Particle Beam Guidance and Focussing

Example:

Setting up a transfer line from

Atrium hotel

to

Slavkofský štít

Atrium hotel

$$\begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \end{pmatrix} =$$

h = 989m

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} =$$

h = 2452m

Frist Step:

definition of the reference path / design orbit (= path of the reference particle)

Second Step:

set-up of your beam-line
→ proper placing of the elements
(= type, position and strengths of your magnets)

Third Step:

calculation of the individual transport matrices $\mathbf{M}_i$ of your elements

$\mathbf{M}_{s_i \rightarrow s_j}$

$\mathbf{M}_{s_i \rightarrow s_j}$

$\mathbf{M}_{s_i \rightarrow s_j}$

$$\mathbf{M}_{s_i \to s_j}$$

Calculation of the Transfer Matrix by multiplying the single matrices

$$\mathbf{M}_{1\to n} = \mathbf{M}_n \cdot \mathbf{M}_{n-1} \cdot \mathbf{M}_{n-2} \cdot \mathbf{M}_{n-3} \cdot \mathbf{M}_{n-4} \cdot \ldots \cdot \mathbf{M}_3 \cdot \mathbf{M}_2 \cdot \mathbf{M}_1$$

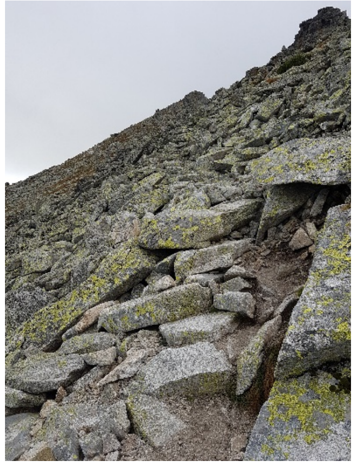




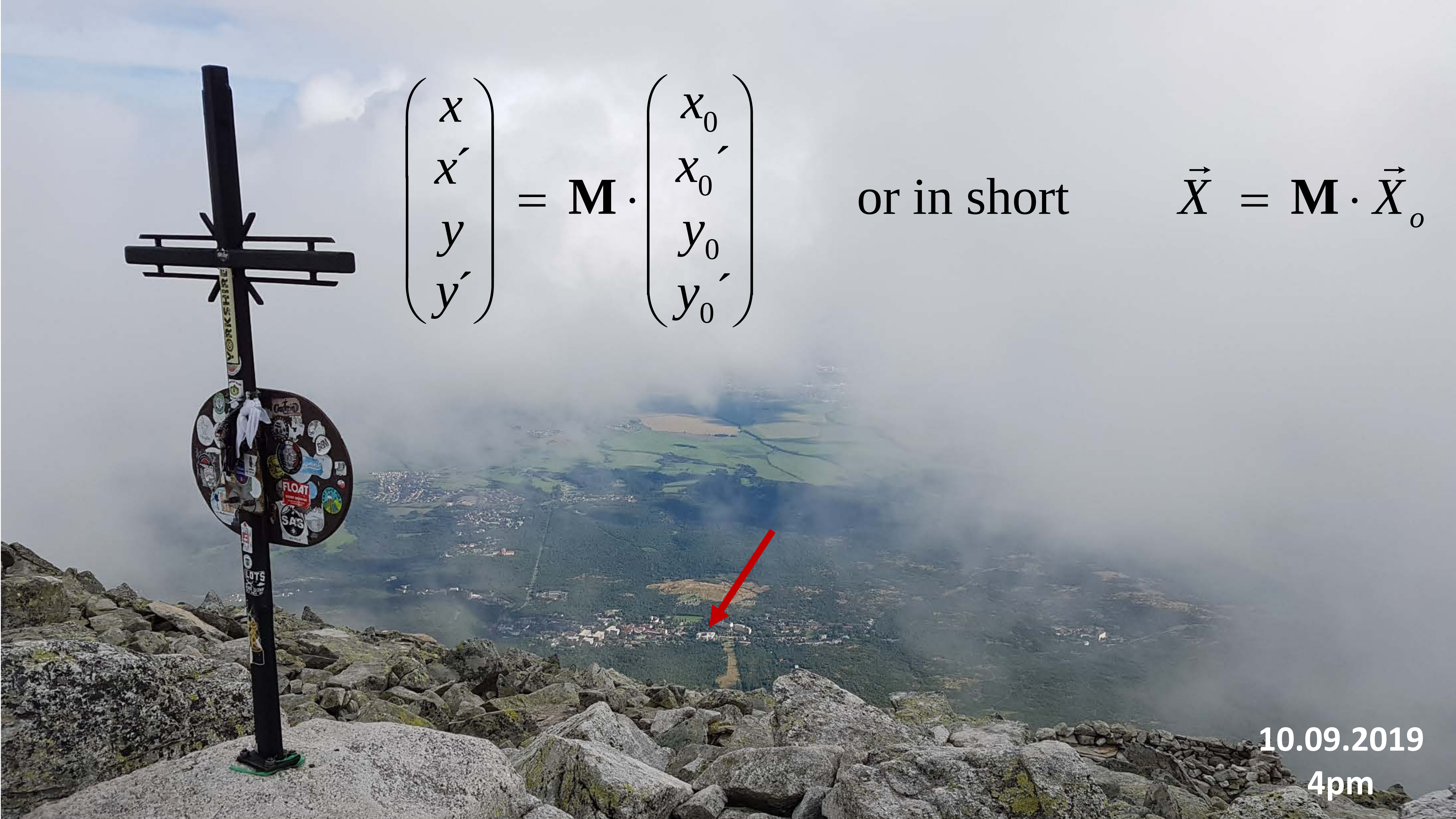









$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \mathbf{M} \cdot \begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \end{pmatrix} \qquad \text{or in short} \qquad \vec{X} = \mathbf{M} \cdot \vec{X}_o$$



10.09.2019
4pm

Individual particles will oscillate around the reference path:

$$\boxed{x(s) = A\sqrt{\beta(s)} \cdot \cos\left(\mu(s) + \varphi_0\right)}$$

Betatron Oscillation

In „linear“ approximation described by equations of motion

$$x''(s) + \left(\frac{1}{\rho(s)} - k(s)\right) \cdot x(s) = \frac{\Delta p}{p_0} \cdot \frac{1}{\rho(s)}$$

$$y''(s) + k(s) \cdot y(s) = 0$$

Macroscopic Beam Parameters determined from Statistical Moments

rms beam size: $$\langle x \cdot x \rangle = \sigma_x^2 = \sqrt{\varepsilon \cdot \beta}$$

rms beam divergence: $$\langle x' \cdot x' \rangle = \sigma_{x'}^2 = \sqrt{\varepsilon \cdot \gamma}$$

correlation: $$\langle x \cdot x' \rangle = r \cdot \sigma_x \cdot \sigma_{x'} = -\varepsilon \cdot \alpha$$

determined by intrinsic beam property (beam „quality“ $\leftrightarrow$ **emittance ε**)
and optics of the magneto-optic system (**beta function β (s)**)

Twiss-Parameters are linked together: $\alpha = -\dfrac{\beta'}{2}, \qquad \gamma = \dfrac{1+\alpha^2}{\beta}, \qquad \mu' = \dfrac{1}{\beta}$

single particle	$A^2 = \beta x'^2 + 2\alpha x x' + \gamma x^2$

$\varepsilon = \beta x'^2 + 2\alpha x x' + \gamma x^2$	area in phase space