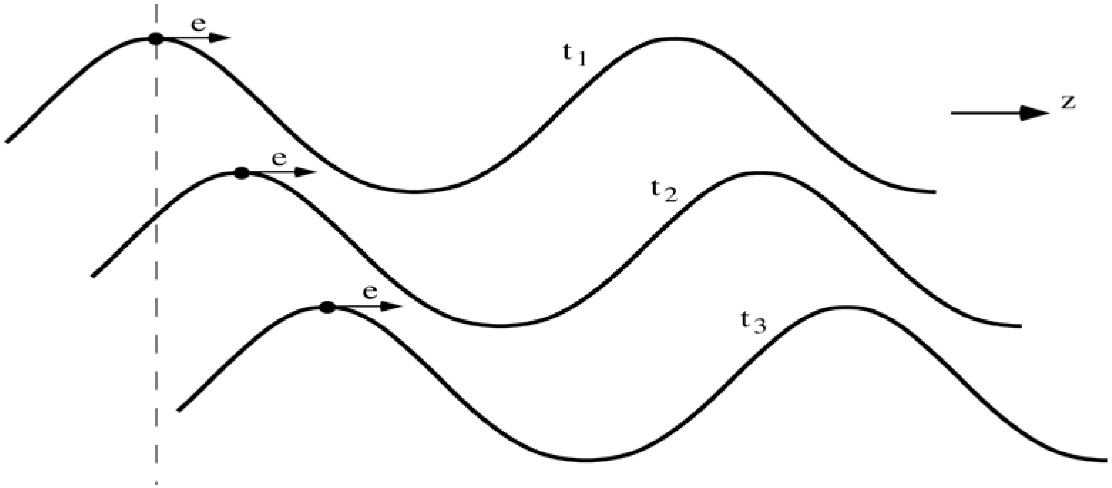
Next came the idea of suppressing the drift tubes using traveling waves. A wave guide has always a phase velocity $v_\varphi > c$. However to get a continuous acceleration the phase velocity of the wave needs to be adjusted to the particle velocity.





solution: slow wave guide with irises ==> iris loaded structure

The Traveling Wave Case



$$E_z = E_0 \cos\left(\omega_{RF} t - kz\right)$$

$$k = \frac{\omega_{RF}}{v_\varphi}$$ wave number

$$z = v(t - t_0)$$

v_φ = phase velocity
v = particle velocity

The particle travels along with the wave, and k represents the wave propagation factor.

$$E_z = E_0 \cos\left(\omega_{RF} t - \omega_{RF}\frac{v}{v_\varphi}t - \phi_0\right)$$

If synchronism satisfied: $v = v_\varphi$ and $E_z = E_0 \cos\phi_0$

where Φ_0 is the RF phase seen by the particle.

Cavity Parameters: Quality Factor Q

The total energy stored is $$W = \iiint_{cavity} \left(\frac{\varepsilon}{2}\left|\vec{E}\right|^2 + \frac{\mu}{2}\left|\vec{H}\right|^2\right) dV.$$

- Quality Factor Q (caused by wall losses) defined as

$$Q_0 = \frac{\omega_0 W}{P_{loss}}$$

Ratio of stored energy W and dissipated power P_{loss} on the walls in one RF cycle

The Q factor determines the maximum energy the cavity can fill to with a given input power.
Larger Q => less power needed to sustain stored energy.

The Q factor is 2π times the number of rf cycles it takes to dissipate the energy stored in the cavity (down by 1/e).

- function of the geometry and the surface resistance of the material:
  - superconducting (niobium) : $Q = 10^{10}$
  - normal conducting (copper) : $Q = 10^4$

Important Parameters of Accelerating Cavities

- Accelerating voltage V_{acc}

$$V_{acc} = \int_{-\infty}^{\infty} E_z e^{-i\frac{\omega z}{\beta c}} dz$$

Measure of the acceleration

- R upon Q

$$\frac{R}{Q} = \frac{|V_{acc}|^2}{2\omega_0 W}$$

Relationship between acceleration V_{acc} and stored energy W

independent from material!

Attention: Different definitions are used!

- Shunt Impedance R

$$R = \frac{|V_{acc}|^2}{2P_{loss}}$$

Relationship between acceleration V_{acc} and wall losses P_{loss}

depends on
- material
- cavity mode
- geometry

Important Parameters of Accelerating Cavities (cont.)

- Fill Time t_F

  - **standing wave cavities**:

$$P_{loss} = -\frac{dW}{dt} = \frac{\omega}{Q} W$$

Exponential decay of the stored energy W due to losses

$$t_F = \frac{Q}{W}$$

time for the field to decrease by 1/e after the cavity has been filled

measure of how fast the stored energy is dissipated on the wall

Several fill times needed to fill the cavity!

  - **travelling wave cavities**:

time needed for the electromagnetic energy to fill the cavity of length L

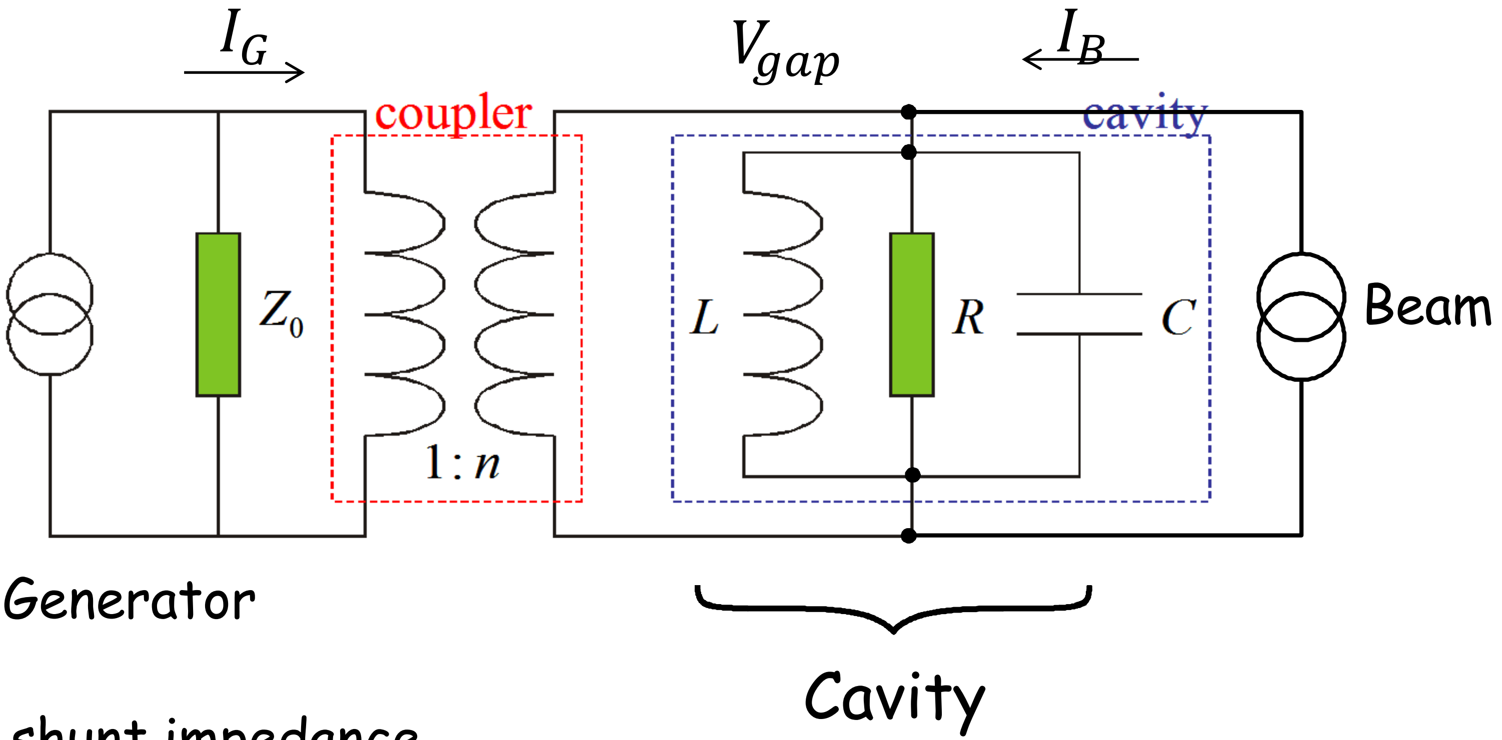
$$t_F = \int_0^L \frac{dz}{v_g(z)}$$

v_g: velocity at which the energy propagates through the cavity

Cavity is completely filled after 1 fill time!

SW Cavity resonator - equivalent circuit

Simplification: single mode

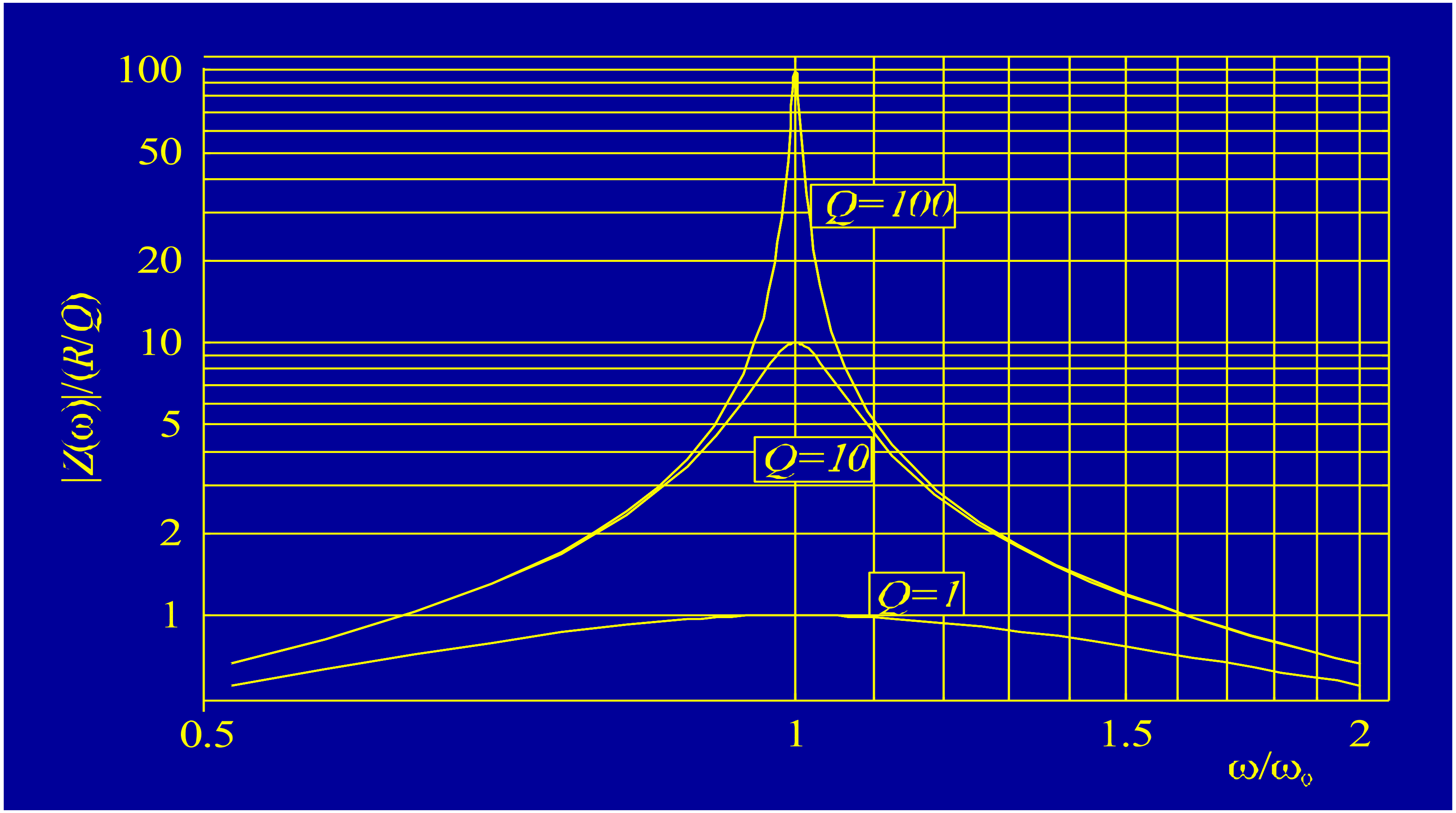


R: shunt impedance

$\sqrt{L/C} = \frac{R}{Q}$: R-upon-Q

$$\omega_0 = \frac{1}{\sqrt{L \cdot C}}$$

Resonance



A high Q_0: small wall losses => less power needed for the same voltage.
But the bandwidth becomes very narrow.
Note: a 1 GHz cavity with a Q_0 of 10^{10} has a natural bandwidth of 0.1 Hz!
… to make this manageable, Q_{ext} is chosen much smaller!

Power coupling - Loaded Q

Note that the generator inner impedance also loads the cavity
- for very large Q_0 more than the cavity wall losses.

To calculate the loaded Q (Q_L), the losses have to be added:

$$\frac{1}{Q_L} = \frac{P_{loss} + P_{ext} + \cdots}{\omega_0\, W} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} + \frac{1}{\ldots}.$$

The coupling factor β is the ratio P_{ext}/P_{loss}.

$$\beta = \frac{P_{ext}}{P_{loss}} = \frac{Q_0}{Q_{ext}}$$

With β, the loaded Q can be written

$$Q_L = \frac{Q_0}{1+\beta}.$$

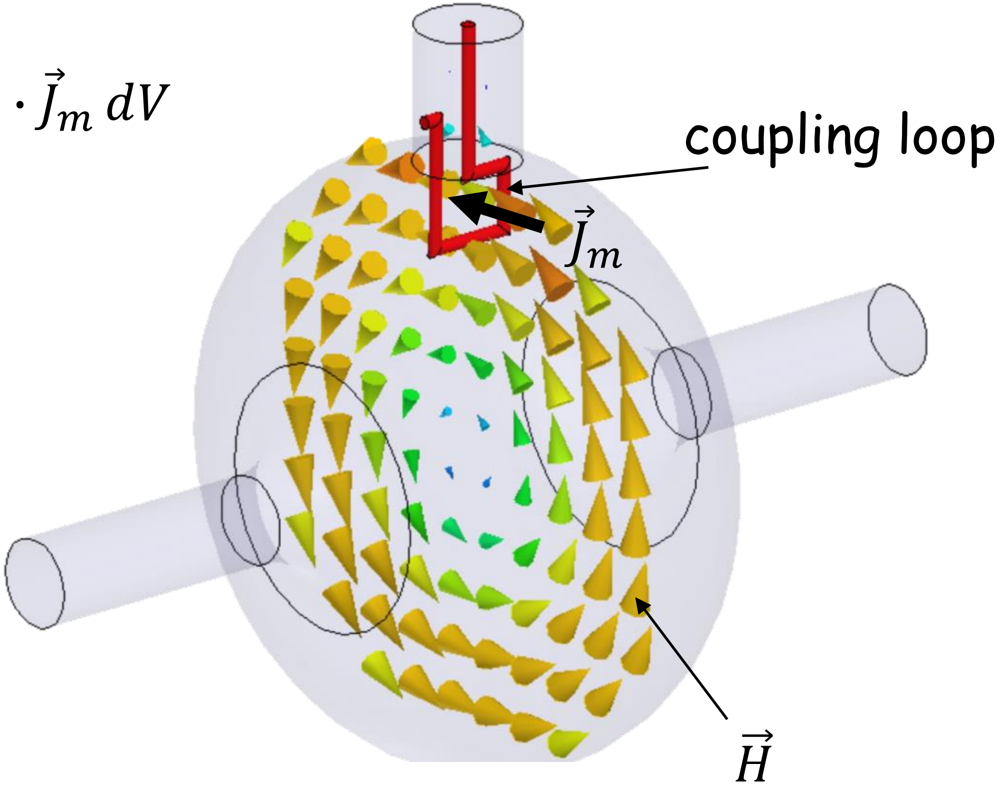
For NC cavities, often $\beta = 1$ is chosen (power amplifier matched to empty cavity); for SC cavities, $\beta = \mathcal{O}(10^4 \ldots 10^6)$.

Magnetic (loop) coupling

The magnetic field of the cavity main mode is intercepted by a coupling loop

The coupling can be adjusted by changing the size or the orientation of the loop.

Coupling: $\propto \iiint \vec{H} \cdot \vec{J}_m \, dV$



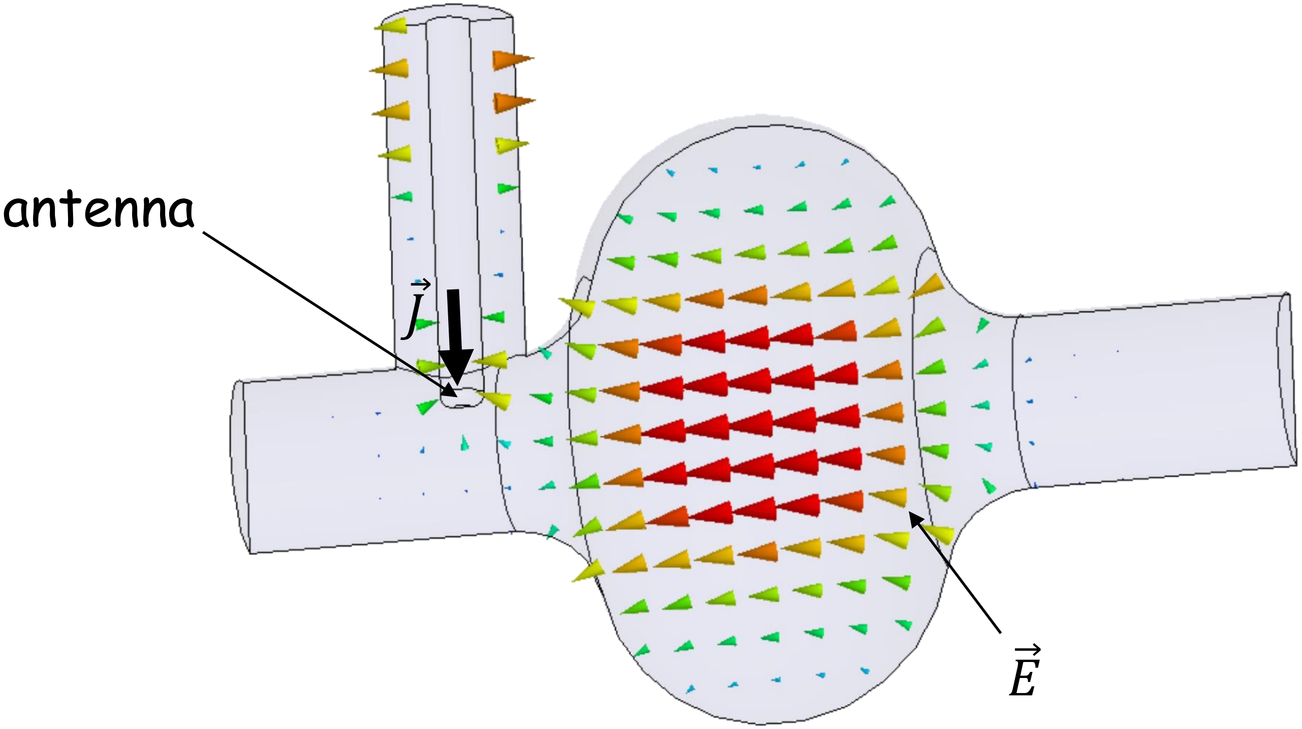
courtesy: David Alesini/INFN

Electric (antenna) coupling

The inner conductor of the coaxial feeder line ends in an antenna penetrating into the electric field of the cavity.

The coupling can be adjusted by varying the penetration.

Coupling $\propto \iiint \vec{E} \cdot \vec{J}\, dV$



courtesy: David Alesini/INFN

Cavity parameters

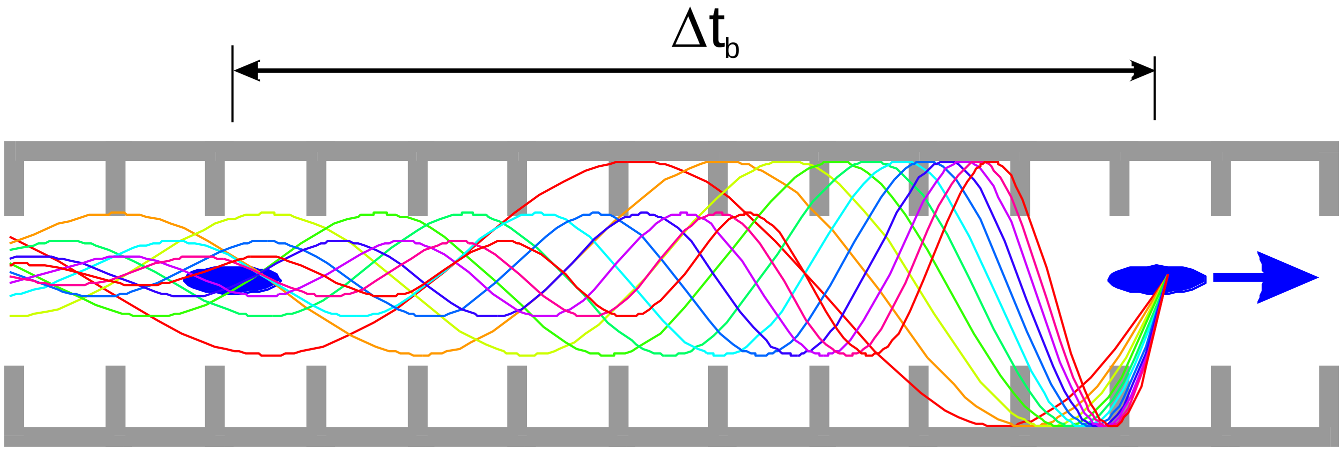
Resonance frequency	$\omega_0 = \frac{1}{\sqrt{L \cdot C}}$	
Transit time factor	$TT = \frac{\left\| \int E_z e^{i\frac{\omega}{\beta c}z}\, dz \right\|}{\left\| \int E_z\, dz \right\|}$	
Q factor	$\omega_0 W = Q\, P_{loss}$	
	Circuit definition	***Linac definition***
Shunt impedance	$\left\|V_{gap}\right\|^2 = 2\, R\, P_{loss}$	$\left\|V_{gap}\right\|^2 = R\, P_{loss}$
R/Q (R-upon-Q)	$\frac{R}{Q} = \frac{\left\|V_{gap}\right\|^2}{2\, \omega_0 W} = \sqrt{L/C}$	$\frac{R}{Q} = \frac{\left\|V_{gap}\right\|^2}{\omega_0 W}$
Loss factor	$k_{loss} = \frac{\omega_0}{2} \frac{R}{Q} = \frac{\left\|V_{gap}\right\|^2}{4W} = \frac{1}{2C}$	$k_{loss} = \frac{\omega_0}{4} \frac{R}{Q} = \frac{\left\|V_{gap}\right\|^2}{4W}$

Wakefields and Beam Loading

The cavities' electric field accelerates the beam.

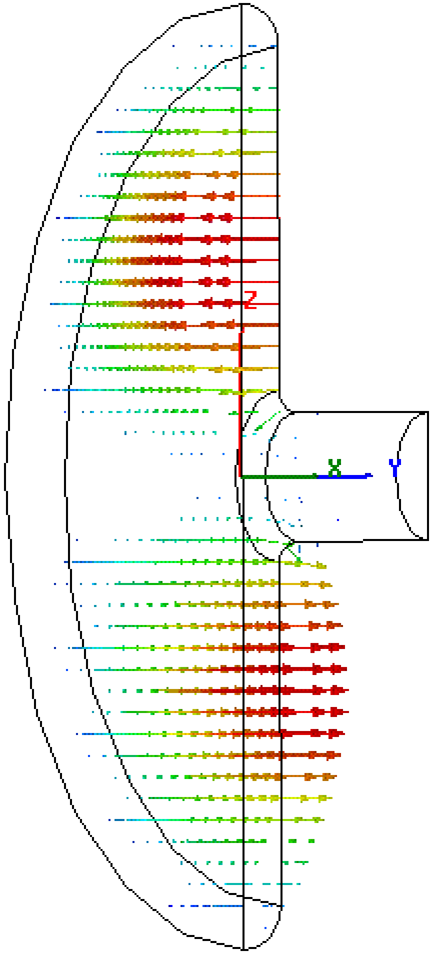
But the beam will also act on the fields inside the cavities

- Accelerating field will be reduced (energy conservation!)
  => Beam Loading (longitudinal wakefield)
- Beam can excite perturbing cavity modes (Higher Order Modes – HOM) and deflect following bunches
  => (transverse) Wakefields

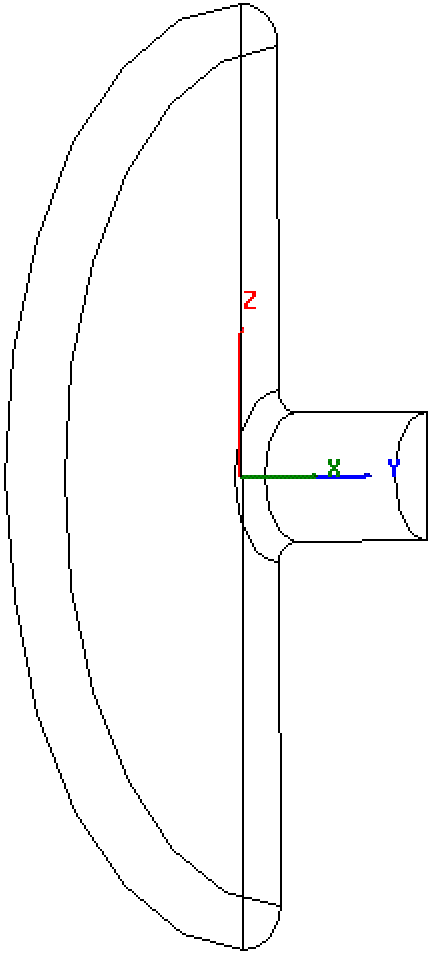


Dipole mode in a pillbox

TM$_{110}$-mode (only 1/4 shown)

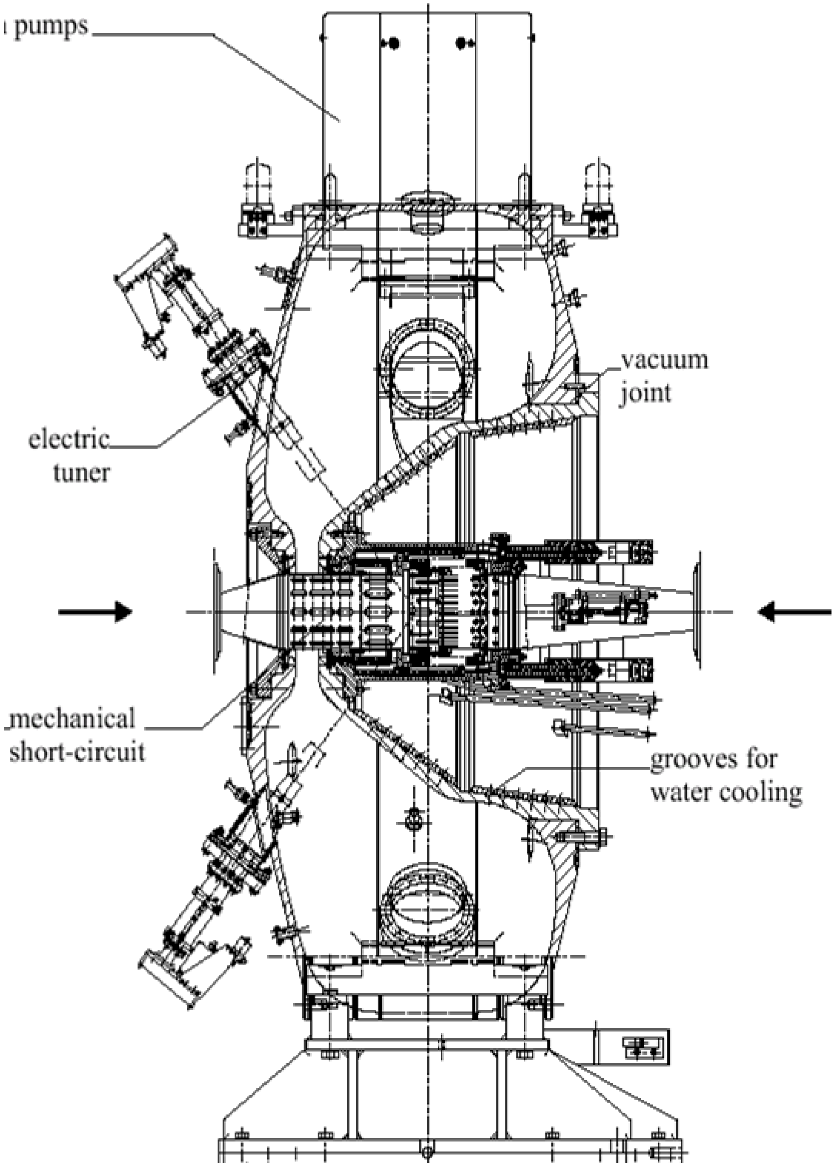


electric field



magnetic field

CERN/PS 80 MHz cavity (for LHC)

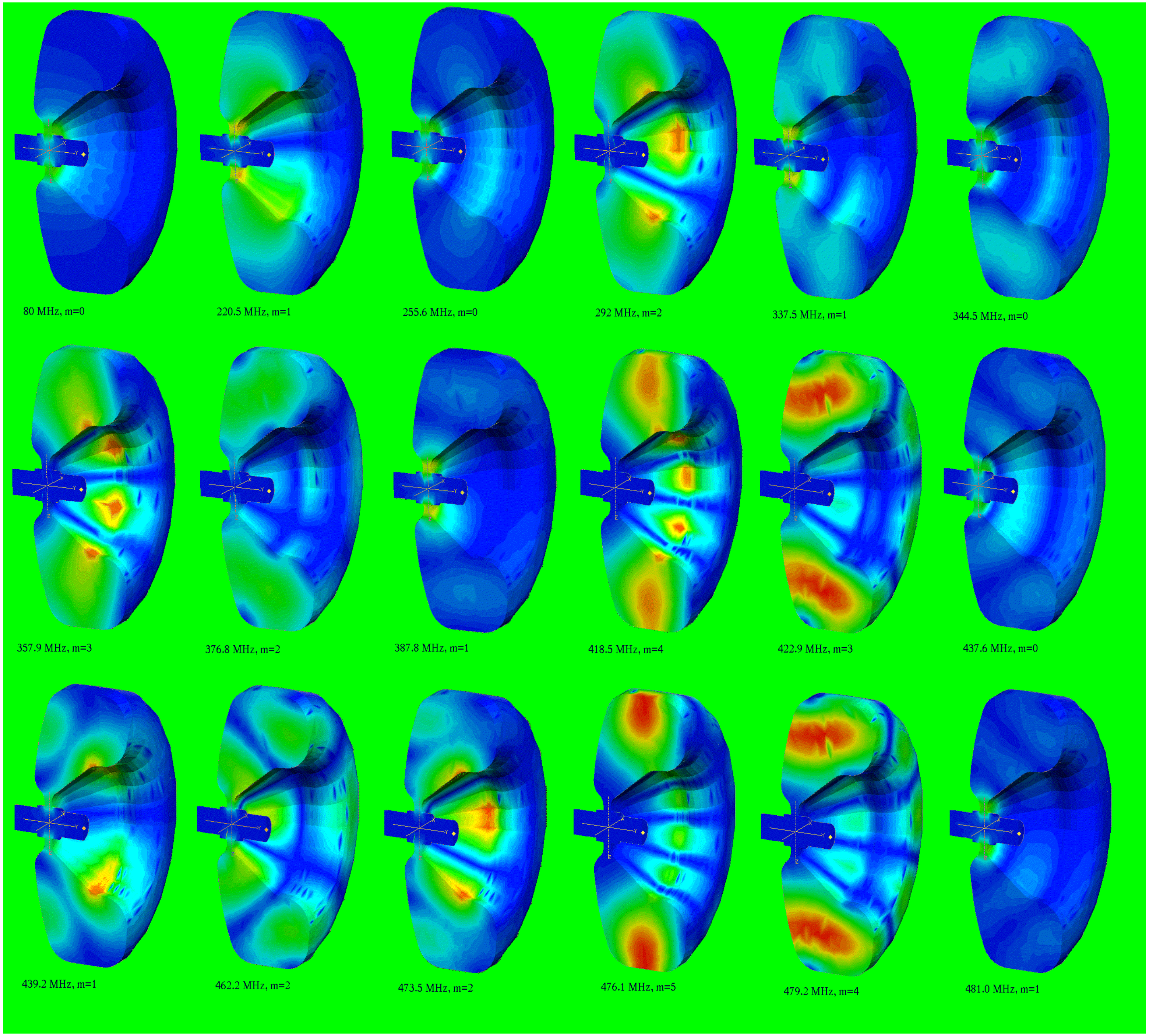


inductive (loop) coupling,
low self-inductance

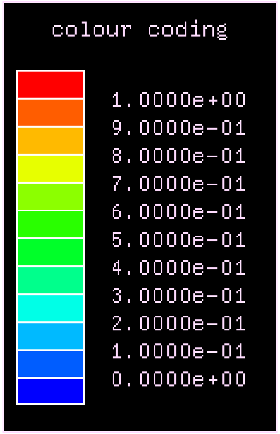
HOM's

Example shown:

CERN/PS 80 MHz cavity

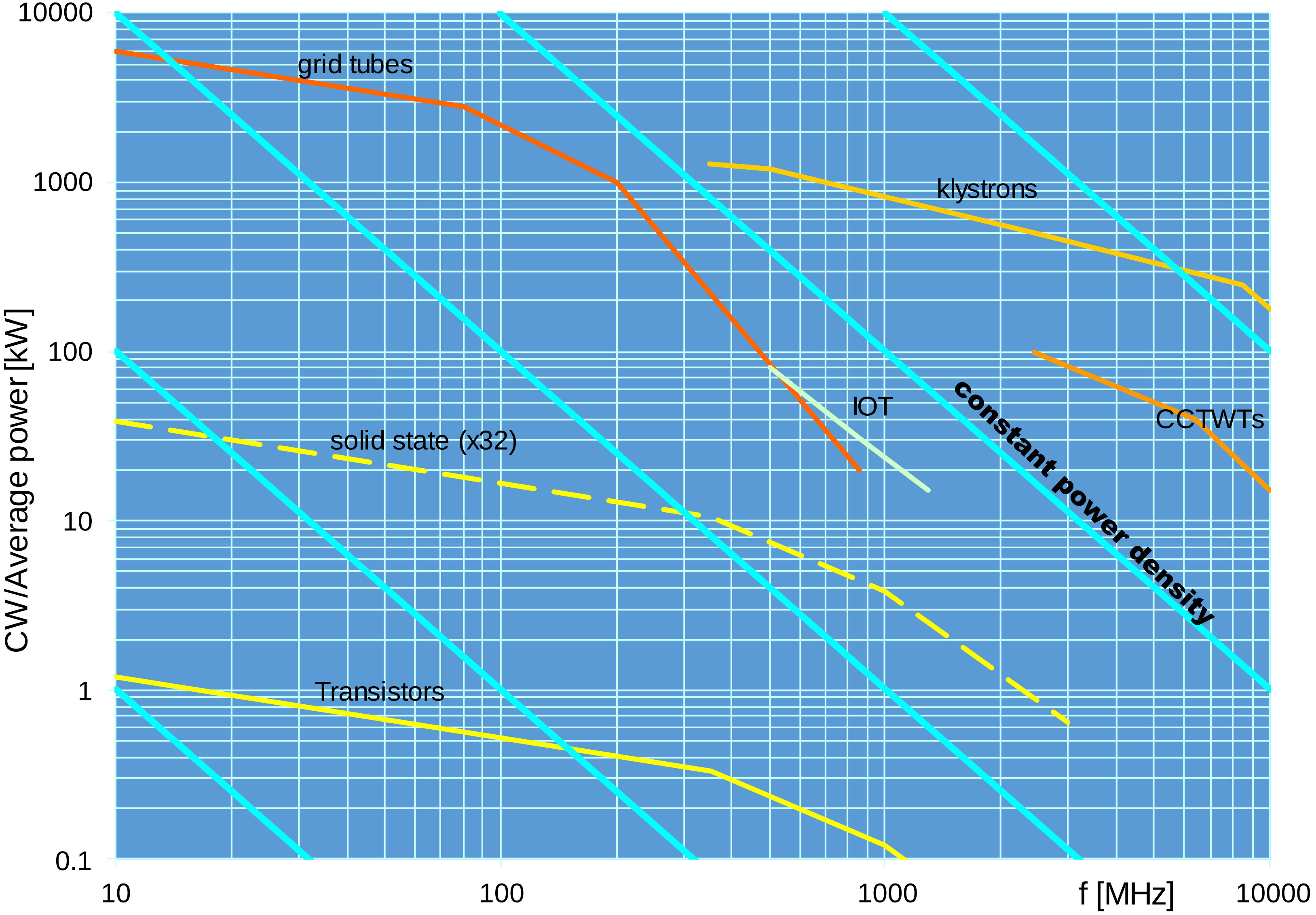


Colour coding: $|\vec{E}|$



RF power sources

Typical ranges (commercially available)

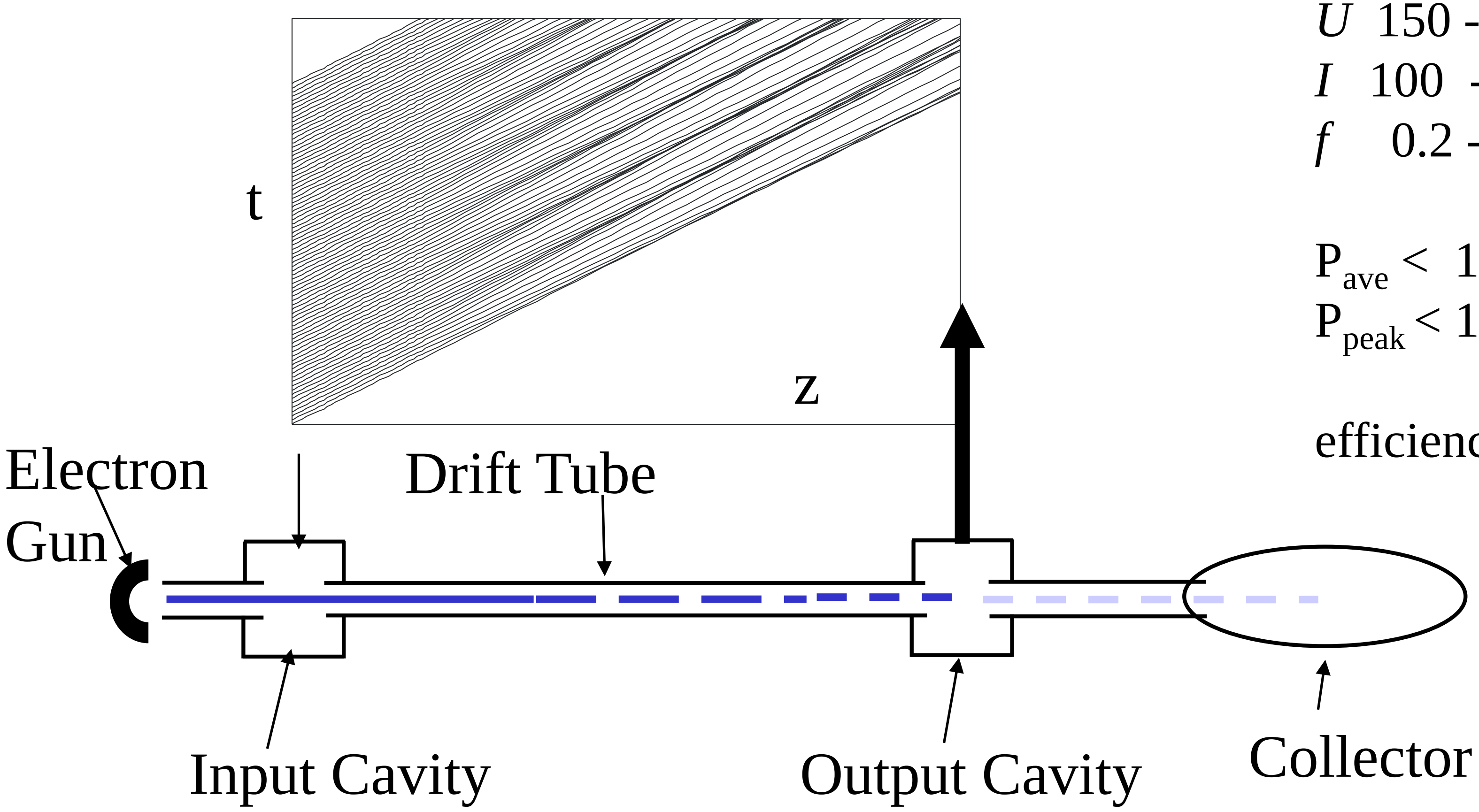


Klystrons

low-power RF signal at the design frequency excites input cavity

Velocity modulation of electron beam -> density modulation

Bunched beam excites output cavity



U 150 -500 kV

I 100 -500 A

f 0.2 -20 GHz

$P_{ave} < 1.5$ MW

$P_{peak} < 150$ MW

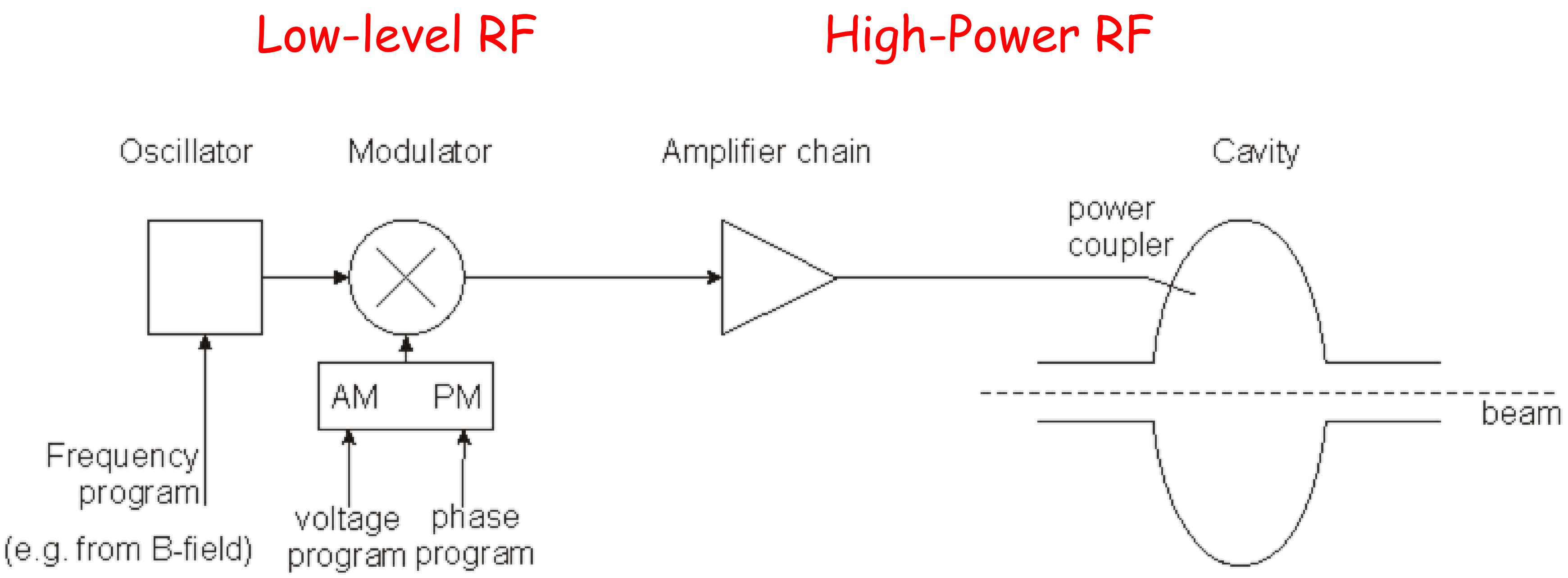
efficiency 40-70%

Klystrons

CERN CTF3 (LIL):
3 GHz, 45 MW,
4.5 μs, 50 Hz, η 45 %

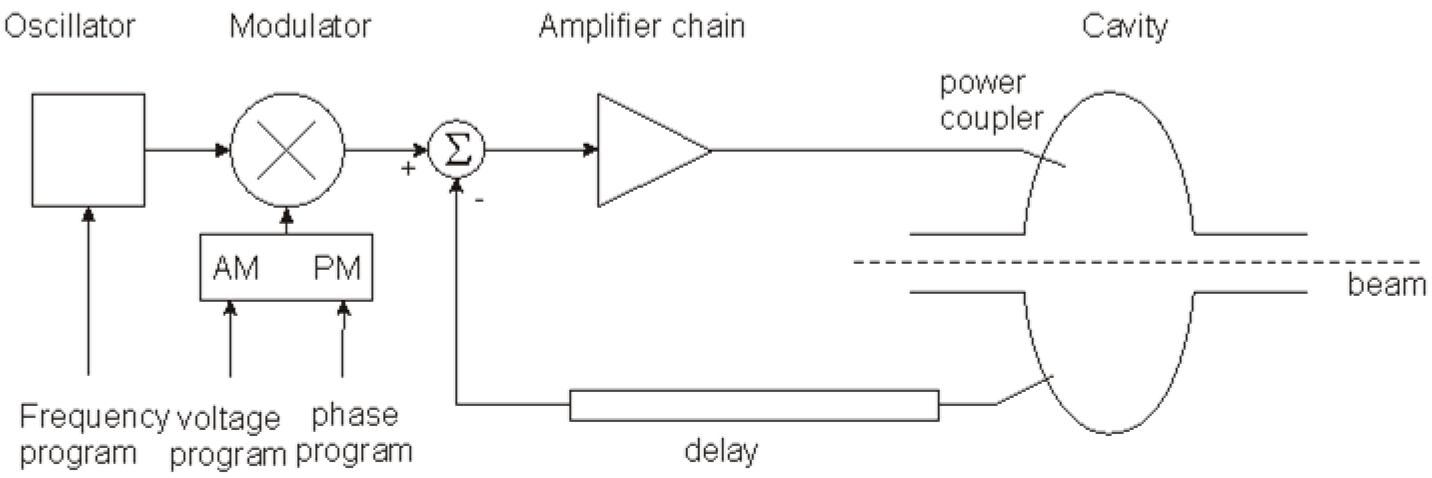
CERN LHC:
400 MHz, 300 kW,
CW, η 62 %

Minimal RF system (of a synchrotron)



- The frequency has to be controlled to follow the magnetic field such that the beam remains in the centre of the vacuum chamber.
- The voltage has to be controlled to allow for capture at injection, a correct bucket area during acceleration, matching before ejection; phase may have to be controlled for transition crossing and for synchronisation before ejection.

RF Feed-back loops

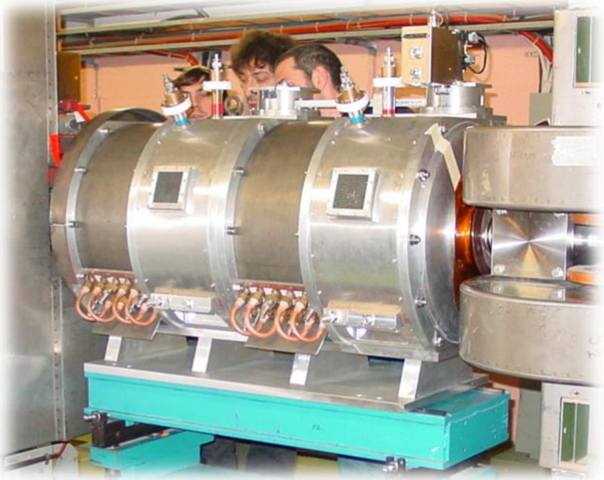


- Compares actual RF voltage and phase with desired and corrects.
- Limited by total group delay (path lengths) (some 100 ns).
- Works also to keep voltage at zero for strong beam loading, i.e. it reduces the beam impedance.

- Voltage control loop (AVC)
- Beam phase loop
- 1-turn feedback
- Radial loop (measure orbit and change f to keep beam centred)
- Synchronisation loop (to other machines at extraction)

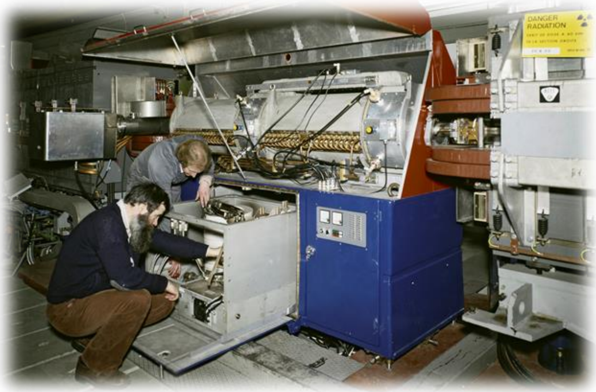
CERN PS RF Systems

10 MHz system, *h*=7…21

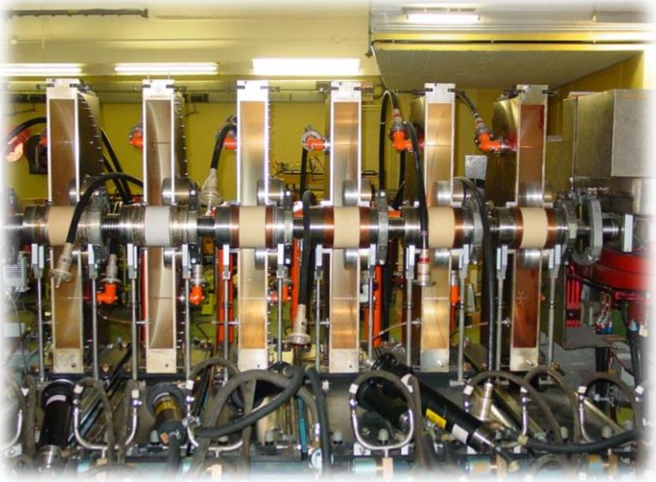


13/20 MHz system, *h*=28/42

40 MHz system, *h*=84



80 MHz system, *h*=168



200 MHz system

Acknowledgements

I would like to thank everyone for the material that I have used.

In particular (hope I don't forget anyone):

- Erk Jensen (from whom I inherited the course)
- Joël Le Duff
- Graeme Burt
- David Alesini
- Fu-Kwun Hwang and Lookang Lawrence Wee

…

Homework:

Try this in your bathtub!