The CERN Accelerator School

Beam Dynamics and Technologies for Future Colliders, 21 February - 6 March 2018, Zurich, Switzerland

Case Study on FCCee

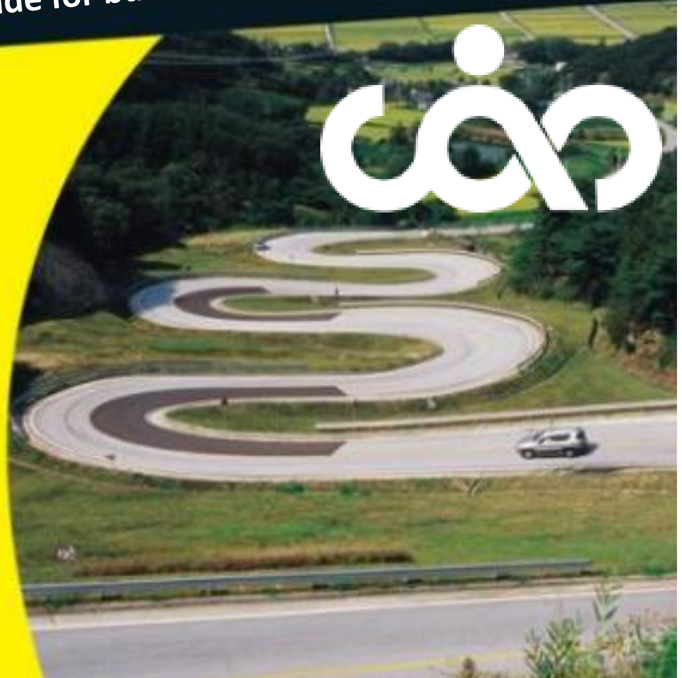
FOR DUMMIES

(A 10-minutes practical guide for building your collider at home)

Linear collider or Circular machine?
Luminosity?
Limit of the stored beam current?
Number of Bunches?

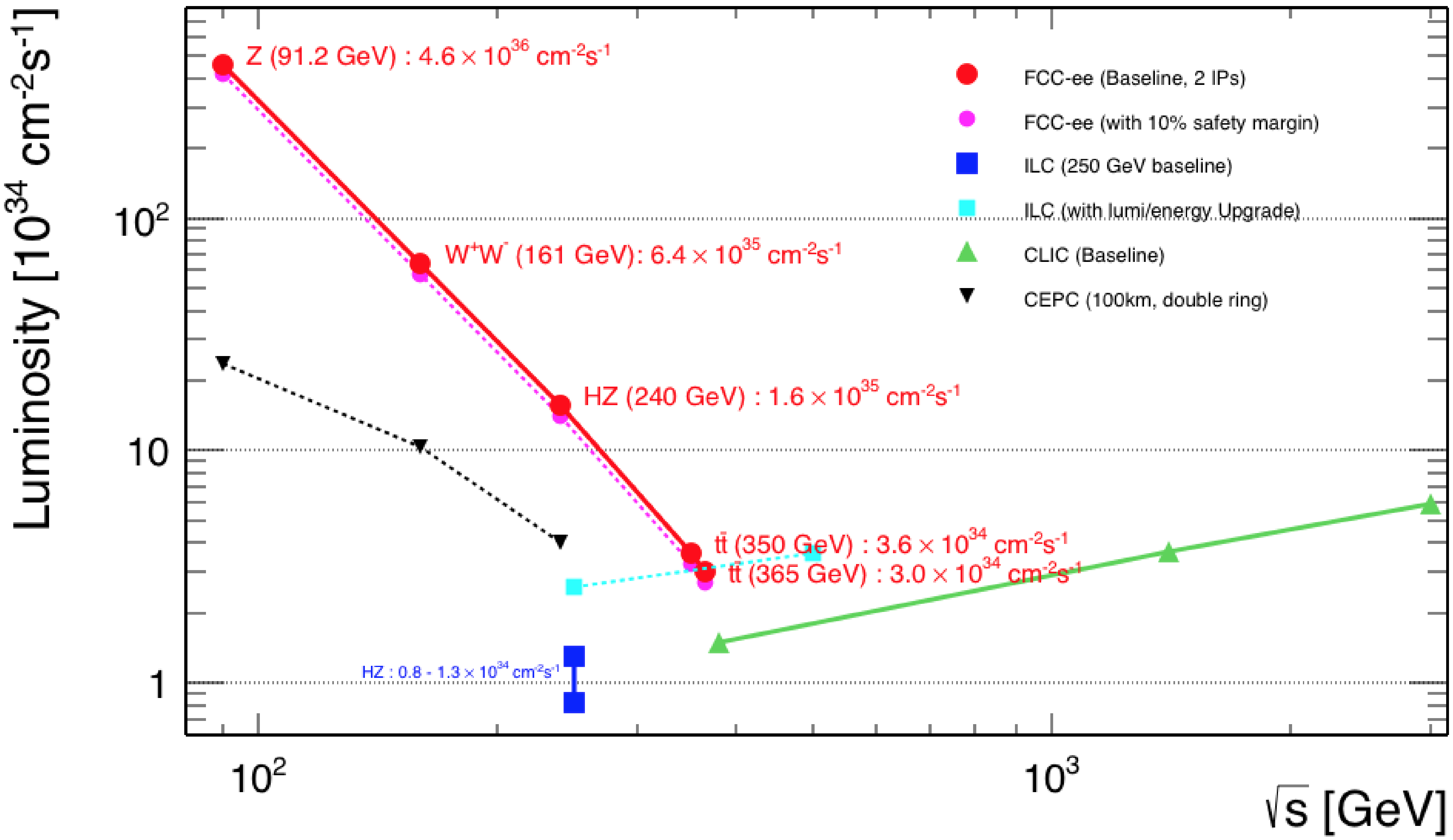
Learn to:

Design a e^+/e^- collider with a beam energy of 45 – 180 GeV, an overall length of 100 km and that can provide in parallel a luminosity of $L=10^{34}$ cm^{-2} s^{-1} to two experiments.



Lower Energy?
RF system?
Arc structure?
Dipole design?
Quadrupole design?

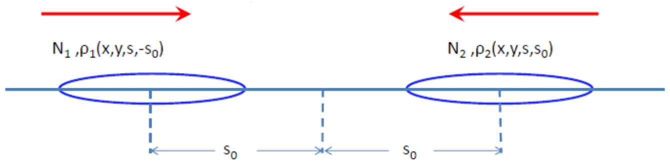
Linear collider or circular machine?



Source : http://tlep.web.cern.ch/content/machine-parameters

Luminosity

For a collider $$\frac{dR}{dt} = L \times \sigma_p$$



Assume beams are Gaussian in all directions and independent of each other

$$L = 2cN_1N_2\cos^2\frac{\phi}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\rho_x^{(1)}(x)\rho_y^{(1)}(y)\rho_s^{(1)}(s-ct)\times\rho_x^{(2)}(x)\rho_y^{(2)}(y)\rho_s^{(2)}(s+ct)dxdydsdt$$

$$L = \frac{N_1N_2fN_b}{4\pi\sigma_x\sigma_y}$$ Several parameters to determine collider Luminosity

N_1&N_2: number of particles per punch in beam 1 & 2 respectively
N_b: number of colliding bunches per beam
σ_x&σ_y: the transverse dimensions
f: revolution frequency

Idealy, increase N1&N2, Nb and decrease σ_x&σ_y

However, we can’t get infinite Luminosity

Energy loss due to SR

$c = 299792458$ m/s
$\epsilon_0 = 8.85418782 \times 10^{-12}$ m^{-3} kg^{-1} s^4 A^2
$\beta = 1$
$q = 1.60217662 * 10^{-19}$ C
$\gamma = 180 \times 10^9/(0.511 \times 10^6)$
$C = 100$ km circumference

From the magnet design lattice group, the proportion of bending in the 100 km is 66 %. Therefore the bending radius is,

$$\begin{aligned} \rho =& \frac{C}{2\pi} * 0.66 \\ =& 10.50 \text{ km} \end{aligned} \tag{1}$$

Energy loss per turn for a single particle:

$$\begin{aligned} U_0 &= \frac{4\pi}{3} \frac{r_e}{(mc^2)^3} \frac{E^4}{\rho} \\ &= 88.5 * 10^3 \frac{E[\text{GeV}^4]}{\rho[\text{m}]} \\ &= 8847.98 \text{ MeV} \end{aligned} \tag{2}$$

Power loss per turn for a single particle:

$$\begin{aligned} P_\gamma &= \frac{(q^2 c)}{6\pi\epsilon_0} \frac{(\beta^4\gamma^4)}{\rho^2} \\ &= 6.4338 * 10^{-6} \text{ W} \end{aligned} \tag{3}$$

The total loss of power from synchrotron radiation should not exceed 50 MW, per beam.

$$
\begin{aligned}
50 * 10^6 &= N \frac{(q^2 c)}{6\pi\epsilon_0} \frac{(\beta^4 \gamma^4)}{\rho^2} \qquad (4) \\
N &= 7.77 * 10^{12}
\end{aligned}
$$

Number of particles, $N = 7.77 * 10^{12}$
The total stored current:

$$
\begin{aligned}
I_{total} &= \frac{qNc}{C} \qquad (5) \\
&= 3.73 \text{ mA}
\end{aligned}
$$

A reasonable total number of particles per bunch was taken to be 2×10^{11} (taken from LEP). Therefore the number of bunches should be,

$$
\begin{aligned}
\frac{N}{N_b} &= \frac{7.77 * 10^{12}}{2 * 10^{11}} \\
&= 39 \text{ bunches}
\end{aligned}
$$

45 GeV: Limits

- Limit of stored current I :

$$P = 88.46 \cdot \frac{E^4 \cdot I}{R}$$

Given in task:

- $P = 50\ \mathrm{MW}$ overall synchotron radiation power
- $E = 45\ \mathrm{GeV}$ beam energy
- $l_{\mathrm{collider}} = 100\ \mathrm{km}$ collider length
- $k_{\mathrm{dipole}} = 0.66$ dipole coverage factor as assumption
- Radius of collider:

$$\rightarrow R = k_{\mathrm{dipole}} \cdot \frac{l_{\mathrm{collider}}}{2\pi} 10.5\ \mathrm{km}$$

- $\rightarrow \boldsymbol{I = 1.45\ \mathrm{A}}$

- Limit of number of bunches per beam N_{b}:

$$I = N_{\mathrm{b}}\, N\, q\, f_{\mathrm{rev}}$$

Given in task:

- $q = 1.602 \cdot 10^{-19}\mathrm{C}$ charge of electron
- $f_{\mathrm{rev}} = \frac{c}{l_{\mathrm{collider}}} \approx 3\ \mathrm{kHz}$ revolution frequency
- $N < 2 \cdot 10^{11}$ number of particles per bunch as assumption

- $\rightarrow \boldsymbol{N_{\mathrm{b}} = 1.5 \cdot 10^4}$

45 GeV: RF system

- RF should at least compensate the energy loss by synchotron radiation
- RF Voltage V_{RF}:

$$V_{RF} = \frac{P}{I}$$

$$\rightarrow \boldsymbol{V_{\mathrm{RF,min}}\ (45\ \mathrm{GeV}) = 32.5\ \mathrm{MV}}$$

- Assumption: Overvoltage of ~50%

$$\rightarrow \boldsymbol{V_{\mathrm{RF}}\ (45\ \mathrm{GeV}) = 50\ \mathrm{MV}}$$

- Harmonic number:

$$h = \frac{f_{\mathrm{RF}}}{f_{\mathrm{rev}}}$$

Assumption: $f_{\mathrm{RF}} = 400\ \mathrm{MHz}$

$$\rightarrow \boldsymbol{h = 133426}$$

- Distance between bunches:

$$\frac{h}{N_{\mathrm{b}}} \cdot \frac{1}{f_{\mathrm{RF}}} = \boldsymbol{22.5\ \mathrm{ns}}$$

- Number of cavities: with assumption input power of each cavity $P_{\mathrm{Input}} \sim 500\ \mathrm{kW}$:

$$\frac{P}{P_{\mathrm{Input}}} = \textbf{100 cavities}$$

- Voltage per cavity:

$$V_{\mathrm{cavity}} = \frac{V_{\mathrm{RF}}}{100} = \boldsymbol{500\ \mathrm{kV}}$$

- Length of 1 cell of the cavity

$$l_{\mathrm{cavity}} = \frac{c}{2\ f_{\mathrm{RF}}} = \boldsymbol{0.37\ \mathrm{m}}$$

- Accelerating field in 1 cavity

$$E_{\mathrm{Acc}} = \frac{V_{\mathrm{cavity}}}{l_{\mathrm{cavity}}} = \boldsymbol{1.3\ \frac{\mathrm{MV}}{\mathrm{m}}}$$

→ low value (normally $E_{\mathrm{Acc}} \sim 10\ \frac{\mathrm{MV}}{\mathrm{m}}$)

- *Input*
- Our assumptions
- Consequences

Equilibrium emittance (1/2)

Balance between synchrotron radiation and quantum excitation

$$\boxed{\epsilon_x = \frac{\sigma_x^2}{\beta_x} = \frac{C_q \gamma^2}{J_x \rho} \langle \mathscr{H} \rangle}$$

$\gamma = E/E_0$ for top energy: $\frac{180 GeV}{511 keV} = 3.52 \times 10^5$

$L = 2\pi R \Rightarrow R = 15.9\,\mathrm{km}$

$L \cdot F = 2\pi\rho \Rightarrow \rho = 10.5\,\mathrm{km}$ (with a filling factor F = 66 %)

$$\langle \mathscr{H} \rangle_{dipole} = \frac{1}{2\pi\rho} \int ds \left(\gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2\right)$$

- *Input*
- Our assumptions
- Consequences

Equilibrium emittance (2/2)

$\langle \mathcal{H} \rangle_{dipole}$ depends on the lattice…Fortunately Teng did the maths for us [1]

$$\epsilon_x = \frac{\sigma_x^2}{\beta_x} = \frac{C_q \gamma^2}{J_x \rho} \langle \mathcal{H} \rangle = \frac{C_q}{J_x} \gamma^2 \theta^3 F \frac{L}{\ell} \longrightarrow \boxed{\begin{array}{c} \varepsilon_x = 1\ nm \\ \varepsilon_y = 0.002 \cdot \varepsilon_x = 2\ pm \end{array}}$$

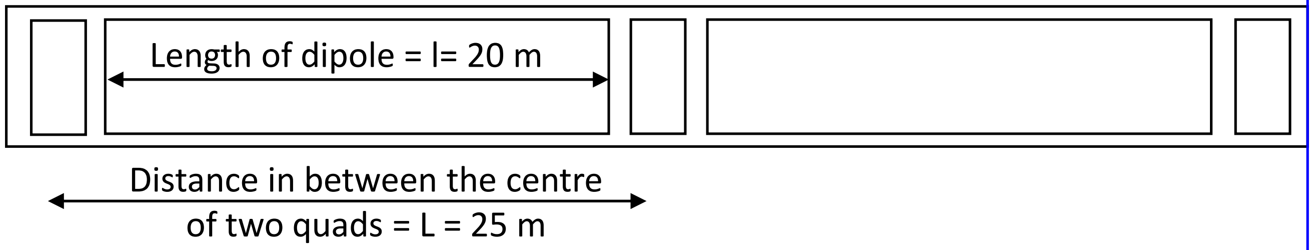
$$\boxed{\mu = 90^\circ \, : \, F = 2.5 \frac{L}{\ell}}$$

$$\mu = 72^\circ \, : \, F = 4.51 \frac{L}{\ell}$$

$$\mu = 60^\circ \, : \, F = 7.51 \frac{L}{\ell}$$

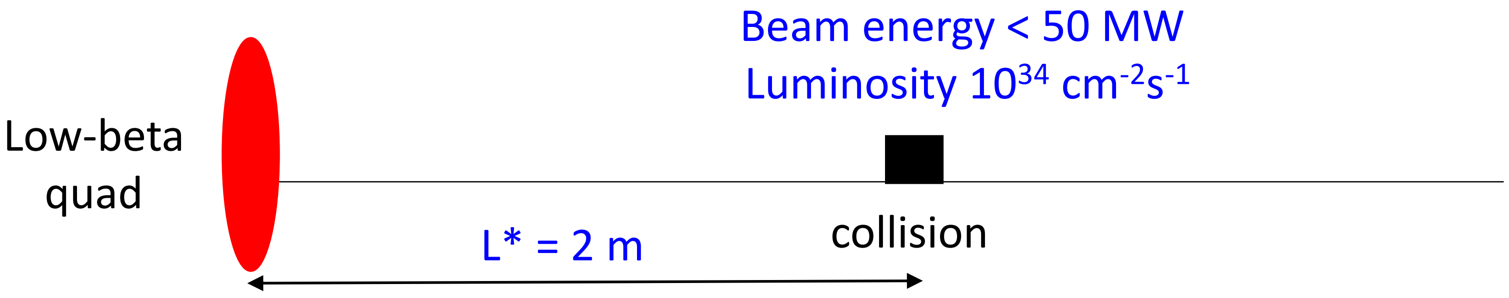
$$\theta = \ell / \rho$$

Our colleges from the lattice:
- 90 degrees phase advance
- L = 25 m; l = 20 m



[1] Fermilab, TM-1269-0102-000. “Minimising the Emittance in Designing the Lattice of an Electron Storage Ring”

Beta-function at the IP (1/2)



Considerations:

The more particles we have per bunch, the better it is for the luminosity

But we need to be careful with the beam-beam force, as it might become un-manageable!

Synchrotron radiation power loss < 50 MW $\longrightarrow$ Beam current 6 mA

$1.18 \cdot 10^{13}$ for 50 MW beam energy

3 kHz

$$L = \frac{N_1 \boxed{N_2 n_B} f_S}{4\pi \beta^* \epsilon}$$

10^{34}

In approximation of flat beams for head-on collisions

$$\xi = \frac{N r_0 \beta *}{4\pi\gamma\sigma^2}$$

$$\boxed{\xi_{x,y} = \frac{N \cdot r_o \cdot \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}}$$

Beta-function at the IP (2/2)

- *Assumption n.1*: the size of the beam in the low-beta quadrupoles is the same in x and y direction to make the best possible use of the magnet aperture.

$$\sigma_x = \sigma_y \longrightarrow \sqrt{\epsilon_x \beta_x} = \sqrt{\epsilon_y \beta_y}$$

- *Assumption n.2* : beam-beam parameter ξ_y = 0,12 from LEP experience [2]
- *Assumption n.3*: horizontal beta function to be 1 m

$$\beta(s) = \beta^* + \frac{s^2}{\beta^*} \qquad s = 2\ \mathrm{m}$$

$$\beta_y^* + \frac{s^2}{\beta_y^*} = \left(\frac{\epsilon_x}{\epsilon_y}\right) \beta_x^* + \frac{s^2}{\beta_x^*}$$

$$\beta_y^* = \frac{s^2}{\frac{\epsilon_x}{\epsilon_y}\left(\beta_x^* + \frac{s^2}{\beta_x^*}\right)}$$

RESULTS

- Bunch population: $1.08 \cdot 10^{11}$
- Number of bunches: 109
- **Luminosity: $1.66 \cdot 10^{34}$**
- Beta Function at the IP

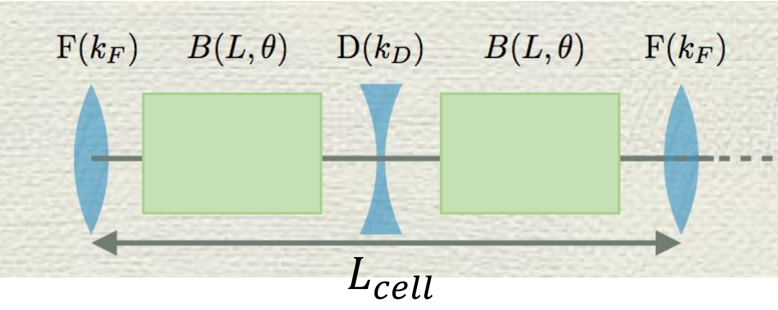
$$\boxed{\begin{aligned} s &= 2\,m \\ \beta_x &= 1\,m \\ \beta_y &= 1.6\,mm \end{aligned}}$$

[2] R. Assmann and K. Cornelis, ”The Beam-Beam Interaction in the Presence of strong Radiation Damping”, CERN-SL-2000-046 OP

Lattice

Various arc lattices

lattice	characteristics	machines
FODO	simplest structure best packing factor of dipoles	high energy machines (LEP, PEP, TRISTAN, etc.)
Multi-bend achromat	small emittance suitable for insertion devices	3G light sources
Theoretical minimum emittance	small emittance fast damping	damping rings (KEK-ATF)
2.5π	non-interleaved sextupole variable emittance/momentum compaction	KEKB/SuperKEKB
Non-periodic	All independent quadrupoles Maximum flexibility	CESR



Why FODO cell?

- **Highest filling factor to minimize SR**
  - Power radiated by a beam of average current I

$$P = 88.46\,\frac{E^4[GeV]\ \ I[A]}{\rho[m]}$$

  - Bending radius

$$\rho = \frac{C}{2\pi}F \qquad F = 10.5km$$

usually F ≈ 66% in high energy rings

  - Number of dipoles
    - $L_{dip} = 20m$

$$N_{dip} = \frac{L_{dip,TOT}}{L_{dip}} = 3300$$

  - Bending angle

$$\theta_{dip} = \frac{2\pi}{N_{dip}} = 1.9\ mrad$$

Emittance

- Minimum emittance in FODO lattice

$$\varepsilon_x = \frac{C_q}{J_x}\gamma^2\theta^3 F \approx \begin{cases} 0.0537\ nm\ (45\ GeV) \\ 0.86\ nm\ (180 GeV) \end{cases}$$

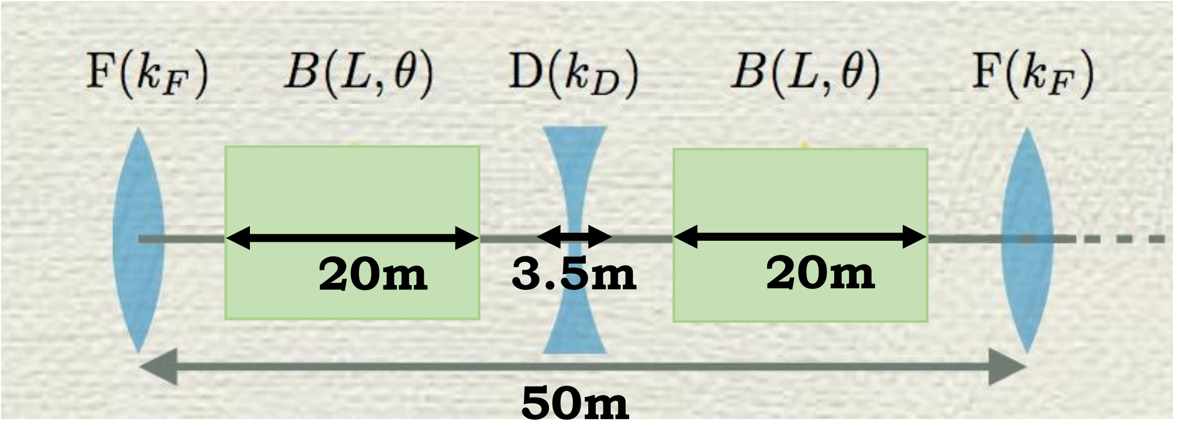
- ❑ $C_q \approx 3.832 \cdot 10^{-13}$ m for electrons
- ❑ Damping partition number $J_x \approx 1$ (no quadrupole component in dipole)
- ❑ $F = \frac{1}{2 sin\mu}\frac{5+3cos\mu}{1-cos\mu}\frac{L_{FODO}}{2L_{dip}} = 0.0625\ L_{FODO}$

Phase advance $\mu = 90°$

$$L_{FODO} \approx 42.2\ m \rightarrow \mathbf{50\ m}$$

$$\varepsilon_x = C_q\,\gamma^2\theta^3 F \approx \begin{cases} 0.063\ nm\ (45\ GeV) \\ 1.09\ nm\ (180 GeV) \end{cases}$$

$$\varepsilon_y = 10^{-3}\varepsilon_x$$



$\beta -$ functions and dispersion

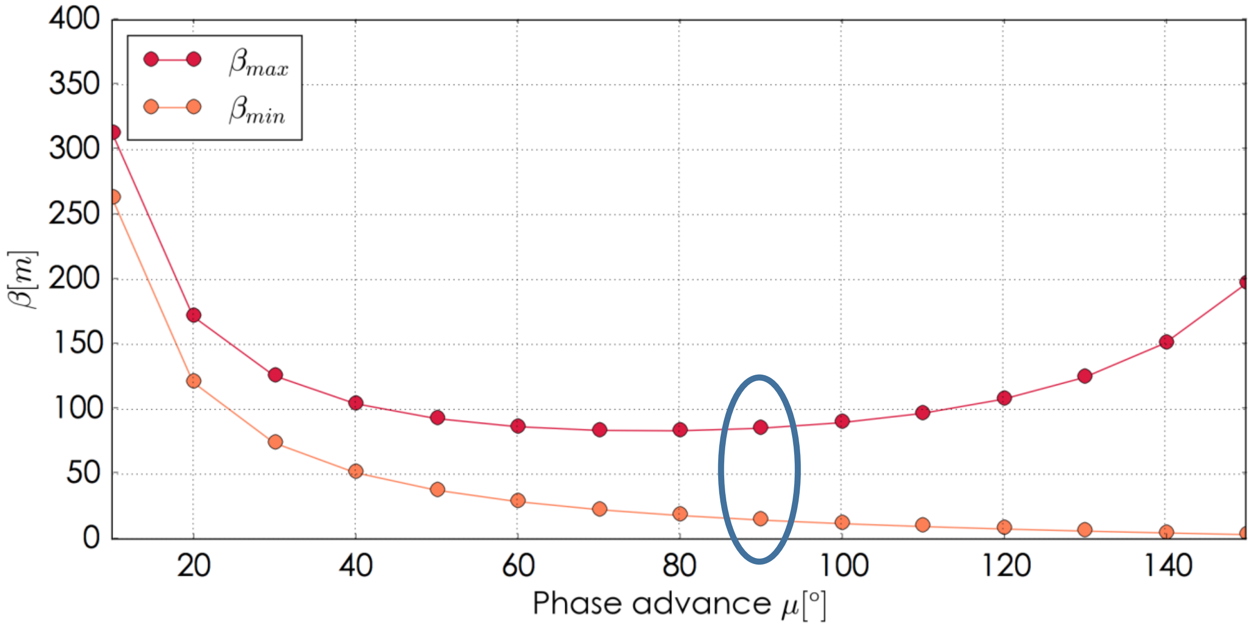
$$\beta_{max} = L_{FODO}\frac{1+\sin\frac{\mu}{2}}{\sin\mu} = 85.35\ m$$

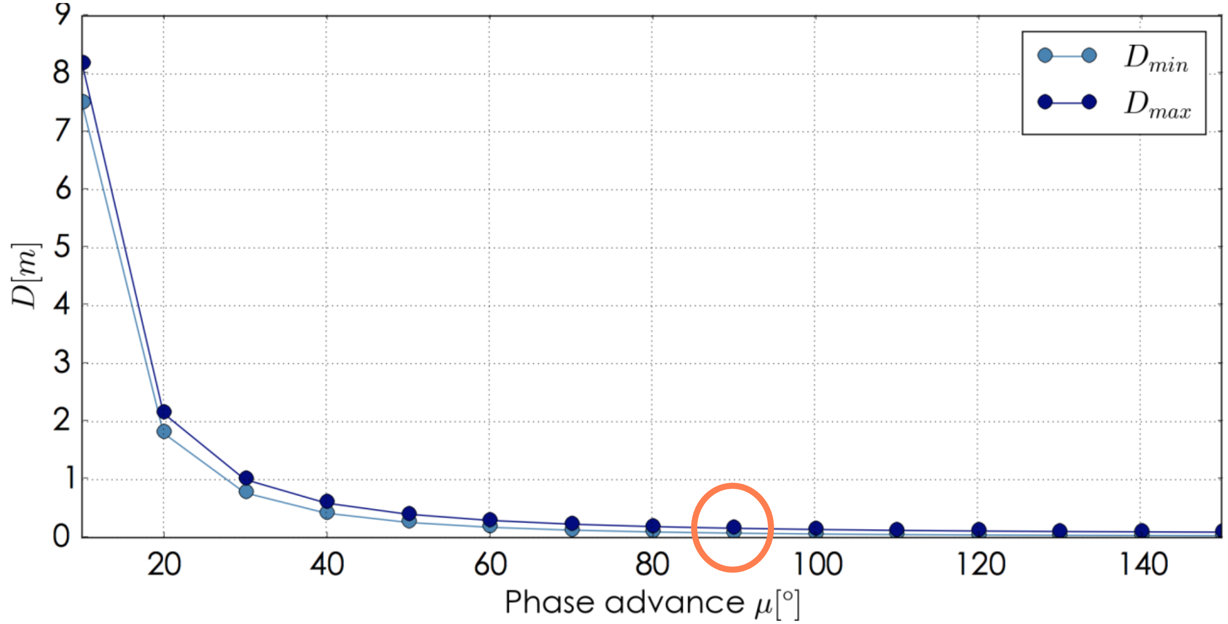
$$\beta_{min} = L_{FODO}\frac{1-\sin\frac{\mu}{2}}{\sin\mu} = 14.64\ m$$

$$\sigma_x = \sqrt{\beta_x\varepsilon_x} = \begin{cases} 0.073\ mm\ (45\ GeV) \\ 0.3\ mm\ (180\ GeV) \end{cases}$$

$$D_{max} = \frac{L_{FODO}^2}{\rho}\frac{1+\frac{1}{2}\sin\frac{\mu}{2}}{4\sin^2\frac{\mu}{2}} = 16\ cm$$

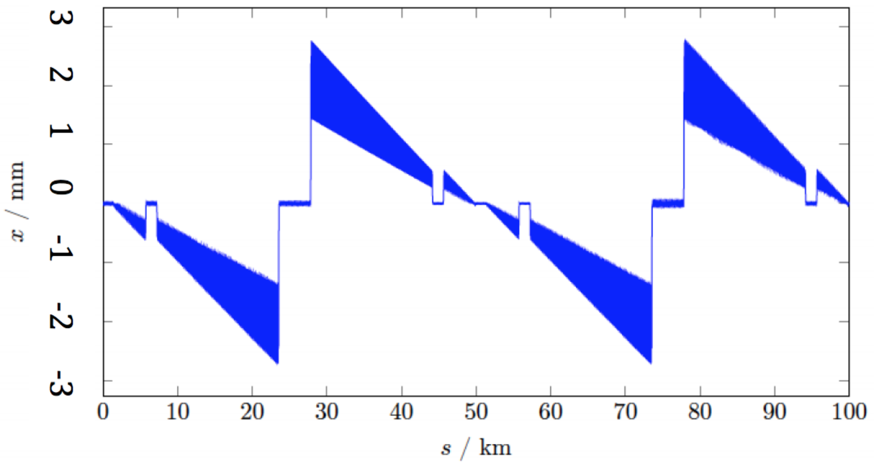
$$D_{min} = \frac{L_{FODO}^2}{\rho}\frac{1-\frac{1}{2}\sin\frac{\mu}{2}}{4\sin^2\frac{\mu}{2}} = 7.6\ cm$$





Saw tooth orbit

- *A particular pattern for the horizontal beam position, due to the energy loss in the arcs and the energy gain in the RF sections*

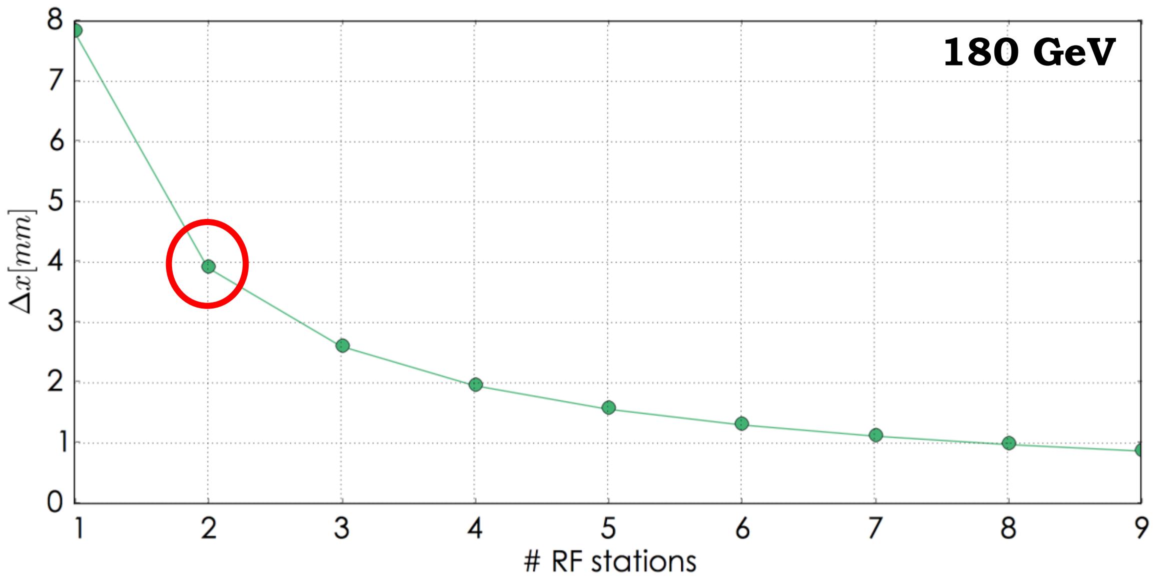


- Energy loss per turn

$$U_0 = 88.46\,\frac{E^4[GeV]}{\rho[m]} = \begin{cases} 0.034\ GeV\ (45\ GeV) \\ 8.8\ GeV\ (180\ GeV) \end{cases}$$

- Orbit offset @ 180GeV

$$\Delta x = D\,\frac{\Delta p}{p} = 4\ mm$$



Quadrupoles (1)

<u>Stability Requirement:</u>

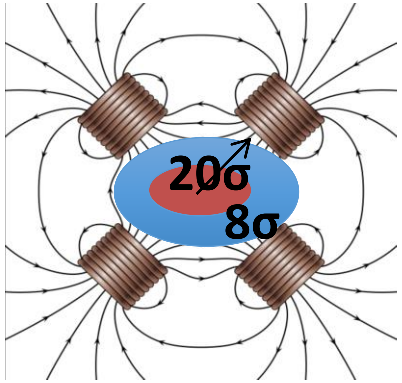
Synchrotron radiation power emitted at 8σ in the quadrupole should not exceed the power emitted in dipoles.

- Beam size σ = 0.4mm
  - *provided by lattice design team:*
    - *hor. equilibrium emittance = 1nm*
    - *beta function arc = 100m*
    - *dispersion =16cm*
  - energy spread from synch. radiation = 0.15%

$$\langle x^2 \rangle = \langle x_\beta^2 \rangle + D^2 \langle \delta^2 \rangle$$

$$\frac{\sigma_{\mathrm{E}}^2}{E^2} = C_{\mathrm{q}}\gamma^2 \frac{\mathcal{I}_3}{J_s \mathcal{I}_2} = C_{\mathrm{q}}\gamma^2 \frac{\mathcal{I}_3}{2\mathcal{I}_2 + \mathcal{I}_{4x} + \mathcal{I}_{4y}}$$

- Minimum aperture $A = 2x\,(20\,\sigma + x_{\text{saw-tooth}}) = 24\text{mm}$
  - consider energy and orbit offsets to estimate sufficient aperture
    - orbit offset from saw-tooth effect = 4mm with 2 RF sections
    - orbit drifts
  - beam pipe thickness



Quadrupole (2)

- Gradient = 6.9 T/m
- Peak pole field = 0.14T
- Strength
- Magnet Length = 3.5m
- Number of windings = 57

$$B_y = g\, x$$

$$k = 0.3\,\frac{g(T/m)}{p(GeV/c)}$$

L = 1/f k

$$g = \frac{2\mu_0 n I}{r^2}$$

Thank you to all the members of the FCCee case study!