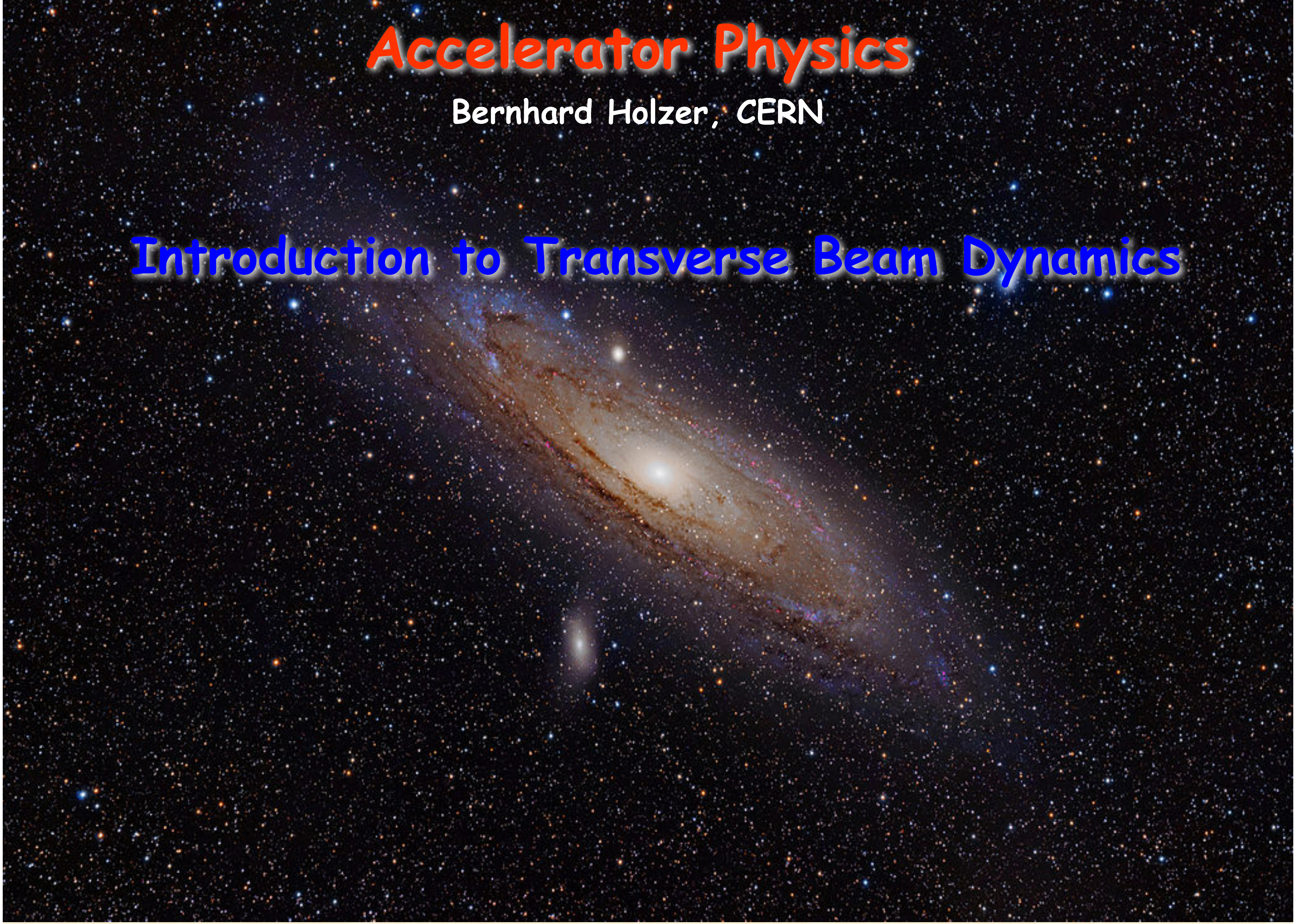
Accelerator Physics

Bernhard Holzer, CERN

Introduction to Transverse Beam Dynamics



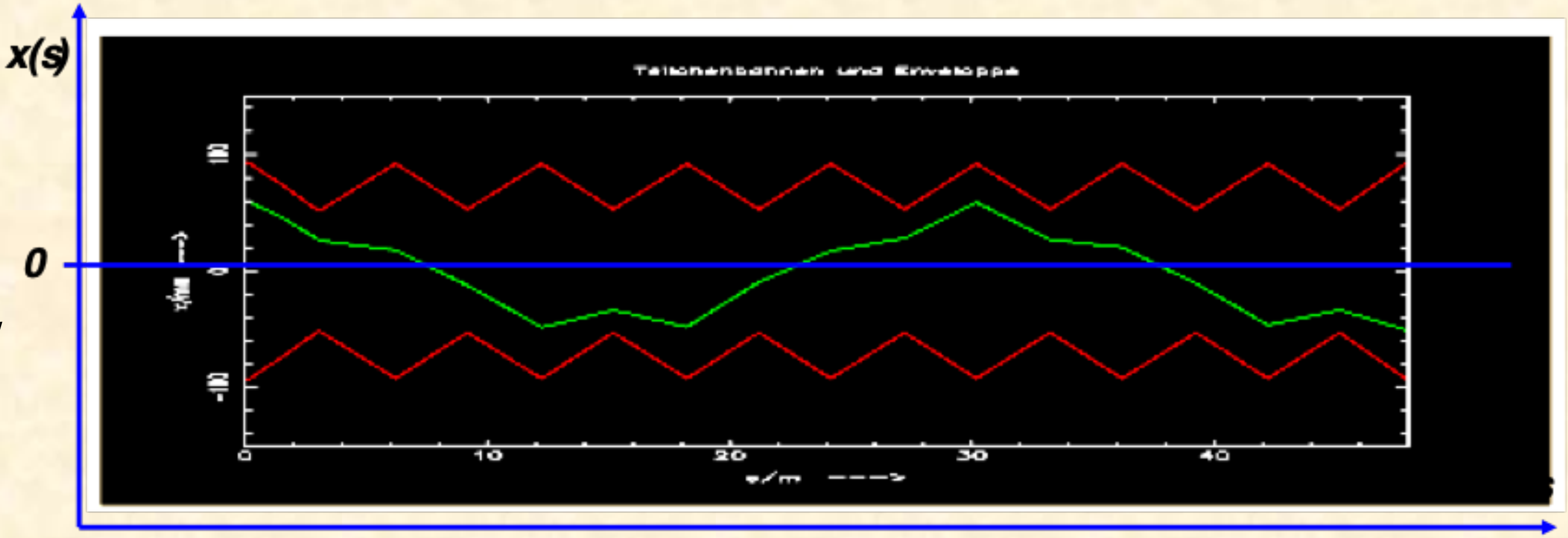
Transverse Beam Dynamics II

The Theory of Synchrotrons: „... how does it work ?"

„...does it ?"

Remember: the "tune" is the oscillation frequency of the beam.

typical values
in a strong
foc. machine:
$x \approx mm,\ x' \leq mrad$



A short advice about "Resonances":

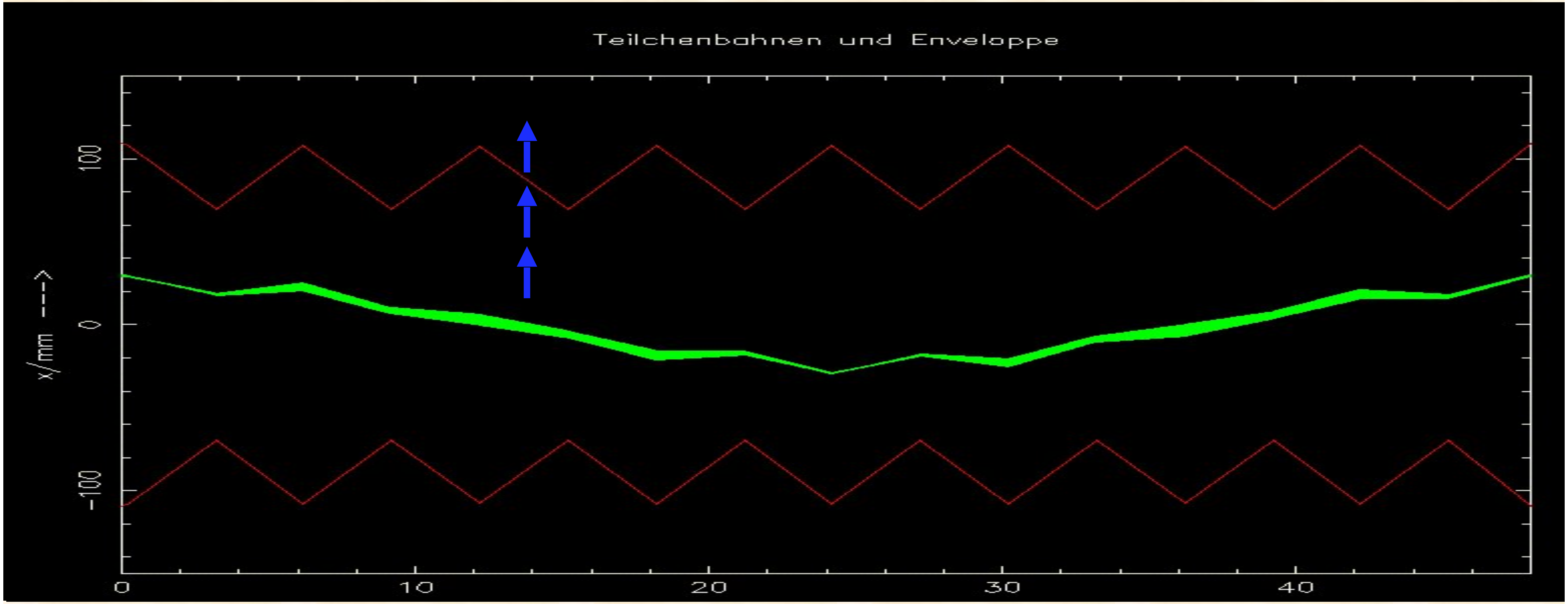
when working with a oscillatory system,
avoid that it "talks" to any (!) external frequency

Most prominent external frequency: Revolution frequency !!

Resonance Problem:

Why do we have so stupid non-integer tunes ?
"Q = 64.0" sounds much better

Qualitatively spoken: *Integer tunes lead to a resonant increase of the closed orbit amplitude in presence of the smallest dipole field error.*



Orbit in case of a small dipole error:

$$x_{co}(s) = \frac{\sqrt{\beta(s)} * \int \frac{1}{\rho_{\tilde{s}}} \sqrt{\beta_{\tilde{s}}} * \cos(\psi_{\tilde{s}} - \psi_s - \pi Q)\, d\tilde{s}}{\underbrace{2 \sin \pi Q}}$$

Assume: Tune = integer $Q = 1 \rightarrow 0$

Tune and Resonances

To avoid resonance conditions the frequency of the transverse motion ***must not be equal to*** *(or a integer multiple of)* ***the revolution frequency***

$1*Q_x = 1 \quad -> \quad Q_x = 1$

$2*Q_x = 1 \quad -> \quad Q_x = 0.5$

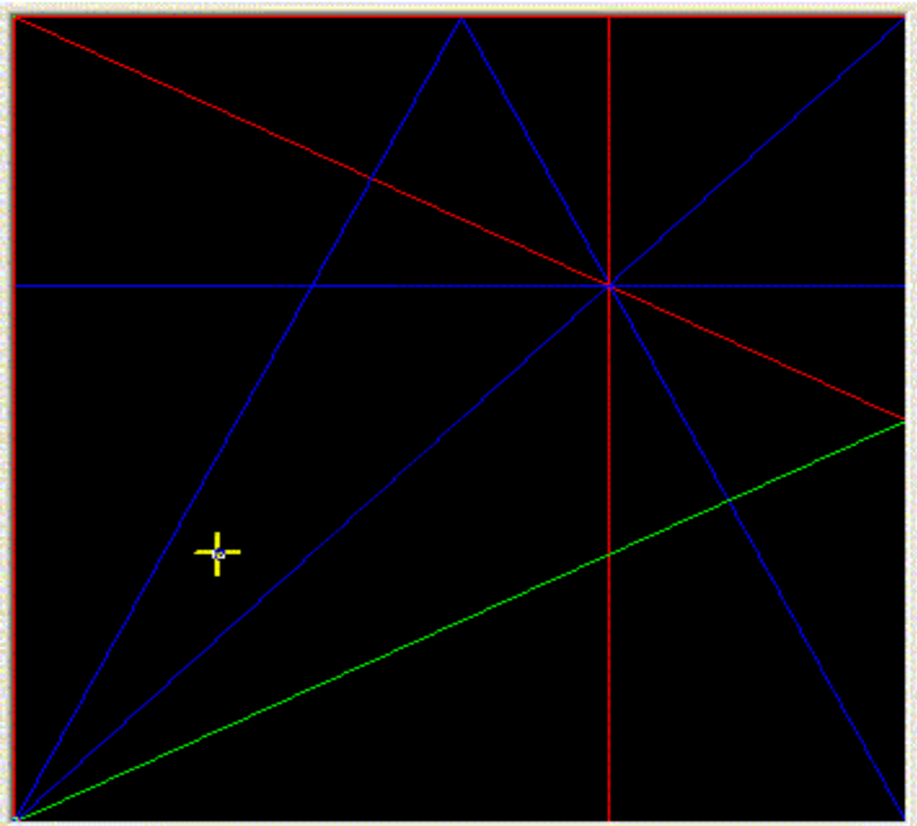
in general:

$m*Q_x + n*Q_y + l*Q_s = integer$

Qy =1.5

Qy =1.3

Qy =1.0

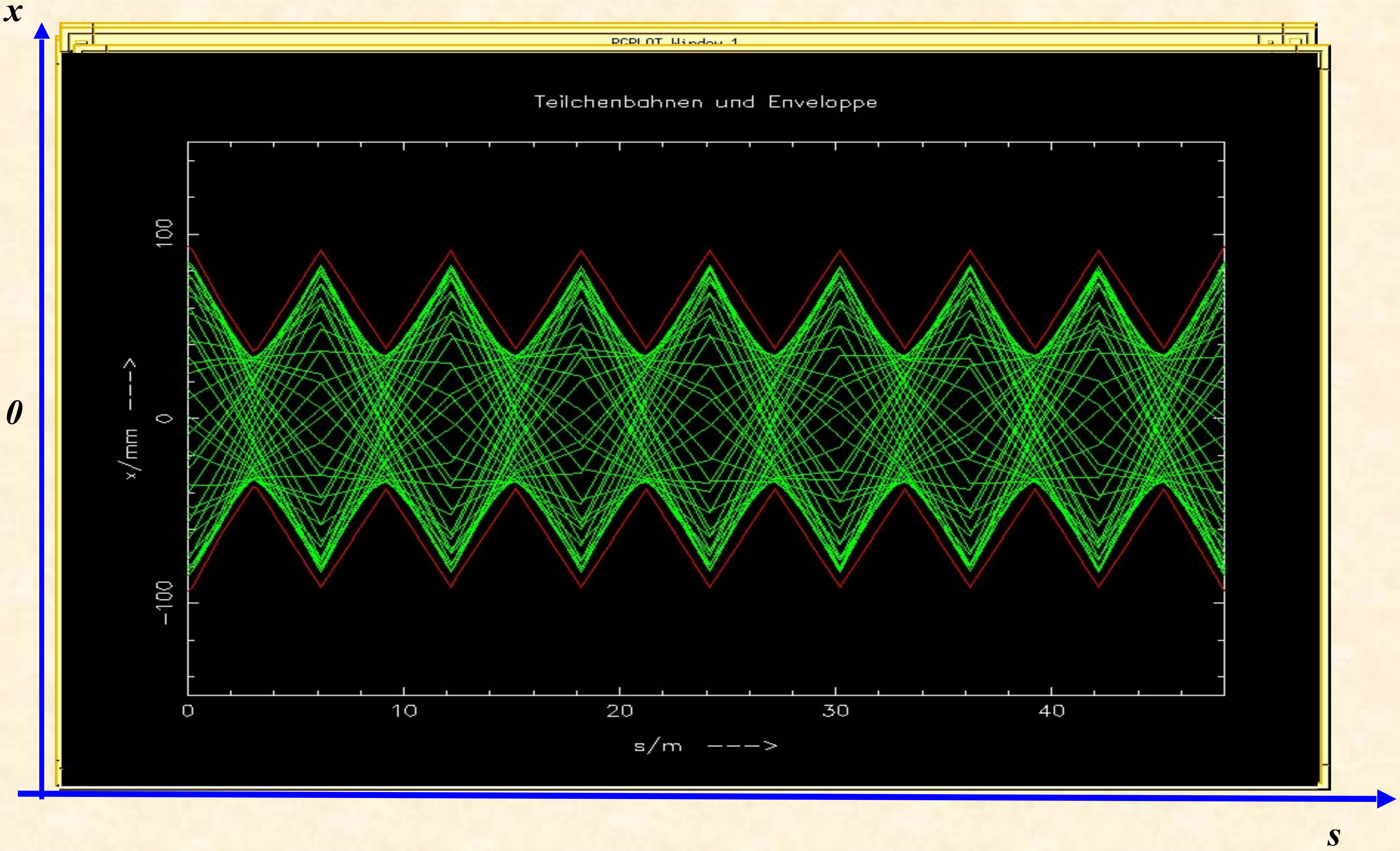


Qx =1.0 Qx =1.3 Qx =1.5

Tune diagram up to 3rd order

Question: ***what will happen, if the particle performs a second turn ?***

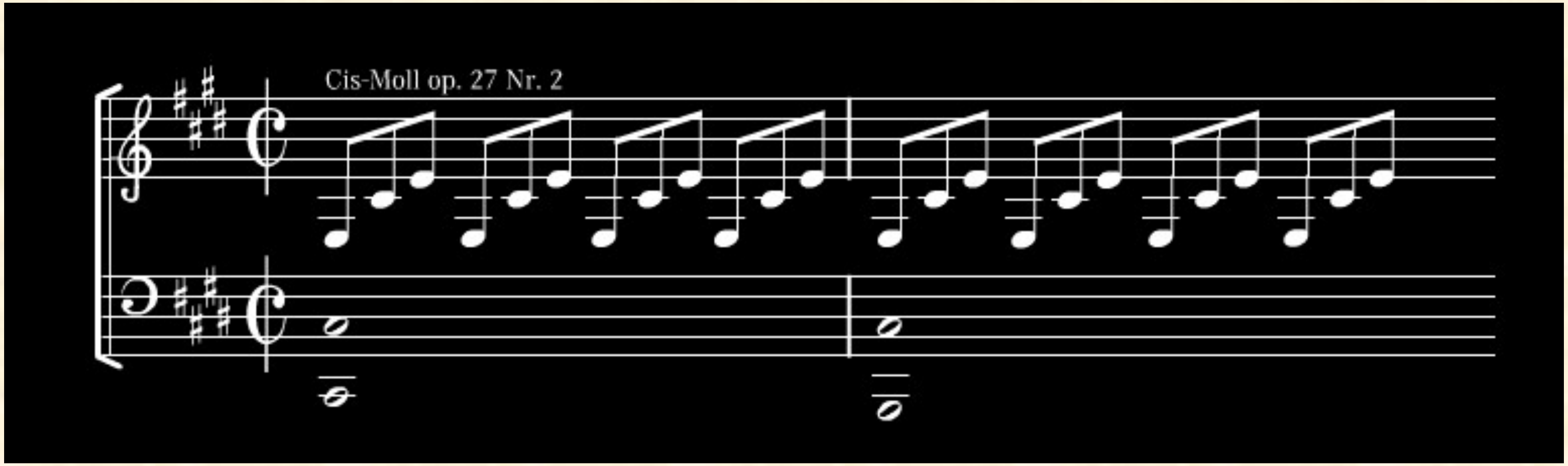
... or a third one or ... 10^{10} turns



19th century:

Ludwig van Beethoven: „Mondschein Sonate“

Sonate Nr. 14 in cis-Moll (op. 27/II,   1801)



Astronomer Hill:

differential equation for motions with periodic focusing properties
„Hill's equation"

Example: particle motion with periodic coefficient

equation of motion: $$x''(s) + k(s) * x(s) = 0$$

restoring force ≠ const,
k(s) = depending on the position s
k(s+L) = k(s), periodic function

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.

7.) The Beta Function

„it is convenient to see"   …   after some beer

... we make two statements:

1.) There exists a mathematical function, that defines the envelope of all particle trajectories and so can act as measure for the beam size. We call it the β – function.

2.) Whow !!

A particle oscillation can then be written in the form

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \phi)$$

ε, Φ = integration constants determined by initial conditions

β(s) periodic function given by focusing properties of the lattice ↔ quadrupoles

$$\beta(s+L) = \beta(s)$$

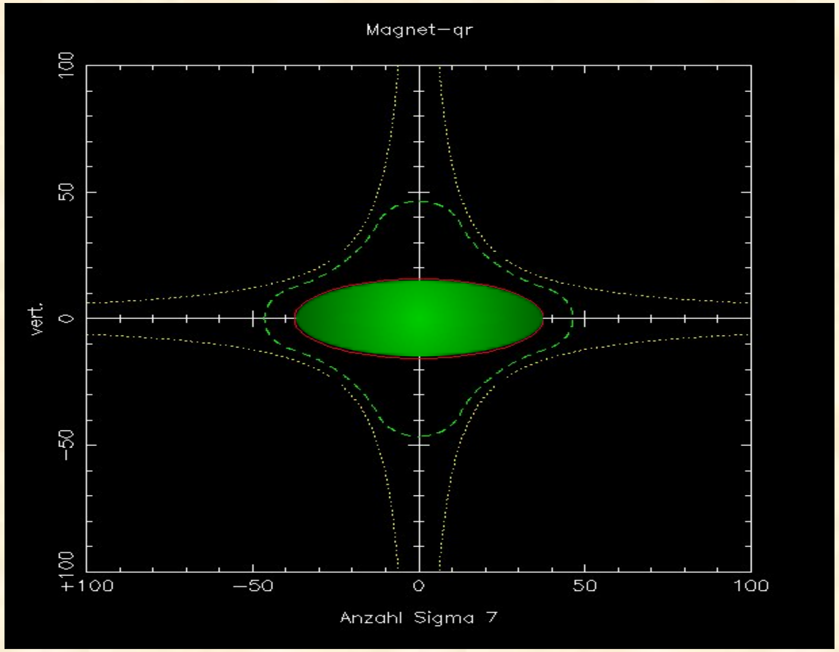
ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter,
cannot be changed by the foc. properties.
scientifiquely spoken: area covered in transverse x, x´ phase space

… and it is constant !!!

The Beta Function

If we obtain the x, x' coordinates of a particle trajectory via

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M_{s_1,s_2} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$

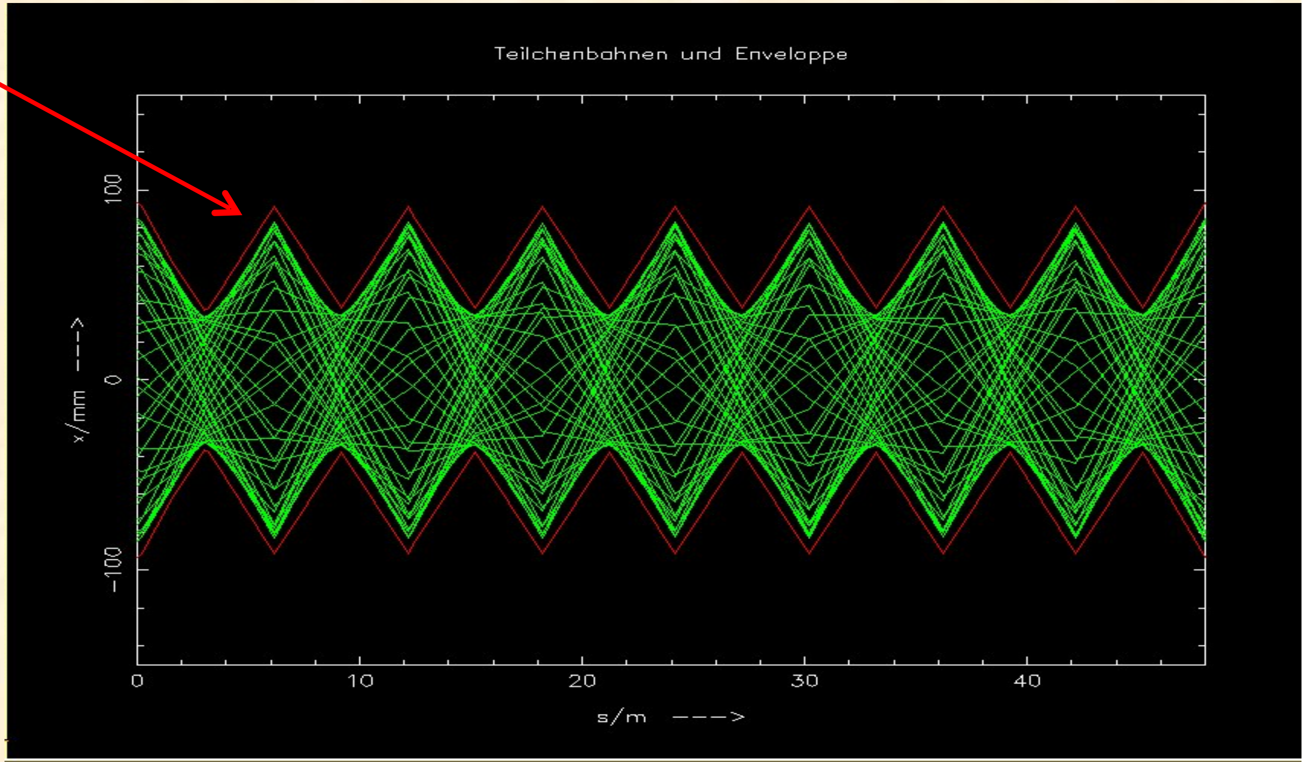


The maximum size of any particle amplitude at a position "s" is given by

$$\hat{x}(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}$$

β determines the beam size
... the envelope of all particle trajectories at a given position "s" in the storage ring.

It reflects the periodicity of the magnet structure.



8.) Beam Emittance and Phase Space Ellipse

general solution of Hill equation

(1) $$x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}\,\cos(\psi(s)+\phi)$$

(2) $$x'(s) = -\frac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}}\left\{\alpha(s)\cos(\psi(s)+\phi)+\sin(\psi(s)+\phi)\right\}$$

from (1) we get

$$\cos(\psi(s)+\phi) = \frac{x(s)}{\sqrt{\varepsilon}\,\sqrt{\beta(s)}}$$

$$\alpha(s) = \frac{-1}{2}\beta'(s)$$

$$\gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$$

Insert into (2) and solve for ε

$$\varepsilon = \gamma(s)\,x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\,x'^2(s)$$

***** *ε* **is a** ***constant*** ***of the motion … it is independent of „s“***
***** ***parametric representation of an ellipse in the x x‘ space***
***** ***shape and orientation of ellipse are given by α, β, γ***

Phase Space Ellipse

particel trajectory: $x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos\{\psi(s)+\phi\}$

max. Amplitude: $\hat{x}(s) = \sqrt{\varepsilon\beta}$ → *x´ at that position …?*

… put $\hat{x}(s)$ *into* $\varepsilon = \gamma(s)\, x^2(s) + 2\alpha(s) x(s) x'(s) + \beta(s)\, x'^2(s)$ *and solve for x´*

$$\varepsilon = \gamma \cdot \varepsilon\beta + 2\alpha\sqrt{\varepsilon\beta}\cdot x' + \beta x'^2$$

→ $x' = -\alpha \cdot \sqrt{\varepsilon / \beta}$

* *A high β-function means a large beam size and a small beam divergence.* **!**
  … et vice versa !!!

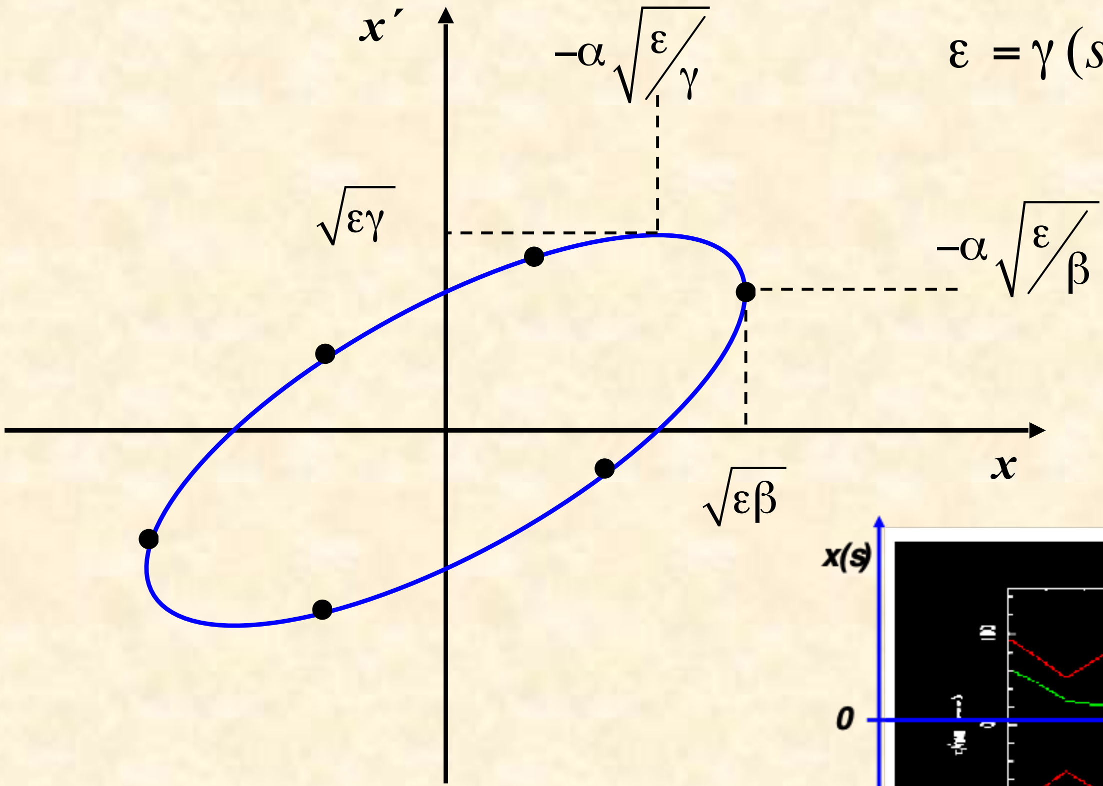
* *In the middle of a quadrupole β = maximum,*
  α = zero } $x' = 0$

… and the ellipse is flat

Beam Emittance and Phase Space Ellipse

***In phase space x, x' a particle* oscillation, observed at a given position "s" in the ring *is running on an ellipse* ... making Q revolutions per turn.**

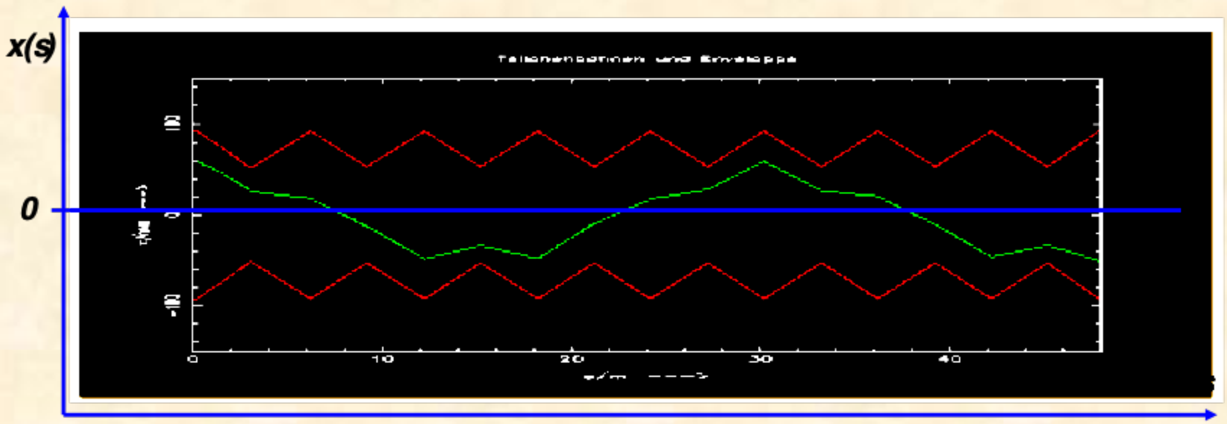
$$x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}\,\cos(\psi(s) + \phi)$$



$$\varepsilon = \gamma(s) * x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$

Liouville:* in reasonable storage rings *area in phase space is constant.

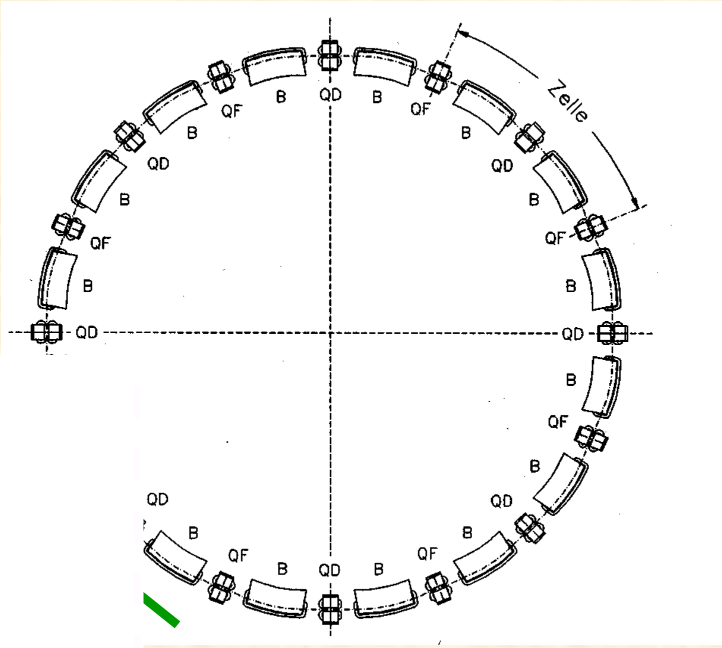
A = π*ε=const

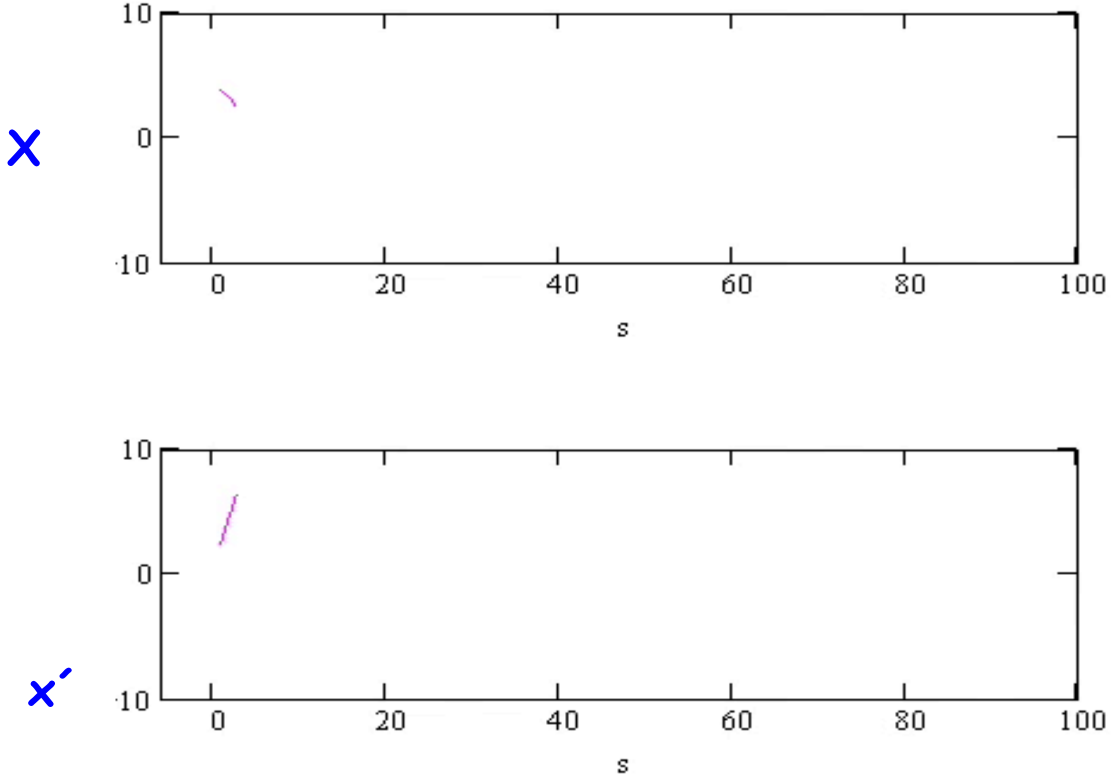


Particle Tracking in a Storage Ring

Calculate x, x´ for each linear accelerator element according to matrix formalism

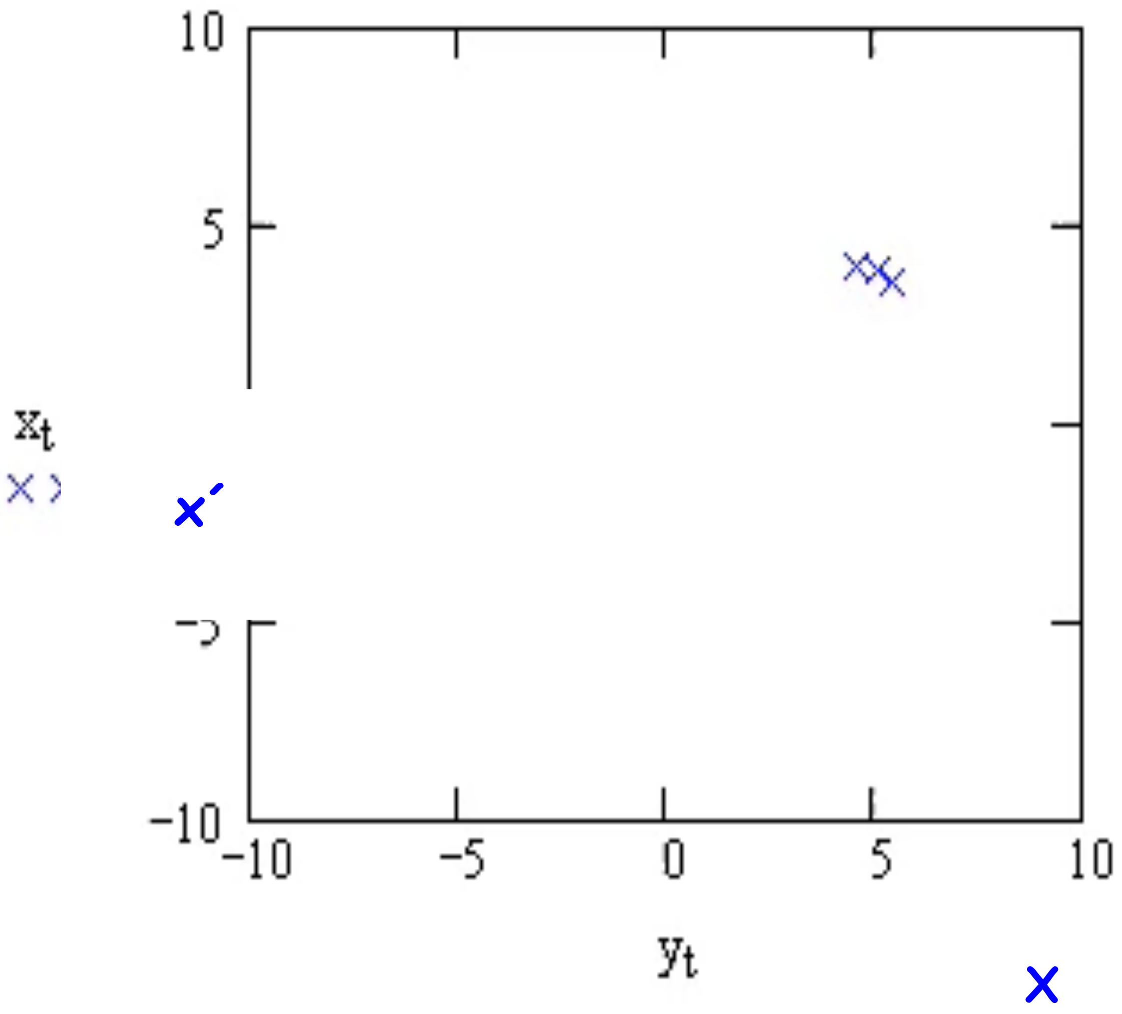
plot x, x´as a function of „s"





… and now the ellipse:

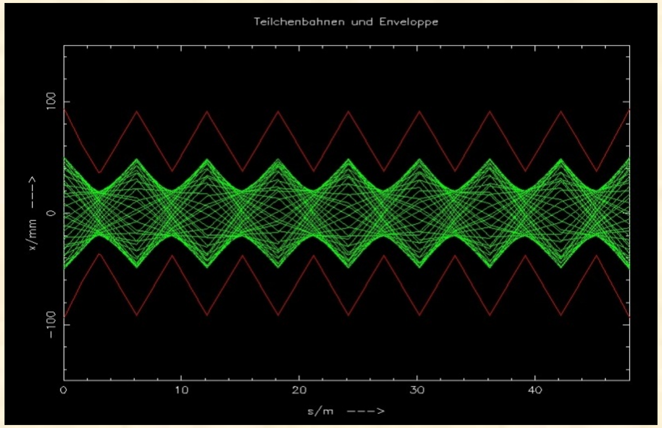
note for each turn x, x´ at a given position „s_1“ and plot in the phase space diagram

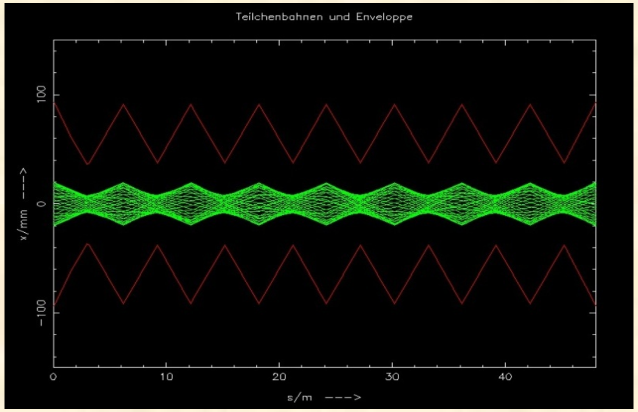


… just as Big Ben

… and just as any harmonic pendulum

Emittance of the Particle Ensemble:



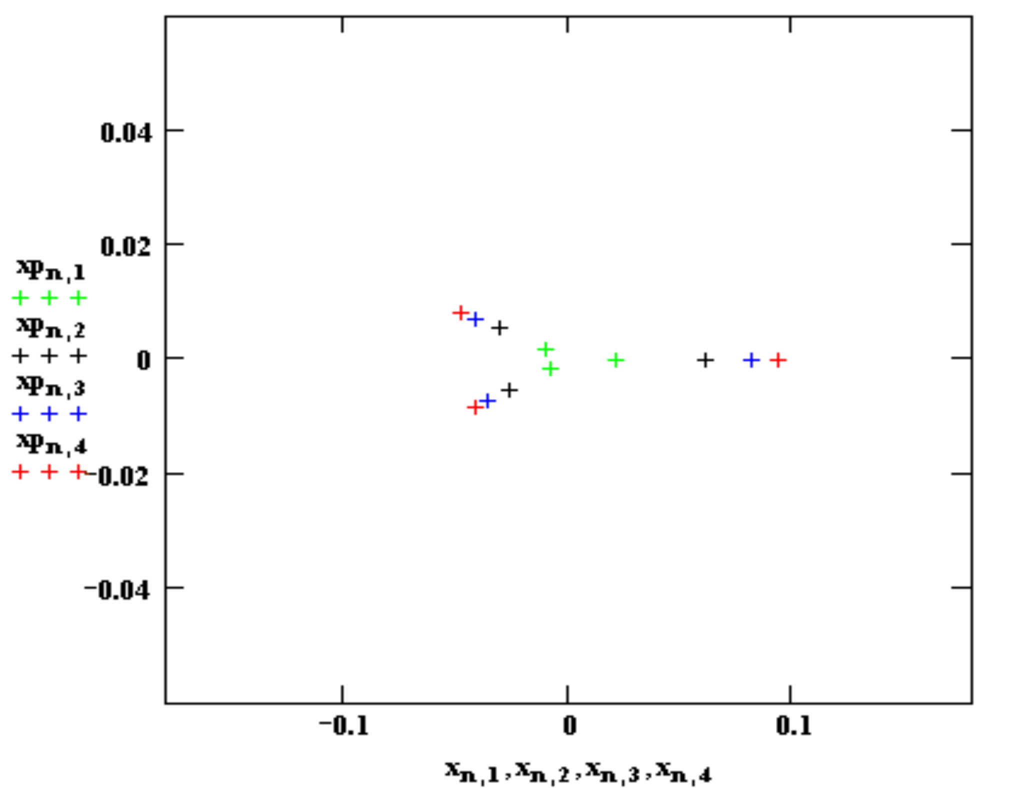


… to be very clear:

as long as our particle is running on an ellipse in x, x' space ...

everything is alright, the beam is stable and we can sleep well at nights.

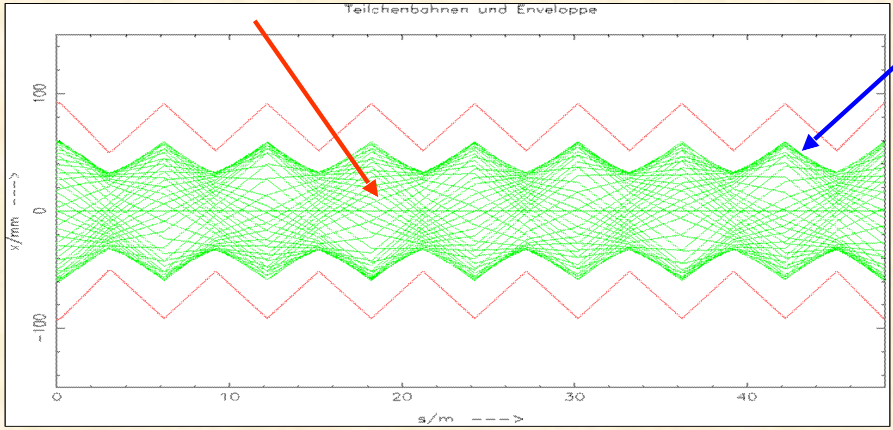
If however we have scattering at the rest gas, or non-linear fields, or beam collisions (!) the particle will perform a jump in x' and ε will increase



Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}\cdot\cos(\Psi(s)+\phi)$$

$$\hat{x}(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}$$



single particle trajectories, N ≈ 10 [11] ***per bunch***

Gauß Particle Distribution:

$$\rho(x) = \frac{N\cdot e}{\sqrt{2\pi}\,\sigma_x}\cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$$

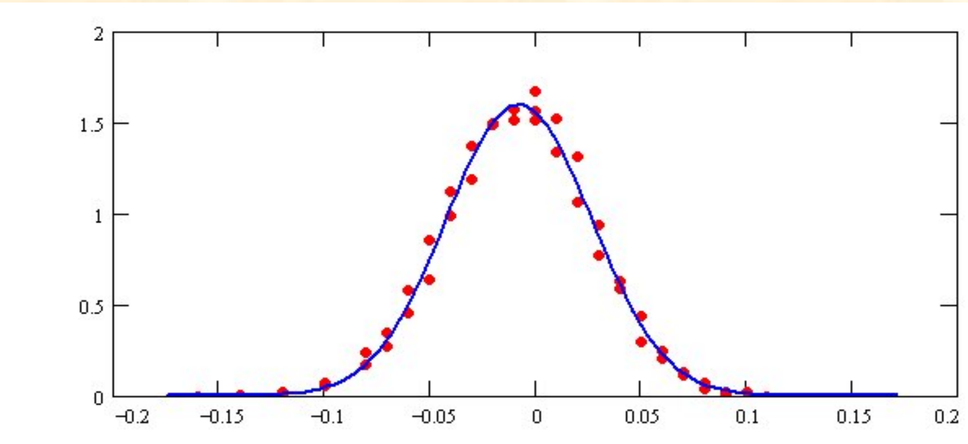
particle at distance 1 σ from centre
↔ 68.3 % of all beam particles

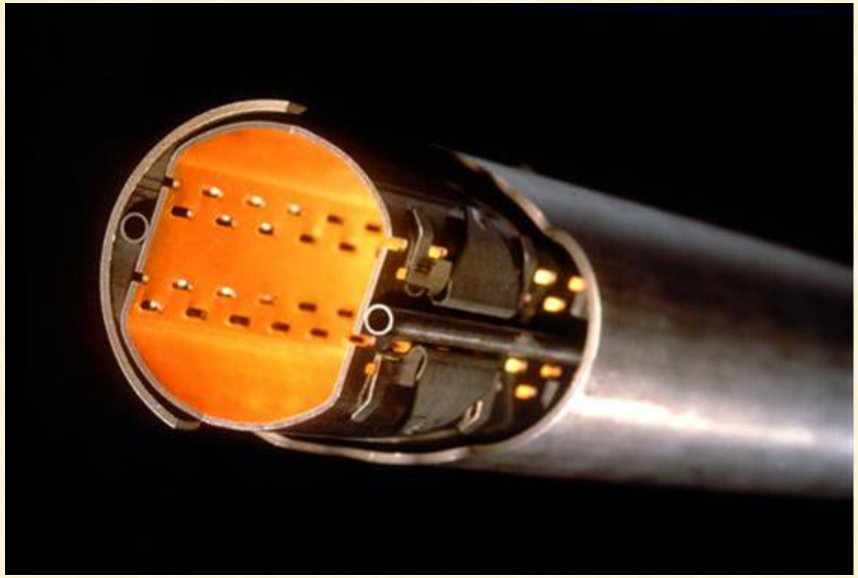
LHC:

$$\beta = 180\,m$$

$$\varepsilon = 5*10^{-10}\,m\,rad$$

$$\sigma = \sqrt{\varepsilon*\beta} = \sqrt{5*10^{-10}\,m*180\,m} = 0.3\,mm$$

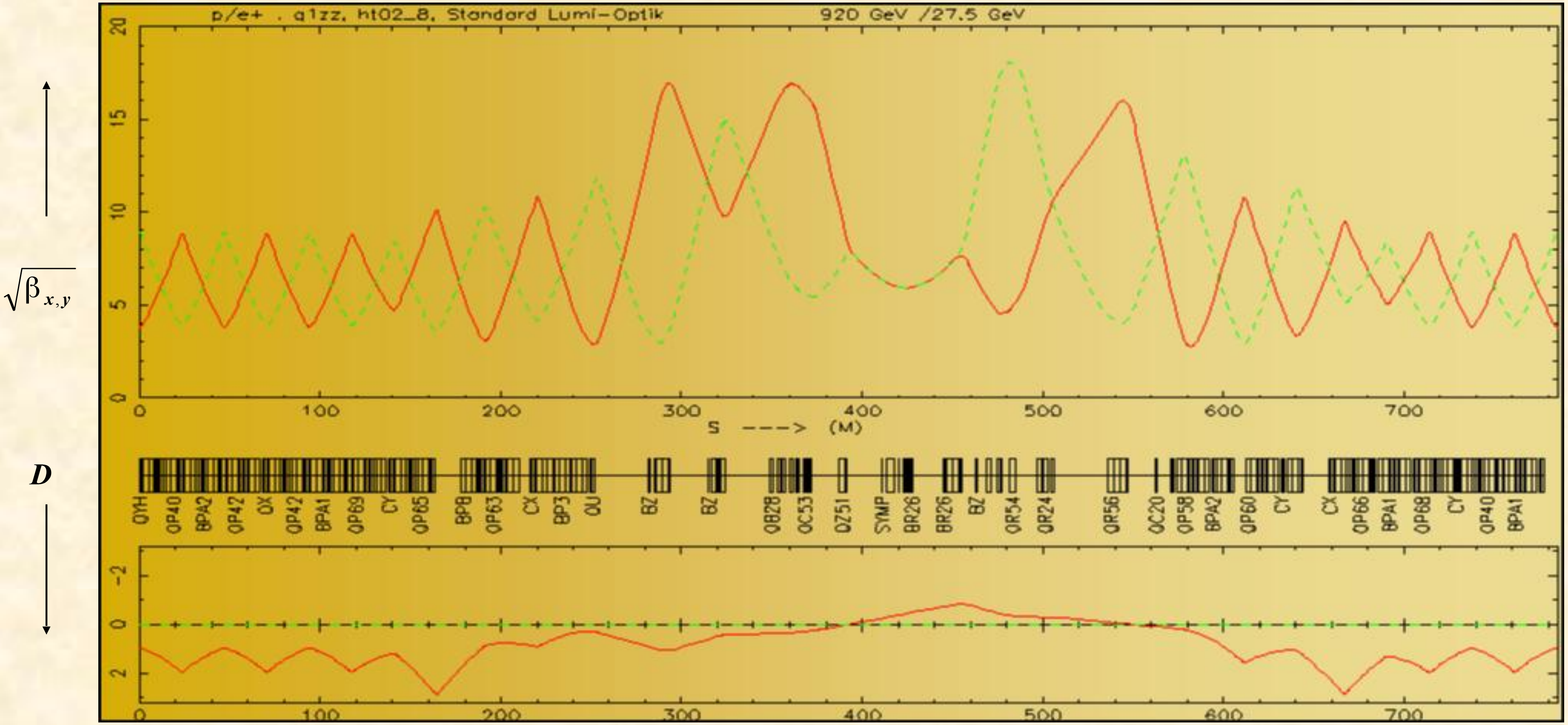




aperture requirements: $r_0 = 18*\sigma$

The „not so ideal“ World

Lattice Design in Particle Accelerators



1952: Courant, Livingston, Snyder:

Theory of strong focusing in particle beams

Recapitulation: ...the story with the matrices !!!

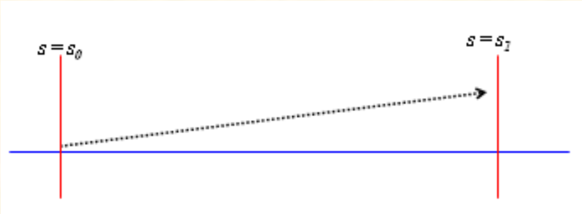
Equation of Motion:

$$x'' + K\,x = 0 \qquad K = 1/\rho^2 - k \;\ldots\; \textit{hor. plane:}$$

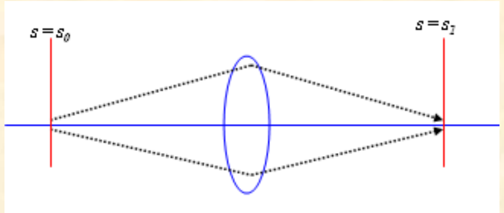
$$K = k \qquad \ldots\; \textit{vert. Plane:}$$

Solution of Trajectory Equations

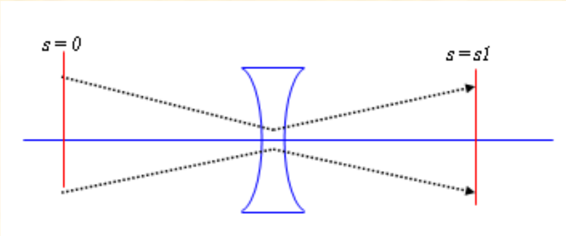
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$



$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}l) & \frac{1}{\sqrt{|K|}}\sin(\sqrt{|K|}l) \\ -\sqrt{|K|}\sin(\sqrt{|K|}l) & \cos(\sqrt{|K|}l) \end{pmatrix}$$



$$M_{defoc} = \begin{pmatrix} \cosh(\sqrt{|K|}l) & \frac{1}{\sqrt{|K|}}\sinh(\sqrt{|K|}l) \\ \sqrt{|K|}\sinh(\sqrt{|K|}l) & \cosh(\sqrt{|K|}l) \end{pmatrix}$$

$$M_{total} = M_{QF} * M_D * M_B * M_D * M_{QD} * M_D * \quad \ldots$$

9.) Lattice Design: „… how to build a storage ring“

Geometry of the ring: $\rightarrow B * \rho = p / e$

p = momentum of the particle,
ρ = curvature radius

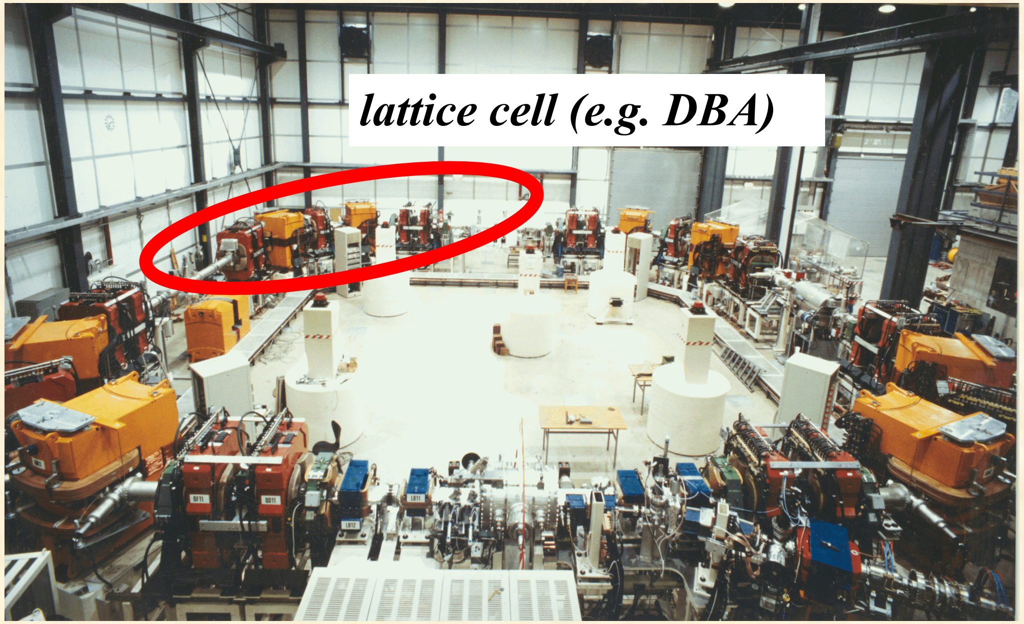
Bρ= beam rigidity

Circular Orbit: bending angle of one dipole

$$\alpha = \frac{ds}{\rho} \approx \frac{dl}{\rho} = \frac{Bdl}{B\rho}$$

The angle defined by one dipole magnet is defined by

$$\frac{B_{field} * length}{momentum}$$



The angle passed through in one revolution must be 2π, so for a full circle

$$\Sigma_{dipoles}(\alpha) = \frac{\oint Bdl}{B\rho} = 2\pi$$

$$\oint Bdl = 2\pi * \frac{p}{e}$$

… defines the integrated dipole field around the machine.

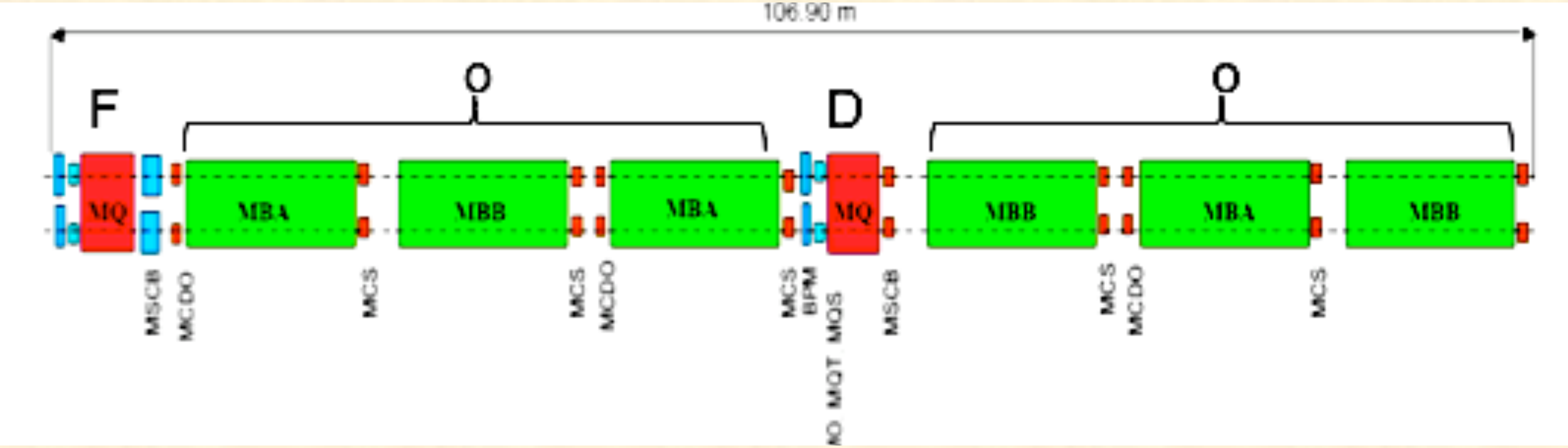
Example LHC:

7000 GeV Proton storage ring
dipole magnets N = 1232
l = 15 m
q = +1 e

$$\int B\,dl \approx N\,l\,B = 2\pi\ p/e$$

$$B \approx \frac{2\pi\ 7000\ 10^{9}\,eV}{1232\ 15\,m\ 3\ 10^{8}\,\frac{m}{s}\ e} = 8.3\ Tesla$$

The Basic Cell of LHC:
… a 90° FoDo lattice



equipped with additional corrector coils

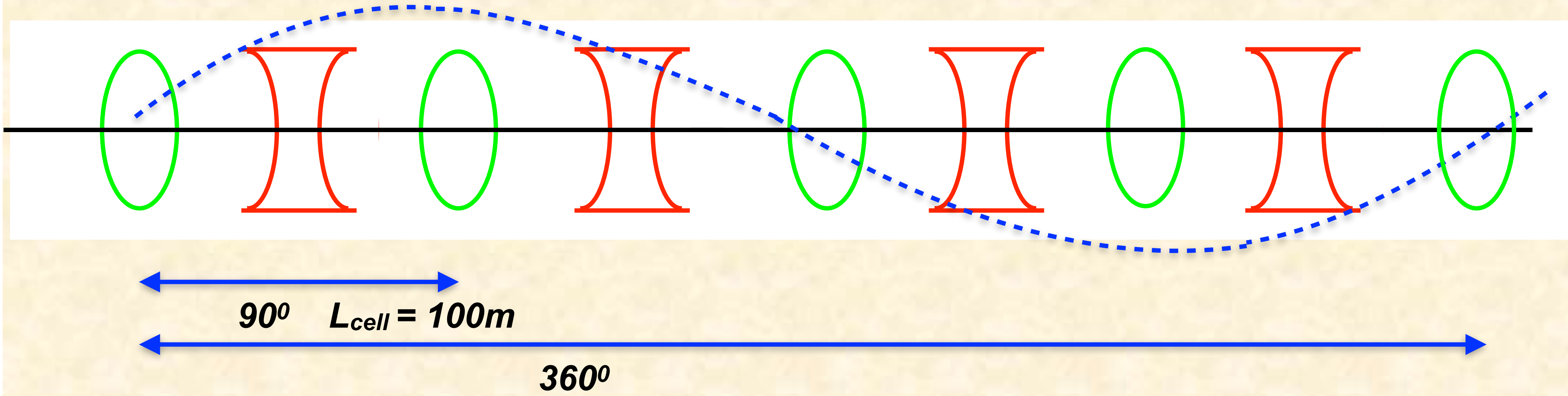
MB: main dipole
MQ: main quadrupole
MQT: Trim quadrupole
MQS: Skew trim quadrupole
MO: Lattice octupole (Landau damping)
MSCB: Skew sextupole
Orbit corrector dipoles
MCS: Spool piece sextupole
MCDO: Spool piece 8 / 10 pole
BPM: Beam position monitor + diagnostics

Dipoles

... The sum of the dipole fileds (in Tesla) multiplied by their length defines the particle momentum that we store in the ring.

Quadrupoles (gradient * length) define the transverse oscillation frequency. In LHC we need 4 cells (100m long each) for a full 360^0 oscillation, which is called a FoDo lattice with 90^0 phase advance.

And just like in playing a guitar, the higher the restoring force (quad gradient) the higher is the frequency (i.e. the phase advance per cell or for the complte ring the tune) … and we could even hear it !!!



The Tune ...

...is the number of transverse oscillations per turn and corresponds to the „Eigenfrequency" or sound of the particle oscillations. As in any oscillating system (e.g. pendulum) we have to avoid resonance conditions between the eigenfrequemcy of the system (= partcicle) and any external frequency that might act on the beam. Most prominent external frequency is the revolution frequemcy itself !! -> avoid integer tunes.

The Beta function

shows the overall effect of all focusing fields; it has a certain value (m) that depends on the actual position in the ring, and is a measure of the transverse beam size.

The beam emittance

describes - independent of the focusing fields - the quality of the particle ensemble. It measures the area in phase space and can be considered like the temperature of a gas.

Small emittance —> high beam quality.

Together with the beta function it defines the beam dimension.

And in between the arcs ???

What about ...

Short Straight Sections

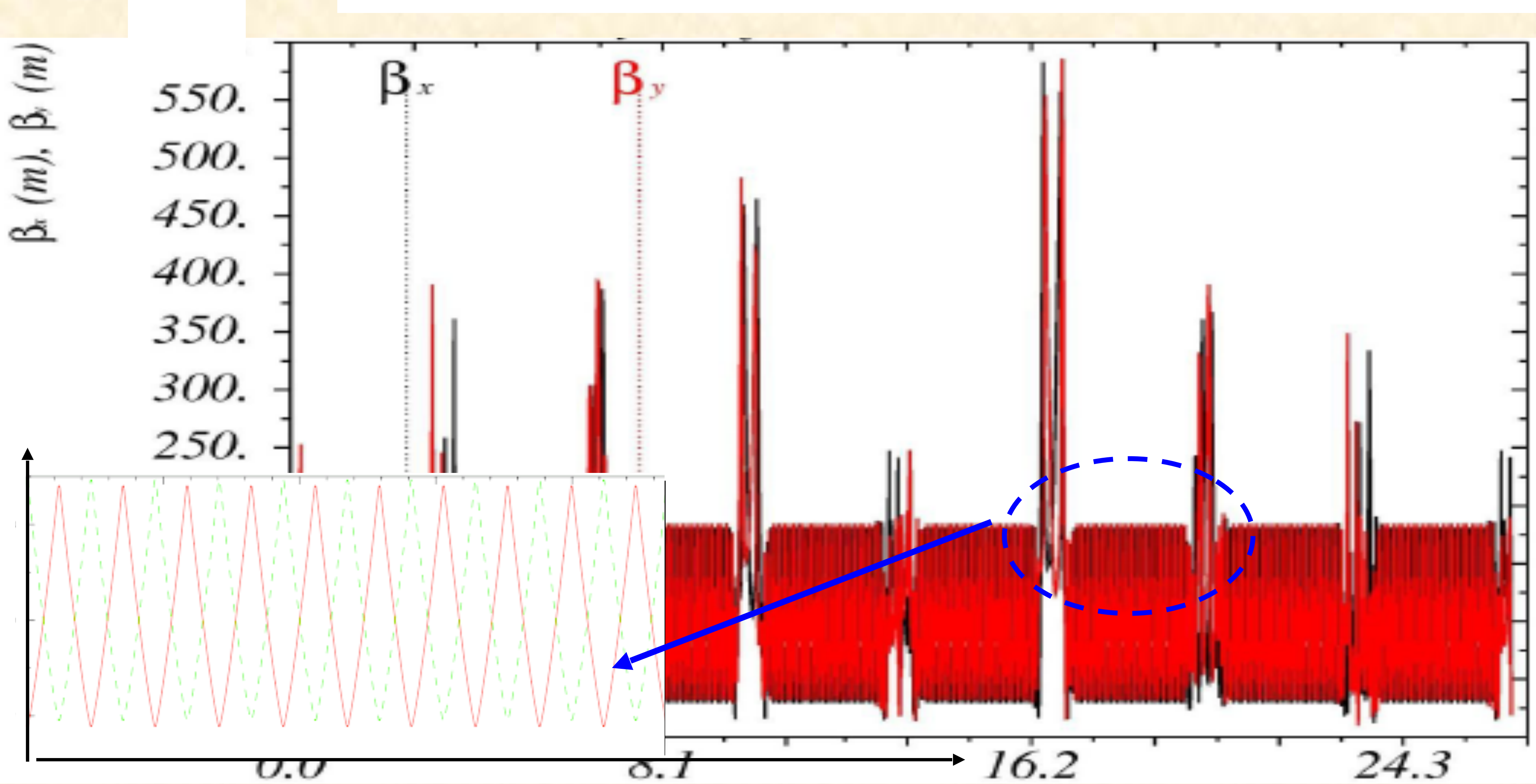
Long Straight Sections

Mini-Beta Insertions

etc etc

FoDo-Lattice

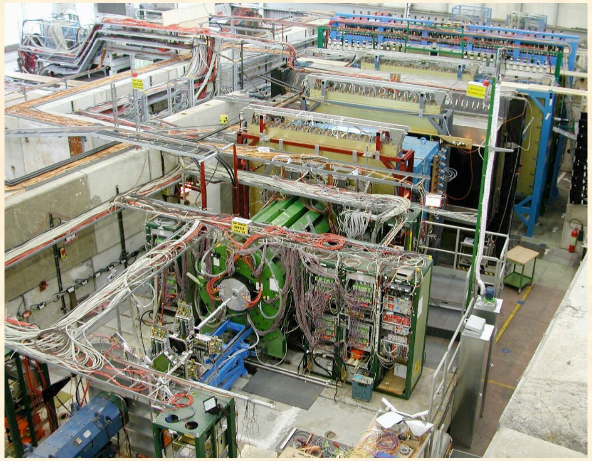
A magnet structure consisting of focusing and defocusing quadrupole lenses in alternating order with nothing in .
(Nothing = elements that can be neglected on first sight: drift, bending magnets, RF structures ... and especially experiments...)



Starting point for the calculation: in the middle of a focusing quadrupole
Phase advance per cell $\mu = 45°$,
→ calculate the twiss parameters for a periodic solution

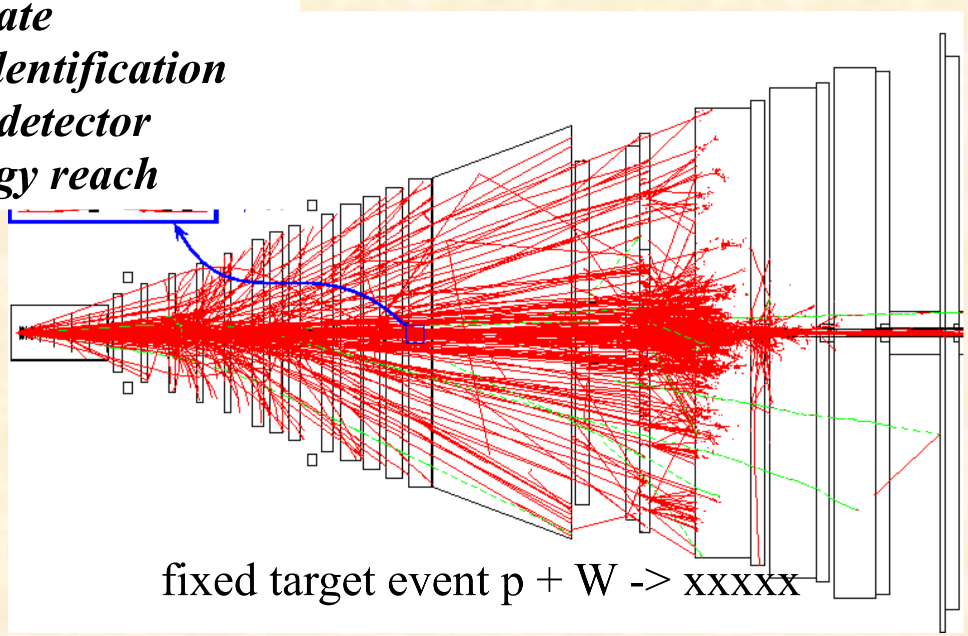
11.) The structure of matter:

Fixed target experiments:



HARP Detector, CERN

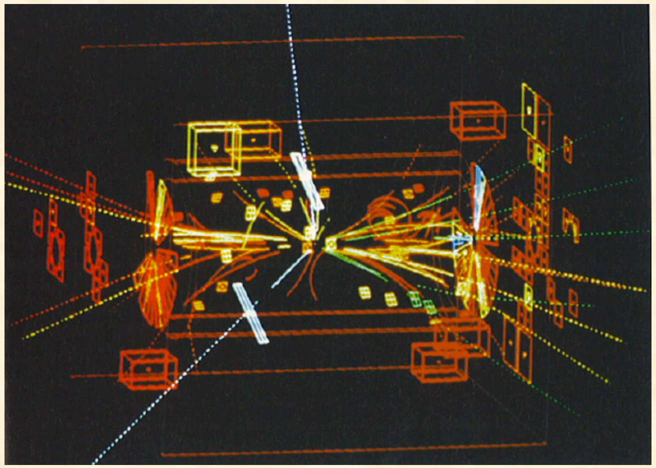
high event rate
easy track identification
asymmetric detector
limited energy reach



fixed target event p + W -> xxxxx

Collider experiments:

$E=mc^2$

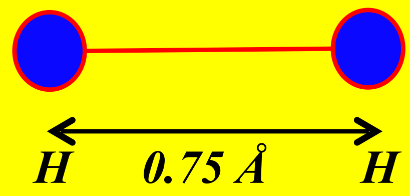


low event rate (luminosity)
but higher energy at IP

$$E_{lab} = E_{cm}$$

Particle Density in matter

Hydrogen molecule

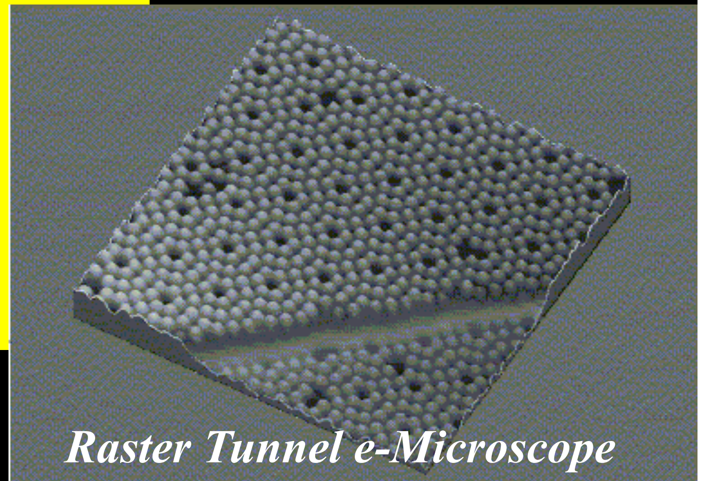


Atomic Distance in Hydrogen Molecule

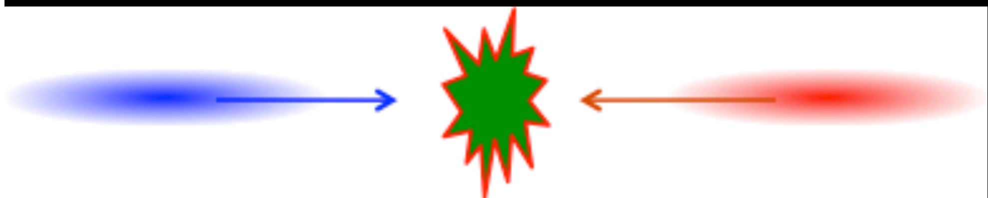
$R_B \approx 0.5$ Å

in solids / fluids $\lambda \approx 2.1 \;\ldots\; 2.9$ Å

in gases $\lambda \approx 33$ Å $= 3.3$ nm

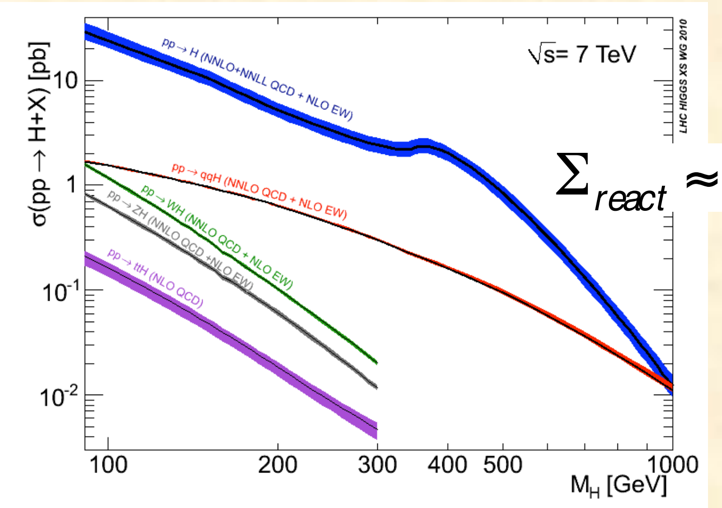


Particle Distance in Accelerators: $\lambda \approx = 6000$ Å $= 600$ nm *(Arc LHC)*

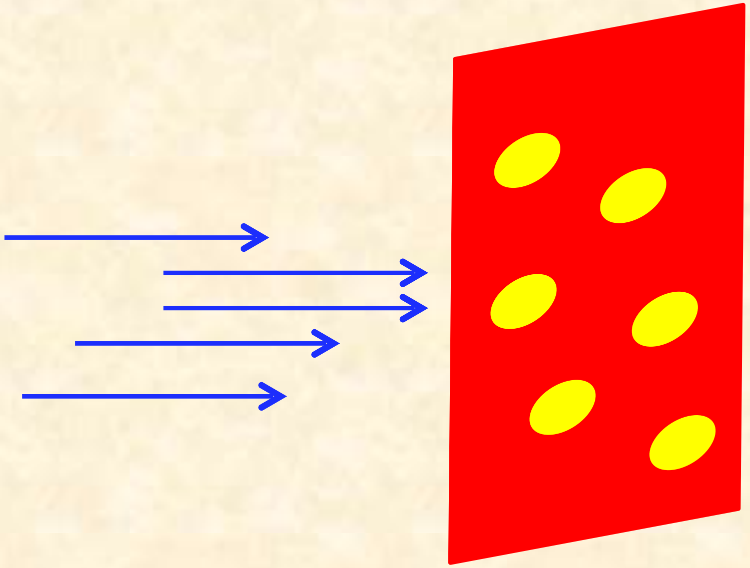


Problem: Our particles are VERY small !!

Overall cross section of the Higgs:



$$\Sigma_{react} \approx 1\, pb$$

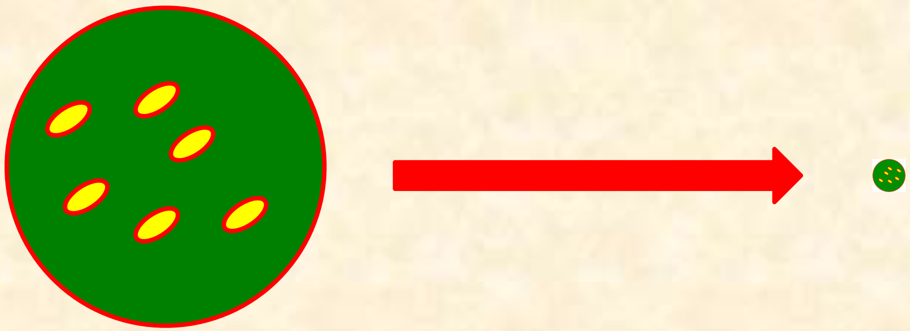


$$1\,b = 10^{-24}\, cm^2$$

$$1\,pb = 10^{-12} * 10^{-24}\, cm^2 = 1/mio * 1/mio * 1/mio * 1/mio * 1/mio * 1/10000\; mm^2$$

The particles are "very small"

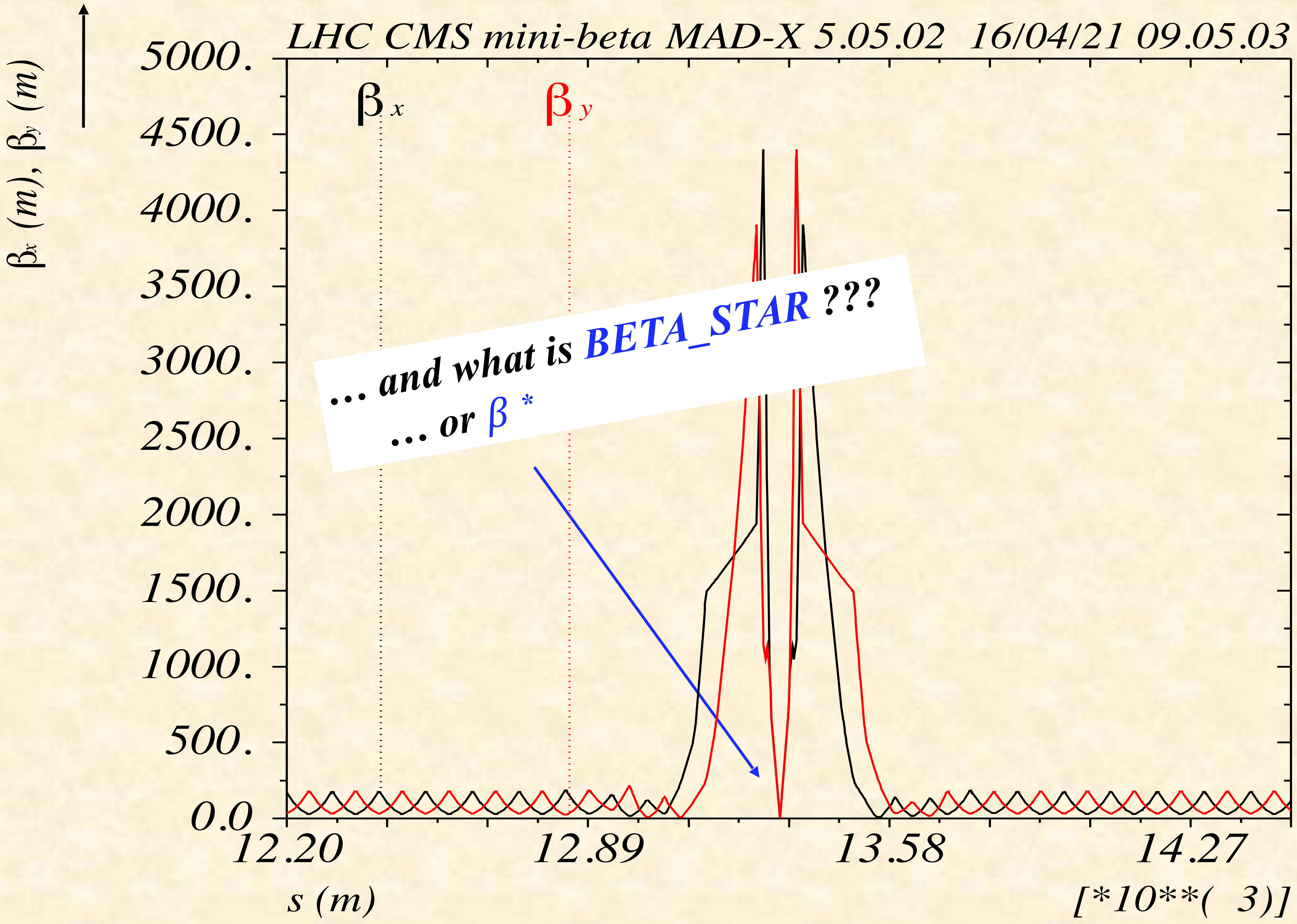
The only chance we have:
compress the transverse beam size … at the IP



LHC typical:

σ = 0.1 mm → 16 μm

12.) Insertions

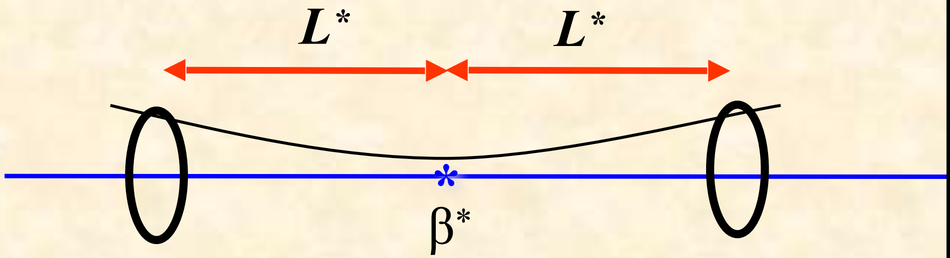


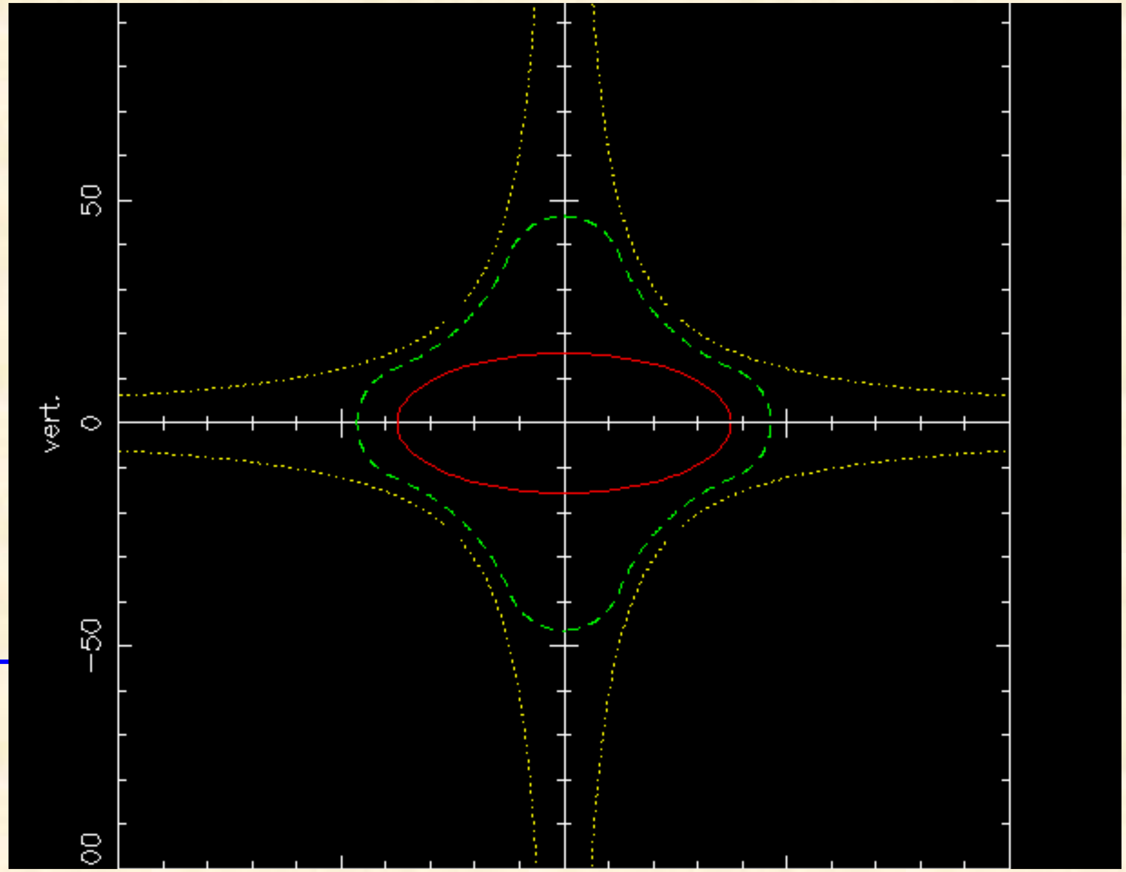
β-Function in a Drift

In a drift, without focusing, the β-function is increasing quadratically.
At the end of a long symmetric drift space the beta function reaches its maximum value in the complete lattice.
-> here we get the largest beam dimension.

-> keep L^* as small as possible

$$\beta(L) = \beta^* + \frac{L^2}{\beta^*}$$

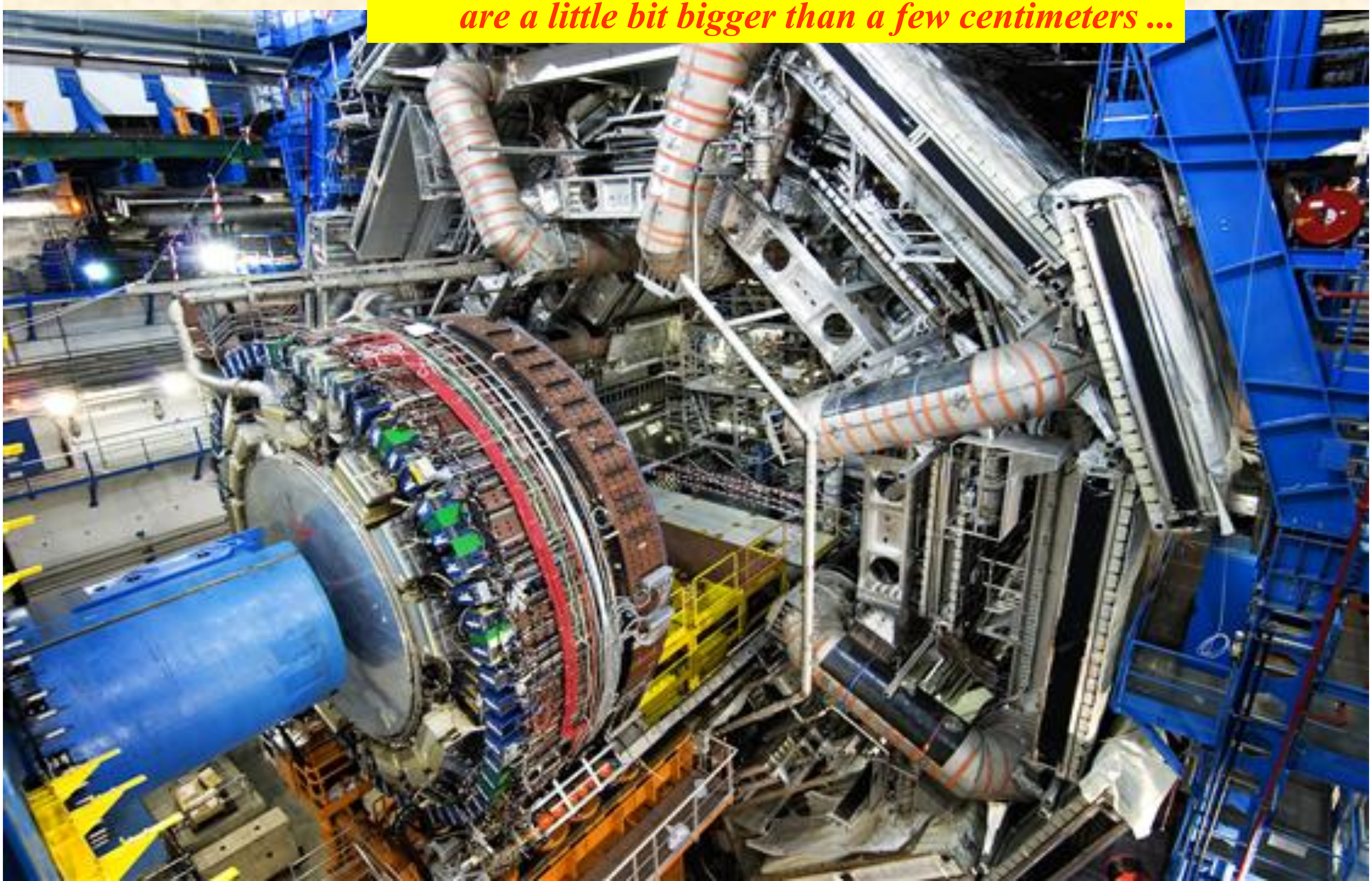




7 sigma beam size inside a mini beta quadrupole

... clearly there is an

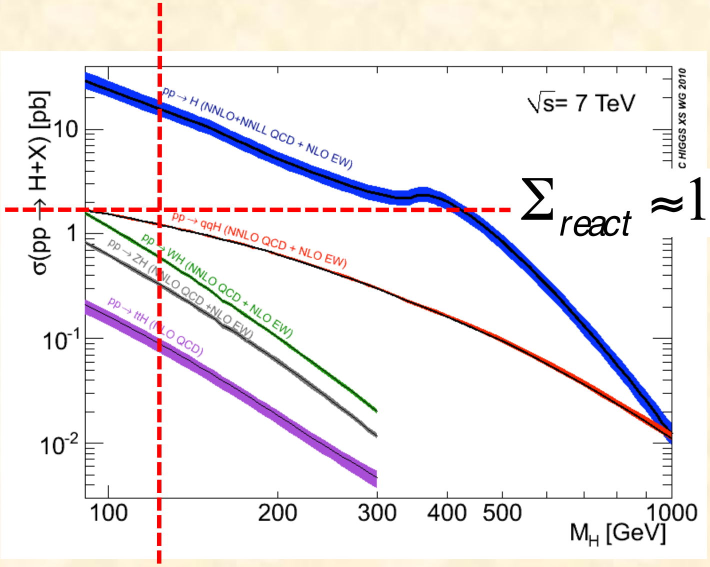
... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...



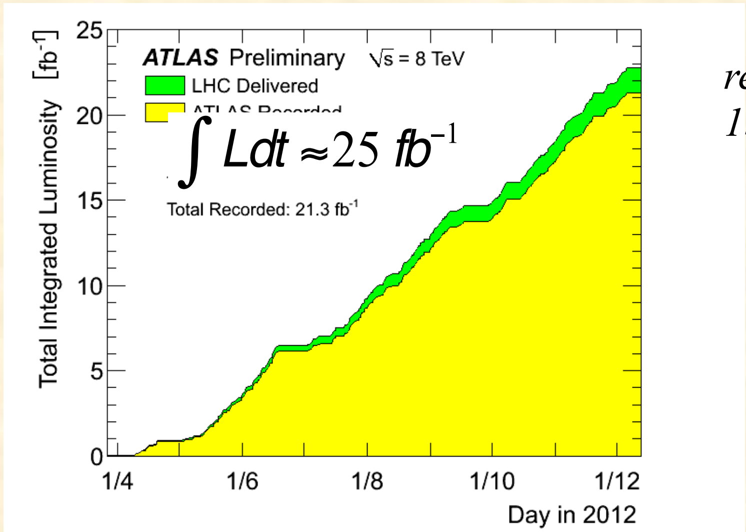
13.) The Mini-β Insertion & Luminosity:

production rate of events is determined by the cross section Σ_{react}
and a parameter L that is given by the design of the accelerator:
… the luminosity

$$R = L * \Sigma_{react} \quad \approx 10^{-12} b \cdot 25 \frac{1}{10^{-15} b} = some\, 1000\, H$$



$$\Sigma_{react} \approx 1\, pb$$



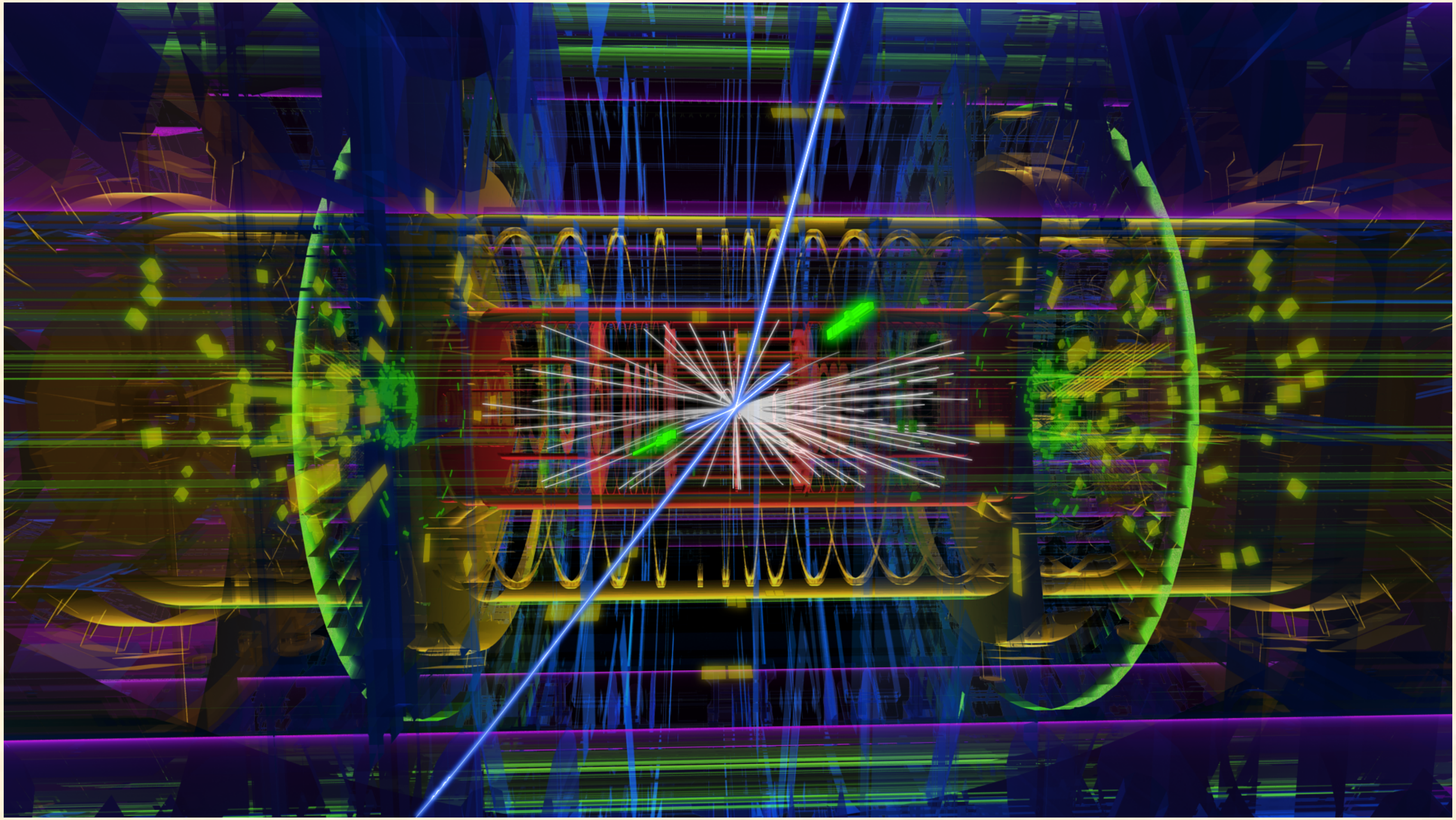
remember:
$1b = 10^{-24}\, cm^2$

The luminosity is a storage ring quality parameter and depends on beam size (β !!) and stored current

$$L = \frac{1}{4\pi e^2 f_0 b} * \frac{I_1 * I_2}{\sigma^*_x * \sigma^*_y}$$

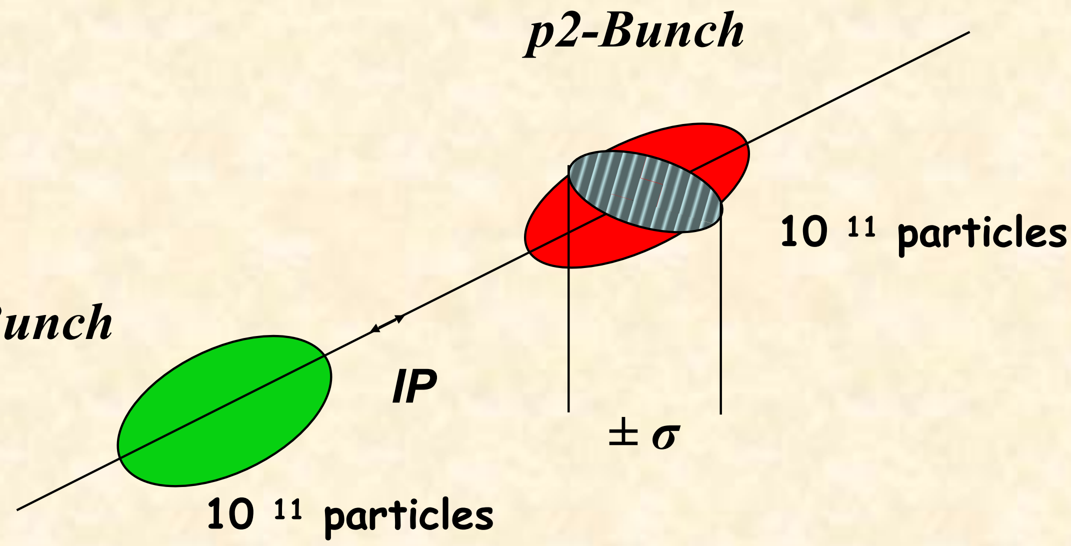
yes ... yes ... there is NO talk without it ...

The Higgs



ATLAS event display: Higgs => two electrons & two muons

Luminosity



Example: Luminosity at LHC

$$f_0 = 11.245\,kHz$$
$$n_b = 2808$$
$$\beta_{x,y} = 0.55\,m$$
$$\varepsilon_{x,y} = 5 * 10^{-10}\,rad\;m$$
$$\sigma_{x,y} = 17\,\mu m$$
$$I_p = 584\,mA$$

$$L = 1.0 * 10^{34}\ \frac{1}{cm^2 s}$$

$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

Number of particles
in the target
in the moving beam
per second
per cm²

➔ *how dense is the moscito cloud*

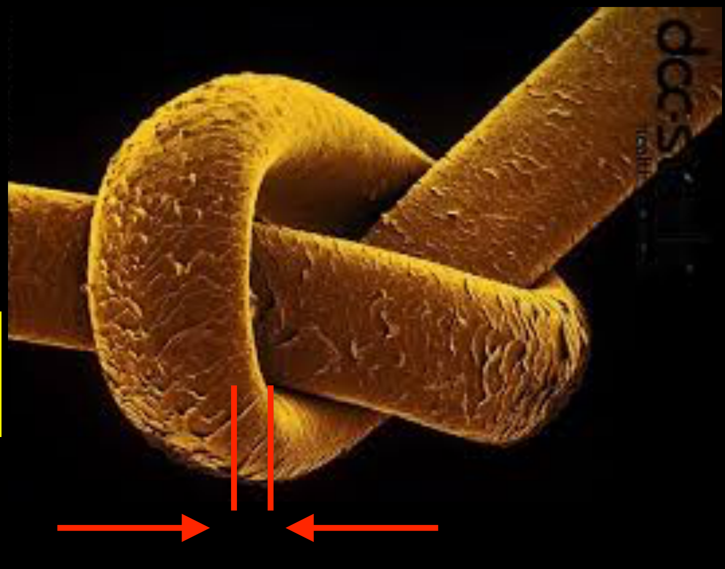
The Luminosity defines the number of "hits". It depends on the particle density at the collision point.

The Beta function at the IP "β*" should be made as small as possible to increase the particle density. In a drift β is growing quadratically and proportional to 1/β*, which sets the ultimate limit to the achievable luminosity.

The distance L* of the focusing magnets from the IP should be as small as possible.
… try to avoid detectors like ATLAS or CMS whenever possible. LOL.

The beam dimensions at the IP are typically a few μm.

Human hair:
$d \approx 70\ \mu m$



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