RF Measurement Techniques I

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Learning Objectives

- **This is an introduction to RF measurement techniques**
  - **To support the practical training in the afternoons**
    - **Hands-on practical training with spectrum and network analyzers**
    - **Numerical simulations (eigenmode) in the computer lab**
  - **Give some basic understanding of RF measurement concepts**
    - **i.e. what so special to measure RF signals?!**
    - **Introduction to transmission-lines, characteristic impedance, forward and reflected waves, *Smith*-chart, S-parameters, signal detection, super-heterodyne concept, RF instruments, etc.**
- **"Pillbox" cavity resonator as example**
  - **Characterization in terms of eigen-modes, Q-values, and R/Q**
  - **How to measure those properties?**
- **A few notes on the beam coupling impedance**
  - **How to measure $Z_{\|}$ with a stretched-wire setup?**

The CAS 2019 RF Playground

Contents

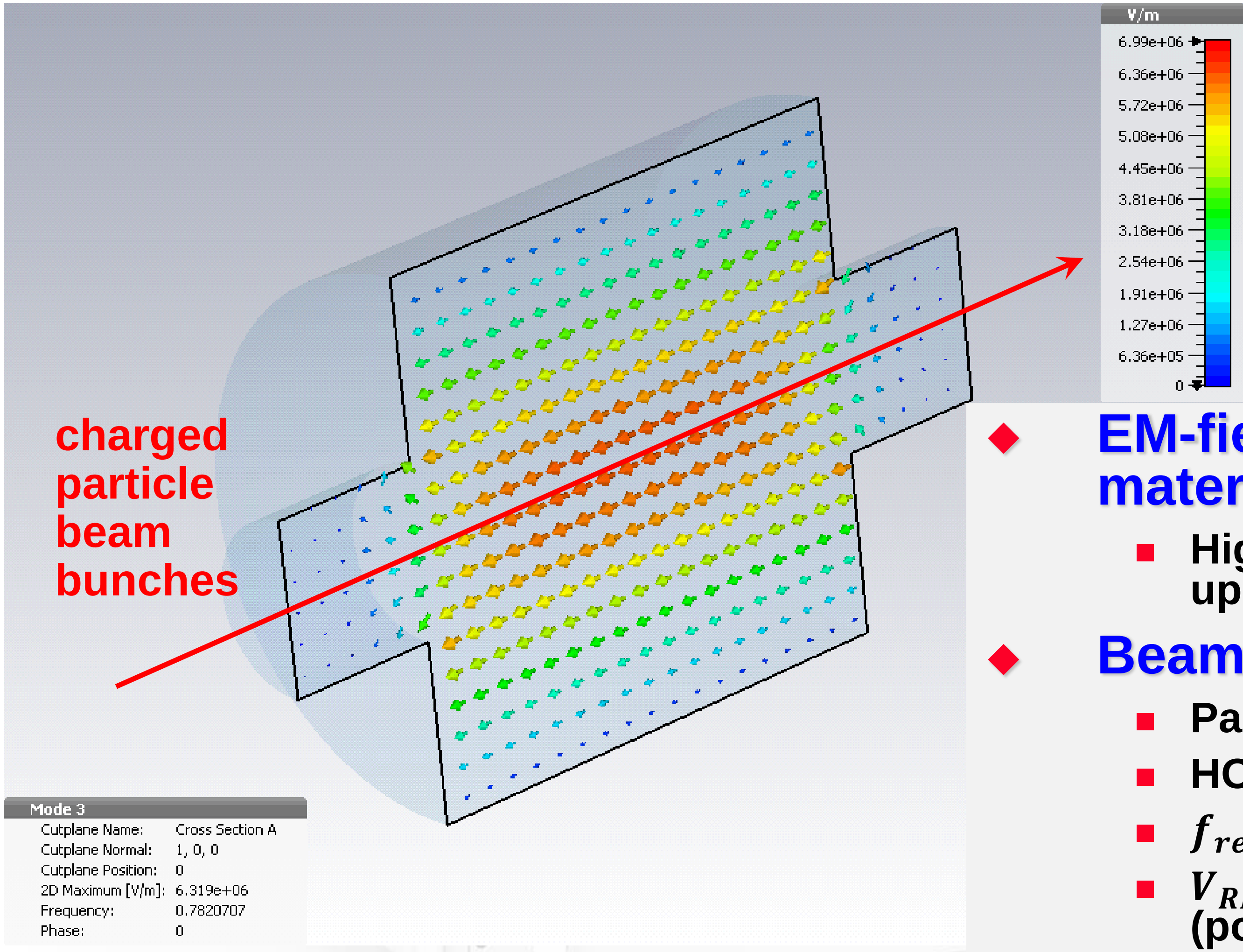
Part I

- Introduction
- RF measurement methods
- Beam and RF signals
- The super-heterodyne concept
- Transmission-lines
- Voltage Standing Wave Ratio (VSWR)
- The *Smith* chart and some examples

Part II

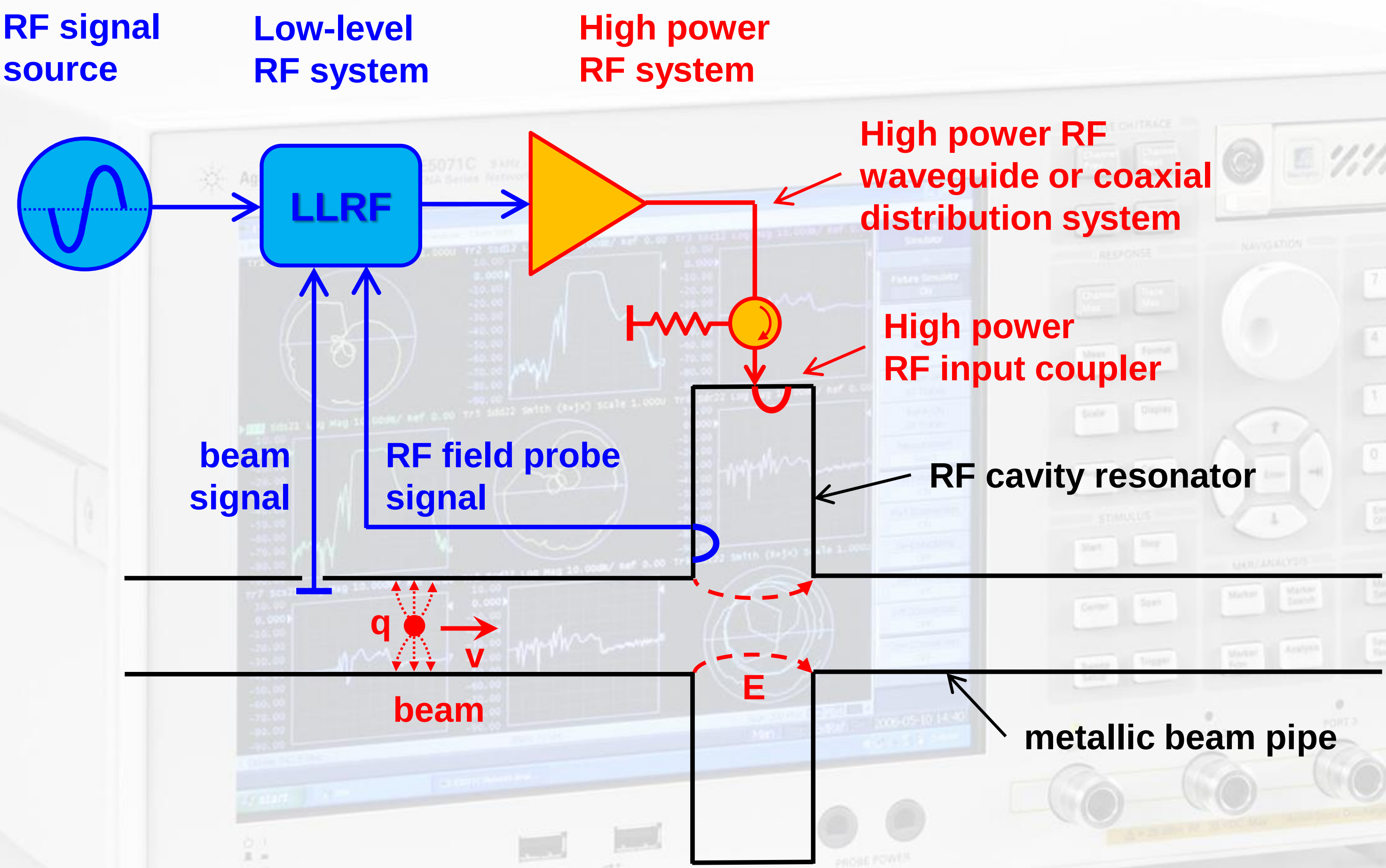
- Introduction to Scattering-parameters (S-parameters)
- The vector network analyzer (VNA)
- RF measurements on a ”pillbox” cavity
- Stretched-wire measurement of the beam coupling impedance
- Appendices

Introduction – A Cavity Resonator



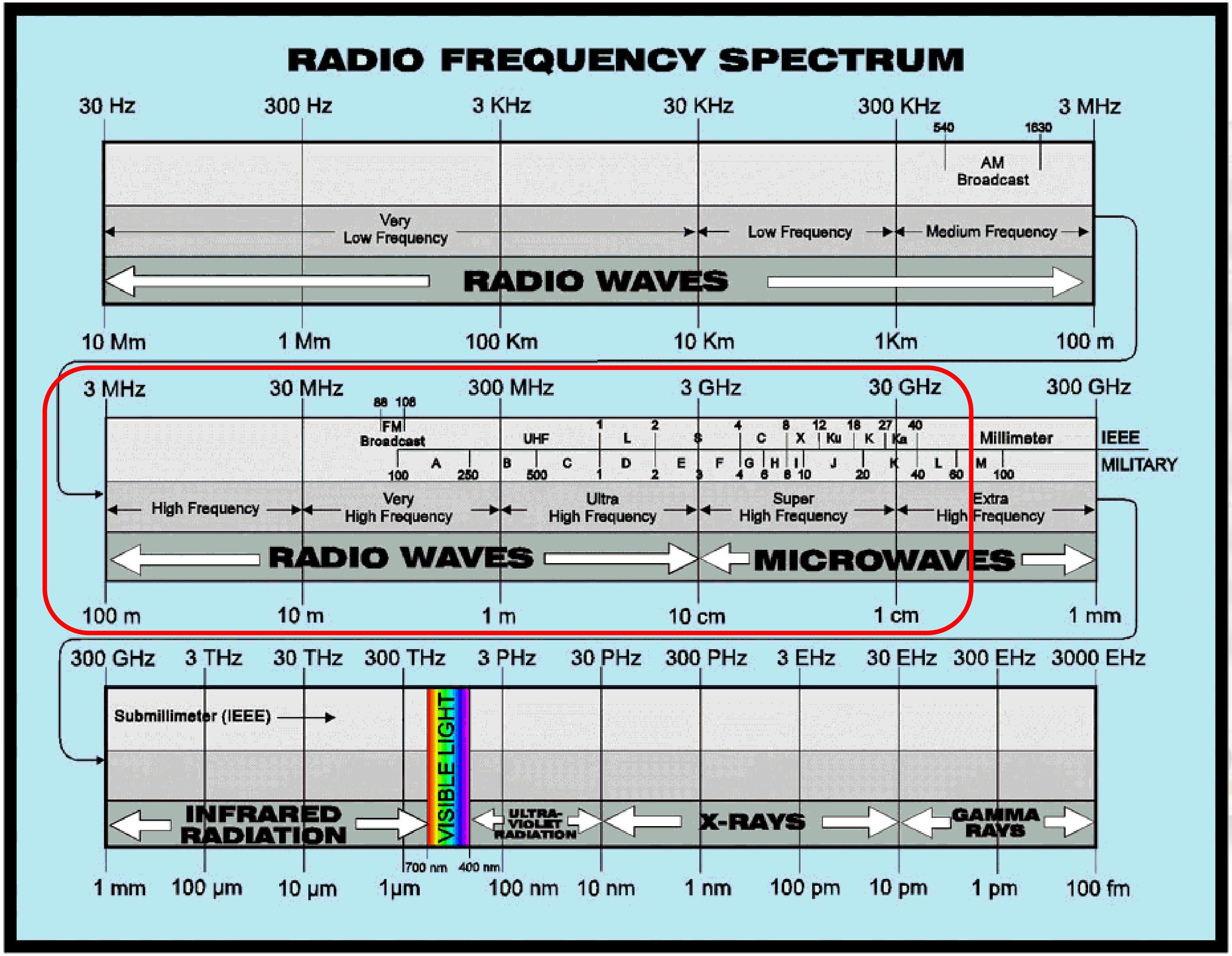
- **EM-fields, RF technology, material science, etc.**
  - **High acceleration gradient, up to 100MV/m**
- **Beam dynamics**
  - **Particle interaction with EM-fields**
  - **HOMs, wakefields**
  - f_{res}, **harmonic number** h
  - V_{RF} **defines a stable RF bucket (potential well)**
    - ***…and therefore*** f_s***, etc.***
  - **Transit time**

Introduction – A simple RF System



Things get a bit more complicated in the real world: pulsed power RF, multi-cell resonators or traveling wave structures, non-relativistic beams, HOM's, SRF, etc.

Introduction – What are Radio Frequencies?



Free space wavelength:

$$\lambda = \frac{c_0}{f}$$

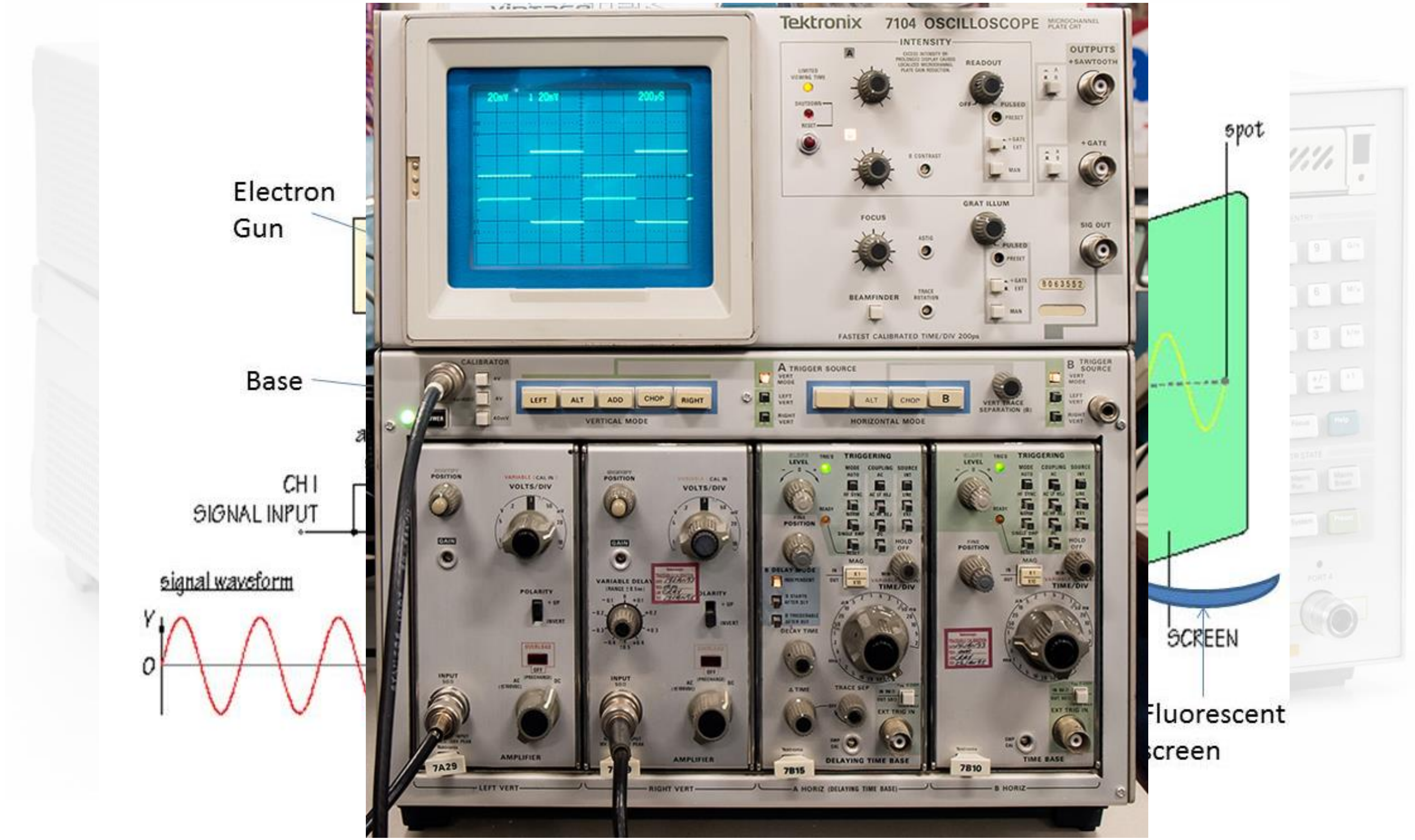
We care about RF concepts if the physical dimensions of an apparatus is $> \lambda/10$

RF Measurements Methods (1)

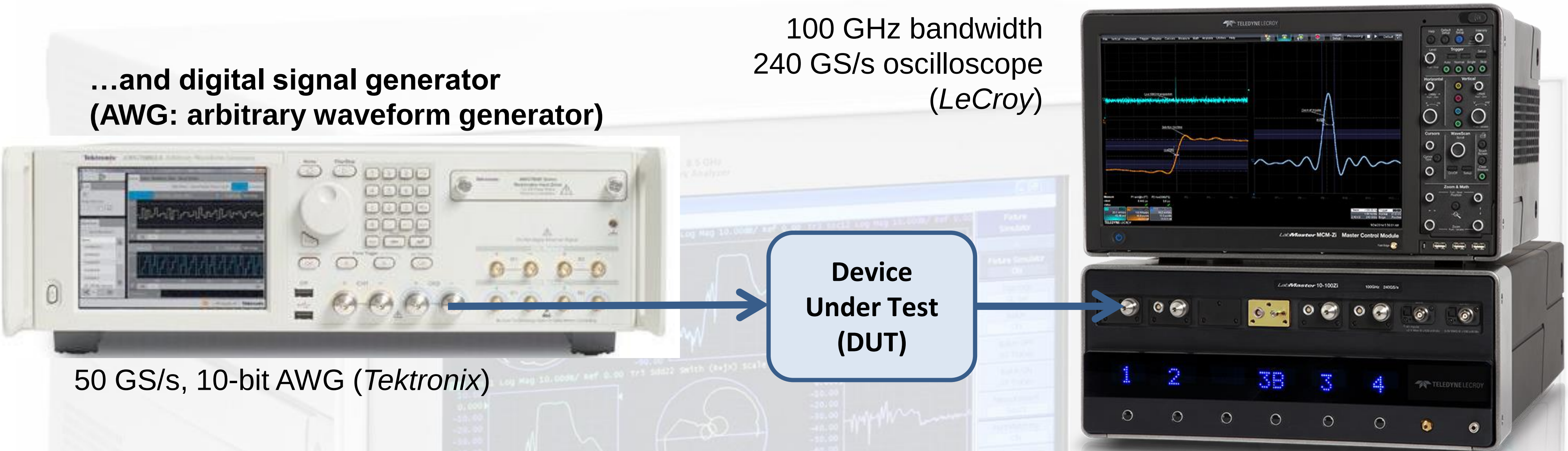
There are different options to observe RF signals
Here some typical measurement tools:

- **Oscilloscope: to observe signals in time domain**
  - **periodic signals**
  - **burst and transient signals with arbitrary waveforms**
  - **application: direct observation of signals from a beam pick-up, from a test generator, or from other sources**
  - **visualizes the shape of a waveform, etc.**
  - **limited performance for the evaluation of non-linear effects.**

Cathode Ray Tube (CRT) Oscilloscope



Today: Digital Storage Oscilloscope (DSO)



- **Signal processing based on fast ADCs and DACs**
  - **Similar "look and feel" as analog oscilloscopes, but better performance**
    - **8…12-bit multi-GS/s ADCs, still, be aware of aliasing effects!**
    - **Fast sampling oscilloscope require sufficient memory resources.**
- **AWG or pulse generator & digital oscilloscope: time domain (TD) test setup**
  - **Device under test (DUT) characterization and trouble shooting**
    - **Impulse, step, or arbitrary waveform (e.g. beam signal) as stimulus signal**
    - **High impedance probe for measurements on the printed circuit board (PCB)**

RF Measurements Methods (2)

- **Spectrum analyzer: to observe signals in frequency domain**
  - **sweeps in equidistant steps through a given frequency range**
  - **application: observation of spectrum from the beam, or from a signal generator or RF source, or the spectrum emitted from an antenna to locate EMI issues in the accelerator tunnel, etc.**
  - **Requires periodic signals**
  - **Large dynamic range!**

- **RF detection (Schottky) diode (RF power meter)**
  - **Supplies a rectified (video) output signal proportional to the RF signal level**
  - **Delivers no frequency or phase information, but operates over a very broad frequency range few MHz to many GHz, and up to 90 dB dynamic range.**

RF Measurements Methods (3)

Vector signal analyzer (VSA), sometimes called FFT analyzer

- Acquires the RF signal, after down-conversion to an intermediate (IF) signal, in time domain by fast sampling
- Further numerical treatment in digital signal processors (DSPs)
- Spectrum calculated using Fast Fourier Transform (FFT)
- Combines features of an oscilloscope and a spectrum analyzer: Signals can be observed directly in time or in frequency domain
- Contrary to the SA, also the spectrum of non-periodic signals and transients can be measured
- Application: Observation of tune sidebands, transient behavior of a phase locked loop, single pass beam signal spectrum, etc.
- Digital oscilloscopes and FFT analyzers share similar technologies, i.e. fast sampling and digital signal processing, and therefore can provide similar measurement options
  - The digital oscilloscope directly digitizes the RF signal
    → limited dynamic range, large instantaneous bandwidth
  - The FFT analyzer digitizes the downconverted IF signal
    → large dynamic range, limited instantaneous bandwidth

RF Measurements Methods (4)

Tools to characterize RF components and sub-systems:

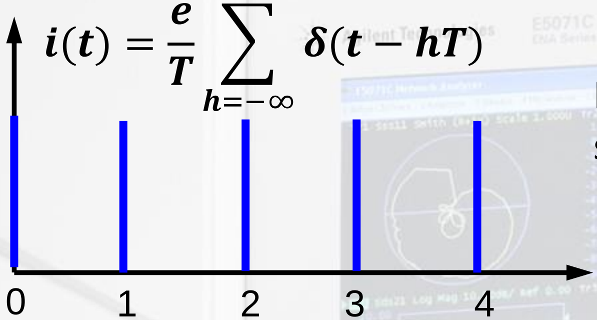
- **Coaxial (or waveguide) measurement transmission-line**
  - **For study and illustration purposes only – not anymore used in today's RF laboratory environment.**
- **Vector Network Analyzer (VNA)**
  - **Combines the functions of a vector spectrum analyzer (FFT analyzer), a RF sweep generator, and a S-parameter test set (directional coupler)**
  - **Excites a *Device Under Test* (DUT, e.g. circuit, antenna, amplifier, etc.) network at a given *Continuous Wave* (CW) frequency, and measures the response in magnitude and phase => determines the S-parameters**
    - **What are S-parameters?!**
  - **Covers a selectable frequency range by measuring step-by-step at subsequent frequency points (similar to the spectrum analyzer)**
  - **Applications: characterization of passive and active RF components, *Time Domain Reflectometry* (TDR) by Fourier transformation of the reflection response, etc.**
  - **The VNA is the most versatile and comprehensive tool in the RF laboratory!**

Beam Current Signals (1)

- **Single charge e in time and frequency domain**
  - **Circulating in a ring accelerator with** $T = \frac{1}{f_{rev}}$

$i(t)$

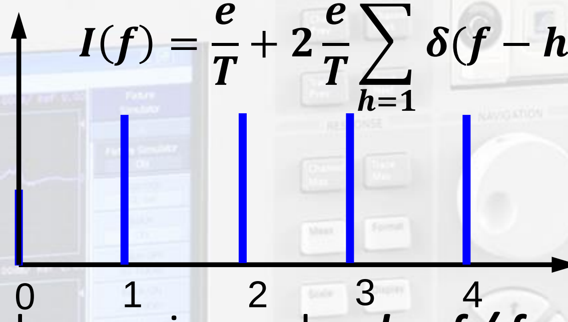
$$i(t) = \frac{e}{T} \sum_{h=-\infty}^{+\infty} \delta(t - hT)$$



Fourier trans. or spectrum analyzer

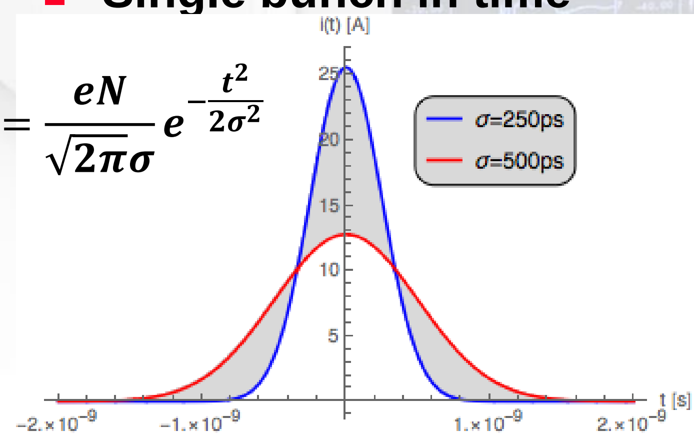
$I(f)$

$$I(f) = \frac{e}{T} + 2\frac{e}{T} \sum_{h=1}^{\infty} \delta(f - hf_{rev})$$

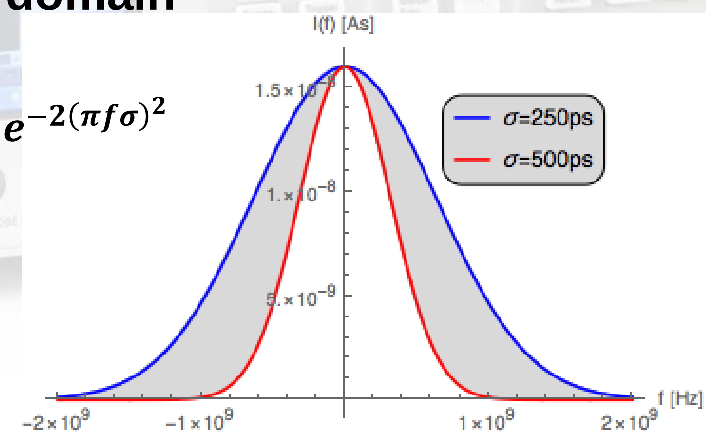


- **Many particles (protons, $N = 10^{11}$) in a Gaussian distribution**
  - **Single bunch in time and frequency domain**

$$i(t) = \frac{eN}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}}$$



$$I(f) = \frac{eN}{2\sqrt{2\pi\sigma^2}} e^{-2(\pi f \sigma)^2}$$



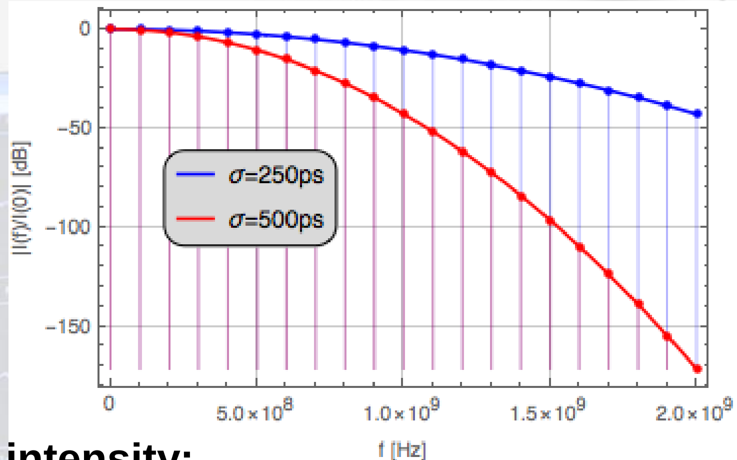
Beam Current Signals (2)

- **Normalized representation in logarithmic amplitude scale**
  - **Typical magnitude spectrum,
    as it would be observed on a spectrum analyzer**

$$\left| {}^{I(f)}\!\big/_{I(0)} \right|_{dB} = 20 \log_{10} \left[e^{-2(\pi f \sigma)^2} \right]$$

$$I(0) = \frac{eN}{2\sqrt{2\pi\sigma^2}}$$

revolution harmonics, here
indicated at: $\frac{1}{T} = f_{rev} = 100\ MHz$



  - **Spectrum of repetitive bunches of same intensity:
    Fourier series expansion**

$$i(t) = \frac{eN}{T} + 2\frac{eN}{T}\sum_{h=1}^{\infty} e^{\frac{(h\omega\sigma)^2}{2}} \cos(h\omega t) \qquad \omega = 2\pi f$$

- **Beam bunches have different distribution functions and bunch length**
  - **Electron bunches are typical 100…1000x shorter than proton bunches**
  - **Ion bunches can be 10…1000 longer than relativistic proton bunches**
  - **Longitudinal particle distributions: Gaussian, parabolic, Cos², Square, etc.**

"dB" [*dee-bee*] or not to be…

- **dezi-Bel: 1 dB = 0.1 B (Bel)**
  - **Logarithmic scaling to compare large, e.g. power ratios:**
  - **or large ratios of other quantities, e.g.:**

$$P_{dB} = 10 \log_{10}\left(\frac{P_1}{P_2}\right)$$

$$V_{dB} = 20 \log_{10}\left(\frac{V_1}{V_2}\right)$$

$$I_{dB} = 20 \log_{10}\left(\frac{I_1}{I_2}\right)$$

dB ratio	P_1/P_2	V_1/V_2
n x 10 dB	10^n	$10^{n/2}$
40 dB	10000	100
20 dB	100	10
10 dB	10	~3.16
6 dB	~4	~2
3 dB	~2	~1.41
0 dB	**1**	**1**
-3 dB	~0.5	~0.71
-20 dB	0.01	0.1

$$\frac{P_1}{P_2} = 10^{\left(\frac{P_{dB}}{10}\right)} \qquad \frac{V_1}{V_2} = 10^{\left(\frac{V_{dB}}{20}\right)}$$

The 3 dB ratio (half power) is a common specification for the bandwidth

"dBm" is not "dB"

- ***dBm*** **is a logarithmic power unit**
  - **based on dB and** $P_{ref} = 1\,mW$
- ***dBm*** **can also be used as logarithmic voltage unit**
  - **e.g. for** $Z_0 = 50\,\Omega$: $V_{ref} = 0.2236\,V$

$$P_{dBm} = 10\log_{10}\left(\frac{P}{P_{ref}}\right)$$

$$V_{dBm} = 20\log_{10}\left(\frac{V}{V_{ref}}\right)$$

dBm	P	V (RMS)
30 dBm	1 W	7.07 V
20 dBm	100 mW	2.24 V
10 dBm	10 mW	707 mV
6 dBm	4.0 mW	446 mV
0 dBm	**1.0 mW**	**224 mV**
-20 dBm	10 µW	22.4 mV
-60 dBm	1.0 nW	224 µV
-120 dBm	1.0 fW	224 nV
- 174 dBm	4.0e-21 W	0.446 nV

$$P = P_{ref}10^{\left(\frac{P_{dBm}}{10}\right)}$$

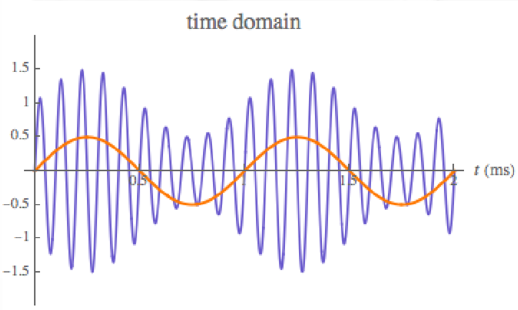
$$V = V_{ref}10^{\left(\frac{V_{dBm}}{20}\right)}$$

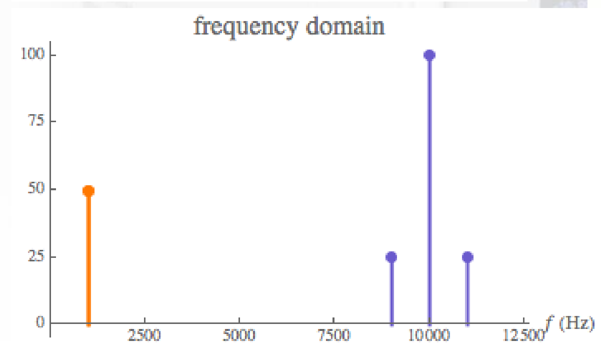
Noise power in a bandwidth *B = 1 Hz* at room temperature

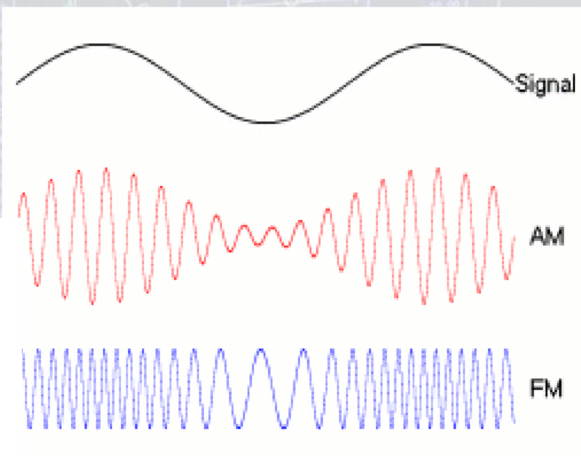
RF Signals & Modulation

- Beam current signals: transient (pulse-like) signals
- RF signals: Sinusoidal signals (CW: continuous wave)
  - High frequency carrier modulated with low frequency information
  - In ring accelerators:

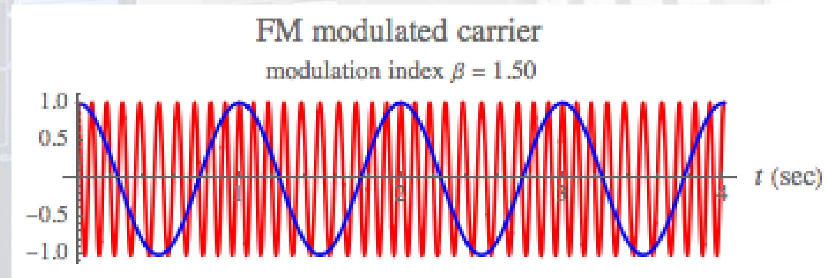
AM: Betatron oscillations

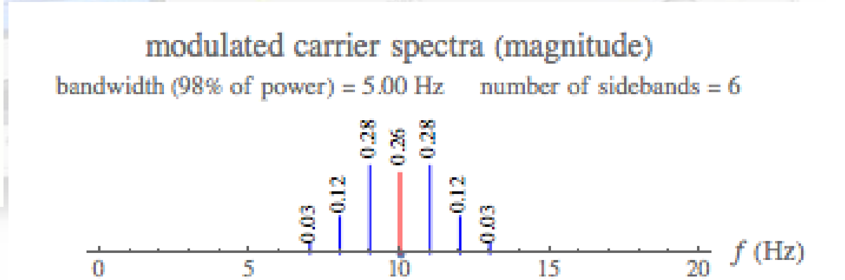




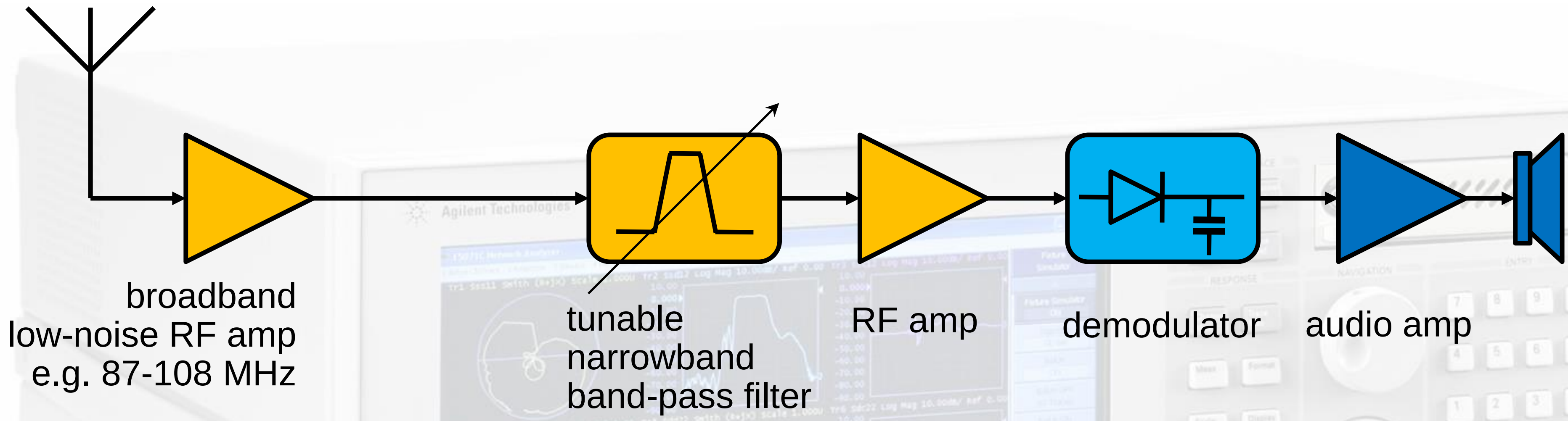


FM: Synchrotron oscillations





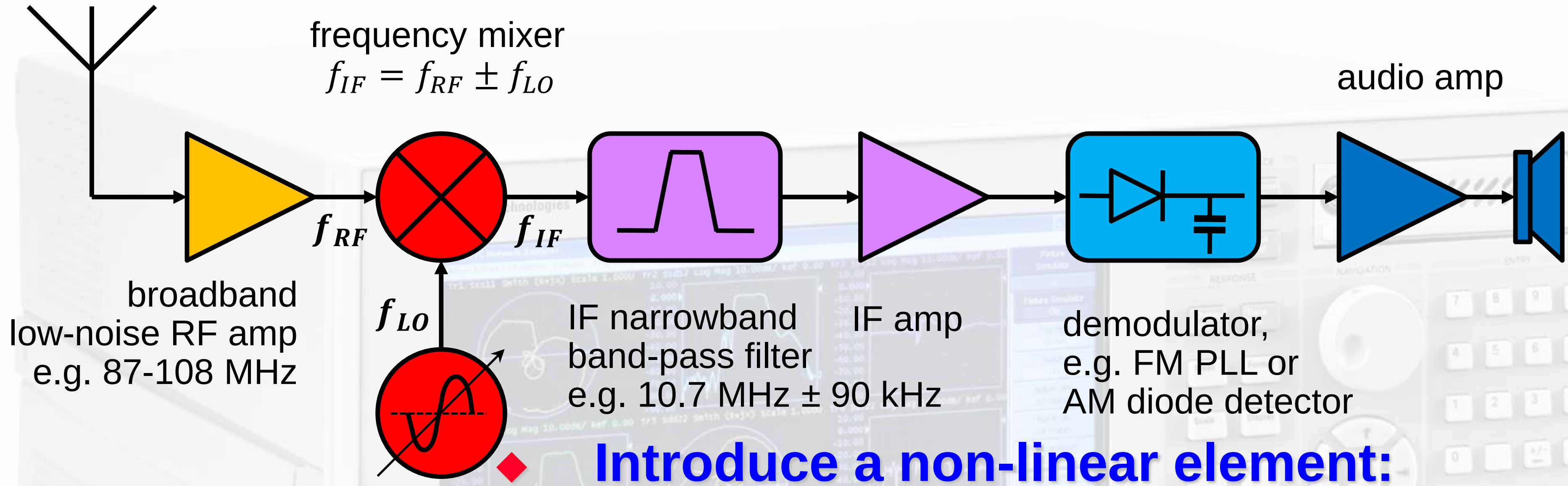
A (too) simple Radio Receiver



- **...or: How does a "traditional" analog radio works?**
  - **It was, and still is, difficult to make precisely tunable narrowband, band-pass filters for high frequencies (~100 MHz)!!**
  - **high frequency low-noise amplifiers are expensive!**
  - **high frequency demodulators are not trivial.**

  - **direct detection of radio and RF signals is challenging!**

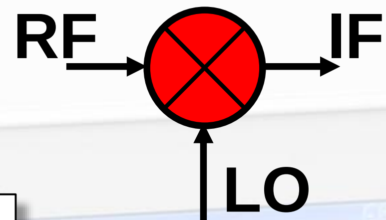
The Super-Heterodyne Receiver



- **Introduce a non-linear element: the (frequency) mixer!**
  - **"down-convert" the RF band to a fixed "intermediate" frequency (IF)** $f_{IF} = f_{RF} \pm f_{LO}$
  - **requires a tunable local oscillator (LO)**
  - **well manageable IF section:**
    - **narrowband band-pass filter(s) (BPF) and amplifier(s)**
  - **RF telecommunication standard**
  - **Often multiple mixing stages are used in modern RF instruments, e.g. spectrum and network analyzers**

The RF Mixer as Downconverter

$y_{RF}(t) = A_{RF}\sin(\omega_{RF}t + \varphi_{RF})$

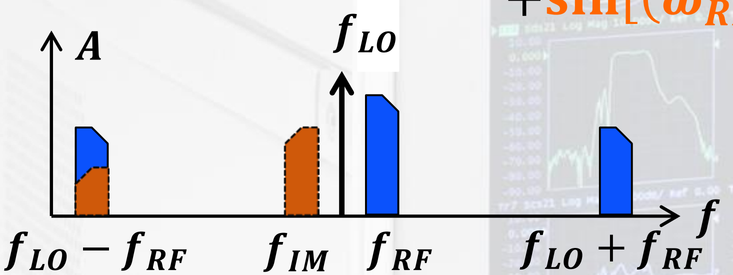


$y_{IF}(t) = y_{RF}(t)\,y_{LO}(t)$

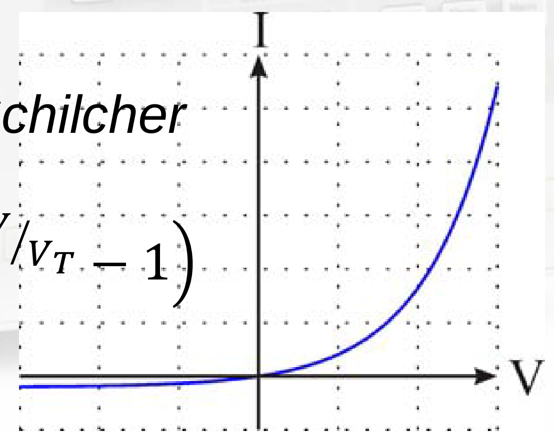
$y_{LO}(t) = A_{LO}\sin(\omega_{LO}t + \varphi_{LO})$

- **Ideal mixer:** $f_{IF} = f_{RF} \pm f_{LO}$

$$y_{IF}(t) = \frac{1}{2}A_{LO}A_{RF}\{\sin[(\omega_{RF} - \omega_{LO})t + (\varphi_{RF} - \varphi_{LO})] \quad \text{upper sideband}$$
$$+\sin[(\omega_{RF} + \omega_{LO})t + (\varphi_{RF} + \varphi_{LO})]\} \quad \text{lower sideband}$$



$I = f(V)$ of a *Schottky* diode



courtesy *T. Schilcher*

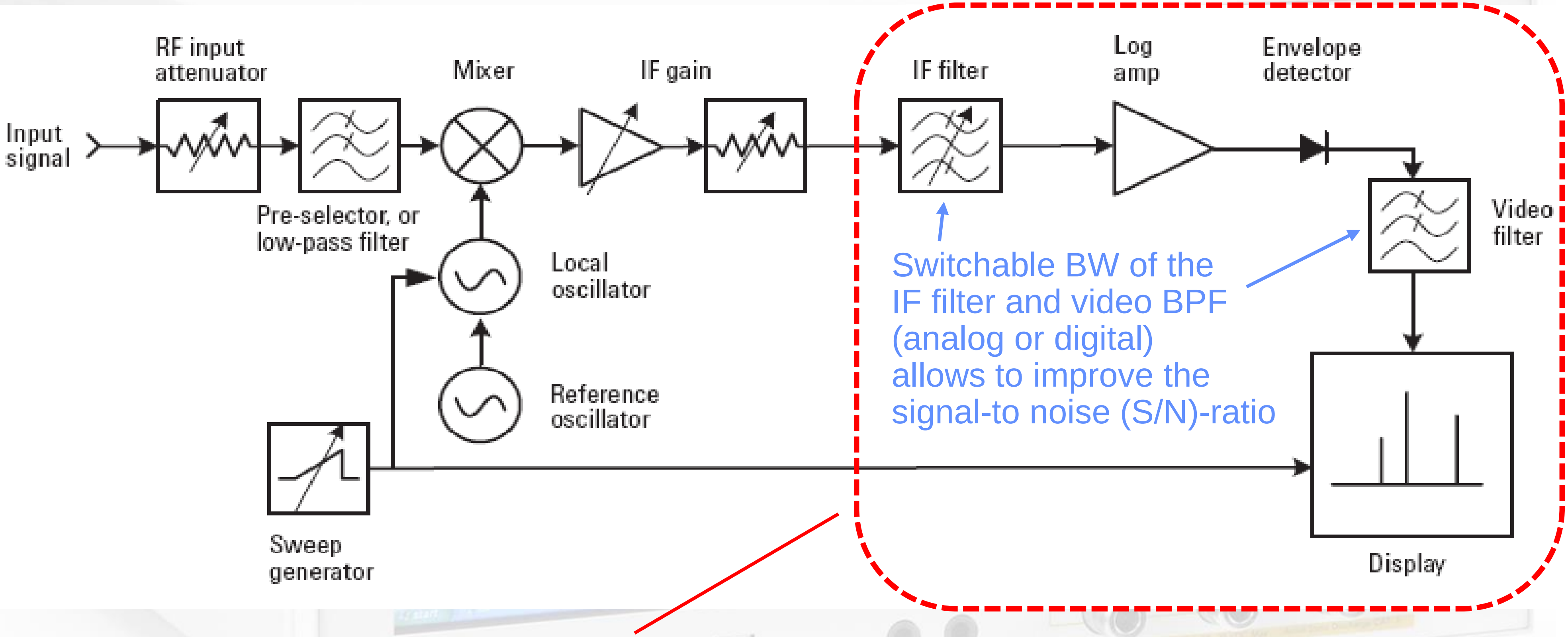
$$I = I_0\left(e^{V/V_T} - 1\right)$$

- **Frequency conversion**
  - **$f_{RF} \neq f_{LO}$: heterodyne receiver**
  - **$f_{RF} = f_{LO}$: homodyne, demodulator**
- **Real-world mixer:** $f_{IF} = mf_{RF} \pm nf_{LO}$
  - **Image frequency:** $f_{IM} = f_{LO} - f_{IF}$

$$\Delta I = I_0 e^{V/V_T}\left[\frac{\Delta V}{V_T} + \frac{1}{2}\left(\frac{\Delta V}{V_T}\right)^2 + \frac{1}{6}\left(\frac{\Delta V}{V_T}\right)^3 + \cdots\right]$$

Simplified Spectrum Analyzer

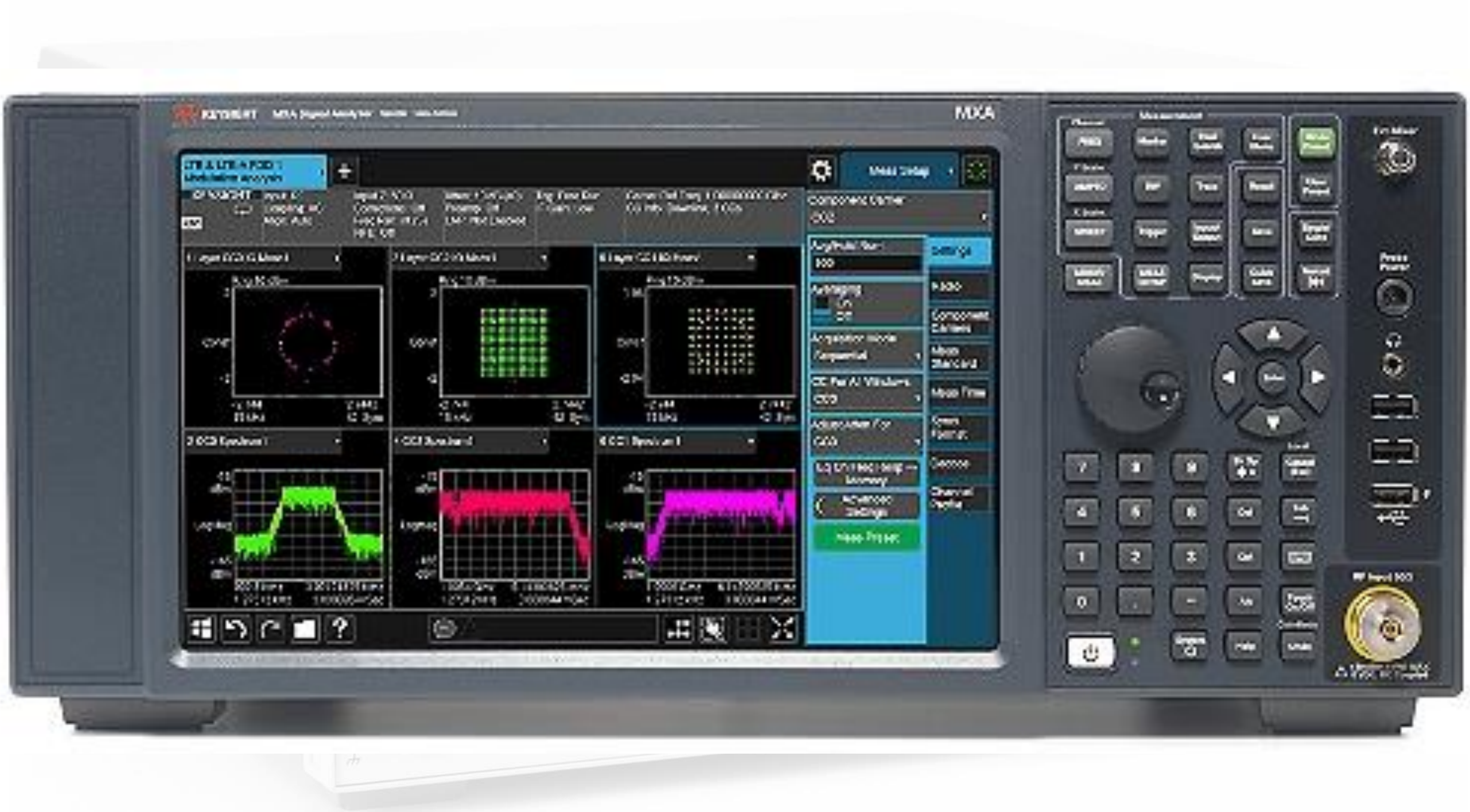
- based on the super-heterodyne principle



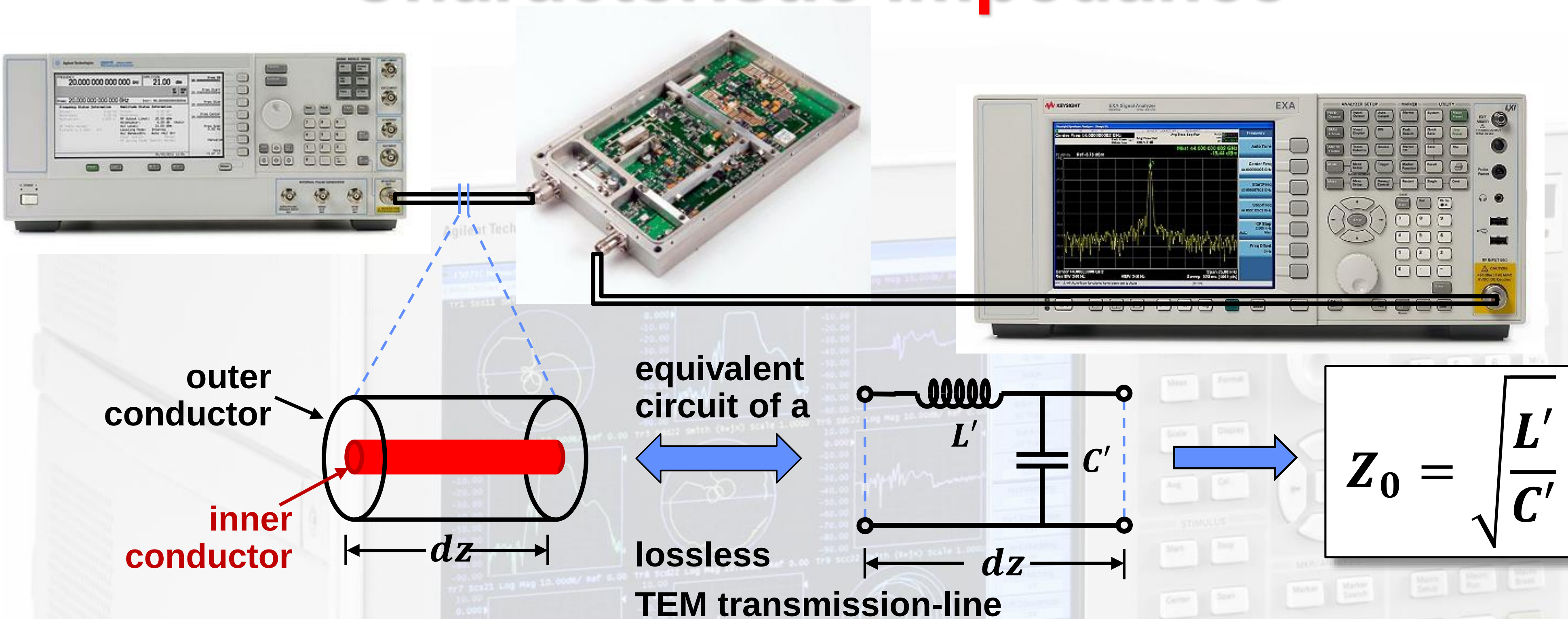
Today, the IF, demodulation, video and display sections of a spectrum analyzer are realized digitally

- **Requires an analog-digital converter (ADC) with sufficient dynamic range**

Modern Spectrum (RF Signal) Analyzer



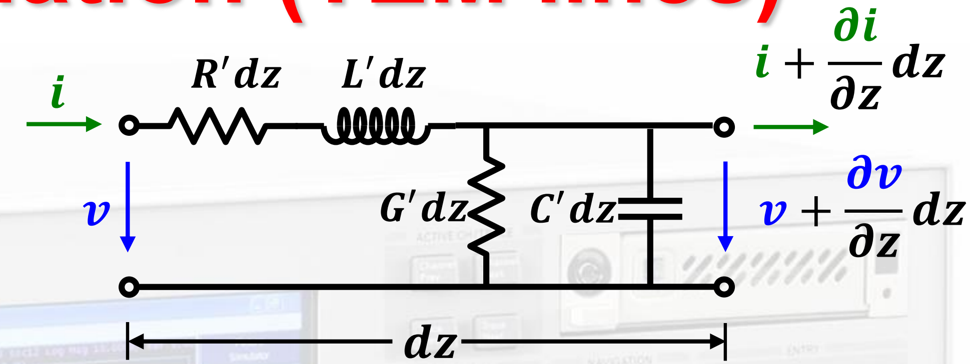
Characteristic Impedance



- **The reference impedance Z_0 in a RF system is defined by the characteristic impedance of the interconnect cables**
  - often coaxial cables of $Z_0 = 50\Omega$ (compromise: high voltage / high power handling)
- **The characteristic impedance of a TEM transmission-line is defined by the cross-section geometry**
  - The ratio of H- and E-field is represented by L' [H/m] and C' [F/m] in the equivalent circuit of a transmission-line segment of length dz
  - The characteristic impedance Z_0 has the unit Ohm [Ω]

Telegrapher's Equation (TEM lines)

A more general approach:



$$\frac{\partial v(z,t)}{\partial z} = -\left(R' + L' \frac{\partial}{\partial t}\right) i(z,t)$$

$$\frac{\partial i(z,t)}{\partial z} = -\left(G' + C' \frac{\partial}{\partial t}\right) v(z,t)$$

in steady state:

$$\frac{dV}{dz} = -(R' + j\omega L')I$$

$$\frac{dI}{dz} = -(G' + j\omega C')V$$

$$\frac{d^2V}{dz^2} = \gamma^2 V$$

voltage and current along a transmission-line:

$$V(z) = V_0 \cosh \gamma z - Z_0 I_0 \sinh \gamma z$$

$$I(z) = I_0 \cosh \gamma z - {V_0}/{Z_0} \sinh \gamma z$$

V_0, I_0: voltage and current at the beginning of the line ($z = 0$)

propagation constant — attenuation constant, phase constant

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta$$

characteristic impedance

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

wave number

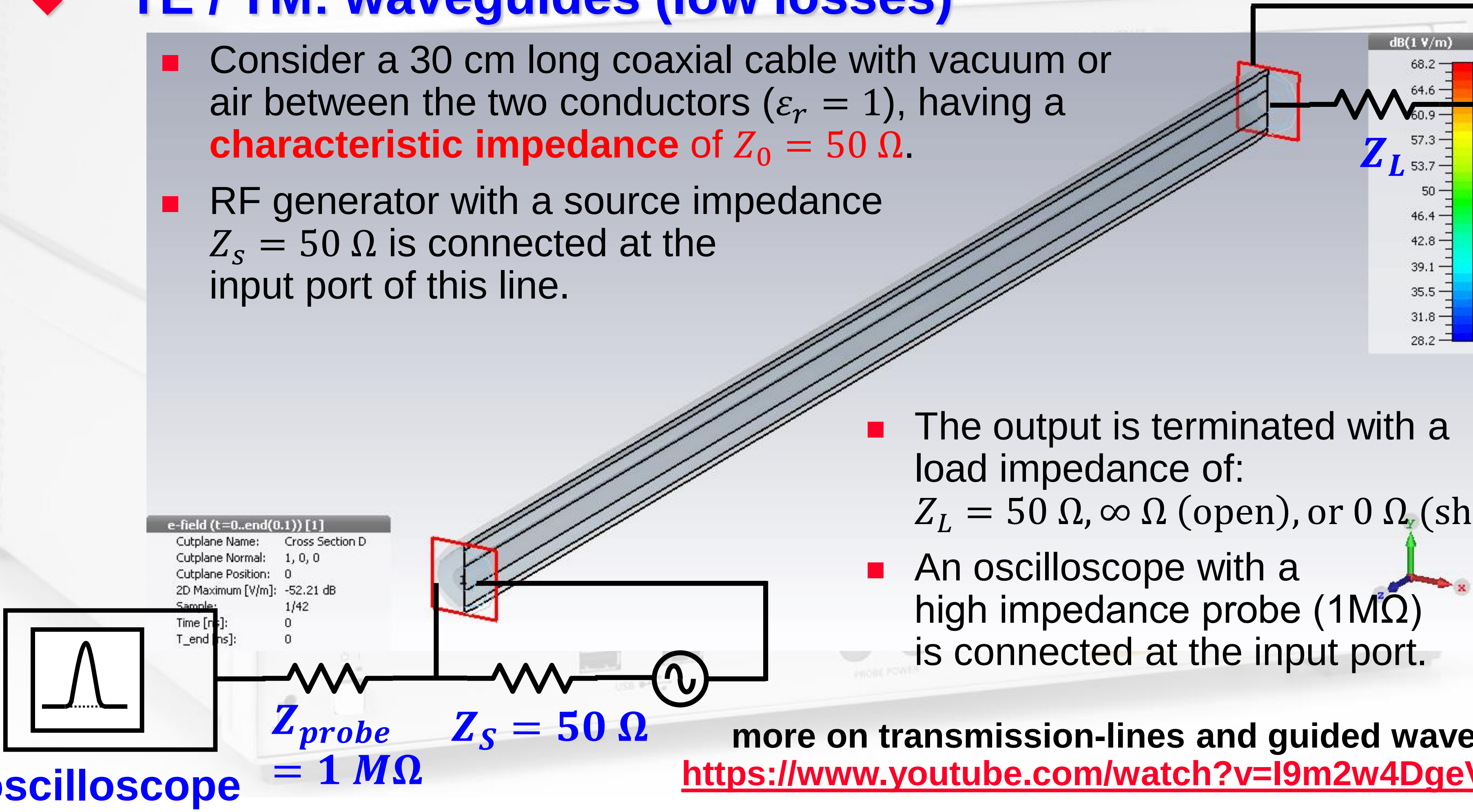
$$k = \frac{2\pi}{\lambda} = \beta$$

phase velocity

$$v_p = \frac{\lambda}{T} = \frac{\omega}{\beta}$$

Transmission-lines in Time Domain (1)

- **TEM: coaxial cables, striplines, micro-striplines, etc.**
- **TE / TM: waveguides (low losses)**

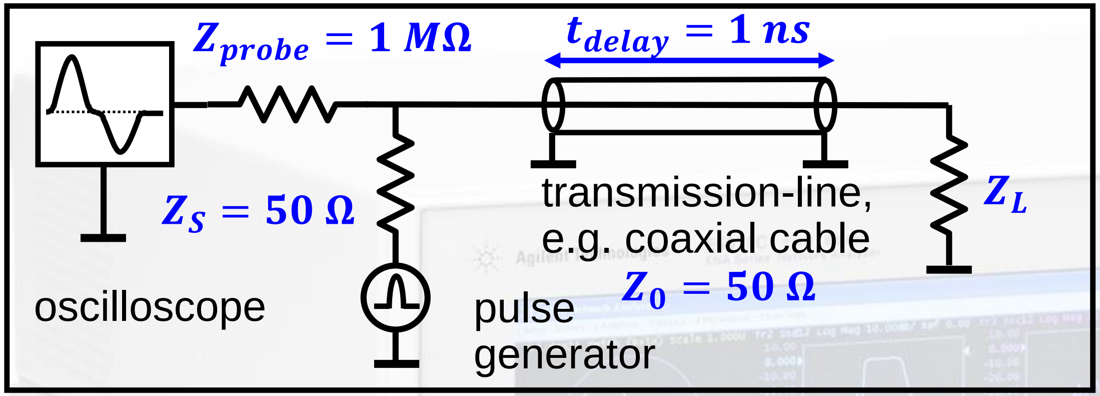


- Consider a 30 cm long coaxial cable with vacuum or air between the two conductors ($\varepsilon_r = 1$), having a **characteristic impedance** of $Z_0 = 50\ \Omega$.
- RF generator with a source impedance $Z_s = 50\ \Omega$ is connected at the input port of this line.
- The output is terminated with a load impedance of: $Z_L = 50\ \Omega, \infty\ \Omega\ \text{(open)}, \text{or}\ 0\ \Omega\ \text{(short)}$
- An oscilloscope with a high impedance probe (1MΩ) is connected at the input port.

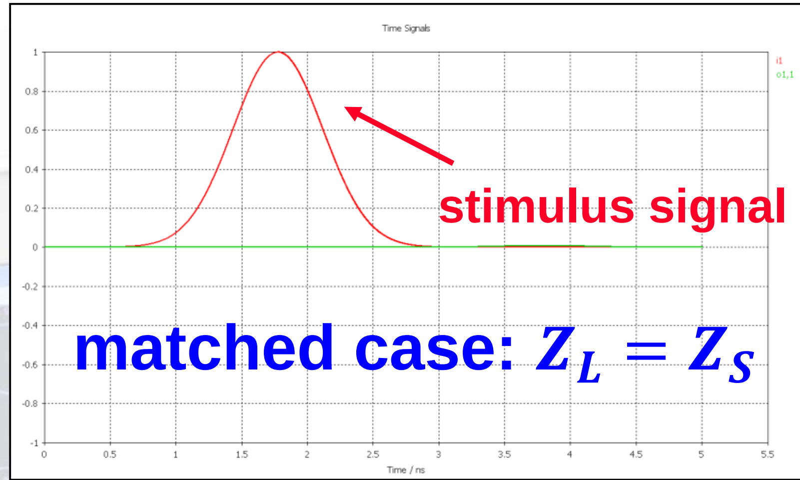
more on transmission-lines and guided waves:
https://www.youtube.com/watch?v=I9m2w4DgeVk

https://www.youtube.com/watch?v=DovunOxlY1k&t=38s

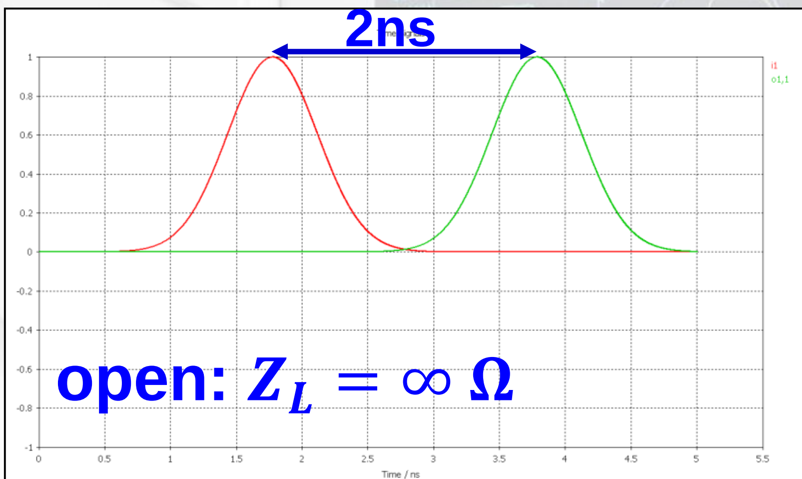
Transmission-lines in Time Domain (2)



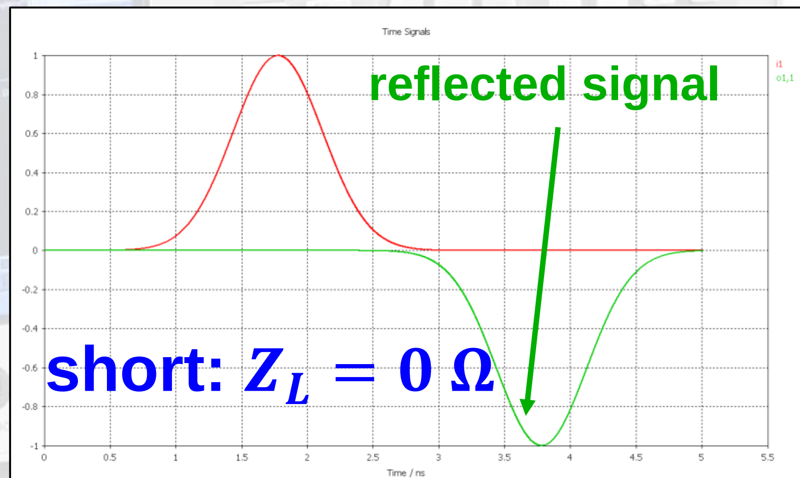
Simple time domain reflectometry (TDR) setup



No reflection

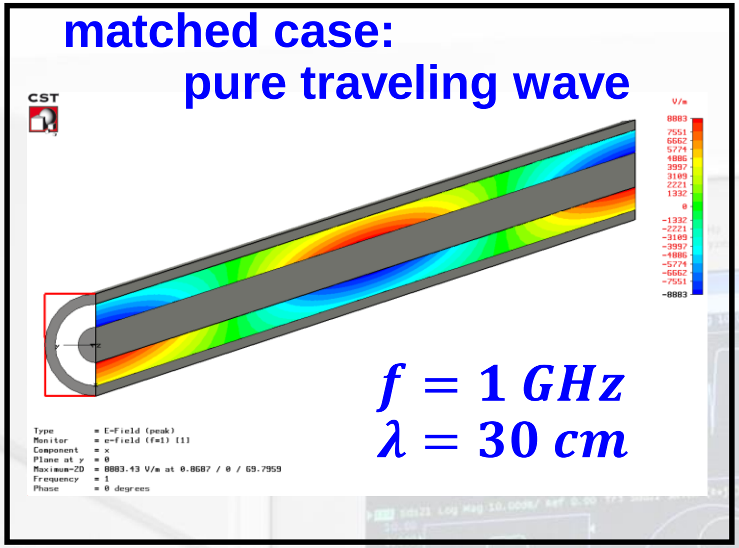


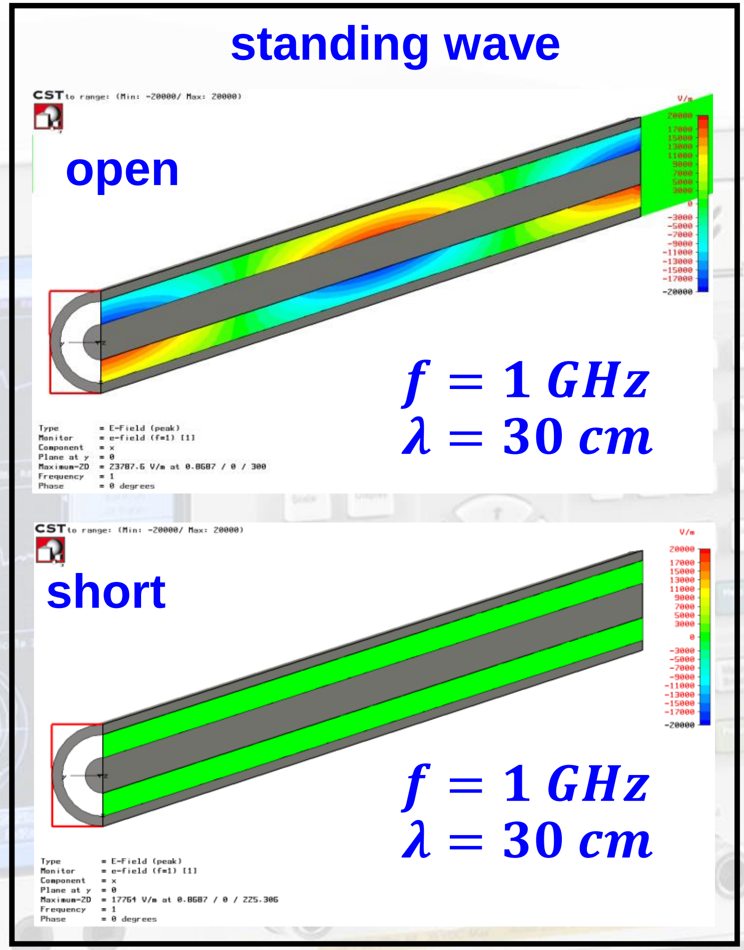
Total reflection:
reflected signal same polarity
delay: $2\ t_{delay} = 2\ ns$



Total reflection:
reflected signal inverse polarity

Transmission-lines in Frequency Domain





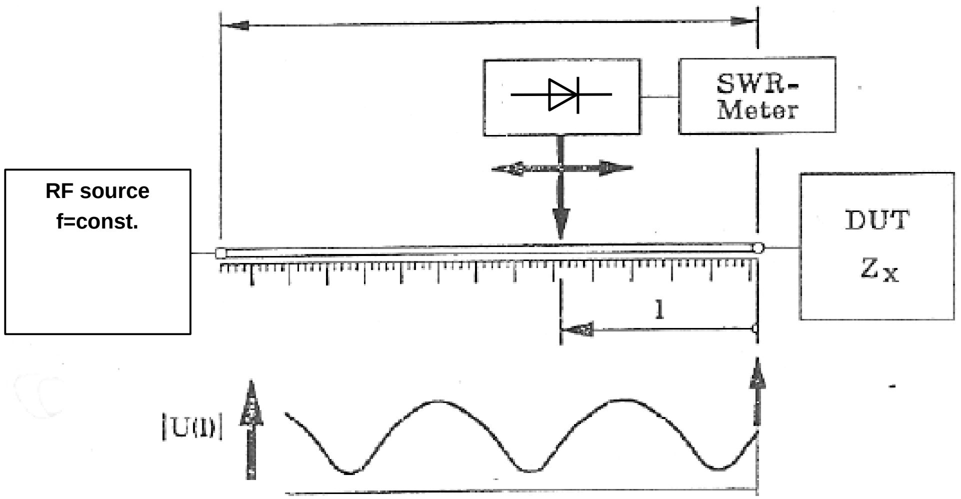
- **Standing and traveling waves:**
  - **The patterns for the short and open case are equal; only the phase is opposite, which correspond to different position of nodes.**
  - **In case of perfect matching:**
    - **traveling wave only.**
  - **Otherwise:**
    - **mixture of traveling and standing waves.**

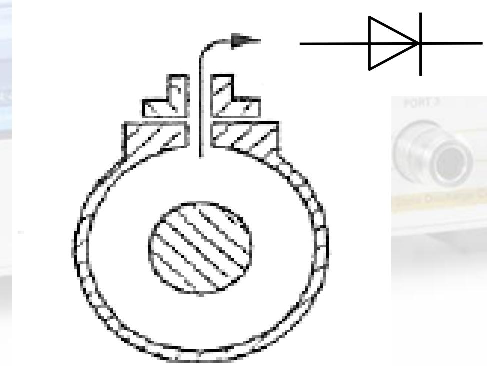
Caution: the color coding corresponds to the radial electric field strength – these are not scalar equipotential lines, which are anyway not defined for time varying fields

Voltage Standing Wave Ratio VSWR (1)

- **On a transmission-line (single frequency, CW):**
  - Superposition of forward *a* (E^{inc}) and backward *b* (E^{refl}) traveling waves $\Longrightarrow$ standing waves
- **Slotted coaxial air-line is used as standing wave detector**
  - Probes the radial electric field along the slotted line.
  - Measurement of E-field minima's E_{min} and maxima's E_{max} with a diode detector, thus detect $|V_{min}|$ and $|V_{max}|$ along the line.
  - Evaluate the reflection coefficient Γ of a DUT of unknown Z_L at the end of the line

$$\Gamma = \frac{E^{\text{refl}}}{E^{\text{inc}}} = \frac{Z_L - Z_0}{Z_L + Z_0}$$





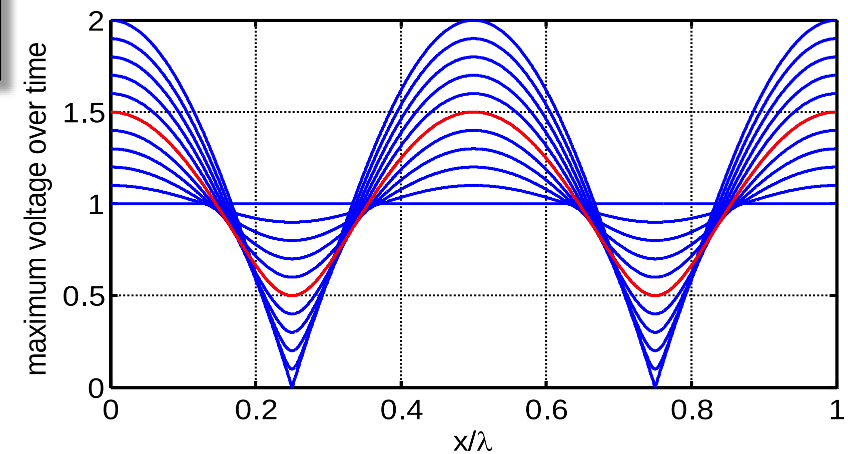
Voltage Standing Wave Ratio VSWR (2)

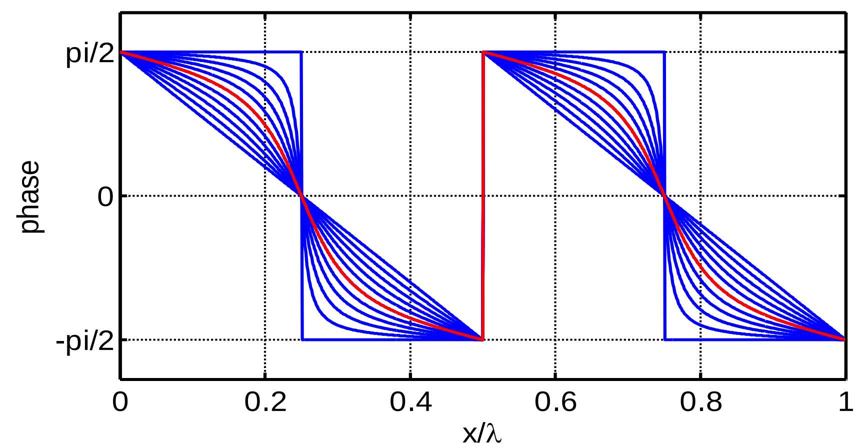
- **The VSWR is defined as:**

$$VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{|a| + |b|}{|a| - |b|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

  - **The phase of the detected E-field along the lossless coaxial line is purged by the diode detection.**
    - **Requires a mixer as detector!**

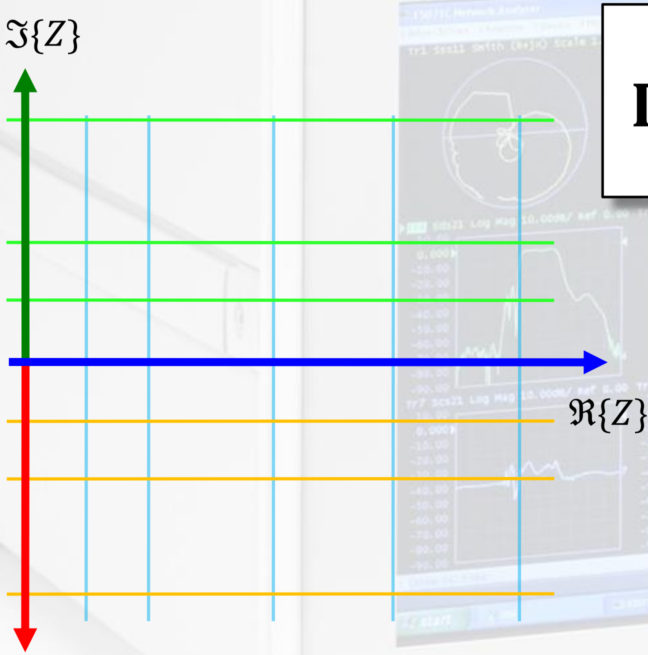
Γ	Return Loss [dB]	*VSWR* = Z_L/Z_0	Refl. Power $1-\|\Gamma\|^2$
0.0	∞	1.00	1.00
0.1	20	1.22	0.99
0.2	14	1.50	0.96
0.3	10	1.87	0.91
0.4	8	2.33	0.84
0.5	6	3.00	0.75
0.6	4	4.00	0.64
0.7	3	5.67	0.51
0.8	2	9.00	0.36
0.9	1	19	0.19
1.0	0	∞	0.00



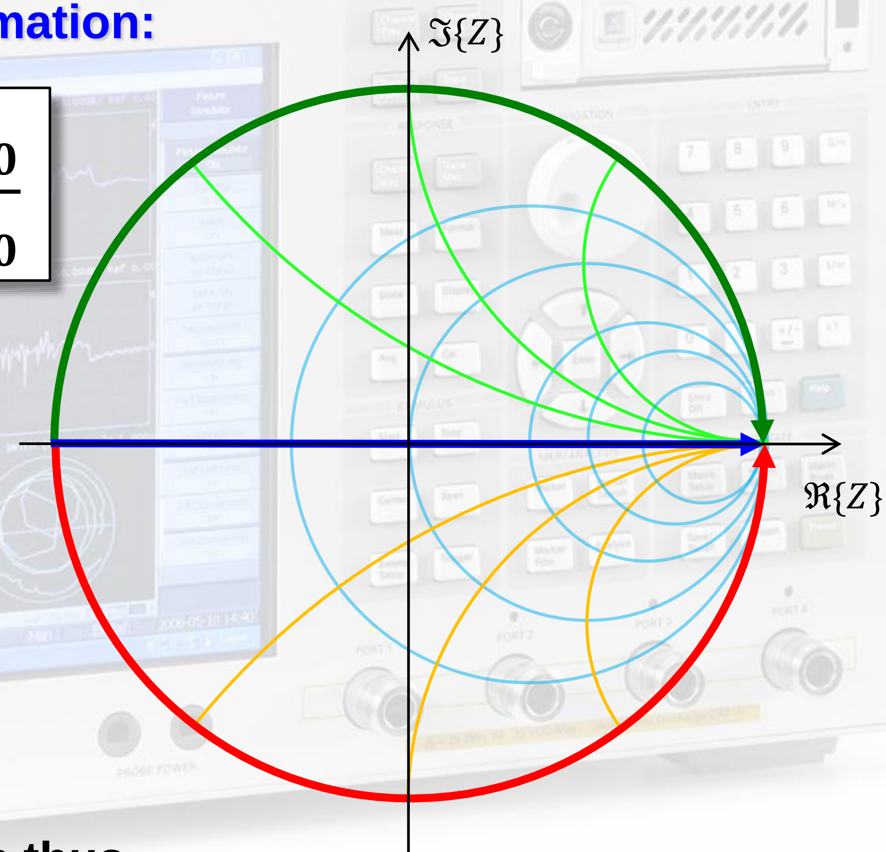


The *Smith* Chart (1)

- The *Smith* Chart (in impedance coordinates) represents the complex Γ-plane (in polar coordinates) within the unit circle.
- It is a conformal mapping of the complex Z-plane on the Γ-plane by applying the transformation:



$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$



- ⇒ the real positive half plane of Z is thus transformed (*Möbius*) into the interior of the unit circle!

The *Smith* Chart (2)

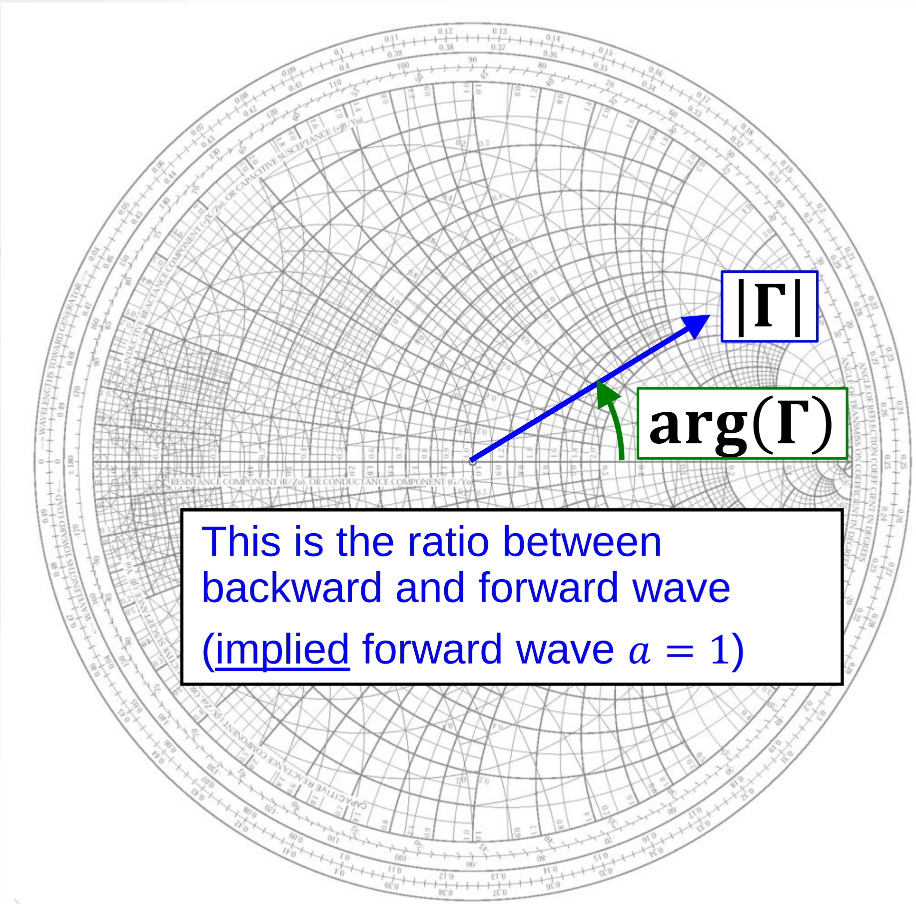
- **The impedance Z is usually normalized** $$z = \frac{Z}{Z_0}$$ **to a reference impedance Z_0, typically the characteristic impedance of the coaxial cables of $Z_0 = 50\ \Omega$.**
- **The normalized form of the transformation follows then as:**

$$\Gamma = \frac{z-1}{z+1} \quad \text{resp.} \quad \frac{Z}{Z_0} = z = \frac{1+\Gamma}{1-\Gamma}$$

<u>This mapping offers several practical advantages</u>:

- The diagram includes all "passive" impedances, i.e. those with positive real part, from zero to infinity in a handy format.
  - Impedances with negative real part ("active device", e.g. reflection amplifiers) would be outside the (normal) *Smith* chart.
- The mapping converts impedances or admittances into reflection factors and vice-versa. This is particularly interesting for studies in the radiofrequency and microwave domain where electrical quantities are usually expressed in terms of "incident" or "forward", and "reflected" or "backward" waves.
  - This replaces the notation in terms of currents and voltages used at lower frequencies.
- Also the reference plane can be moved very easily using the *Smith* chart.

The *Smith* Chart (3)



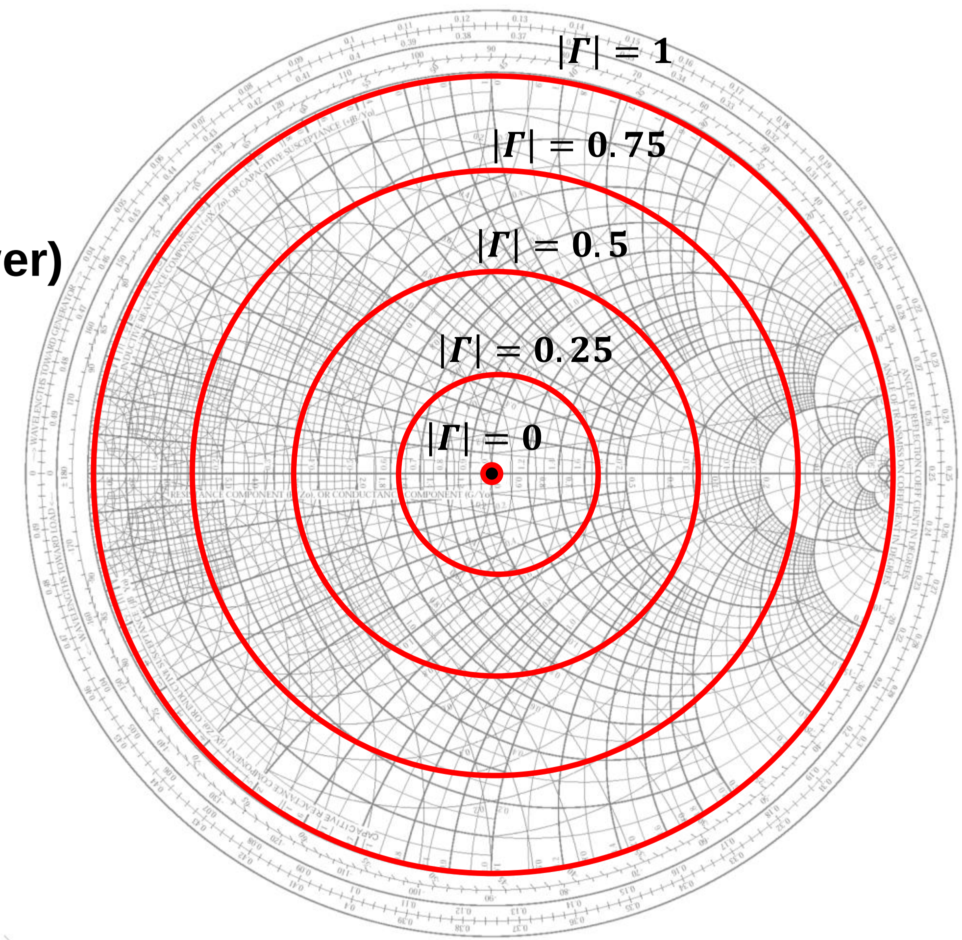
- **In the *Smith* chart, the complex reflection factor**

$$\Gamma = |\Gamma| e^{j\varphi} = \frac{b}{a}$$

is expressed in linear cylindrical coordinates, representing the ratio of backward vs. forward traveling waves.

The *Smith* Chart (4)

- **The distance from the center of the directly proportional to the magnitude of the reflection factor $|\Gamma|$, and permits an easy visualization of the matching performance.**
  - **In particular, the perimeter of the diagram represents total reflection: $|\Gamma| = 1$.**
  - **(power dissipated in the load) = (forward power) – (reflected power)**



$$\mathbf{P} = |a|^2 - |b|^2$$
$$= |a|^2 \left(1 - |\Gamma|^2\right)$$

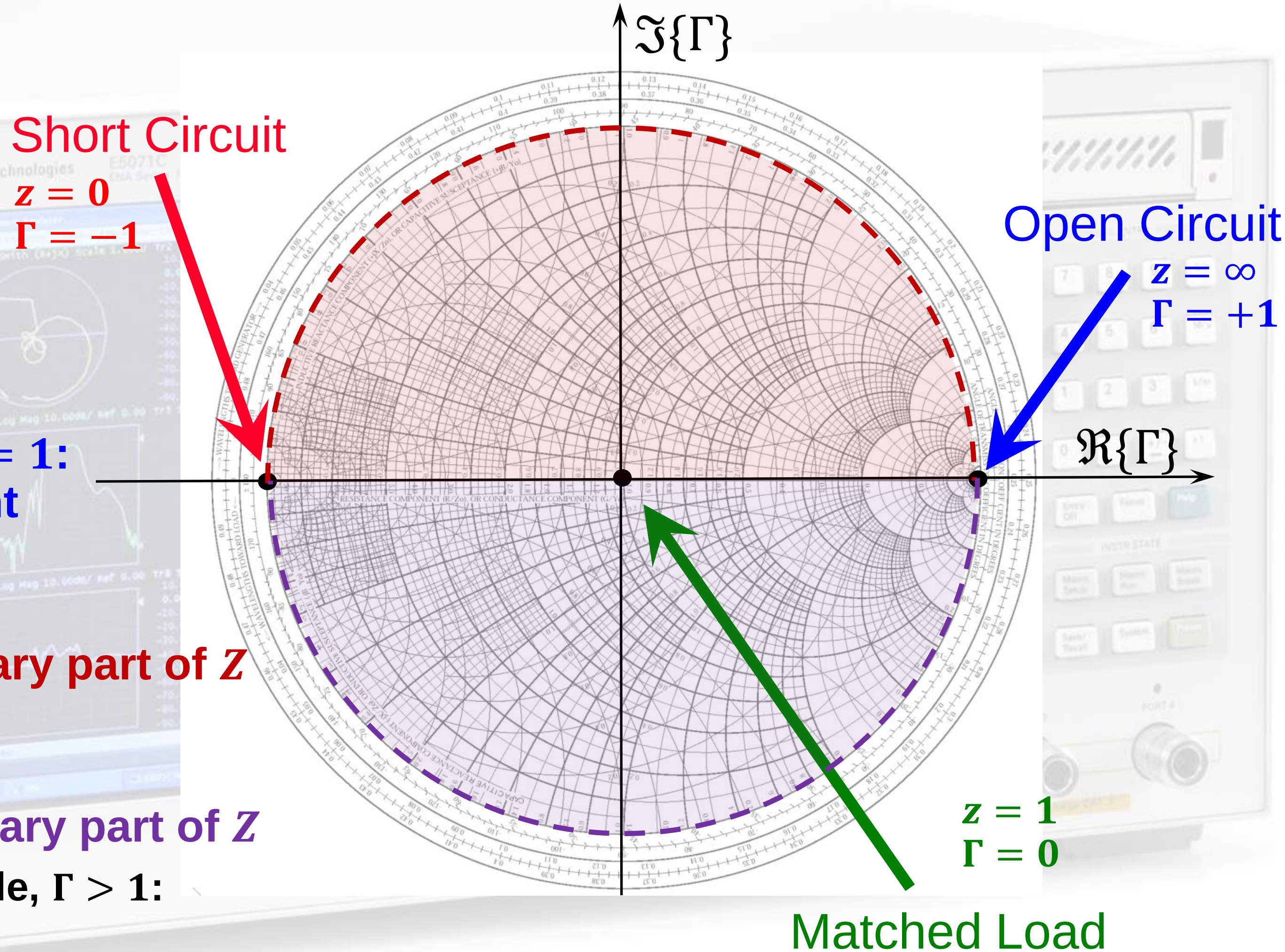
available source power

mismatch losses

The *Smith* Chart – “Important Points”

Important Points:

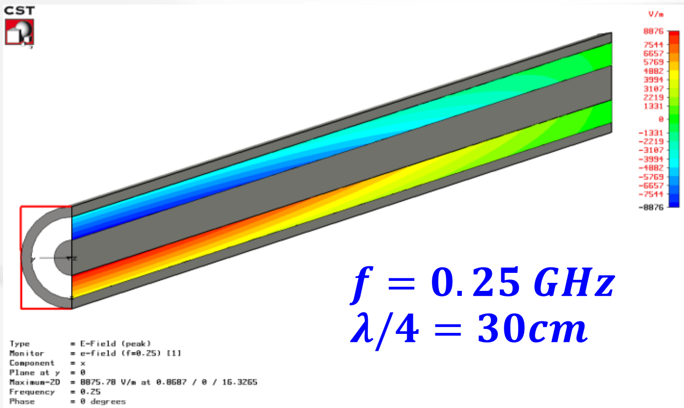
- **Short Circuit**
  $\Gamma = -1, z = 0$
- **Open Circuit**
  $\Gamma = +1, z \rightarrow \infty$
- **Matched Load**
  $\Gamma = 0, z = 1$
- **On the circle $\Gamma = 1$: lossless element**
- **Upper half: ”inductive” = positive imaginary part of Z**
- **Lower half: ”capacitive” = negative imaginary part of Z**
  - **Outside the circle, $\Gamma > 1$: active element, for instance tunnel diode reflection amplifier**



Coming back to our Example…

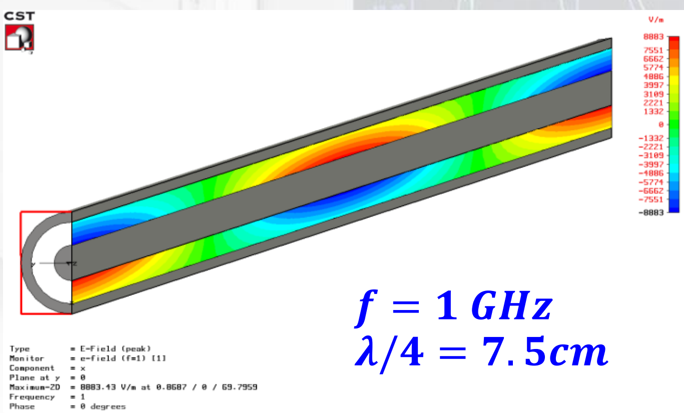
matched case:

pure traveling wave=> no reflection



$f = 0.25\ GHz$

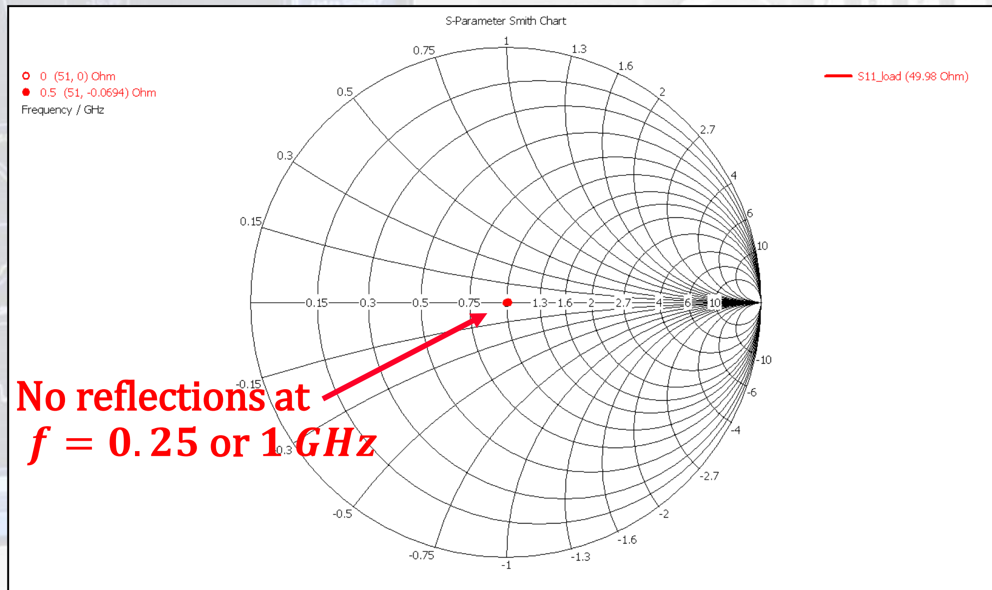
$\lambda/4 = 30cm$



$f = 1\ GHz$

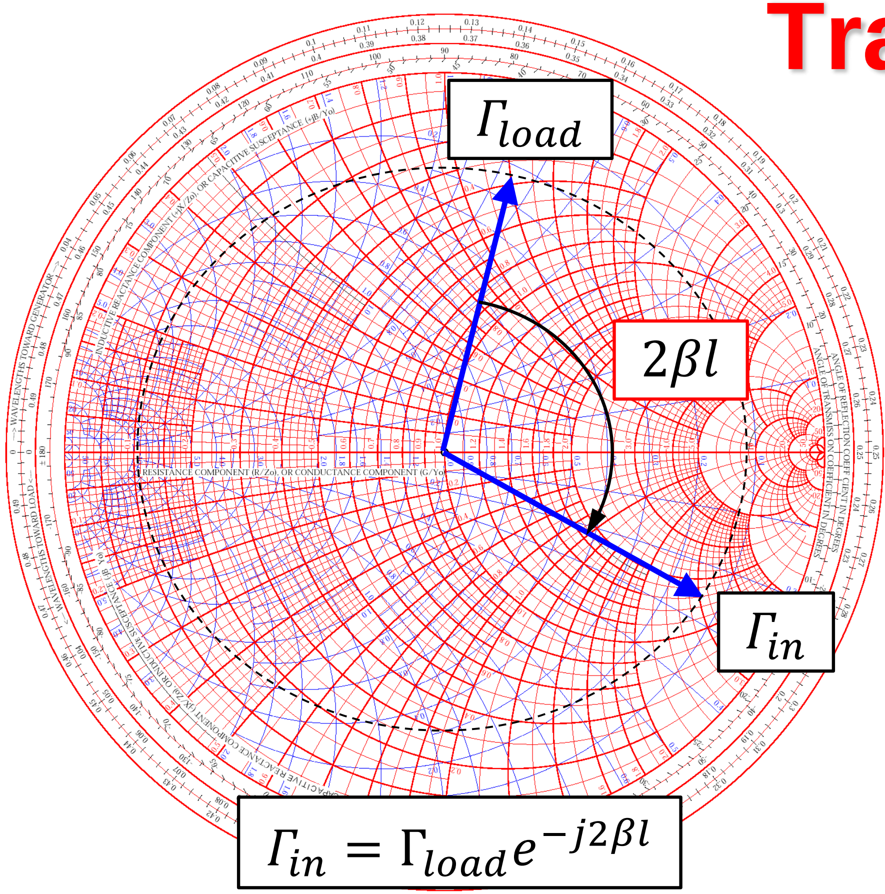
$\lambda/4 = 7.5cm$

Coax cable with vacuum or air with a lenght of 30 cm



No reflections at $f = 0.25$ or $1\ GHz$

Impedance Transformation using Transmission-lines

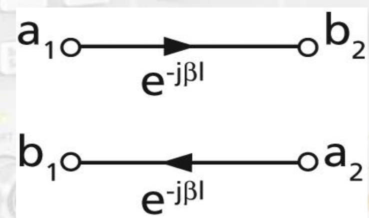


The S-matrix for an ideal, lossless transmission line of length l is given by

$$\boldsymbol{S} = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix}$$

where $\beta = 2\pi/\lambda_g$

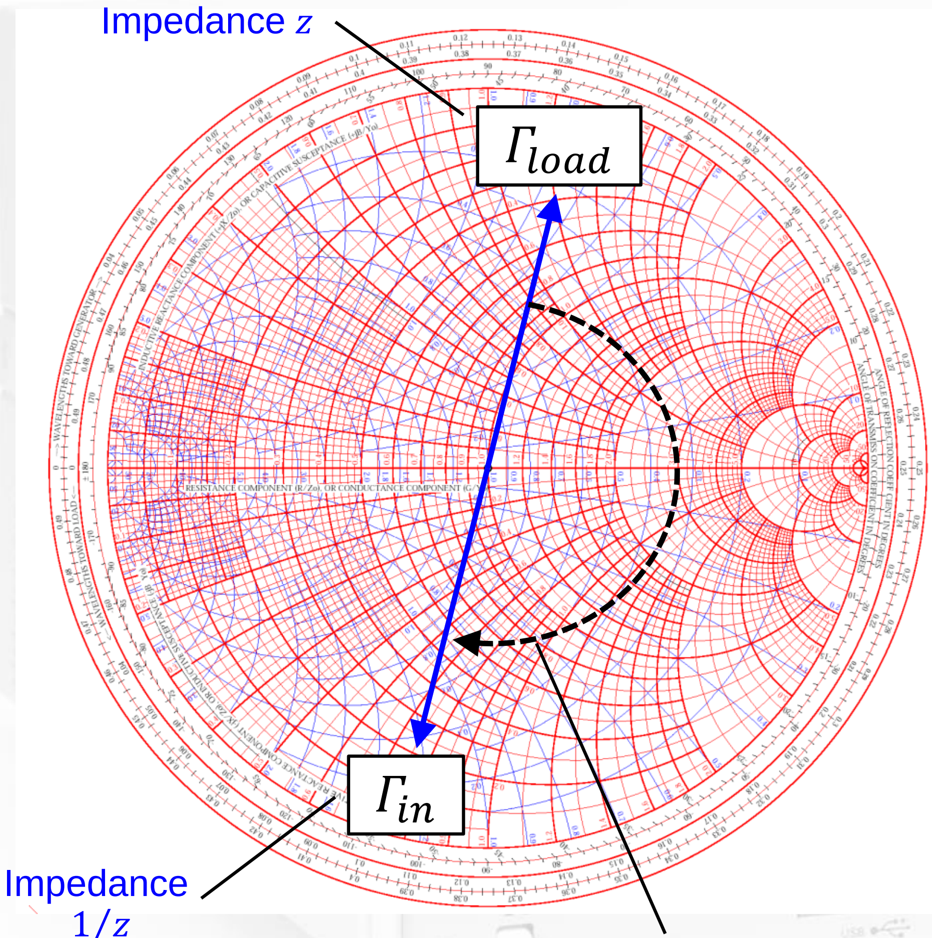
is the propagation coefficient at the guide wavelength λ_g (this refers to the wavelength on the line containing some dielectric).



How to remember when adding a section of transmission-line?
We have to turn clockwise: assume we are at $\Gamma = -1$ (short circuit) and add a short piece of e.g. coaxial cable.
We actually introduced an inductance, thus we are in the upper half of the *Smith*-Chart.

N.B.: The reflection factors are evaluated with respect to the characteristic impedance Z_0 of the line segment.

$\lambda/4$-line Transformations



A transmission line of length

$$l = \lambda/4$$

transforms a load reflection Γ_{load} to its input as

$$\Gamma_{in} = \Gamma_{load} e^{-j2\beta l} = \Gamma_{load} e^{-j\pi} = -\Gamma_{load}$$

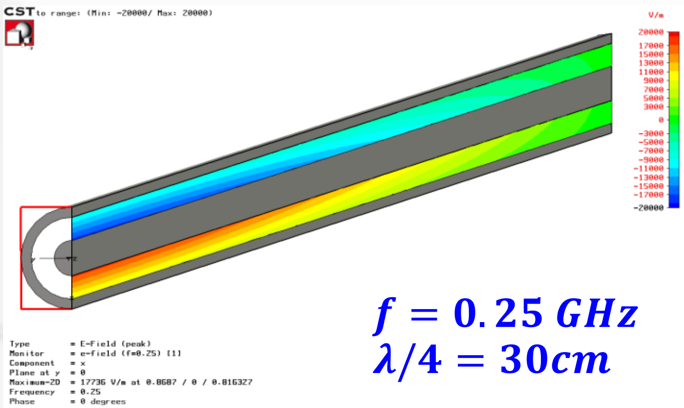
This results, a normalized load impedance z is transformed into $1/z$.

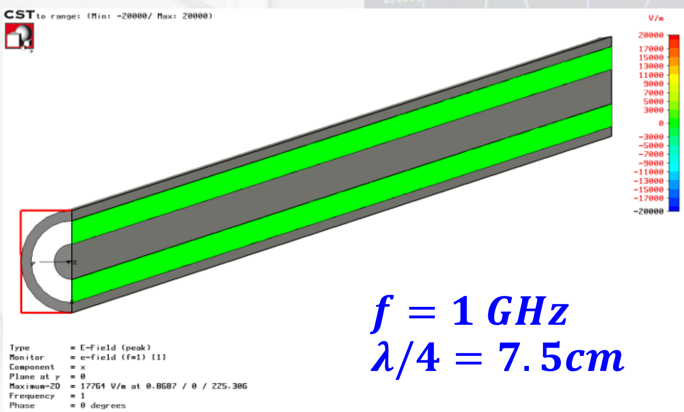
In particular, a short circuit at one end is transformed into an open circuit at the other. This is the principle of $\lambda/4$-resonators.

when adding a transmission line to some terminating impedance we rotate clockwise through the *Smith*-Chart.

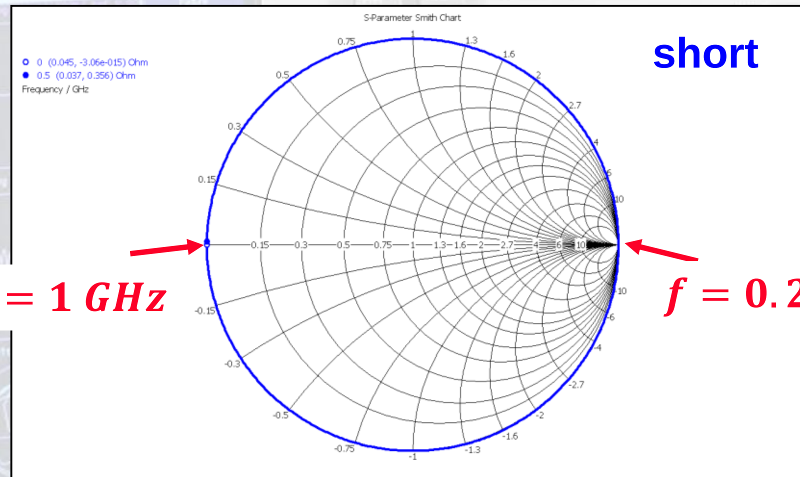
Again our Example: Short at the end

short : standing wave





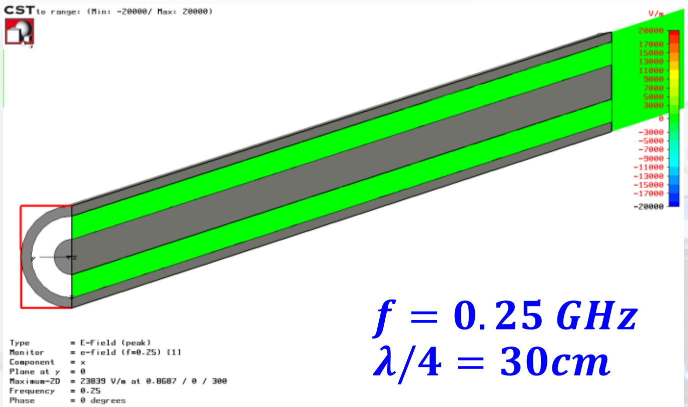
Coax cable with vacuum or air with a length of $l = 30\ cm$

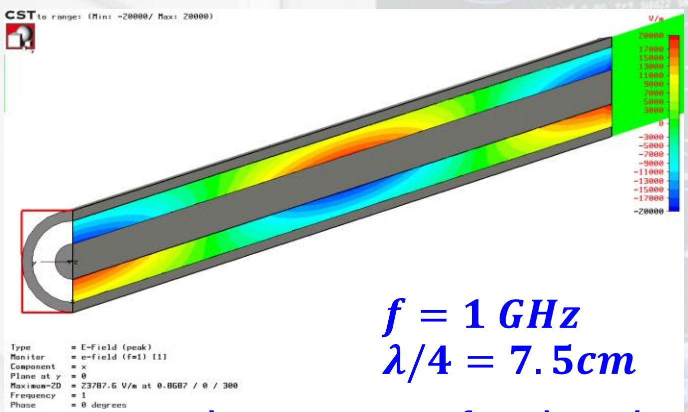


- If length of the transmission line changes by $\lambda/4$ a short circuit at one side is transformed into an open circuit at the other side.

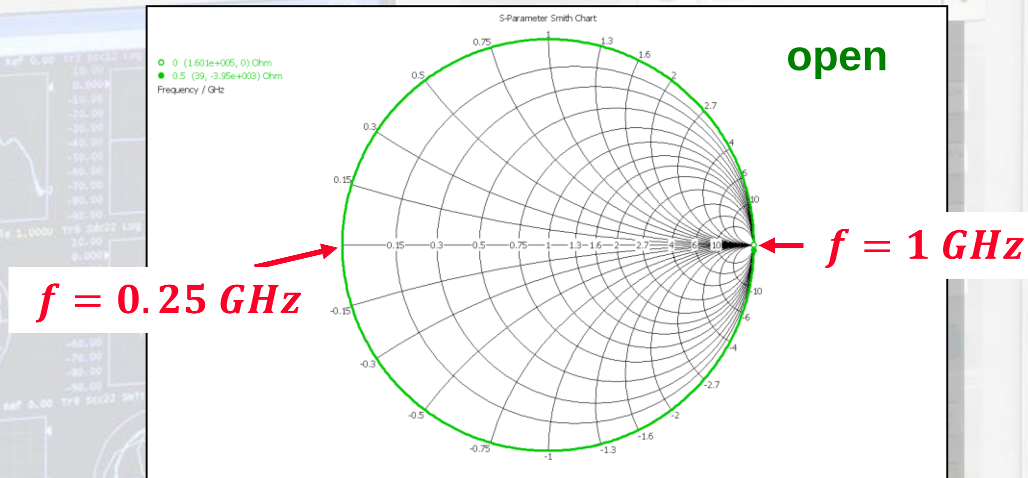
Again our Example: Open end

open : standing wave





Coax cable with vacuum with a length of 30 cm



- The patterns for the short and open terminated case appear similar; However, the phase is shifted which correspond to a different position of the nodes.
- If the length of a transmission line changes by $\lambda/4$, an open becomes a short, and vice versa!

Fun with the *Smith* Chart…

- **Download the Smith 4.1 software (Windows)**
  - http://www.fritz.dellsperger.net/smith.html
- **Home exercise:**
  - **Find the values of two elements to match to $Z_{in} = 50\ \Omega$ at $f = 500\ MHz$**

Z_L	C Series	L Series	R Series
Z = (50 + j25) Ω			
Z = (50 - j25) Ω			
Z = (4 + j21) Ω			
Z = (20 – j50) Ω			

Z_L	C Shunt	L Shunt	R Shunt
Z = (50 + j25) Ω			
Z = (50 - j25) Ω			
Z = (4 + j21) Ω			
Z = (20 – j50) Ω			

  - **Find two reactive elements to match: (two solutions are possible)**

Z_L	C Series	L Series	C Shunt	L Shunt
Z = (32 – j66) Ω				
Z = (13 – j9) Ω				
Z = (37 + j34) Ω				
Z = (78 + j78) Ω				

End of Part I

Thank you!