Plasma wake generation (non linear) + blowout regime

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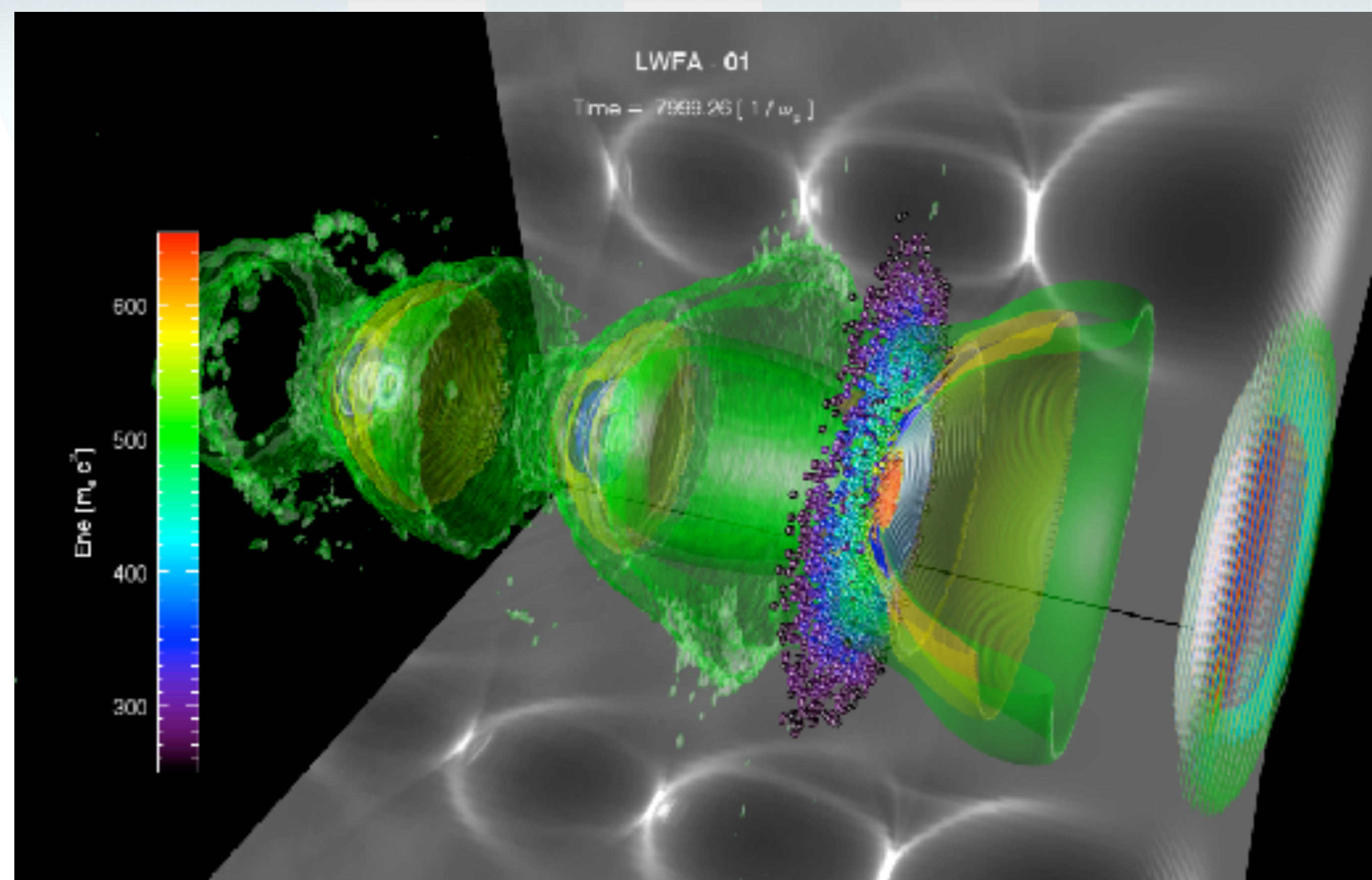

Accelerates ERC-2010-AdG 267841

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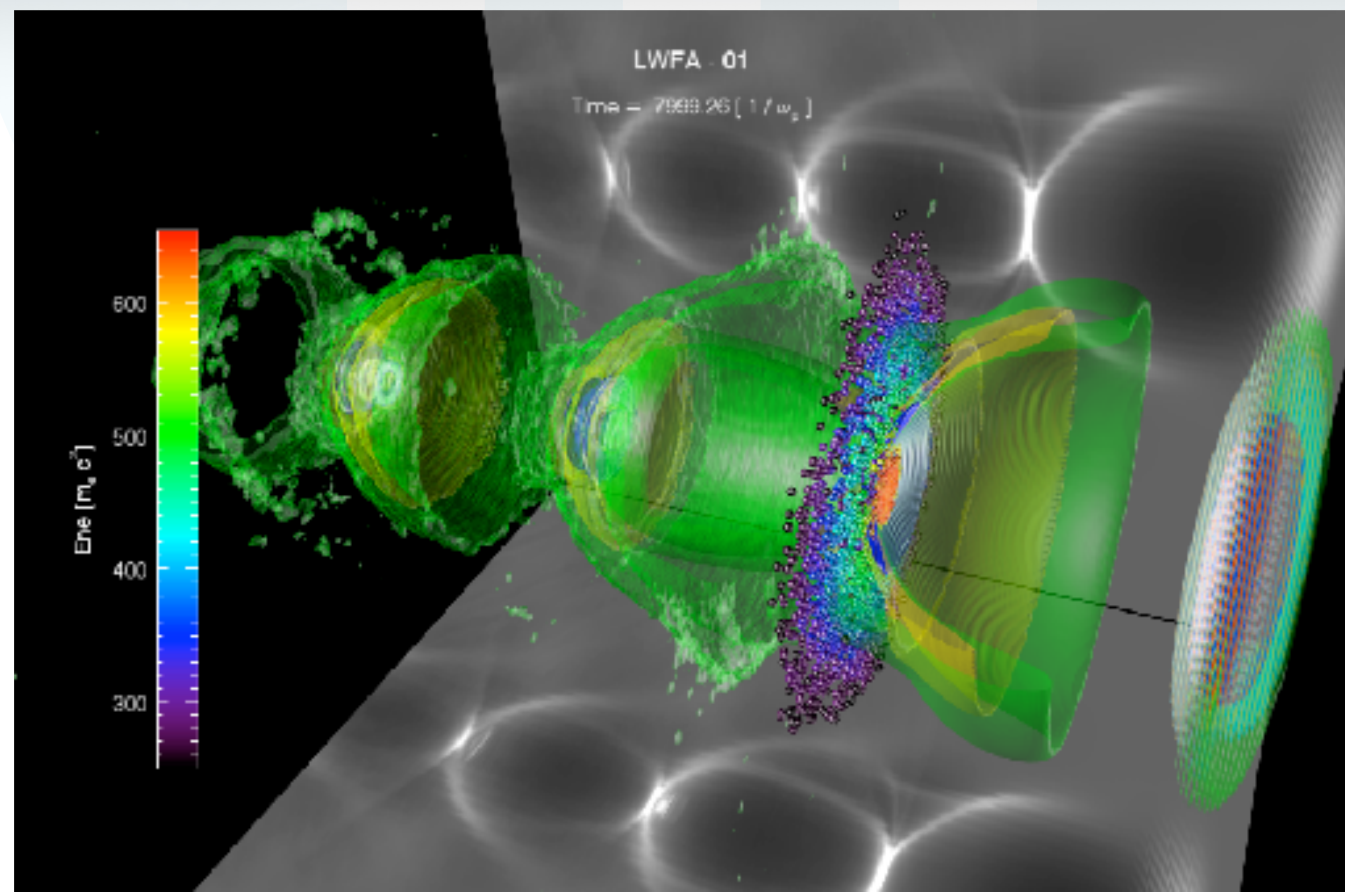

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- F. Fiúza, J. Martins, S. F. Martins, R. A. Fonseca
- Work in collaboration with:
  - **W. B. Mori**, C. Joshi (UCLA), W. Lu (Tsinghua) R. Bingham (RAL)
- Simulation results obtained at **epp and IST Clusters (IST), Hoffman (UCLA), Franklin (NERSC), Jaguar (ORNL), Intrepid (Argonne), and Jugene (FZ Jülich)**

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SEVENTH FRAMEWORK PROGRAMME

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VOLUME 43, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JULY 1979

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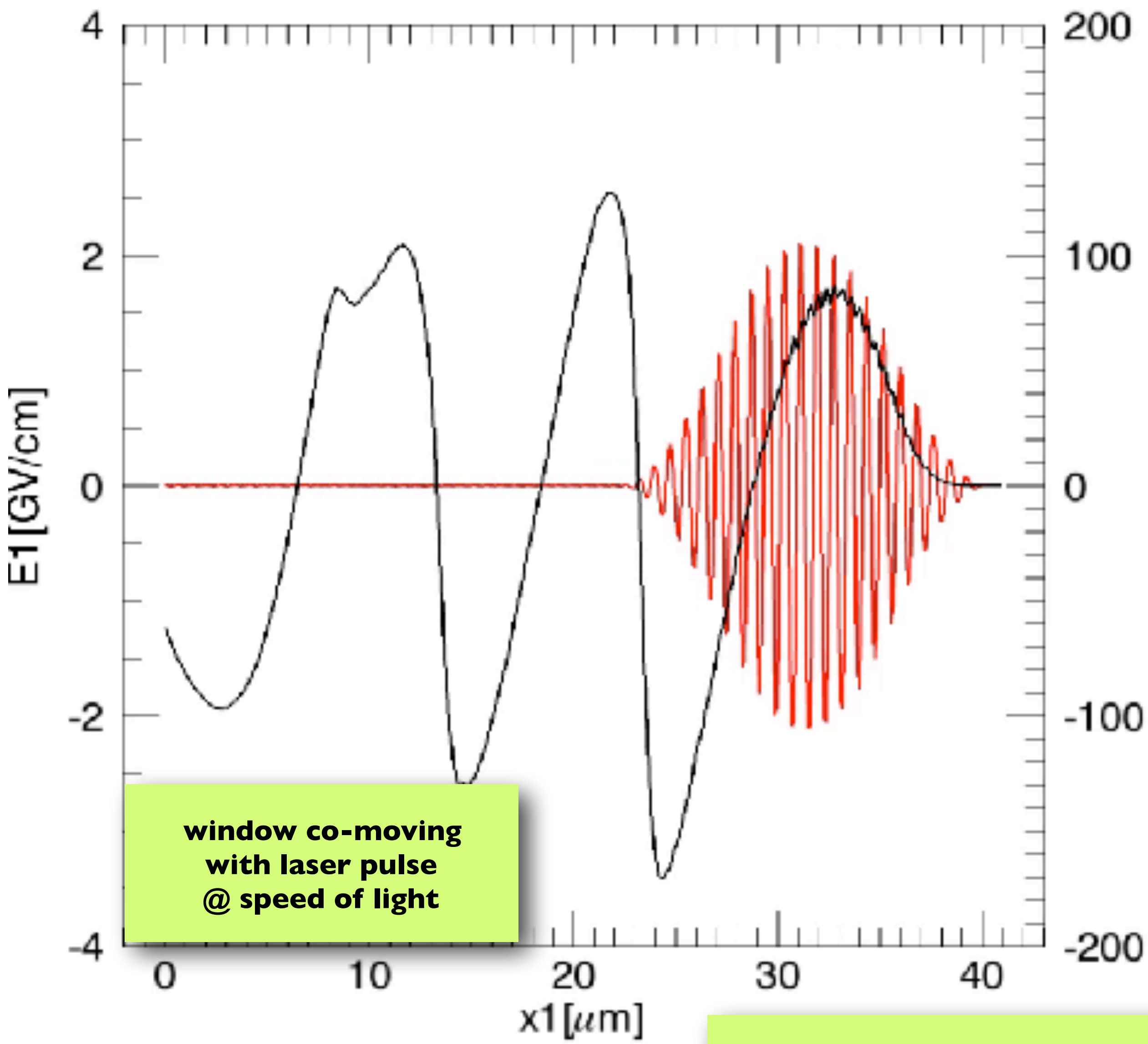

VOLUME 54, NUMBER 7 PHYSICAL REVIEW LETTERS 18 FEBRUARY 1985

Acceleration of Electrons by the Interaction of a Bunched Electron Beam with a Plasma

Pisin Chen[a]
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
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Department of Physics, University of California, Los Angeles, California 90024
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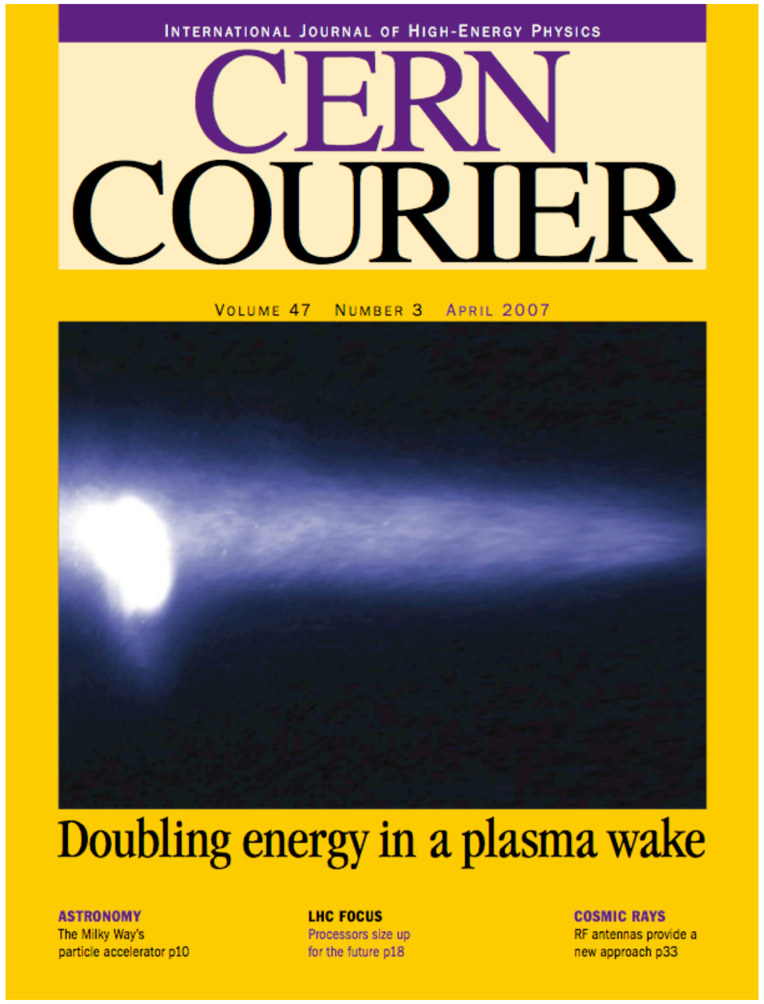

$$E_0[\mathrm{V/cm}] \approx 0.96\, n_0^{1/2}\, [\mathrm{cm}^{-3}]$$

$$n_0 = 10^{18}\ \mathrm{cm}^{-3} \rightarrow E_0 \approx 1\ \mathrm{GV/cm}$$

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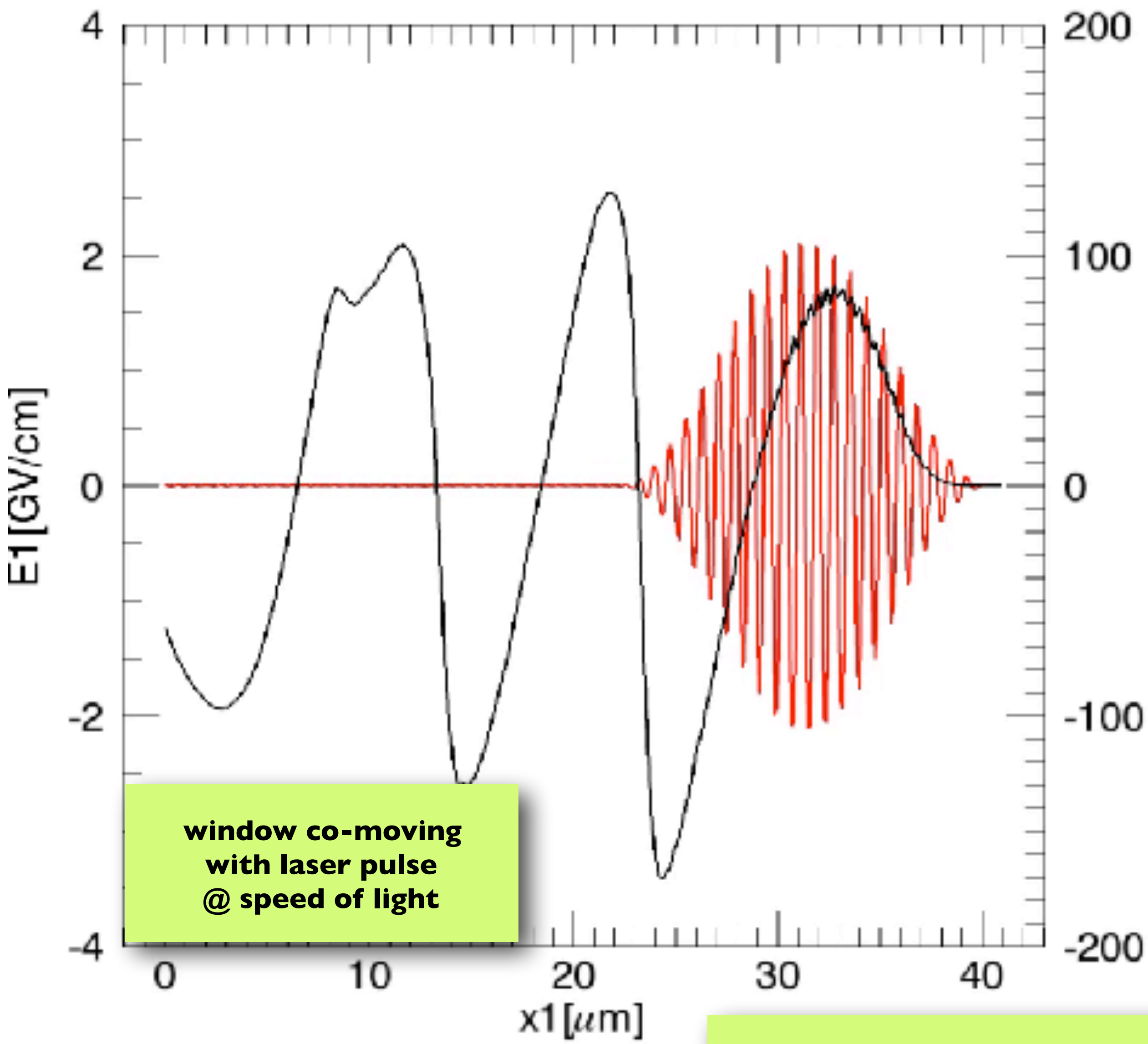

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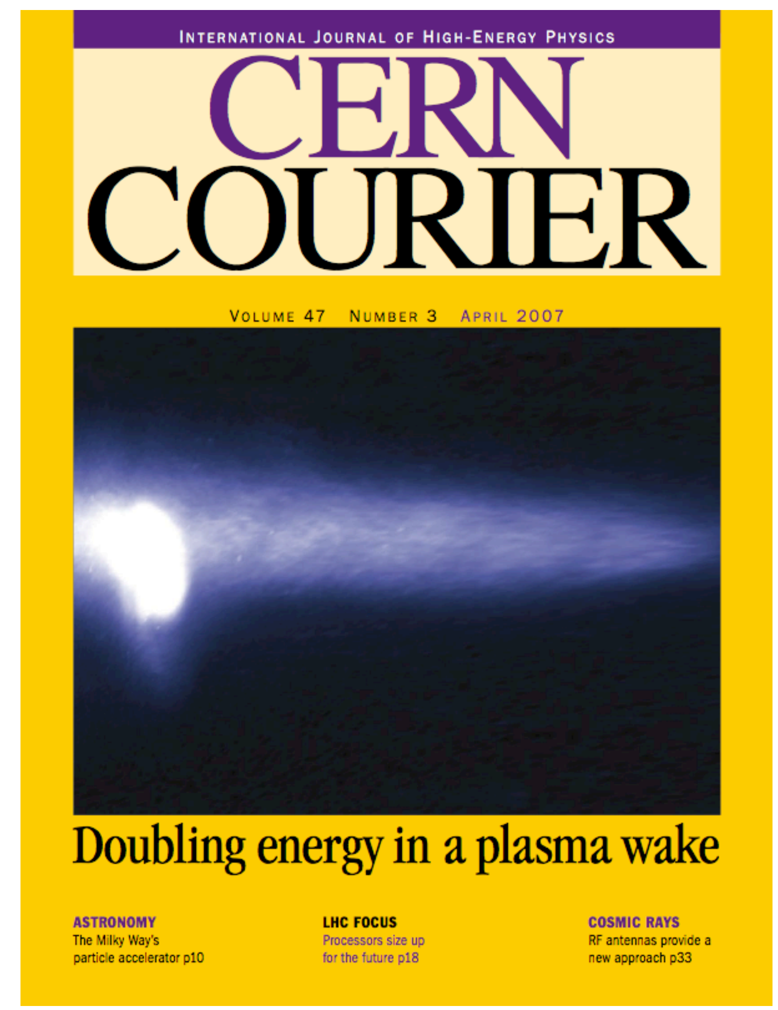

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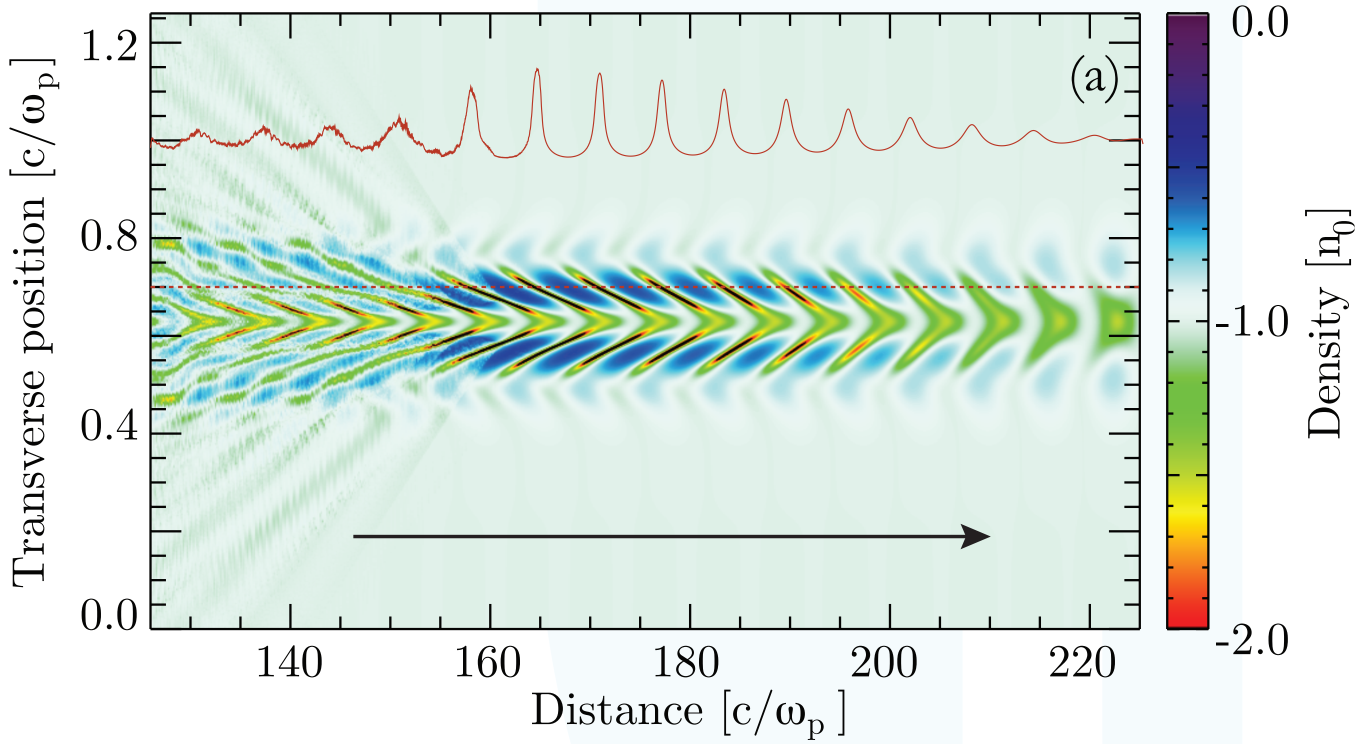
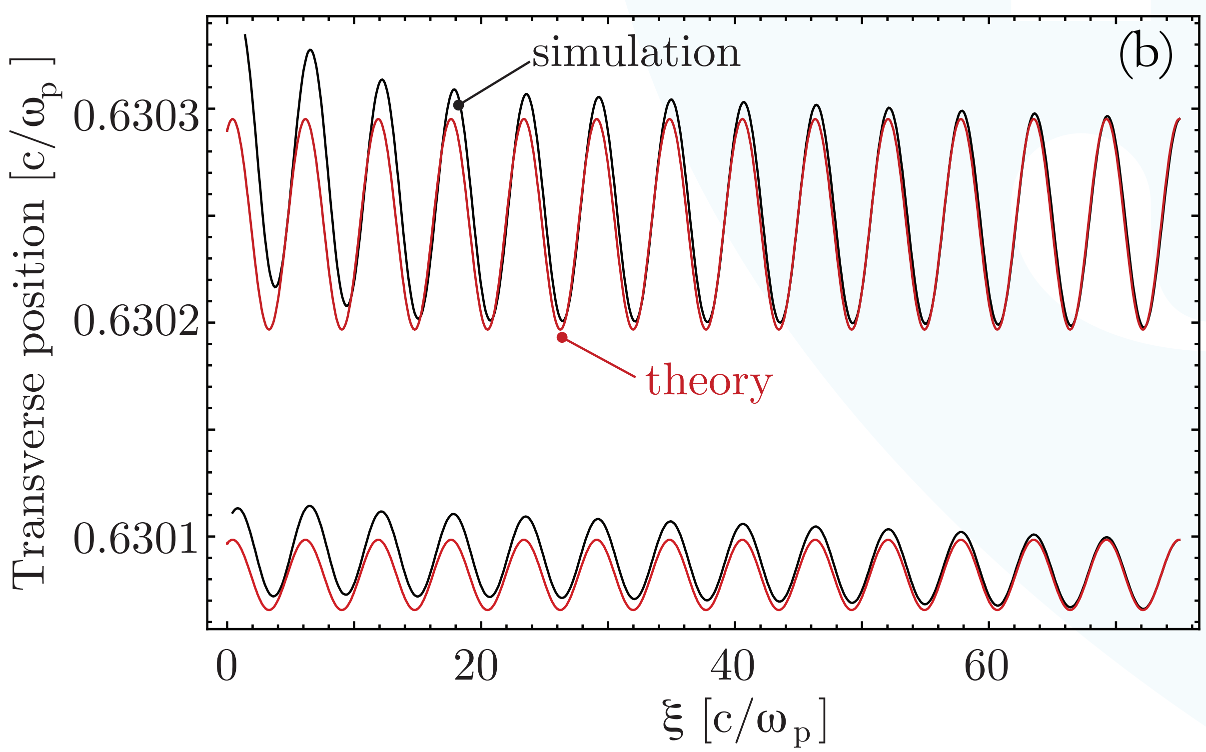
Multidimensional plasma waves are nonlinear





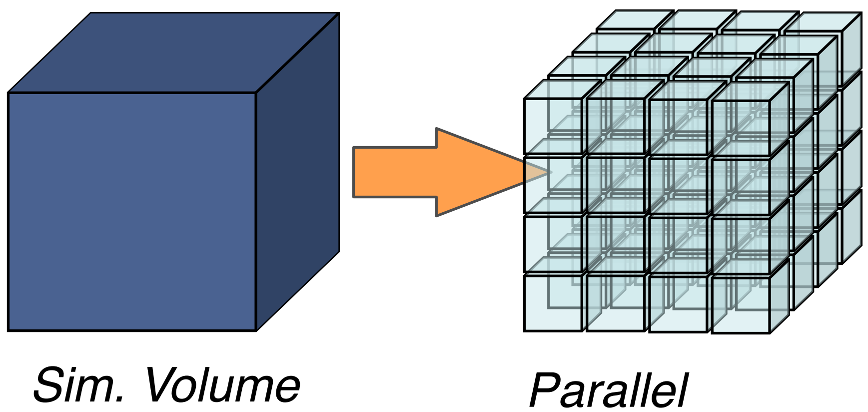
J. M. Dawson, PR **113** 383 (1959); J. Vieira et al, PRL **106** 225001 (2011); J. Vieira et al, PoP **21** 056705 (2014)

Lasers and intense beams drive large waves

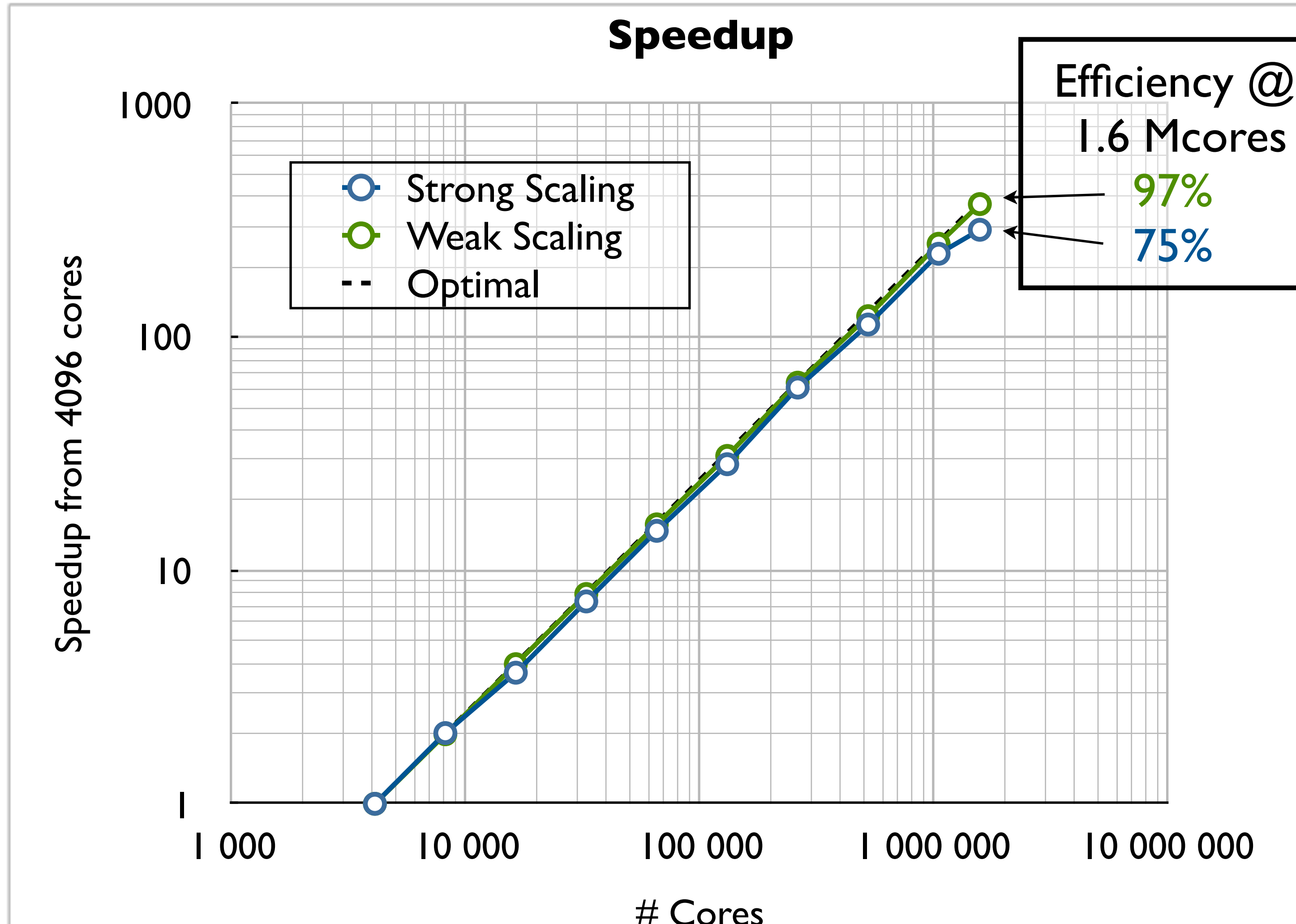
Nazaré, Portugal, Feb 2013

Simulations play an important role

Scaling Tests

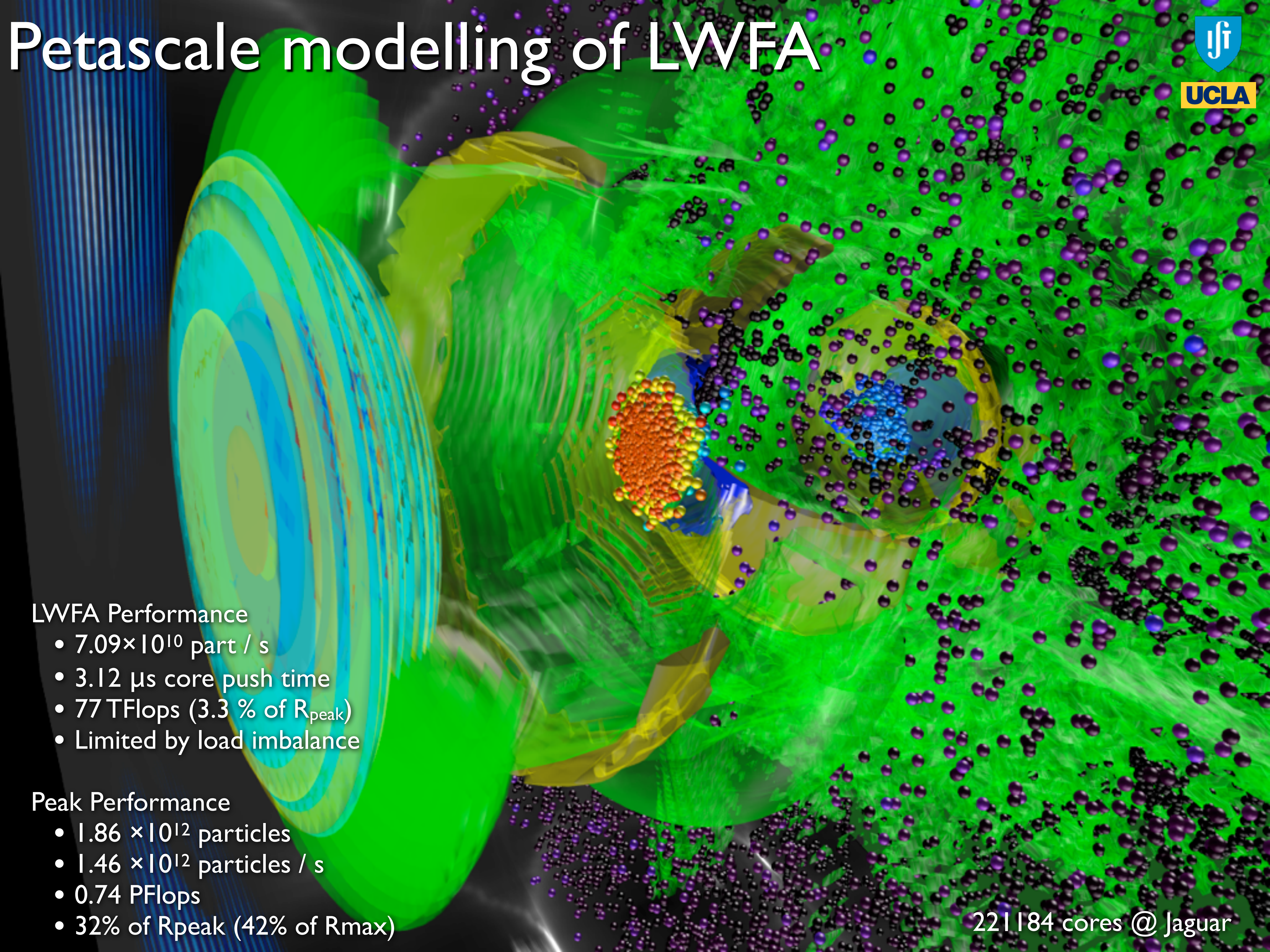


- Scaling tests on LLNL Sequoia
  - 4096 → 1572864 cores (full system)
- Warm plasma tests
  - Quadratic interpolation
  - u_{th} = 0.1 c
- Weak scaling
  - Grow problem size
  - cells = $256^3 \times (N_{cores} / 4096)$
  - 2^3 particles/cell
- Strong scaling
  - Fixed problem size
  - cells = 2048^3
  - 16 particles / cell



F. Fiúza *et al.* (2013)

Petascale modelling of LWFA



LWFA Performance

- 7.09×10^{10} part / s
- 3.12 µs core push time
- 77 TFlops (3.3 % of R_{peak})
- Limited by load imbalance

Peak Performance

- 1.86×10^{12} particles
- 1.46×10^{12} particles / s
- 0.74 PFlops
- 32% of Rpeak (42% of Rmax)

221184 cores @ Jaguar

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Plasmas waves always demonstrate nonlinear behavior

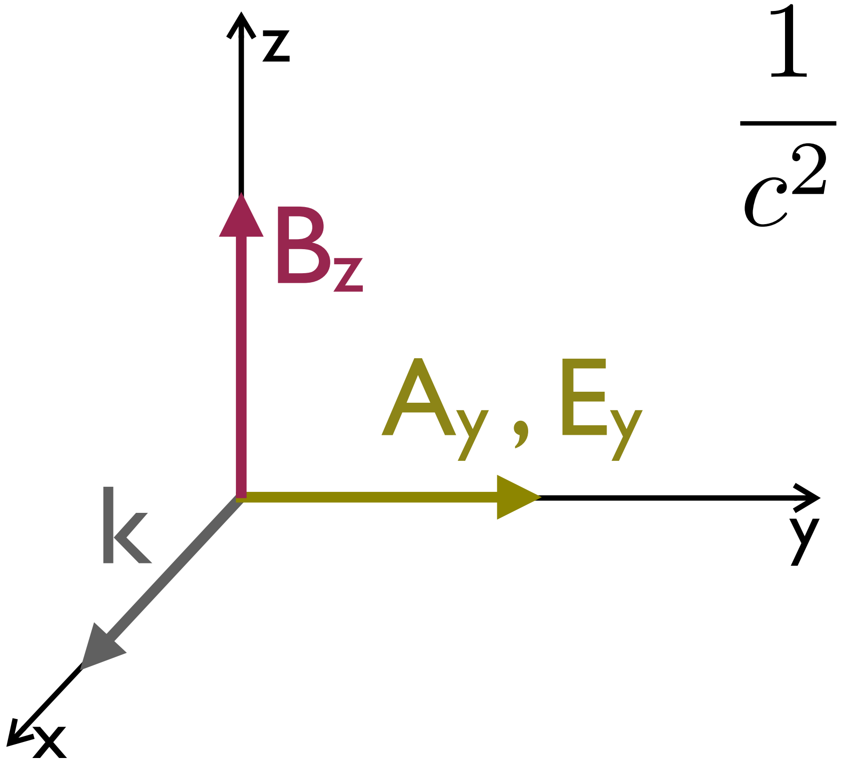
General formalism
Master equation: relativistic fluid + Maxwell's equations

"Short" pulses
Quasi-static equations, Wakefield generation

Summary

The wave equation for e.m. waves

The more standard approach



From Maxwell's equations in Coulomb gauge

$$\frac{1}{c^2}\partial_t^2\vec{A} + \nabla\times\nabla\times\vec{A} = \frac{4\pi}{c}\vec{J} - \frac{1}{c}\partial_t\nabla\phi$$

$$p_y = \frac{eA_y}{c}$$

Conservation of canonical momentum

$$\frac{1}{c^2}\partial_t^2 A_y - \partial_x^2 A_y = \frac{4\pi}{c}J_y = -\frac{4\pi e^2}{mc^2}\frac{n}{\gamma}A_y$$

$$\gamma = \sqrt{1 + \frac{p_x^2}{m^2c^2} + \frac{e^2A_y^2}{m^2c^4}}$$

Linearized wave equation for e.m. waves

Ordering

$$\frac{p_x}{mc} \qquad \frac{e^2 A_y^2}{m^2 c^4} \qquad \frac{\delta n}{n_0} = \frac{n}{n_0} - 1$$

All the same order, and << 1

$$\frac{1}{\gamma} \simeq 1 - \frac{1}{2}\frac{e^2 A_y^2}{m^2 c^4} - \frac{1}{2}\frac{p_x^2}{m^2 c^2}$$

Wave equation for vector potential of e.m. wave

$$\frac{1}{c^2}\partial_t^2 A_y - \partial_x^2 A_y \simeq -\frac{\omega_{p0}^2}{c^2}\left(1 + \frac{\delta n}{n_0} - \frac{1}{2}\frac{e^2 A_y^2}{m^2 c^4}\right) A_y$$

Evolution of the electron density

Equation for the evolution of the electron density in the presence of Ay

Linearizing the continuity equation + time derivative

$$\partial_t \delta n + n_0 \nabla \delta \vec{v} = 0 \qquad \partial_t^2 \delta n + n_0 \nabla \partial_t \delta \vec{v} = 0$$

Linearized Euler's equation

$$\partial \delta \vec{v} = -\frac{e}{m} \delta \vec{E} - c^2 \nabla \left(1 + \frac{1}{2} \frac{e^2 A_y^2}{m^2 c^4} \right)$$

Equation for driven electron plasma waves

$$\partial_t^2 \frac{\delta n}{n_0} + \frac{4\pi e^2 n_0}{m_e} \frac{\delta n}{n_0} = c^2 \nabla^2 \frac{1}{2} \frac{e^2 A_y^2}{m^2 c^4}$$

Coupling of light with plasma electrons

Driven electron plasma waves

$$\left(\partial_t^2 + \omega_{p0}^2\right) \frac{\delta n}{n_0} = \frac{c^2}{2} \nabla^2 \frac{e^2 A_y^2}{m^2 c^4}$$

E.m. waves coupled with plasma + relativistic mass correction

$$\frac{1}{c^2} \partial_t^2 A_y - \partial_x^2 A_y = -\frac{\omega_{p0}^2}{c^2} \left(1 + \frac{\delta n}{n_0} - \frac{1}{2} \frac{e^2 A_y^2}{m^2 c^4}\right) A_y$$

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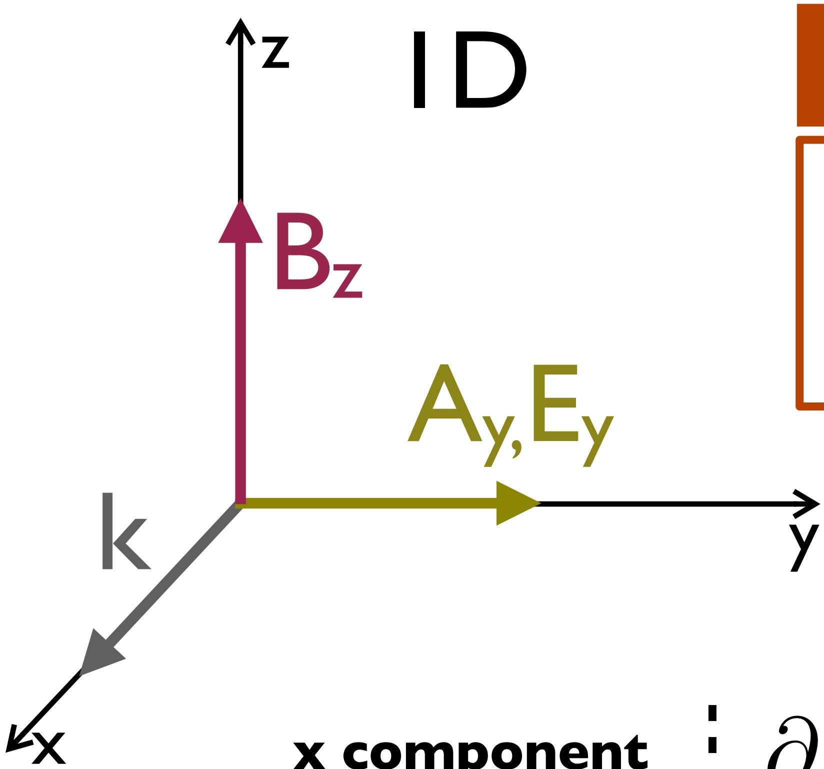
Starting point: the master equation



Master equation

$$\partial_t^2 \vec{p} + c^2 \nabla \times \nabla \times \vec{p} = -\left[\omega_{p0}^2 + \frac{1}{m}\nabla \cdot \left(\partial_t \vec{p} + mc^2 \nabla \gamma\right)\right]\frac{\vec{p}}{\gamma} - mc^2 \partial_t \nabla \gamma$$

Normalized Units

x component

$$\partial_t^2 p_x + \left(1 + \partial_t \partial_x p_x + \partial_x^2 \gamma\right)\frac{p_x}{\gamma} + \partial_t \partial_x \gamma = 0$$

y component

$$\partial_t^2 p_y - \partial_x^2 p_y + \left(1 + \partial_t \partial_x p_x + \partial_x^2 \gamma\right)\frac{p_y}{\gamma} = 0$$

Remember: from canonical momentum conservation $p_y = a_y$

Normalisations

Electric field normalised to the cold wave breaking limit

$$E \simeq \frac{m_e c \omega_p}{e} \simeq 0.96\sqrt{n_0[\mathrm{cm}^{-3}]}\mathrm{V/cm}$$

Magnetic field normalised to the cold wave breaking limit multiplied by c

$$B \simeq \frac{m_e c^2 \omega_p}{e} \simeq 32\sqrt{n_0[10^{16}\mathrm{cm}^{-3}]}\mathrm{T}$$

Scalar and vector potentials normalised to electron rest energy divided by the elementary charge

$$\phi \simeq A \simeq \frac{m_e c^2}{e} \simeq \frac{0.511\mathrm{MeV}}{e}$$

Space and time normalised to the plasma skin depth and inverse of plasma frequency

$$d \simeq \frac{1}{k_p} \simeq \frac{5.32\ \mu\mathrm{m}}{\sqrt{n_0[10^{18}\mathrm{cm}^{-3}]}} \qquad t \simeq \frac{1}{\omega_p} \simeq \frac{17\ \mathrm{fs}}{\sqrt{10^{18}\ \mathrm{cm}^{-3}}}$$

Charge, mass and velocity normalised to the elementary charge, electron mass and speed of light. Momenta normalised to m_e c

Everything at c: Speed of light variables

and the envelope approximation

- Waves driven by short laser pulses with $v_{ph} \sim c$

$$\psi = t - x \quad \tau = x \qquad p_x \propto e^{-\omega_{p0}\psi} \qquad p_y \propto e^{-\omega_0 \psi}$$

In speed of light variables

$$\partial_t = \partial_\psi$$

$$\partial_x = \partial_\tau - \partial_\psi$$

One further approximation: the envelope approximation $\partial_\tau \ll \partial_\psi \qquad \partial_\tau \sim \left(\omega_{p0}/\omega_0\right)^2$

$$\partial_\psi^2 p_x + \left(1 - \partial_\psi^2 p_x + \partial_\psi^2 \gamma\right)\frac{p_x}{\gamma} - \partial_\psi^2 \gamma \simeq 0$$

$$2\partial_\tau \partial_\psi p_y + \left(1 - \partial_\psi^2 p_x + \partial_\psi^2 \gamma\right)\frac{p_y}{\gamma} \simeq 0$$

P. Sprangle et al (1990)

1D Quasi Static equations

Using the definition

$$\gamma - p_x \equiv \chi$$

$$\left(\frac{p_x}{\gamma} - 1\right)\partial_\psi^2 \chi = -\frac{p_x}{\gamma}$$

$$2\partial_\tau \partial_\psi p_y + \left(1 + \partial_\psi^2\right)\frac{p_y}{\gamma} = 0$$

1D quasi-static equations

$$\partial_\psi^2 \chi = -\frac{1}{2}\left(1 - \frac{1 + p_y^2}{\chi^2}\right)$$

$$2\partial_\tau \partial_\psi p_y + \frac{p_y}{\chi} = 0$$

- $1/\chi$ is the plasma susceptibility
- Physically, quasi-static means the laser pulse envelope changes in a much longer time scale than the phase *or* laser pulse envelope does not evolve in the time it takes for an electron to go across the laser pulse (~ pulse duration)
- The basis for reduced numerical models (WAKE & QuickPIC & HiPACE)

P. Sprangle et al (1990)

Physical interpretation

Plasma susceptibility

$$\frac{1}{\chi} \equiv \frac{n}{\gamma}$$

With $\chi = 1 + \phi$

Also written as:

$$\partial_\psi^2 \phi + \frac{1}{2}\left[1 - \frac{1 + p_y^2}{(1+\phi)^2}\right] = 0$$

$$2\partial_\tau \partial_\psi p_y + \frac{p_y}{1+\phi} = 0$$

$p_y = a_y$!

Simplified Euler's equation

$$\partial_t p_x = -E_x - \partial_x \gamma \qquad E_x = -\partial_x \phi \qquad \partial_t p_x = \partial_x(\phi - \gamma)$$

In speed of light variables

$$-\partial_\psi \left(\gamma - p_x - \phi\right) = \partial_\tau \left(\phi - \gamma\right) \simeq 0$$

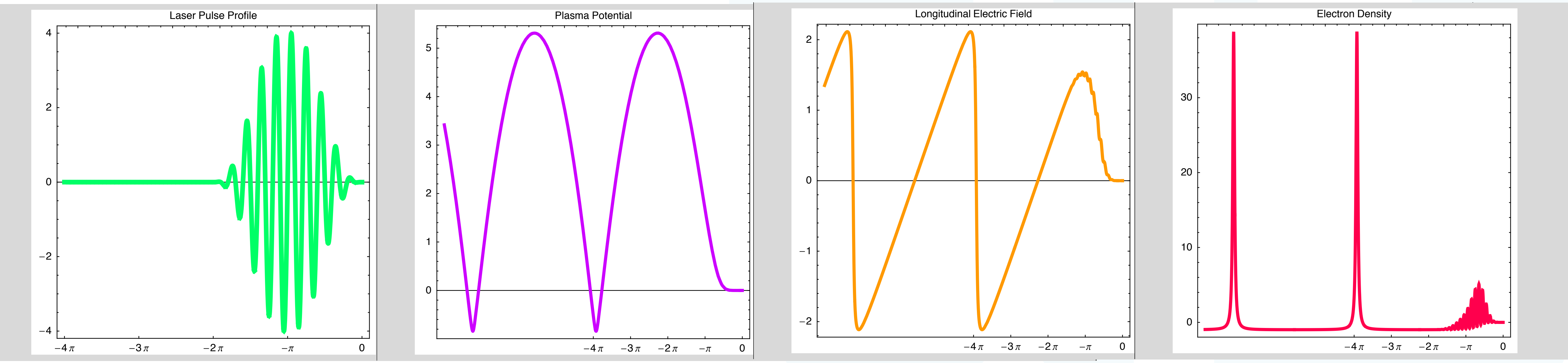
Plasma Potential

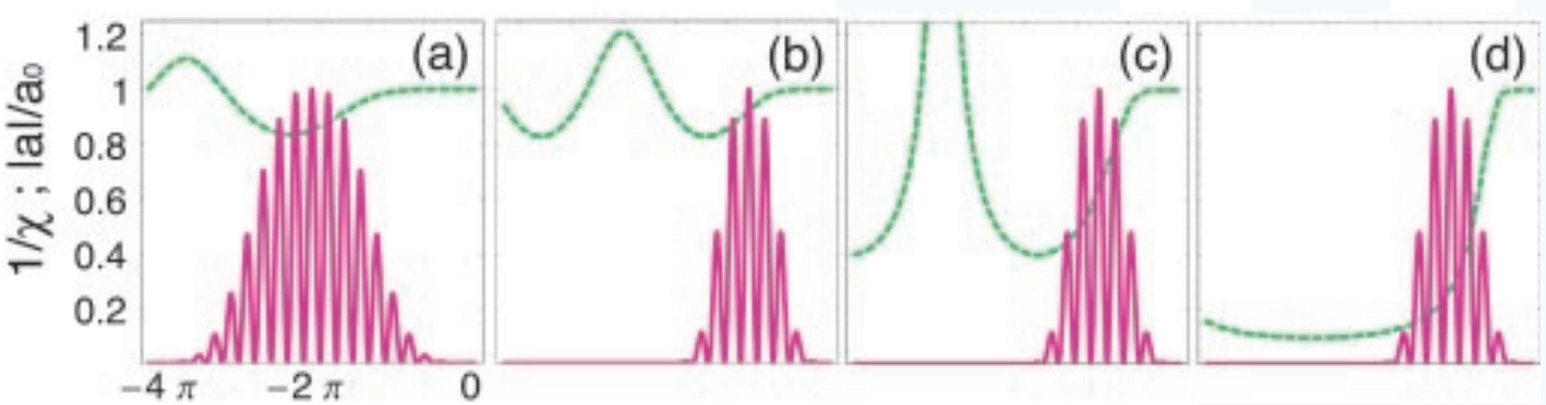
$$\chi = \gamma - p_x = \phi + \text{const.} = 1 + \phi$$

Wakefield generation

Quasi-static equations at the basis of many theoretical developments on laser wakefield

a0=4, L = $\lambda_p/2$





Increasing a0

Wakefield structure and wavebreaking

Analytical results can be obtained for specific laser pulse shapes (e.g. square pulse Berezhiani & Muruzidze, 90**)**

$$\gamma_\perp = \sqrt{1 + a_{y0}^2} \qquad \phi_{\max} \sim \gamma_\perp^2 - 1 \qquad E_{\max} \sim \frac{\gamma_\perp^2 - 1}{\gamma_\perp}$$

$$p_{\max} \sim \frac{\gamma_\perp^4 - 1}{2\gamma_\perp^2}$$

Peak electric field ~ a_{y0}

Optimal pulse length for wakefield excitation $\lambda_p/2$ Depends on pulse shape

- Quasi-static approximation breaks down when plasma wave breaks

plasma sheaths cross

$$v_{\text{fluid}} \sim v_\phi \qquad \frac{\delta n}{n_0} \to \infty \qquad \partial_x E_x \to \infty$$

Wavebreaking limit (cold)

Non relativistic $$\frac{eE_{pw}}{mv_\phi\omega_{p0}} = 1$$

Relativistic $$\frac{eE_{pw}}{mc\omega_{p0}} = \sqrt{2}\sqrt{\gamma_\phi - 1}$$

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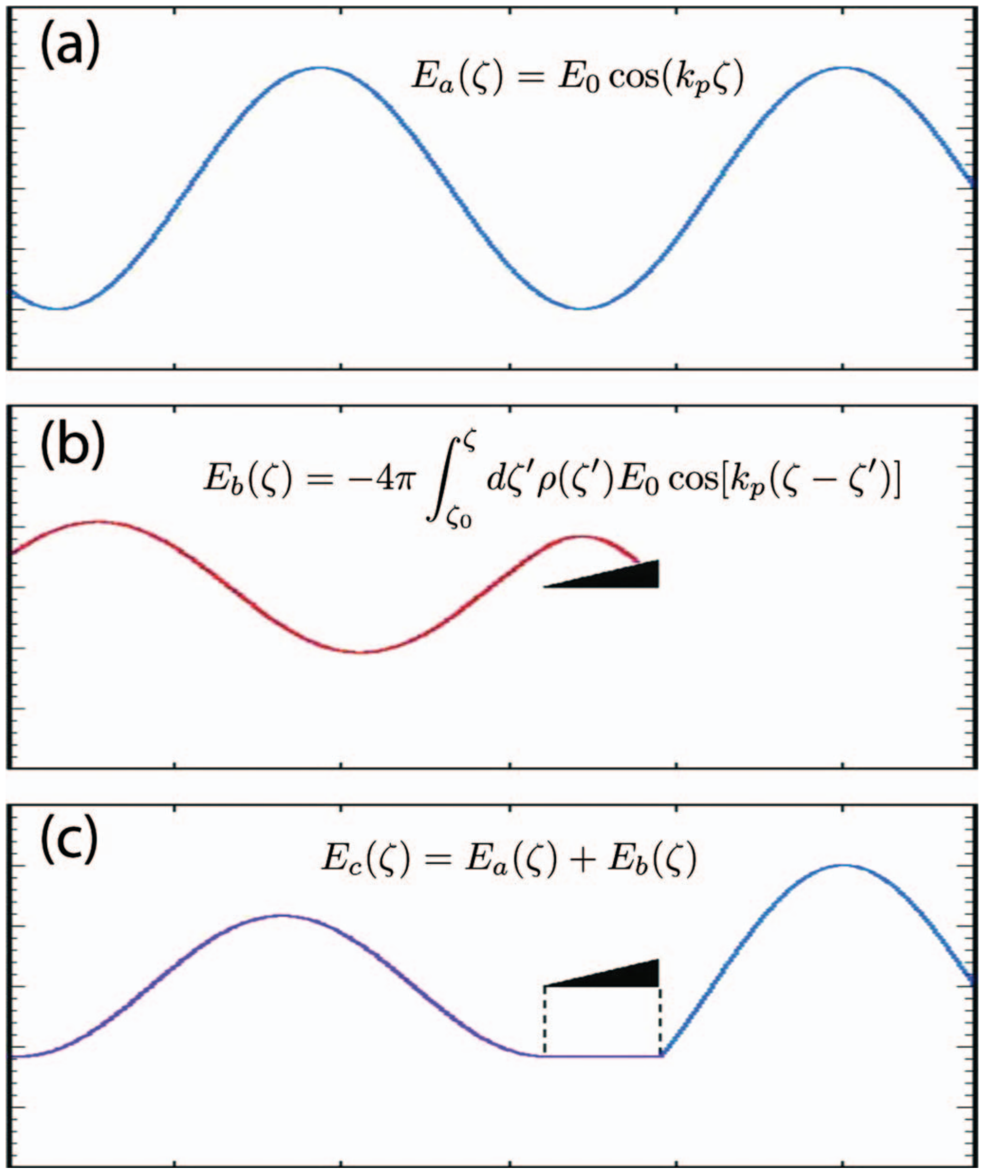
Relativistic $\dfrac{eE_{pw}}{mc\omega_{p0}} = \sqrt{2}\sqrt{\gamma_\phi - 1}$

Naïve 1D estimate for breakdown of quasi-static (square pulse)

$$a_{y0\,\max} \sim \frac{4.6}{\lambda[1\mu\text{m}]n[10^{19}\text{cm}^{-3}]}$$

Beam loading in the linear regime

Beam loading concept



Properly tailored witness electron bunch flattens accelerating wakefield: no energy spread growth!

T. Katsouleas et al (1987)

Optimal scenario: wakefield due to beam cancels plasma wave field exactly

$$N_0 = 5 \times 10^5 \left(\frac{n_1}{n_0}\right) \sqrt{n_0} A$$

Energy spread: as particle energy spread becomes 100% number (N) approaches N_0:

$$\frac{\Delta\gamma_{\max} - \Delta\gamma_{\min}}{\Delta\gamma_{\max}} = \frac{E_i - E_f}{E_i} = \frac{N}{N_0}$$

Efficiency: tends to 100% when N approaches N_0.

$$\eta_b = \frac{N}{N_0}\left(2 - \frac{N}{N_0}\right)$$

Key trade off

The energy gain is less than twice the energy per particle of the driving bunch (transformer ratio)

$$R = \frac{\Delta E_b}{E_d} = 2 - \frac{N}{N_d}$$

Blow out regime (or the bubble regime)

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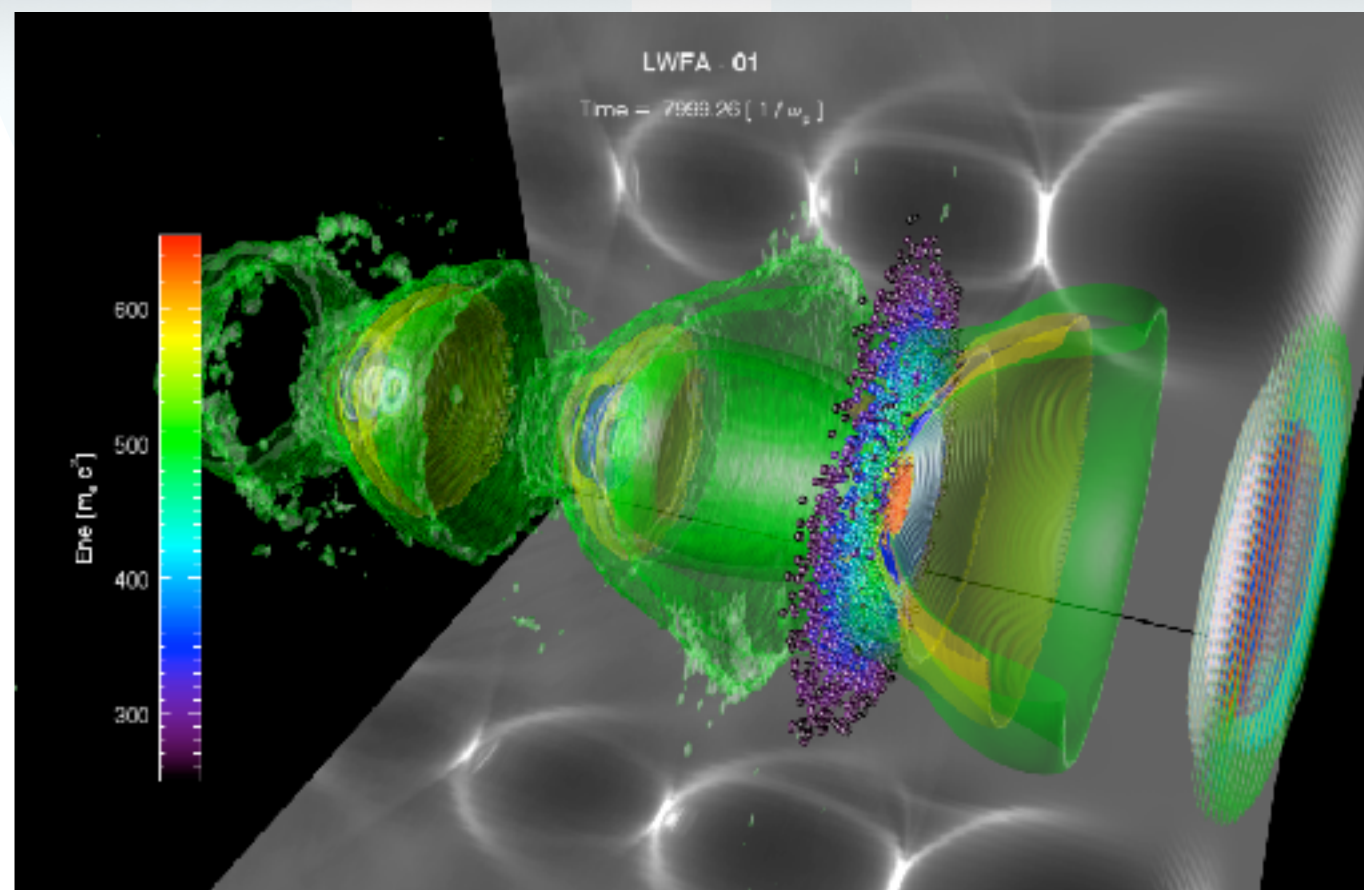

Can laser plasma accelerators reach the energy frontier?

From the 80s to last week, and beyond ...

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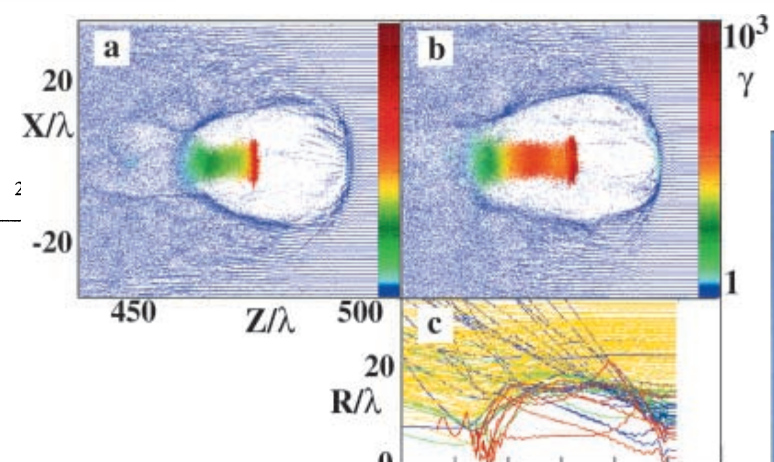
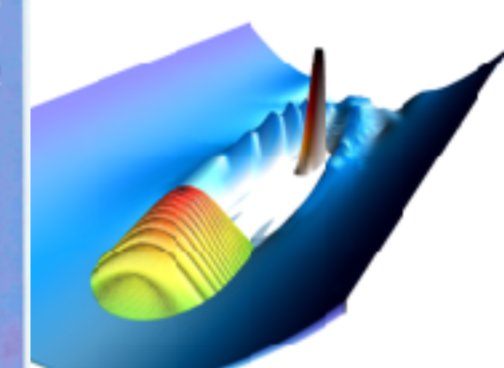


Ultrahigh gradient particle acceleration by intense laser-driven plasma density waves

C. Joshi*, W. B. Mori*, T. Katsouleas*, J. M. Dawson*,
J. M. Kindel† & D. W. Forslund†

* University of California Los Angeles, California 90024, USA
† Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA



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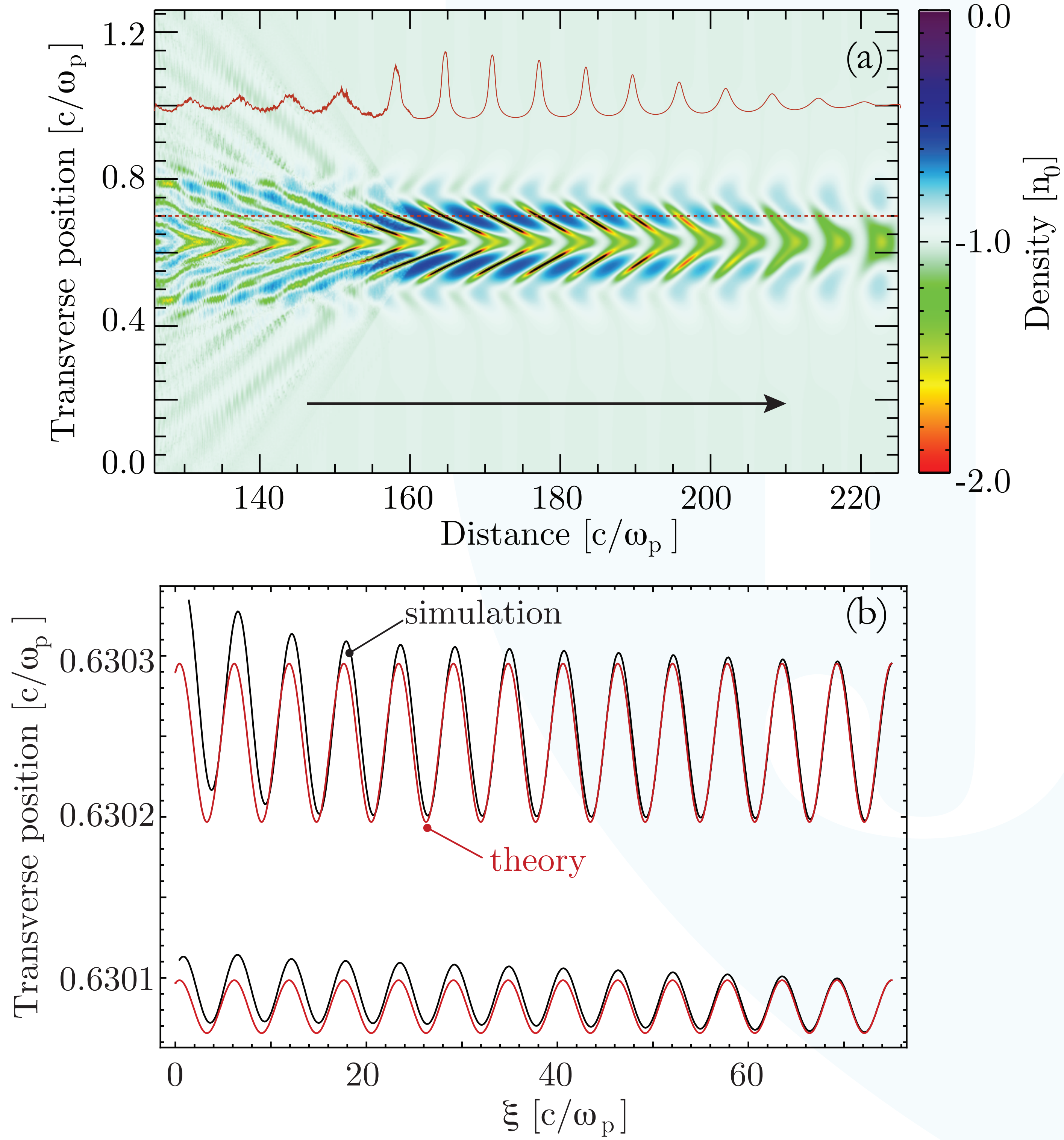
Wakefields are multidimensional



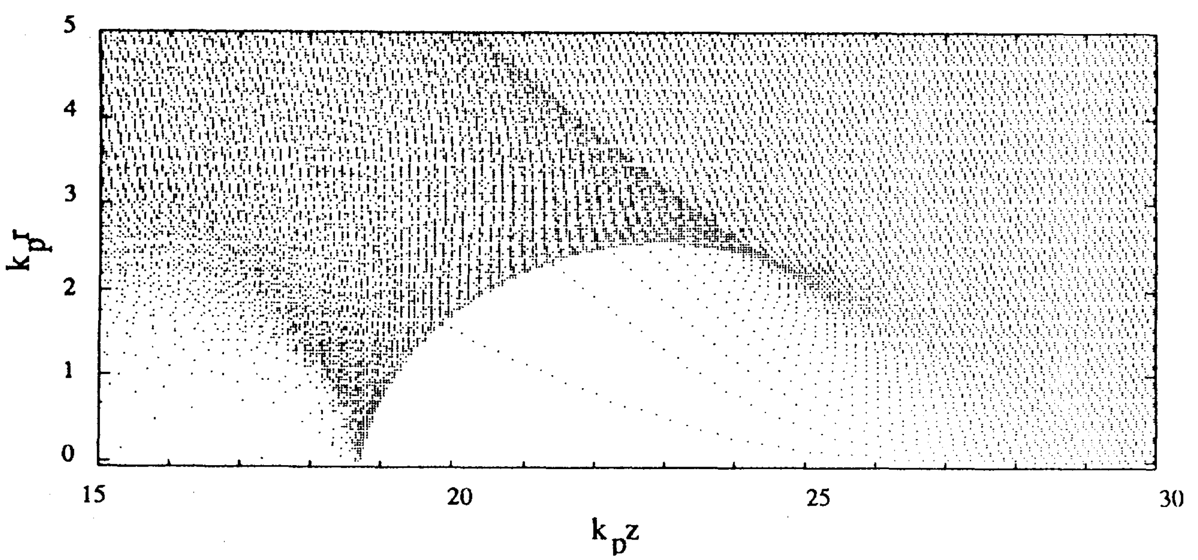
J. M. Dawson, PR **113** 383 (1959); J. Vieira et al, PRL **106** 225001 (2011); J. Vieira et al, PoP **21** 056705 (2014)

3D non-linear laser and beam driven wakes

Beam driven	Laser driven

Beam driven

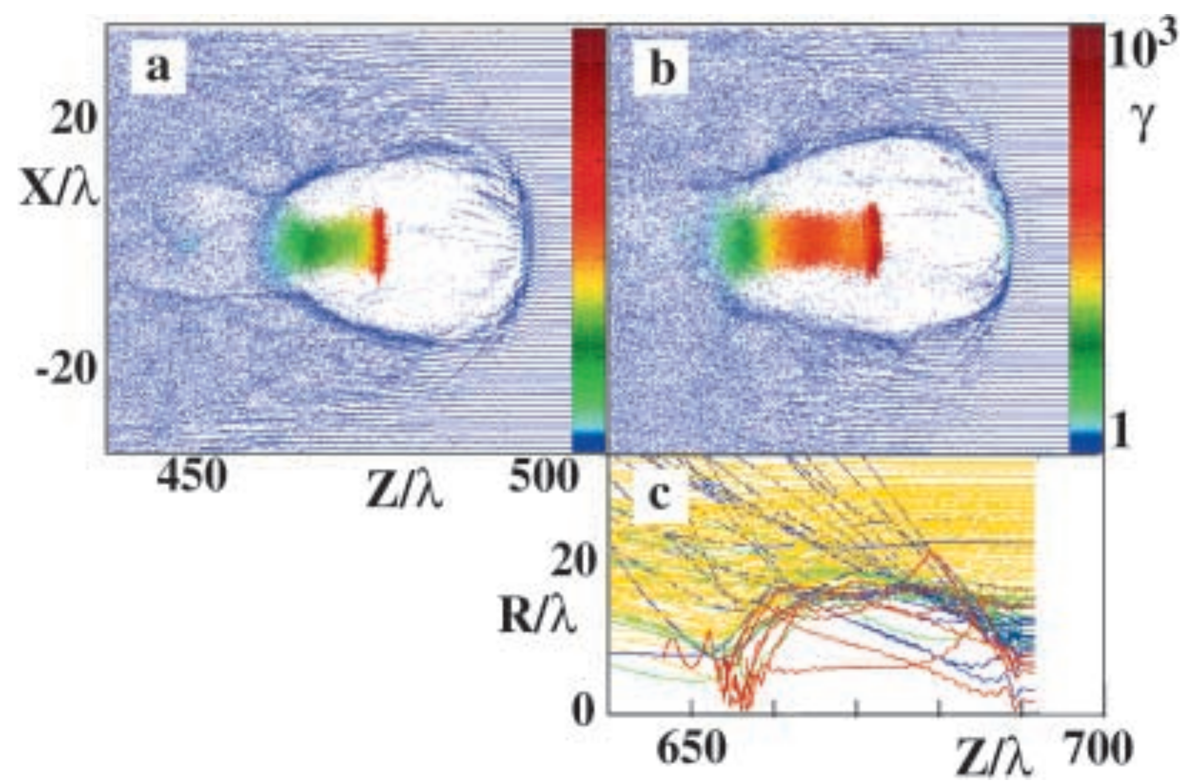
Non-linear plasma wave generation by an electron bunch with $n_b/n_0>1$. Electron cavitation is a distinctive signature of the blowout regime.



J.B. Rosenzweig et al, Phys. Rev. A 44, R6189 (1991)

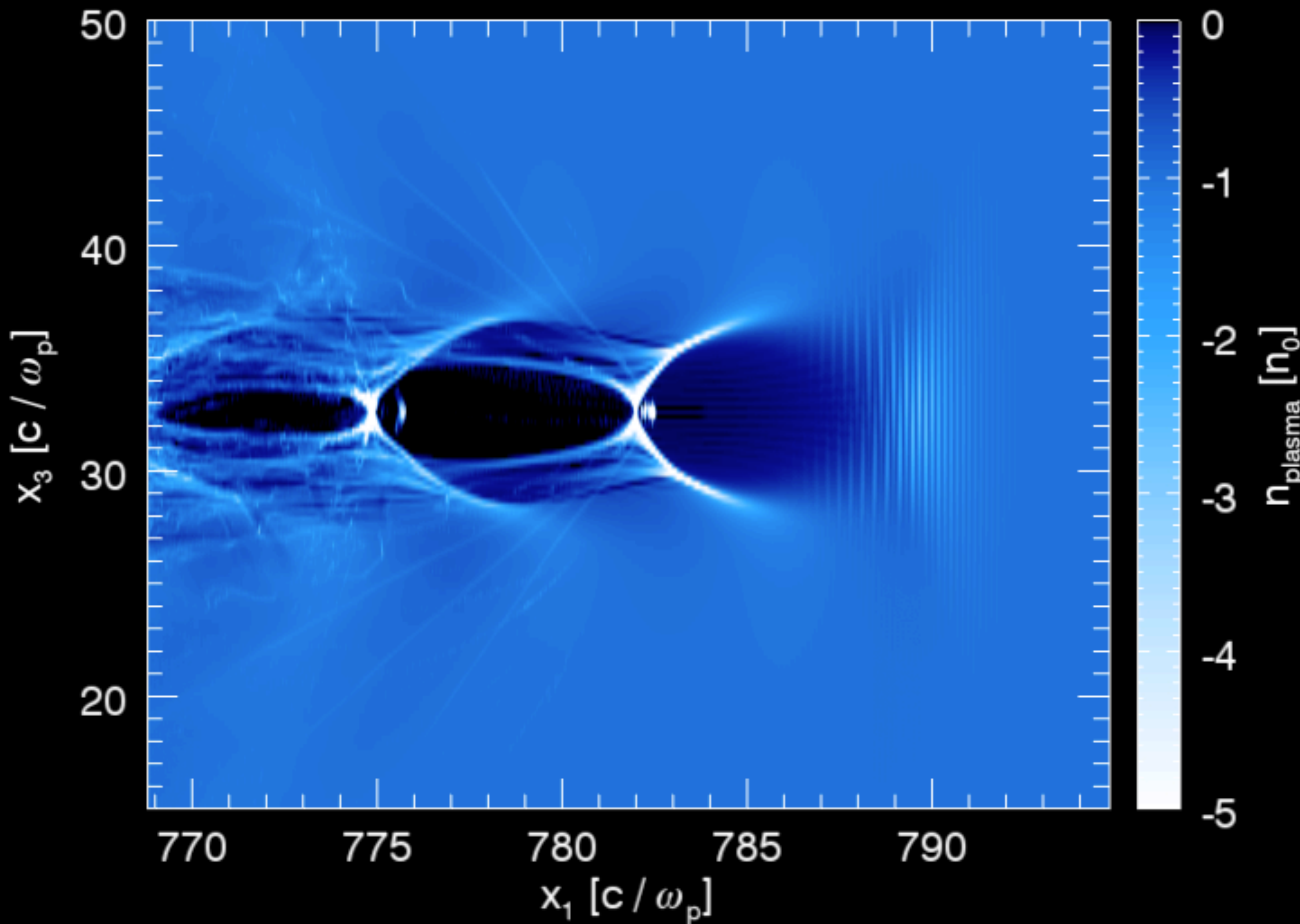
Laser driven

Plasma wave generation and electron acceleration driven by ultra-high intensity laser with $a_0>>1$



A. Pukhov, J.Meyer-Ter-Vehn, Appl. Phys. B 74, 355 (2002)

Laser blowout



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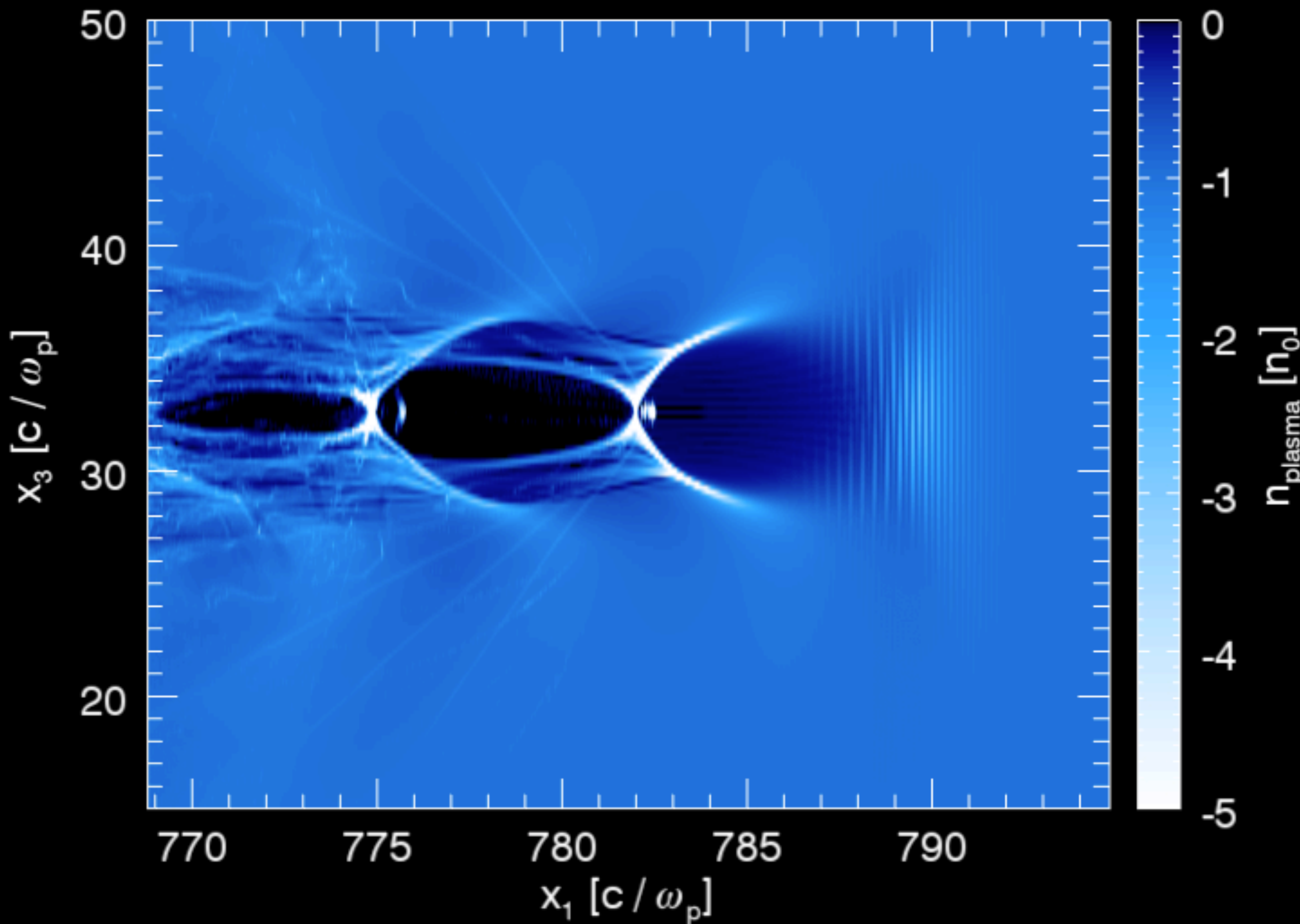
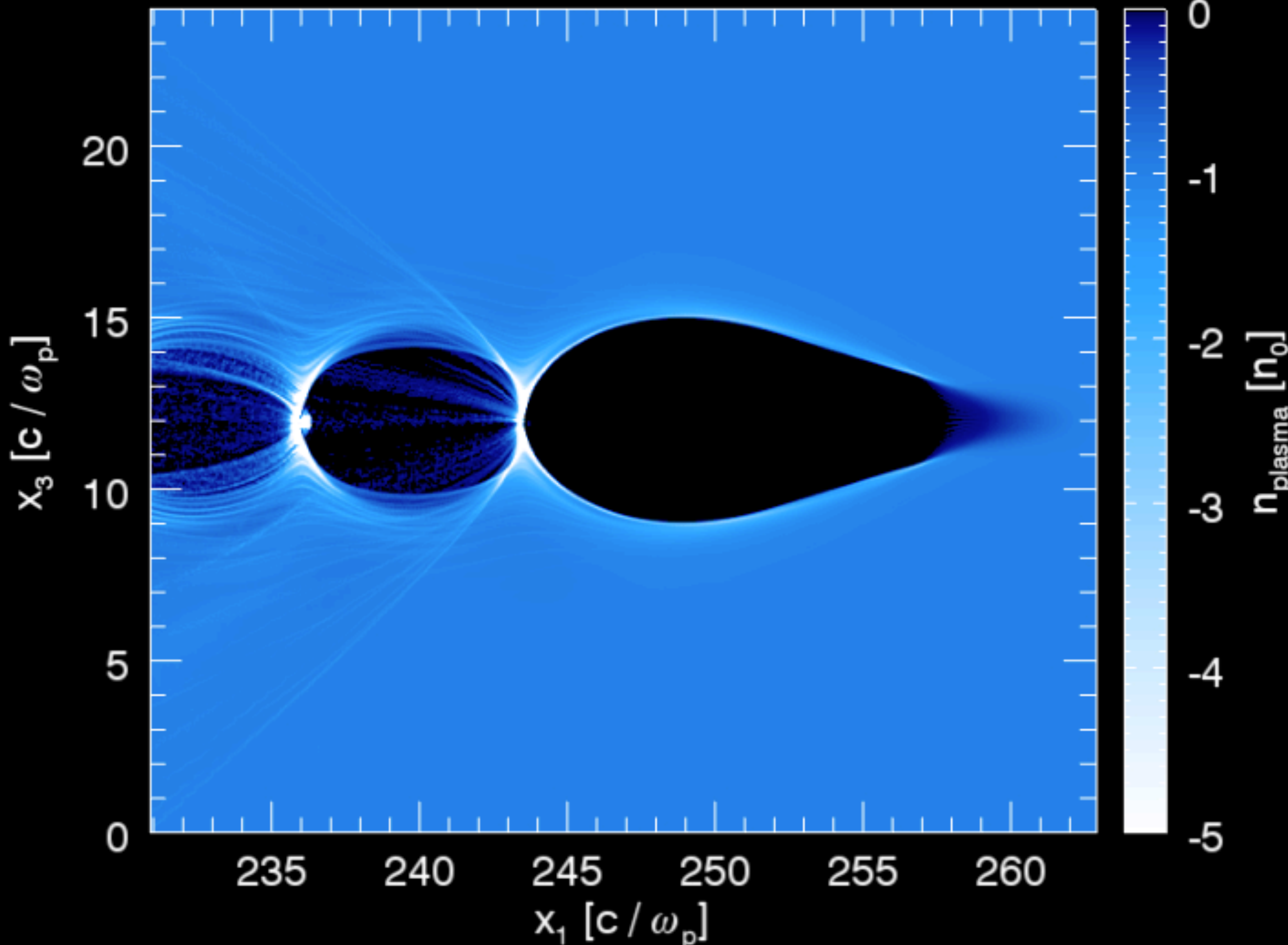


Electron beam blowout



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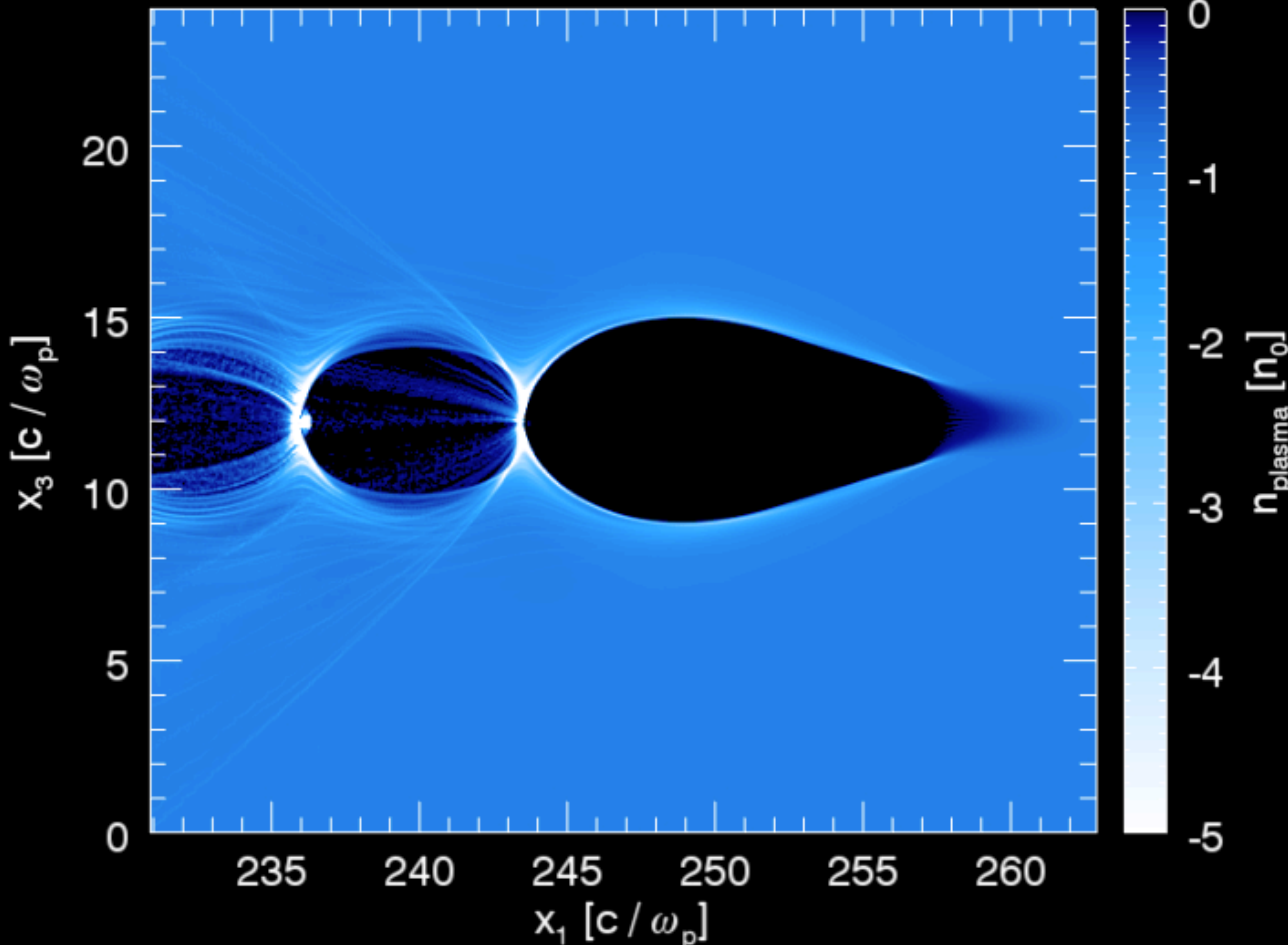
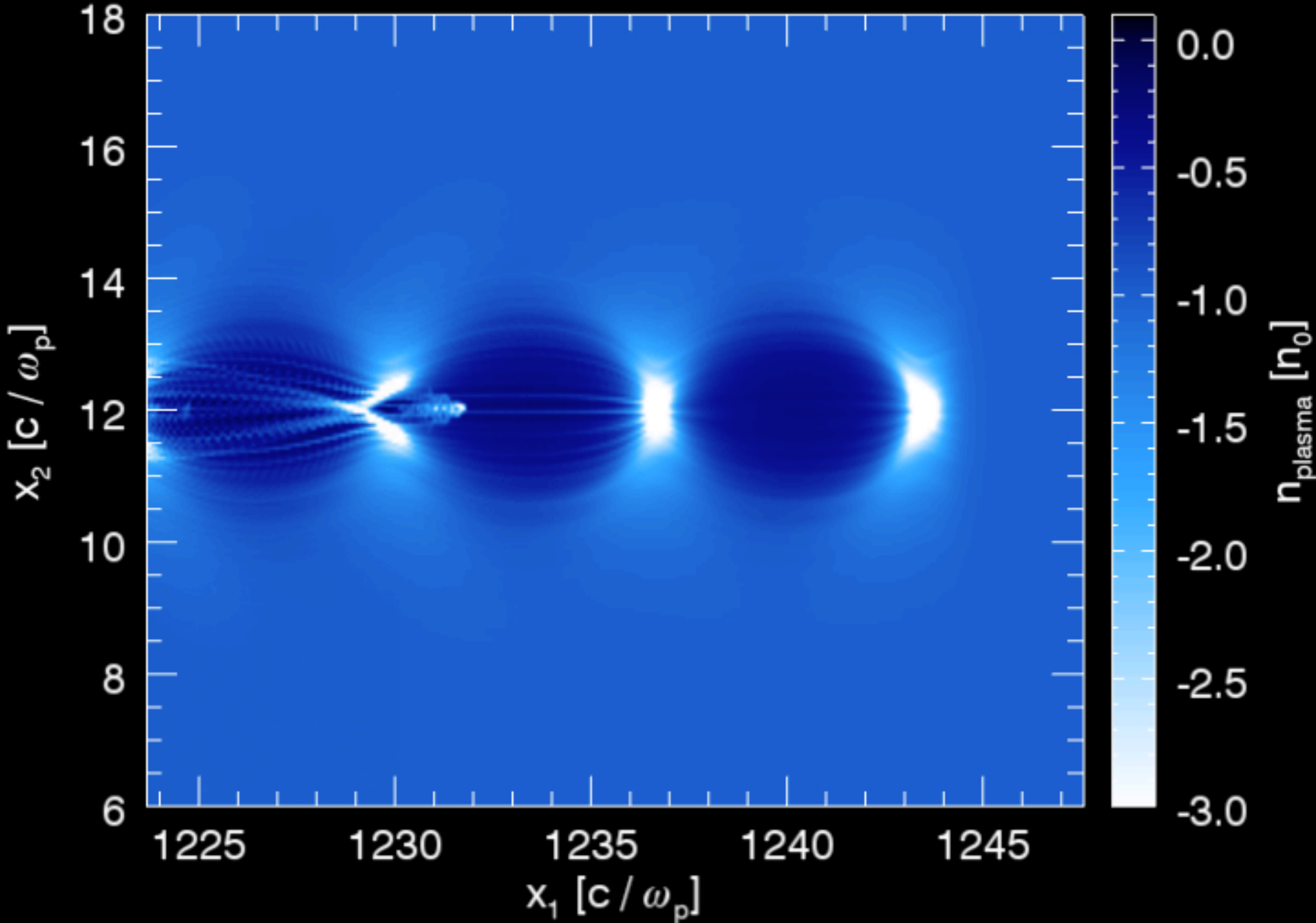
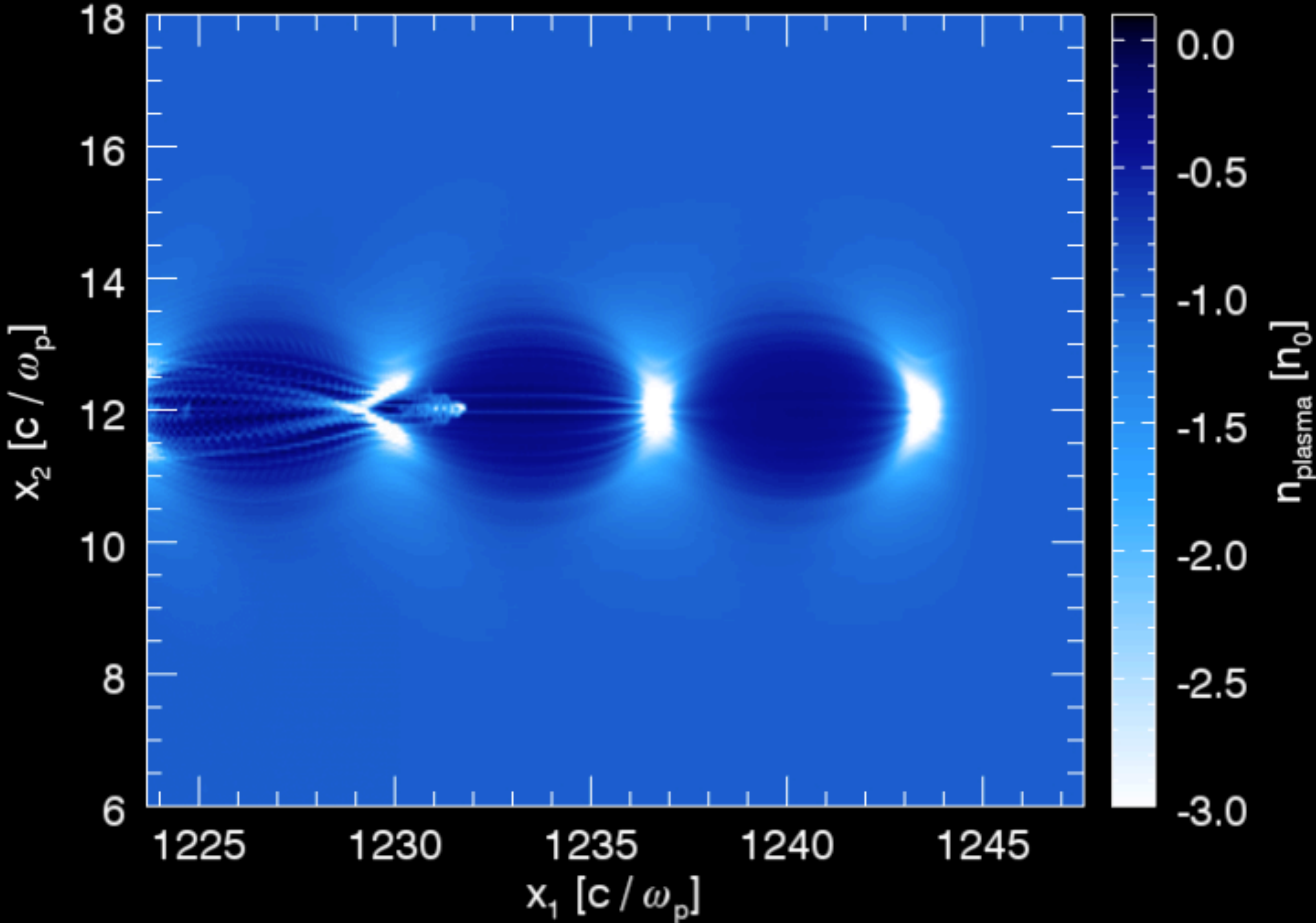


And for positron drivers?



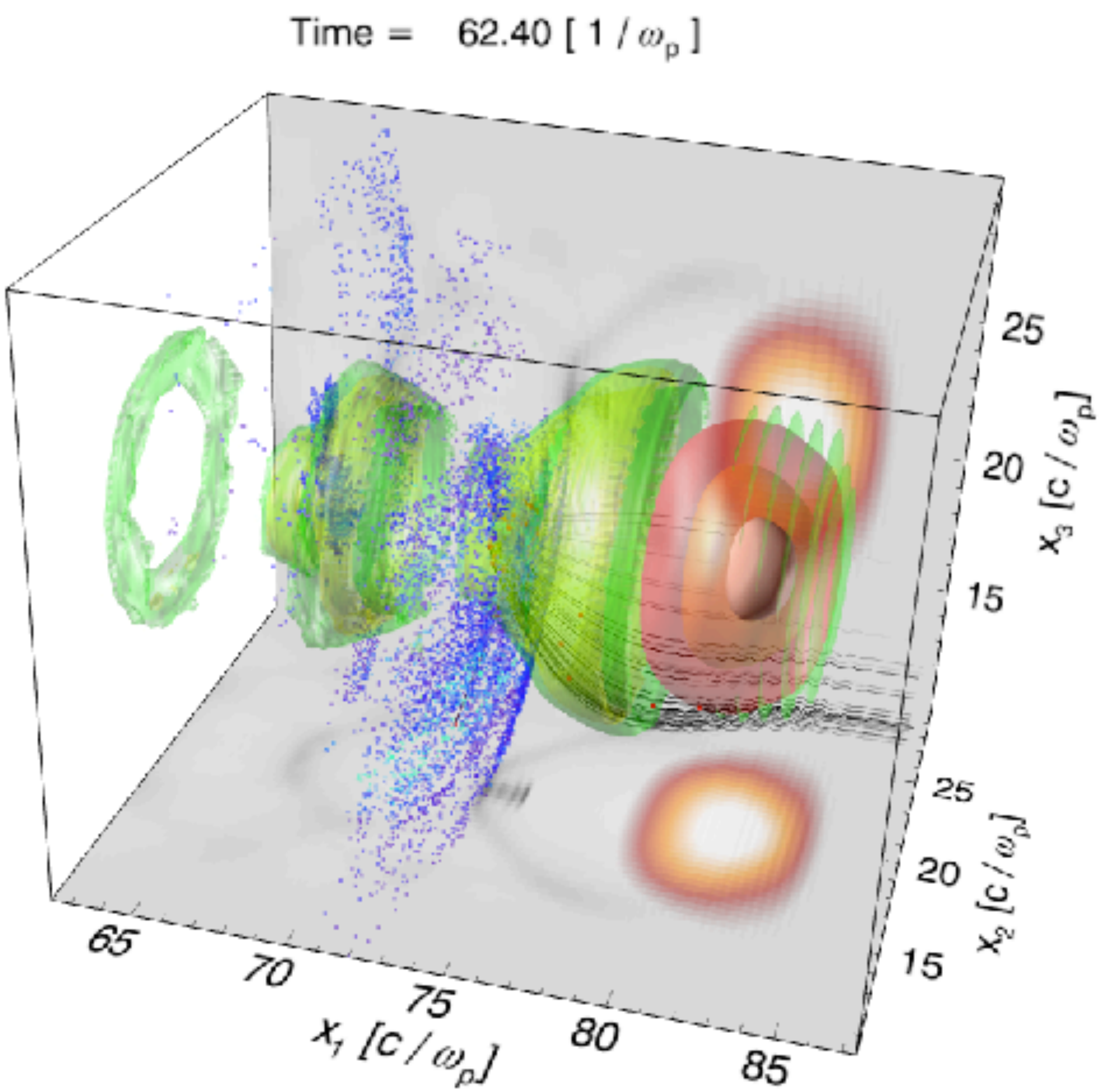
And for positron drivers?

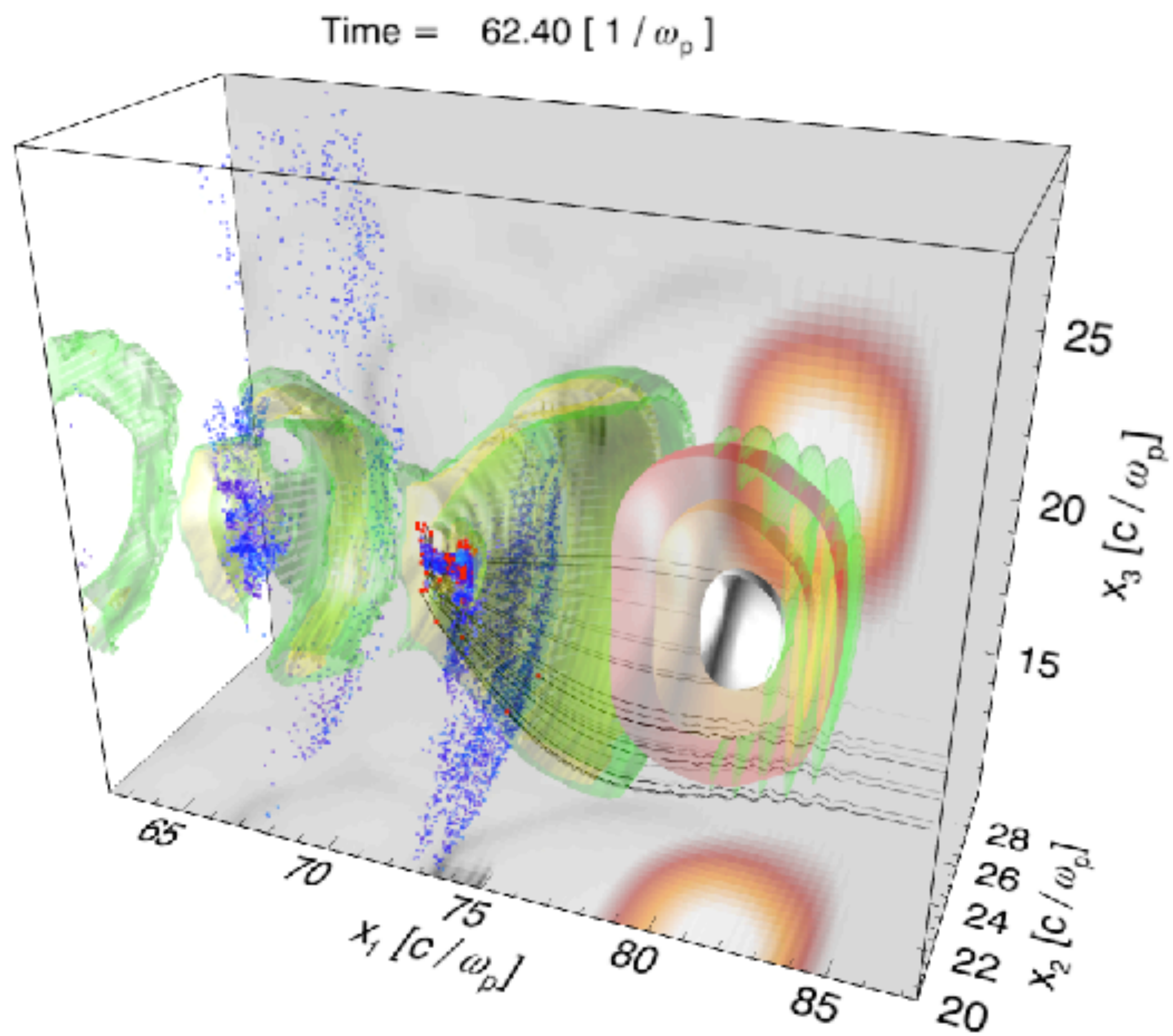


Contents

Structure of laser driven wakefield

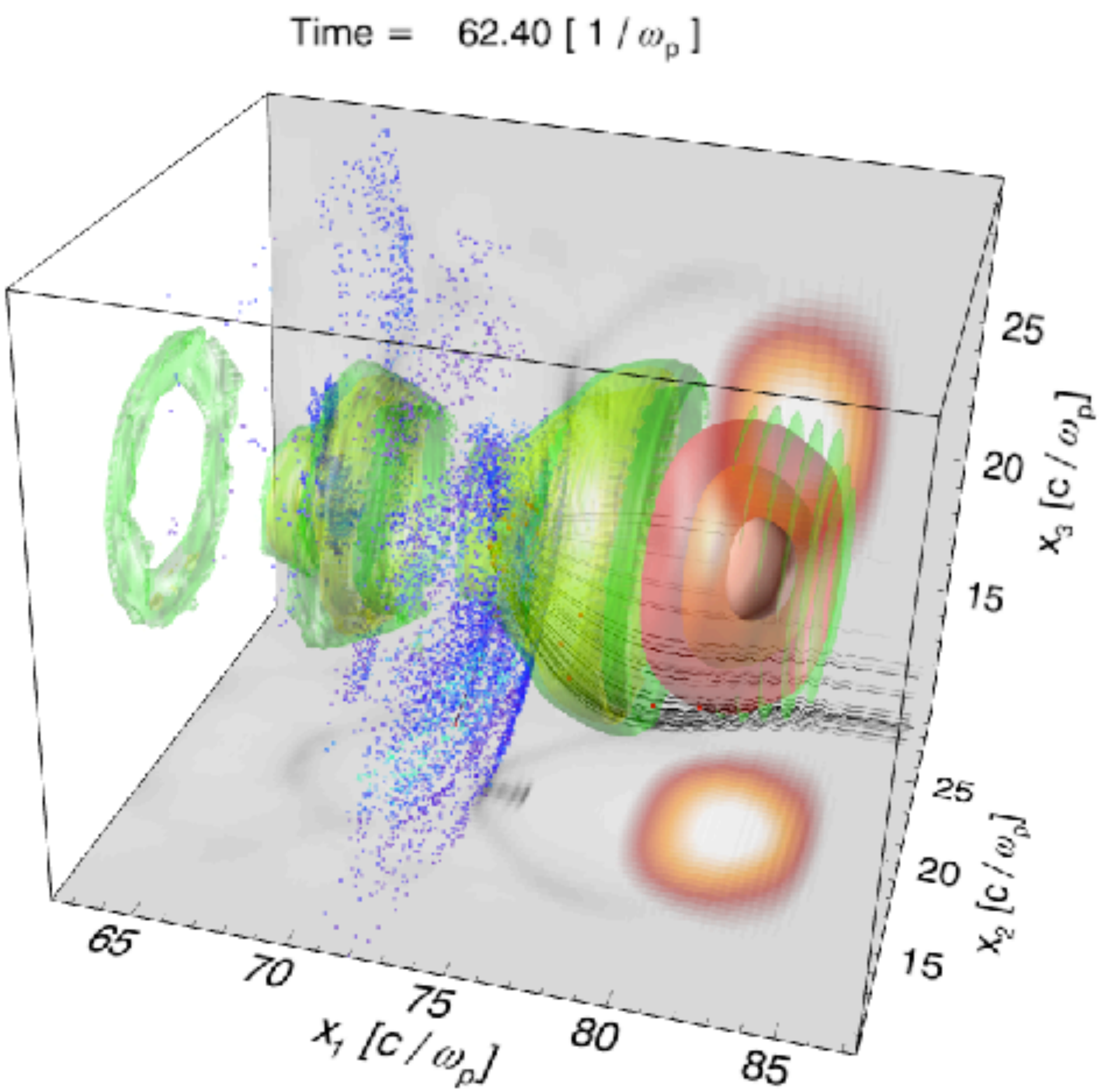
Self-injection provides electrons for acceleration

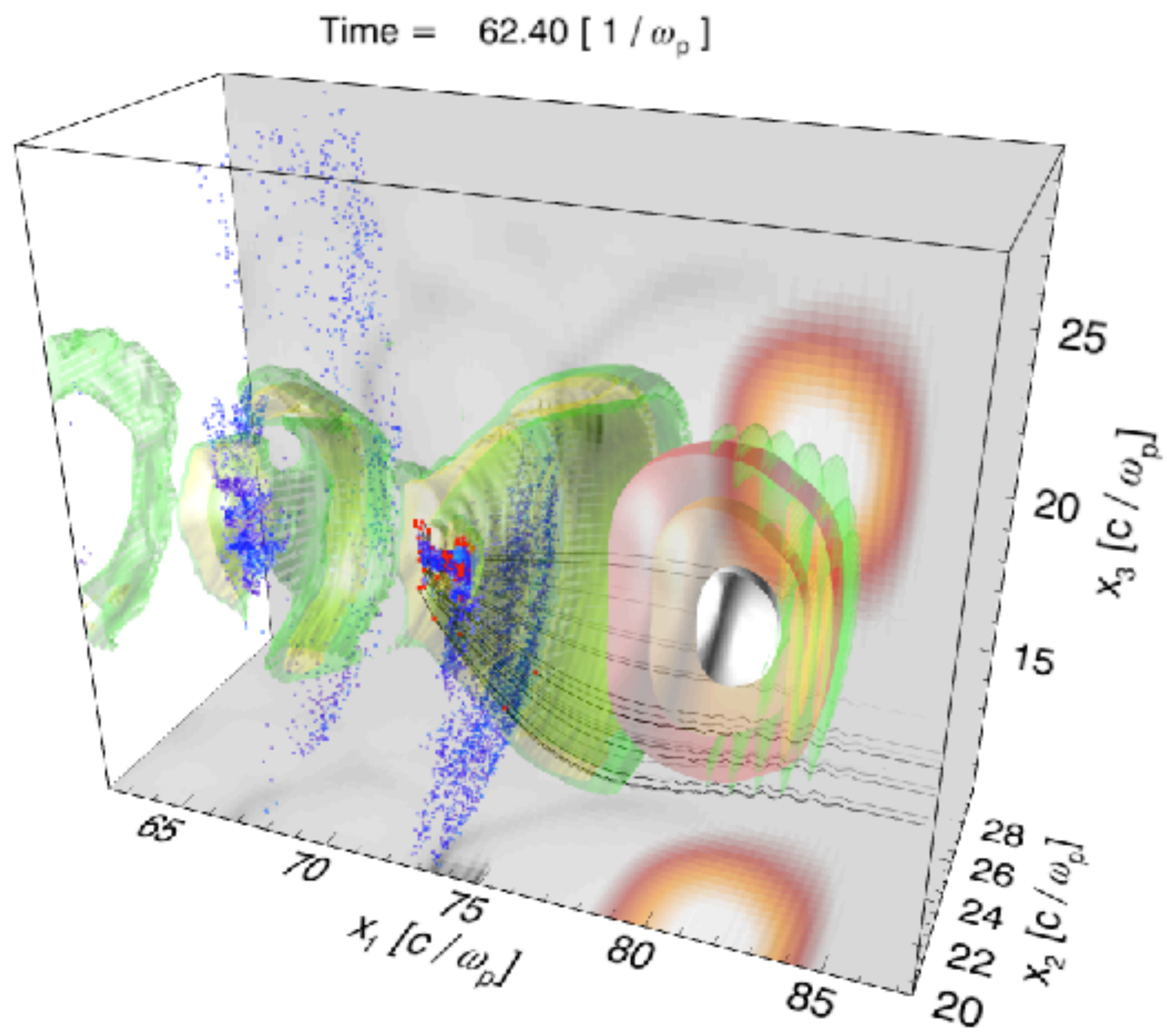




Structure of laser driven wakefield

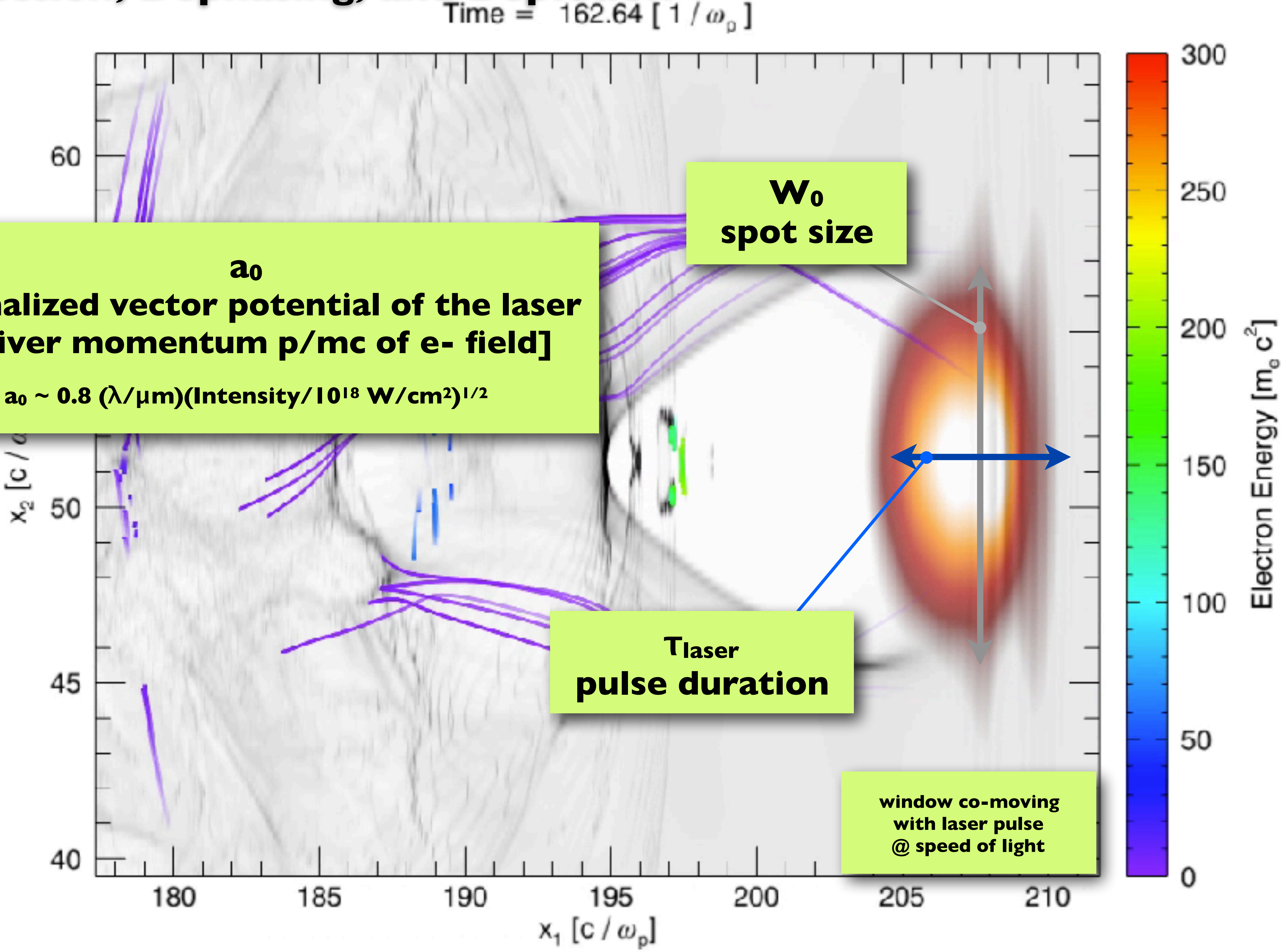
Self-injection provides electrons for acceleration





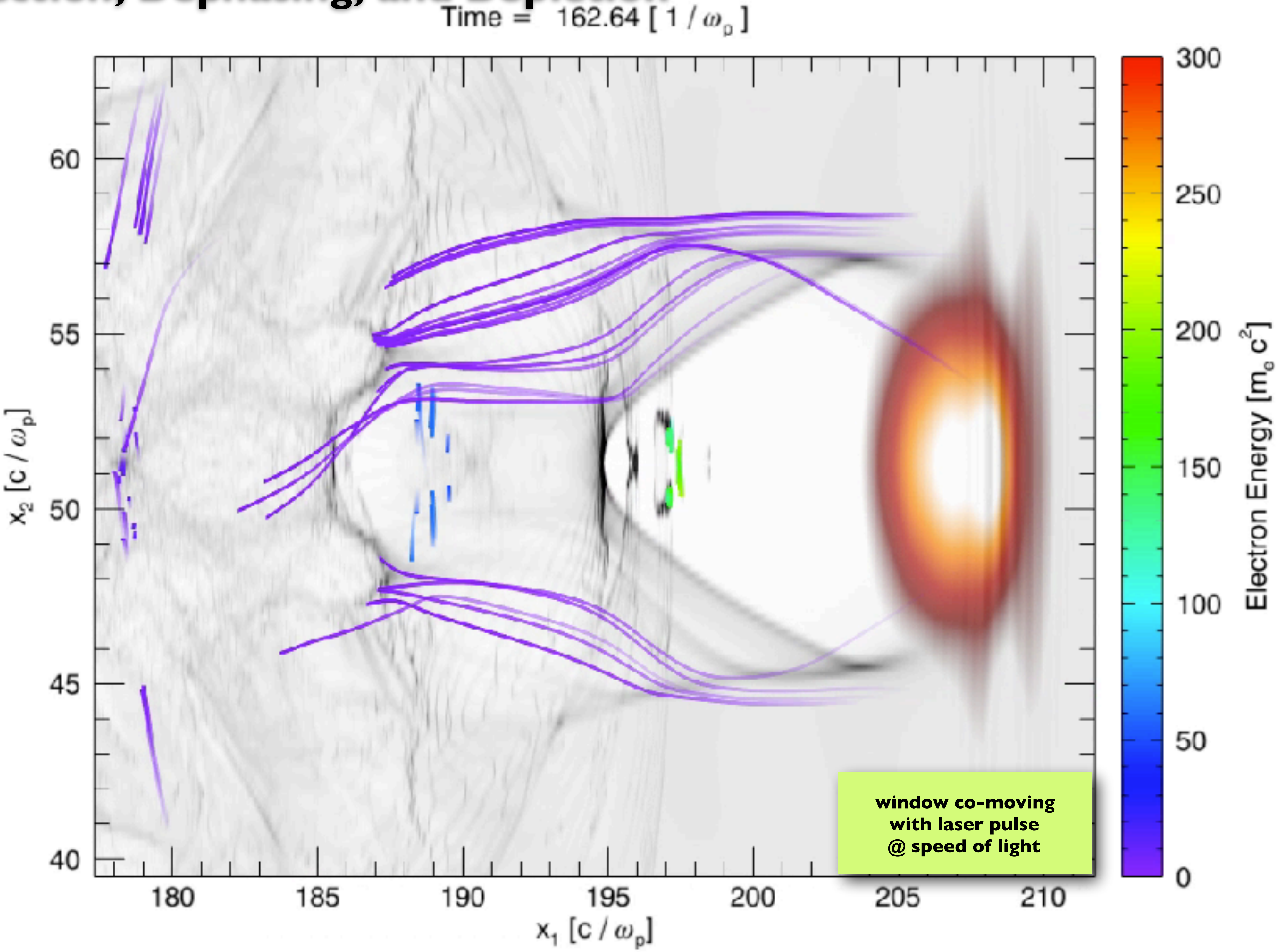
Blow-out regime of laser wakefield acceleration

Self-injection, Dephasing, and Depletion



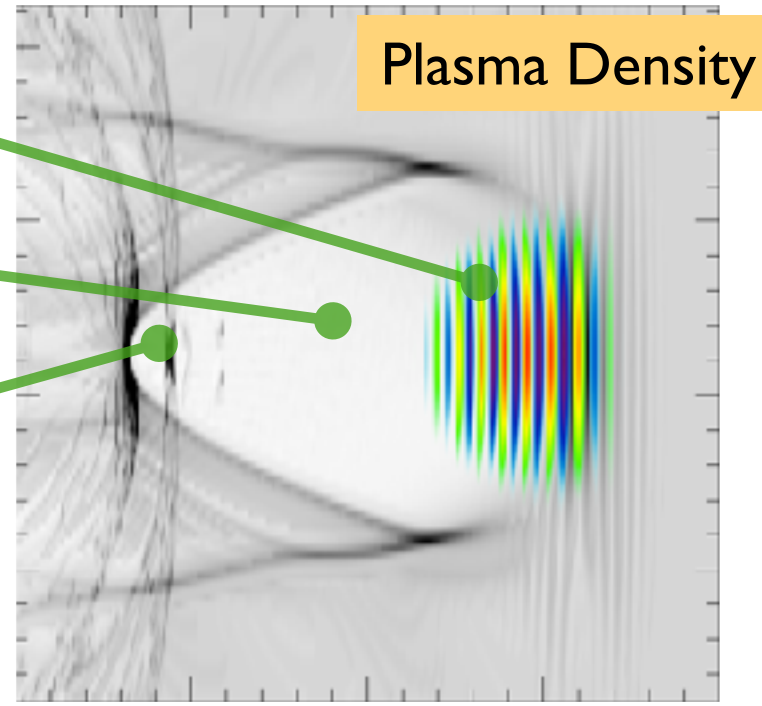
Blow-out regime of laser wakefield acceleration

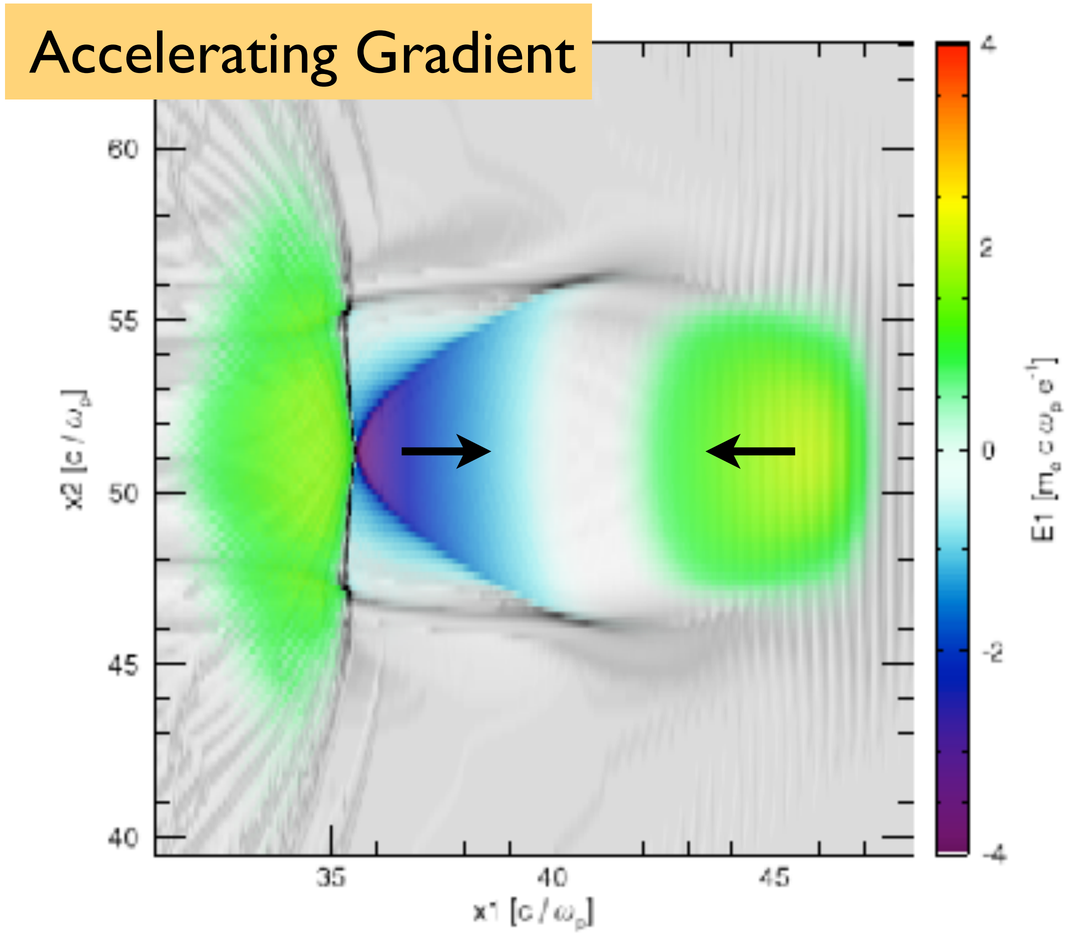
Self-injection, Dephasing, and Depletion

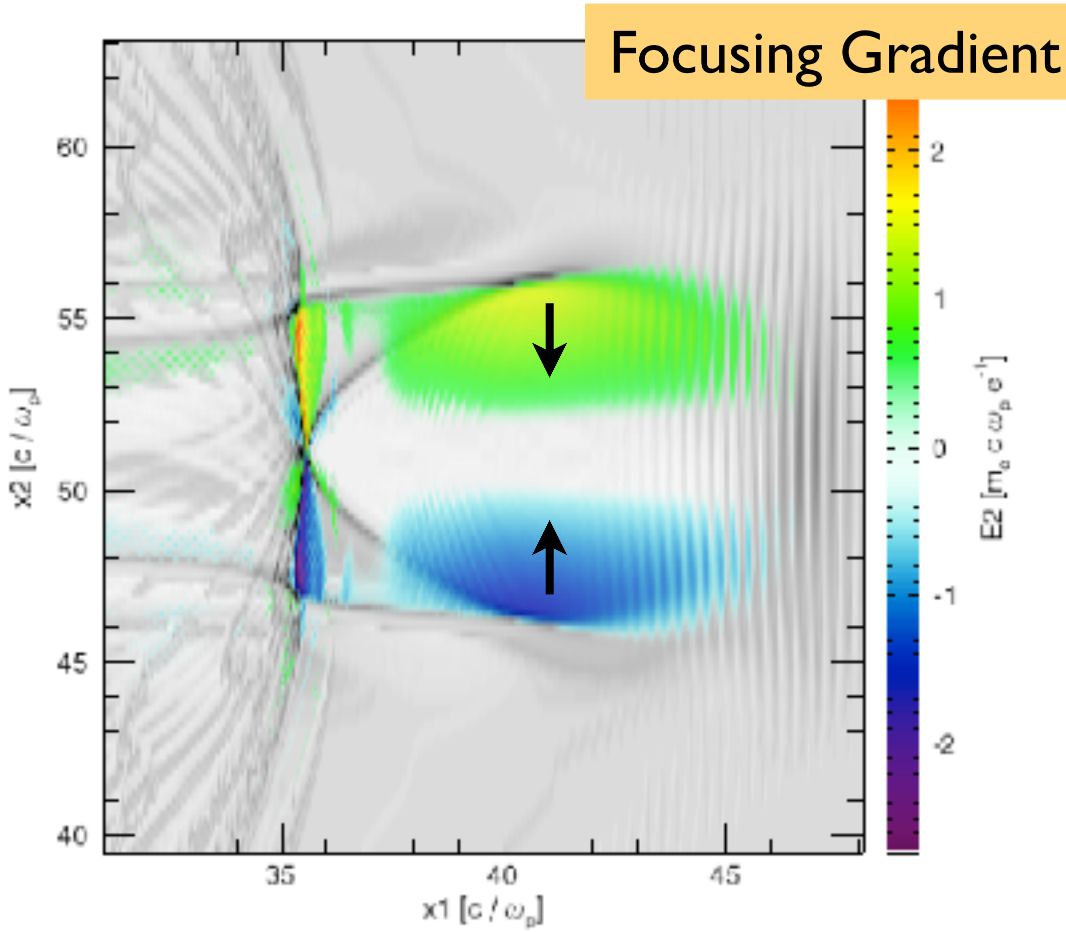


Blow-out regime of laser wakefield acceleration

- Intense laser pulse pushes electrons away from axis
- Electron void is formed behind laser
  - **Blowout-regime/ bubble regime**
- Electrons return to axis due to ion channel force
- Trajectory crossing leads to self injection when outer sheet near spot-size reaches axis
- Ion column creates strong accelerating and focusing gradients



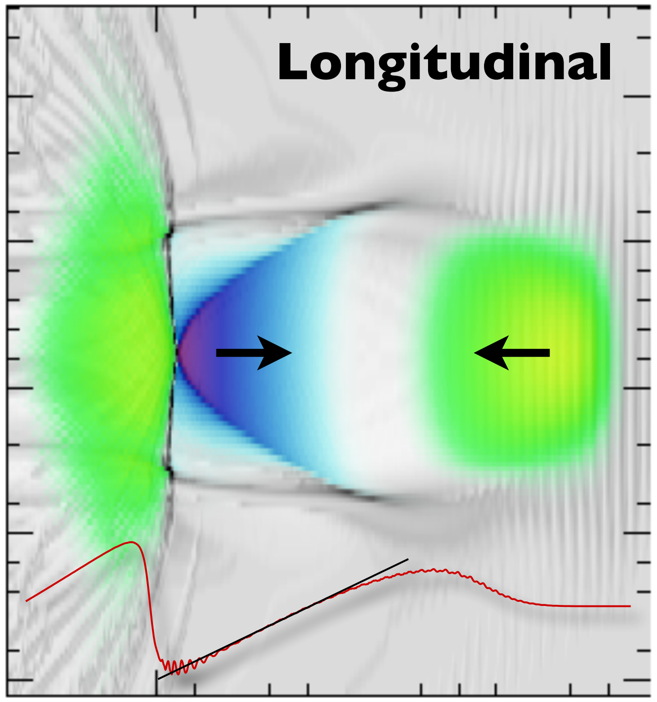




Phenomenological theory based on physical picture

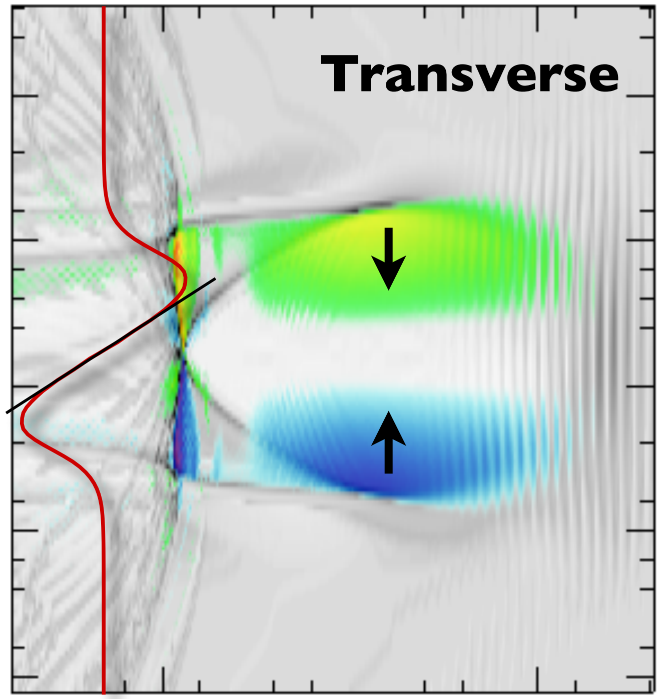
Dynamics of the laser and e- define key parameters

Electric fields created by laser pulse



Linear accelerating gradient

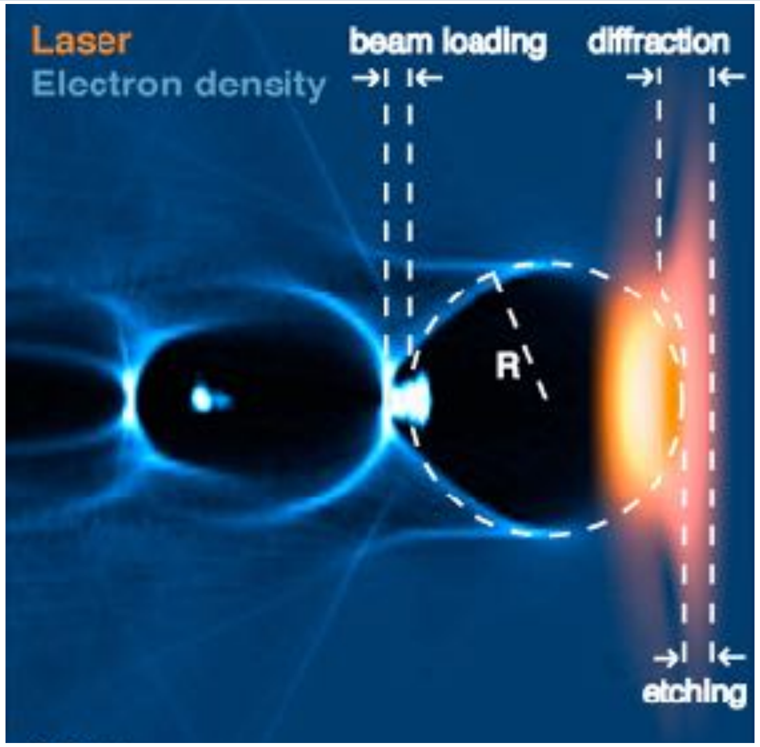
$$E_{z\,\max} \approx \sqrt{a_0}$$



Linear focusing force

$$k_p R \simeq 2\sqrt{a_0}$$

Matched laser parameters



Match laser spot size to bubble radius

$$k_p R \simeq k_p W_0 = 2\sqrt{a_0}$$

For maximum energy gain: trapped e- dephasing before pump depletion

$$L_{\rm etch} \simeq c\omega_0^2/\omega_p^2 \tau_{\rm FWHM} \quad L_{\rm etch} > L_d \quad L_d \simeq \frac{2}{3}\frac{\omega_0^2}{\omega_p^2} R$$

$$c\tau_{\rm FWHM} > 2R/3$$

W. Lu *et al.* PR-STAB (2007)

Different regimes for LWFA

Maximum electron energy →

	Self-guiding		External-guiding	
	Self Injection I*	Self Injection II**	Self Injection**	External Injection**
Main goal	Maximize Charge	←	Maximize electron energy	→
Efficiency	19%	←	$\sim 0.52/a_0$	→
Typical a_0	$\gtrsim \sqrt{2n_c/n_p}$	PW range $\sim (n_c/n_p)^{1/5}$	$\gtrsim 3$	~ 2

Laser pulse

$$\tau_{\rm FWHM}[\rm fs] \simeq 53.22 \left(\frac{\lambda_0[\mu \rm m]}{0.8}\right)^{2/3} \left(\frac{\epsilon[\rm J]}{a_0^2}\right)^{1/3}$$

$$W_0 = \frac{3}{2} c\tau_{\rm FWHM}$$

Plasma

$$n_p[10^{18}\ \rm cm^{-3}] \simeq 3.71 \frac{a_0^3}{P[\rm TW]} \left(\frac{\lambda_0[\mu \rm m]}{0.8}\right)^{-2}$$

$$L_{\rm acc}[\rm cm] \simeq 14.09 \frac{\epsilon[\rm J]}{a_0^3}$$

Injected bunch

$$\Delta E[\rm GeV] \simeq 3 \left(\frac{\epsilon[\rm J]}{a_0^2} \frac{0.8}{\lambda_0[\mu \rm m]}\right)^{2/3}$$

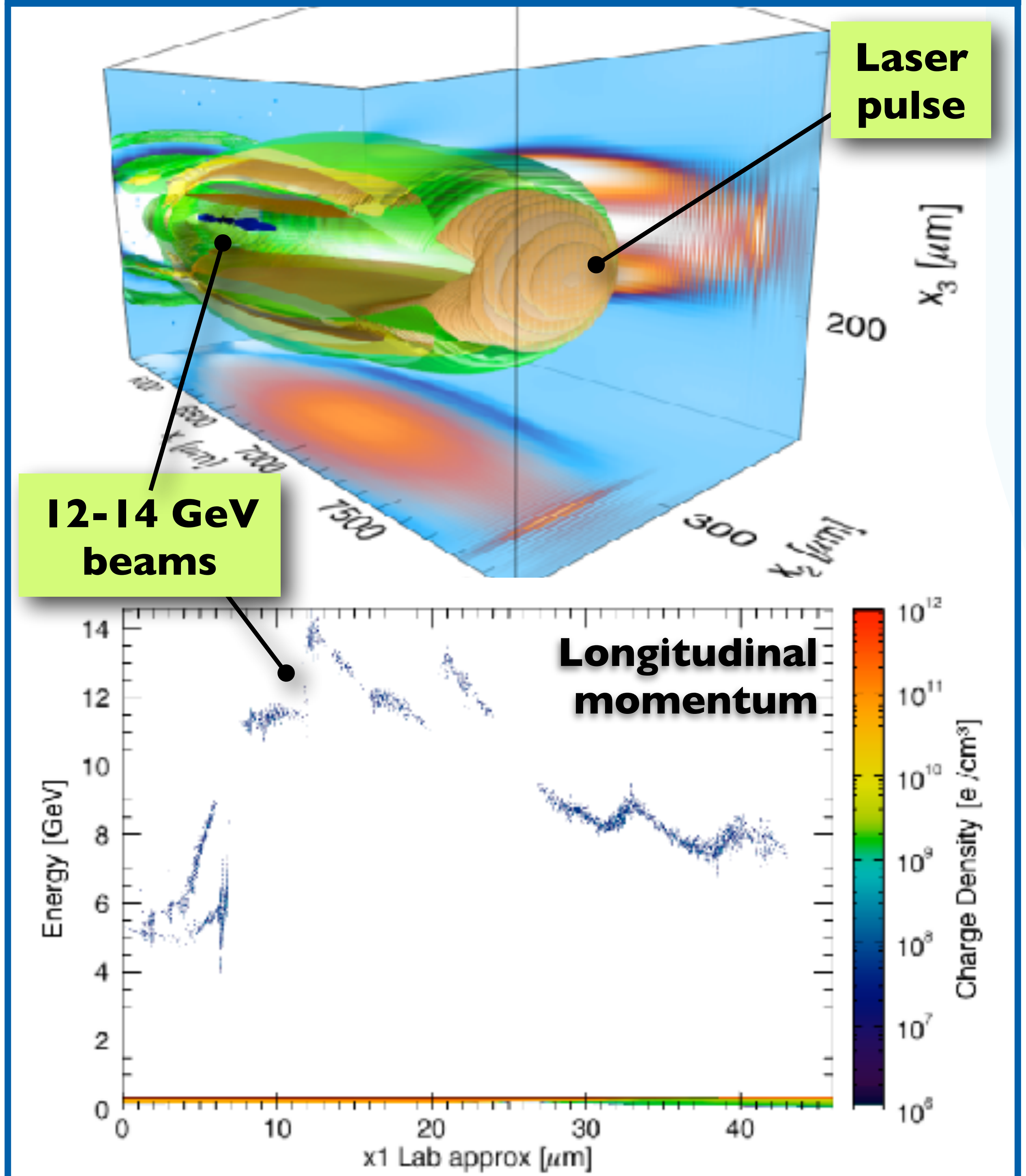
$$q[\rm nC] \simeq 0.17 \left(\frac{\lambda_0[\mu \rm m]}{0.8}\right)^{2/3} (\epsilon[\rm J]\ a_0)^{1/3}$$

* S. Gordienko and A. Pukhov PoP (2005)
** W. Lu *et al.* PR-STAB (2007)

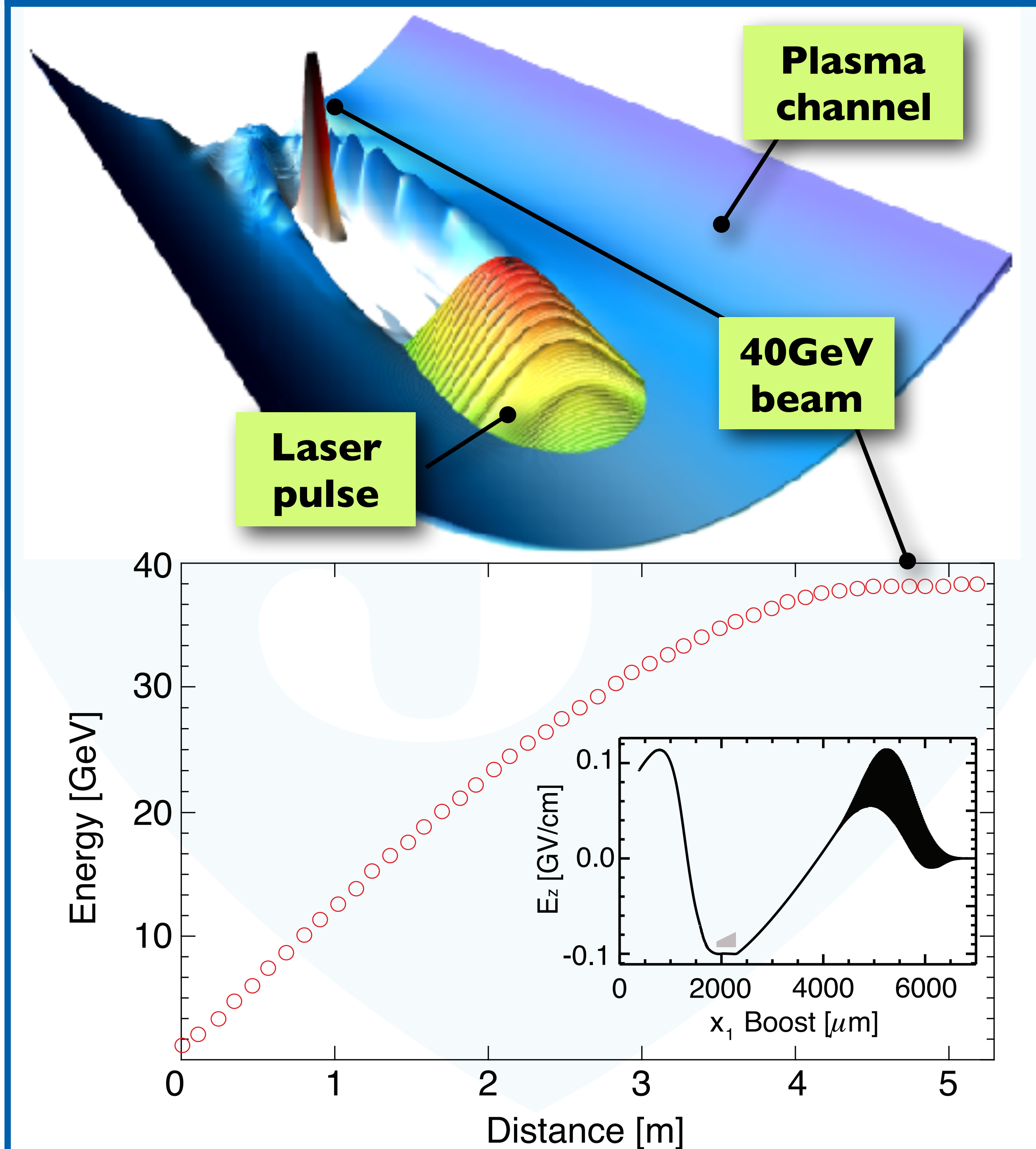
For the correct pre-factors in all the equations check Silva et al, (2009) Comptes Rendus Physique, 10(2-3), 167–175.

Acceleration distances can be reduced by orders of magnitude

Self-injection: >10 GeV



External-injection w/ beam loading: 40GeV



S.F. Martins et al Nat. Physics **6** 311 (2010)

Parameter range for 300J laser system

	Self-guiding		External-guiding
	Self Injection I*	Self Injection II**	External Injection**
Laser			
a0	53	5.8	2
Spot [μm]	10	50	101
Duration [fs]	33	110	224
Plasma			
Density [cm^{-3}]	1.5×10^{19}	2.7×10^{17}	2.2×10^{16}
Length [cm]	0.25	22	500
e- Bunch			
Energy [GeV]	**3**	**13**	**53**
Charge [nC]	14	2	1.5

For the correct pre-factors in all the equations check Silva et al, (2009) Comptes Rendus Physique, 10(2-3), 167–175.

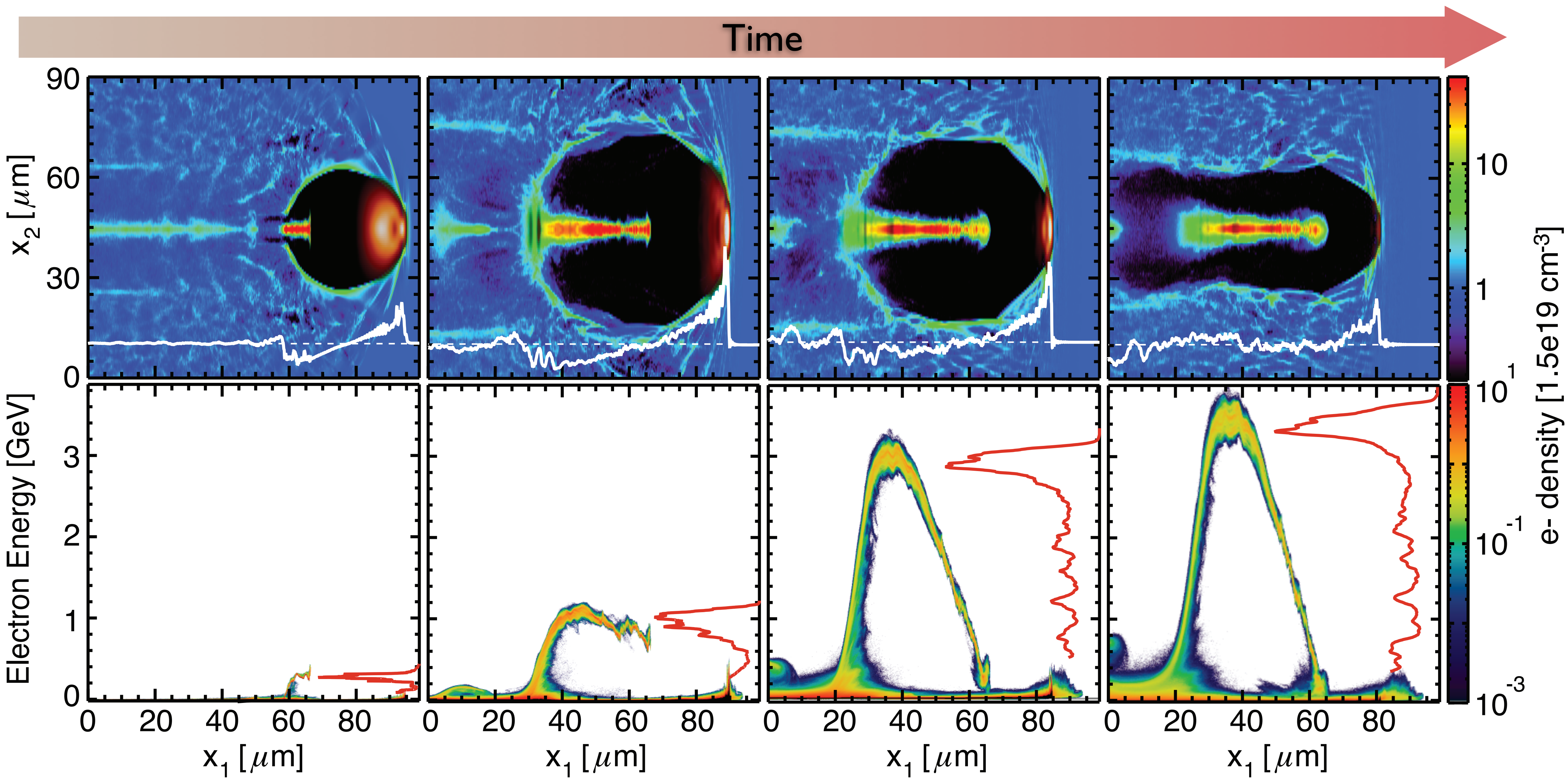
* S. Gordienko and A. Pukhov PoP (2005)

** W. Lu et al. PR-STAB (2007)

+3GeV self-injection in strongly nonlinear regime

Extreme blowout $a_0=53$

S.F. Martins et al, Nature Physics (April 2010)

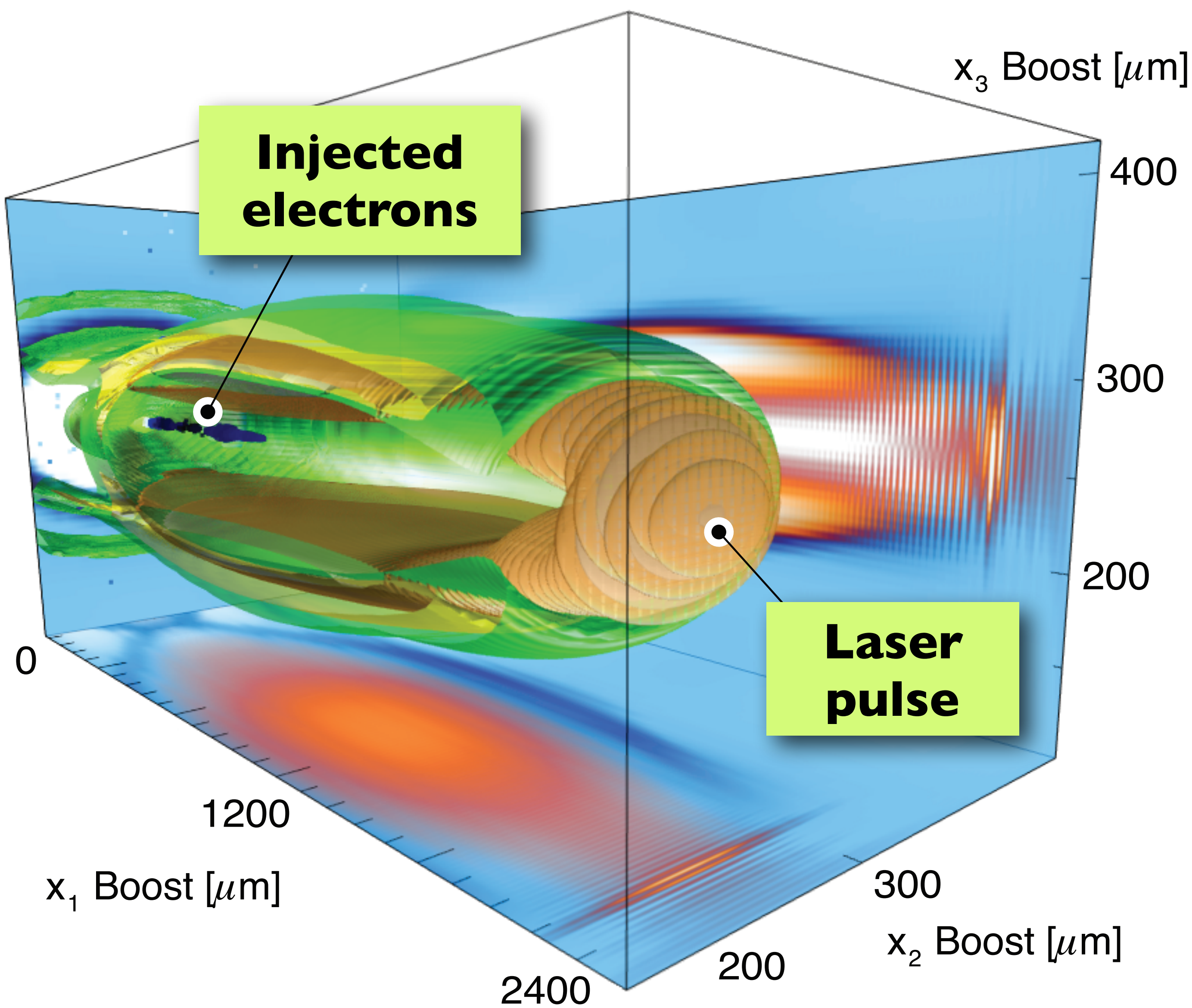


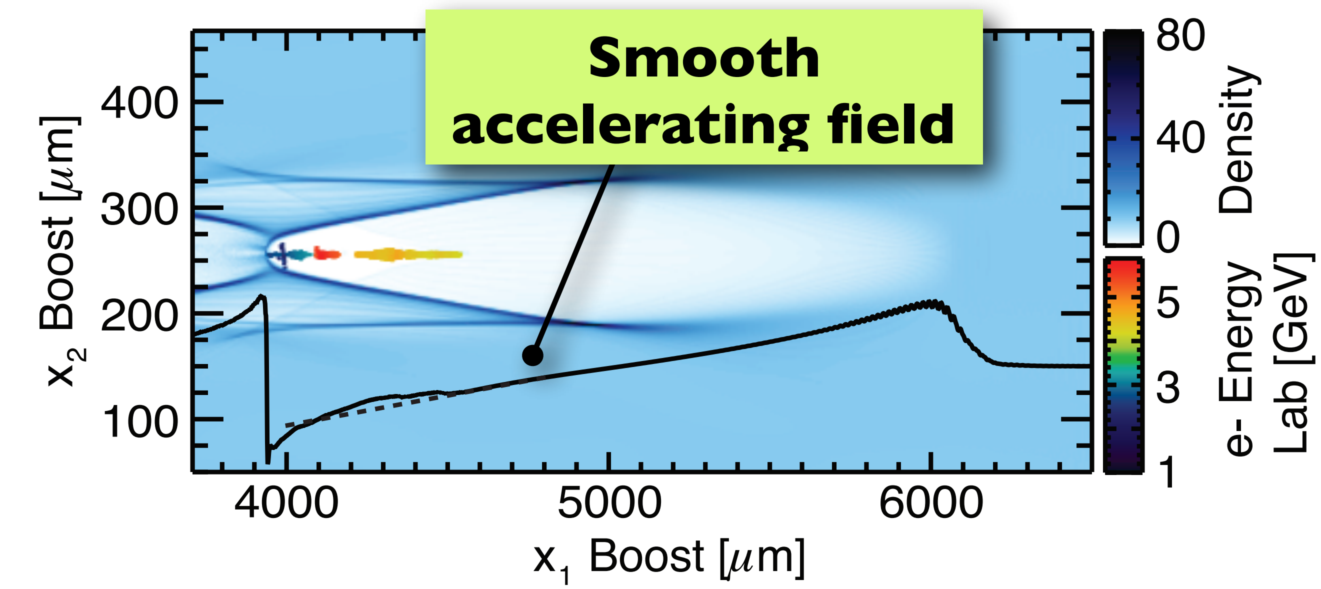
Laboratory frame
3000x256x256 cells
$\sim 10^9$ particles
10^5 timesteps

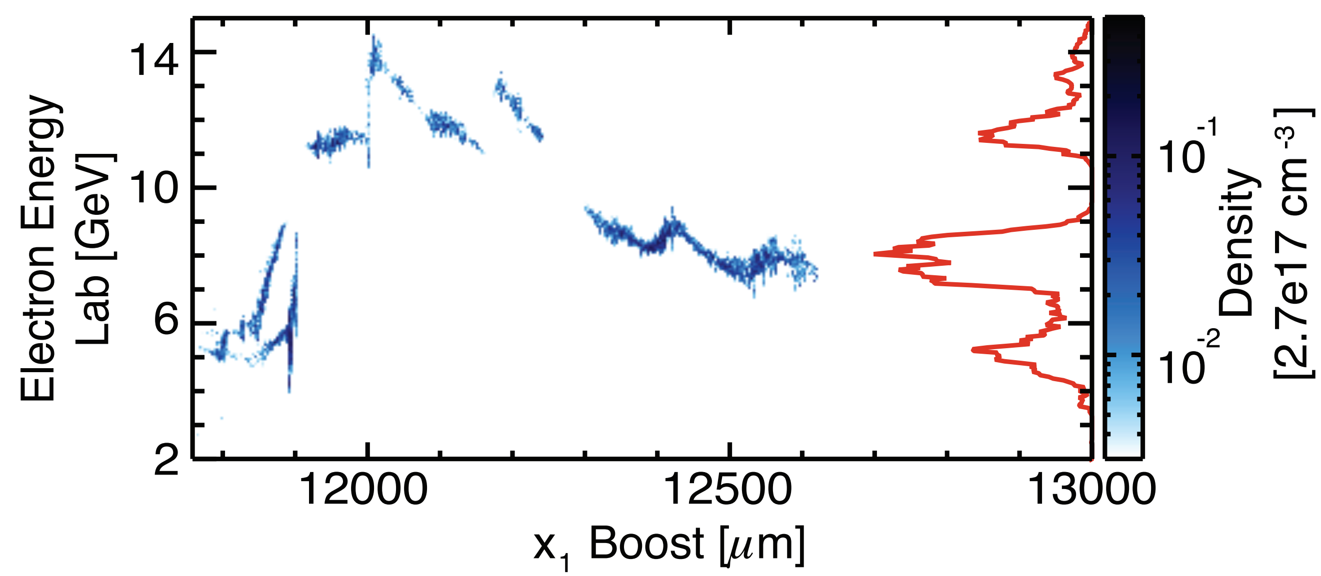
3.4 GeV
17 nC

+10GeV self-injection in nonlinear regime

Controlled self-guided $a_0=5.8$





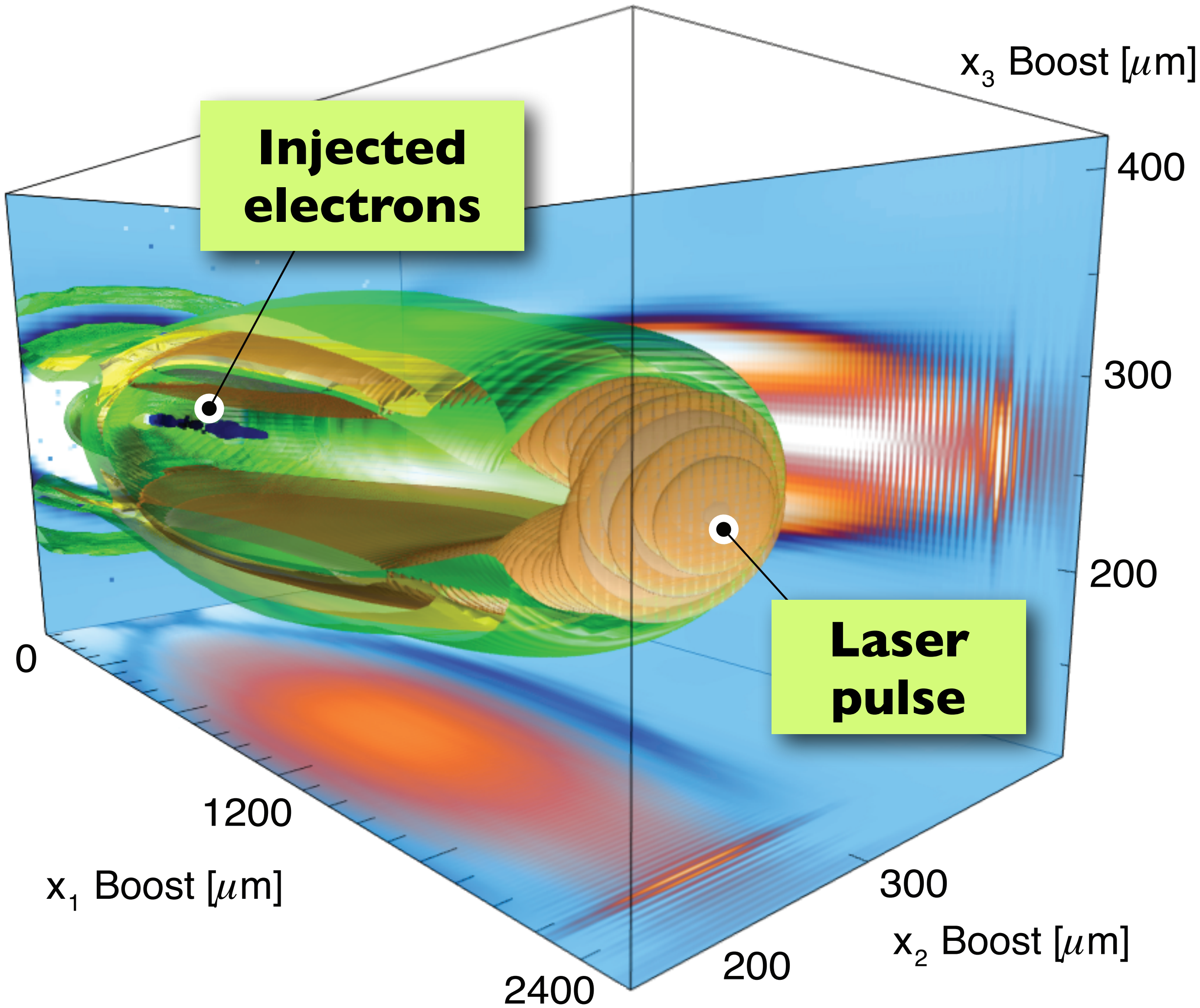


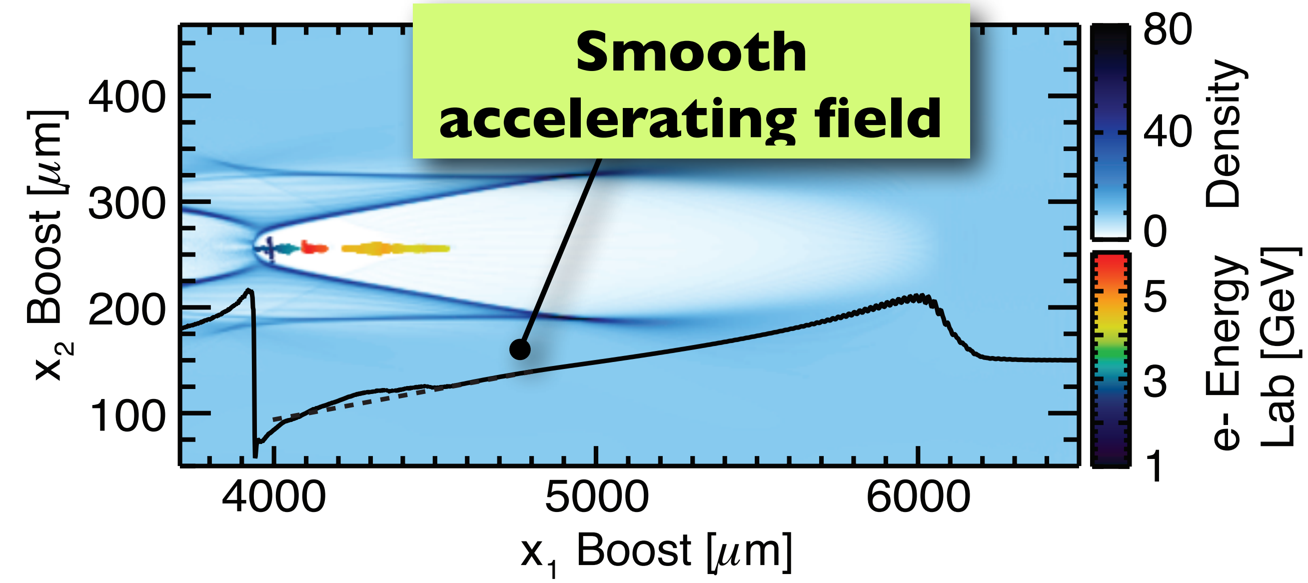
Boosted frame
7000x256x256 cells
~10^9 particles
3×10^4 timesteps
$\Upsilon=10$

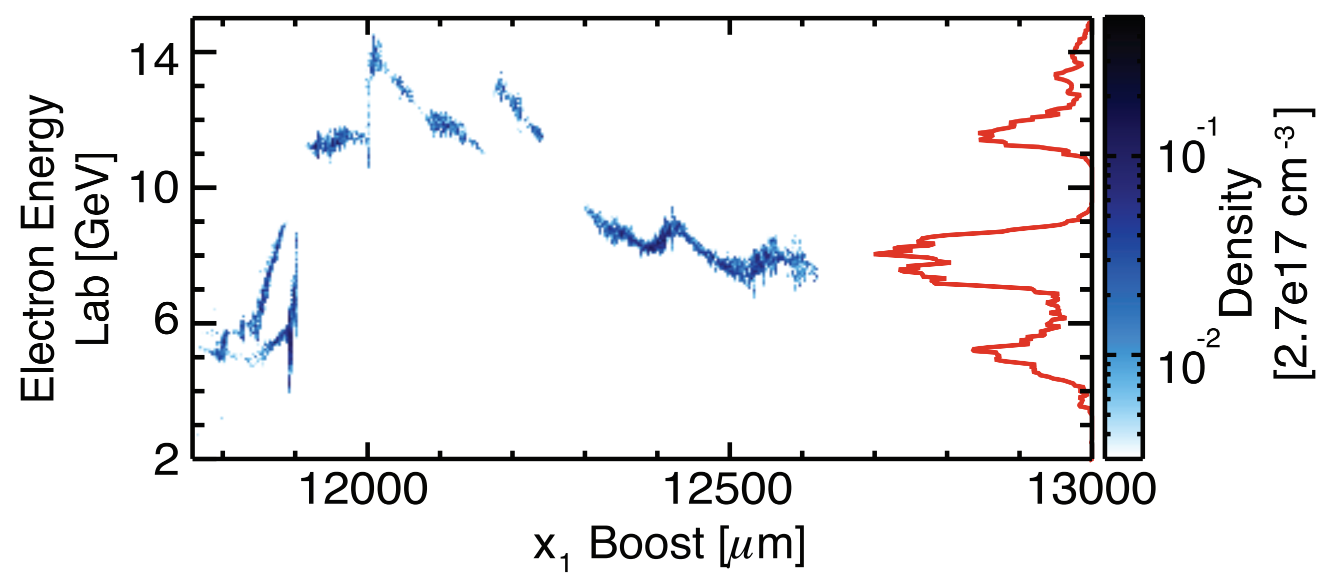
7-12 GeV
1-2 nC

+10GeV self-injection in nonlinear regime

Controlled self-guided a_0=5.8







Boosted frame
7000x256x256 cells
~10^9 particles
3×10^4 timesteps
Υ=10

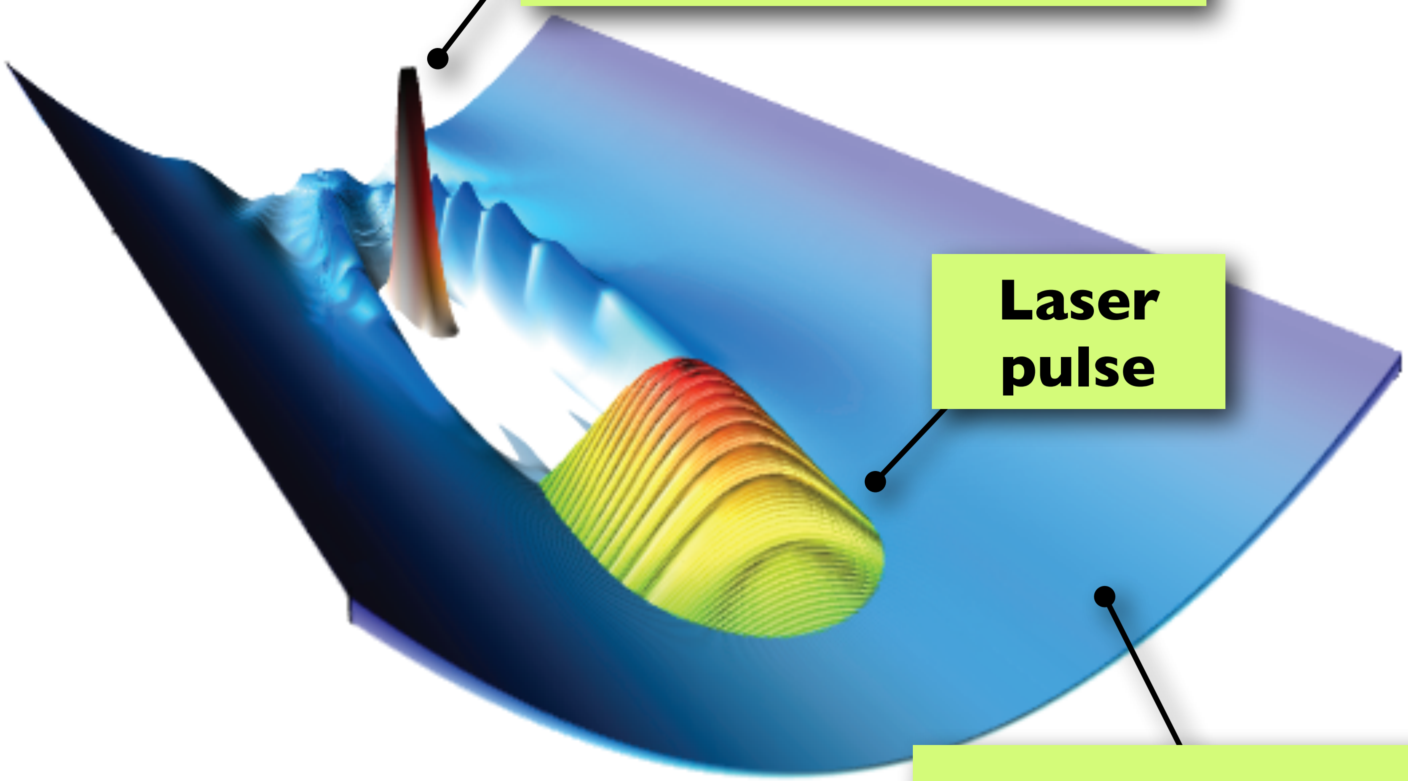
~300x faster
than lab simulation

7-12 GeV
1-2 nC

+40GeV with externally injected beams

Channel guided $a_0=2$

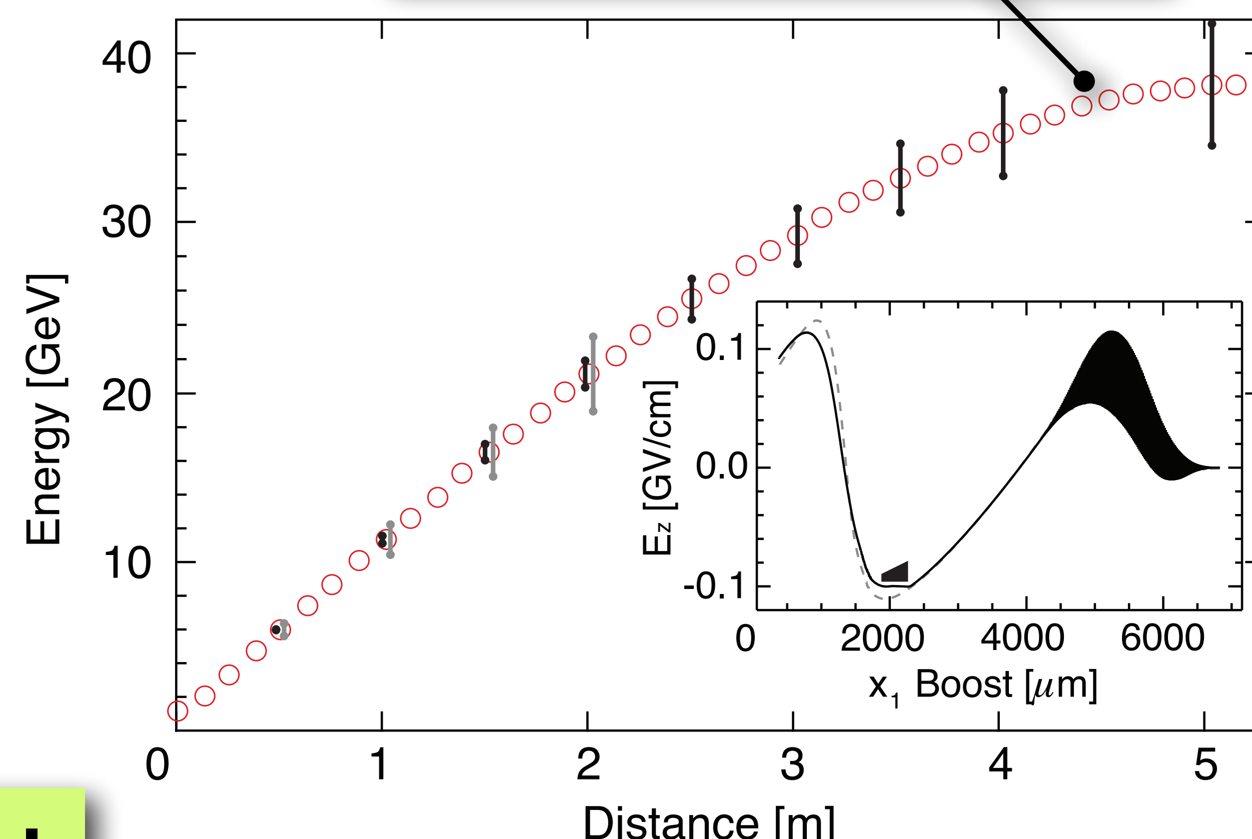
Tailored injected beam to minimize final energy spread



Laser pulse

Guiding channel
Length: 5.28m
Density: 2.2e16 cm^{-3}

Stable accelerating field for over 5 meters



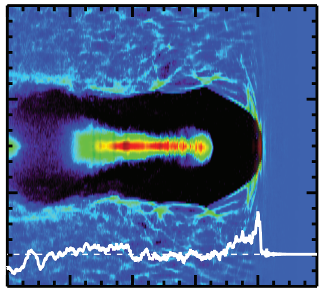
Boosted frame
8000x128x128 cells
~5×10^8 particles
2×10^5 timesteps
$\Upsilon=10$

~300x faster than lab simulation

40 GeV
~1 nC

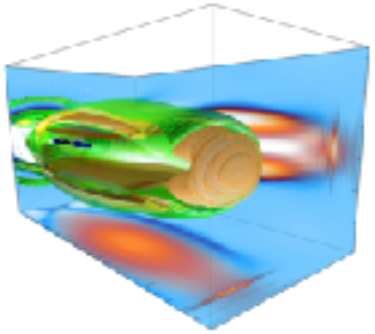
Energy frontier LWFA modeling

Extreme blowout :: a_0=53



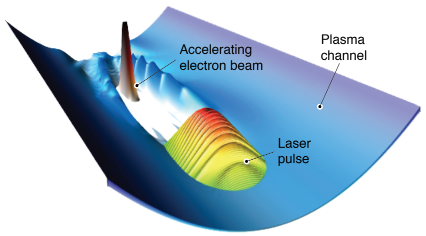
- Very nonlinear and complex physics
- Bubble radius varies with laser propagation
- Electron injection is continuous ⇒ very strong beam loading
- Wakefield is noisy and the bubble sheath is not well defined

Controlled self-guided :: a_0=5.8



- Lower laser intensity ⇒ cleaner wakefield and sheath
- Loaded wakefield is relatively flat
- Blowout radius remains nearly constant
- Three distinct bunches ⇒ room for tuning the laser parameters

Channel guided :: a_0=2



- Lowest laser intensity ⇒ highest beam energies (less charge)
- External guiding of the laser ⇒ stable wakefield
- Tailored electron beam that initially flattens the wake
- Controlled acceleration of an externally injected beam to very high energies

Contents

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The plan for a theoretical model for the blowout regime

Determine the equation of motion for a fluid element in the quasi-static approximation and assuming no sheath crossing

Determine the structure of the fields (cylindrically symmetric) for a model of the current/charge system in the bubble/blowout

Determine the equation of motion for the inner surface of the blowout region ($r = r_b$)

Generic particle Hamiltonian in 3D

Hamiltonian for a charged particle:

$$H = \sqrt{m_e^2 c^4 + (\mathbf{P} + e\mathbf{A}/c)^2} - e\phi$$

Canonical momentum (**P=p**-e**A/c**)

Vector potential

scalar potential

New co-moving frame variables:

$$\xi = v_\phi t - x$$

Distance to the head of a beam moving at v_Φ

$$\tau = x$$

Propagation distance

Hamiltonian in the co-moving frame

$$\mathcal{H} = H - v_\phi P_{||}$$

Hamilton's equations in the co-moving frame variables

Chain rule for co-moving frame variables

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} - \frac{\partial}{\partial \tau}$$

$$\frac{\partial}{\partial t} = v_\phi \frac{\partial}{\partial x}$$

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} = \left(v_\phi - v_\parallel\right)$$

Hamilton's equations in co-moving frame

$$\frac{\mathrm{d}P_\parallel}{\mathrm{d}t} = -\frac{\partial H}{\partial x} = \frac{\partial H}{\partial \xi} - \frac{\partial H}{\partial \tau}$$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\partial H}{\partial t} = v_\phi \frac{\partial H}{\partial \xi}$$

Evolution of the Hamiltonian in the co-moving frame

General evolution of the co-moving frame Hamiltonean

$$\left(v_\phi - v_\parallel\right)\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}\xi} = \left[\mathbf{v}\cdot\frac{\partial \mathbf{A}}{\partial \tau} - \frac{\partial \phi}{\partial \tau}\right]$$

=d/dt

use the chain rule

$\Delta\mathcal{H}=\mathcal{H}(t_f)-\mathcal{H}(t_i)$ depends on initial and final positions only:

$$\Delta\mathcal{H} = \int \frac{\mathrm{d}\mathcal{H}}{\mathrm{d}t}\mathrm{d}t = \int \frac{\mathrm{d}\xi}{v_\phi - v_\parallel}\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}\xi}$$

Integration over the particle's trajectory

~0 for a non-evolving wake/driver (**quasi-static approximation**)

Constants of motion under quasi-static approximation

General constant of motion under quasi-static approximation

$$\Delta H = \Delta\gamma - v_\phi \Delta p_{||} - \left(\Delta\phi - v_\phi \Delta A_{||}\right)$$
$$= \Delta\gamma - v_\phi \Delta p_{||} - \Delta\psi$$

pseudo potential
$\Psi=\Phi-v_\phi A_{||}$

Constant of motion for a particle initially at rest in region of vanishing fields

$$\gamma\left(1-\beta_{||}\right) = 1+\psi$$

For $\beta_{||} \rightarrow 1 \Rightarrow \Psi \rightarrow -1$

For $\beta_{||} \rightarrow -1 \Rightarrow \Psi \rightarrow \infty$

$$-1 < \psi < +\infty$$

Lorentz force equation for the radial motion of a plasma electron under the quasi-static approximation

Goal: write Lorentz force in the co-moving frame (v_ϕ=c=1)

Use constant of motion to write total time derivative:

$$\frac{\mathrm{d}}{\mathrm{d}t} = (1 - v_{\|})\frac{\mathrm{d}}{\mathrm{d}\xi} = \frac{1+\psi}{\gamma}\frac{\mathrm{d}}{\mathrm{d}\xi}$$

velocity normalised to c

Use constant of motion to write total time derivative:

$$p_\perp = \gamma v_\perp = (1+\psi)\frac{\mathrm{d}r_\perp}{\mathrm{d}\xi} \qquad \longrightarrow \qquad \frac{\mathrm{d}p_\perp}{\mathrm{d}t} = \frac{1+\psi}{\gamma}\frac{\mathrm{d}}{\mathrm{d}\xi}\left[(1+\psi)\frac{\mathrm{d}}{\mathrm{d}\xi}\right]$$

Recast γ using constant of motion

$$\gamma = \frac{1 + p_\perp^2 + (1+\psi)^2}{2(1+\psi)}$$

W. Lu, MsC thesis, UCLA (2004)

Lorentz force equation for the radial motion of a plasma electron under the quasi-static approximation

$$\frac{2\left(1+\psi\right)^2}{1+\left(1+\psi\right)^2\left(\frac{\mathrm{d}r}{\mathrm{d}\xi}\right)^2+\left(1+\psi\right)^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left[\left(1+\psi\right)\frac{\mathrm{d}r}{\mathrm{d}\xi}\right]=F_\perp$$

$$F_\perp=-\left(E_r-v_{||}B_\theta\right)$$

particles do not move in ξ under the q.s.a.

Potentials associated with electromagnetic fields under q.s.a.:

All other fields vanish for a cylindrically symmetric configuration

$$E_z=\frac{\partial\psi}{\partial\xi}$$

accelerating field

$$E_r=-\frac{\partial\phi}{\partial r}-\frac{\partial A_r}{\partial\xi}$$

radial electric field

$$B_\theta=-\frac{\partial A_z}{\partial r}-\frac{\partial A_r}{\partial\xi}$$

azimuthal magnetic field

W. Lu, MsC thesis, UCLA (2004)

The plan for a theoretical model for the blowout regime

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Electromagnetic field equations for cylindrically symmetric plasma waves

Equations for potentials under q.s.a.:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_r}{\partial r}\right) - \frac{A_r}{r^2} = n_e v_\perp$$

plasma density normalised to background density (n_0)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_\parallel}{\partial r}\right) = n_b + n_e v_\parallel$$

particle beam driver density normalised to n_0

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) = n_e + n_e v_\parallel - 1$$

immobile ion density normalised to n_0

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) = n_b + n_e - 1$$

$$\frac{1}{r}\frac{\partial}{\partial r} r A_r = -\frac{\partial \psi}{\partial \xi}$$

Gauge condition

W. Lu, MsC thesis, UCLA (2004)

General solutions to wakefield potentials

Right hand side of Lorentz force:

$$F_\perp = -\left(E_r - v_\parallel B_\theta\right) = \left(\frac{\partial \phi}{\partial r} - v_\parallel \frac{\partial A_\parallel}{\partial r}\right) + \left(1 - v_\parallel\right) \frac{\partial A_r}{\partial \xi} - \frac{1}{\gamma} \nabla_\perp \left|\frac{a_L}{2}\right|^2$$

General solutions for potentials:

ion contribution (no electrons in blowout)

beam shape

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) = n_b + n_e - 1 \longrightarrow \phi = \phi_0(\xi) - \frac{r^2}{4} + \lambda(\xi)\ln(r)$$

$$\lambda(\xi) = \int_0^\infty r n_b \mathrm{d}r$$

ξ dependence: blowout shape

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_\parallel}{\partial r}\right) = n_b + n_e v_\parallel \longrightarrow A_\parallel = A_{\parallel 0}(\xi) + \lambda(\xi)\ln r$$

From gauge condition:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) = n_e + n_e v_\parallel - 1 \longrightarrow A_r = A_{r0}(\xi)\, r \longrightarrow A_{r0}(\xi) = -\frac{1}{2}\frac{\mathrm{d}\psi_0}{\mathrm{d}\xi}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) = n_e + n_e v_\parallel - 1 \longrightarrow \psi = \psi_0(\xi) - \frac{r^2}{4}$$

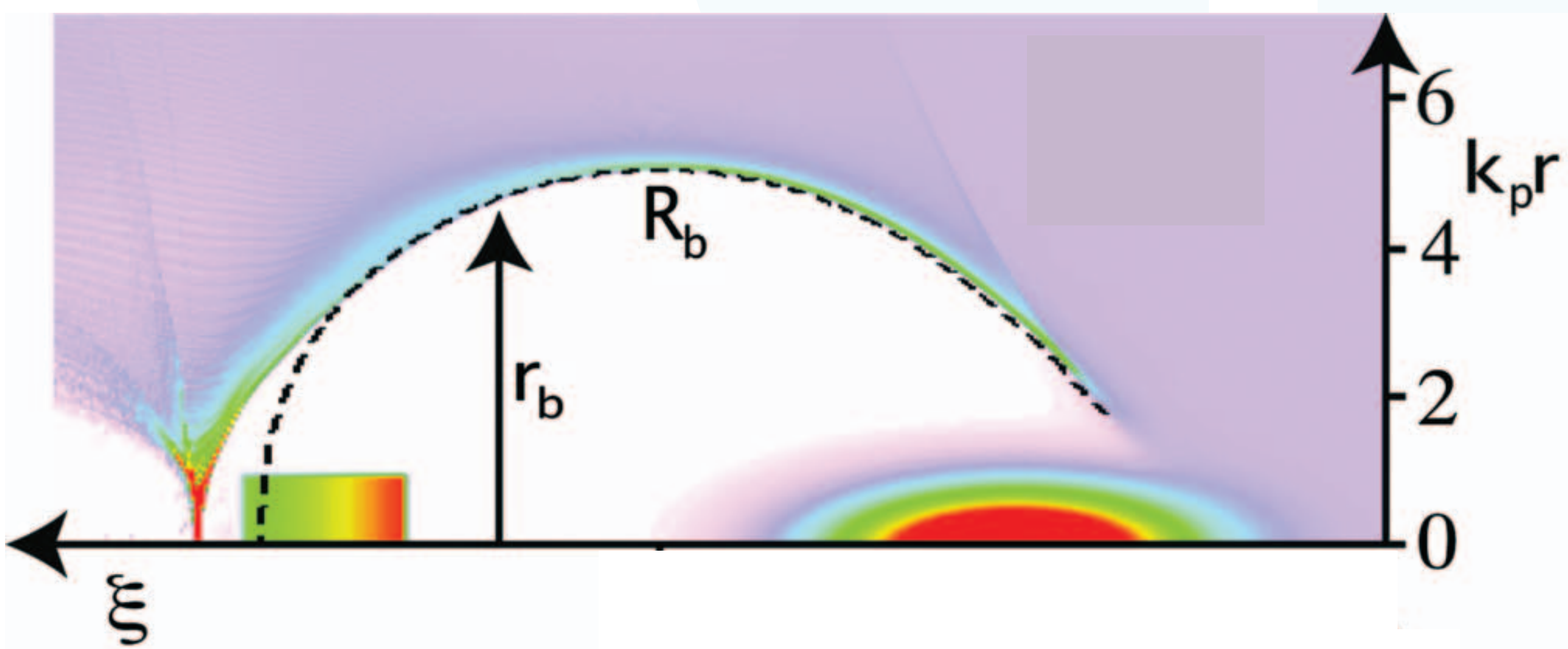
W. Lu, MsC thesis, UCLA (2004)

Find equation of motion for the electron layer defining the blowout region

Right hand side of Lorentz force re-written

$$F_\perp = -\frac{r}{2} + \left(1 - v_\parallel\right)\frac{\lambda(\xi)}{r} + \left(1 - v_\parallel\right)\frac{\mathrm{d}A_{r0}}{\mathrm{d}\xi} r - \frac{1}{\gamma}\nabla_\perp \left|\frac{a_L}{2}\right|^2$$

Goal: write the Lorentz force for the motion of the thin electron sheath that defines the blowout:



W. Lu, MsC thesis, UCLA (2004)

Equation of motion for the blowout radius

$$F_\perp = -\frac{r}{2} + (1 - v_\parallel)\frac{\lambda(\xi)}{r} + (1 - v_\parallel)\frac{\mathrm{d}A_{r0}}{\mathrm{d}\xi} r - \frac{1}{\gamma}\nabla_\perp\left|\frac{a_L}{2}\right|^2$$

Recall:

$$(1 - v_\parallel) = \frac{1 + \psi}{\gamma} \qquad \gamma = \frac{1 + p_\perp^2 + (1 + \psi)^2}{2(1 + \psi)}$$

The pseudo potential Ψ (see how important it is!) fully determines the motion of the blowout region

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\left[(1 + \psi)\frac{\mathrm{d}r_b}{\mathrm{d}\xi}\right] = r_b\left\{-\frac{1}{4}\left[1 + \frac{1}{(1 + \psi)^2} - \left(\frac{\mathrm{d}r_b}{\mathrm{d}\xi}\right)^2\right]\right\} - \frac{1}{2}\frac{\mathrm{d}^2\psi_0}{\mathrm{d}\xi^2} + \frac{\lambda(\xi)}{r_b^2} - \frac{1}{\left(\psi_0 - \frac{r_b^2}{4}\right)}\nabla_\perp\left|\frac{a_L}{2}\right|^2$$

W. Lu, MsC thesis, UCLA (2004)

General expressions to calculate the pseudo-potential Ψ

Recall differential equation for Ψ

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) = n_e + n_e v_{\|} - 1$$

Use Green's function method to find an integral solution

$$\psi(r,\xi) = \ln r \int_0^r r' \left[n_e(r',\xi)\left(1 - v_{\|}(r',\xi)\right) - 1\right] \mathrm{d}r' + \int_r^\infty r' \ln r' \left[n_e(r',\xi)\left(1 - v_{\|}(r',\xi)\right) - 1\right] \mathrm{d}r'$$

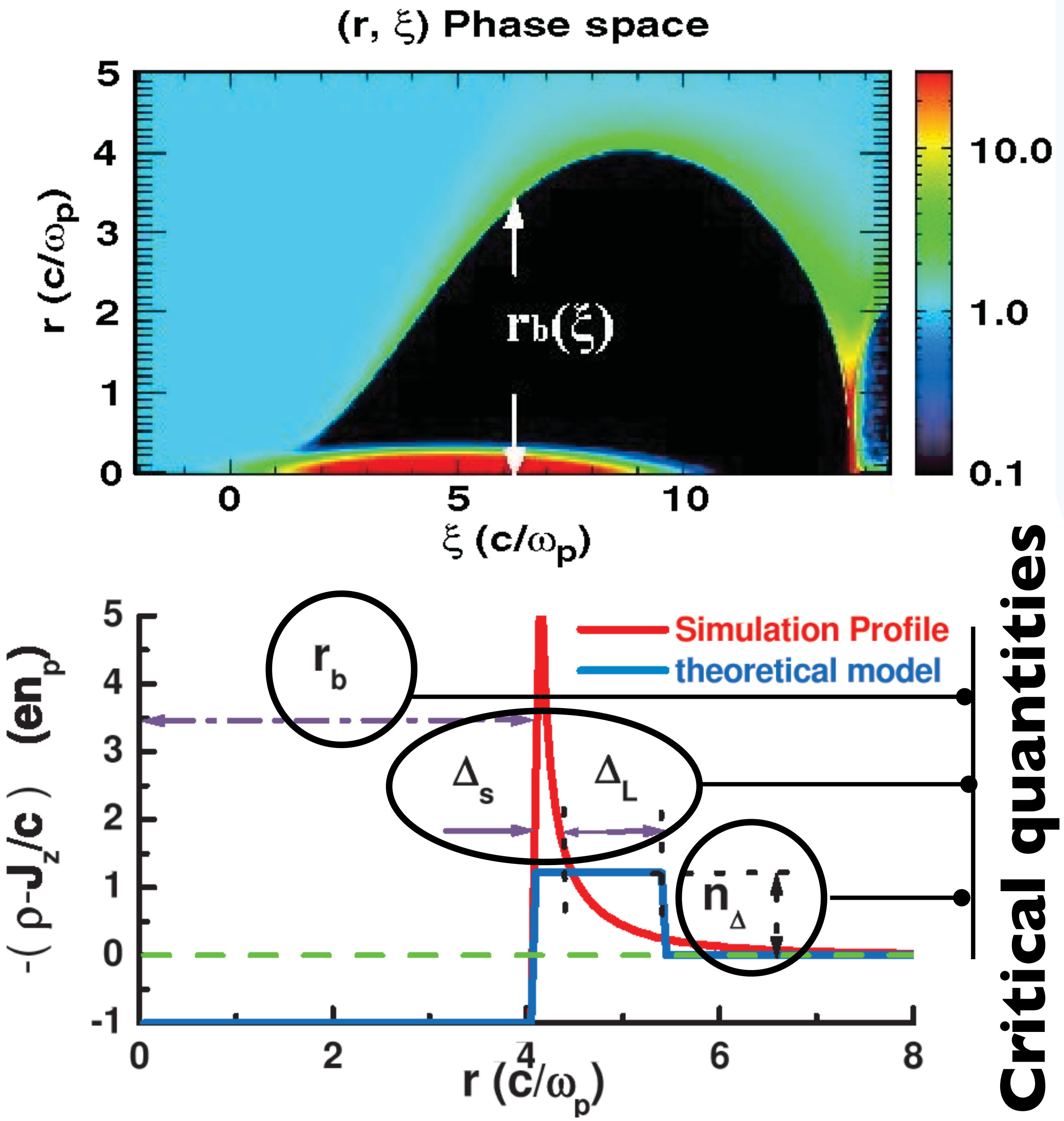
Boundary condition: Ψ vanishes away from the blowout region

$$\int_0^r r' \left[\underbrace{n_e(r',\xi)\left(1 - v_{\|}(r',\xi)\right)} - 1\right] \mathrm{d}r' = 0$$

Need model for $n_e(1-v_{\|})$

W. Lu, MsC thesis, UCLA (2004)

Source term model for Ψ in the blowout regime



W. Lu et al, PRL 96 165002 (2006)

Boundary condition:

$$\int_0^r r' \left[n_e\left(r',\xi\right)\left(1-v_{\parallel}\left(r',\xi\right)\right)-1\right] \mathrm{d}r' = 0$$

leads to:

height of the blowout sheath

$$n_\Delta\left(\xi\right) = \frac{r_b^2}{\left(r_b+\Delta\right)^2 - r_b^2}$$

width of the blowout sheath

$$\Delta = \Delta_s + \Delta_L$$

Non-relativistic blowout

$$\alpha(\xi) = \frac{\Delta}{r_b} \gg 1$$

Relativistic blowout

$$\alpha(\xi) = \frac{\Delta}{r_b} \ll 1$$

Pseudo-potential in the blowout regime: from non-relativistic to ultra-relativistic plasma responses

General expression for Ψ

$$\psi\left[r_b\left(\xi\right)\right] = \frac{r_b^2}{4}\underbrace{\left(\frac{\left(1+\alpha\right)^2\ln\left(1+\alpha\right)^2}{\left(1+\alpha\right)^2} - 1\right)}_{\equiv\beta}$$

Non-relativistic blowout regime

$$\psi\left(r,\xi\right) \simeq \frac{r_b^2}{4}\ln\frac{1}{r_b} - \frac{r^2}{4}$$

Ultra-relativistic blowout regime

$$\psi\left(r,\xi\right) \simeq \left(1+\alpha\right)\frac{r_b^2}{4} - \frac{r^2}{4}$$

W. Lu et al, PRL 96 165002 (2006)

The plan for a theoretical model for the blowout regime

Determine the equation of motion for a fluid element in the quasi-static approximation and assuming no sheath crossing

Determine the structure of the fields (cylindrically symmetric) for a model of the current/charge system in the bubble/blowout

Determine the equation of motion for the inner surface of the blowout region ($r = r_b$)

Full equation of motion for the blowout radius

Equation describing the motion of the blowout region

$$A(r_b)\frac{\mathrm{d}^2 r_b}{\mathrm{d}\xi^2} + B(r_b) r_b \left(\frac{\mathrm{d}r_b}{\mathrm{d}\xi}\right)^2 + C(r_b) r_b = \frac{\lambda(\xi)}{r_b} - \frac{1}{4}\frac{\mathrm{d}|a|^2}{\mathrm{d}r}\frac{1}{(1+\beta r_b^2/4)^2}$$

$$A(r_b) = 1 + \left(\frac{1}{4} + \frac{\beta}{2} + \frac{1}{8} r_b \frac{\mathrm{d}\beta}{\mathrm{d}r_b}\right) r_b^2$$

$$B(r_b) = \frac{1}{2} + \frac{3}{4}\beta + \frac{3}{4} r_b \frac{\mathrm{d}\beta}{\mathrm{d}r_b} + \frac{1}{8} r_b^2 \frac{\mathrm{d}^2\beta}{\mathrm{d}r_b^2}$$

$$C(r_b) = \frac{1}{4}\left(1 + \frac{1+|a|^2/2}{1+\beta r_b^2/4}\right)$$

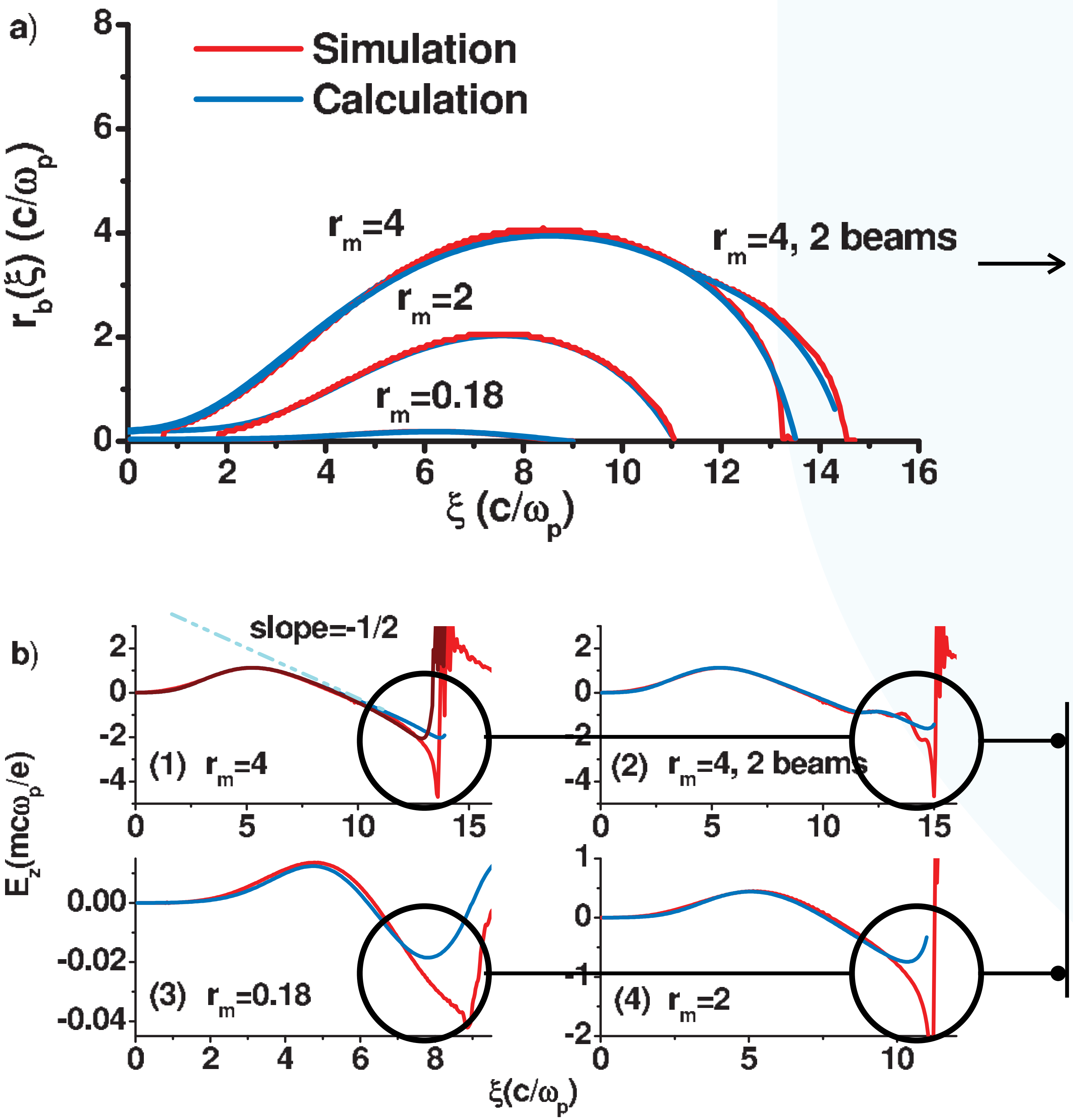
→

Assume that Δ does not depend on ξ.

Does not hold at the back of the bubble where $\Delta \sim r_b$

W. Lu et al, PRL **96** 165002 (2006);
W. Lu et al, PoP **13** 056709 (2006)

Theory compares very well with computer simulations



Very good agreement for a wide range of conditions

From weakly-relativistic to strongly relativistic blowouts

Perfect match except at the back of the bubble where $\Delta \sim r_b$

W. Lu et al, PRL 96 165002 (2006)

The blowout is close to a sphere regardless of the nature of the driver (laser or particle bunch)

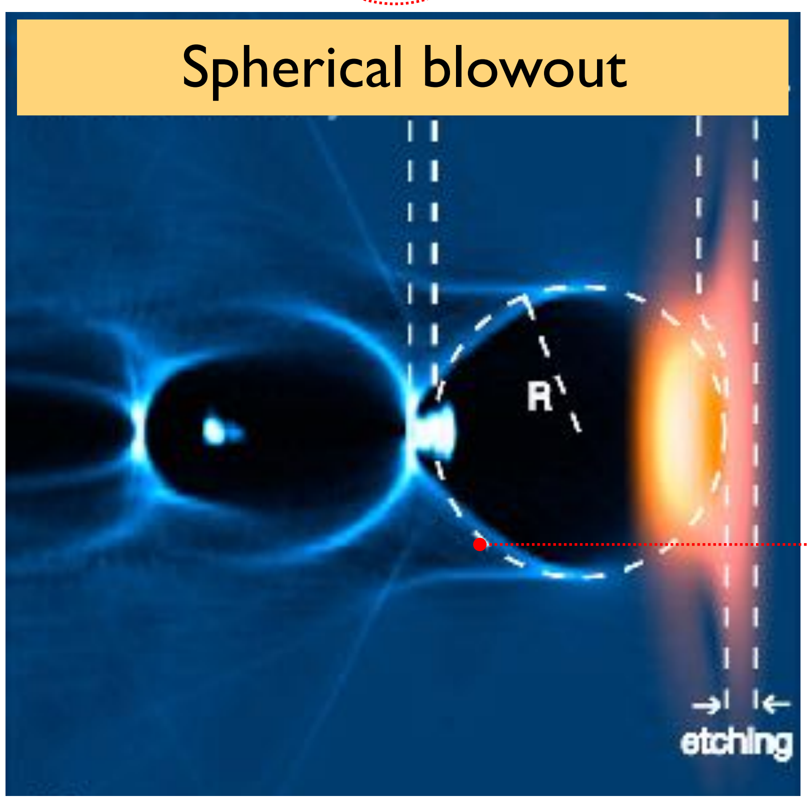
Ultra-relativistic blowout:

$$r_b \frac{\mathrm{d}^2 r_b}{\mathrm{d}\xi^2} + 2\left(\frac{\mathrm{d}r_b}{\mathrm{d}\xi}\right)^2 + 1 = \frac{4\lambda(\xi)}{r_b} - \frac{\mathrm{d}|a|^2}{\mathrm{d}r}\frac{1}{\left(1+\beta r_b^2/4\right)^2}$$

=0 right after the driver

Equation for surface of a sphere:

$$r_b \frac{\mathrm{d}^2 r_b}{\mathrm{d}\xi^2} + \left(\frac{\mathrm{d}r_b}{\mathrm{d}\xi}\right)^2 + 1 = 0$$



The factor '2' leads to stronger bending of r_b at the back of the bubble

W. Lu et al, PRL 96 165002 (2006)

Accelerating field in the blowout regime

Recall field expressions

$$E_z = \frac{\partial \psi}{\partial \xi}$$

Ultra-relativitic blowout (α≪1):

$$\psi(r,\xi) \simeq (1+\alpha)\frac{r_b^2}{4} - \frac{r^2}{4}$$

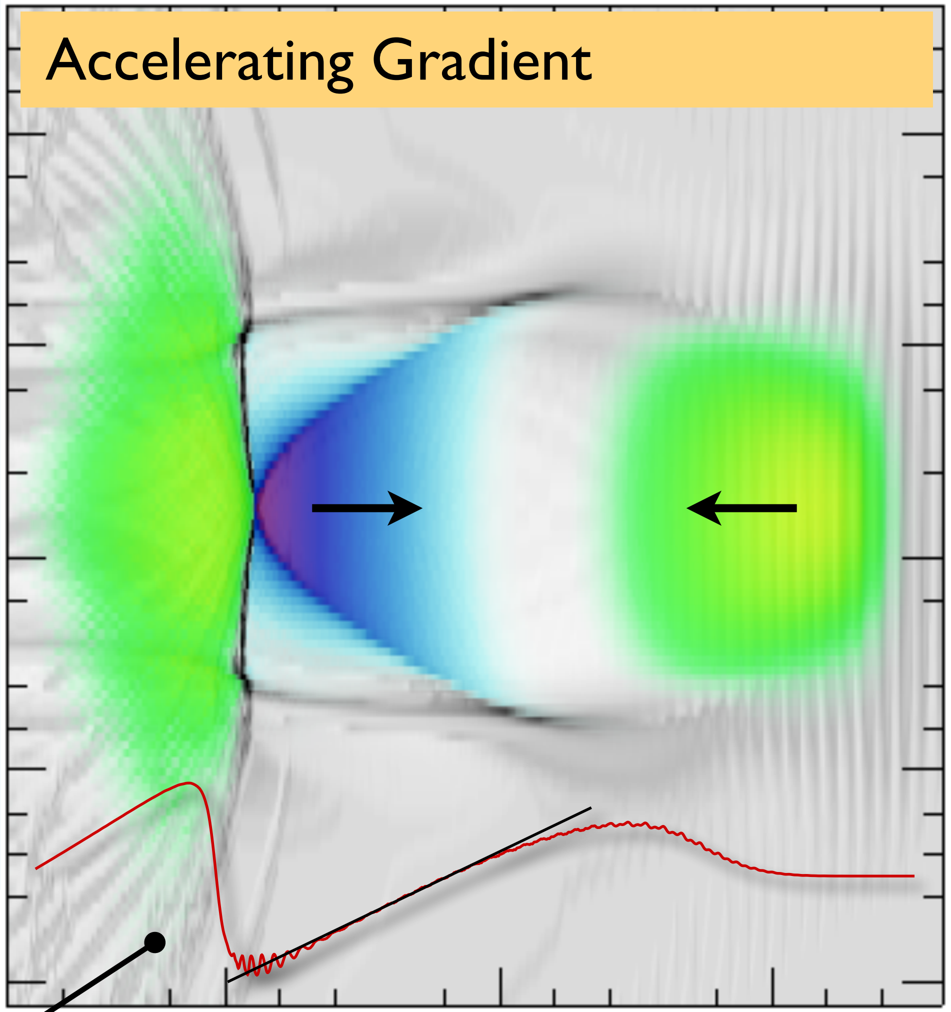
Ultra-relativitic blowout (α≪1):

$$E_z \simeq \frac{1}{2}\frac{\mathrm{d}r_b}{\mathrm{d}\xi}$$

Integration of the equation for $r_b(\xi)$ yields at the center of the bubble:

$$E_z \simeq \frac{\xi}{2} \qquad E_z^{\max} \simeq \frac{R_b}{2}$$

W. Lu et al, PRL 96 165002 (2006)



Focusing force in the blowout regime

Recall field expressions

$$E_r = -\frac{\partial \phi}{\partial r} - \frac{\partial A_r}{\partial \xi} \qquad B_\theta = -\frac{\partial A_z}{\partial r} - \frac{\partial A_r}{\partial \xi}$$

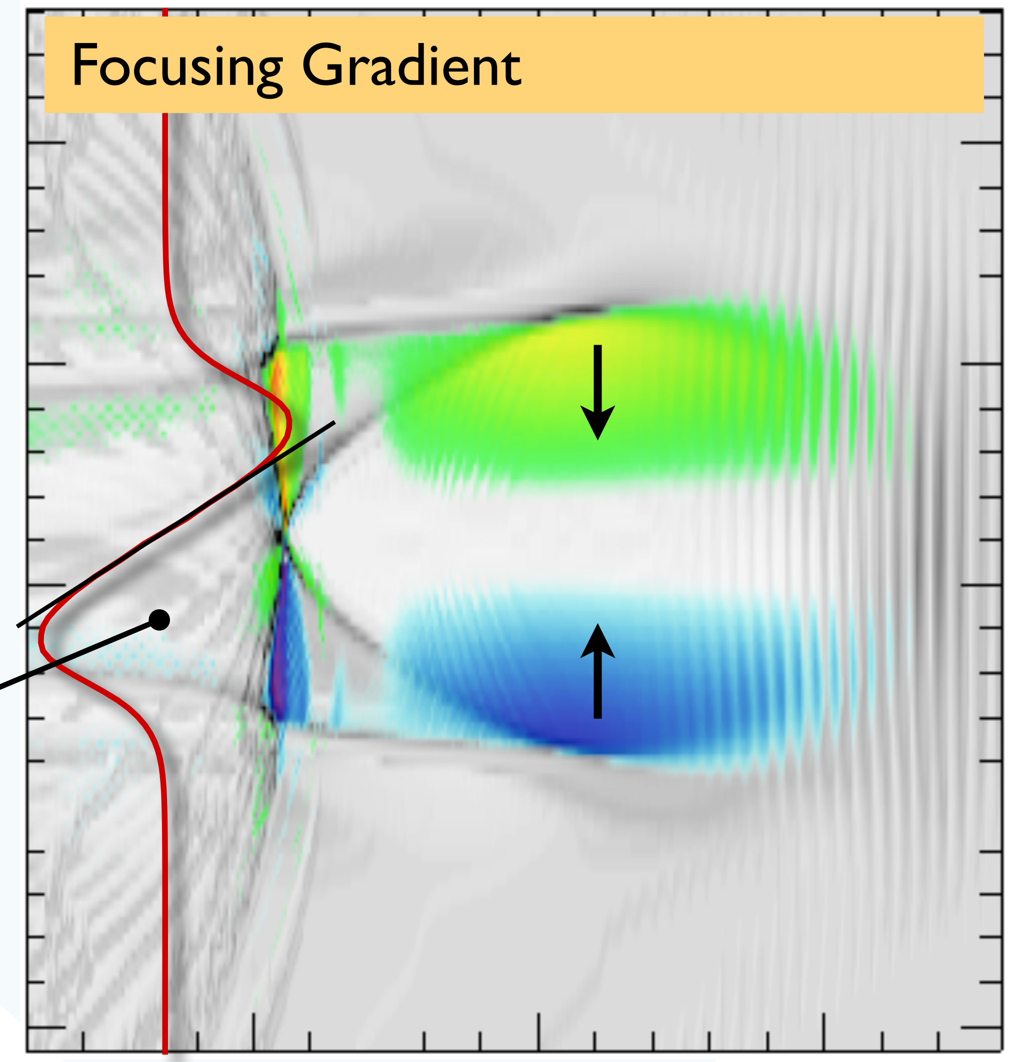
Focusing for relativistic particle

$$E_r - B_\theta = -\frac{\partial \left(\phi - A_\parallel\right)}{\partial r} = -\frac{\partial \psi}{\partial r}$$

v = c = 1

Linear focusing force:

$$E_r - B_\theta = \frac{r}{2}$$



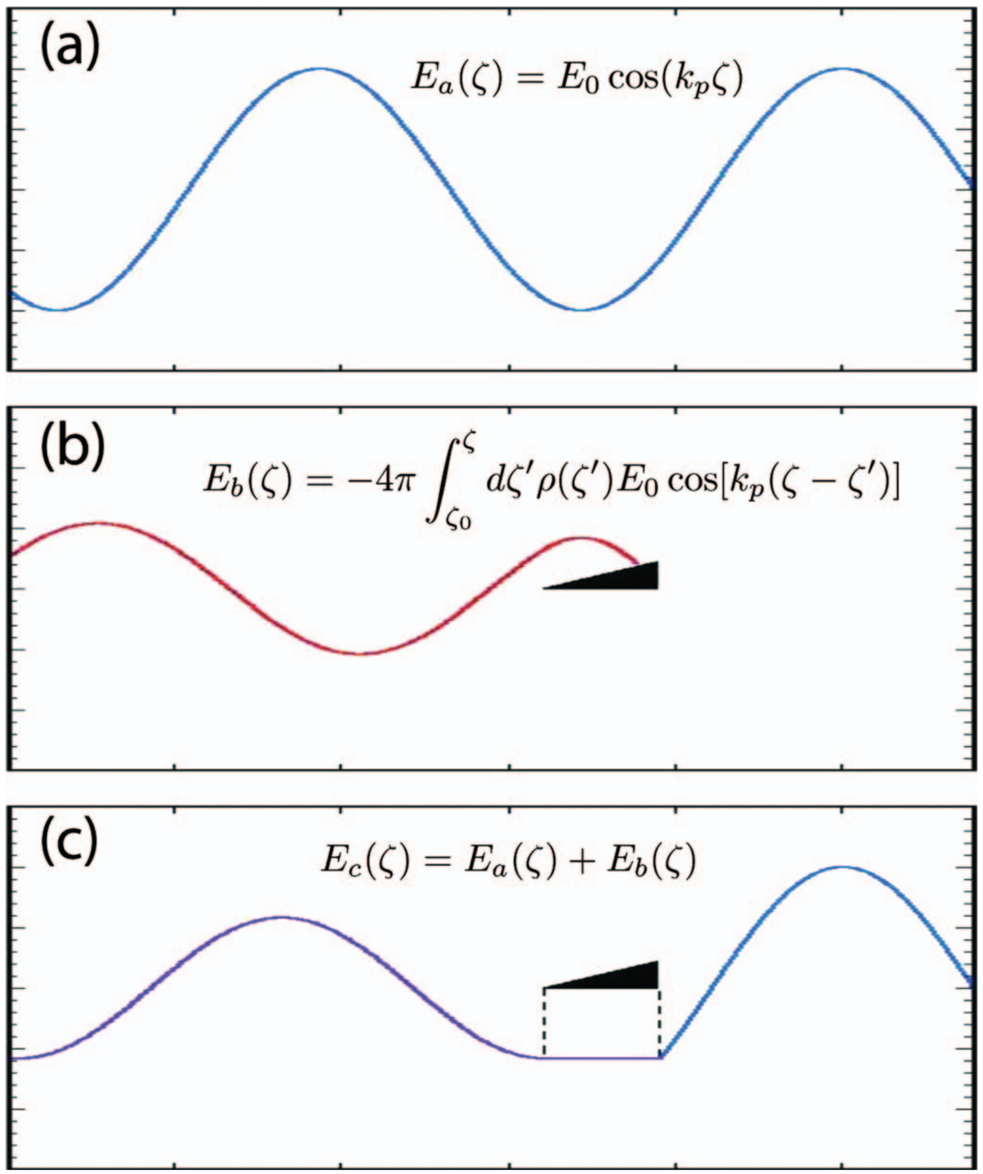
W. Lu et al, PRL 96 165002 (2006)

Contents

Beam loading: achieving high quality bunches with low energy spreads

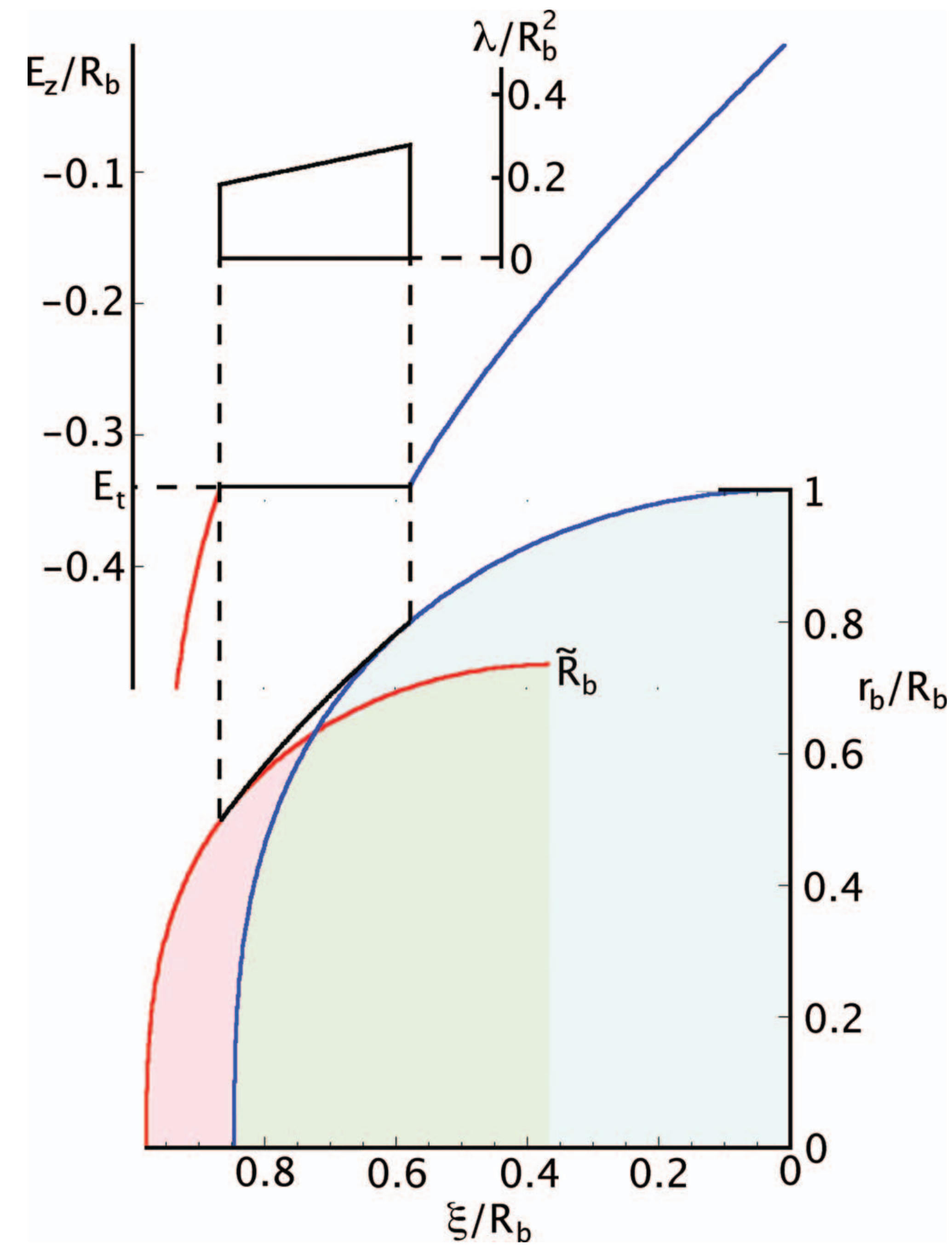
Goal: find the optimal beam profile that flattens accelerating fields

Linear regime



Properly tailored witness electron bunch flattens accelerating wakefield: no energy spread growth!

Blowout regime



M.Tzoufras et al (2008)

Optimal shape for witness electron bunch

Goal: find an exact solution for E_z at any position after the driver

Beam loading in the blowout

$$E_z = \frac{\partial \psi}{\partial \xi}$$

+

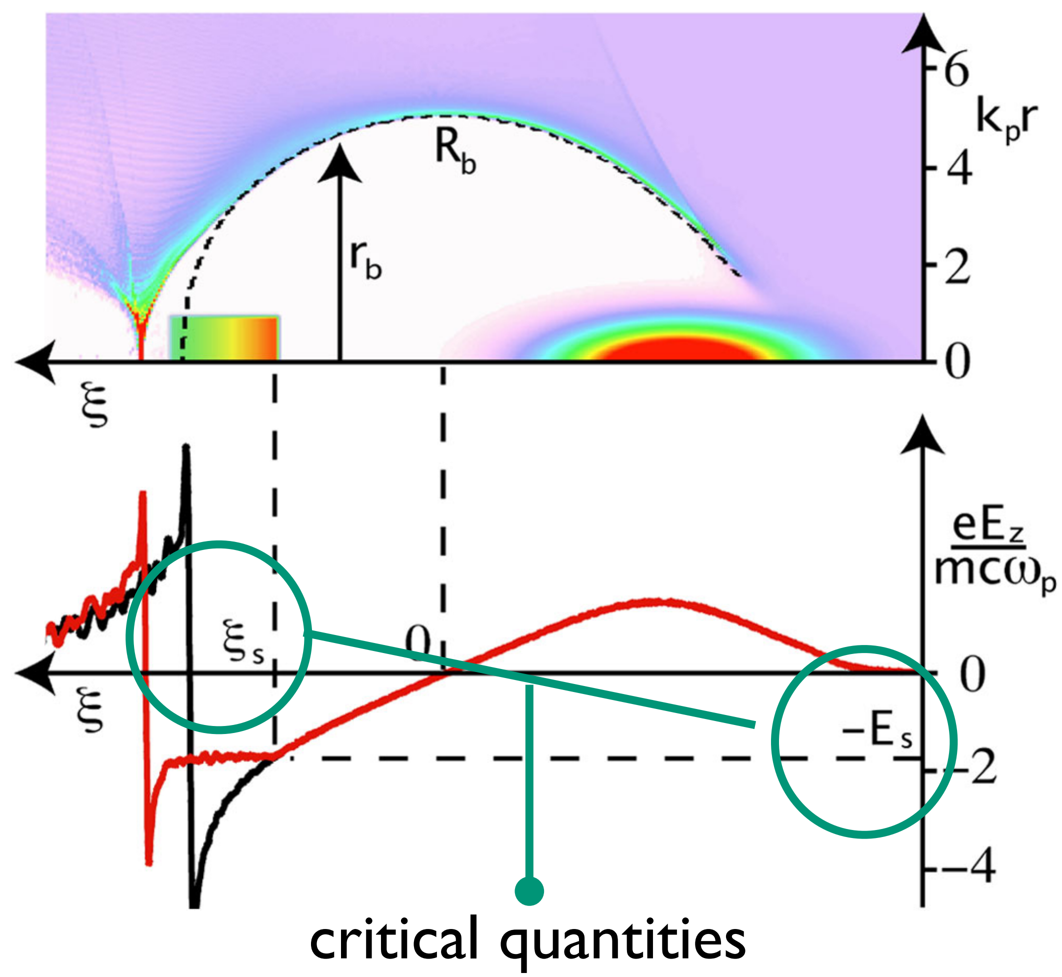
$$r_b \frac{d^2 r_b}{d\xi^2} + 2\left(\frac{dr_b}{d\xi}\right)^2 + 1 = \frac{4\lambda(\xi)}{r_b^2}$$

=

$$E_z = \frac{1}{2} r_b \frac{\mathrm{d}r_b}{\mathrm{d}\xi} = -\frac{r_b}{2\sqrt{2}} \sqrt{\frac{16 \int l(\xi)\xi \mathrm{d}\xi + C}{r_b^4} - 1}$$

l is the current density of the witness beam

Trapezoidal bunches lead to ideal beam-loading



$$l(\xi_s) = \sqrt{E_s^4 + \frac{R_b^4}{16}}$$

$$l(\xi) = \sqrt{E_s^4 + \frac{R_b^4}{16}} - E_s(\xi - \xi_s)$$

trapezoidal bunch

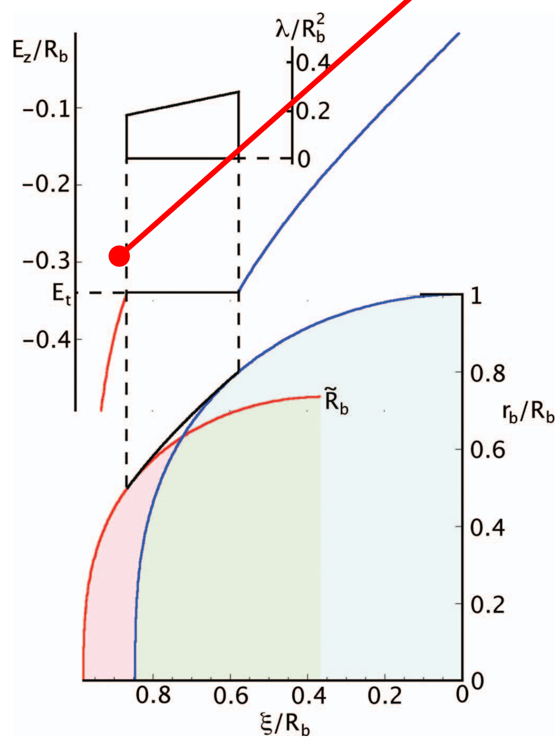
M. Tzoufras et al, PRL **101** 145002 (2008);
M. Tzoufras et al, PoP **16** 056705 (2009);

Total charge and efficiency in the blowout regime

Maximum charge in the blowout

Witness goes all the way until the bubble closes (r_b=0)

$$Q_{tr} = \frac{\pi}{16}\frac{R_b^4}{E_t}$$



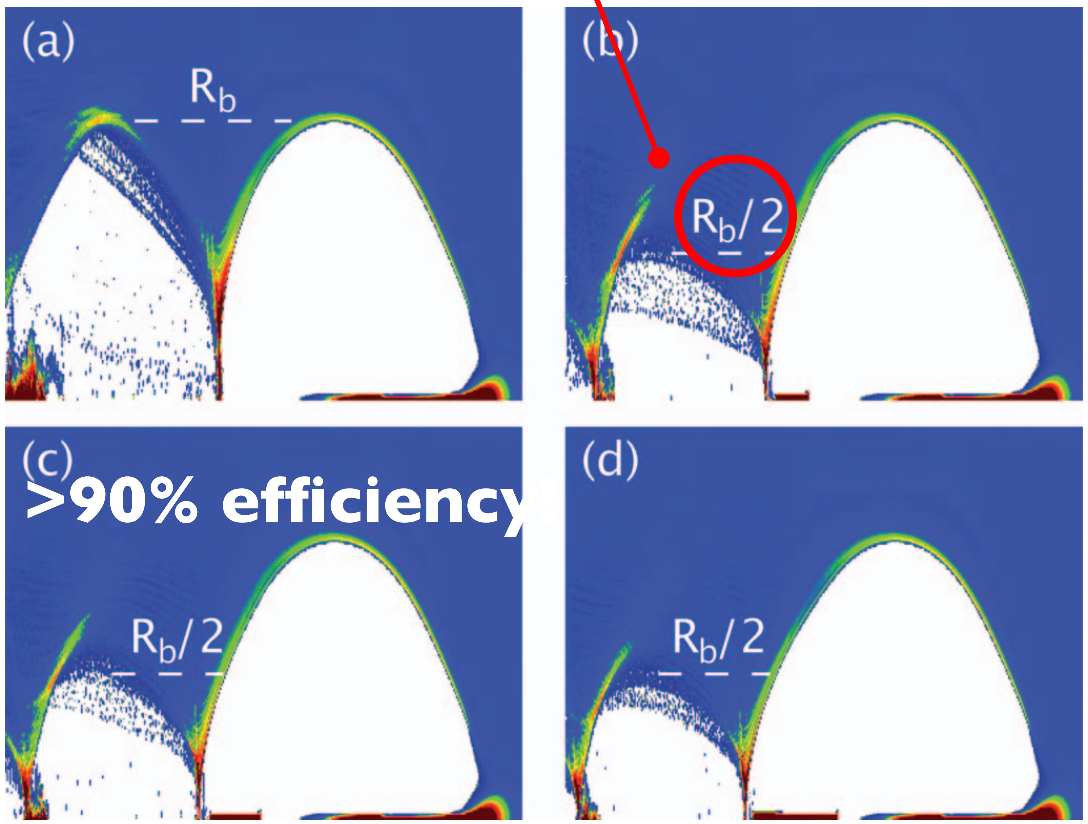
Smaller E_t: increases but final energy gain lowers

Efficiency

Efficiency: ratio between absorbed energy and total wakefield energy

$$\eta = 1 - \left(\frac{\tilde{R}_b}{R_b}\right)^4 = \frac{Q_F}{Q_{tr}}$$

actual beam charge



M.Tzoufras et al, PRL **101** 145002 (2008);
M.Tzoufras et al, PoP **16** 056705 (2009);

Engineering formulas for the maximum injected charge

Scaling for maximum number of particles

Energy in longitudinal ($\epsilon_{||}$) and focusing ($\epsilon_{\perp}$) wakefields:

$$\epsilon_{||} \simeq \epsilon_{\perp} \simeq \frac{1}{120} (k_p R_b^5) \left(\frac{m_e^2 c^5}{e^2 \omega_p} \right)$$

Energy absorbed by N particles (average accelerating field $E_z \propto R_b/2$:

$$\epsilon_{e-} \simeq \frac{m_e c^2 N R_b}{4}$$

Estimate for total particle number (r_e is the classical electron radius):

$$N \simeq \frac{1}{30} (k_p R_b)^3 \frac{1}{k_p r_e}$$

M. Tzoufras et al, PRL **101** 145002 (2008); W. Lu PRSTAB **10** 0301061 (2007)

Formulas

Number of particles as a function of laser parameters:

$$N \simeq 2.5 \times 10^9 \frac{\lambda_0[\mu\mathrm{m}]}{0.8} \sqrt{\frac{P[\mathrm{TW}]}{100}}$$

Efficiency is N x ΔE / Laser energy:

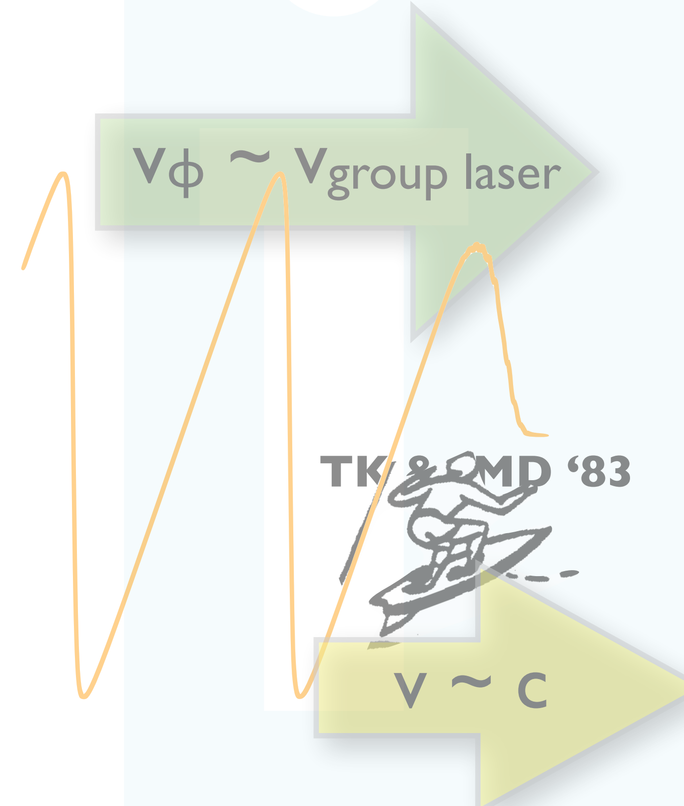
$$\Gamma \simeq 1/a_0$$

Higher efficiencies using more moderate laser intensities but still in the blowout.

Limits to energy gain in LWFA

Dephasing, Diffraction, Depletion

$$\Delta E = eE_z L_{\mathrm{acc}}$$



Dephasing

electrons overtake accelerating structure in $\mathbf{L_{dph}}$ ~ $10\ \mathrm{cm}/n_0\ [10^{16}\ \mathrm{cm}^{-3}]$

Diffraction

laser pulse diffracts in scale of $\mathbf{Z_r}$ (Rayleigh length) ~ few mm

Depletion

laser pulse looses its energy to the plasma in $\mathbf{L_{depl}}$

for small a_0, $\mathbf{L_{depl}} \gg \mathbf{L_{dph}}$; for $a_0 > 1$, $\mathbf{L_{depl}} \sim \mathbf{L_{dph}}$

Stable propagation in a plasma wakefield accelerator

Stable wakefields are critical to provide high quality bunches with high energies

Beam waist evolution in blowout

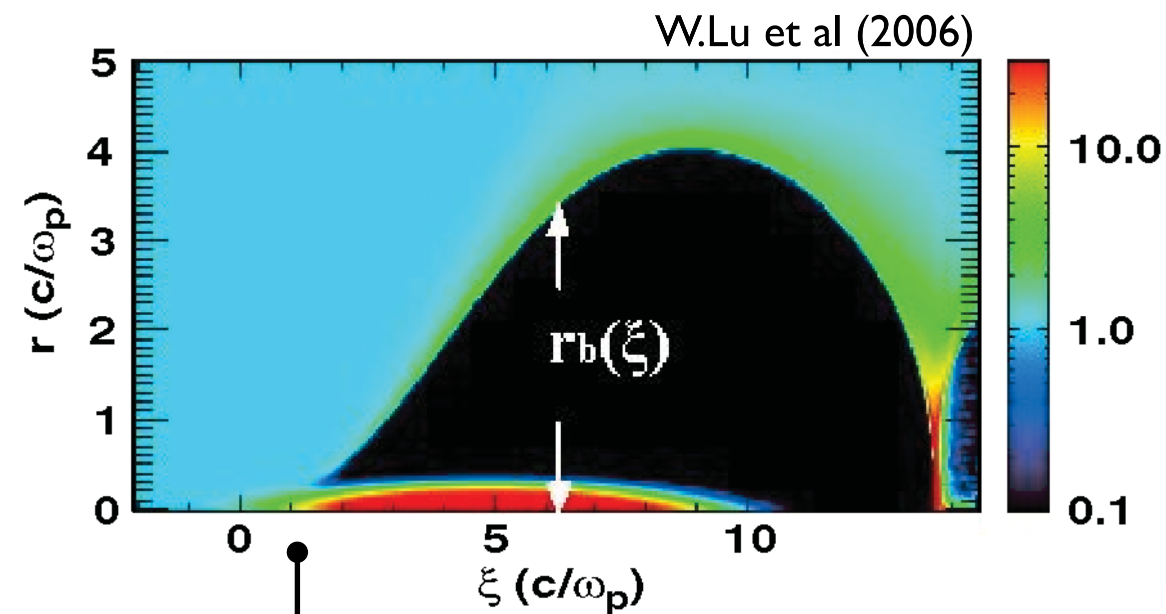
waist

$$K = \frac{\omega_p}{c\sqrt{2\gamma}}$$

$$\frac{d^2\sigma_r}{dz^2} + \left(K^2 - \frac{\epsilon_N^2}{\gamma^2\sigma_r^4}\right)\sigma_r = 0$$

=0 for matched propagation

linear focusing forces lead to extremely stable beam propagation



beam head can erode as it ionises the plasma and/or is not travelling in the blowout

C. Clayton et al, PRL **88** 154801 (2002)

Stable propagation in a laser wakefield accelerator

Laser pulse body guiding

Blowout radius:

$$F_p \sim \frac{a_0}{w_0} \sim F_{ion} \sim \frac{r_b}{2}$$

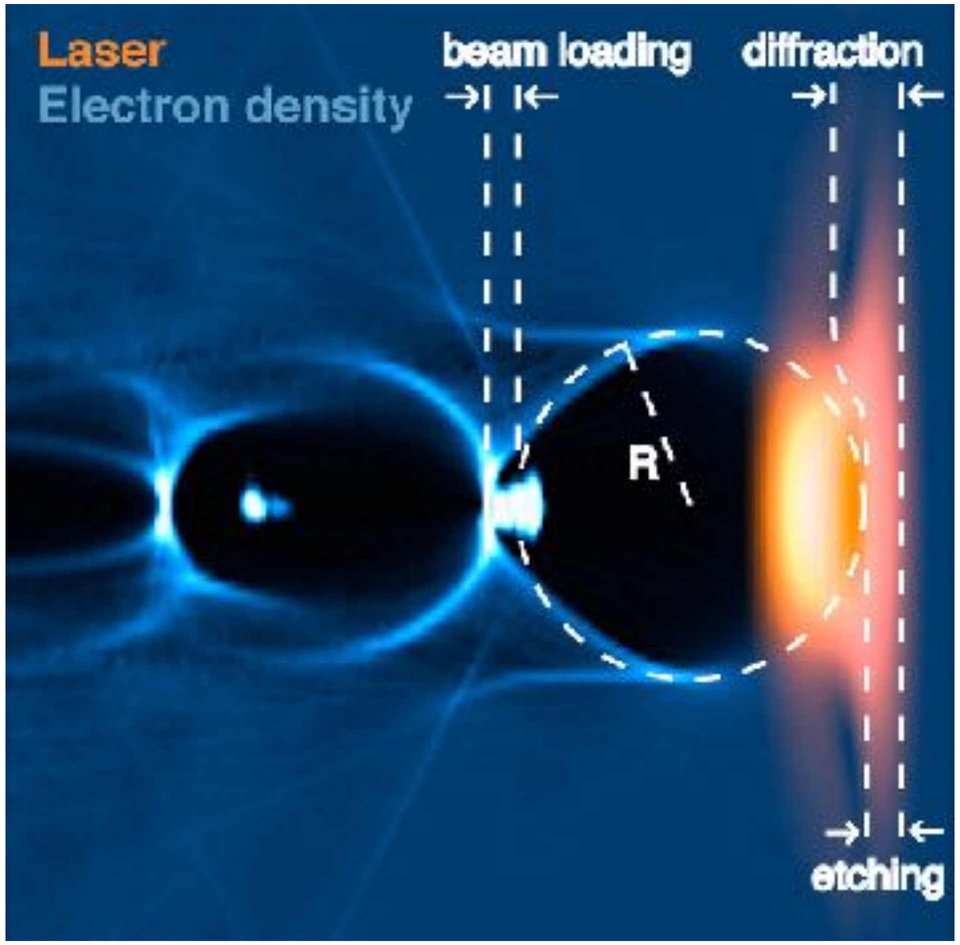
spot-size (normalised to $1/k_p$)

Guiding condition:

$$k_p w_0 \sim k_p R_b \sim 2\sqrt{a_0}$$

spot-size matched to the blowout radius

Laser pulse front guiding



etching rate higher than diffraction rate

$$a_0 \sim (n_c/n_p)^{1/5}$$

W. Lu et al. PR-STAB (2007)

For the correct pre-factors in all the equations check Silva et al, (2009) Comptes Rendus Physique, 10(2-3), 167–175.

Scalings for the acceleration distance in the blowout regime

Acceleration length

Pump depletion:

$$\frac{v_{\rm etch}}{c} L_{\rm etch} \simeq c\tau_{\rm FWHM}$$

$$\frac{v_{\rm etch}}{c} = \frac{\omega_p^2}{\omega_0^2}$$

$$L_{\rm etch} \sim c\tau_{\rm FWHM} \frac{\omega_0^2}{\omega_p^2}$$

Dephasing:

$$\frac{(c - v_\phi)}{c} L_d = R_b$$

$$v_\phi = v_g - v_{\rm etch} = 1 - \frac{3}{2}\frac{\omega_p^2}{\omega_0^2}$$

$$L_d = \frac{2}{3}\frac{\omega_0^2}{\omega_p^2} R_b$$

W. Lu et al. PR-STAB (2007)

Minimum pulse duration

De-phasing larger or equal to pump depletion:

$$\tau_{\rm FWHM} \geq \frac{2R_b}{3}$$

Optimal condition: no energy left in the driver after dephasing:

$$\tau_{\rm FWHM} = \frac{2R_b}{3}$$

For the correct pre-factors in all the equations check Silva et al, (2009) Comptes Rendus Physique, 10(2-3), 167–175.

Scalings for the maximum energy in a LWFA

Average accelerating field

$$E_z \simeq \frac{\xi}{2} \qquad E_z^{\max} \simeq \frac{R_b}{2} \qquad R_b \simeq 2\sqrt{a_0}$$

$$\downarrow$$

$$\langle E_z \rangle \sim \frac{\sqrt{a_0}}{2}$$

Maximum energy

$$\Delta E = m_e c^2 \langle E_z \rangle L_{\text{accel}}$$

$$\downarrow$$

$$\Delta E = \frac{2}{3} m_e c^2 \left(\frac{\omega_0}{\omega_p}\right)^2 a_0$$

For the correct pre-factors in all the equations check Silva et al, (2009) Comptes Rendus Physique, 10(2-3), 167–175.

W. Lu et al. PR-STAB (2007)

Blowout regime vs linear regime

Maximum charge

The blowout regime maximizes the charge that can be accelerated. Thus the number of energetic particles can be much larger in the blowout regime.

Maximum energy

The maximum energy is larger in the linear regime than in the non-linear regime as it implies the use of lower densities where electrons take longer to dephase and the laser takes longer to deplete.

Beam quality

Focusing foces are linear in the blowout regime. Thus, particle bunches can accelerate with little emittance growth. This is generally not possible in the linear regime as the focusing force is non-linear.

Stability

In the laser case, external guiding structures are required to focus the laser pulse in the linear regime. In the blowout regime, the laser can be self-guided by the plasma wave it creates. This leads to very stable accelerating and focusing fields.

Positron acceleration for a linear collider

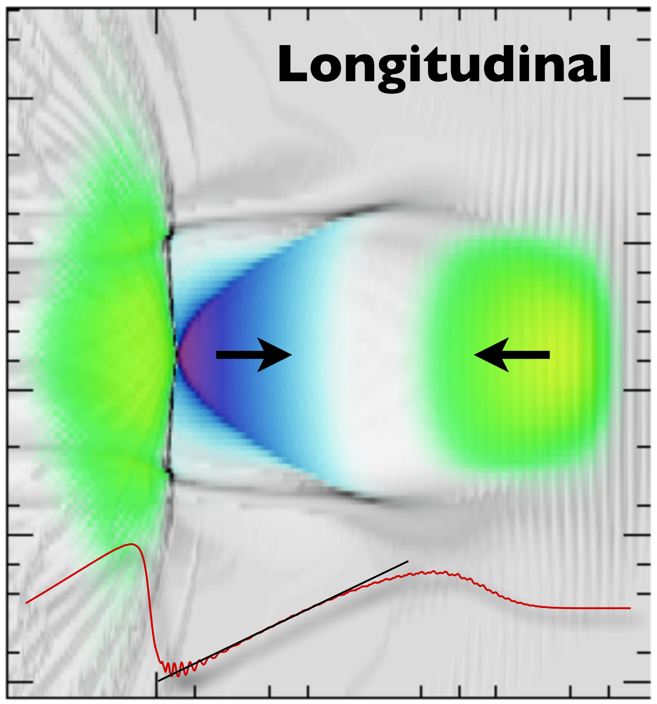
Recent work shows that positrons can accelerate in non-linear regimes. Until recently this was thought to be impossible.

Contents

Acceleration + focusing for positrons is limited

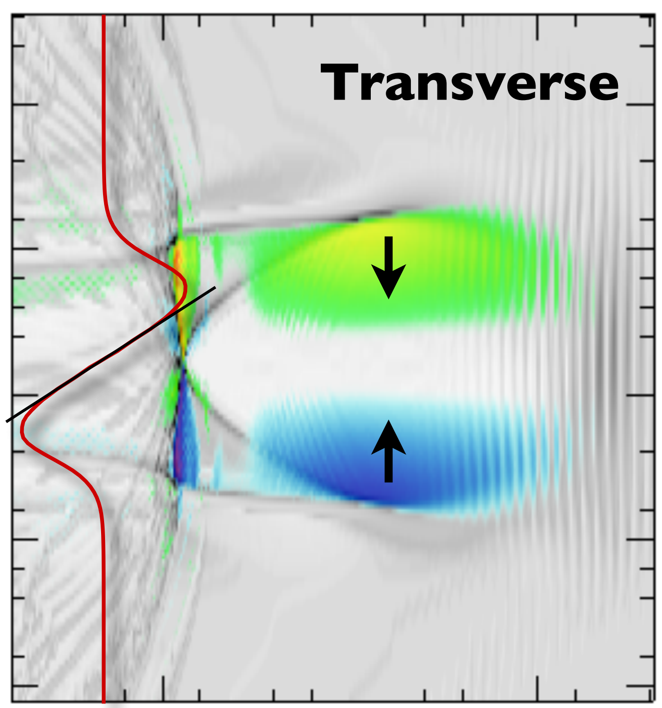
Dynamics of the laser and e- define key parameters

Electric fields created by laser pulse



Linear accelerating gradient

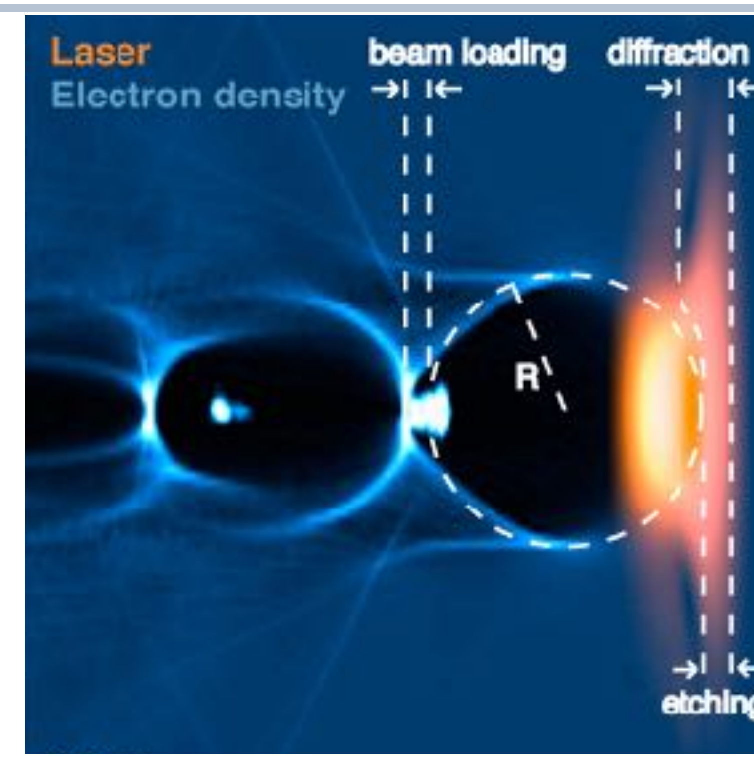
$$E_{z\,\max} \approx \sqrt{a_0}$$



Linear focusing force

$$k_p R \simeq 2\sqrt{a_0}$$

Matched laser parameters



Match laser spot size to bubble radius

$$k_p R \simeq k_p W_0 = 2\sqrt{a_0}$$

For maximum energy gain: trapped e- dephasing before pump depletion

$$L_{\text{etch}} \simeq c\omega_0^2/\omega_p^2 \tau_{\text{FWHM}} \quad L_{\text{etch}} > L_d \quad L_d \simeq \frac{2}{3}\frac{\omega_0^2}{\omega_p^2} R$$

$$c\tau_{\text{FWHM}} > 2R/3$$

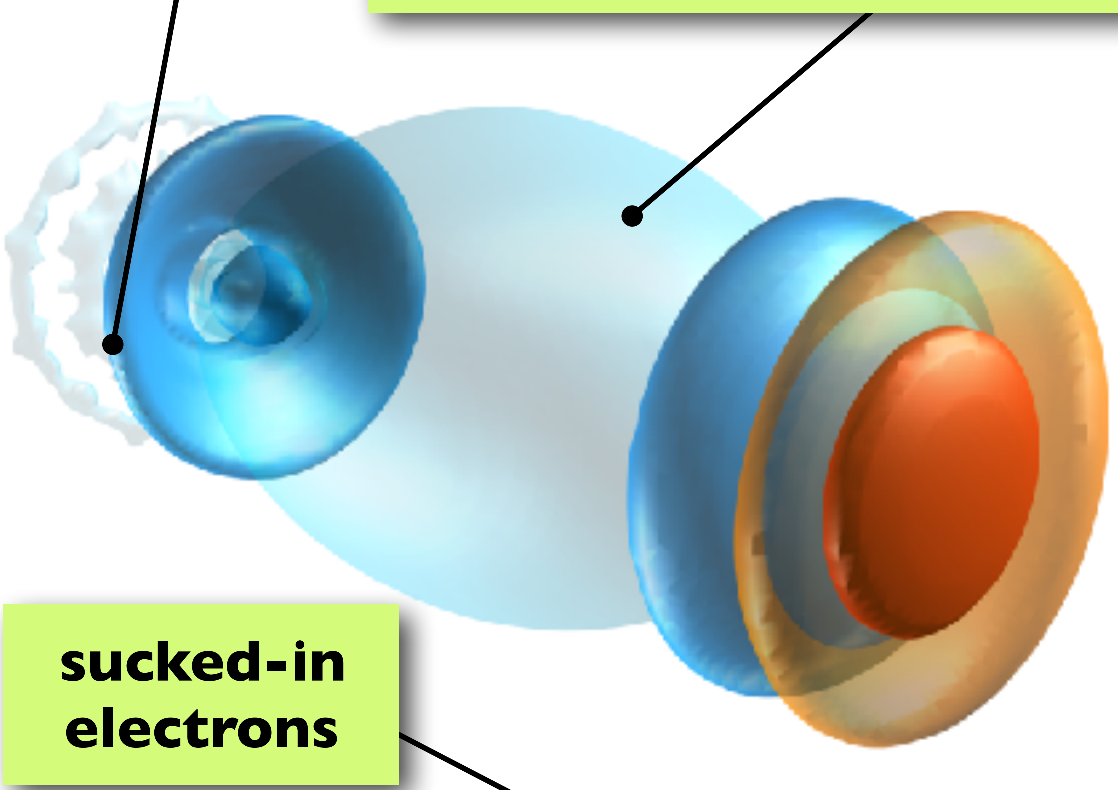
W. Lu *et al.* PR-STAB (2007)

Positrons can not ride large amplitude plasma waves because they are quickly defocused away from the plasma wave.

Positrons can not ride large amplitude plasma waves because they are quickly defocused away from the plasma wave.

Suck-in regime for positron beam and electron acceleration

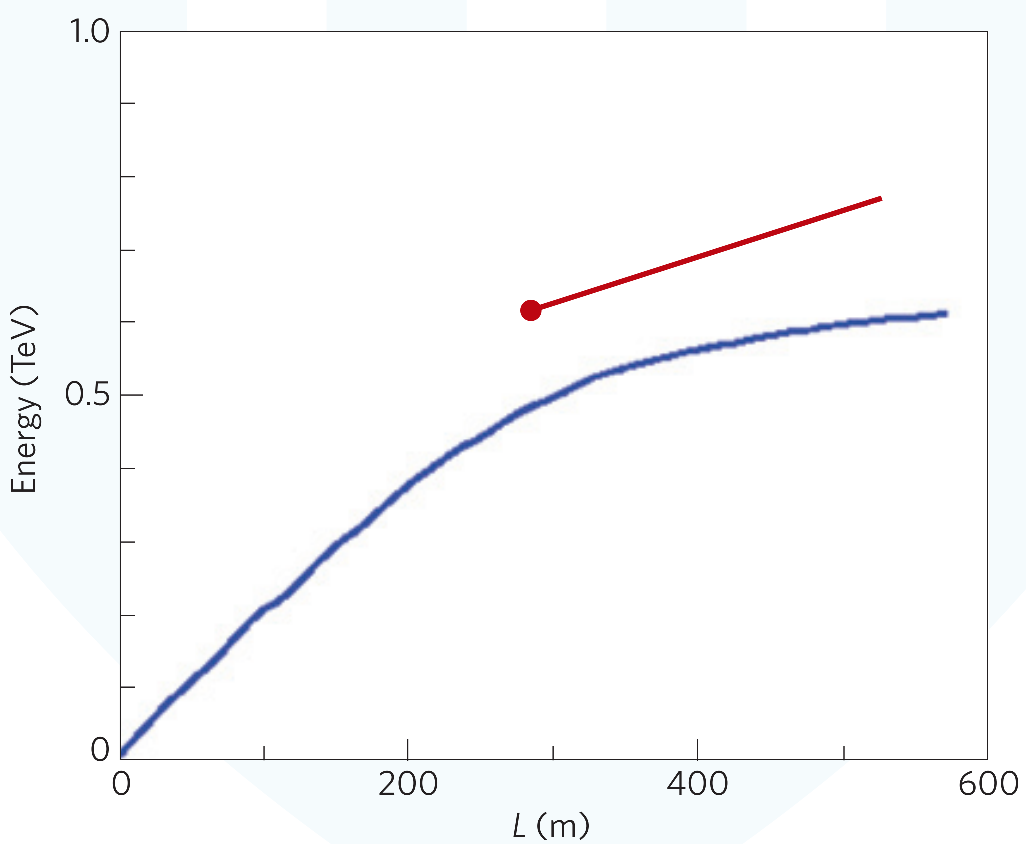
Model for suck-in regime



Onset of Suck-in regime - scaling determined from equation of motion for plasma electrons

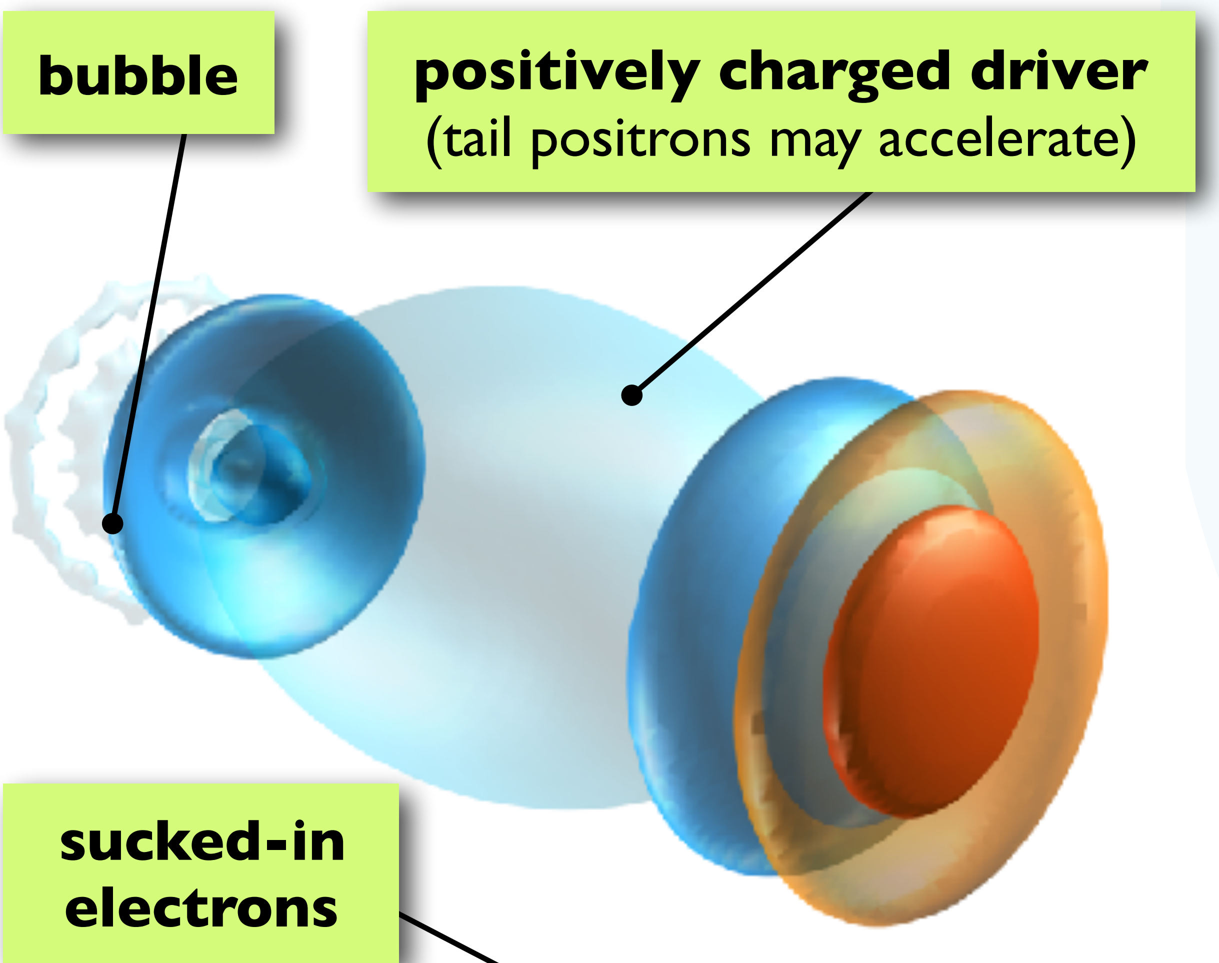
$$\tau_{\mathrm{col}} \simeq \sqrt{\pi}\left(\frac{r_0}{\sigma_r}\sqrt{\frac{m_b}{4\pi n_b e^2}}\right) \ll \lambda_p/c$$

- p^+ plasma wake similar to e^+
- beam loading is also identical
- requires p^+ bunches shorter than c/ω_p



Suck-in regime for positron beam and electron acceleration

Model for suck-in regime



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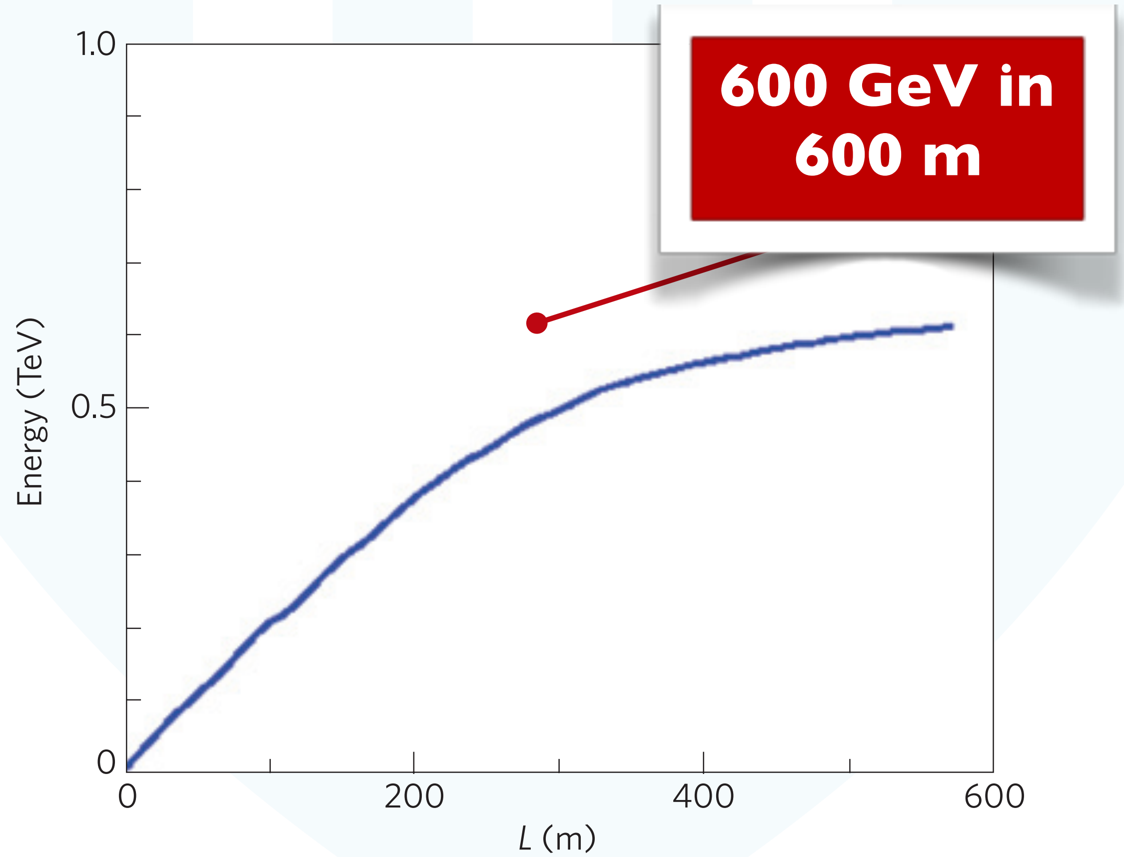
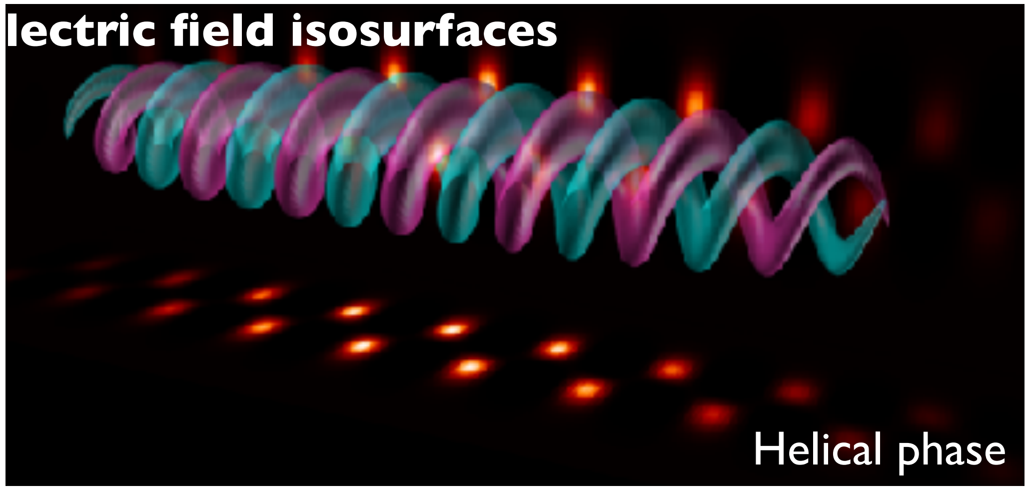
Proton driven plasma wakefield accelerator

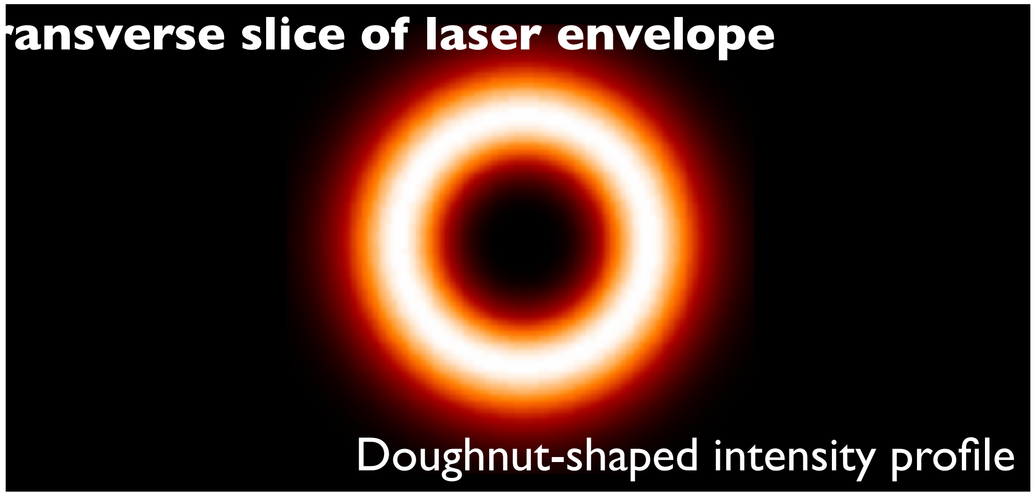
- p^+ plasma wake similar to e^+
- beam loading is also identical
- requires p^+ bunches shorter than c/ω_p



Positron acceleration using lasers with Orbital Angular Momentum

LG lasers have doughnut intensity profiles



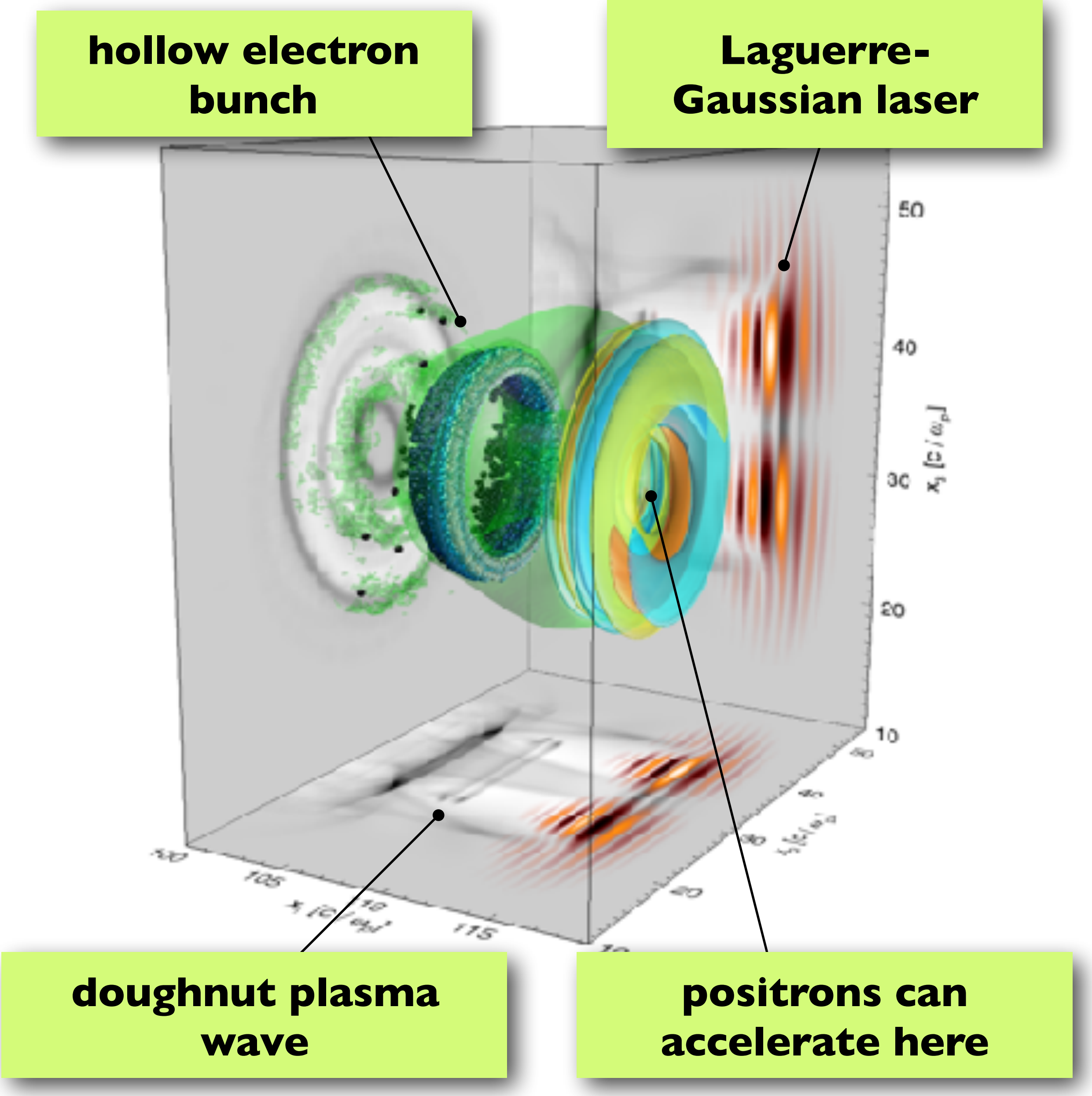


Laguerre Polynomial

$$a_r(r) = c_{l,p}\left(\frac{r}{w_0}\right)^{|l|}\exp\left(-\frac{r^2}{w_0^2}+il\theta\right)L_p^{|l|}\left(\frac{2r^2}{w_0^2}\right)$$

Helical phase

LG lasers drive doughnut plasma waves

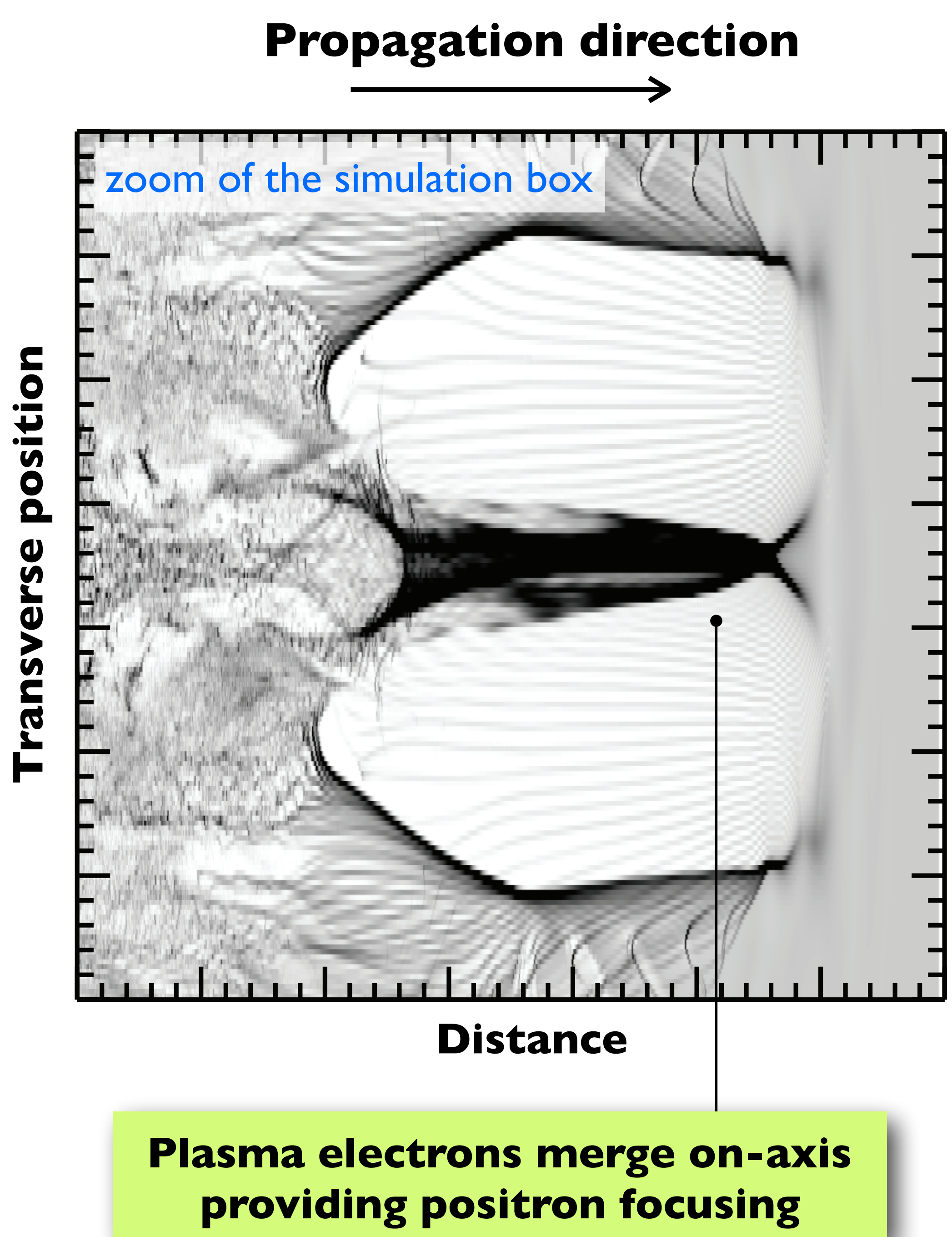


J. Vieira and J. T. Mendonça PRL **112**, 215001 (2014)

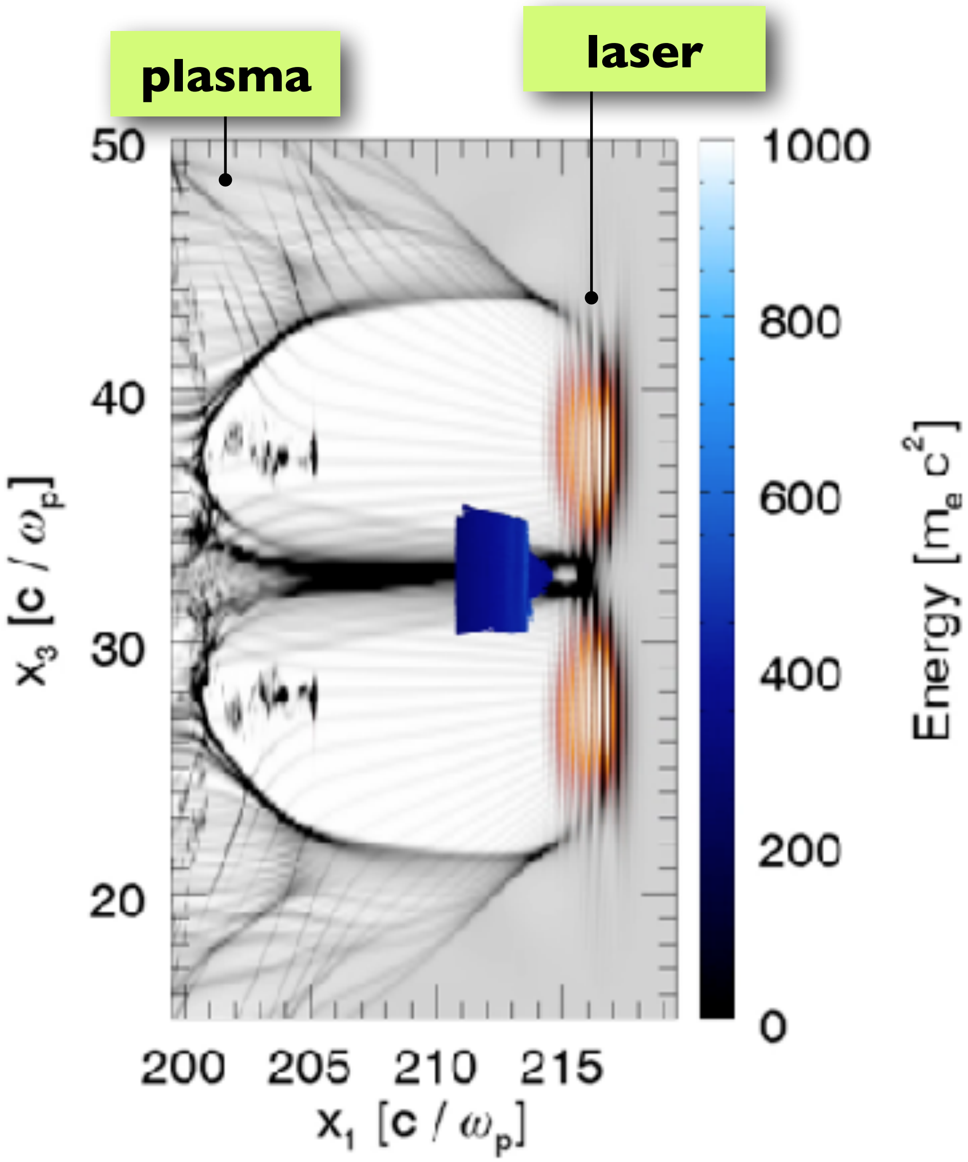
Three dimensional simulations confirm positron acceleration mechanism in strongly non-linear regimes

J. Vieira and J. T. Mendonça PRL **112**, 215001 (2014)

Onset of positron focusing and acceleration



Demonstration of positron acceleration



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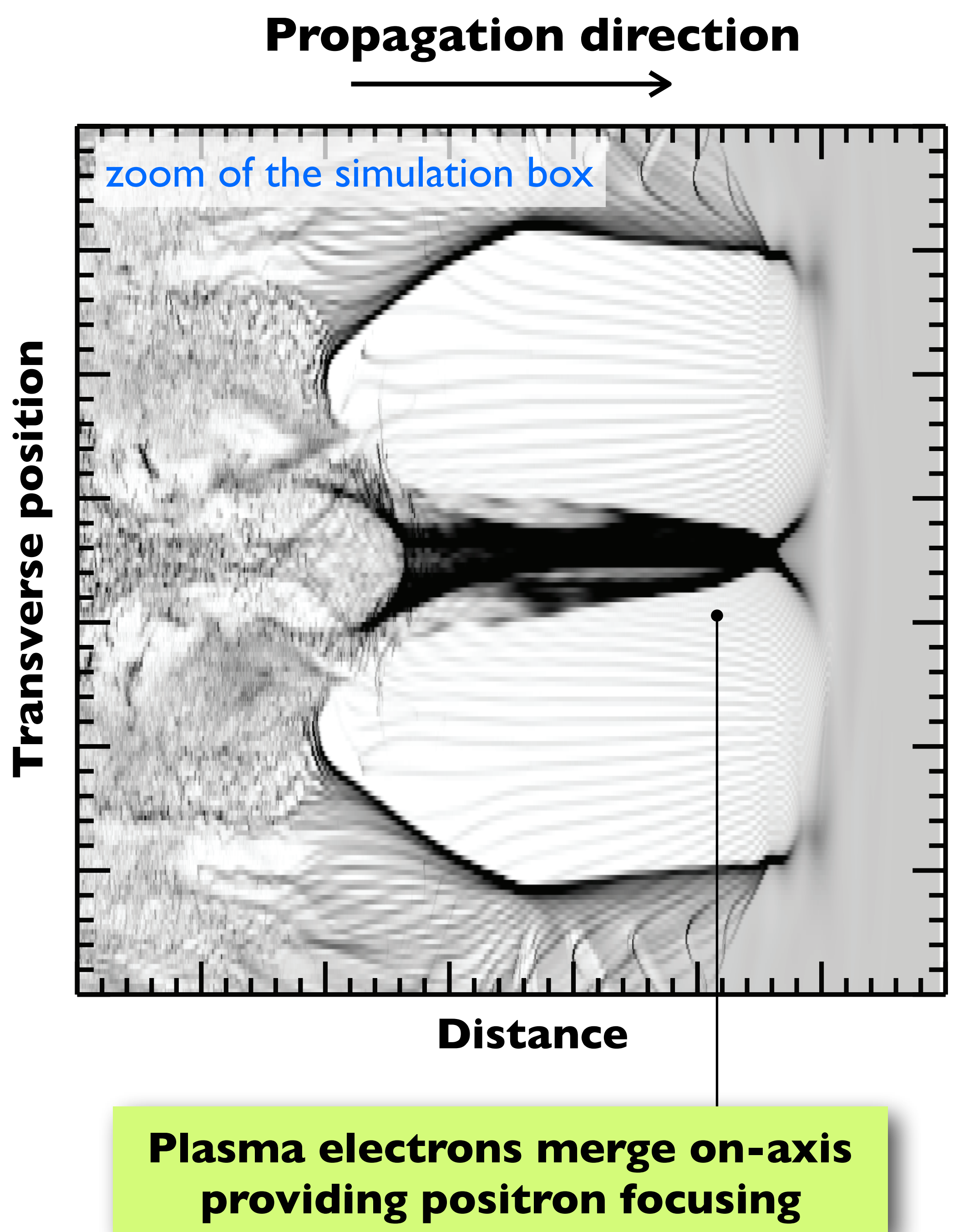

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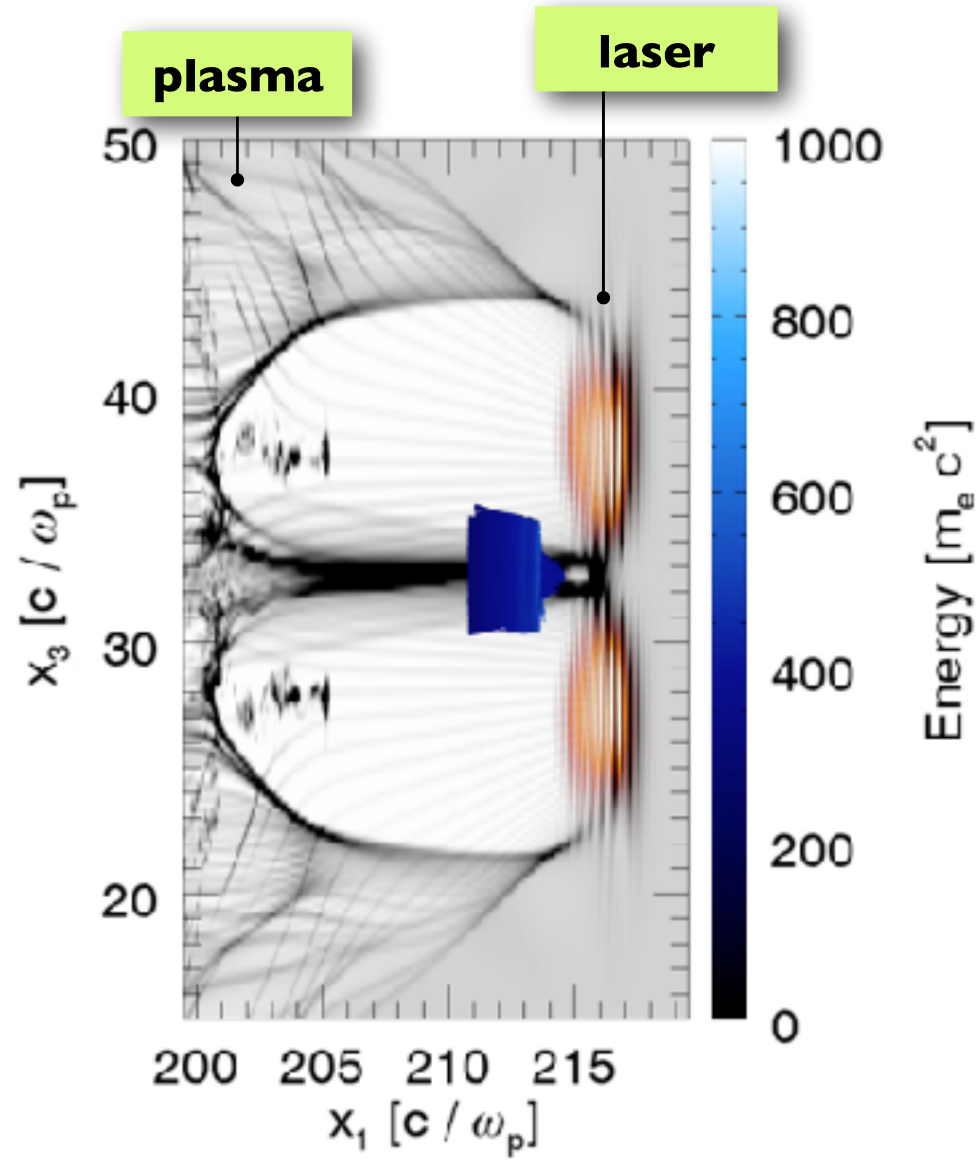

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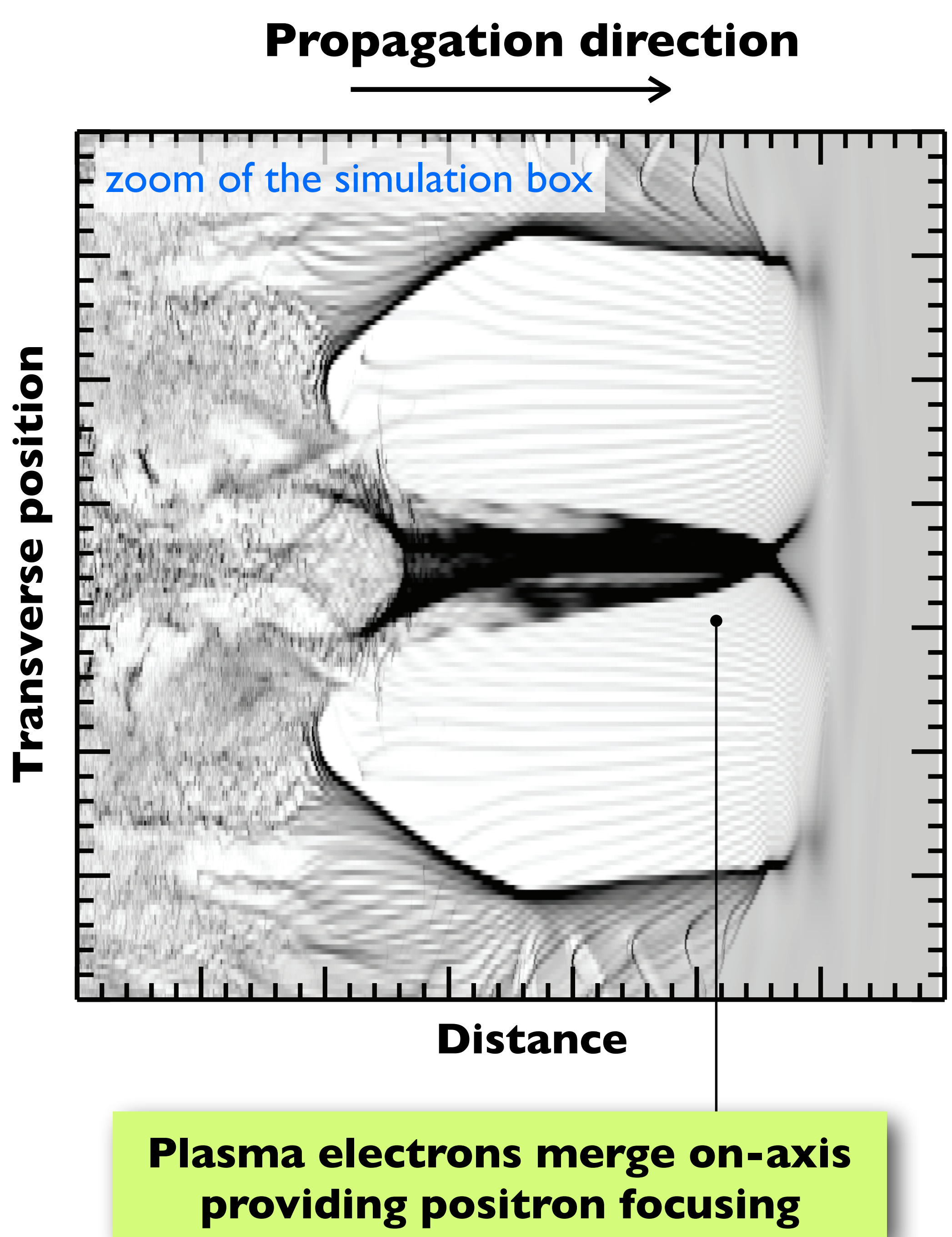

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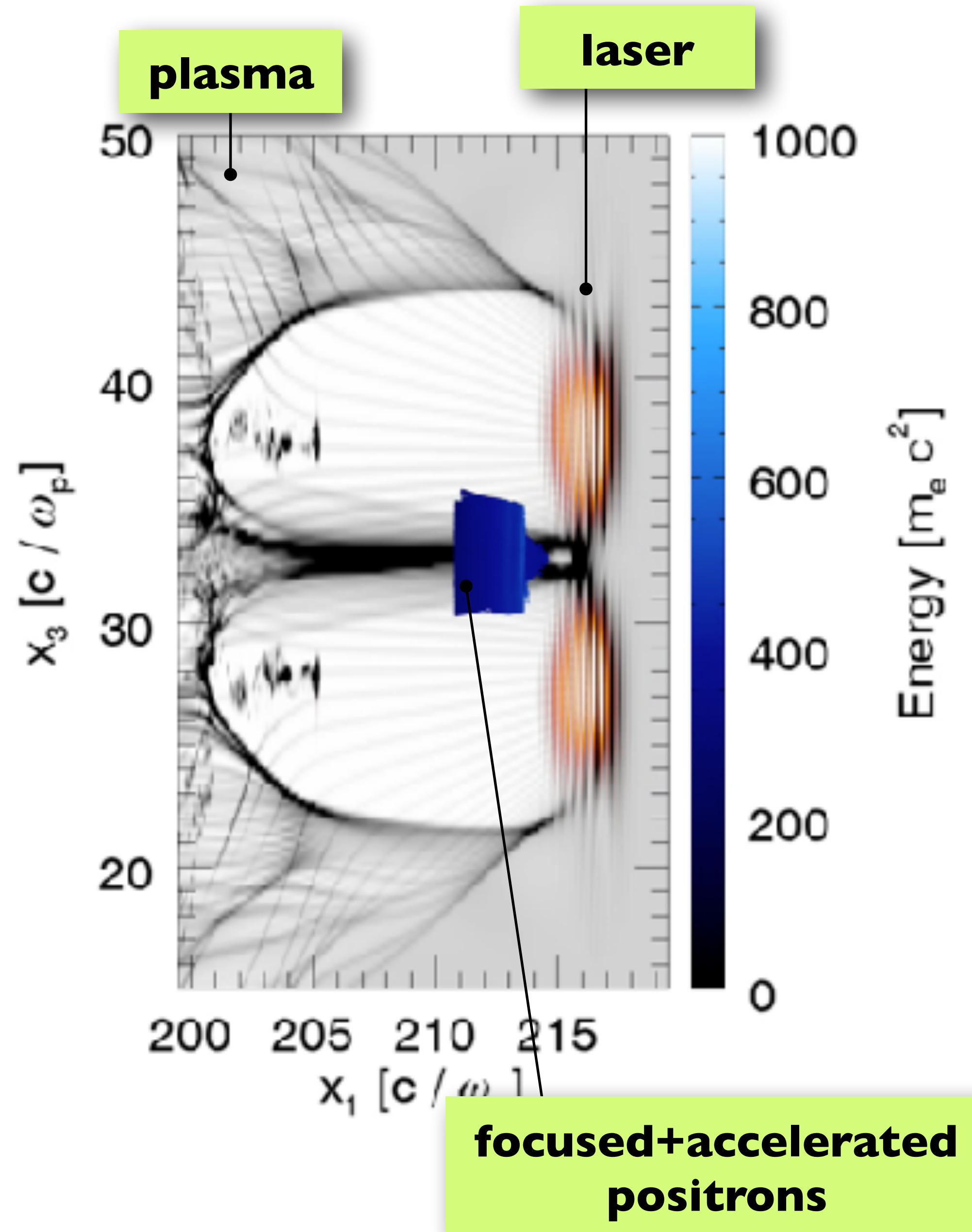
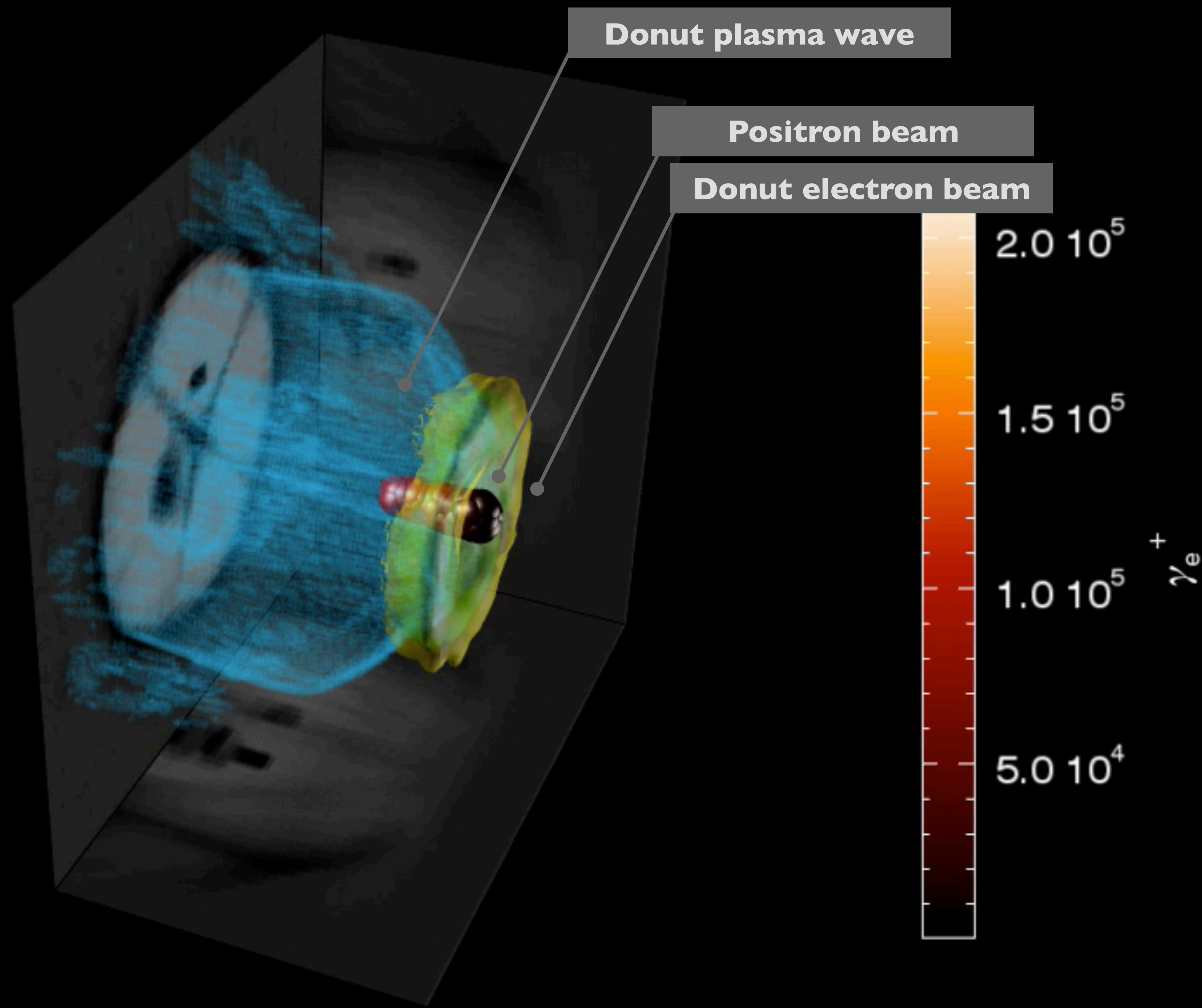


Positron acceleration using SLAC type ring electron bunches



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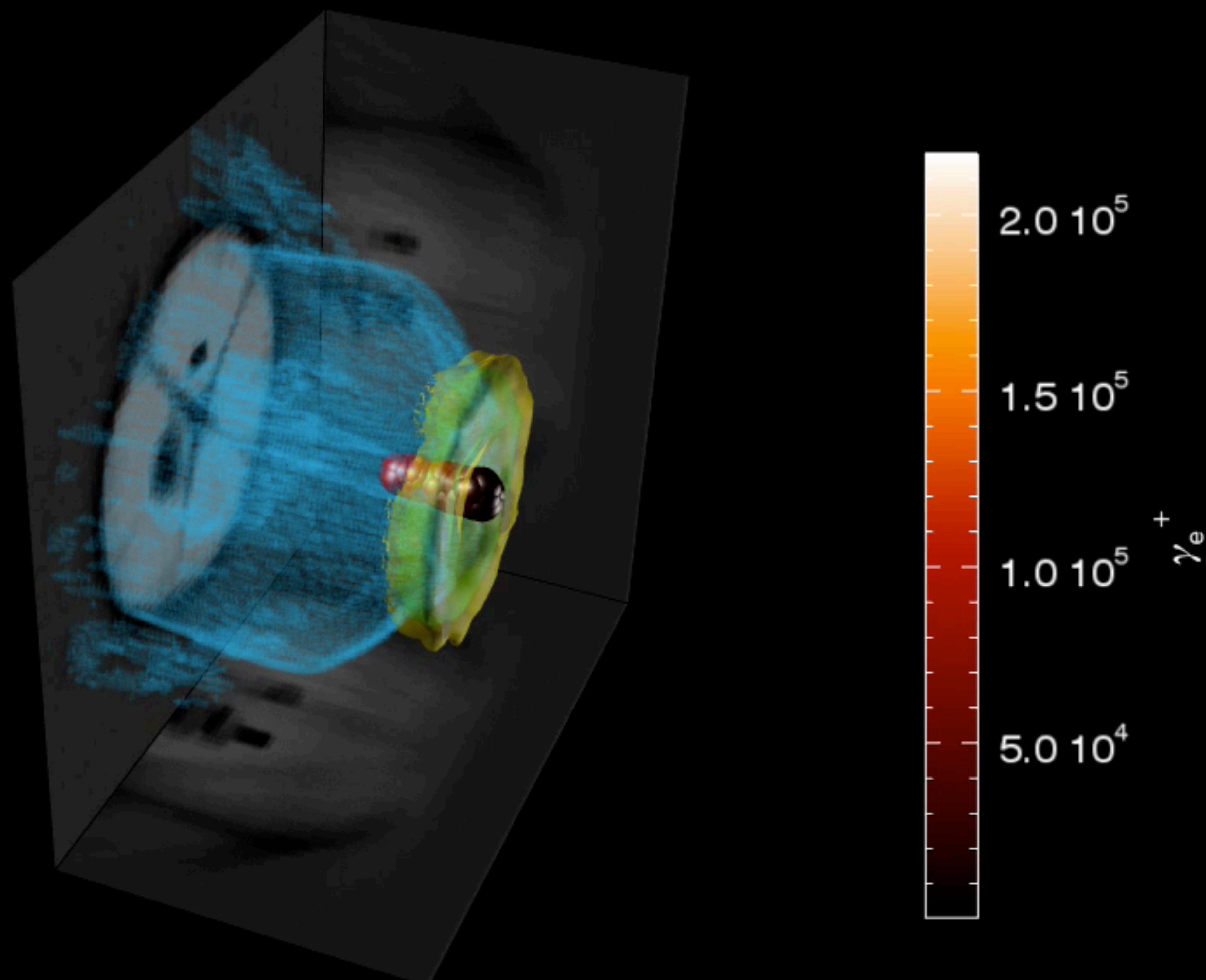
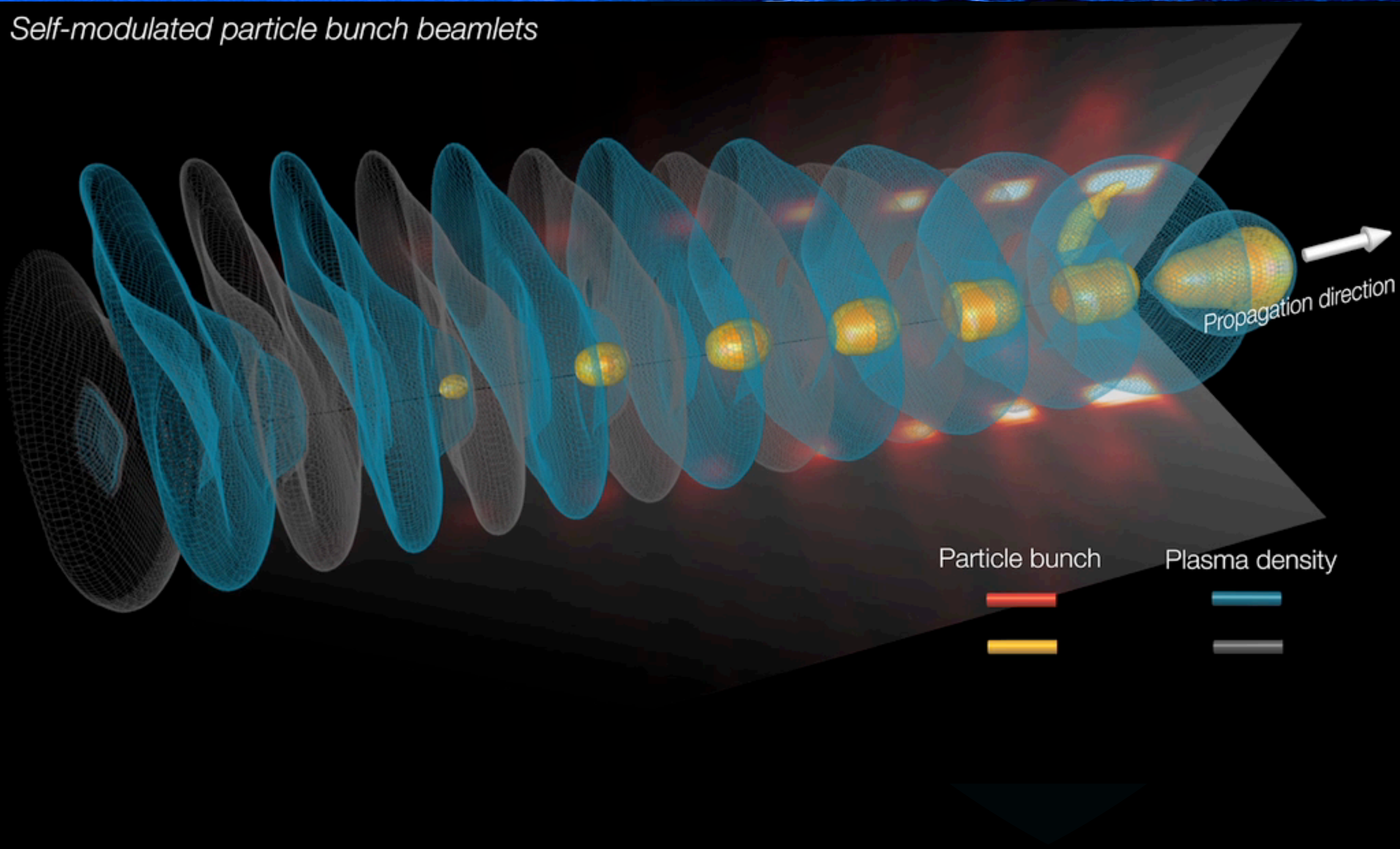
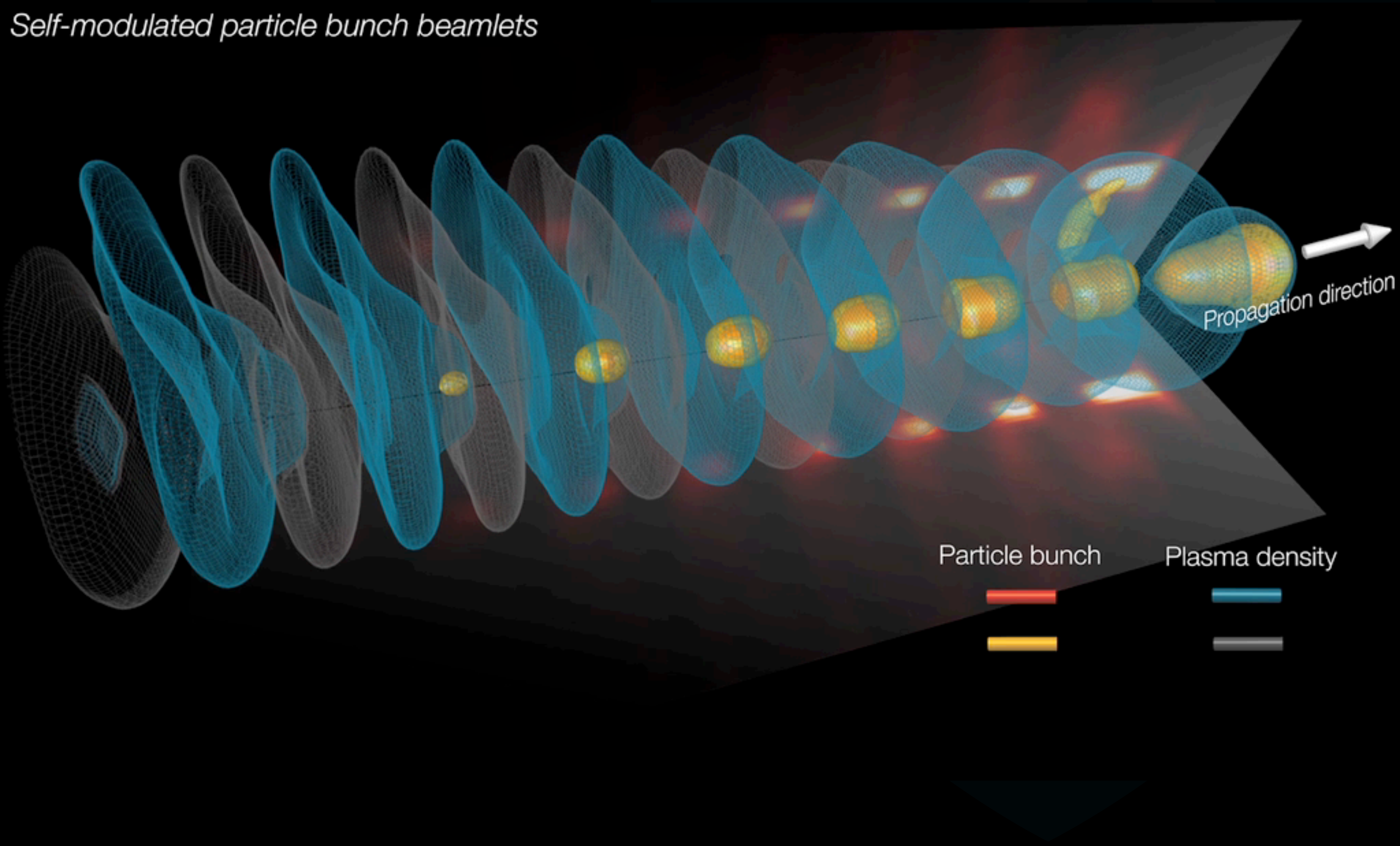


What about long beams?



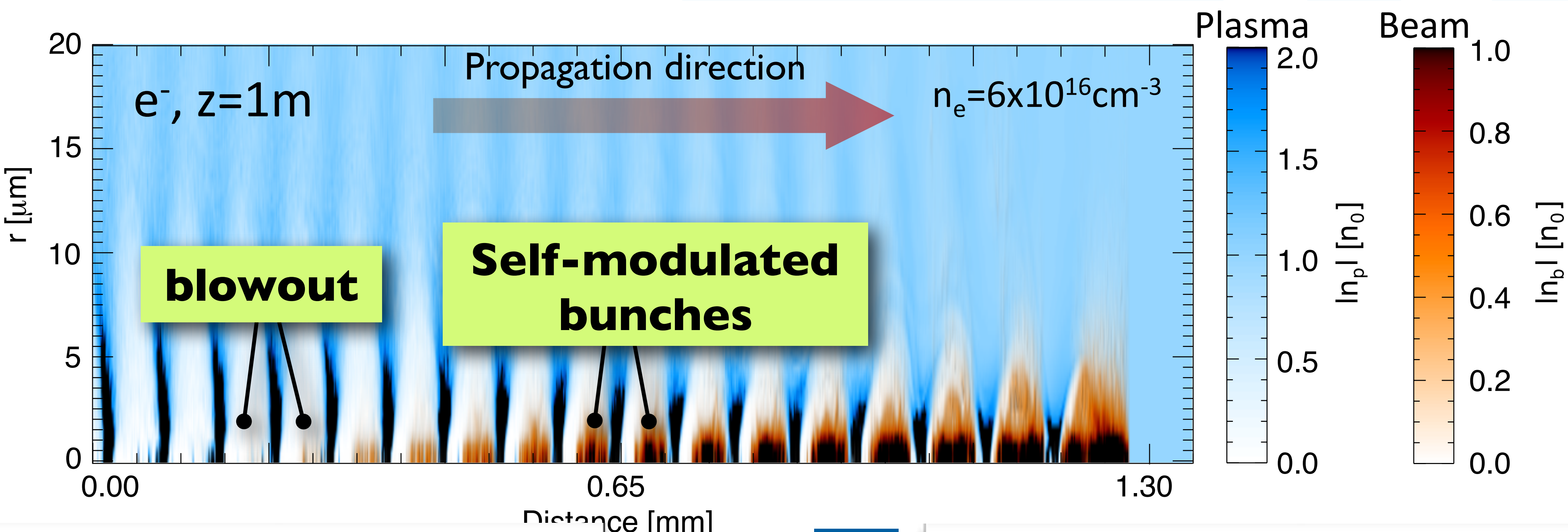
J. Vieira et al PRL **112**, 205001 (2014)

What about long beams?

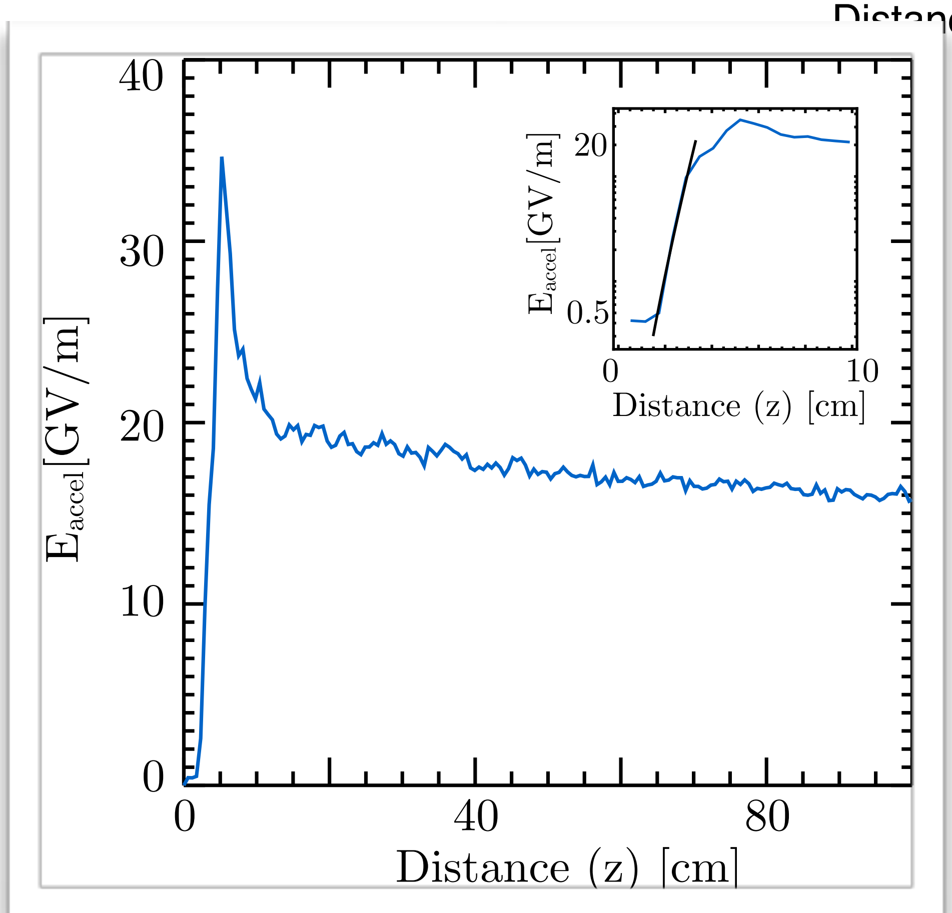


J.Vieira et al PRL **112**, 205001 (2014)

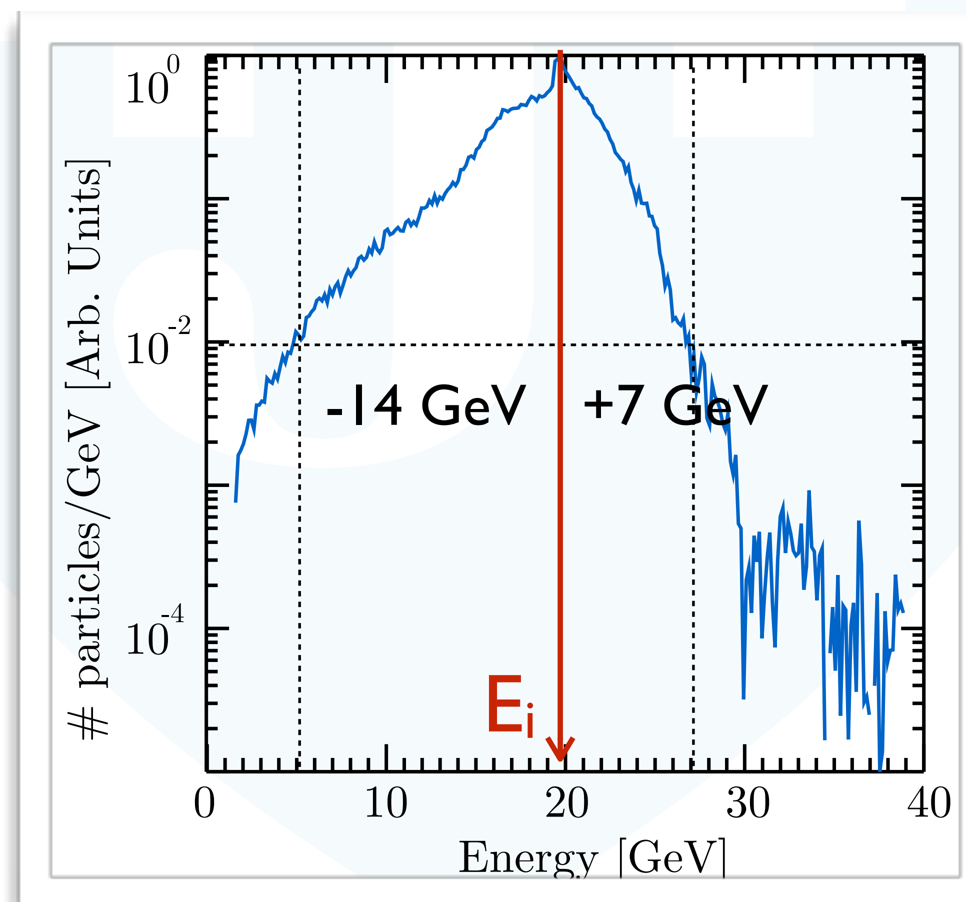
The self-modulation drives plasma wakefields in the blowout



20 GeV/m after 10 cm

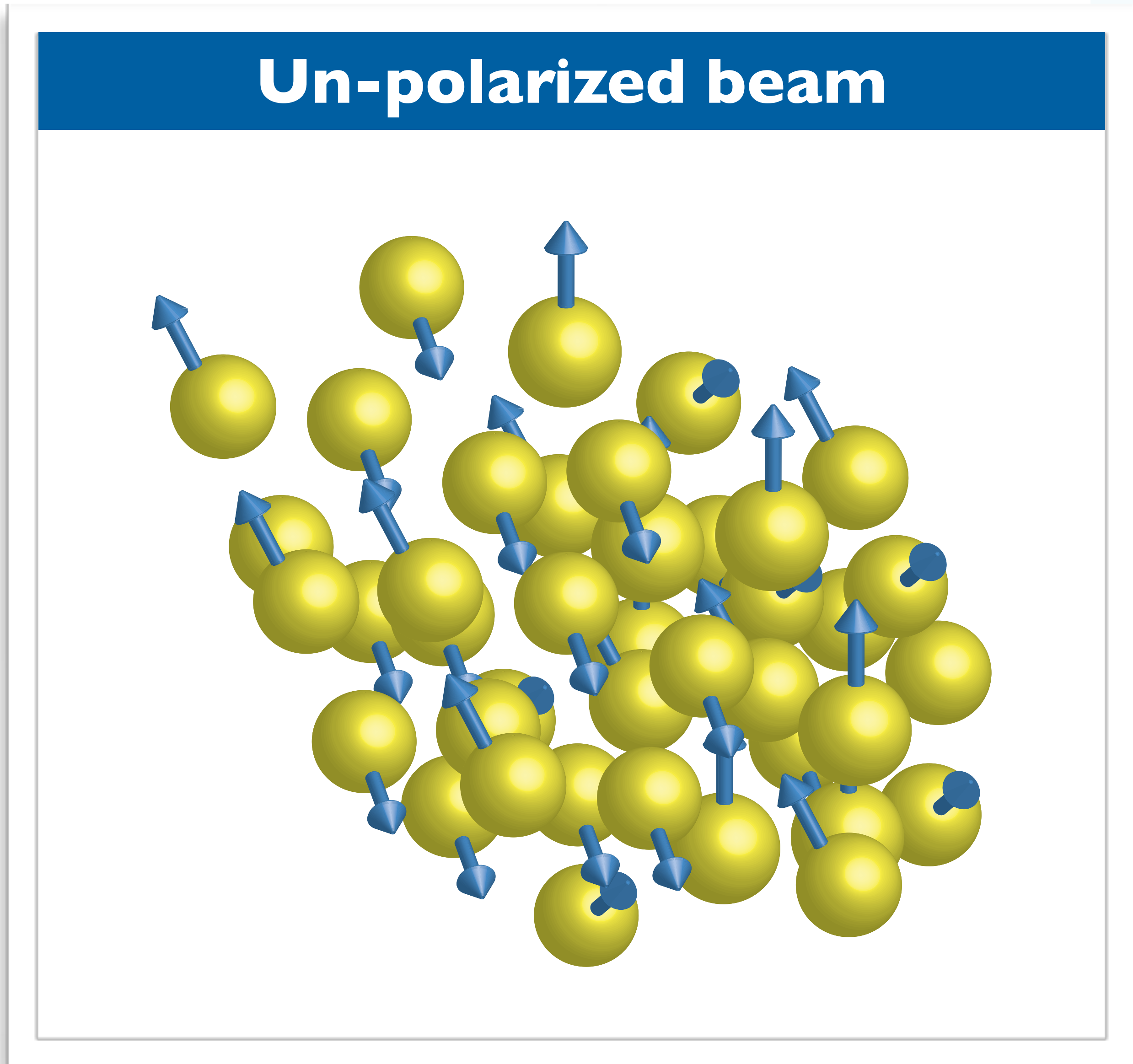


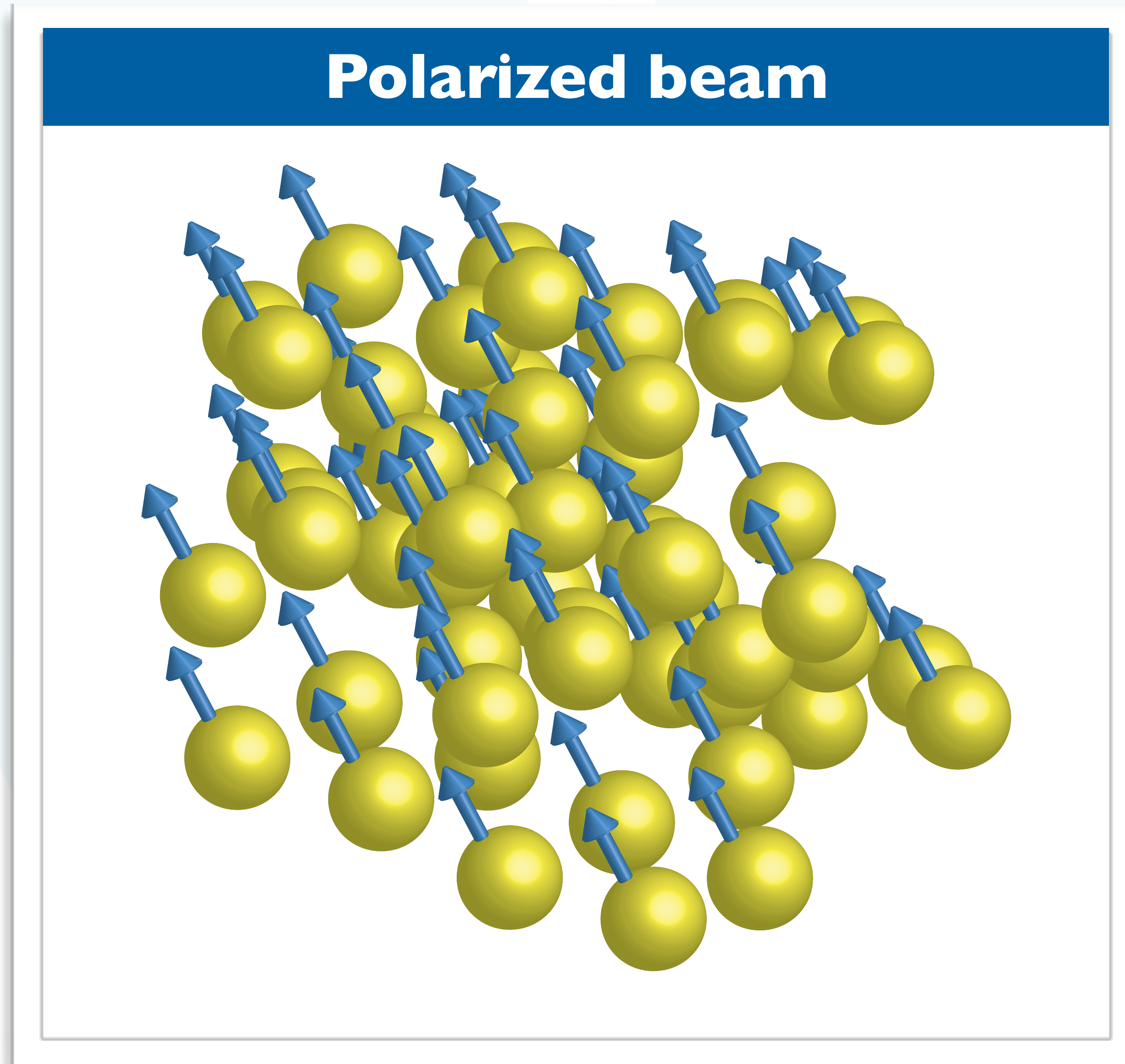
10 GeV @ 1% energy level



J.Vieira et al PoP 19 063105 (2012).

Spin precession in plasma waves

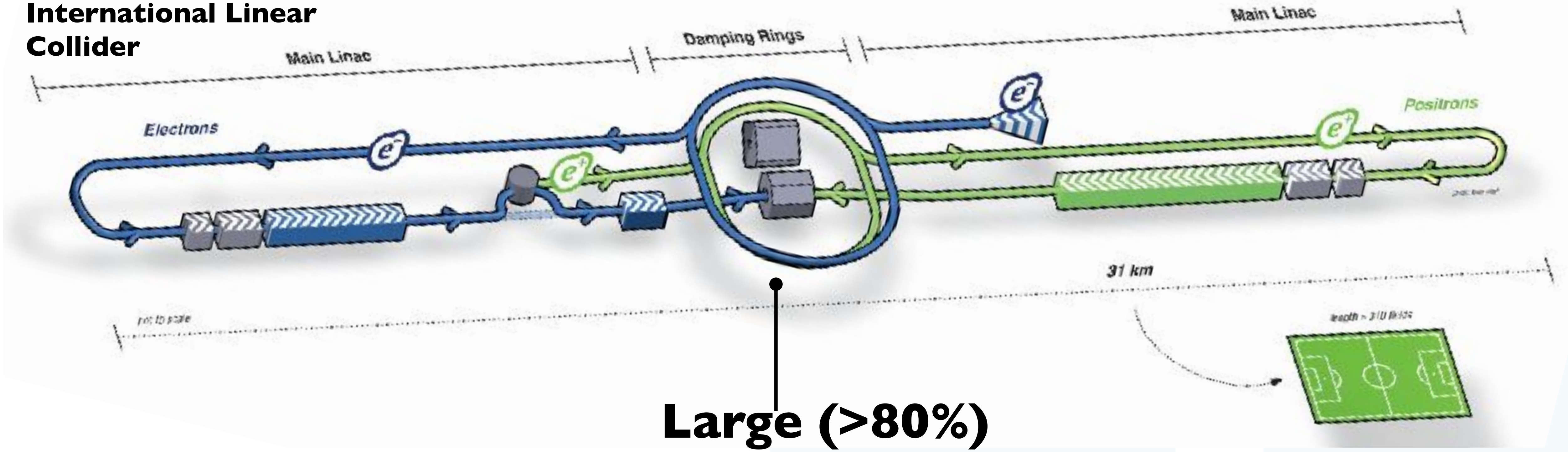




Beam polarization is the average spin vector including the contributions from all beam particles

T-BMT equations define the spin precession dynamics

International Linear Collider



Large (>80%) polarisations at interaction point

Relativistic spin-precession equation

$$\frac{d\mathbf{s}}{dt} = -\left[\left(a+\frac{1}{\gamma}\right)(\mathbf{B}-\mathbf{v}\times\mathbf{E})-\mathbf{v}\frac{a\gamma}{\gamma+1}\mathbf{v}\cdot\mathbf{B}\right]\times\mathbf{s}=\mathbf{\Omega}\times\mathbf{s}.$$

Can plasmas provide polarised beam sources?

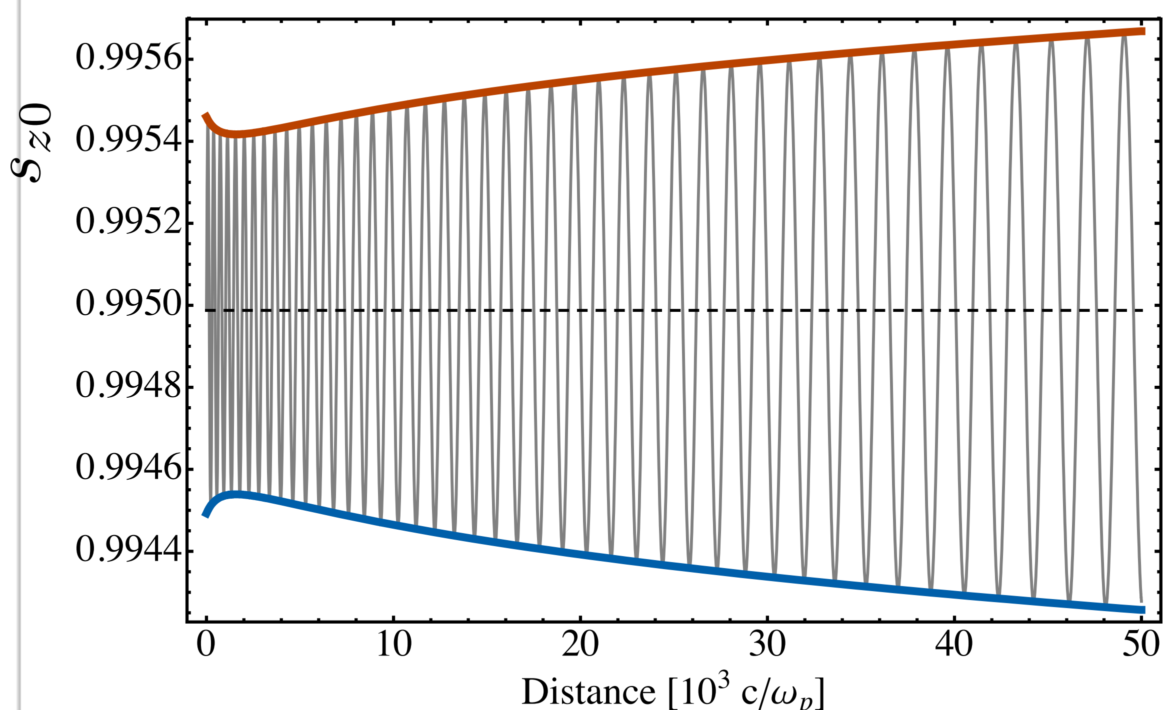
J.Vieira et al PR-STAB **14** 071303 (2011)

Spin precession is very small in plasma waves in the blowout regime

$$s_z(t) = \sqrt{1 - s_{\phi 0}^2}\sin\left[-\int_0^t \left(a + \frac{1}{\gamma}\right) F_r dt + \arctan\left(\frac{s_{z0}}{s_{\phi 0}}\right)\right]$$

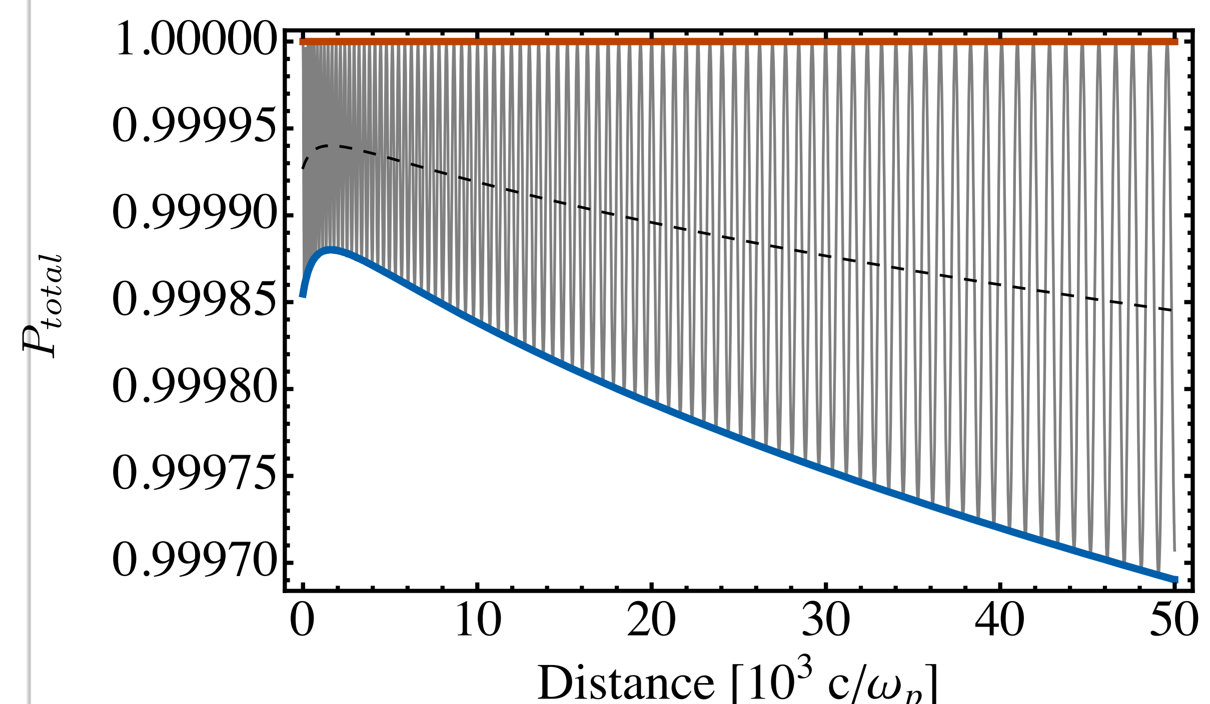
$\ll 1$

Single electron



The **individual beam particle spin variations are very small** even for the standards to conventional accelerators

Electron beam - Polarization



The total beam polarisation variations **are also very small and are on the order 0.01 % for very high accelerations**

J.Vieira et al PR-STAB **14** 071303 (2011)

Contents

Motivation
Plasmas waves are multidimensional

Blowout regime
Phenomenological model

Theory for blowout
Field structure and beam loading

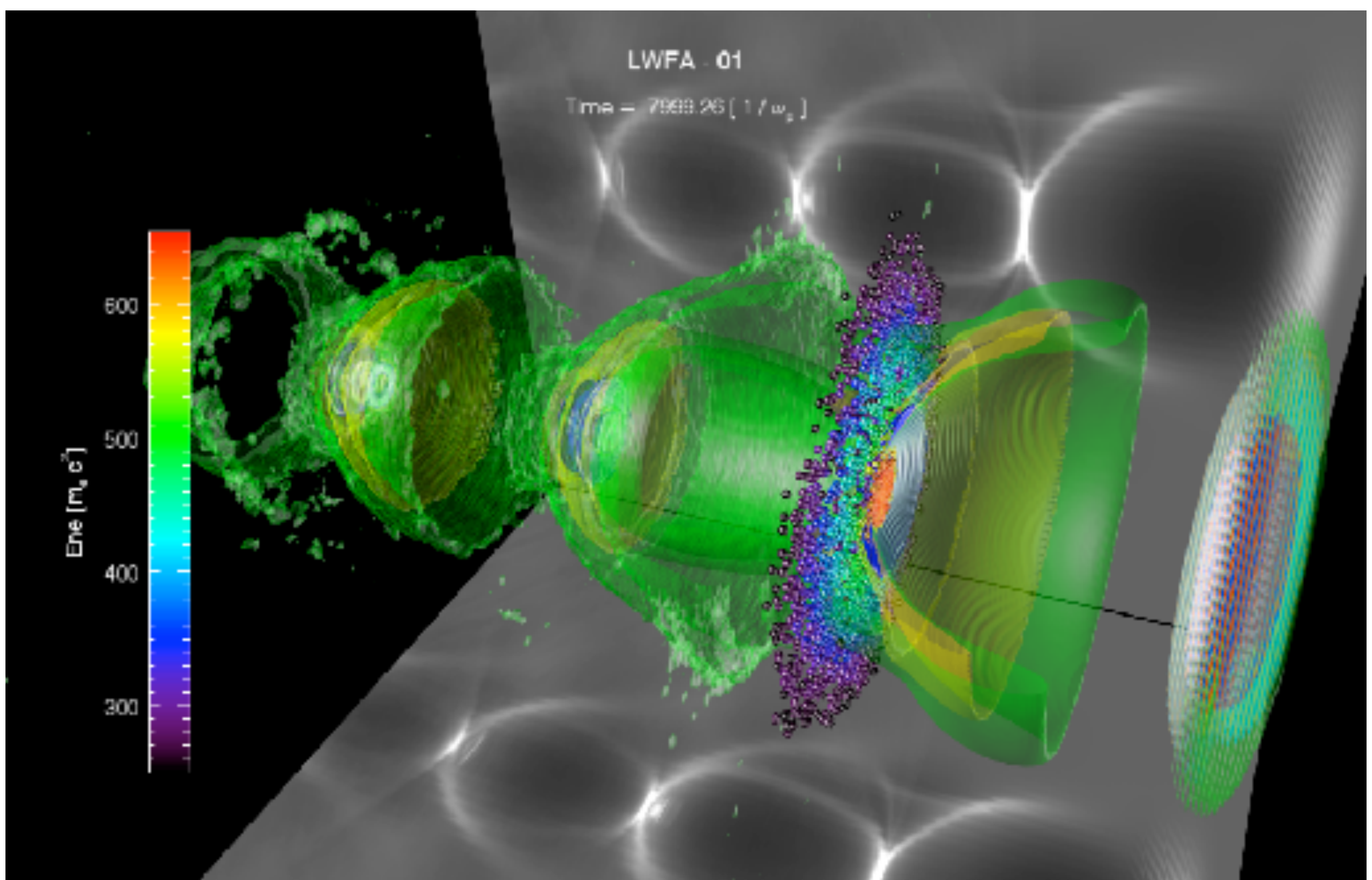
Challenges
Positron acceleration, long beams, polarized beams

Summary

Summary and take home messages

Plasma waves are intrinsically nonlinear (even when driven in the linear regime!)

Blowout regime suitable for electron acceleration



Challenges

Blowout/suck-in theory for more complex drivers (e.g. positrons/protons, ring drivers)

Positron acceleration in the blowout regime

Reduced models to capture self-injection

Towards a master equation

Deriving equation that depends only on a fluid quantity (p) [W.B. Mori]

Time derivative of Euler's equation

$$\partial_t^2 \vec{p} = -e\partial_t \vec{E} - mc^2 \partial_t \nabla \gamma$$

Ampère's Law and using conservation of the canonical momentum + definition of current

$$\vec{J} = -en\vec{v} = -en\frac{\vec{p}}{\gamma}$$

$$\partial_t^2 \vec{p} + c^2 \nabla \times \nabla \times \vec{p} = -\frac{4\pi e^2}{m} n \frac{\vec{p}}{\gamma} - mc^2 \partial_t \nabla \gamma$$

Density from Poisson's equation (also using simplified Euler's equation)

$$n = n_0 + \frac{1}{4\pi e^2} \nabla \cdot \left(\partial_t \vec{p} + mc^2 \nabla \gamma\right)$$

The master equation and waves

Master equation

$$\partial_t^2 \vec{p} + c^2 \nabla \times \nabla \times \vec{p} = -\left[\omega_{p0}^2 + \frac{1}{m}\nabla \cdot \left(\partial_t \vec{p} + mc^2 \nabla \gamma\right)\right] \frac{\vec{p}}{\gamma} - mc^2 \partial_t \nabla \gamma$$

$$\partial_t^2 \vec{p} + c^2 \nabla \times \nabla \times \vec{p} + \left[1 + \nabla \cdot (\partial_t \vec{p} + \nabla \gamma)\right] \frac{\vec{p}}{\gamma} + \partial_t \nabla \gamma = 0$$

Longitudinal waves

$$\nabla \times \vec{p} = 0 \qquad \left(\partial_t^2 + \omega_{p0}^2\right) \vec{p} = 0 \qquad \omega = \omega_{p0}$$

Transverse waves

$$\nabla \cdot \vec{p} = 0 \qquad \left(\partial_t^2 - \nabla^2 + \frac{1}{\gamma_0}\right) \vec{p} = 0 \qquad \omega^2 = k^2 c^2 + \frac{\omega_{p0}^2}{\gamma}$$