Laser propagation in plasma

Stuart Mangles

Learning Objectives

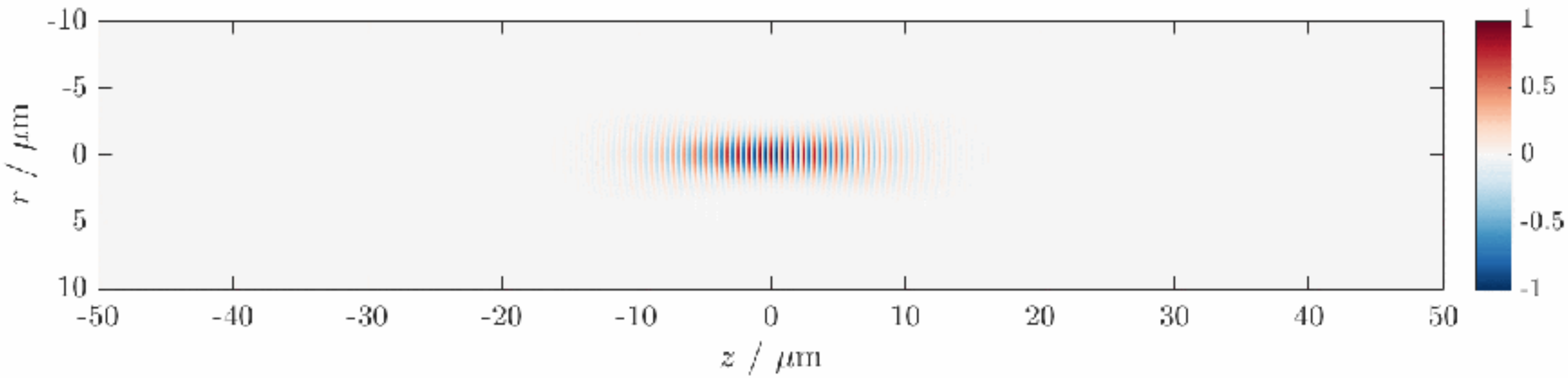
- By the end of this lecture you should
  - be familiar with basic concepts of laser propagation in vacuum
  - be able to describe the non-linear refractive index of plasmas
  - be able to use the non-linear refractive index to study key phenomena including:
    - » self-focussing; guiding, pulse-compression, photon “deceleration”
    - » self-modulation instability; hosing; wakefield evolution

Laser propagation in vacuum

- Gaussian beam *approximation*
  - *Electric field of propagating laser pulse in vacuum:*

transverse — longitudinal — phase

$$\mathbf{E}(r,z,t) = E_0\hat{\mathbf{x}} \frac{w_0}{w(z)} \exp\left(-\frac{r^2}{w(z)^2}\right) \exp\left[-\frac{(z-v_g t)^2}{(c\tau)^2}\right] \exp\left[-i\left(kz - \omega t + k\frac{r^2}{2R(z)} - \psi(z)\right)\right]$$



– *Rayleigh range*

$$z_{\mathrm{R}} = \frac{\pi w_0^2}{\lambda}$$

– transverse beam size

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_{\mathrm{R}}}\right)^2}$$

– *wavefront curvature*

$$R(z) = z\left[1+\left(\frac{z_{\mathrm{R}}}{z}\right)^2\right]$$

Laser propagation in vacuum

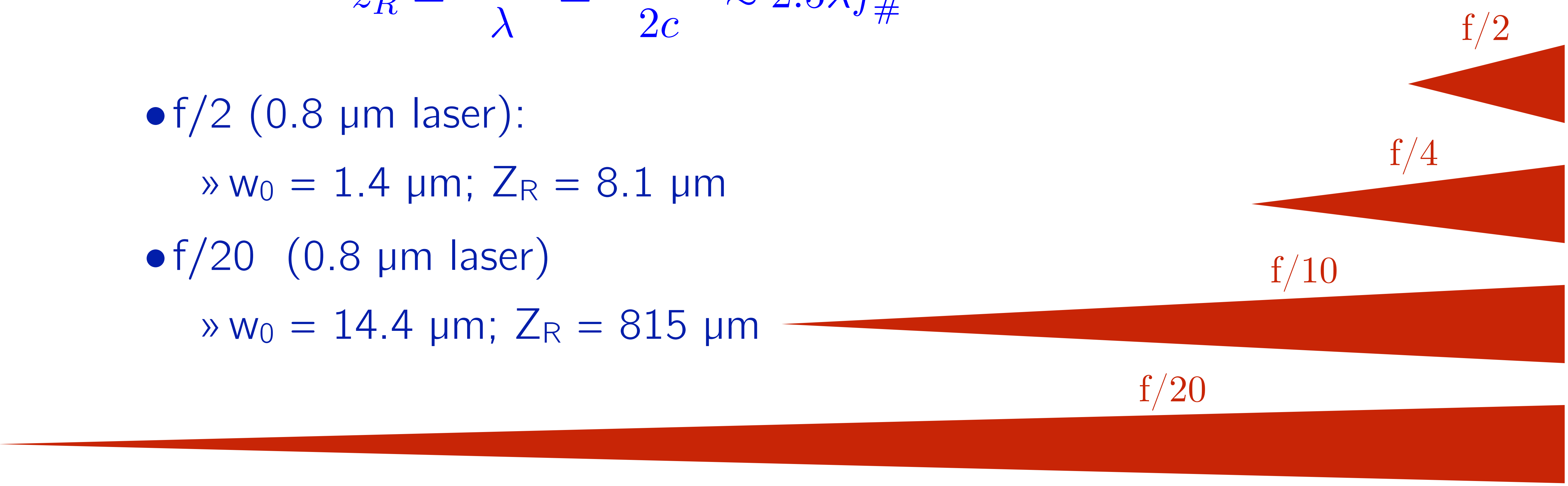
- The f-number f#
  - Ratio of focal length to (collimated) beam diameter
    - controls how tightly focussed the laser is

$$w_0 = \frac{2\sqrt{2}}{\pi}\lambda f_{\#} \approx 0.9\lambda f_{\#}$$

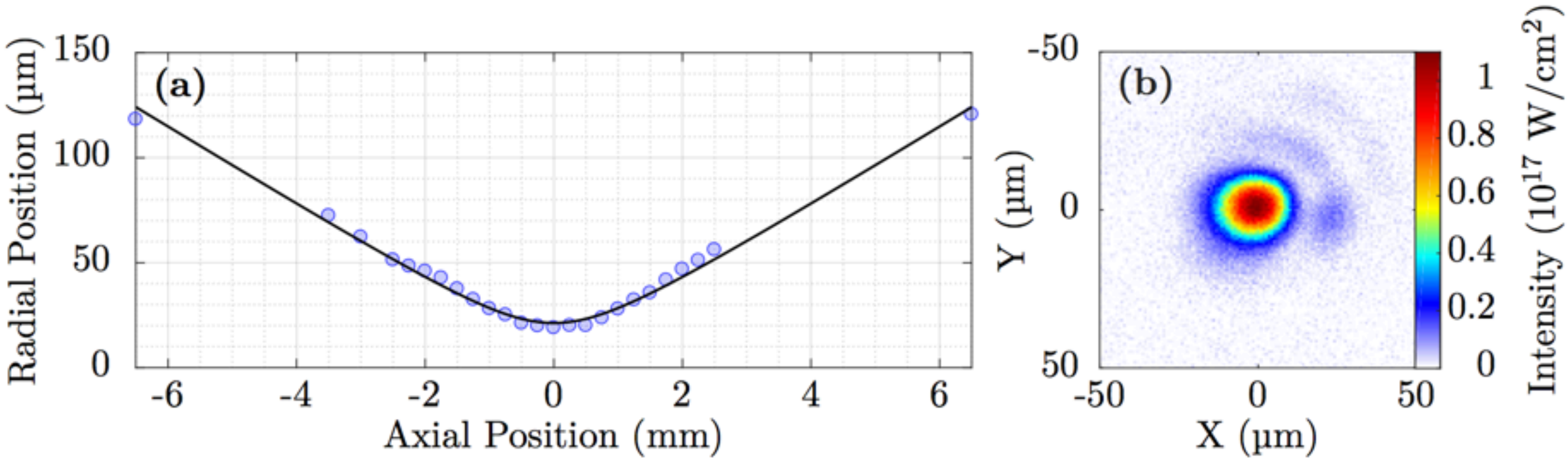
    - controls distance over which laser stays intense

$$z_R = \frac{\pi w_0^2}{\lambda} = \frac{\omega_0 w_0^2}{2c} \approx 2.5\lambda f_{\#}^2$$

    - f/2 (0.8 μm laser):
      » w_0 = 1.4 μm; Z_R = 8.1 μm
    - f/20 (0.8 μm laser)
      » w_0 = 14.4 μm; Z_R = 815 μm



Laser propagation in vacuum



R Shalloo PhD Thesis 2019

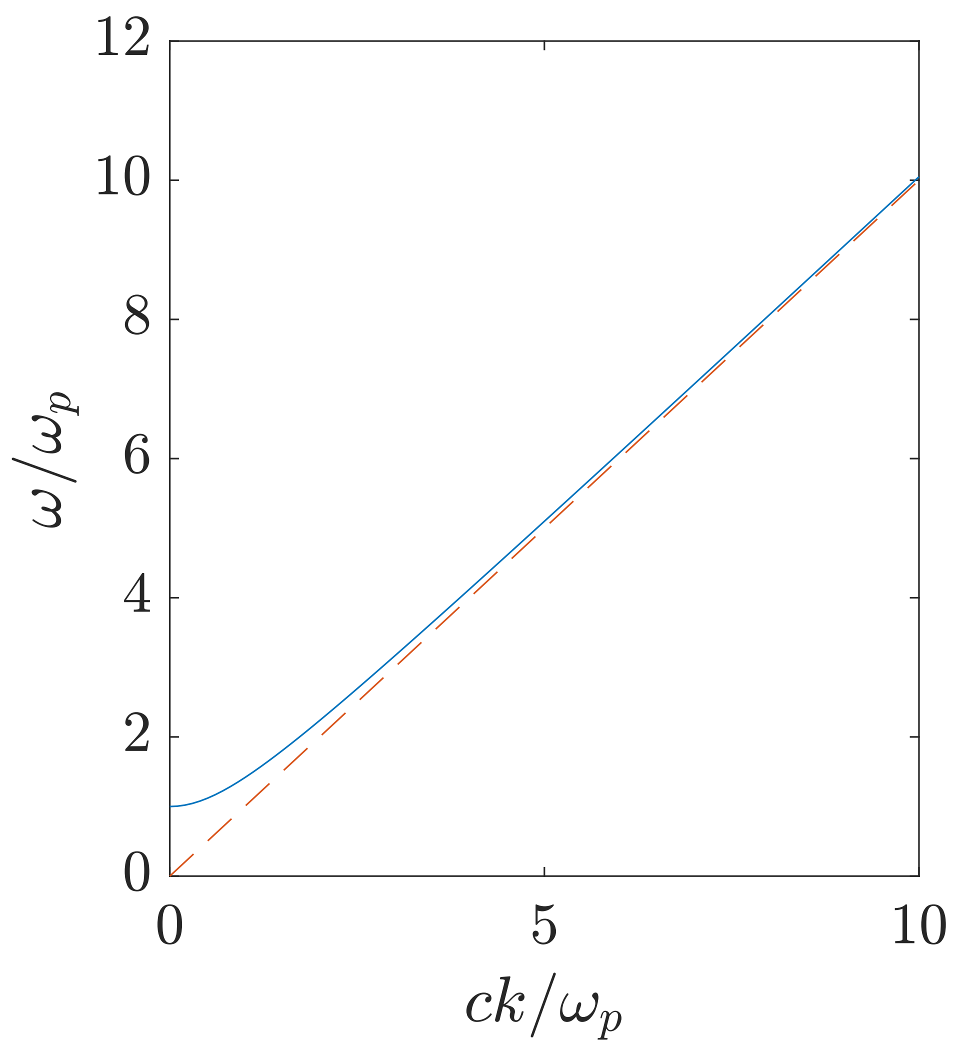
- **NB real high power lasers are not gaussian!!**
  - near field beam is closer to flat top
  - wavefront curvature and intensity profile are not perfect
  - spatio-temporal couplings: errors in chirped pulse amplification system
    - » all affect the propagation of lasers, especially due to non-linear effects in plasma

Laser propagation in plasma

- dispersion relation for low intensity EM waves in plasma

$$\omega^2 = \omega_p^2 + k^2 c^2$$

$$\omega_p = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$



- phase velocity

$$v_p = \frac{\omega}{k}$$
$$= c\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{-\frac{1}{2}} \approx c\left(1 + \frac{1}{2}\frac{\omega_p^2}{\omega^2}\right)$$

- group velocity

$$v_g = \frac{\partial \omega}{\partial k}$$
$$= c\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{\frac{1}{2}} \approx c\left(1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2}\right)$$

Non-linear refractive index in plasma

- High intensity laser modifies the refractive index

$$\eta = \frac{c}{v_p} = \left(1 - \frac{\omega_p^2}{\gamma\omega_0^2}\right)^{\frac{1}{2}}$$

- Depends on
  - » local plasma density
  - » local laser frequency
  - » local laser intensity

$$n = n_0 + \frac{\delta n}{n_0} n_0$$

$$\omega_L = \omega_0 + \frac{\delta\omega_L}{\omega_0}\omega_0$$

$$\langle\gamma\rangle = 1 + \frac{a_0^2}{4}$$

- Ignoring 2nd order terms this gives

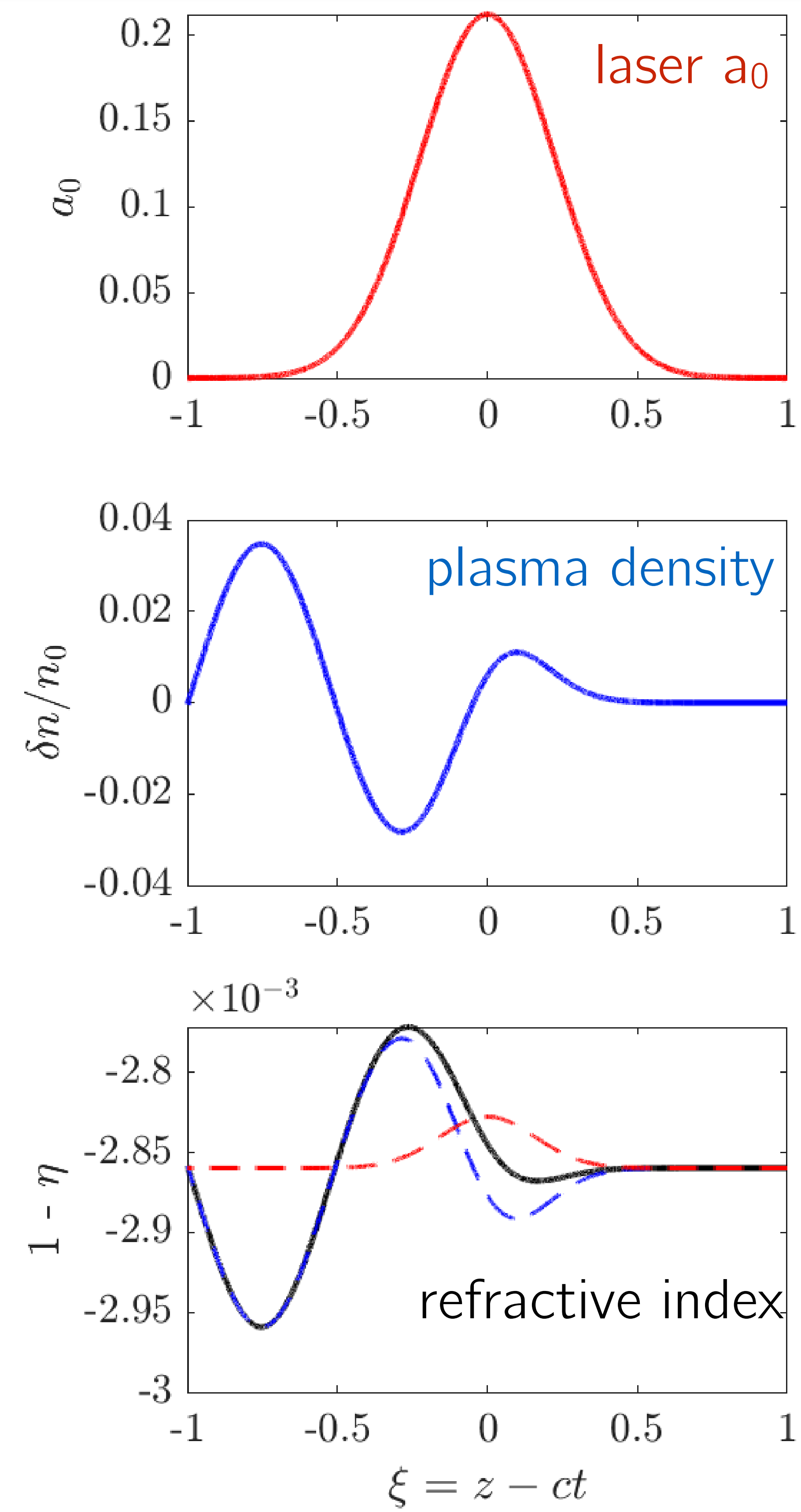
$$\eta = 1 - \frac{1}{2}\frac{\omega_p^2}{\omega_0^2}\left(1 + \frac{\delta n}{n_0} - \frac{2\delta\omega_L}{\omega_0} - \frac{a_0^2}{4}\right)$$

WB Mori IEEE J Quantum Elec. **33**, 1942 (1997)

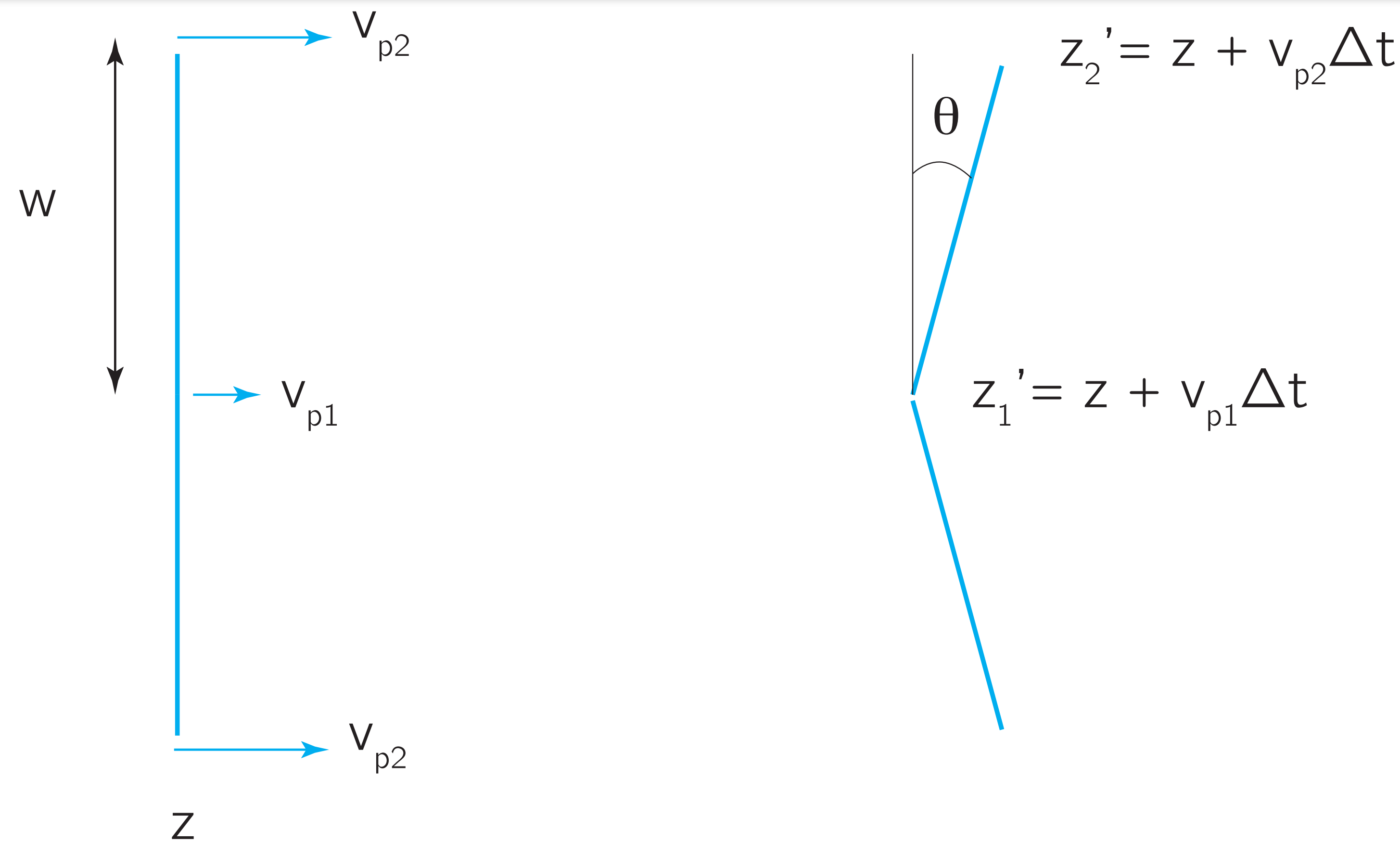
Wave frame

- Non-linear refractive index due to high intensity laser pulse moves with the laser
  - change in density due to ponderomotive force
  - change in electron "mass" due to quiver motion in laser field
- Introduce "wave frame" variables

$$\xi = z - ct$$
$$\tau = t$$



Self-focusing



- Transverse gradient of refractive index leads to tilting of wavefront

$$\theta \simeq \Delta v_p \Delta t / w$$

$$\Delta v_p \approx w \frac{\partial v_p}{\partial r} = -w \frac{c}{\eta^2} \frac{\partial \eta}{\partial r} \qquad \theta \simeq -\frac{c}{\eta^2} \frac{\partial \eta}{\partial r} \Delta t$$

WB Mori IEEE J Quantum Elec. **33**, 1942 (1997)

Self-focusing

- Energy flows perpendicular to the wavefront
  - rate of energy flow inwards (or outwards) is equal to rate of change of spot size

$$\frac{\partial w}{\partial \tau} = -v_{g,r} = -v_g \sin\theta \approx -v_g \theta$$

  - in plasma v_g is related to refractive index through: $v_g = c\eta$
  - so we get: $$\frac{\partial w}{\partial \tau} = \frac{c^2}{\eta}\frac{\partial \eta}{\partial r}\Delta t$$
  - 'acceleration' of spot size due to radial variation in refractive index is therefore

$$\frac{\partial^2 w}{\partial \tau^2} = \frac{c^2}{\eta}\frac{\partial \eta}{\partial r}$$

WB Mori IEEE J Quantum Elec. **33**, 1942 (1997)

Relativistic self-focussing

- High laser intensity on axis creates focusing effect through the relativistic (a_0) term in the non-linear refractive index

- To get self-focusing need rate of focussing to be faster than the rate of defocussing from diffraction

$$\left.\frac{\partial^2 w}{\partial \tau^2}\right|_{\text{plasma}} + \left.\frac{\partial^2 w}{\partial \tau^2}\right|_{\text{diffraction}} \leq 0$$

Relativistic self-focusing

- The diffraction term can be found from gaussian waist equation

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_\mathrm{R}}\right)^2}$$

  - near focus ($z < z_\mathrm{R}$), can differentiate this to get

$$\begin{aligned}\frac{\partial w}{\partial \tau} &= \frac{\partial z}{\partial \tau}\frac{\partial w}{\partial z} \approx c\frac{\partial w}{\partial z} \\ &= c\frac{\partial}{\partial z}\left(1+\frac{z^2}{2z_R^2}\right) \\ &= \frac{cz}{z_R^2} = \frac{4c^3 z}{w_0^3\omega_0^2}\end{aligned}$$

$$\left.\frac{\partial^2 w}{\partial \tau^2}\right|_{\mathrm{diffraction}} = \frac{4c^4}{\omega_0^2 w_0^3}$$

Relativistic self-focusing

- The relativistic term from plasma refractive index becomes

$$\eta = 1 - \frac{1}{2}\frac{\omega_p^2}{\omega_0^2}\left(1 - \frac{a^2(r,z)}{4}\right)$$

$$\frac{\partial^2 w}{\partial \tau^2} = \frac{c^2}{\eta}\frac{\partial \eta}{\partial r}$$

$$\frac{\partial^2 w}{\partial \tau^2} = -\frac{c^2}{8}\frac{\omega_p^2}{\omega_0^2}\frac{\partial}{\partial r}a^2(r,z)$$

– approximating the transverse gradient of a^2 as: $\frac{\partial}{\partial r}a^2 \approx \frac{a_0^2}{w_0}$

$$\left.\frac{\partial^2 w}{\partial \tau^2}\right|_{\text{plasma}} = -\frac{1}{8}\frac{\omega_p^2}{\omega_0^2}\frac{a_0^2}{w_0}c^2$$

Relativistic self-focusing

$$\left.\frac{\partial^2 w}{\partial \tau^2}\right|_{\text{diffraction}} = \frac{4c^4}{\omega_0^2 w_0^3} \qquad \left.\frac{\partial^2 w}{\partial \tau^2}\right|_{\text{plasma}} = -\frac{1}{8}\frac{\omega_p^2}{\omega_0^2}\frac{a_0^2}{w_0}c^2$$

– Balancing the rate of relativistic focusing with diffraction:

$$\left.\frac{\partial^2 w}{\partial \tau^2}\right|_{\text{plasma}} + \left.\frac{\partial^2 w}{\partial \tau^2}\right|_{\text{diffraction}} = 0$$

$$w_0^2 a_0^2 = 32\frac{c^2}{\omega_p^2}$$

– Noting that $w_0{}^2 a_0{}^2$ is related to the laser power (area x intensity) it can be shown....

$$P[\mathrm{GW}] \geq 17.3\frac{\omega_0^2}{\omega_p^2}$$

Plasma channel guiding

- A similar treatment can be applied to estimate the effect of a parabolic plasma channel:

$$n_e(r) = n_{e0} + \Delta n_e \frac{r^2}{r_{\text{ch}}^2}$$

  – balancing focusing due to the channel with diffraction produces a "matched spot size"

$$w_m = \left(\frac{r_{\text{ch}}^2}{\pi r_e \Delta n_e} \right)^{\frac{1}{4}}$$

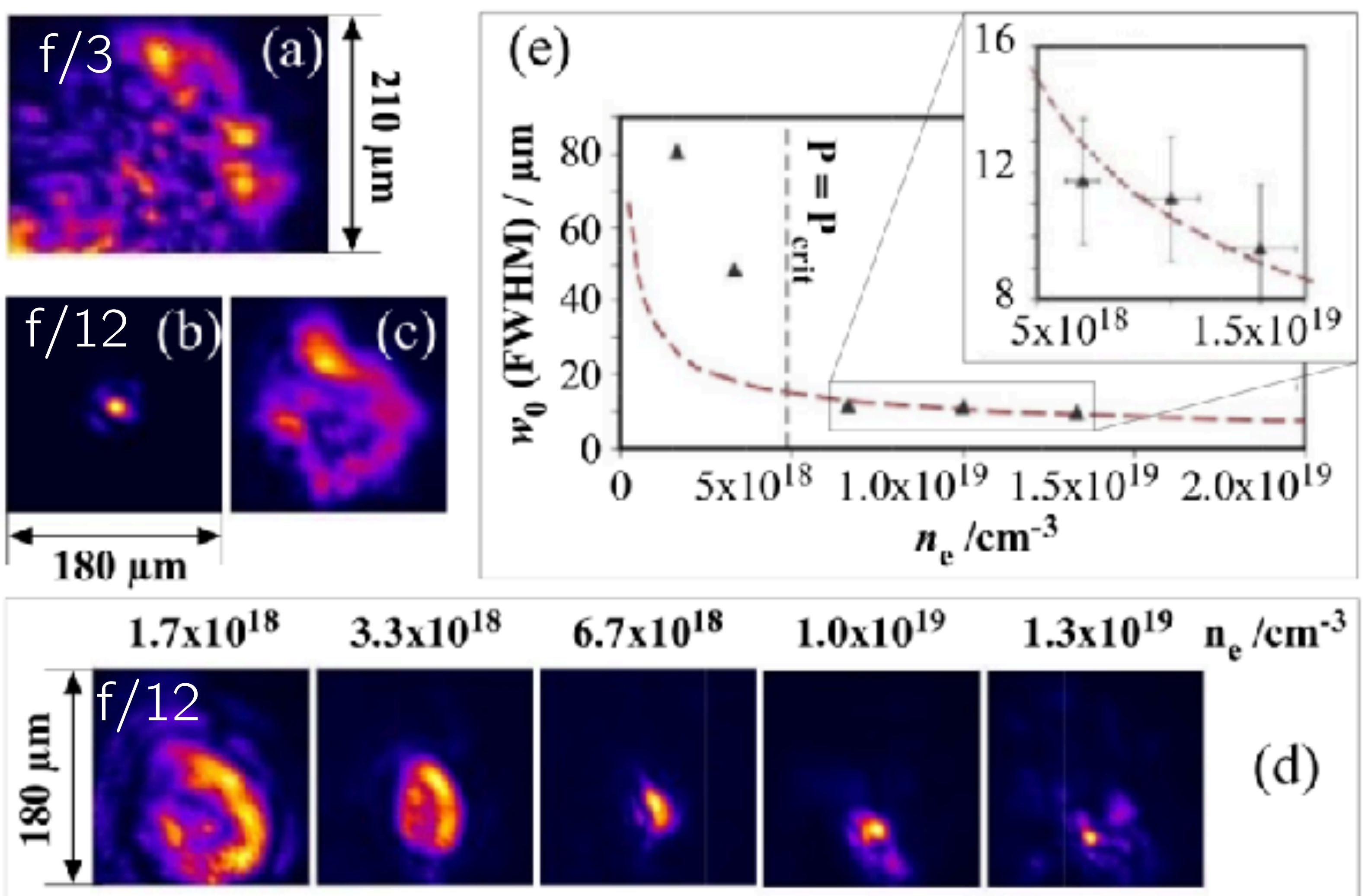
Self-guiding

- In LWFA we have a lot going on

$$\left.\frac{\partial^2 w}{\partial \tau^2}\right|_{\text{relativistic}} + \left.\frac{\partial^2 w}{\partial \tau^2}\right|_{\text{ponderomotive}} + \left.\frac{\partial^2 w}{\partial \tau^2}\right|_{\text{pre-formed}} + \left.\frac{\partial^2 w}{\partial \tau^2}\right|_{\text{diffraction}} \leq 0$$

  - ***relativistic***: variation in a_0
  - ***ponderomotive***: variation in n_e due to laser pulse
  - ***pre-formed***: variation in n_e due to pre-formed channel

Self-guiding

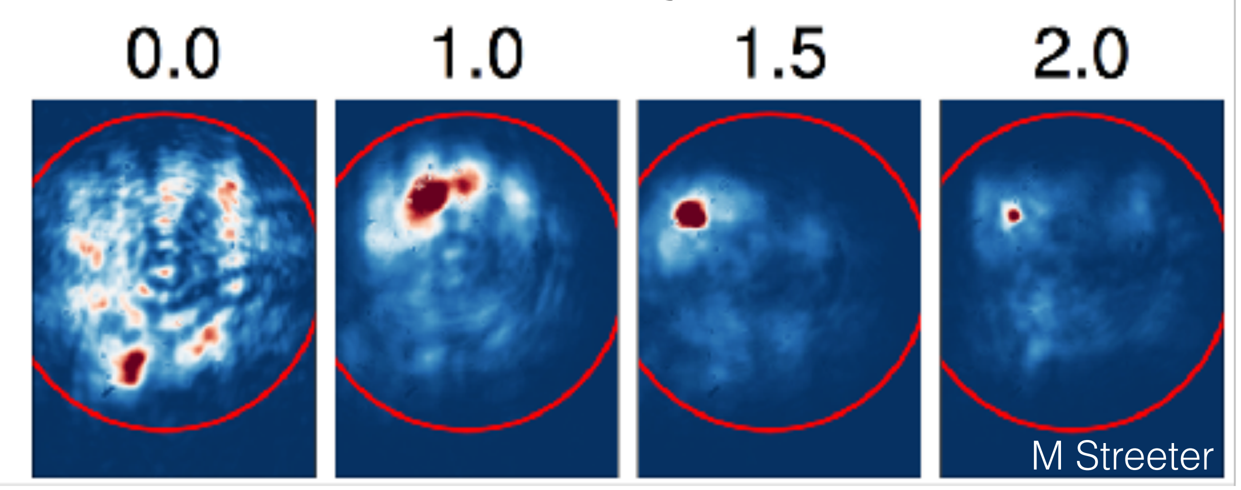


- Experimental measurements of self-guiding
- Laser profile at exit of 2 mm gas jet shows guided spot with 14 TW pulse for $P > P_c$
- guiding over approx $5–7\ z_R$

AGR Thomas PRL 2007

Self-guiding

plasma density x10^{18} cm^{-3}



images of laser spot at exit 15 mm laser wakefield accelerator

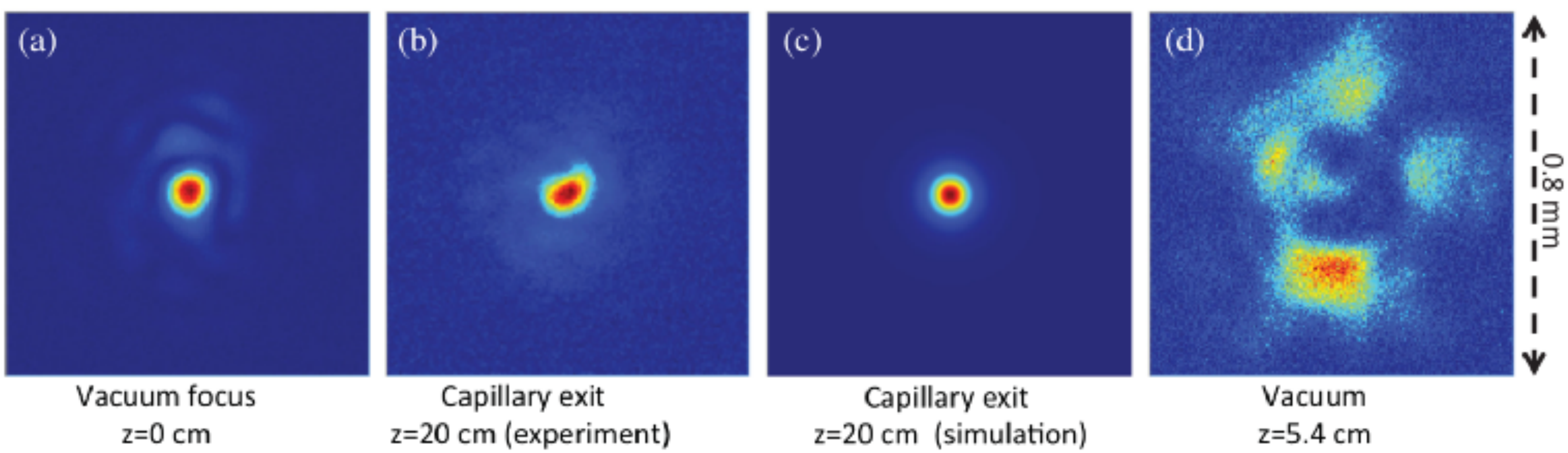
MJV Streeter PhD Thesis 2013

- Experimental measurements of self-guiding
- Laser profile at exit of 15 mm gas jet shows guided spot with 180 TW pulse for $P > P_c$
- guiding over approx 15 z_R

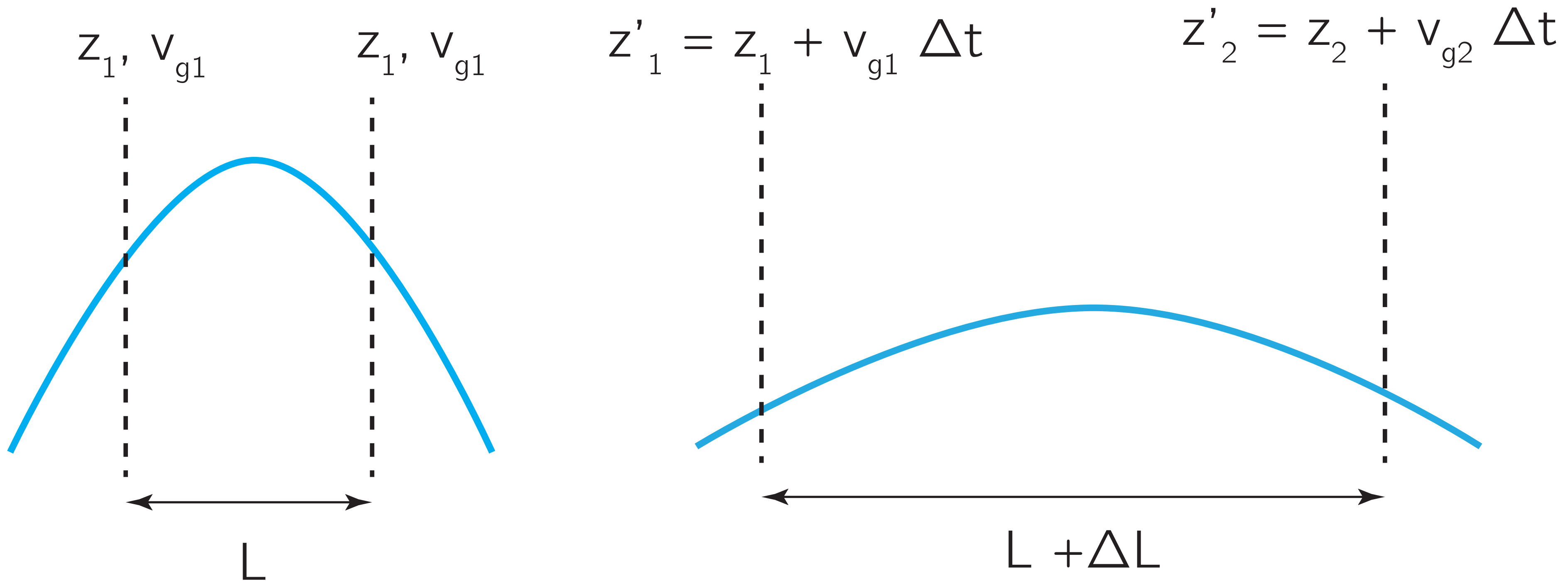
Pre-formed plasma channel guiding

- Pre-formed plasma channels very successful at guiding high power lasers over long distances:
  - e.g LBNL successfully guided 0.85 PW laser pulse over 20 cm (15 Z_R)

AJ Gonsalves PRL 2019



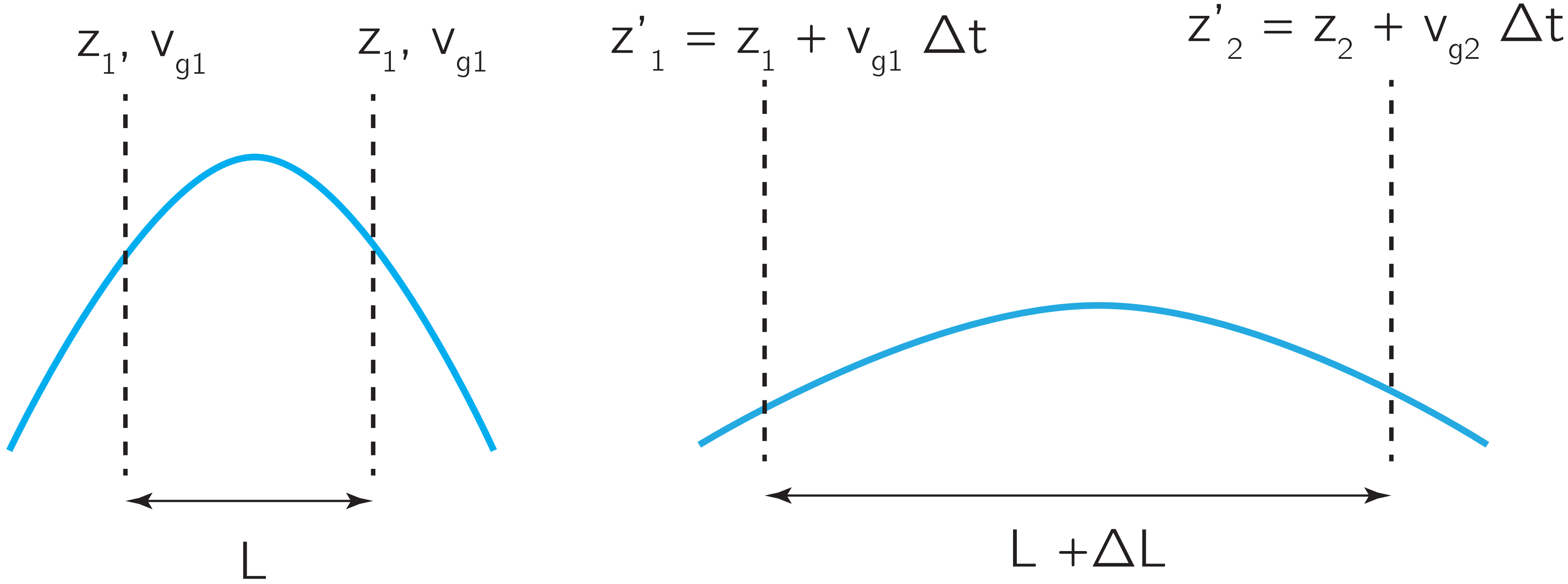
Pulse Compression



- Longitudinal variation in group velocity changes shape of pulse envelope as it propagates
  - compression / stretching

WB Mori IEEE J Quantum Elec. **33**, 1942 (1997)

Pulse Compression



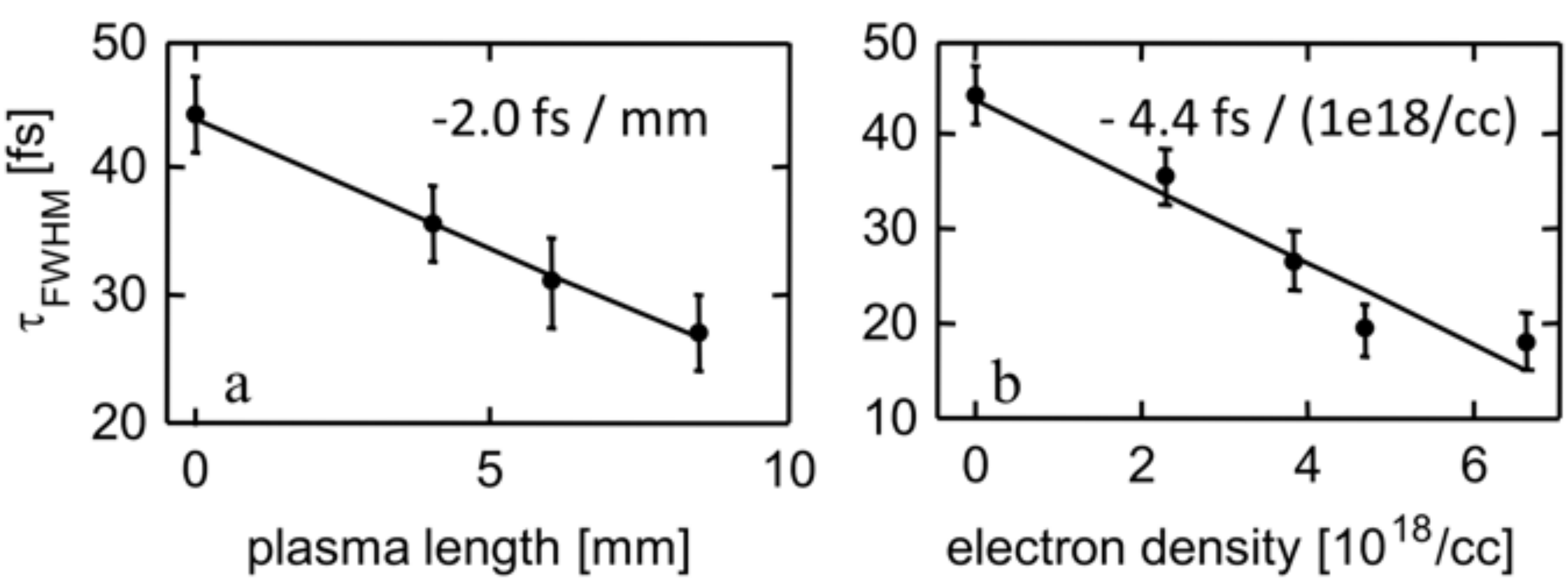
- consider point at front, z_1 and back, z_2, of laser pulse initially separated by distance L
- change in separation in time Δt: $\Delta L = (v_{g2} - v_{g1})\,\Delta t$
- relate change in group velocity to gradient in group velocity $\Delta v_g \approx \frac{\partial v_g}{\partial z} L$
- in wave frame rate of compression is

$$\frac{1}{L}\frac{\partial L}{\partial t} = -c\frac{\partial \eta}{\partial \xi}$$

WB Mori IEEE J Quantum Elec. **33**, 1942 (1997)

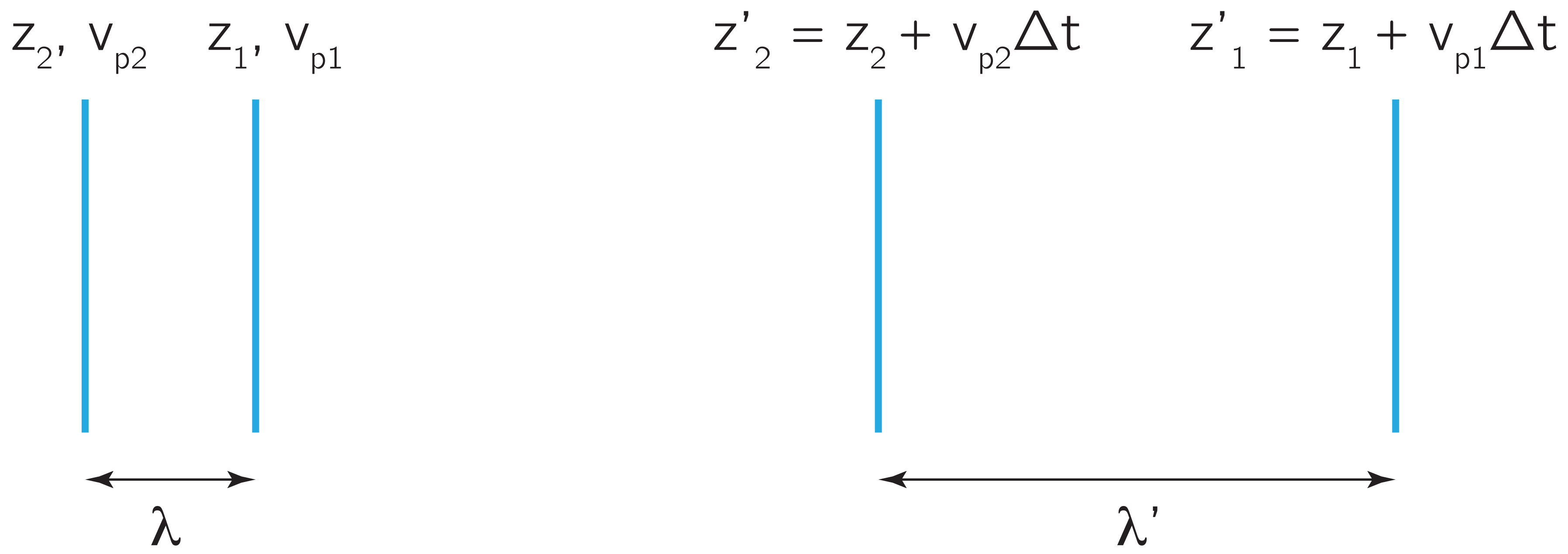
Pulse Compression in the Bubble Regime

- Simple model for compression in the bubble regime
  - front of pulse in plasma $n_e = n_0$; back of pulse $n_e = 0$
  - gives rate of compression $$\tau = \tau_0 - \frac{n_0 l}{2 c n_c}$$
  - measured rates of compression in non-linear plasma wave very close to this prediction



JS Schreiber PRL **105**, 235003 (2010)

Photon "acceleration"



- longitudinal variation in refractive index also means phase velocity varies, leading to a change in the wavelength

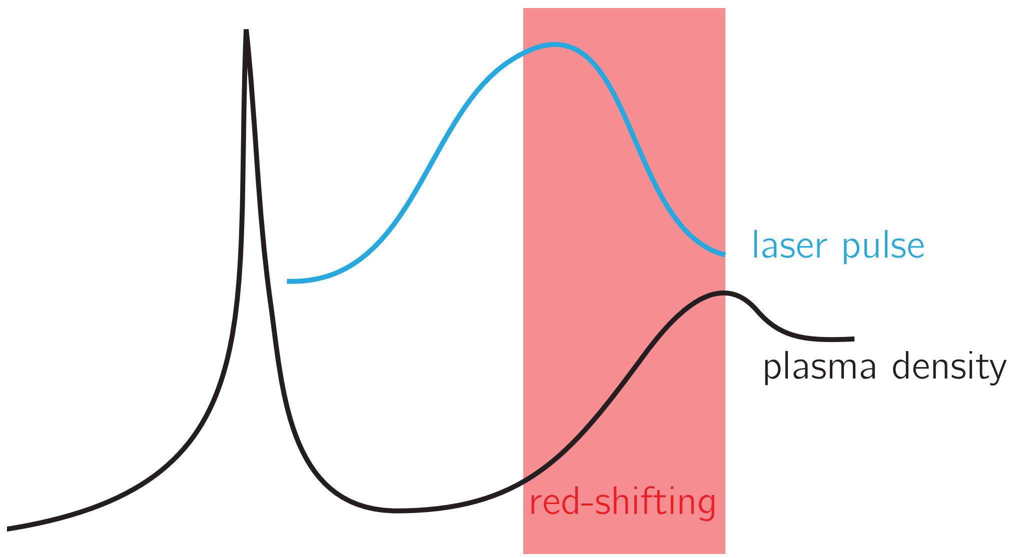
$$\Delta v_\phi = \frac{\partial v_\phi}{\partial z}\lambda_0 \qquad \frac{\partial \lambda}{\partial t} = \frac{\partial v_\phi}{\partial z}\lambda_0$$

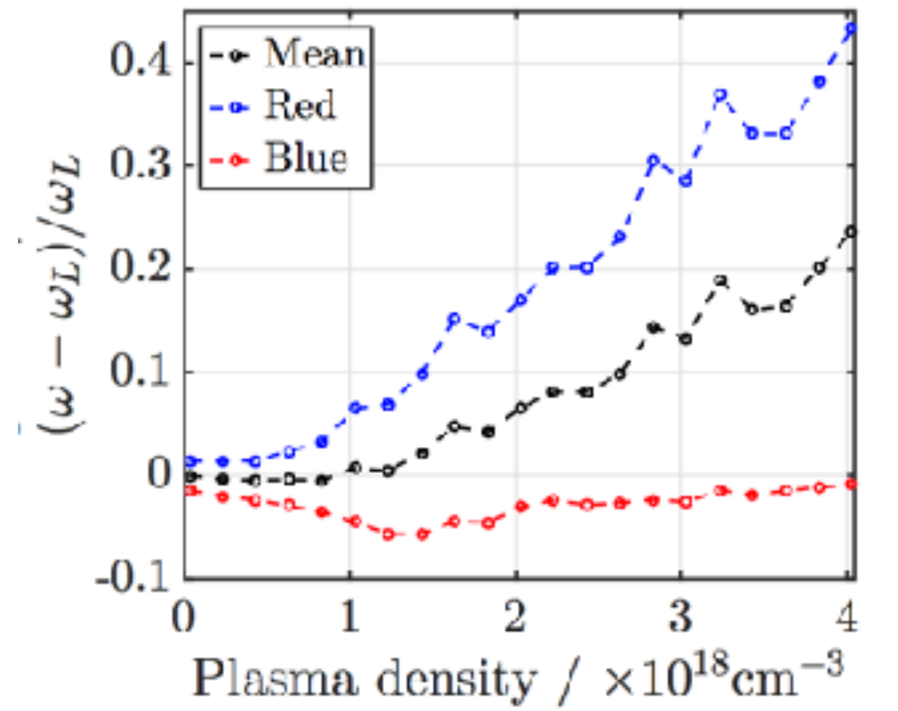
- For refractive index gradient caused by the laser pulse, the rate of change of frequency in the wave frame is:

$$\frac{1}{\omega}\frac{\partial \omega}{\partial \tau} = \frac{c}{\eta^2}\frac{\partial \eta}{\partial \xi}$$

WB Mori IEEE J Quantum Elec. **33**, 1942 (1997)

Photon "deceleration" in the Bubble Regime





K Poder PhD Thesis 2016

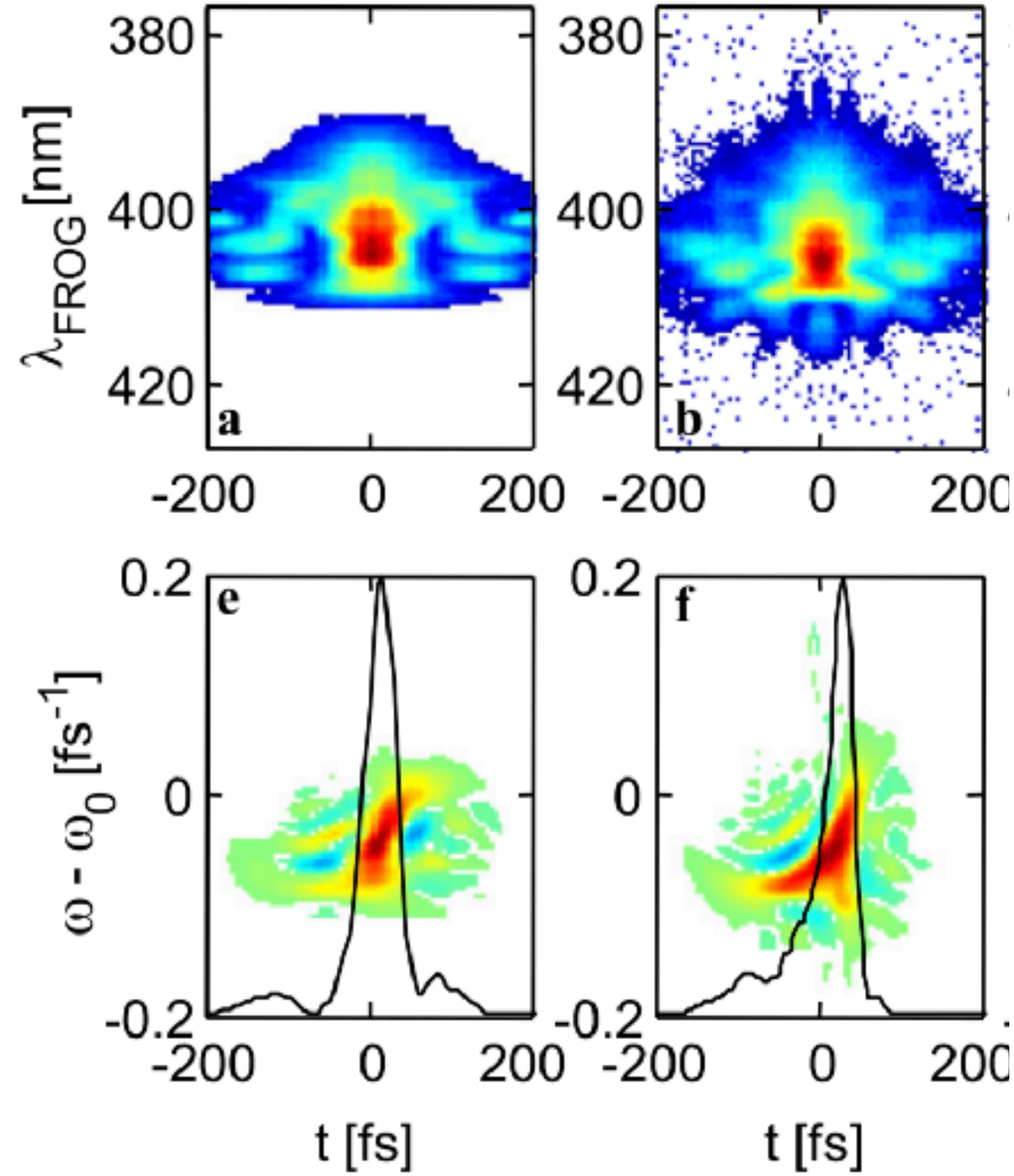
- Refractive index at front of bubble causes red-shifting of photons
  - these then slip back inside the pulse (lower group velocity)
  - leads to pulse front etching at $v_{\text{etch}} = \frac{\omega_p^2}{\omega_0^2} c$
  - non-linear group velocity is approximately

$$v_{\text{g,nl}} = v_{\text{g}} - v_{\text{etch}}$$

$$\approx c \left(1 - \frac{3}{2} \frac{\omega_p^2}{\omega_0^2} \right)$$

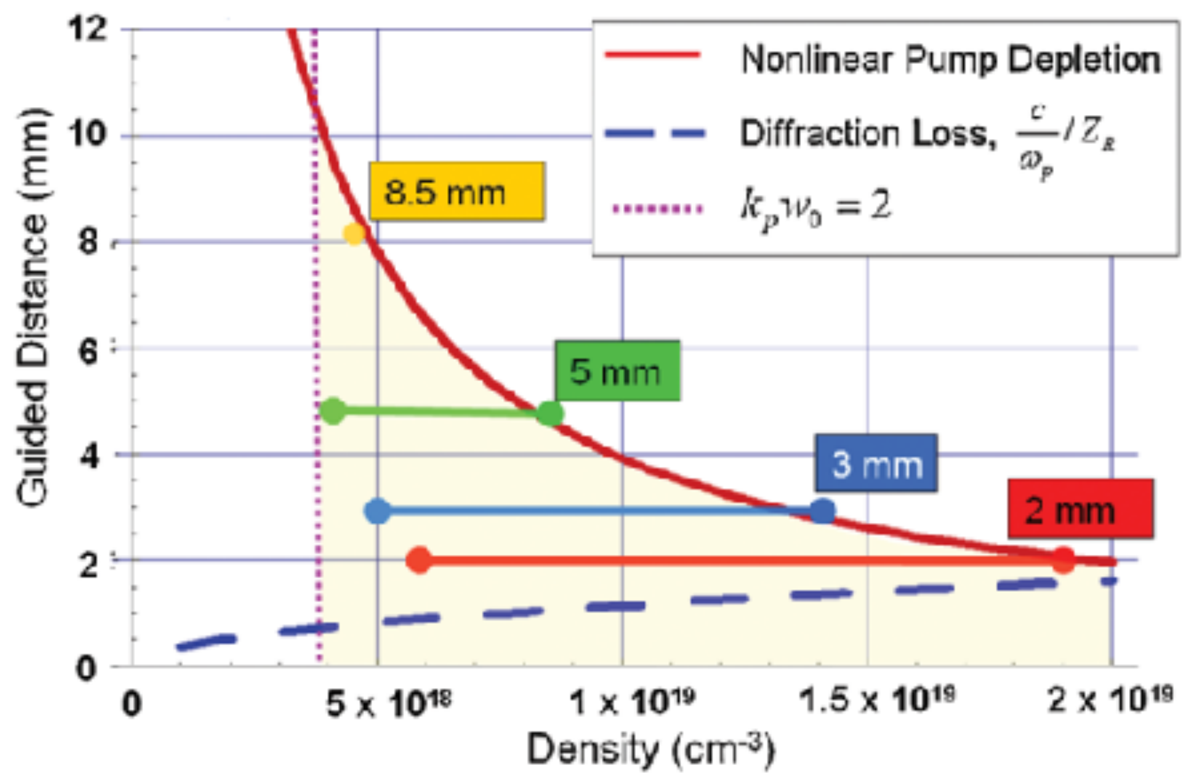
Photon "deceleration" in the Bubble Regime

- Frequency Resolved Optical Gating (FROG) measurements of pulse shape show red-shifting at the front of the pulse



JS Schreiber PRL **105**, 235003 (2010)

Photon "deceleration" in the Bubble Regime

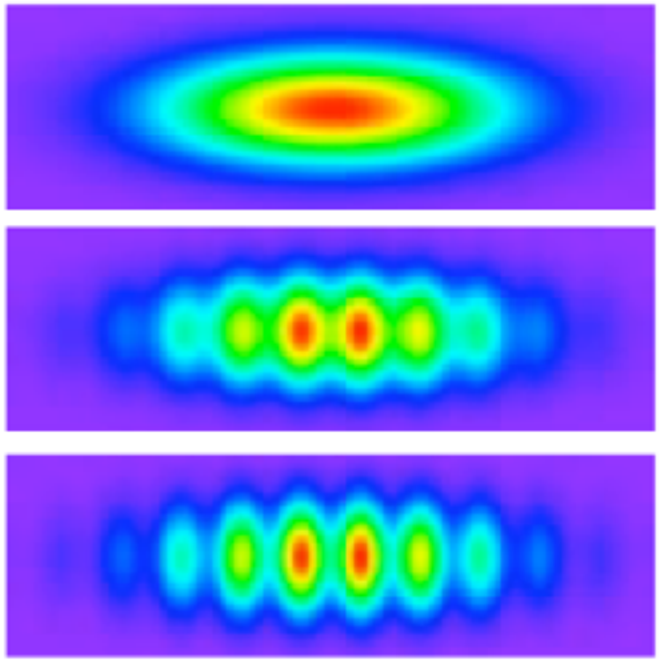


J Ralph PRL **102**, 175003 (2009)

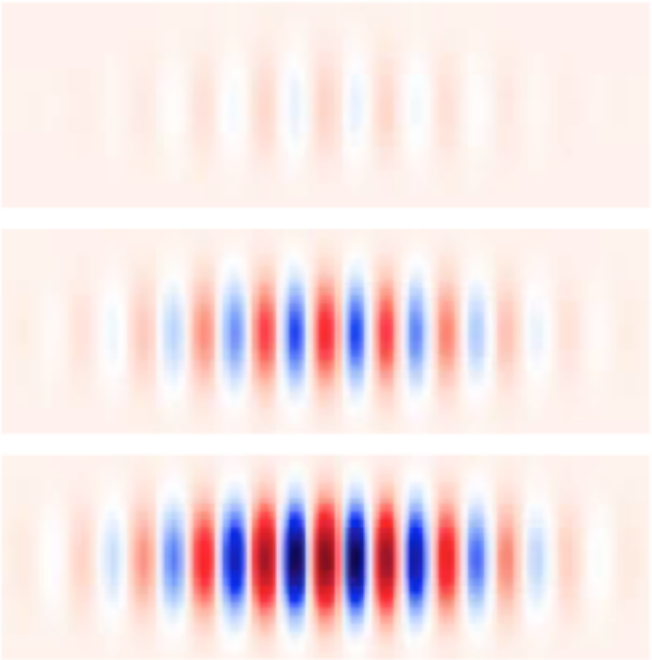
- Photon deceleration / etching determines the pump depletion length in non-linear wakes

$$L_{\rm etch} \simeq \frac{c}{v_{\rm etch}} c\tau_{\rm FWHM} \simeq \frac{\omega_p^2}{\omega_0^2} c\tau_{\rm FWHM}$$

Propagation instabilities: SM-LWFA



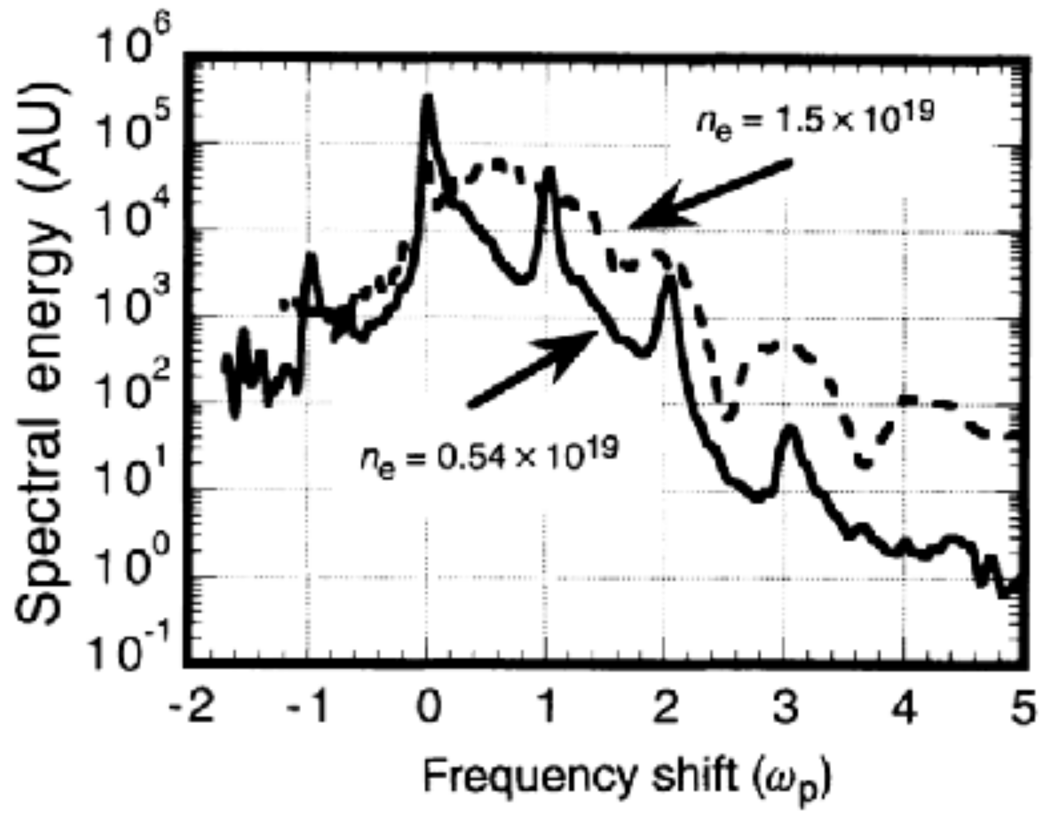
laser intensity

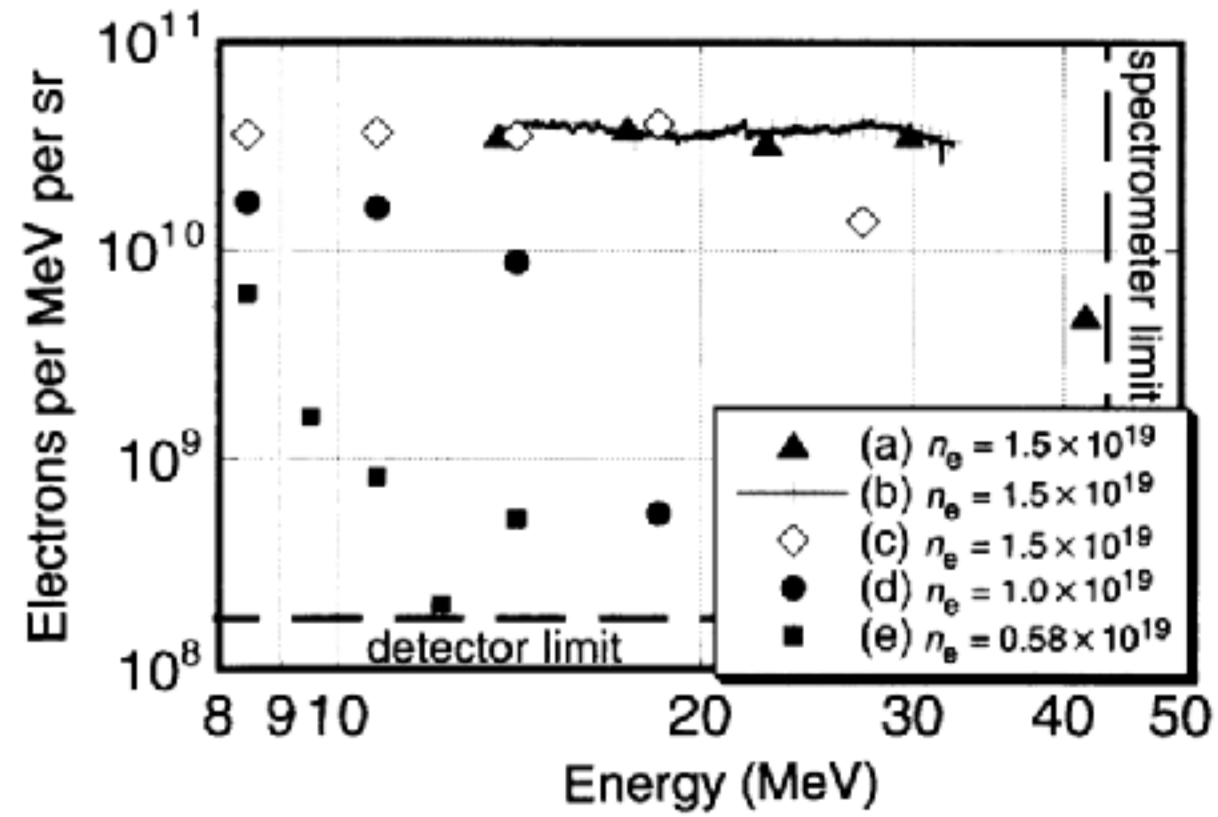


plasma density perturbation

- Self-modulation instability
  - laser pulse longer than plasma wavelength $c\tau > \lambda_p$
  - drives a low amplitude plasma wave
    - » compression / photon "acceleration" in longitudinal direction
    - » *and* self focusing in transverse direction
      - » increases plasma wave amplitude in positive feedback look
      - » produces very large amplitude waves

Propagation instabilities: SM-LWFA

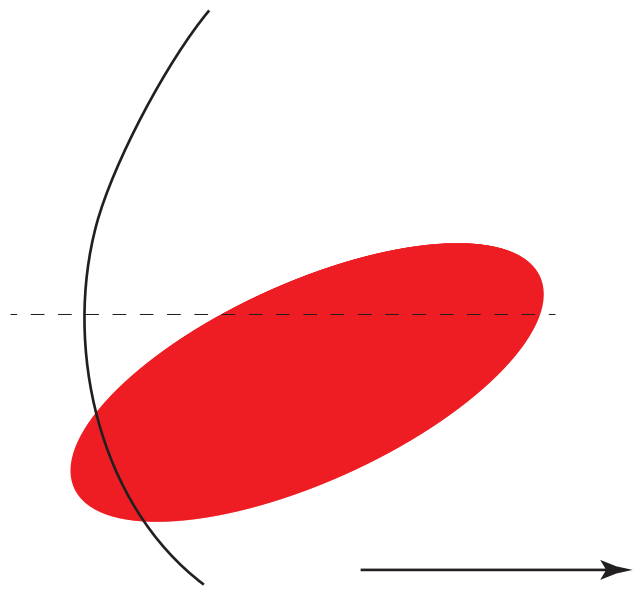




- Signature of SM-LWFA is appearance of peaks at $\omega = \omega_0 \pm n\omega_p$
- 1995: Modena et al, observed waves driven to breaking point for first time — self-injection of electrons into a wakefield accelerator

Modena Nature 1995

Propagation instabilities: Hosing



laser propagation direction

– Consider a laser pulse with spatio-temporal coupling issue

» back of pulse sits in plasma wave created by front of pulse

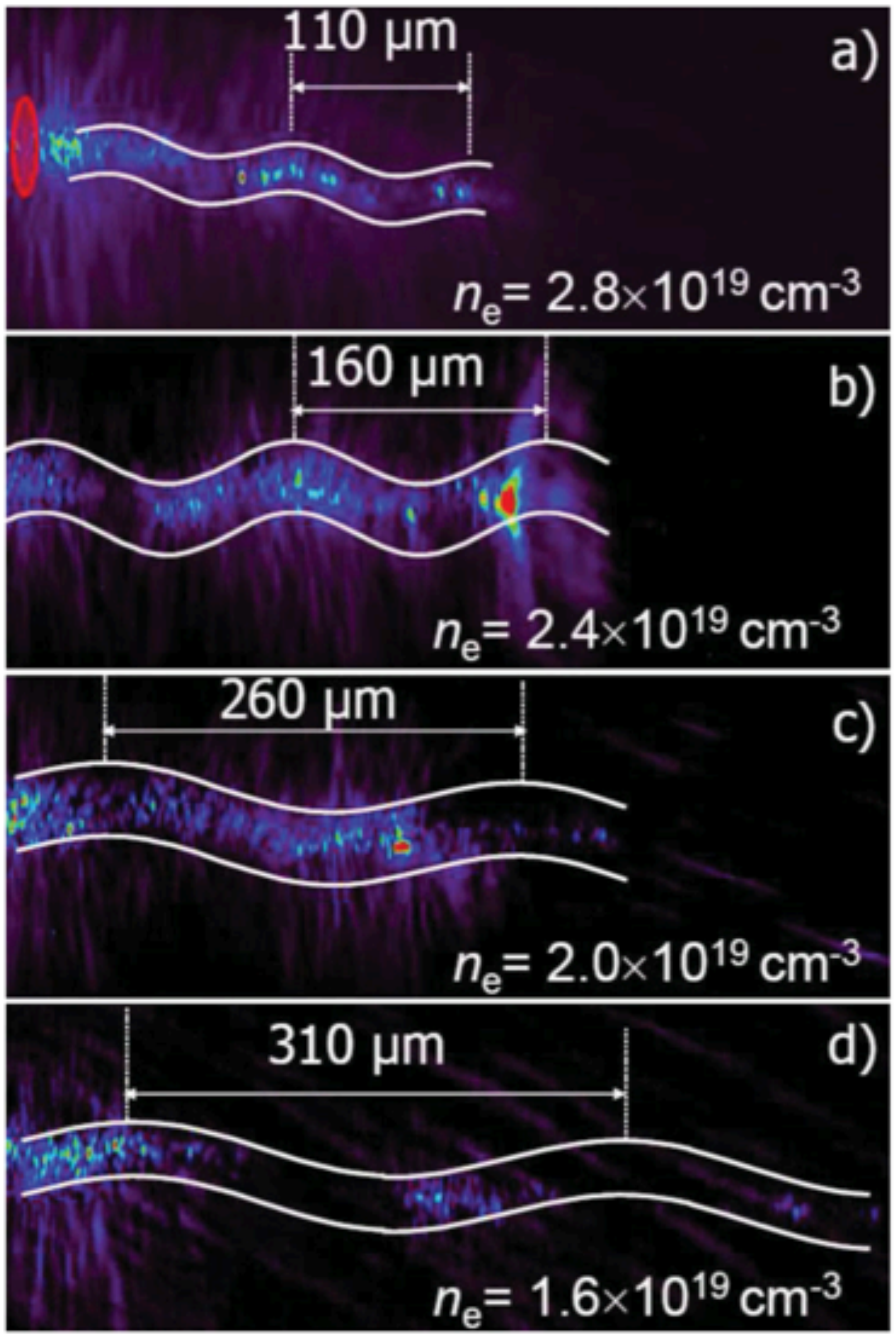
» but off-axis so feels focusing “force” due to transverse density gradient

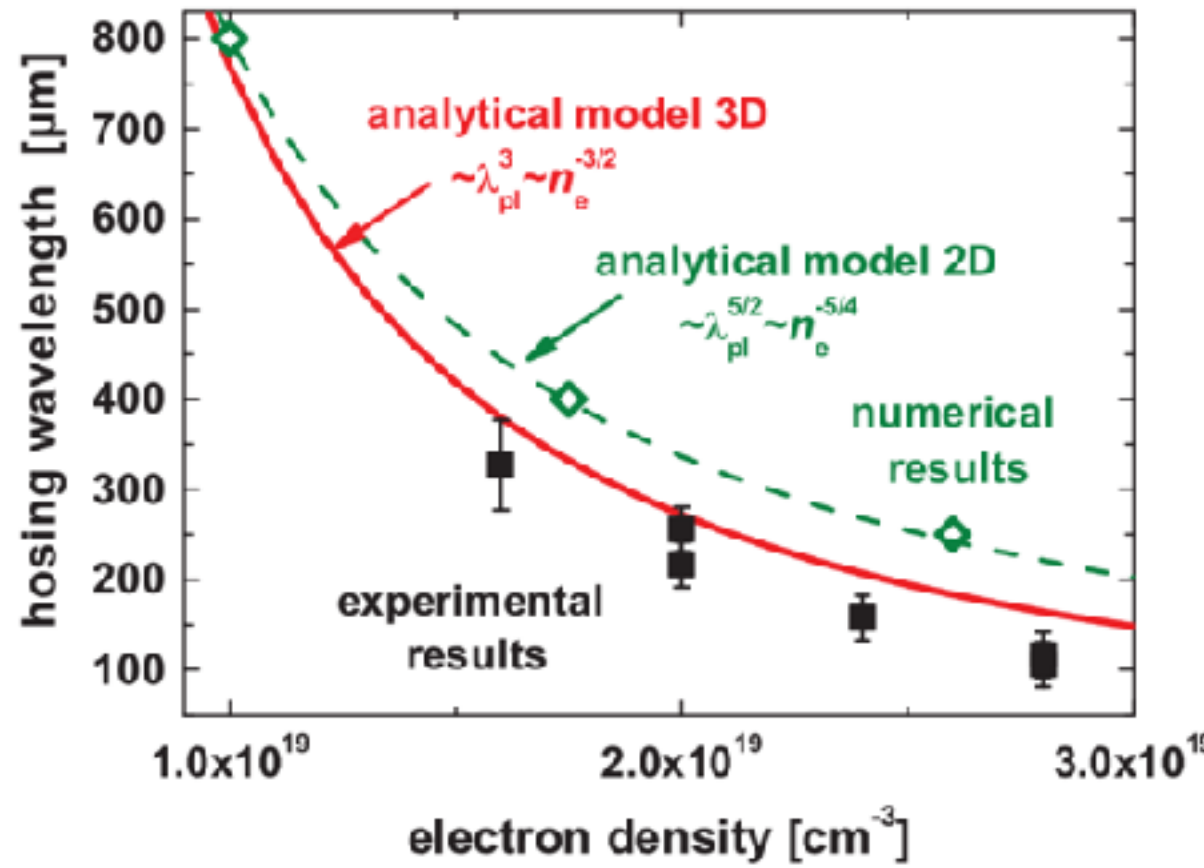
» back of pulse will overshoot and oscillate - laser “hoses” as it propagates.

$$\lambda_{\text{hosing}} \approx \frac{4\sqrt{2}\lambda_L^2}{a_0 w_0}\left(\frac{n_{\text{cr}}}{n_e}\right)^{3/2}$$

Kaluza PRL 2010

Propagation instabilities: Hosing

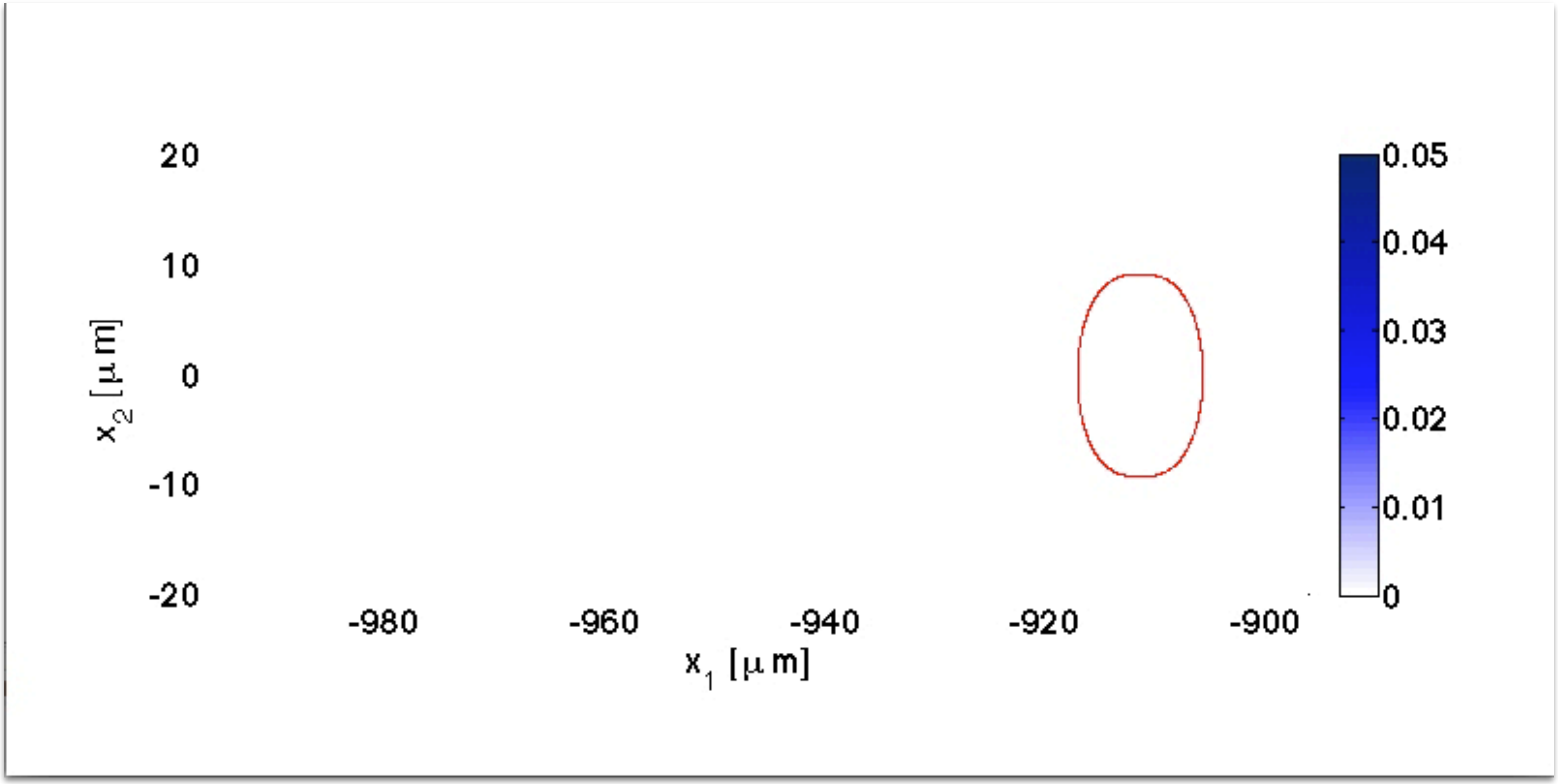




- Hosing has been observed by imaging the side-scatter/ self emission
- Hosing wavelength matches the theory

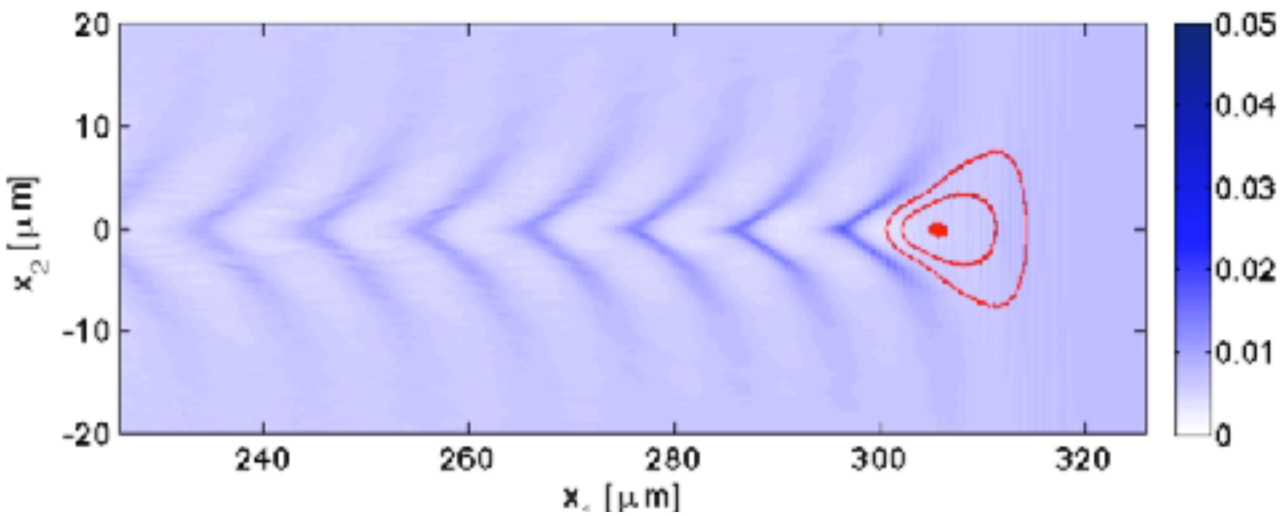
Kaluza PRL 2010

Evolution of plasma waves due to non-linear plasma optics

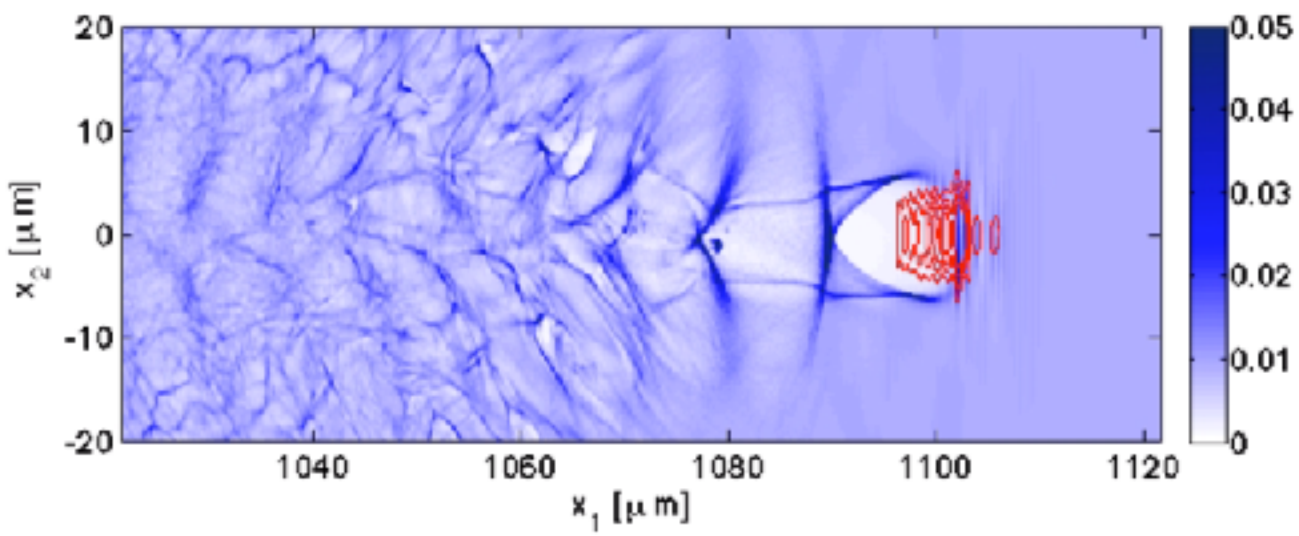


- Combined effects of non-linear plasma optics lead to co-evolution of laser pulse and plasma wave
- This is crucial for injection in many LWFAs

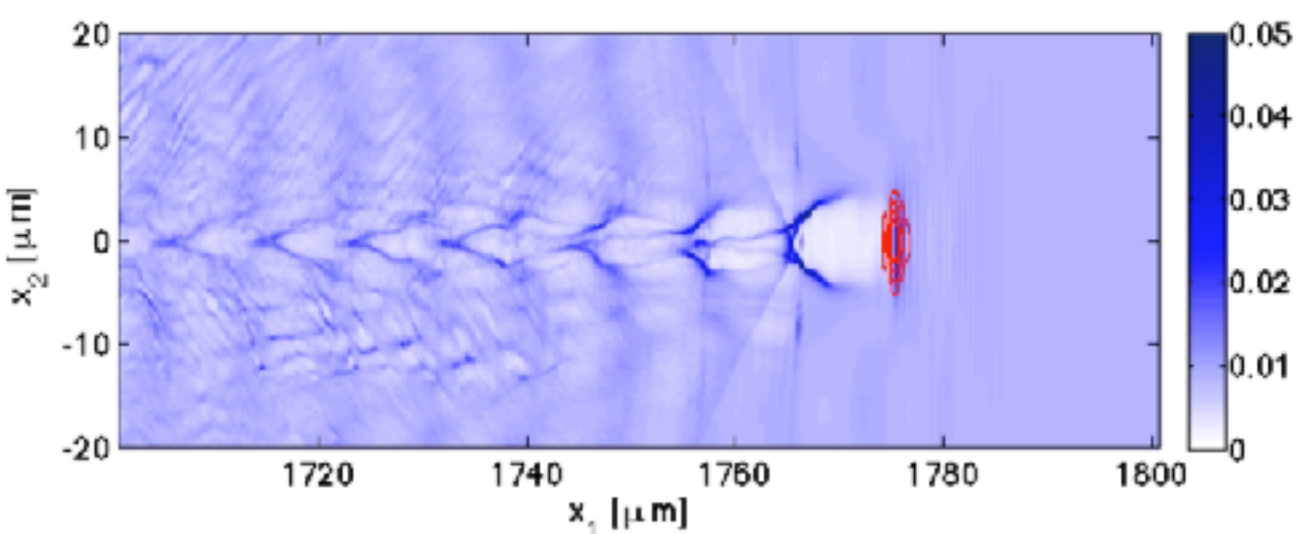
Evolution of plasma waves due to non-linear plasma optics



– ponderomotive + relativistic self-focusing at back of pulse



– pulse front etching/ compression



– power amplification leading to injection (see Streeter PRL 2018, Sävert PRL 2015)

Summary

- Introduced concepts needed to understand how lasers propagate inside a LWFA
  - self-focussing / guiding
  - pulse compression
  - photon “deceleration”
- Introduced propagation instabilities
  - self-modulation
  - hosing
- Discussed role of pulse evolution in self-injection

stuart.mangles@imperial.ac.uk @plasmaStu