Electromagnetism

Piotr Skowronski

Based on slides of

Andrea Latina https://indico.cern.ch/event/

The CERN Accelerator School

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Electric force

- Common life examples
  - Kid sliding on a plastic surface

Electric force

- Common life examples
  - Polystyrene on cat

Electric force

- The force that
  - repels the hairs
  - attracts polystyrene to cat's fur

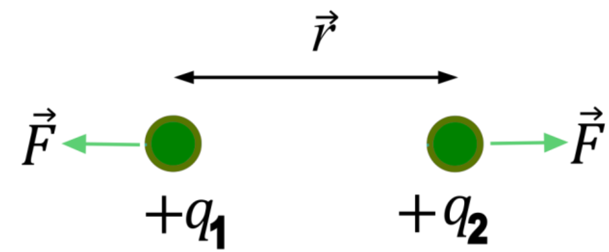
  is due to **electric charge**

- If electric charges are at rest then we call it

  electrostatic force

Coulomb law

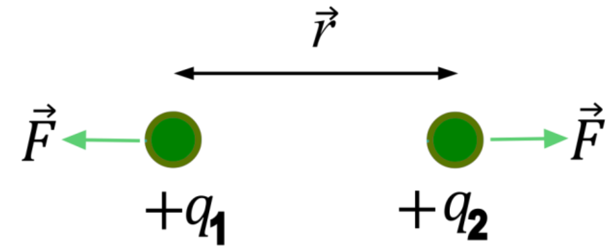
$$F_E = K \cdot \frac{\boldsymbol{q_1 \cdot q_2}}{r^2}$$



- Electrostatic force between point-like objects is
  - Proportional to electric charge of each of the two interacting objects

Coulomb law

$$F_E = K \cdot \frac{q_1 \cdot q_2}{\boldsymbol{r}^2}$$



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  - Proportional to electric charge of each of the two interacting objects
  - Inversely proportional to square of the distance

Coulomb law

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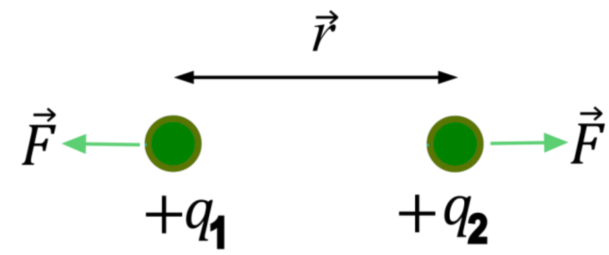

- Electrostatic force between point-like objects is
  - Proportional to electric charge of each of the two interacting objects
  - Inversely proportional to square of the distance
  - Proportional to Coulomb constant K
    - Which depends on medium type (vacuum, air, water)

Distance

$$F_E = \boldsymbol{K} \cdot \frac{q_1 \cdot q_2}{r^2}$$

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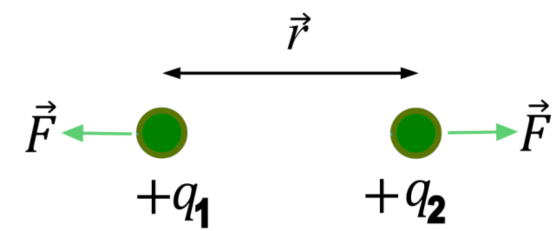
- Electrostatic force between point-like objects is
  - Inversely proportional to square of the distance
  - If we increase the distance 2x then the force is 4x smaller
  - If we increase the distance 10x then the force is 100x smaller

Electric charge

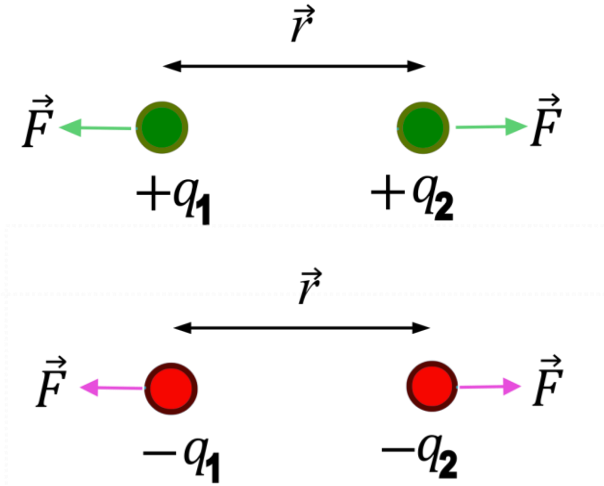
$$F_E = K \cdot \frac{\boldsymbol{q_1 \cdot q_2}}{r^2}$$



- Electrostatic force between point-like objects is
  - Proportional to electric charge of each interacting objects
  - **It means that if one of the objects has 2x more charge then force is 2x stronger**

Electric charge

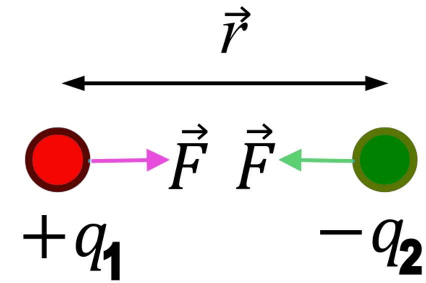
$$F_E = \boldsymbol{K} \cdot \frac{q_1 \cdot q_2}{r^2}$$



- Electric charge can be negative or positive
- If charge of both objects is the same then the force is repelling

Electric charge

$$F_E = K \cdot -\frac{\boldsymbol{q_1} \cdot \boldsymbol{q_2}}{r^2}$$



- Electric charge can be negative or positive
- If charge of both objects is opposite then the force is attracting

What is electric charge?

- It is a fundamental property of some elementary particles

- It has unit of Coulomb [C]

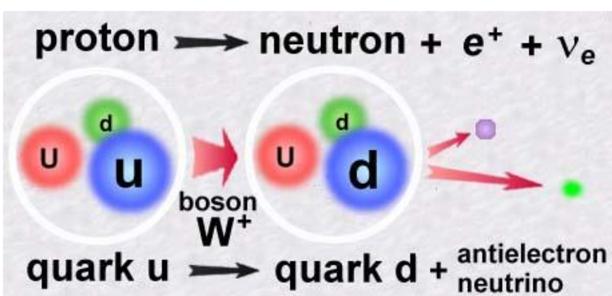
What is electric charge?

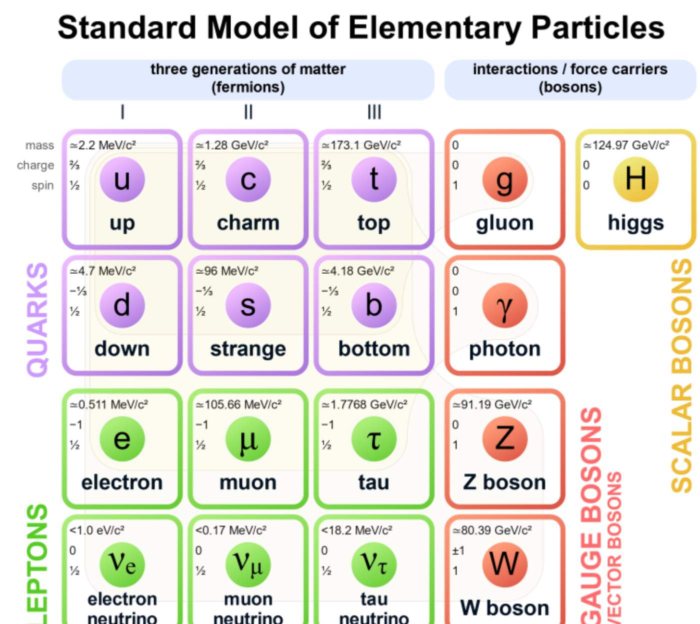
- Charge can be negative, for example for electron, or positive, for example for proton
- Electron charge, e = -1.602·10^{-19} C, is exactly opposite of proton charge
  - Why? Because proton can decay to positron, i.e. anti-electron, plus neutral stuff

$$ {}^{11}_{6}\mathrm{C} \rightarrow {}^{11}_{5}\mathrm{B} + e^{+} + \nu_{e} + 0.96\ \mathrm{MeV} $$

Electric charge

- All free particles have charge that is multiple of *e*
- Quarks have charge of 2/3 or -1/3 of *e*
  - But they are bound to exist only in triplets such that the total charge is 0, e, 2e
  - N.B. beta decay is in fact a decay of up quark to down quark

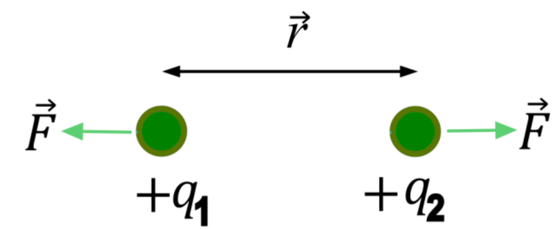




Electrostatic force

- The force acts in direction of the 2 objects

$$F_E = K \cdot \frac{q_1 \cdot q_2}{\boldsymbol{r}^2}$$

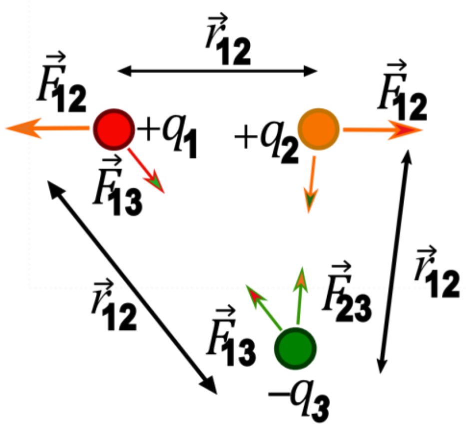


- In vector notation the equation is written

$$\overrightarrow{F_E} = K \cdot \frac{q_1 \cdot q_2 \cdot \vec{r}}{\|\boldsymbol{r}\|^3}$$

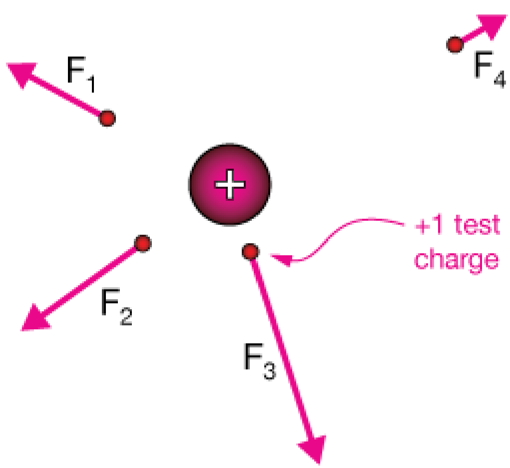
Multibody interaction

- If there is more than 2 charges interacting then we can calculate force of each pair and add the resulting forces as vectors: principle of superposition



Electric field

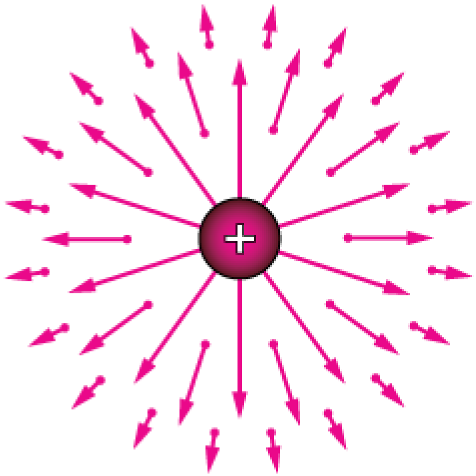
- It is much easier to do multibody calculations if we introduce the **electric field**
- To every point in space we assign a vector
- Its length corresponds to the force that the charged object would exert on a 1 Coulomb point like charge



Electric field of a positively charged sphere e.g., a proton

$$\vec{E} = K \cdot \frac{q}{\|\boldsymbol{r}\|^3} \vec{r}$$

$$E = K \cdot \frac{q}{r^2}$$



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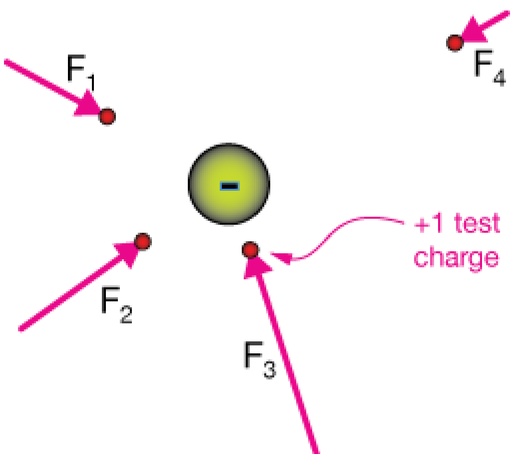
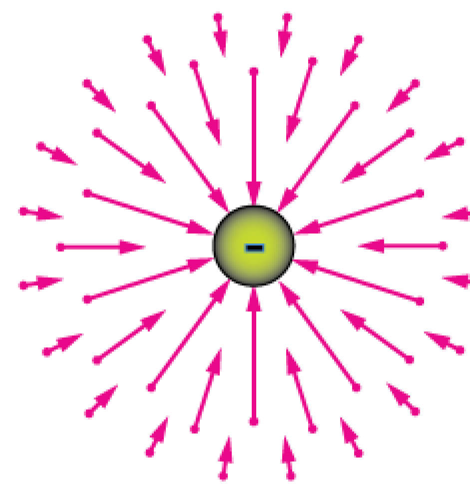


Electric field of a negatively charged sphere e.g., an electron

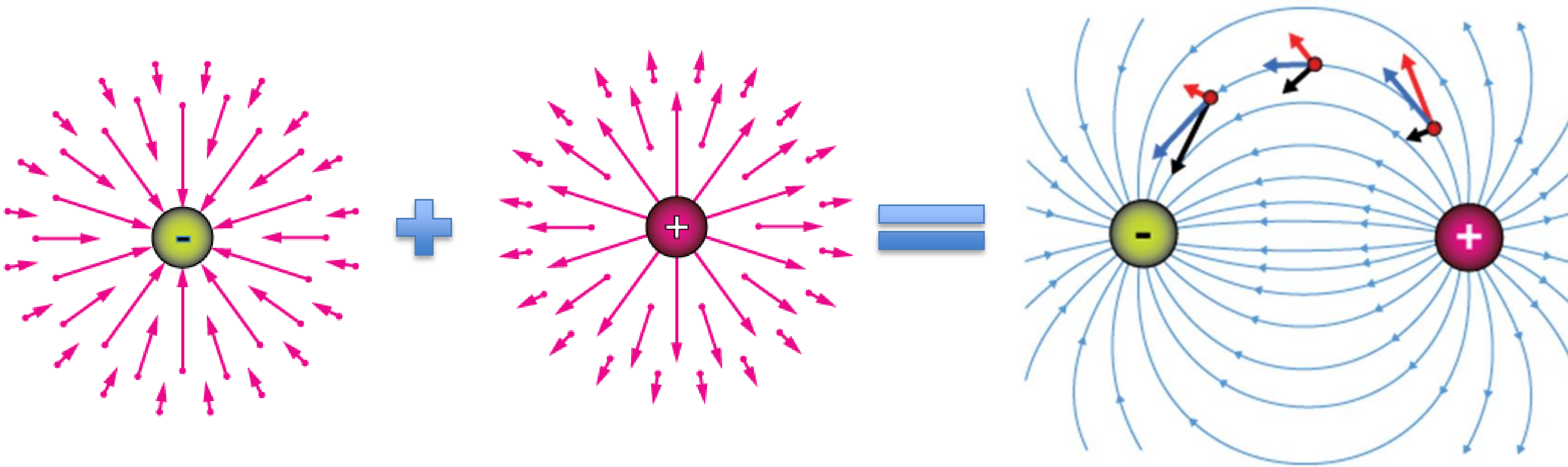
$$\vec{E} = -K \cdot \frac{q}{\|\boldsymbol{r}\|^3}\vec{r}$$

$$E = -K \cdot \frac{q}{r^2}$$



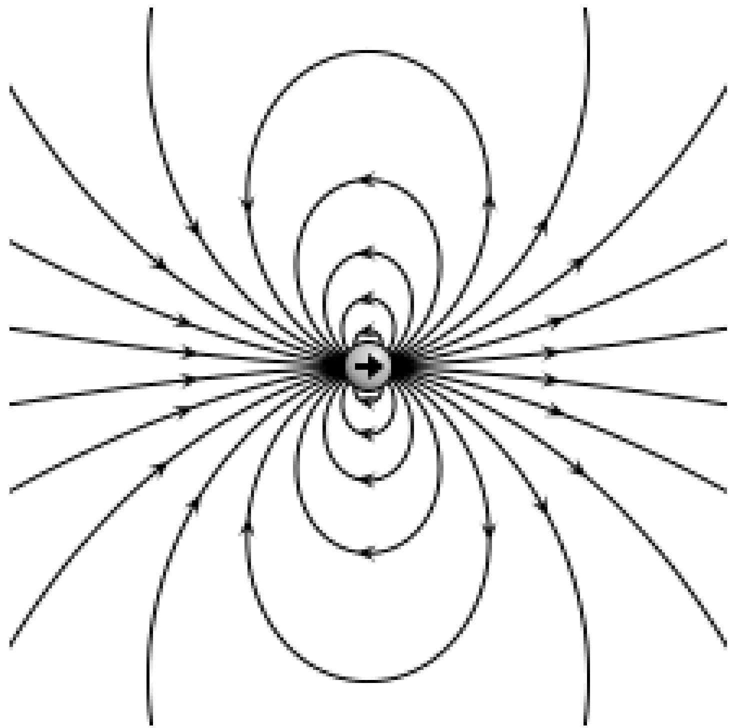
Superposition of electric fields

- The fields can be simply added
- Having electric field $\vec{E}$ we can calculate force $\vec{F} = q\vec{E}$



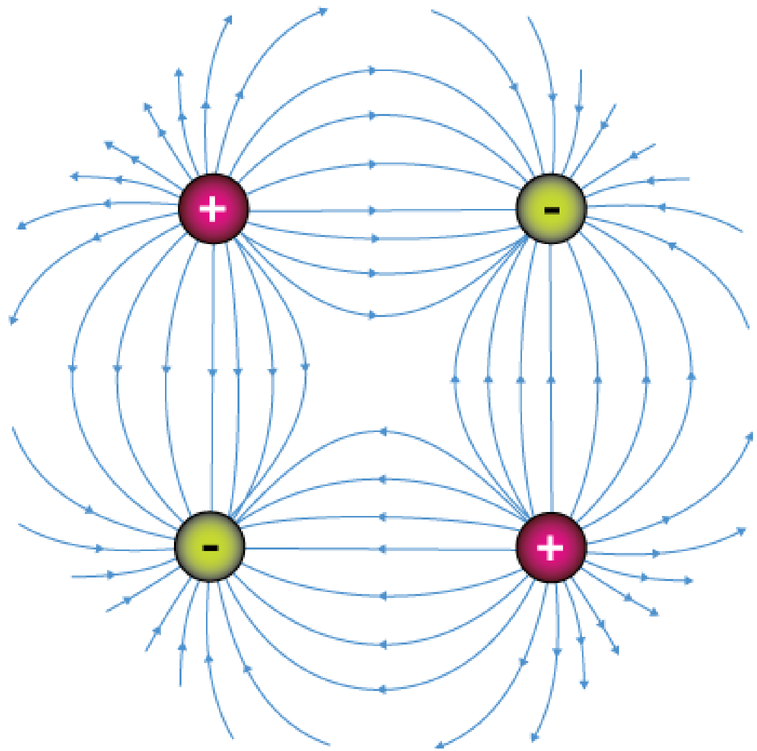
Electric fields

- Field of an electric dipole



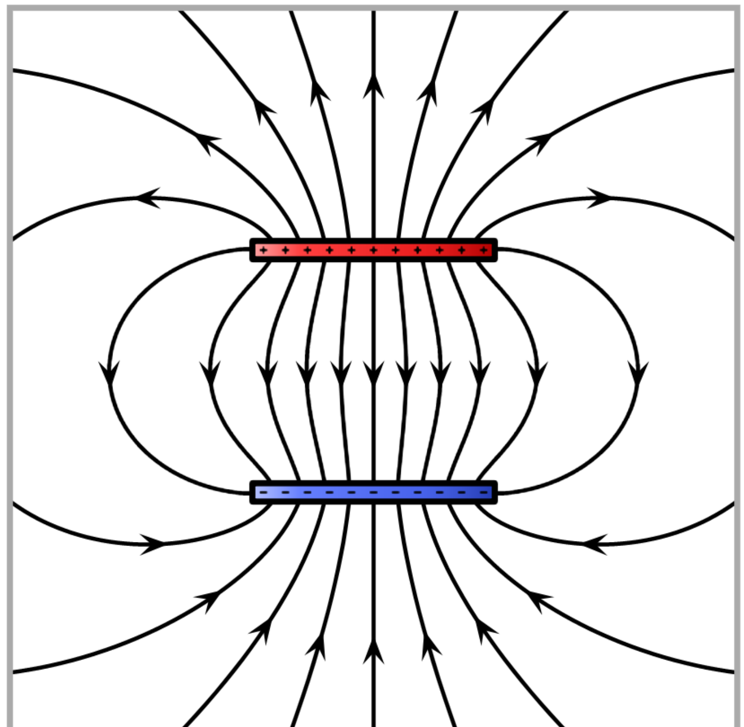
Electric fields

- Field of an electric quadrupole



Electric fields

- Field between charged plates



Electrostatic forces and object shapes

- The electric field distribution depends on the shape of the charged objects
- The same way the electrostatic force between arbitrary objects depends on their shape

Influence of medium

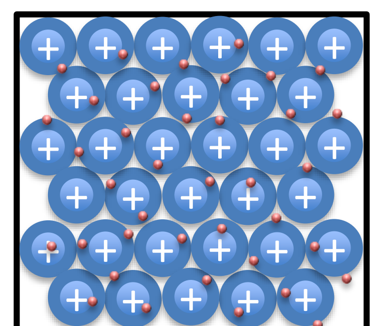
$$F_E = \boldsymbol{K} \cdot \frac{q_1 \cdot q_2}{r^2}$$

- The force changes depending on the medium that the objects are immersed in
  - Why? Electric charge stays the same …
  - Because the medium is made of charged particles

Influence of medium

$$F_E = \boldsymbol{K} \cdot \frac{q_1 \cdot q_2}{r^2}$$

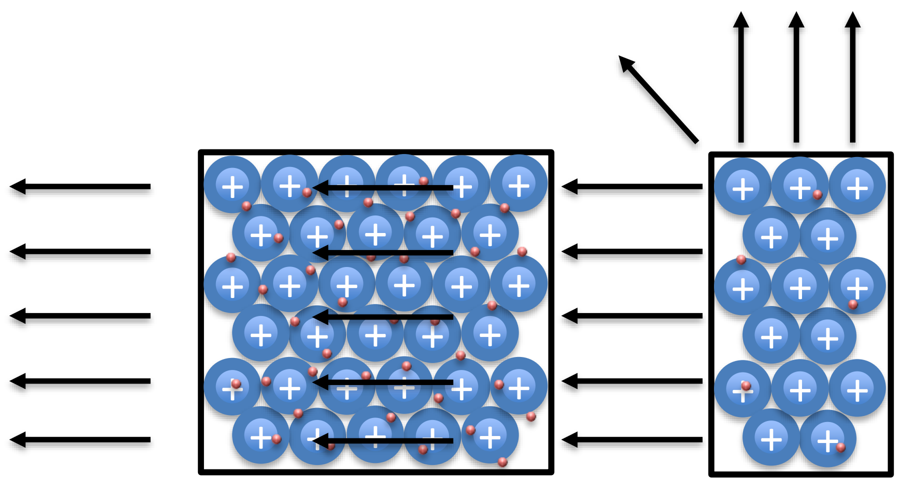
- In metals electrons can freely move within volume



Influence of medium

$$F_E = \boldsymbol{K} \cdot \frac{q_1 \cdot q_2}{r^2}$$

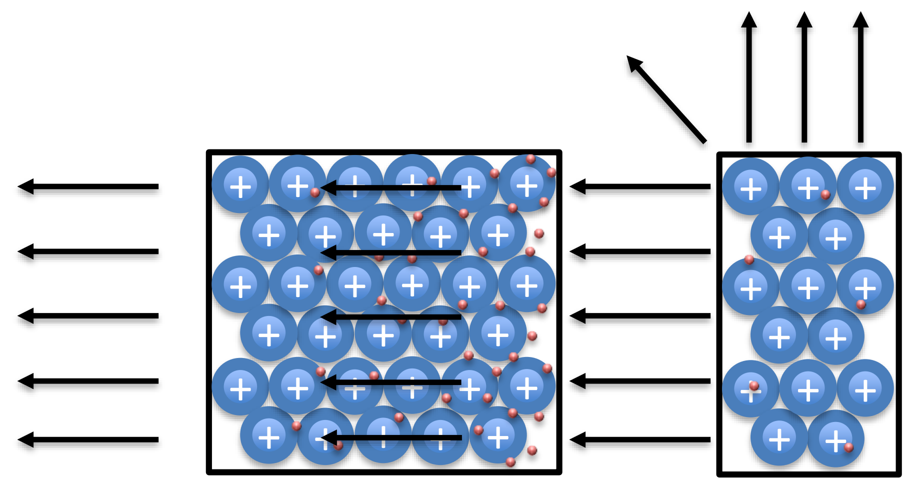
- In metals electrons can freely move within volume
- External charge exerts force on the electrons and protons



Influence of medium

$$F_E = K \cdot \frac{q_1 \cdot q_2}{r^2}$$

- In metals electrons can freely move within volume
- External charge exerts force on the electrons and protons
  - Electrons are attracted towards a positive charge and are repelled from a negative one
- Their displacement creates uneven distribution within the volume



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- In metals electrons can freely move within volume
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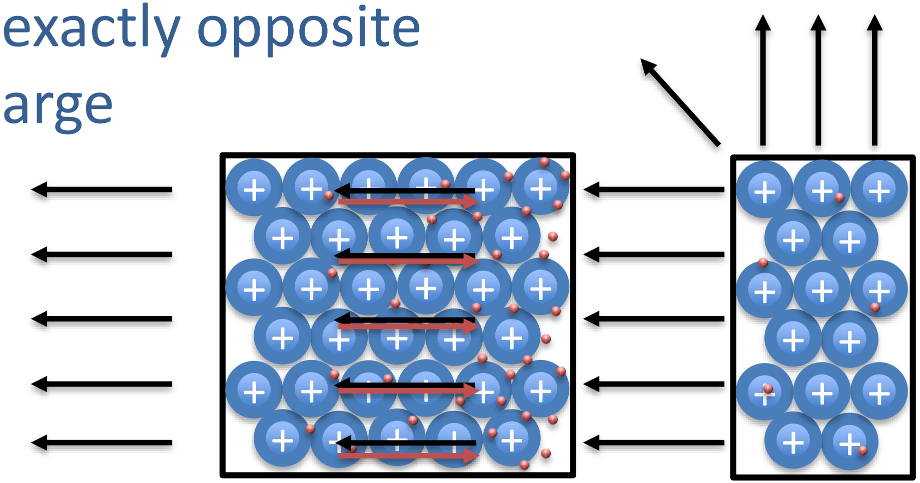
  - Their displacement creates uneven distribution within the volume
  - The resulting electric field is exactly opposite to the one of the external charge



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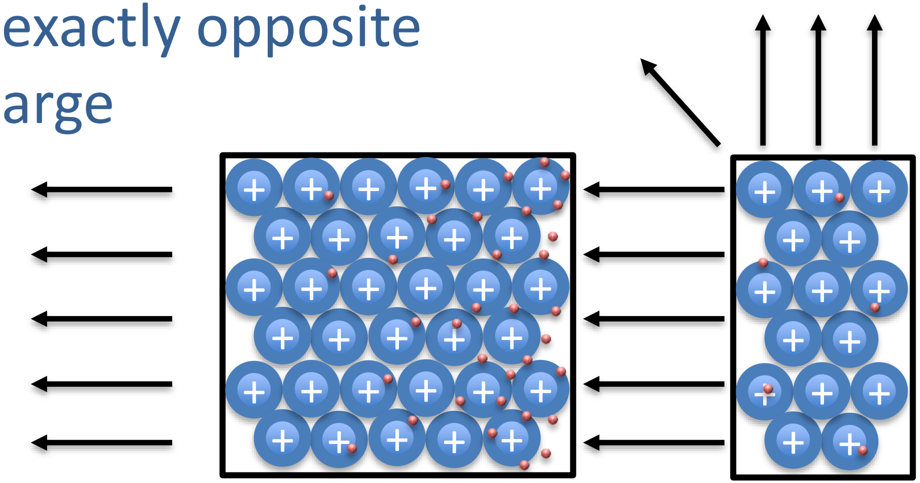
  - Electrons are attracted towards positive charge and are repelled from a negative one
  - Their displacement creates uneven distribution within the volume
  - The resulting electric field is exactly opposite to the one of the external charge
  - The electron motion continues until there is no electric field in the volume

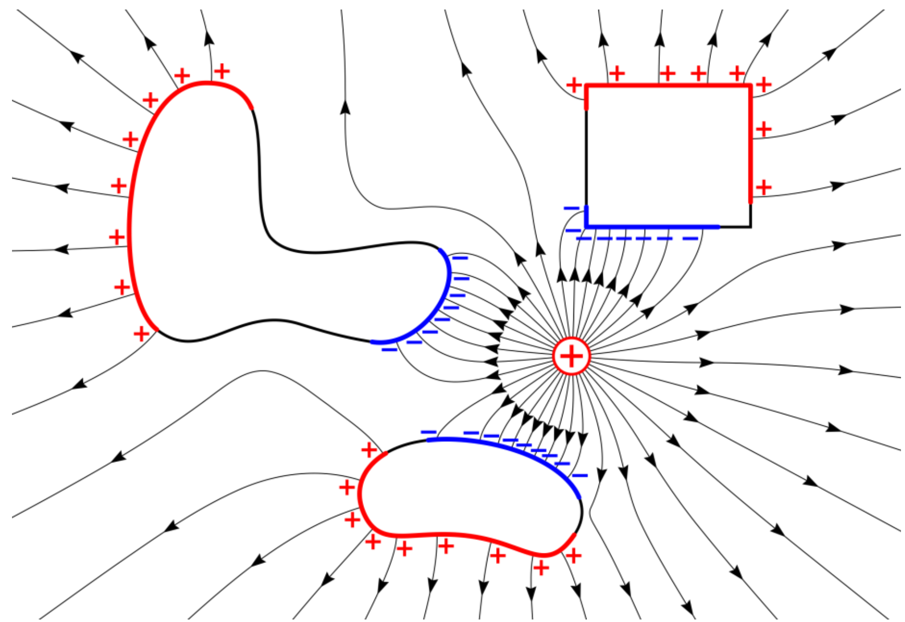


Influence of medium

$$F_E = \boldsymbol{K} \cdot \frac{q_1 \cdot q_2}{r^2}$$

- In metals electrons can freely move within volume

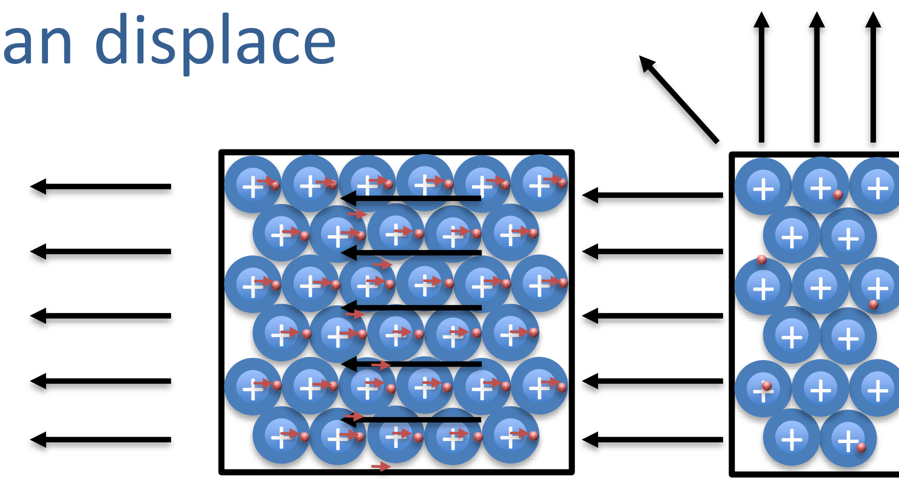
→ **Electric fields cannot penetrate metallic volumes**



Influence of medium

$$F_E = K \cdot \frac{q_1 \cdot q_2}{r^2}$$

- In non-metallic materials electrons cannot freely move
- Still upon external electric field electrons displace within their molecules and the material becomes polarized
- Induced electric field reduces the external field by the amount that depends on how much the electrons can displace



Influence of medium $F_E = \boldsymbol{K} \cdot \frac{q_1 \cdot q_2}{r^2}$

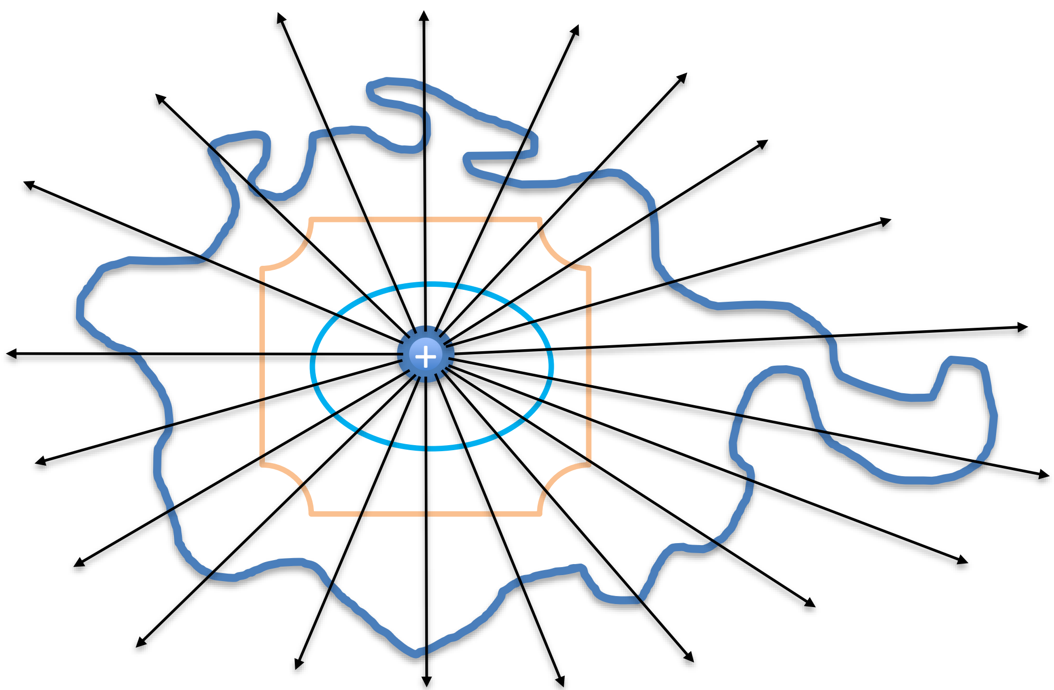
- Coulomb constant depends on the medium
- For vacuum $K = \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 \frac{C}{N \cdot m}$
  - where ε_0 is the vacuum permittivity
- For dielectrics $K = \frac{1}{4\pi\varepsilon}$, where ε is the material permittivity
- $\varepsilon = \varepsilon_r \varepsilon_0 = (1 + \chi)\varepsilon_0$, where
  - ε_r is the relative permittivity of the material
  - χ is susceptibility of the material

Influence of medium $F_E = \boldsymbol{K} \cdot \frac{q_1 \cdot q_2}{r^2}$

- The material permittivity $\varepsilon = \varepsilon_r \varepsilon_0 = (1 + \chi)\varepsilon_0$, where
  - ε_r is the relative permittivity of the material
  - χ is the susceptibility of the material
- Material permittivity in general depends on many factors
  - Temperature, pressure, if external electric field is time varying then on its frequency, …
  - One needs to take into account multiple phenomena to calculate correctly the electric field in dielectric
    - Sound waves, heat waves, ….

Gauss Law

Field flux out of an arbitrary closed surface
is proportional to the charge enclosed by the surface
irrespective of how that charge is distributed



$$\Phi_E = \frac{q}{\varepsilon_0}$$

or

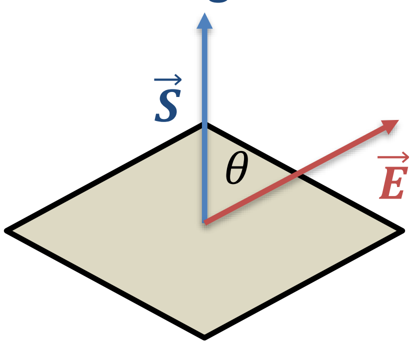
$$\oint\!\!\!\oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

Gauss Law

Field flux out of an arbitrary closed surface
is proportional to the charge enclosed by the surface
irrespective of how that charge is distributed

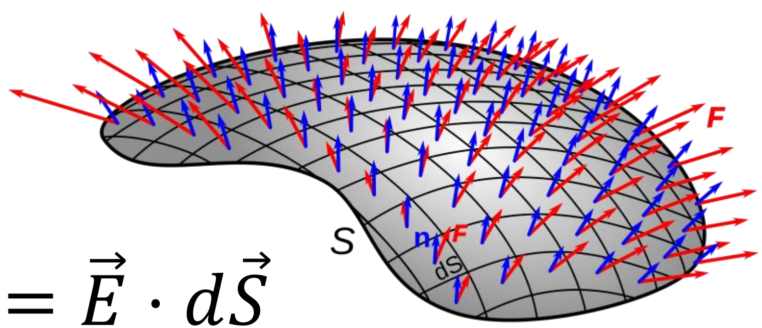
How much field $\vec{E}$ crosses area $\vec{S}$

S is a vector sticking out of the surface.
The vector length is the area of the surface



$$\Phi_E = \mathbf{E} \cdot \mathbf{S} = ES\cos\theta,$$

For and infinitesimally small area
we get a differential equation



$$d\Phi_E = \vec{E} \cdot d\vec{S}$$

$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{S}$$

Gauss Law

Field flux out of an arbitrary closed surface
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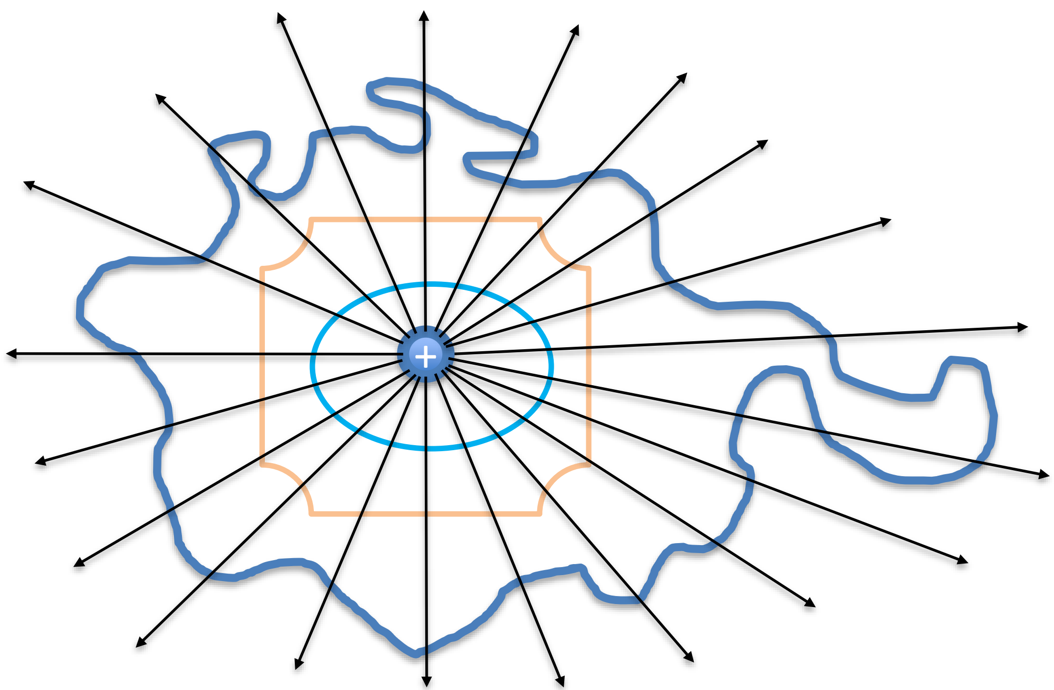

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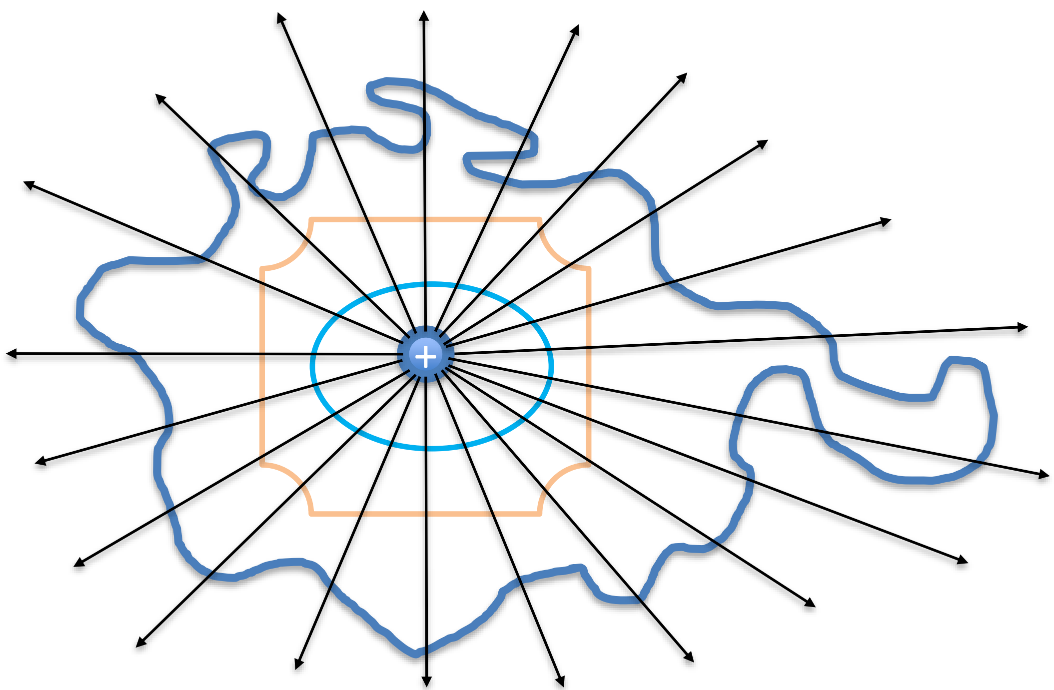
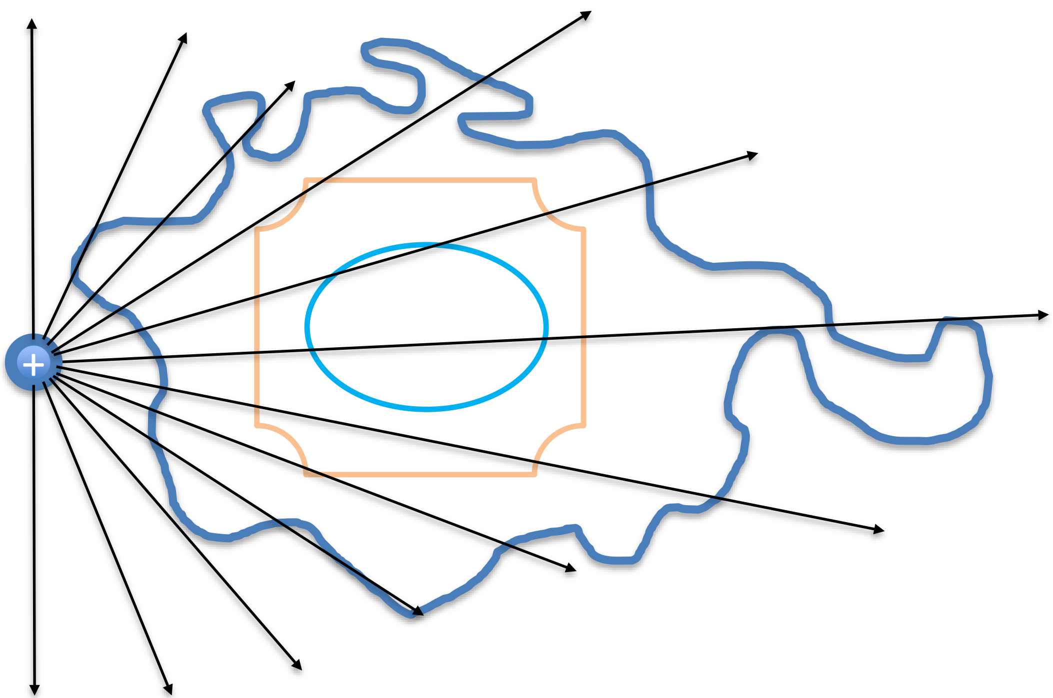


Only electric charges can create field lines

$$\oiint_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

Gauss Law

If there is no charge inside the volume, then the total flux is zero, because the same amount of field enters the volume as leaves it



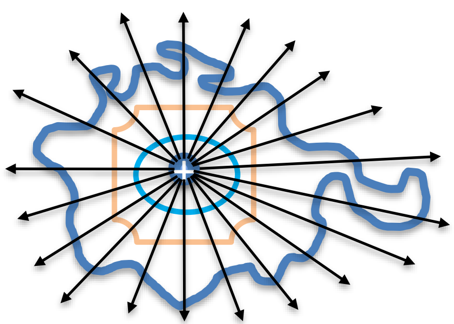
Gauss Law

- It has another form using divergence operator

$$\nabla \cdot \vec{E} = \frac{\partial \vec{E}_x}{\partial x} + \frac{\partial \vec{E}_y}{\partial y} + \frac{\partial \vec{E}_z}{\partial z} = \frac{\rho}{\varepsilon_0}$$

  - Where ρ is the volume charge density
- Divergence tells how much field is created at a given point
- Only electric charges can create electric field lines

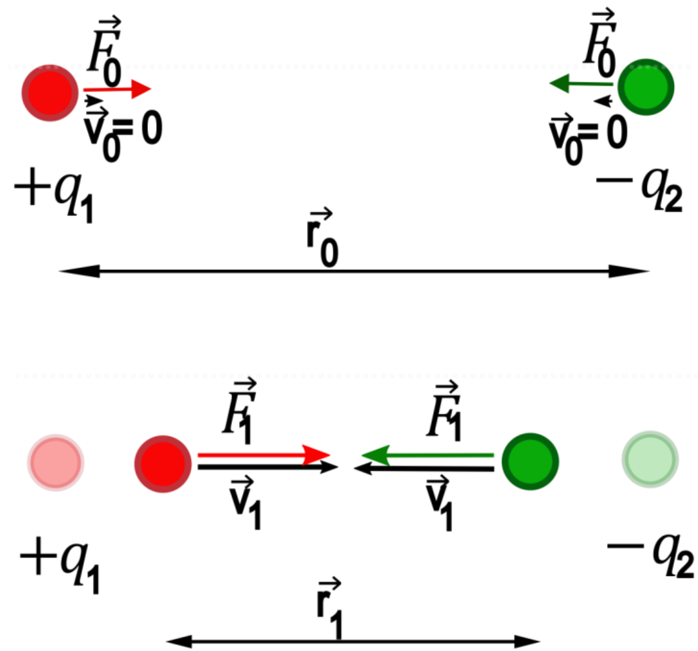
$$\oiint_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$



Field flux out of an arbitrary closed surface is proportional to the charge enclosed by the surface irrespective of how that charge is distributed

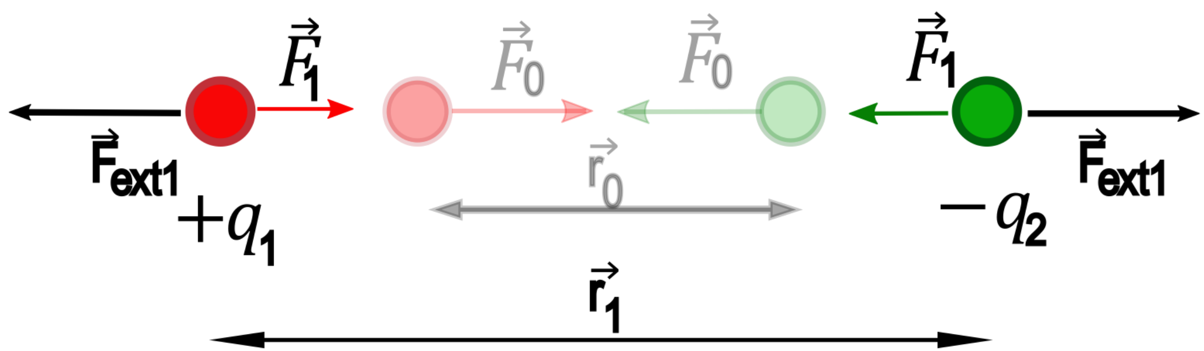
Electrostatic potential energy

- If we let the charges move upon electrostatic force, then they start accelerating → they gain kinetic energy



Electrostatic potential energy

- If we want to separate opposite sign charges, then we need to put work into it



Electrostatic potential energy

- Work needed to bring 2 point-like charges to a distance r

$$W = \int_{\infty}^{r} \vec{F} \cdot d\vec{r} = q_1 \int_{\infty}^{r} \vec{E} \cdot d\vec{r} = K q_1 q_2 \int_{\infty}^{r} \frac{dr}{r^2} = K q_1 \, q_2 \frac{1}{r}$$

Electrostatic potential energy

- If we let the charges move upon electrostatic force, then they start accelerating → they gain kinetic energy
- If we want to separate opposite sign charges, then we need to put work into it
- Electric field has potential energy
  - For example, potential energy of 2 point-like charges of 1 C brought together to a distance of 1 cm is

$$U_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 \cdot q_2}{r} = 9 \cdot 10^9 \cdot \frac{1 \cdot 1}{0.01} = 9 \cdot 10^{11}\, J$$

Electrostatic potential energy

- Potential energy of electric field
  - For example, potential energy of 2 point-like charges of +1 C brought together at distance of 1cm is

$$U_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 \cdot q_2}{r} = 9 \cdot 10^9 \cdot \frac{1 \cdot 1}{0.01} = 9 \cdot 10^7 \, J$$

  - If one of the charges has mass of 1 kg and we let it go, then all the potential energy will be converted to kinetic energy

$$U_E = E_K = \frac{mv^2}{2} \Rightarrow$$

$$v = \sqrt{2U_E/m} = 1'341.6 \, km/s = 4'829'907 \, km/h$$

  - 1 Coulomb it is a lot of charge!

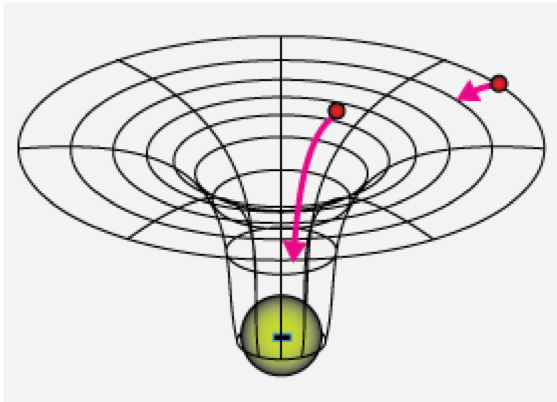
Electric potential

- Potential energy per 1 Coulomb is called **potential**

$$V = \frac{U_E}{q}$$

- It corresponds to the energy needed to bring 1 C charge from infinity to a given point
- Unit is called Volt [V]
- For point-like charges

$$V = K\frac{q}{r}$$



Electric potential

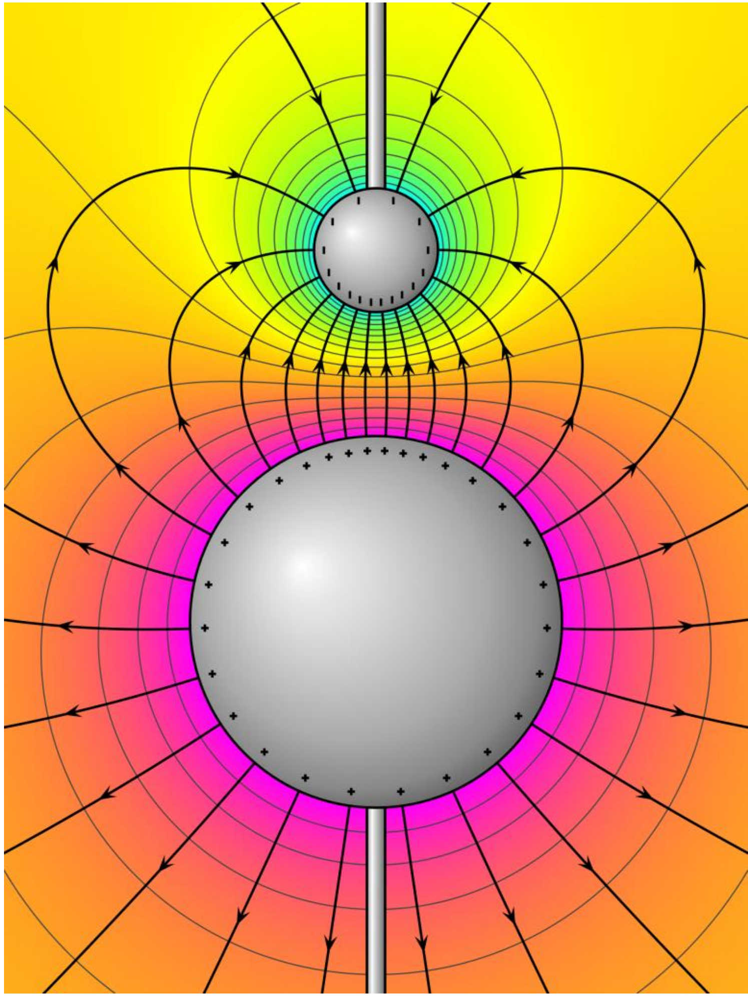
- Usually it is much easier to solve equations using potentials than using fields and forces
  - Potential is a scalar, single value at each point in space
  - Field is a vector, it has 3 values for each point in space, so normally 3 equations are needed
  - Field can be easily obtained from potential, namely, field is equal to gradient of potential:

    $$\vec{E} = \left(E_x, E_y, E_z\right) = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$

Field and potential

$$\vec{E} = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$

- Thin lines = equipotential lines
- Thick lines = electric field lines
- Electric field lines are always perpendicular to equipotential lines



Capacitance

- It’s the ratio between charge and produced voltage

$$C = \frac{q}{V}$$

- Unit is Farad [F]

Magnetic Force

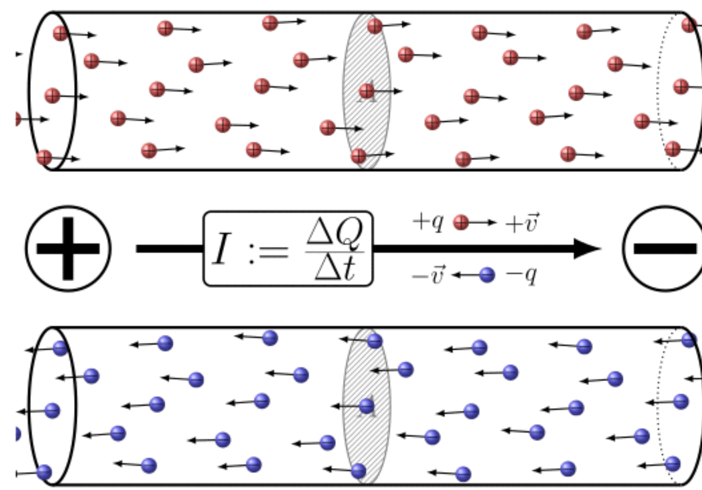
- Real life examples
  - Compass
  - Magnets
  - Attracted pair of wires

Electric current

- Magnetic force is due to moving electric charges
- Flow of charges is called electric current

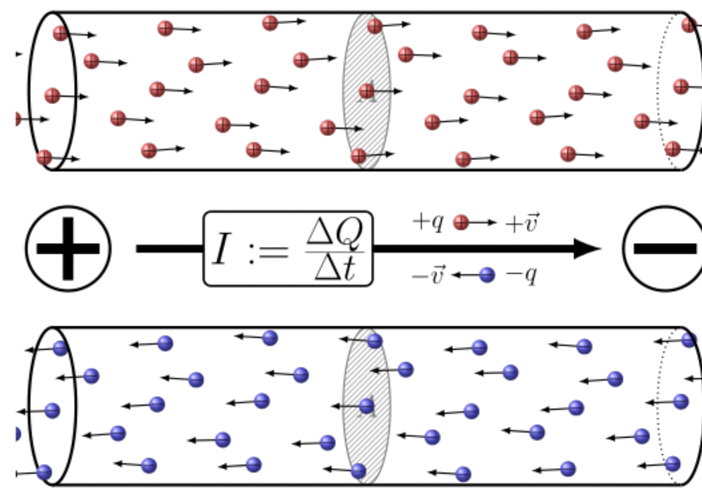
$$I = \frac{dq}{dt}$$

- It measures how much charge flows through a surface in a unit of time
- Unit is Ampere [A]



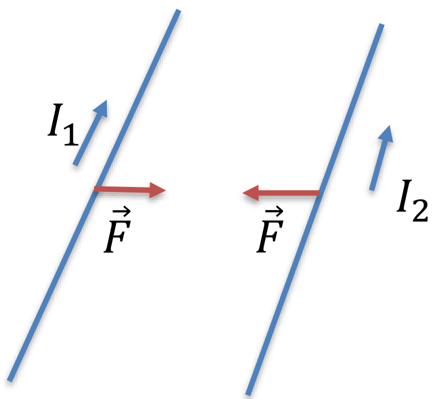
Electric current

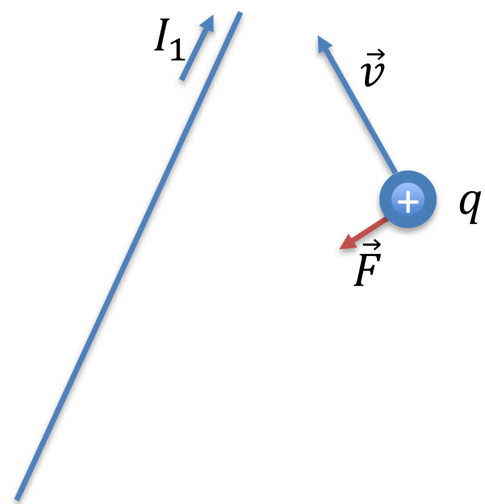
- Positive current when
  - Positive charge moves towards positive direction
  - Negative charge moves towards negative direction



Magnetic Force

- Magnetic force occurs only when both charges are moving

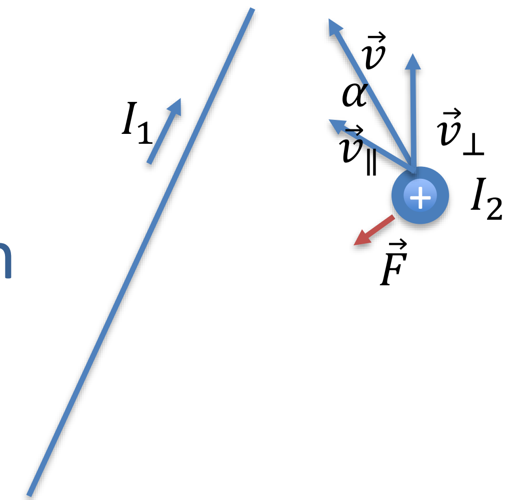




Magnetic Force charge and wire

- Only velocity component in plane with the wire and charge is important

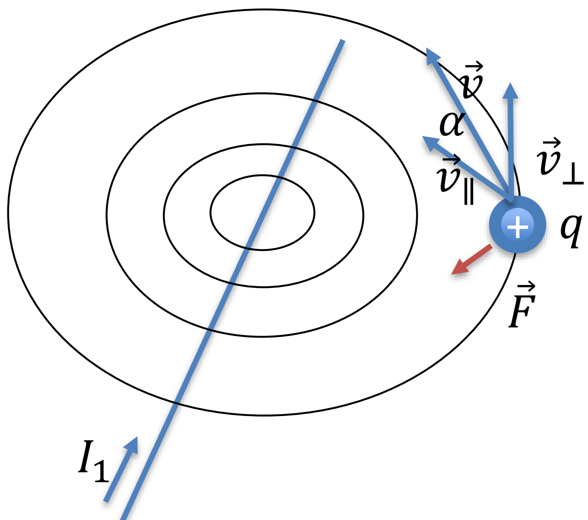
$$\vec{F} = \frac{\mu_0}{2\pi r} q \vec{v}_{\parallel} I_1$$



- If current and charge are positive then the force is always towards the wire
- As closer to the wire as stronger the force
- Is proportional to charge and current
- μ_0 is the magnetic vacuum permeability (physical const.)

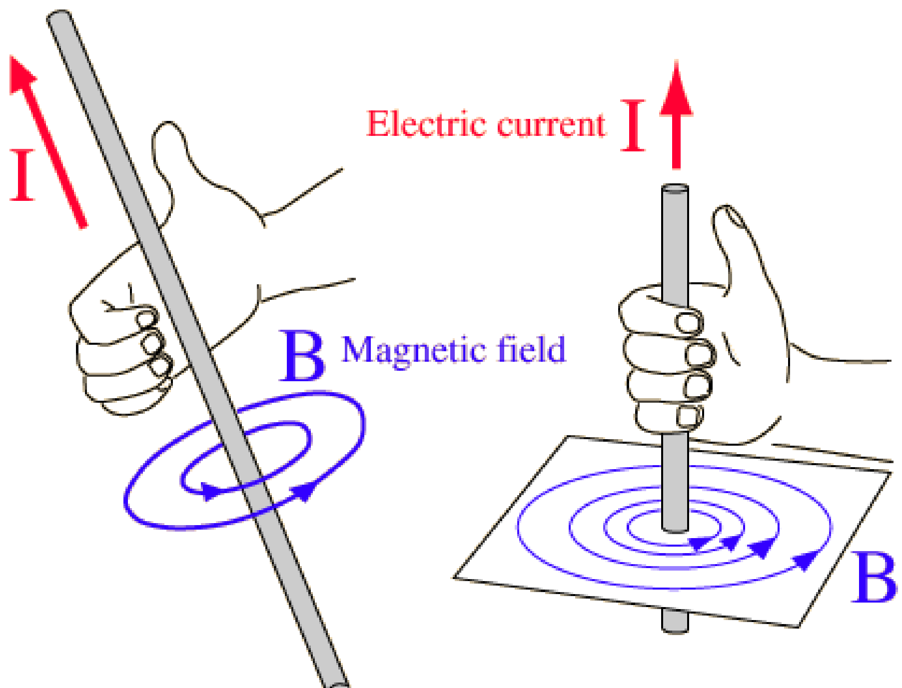
Magnetic Field

- The force is the same for the same r: $\vec{F} = \frac{\mu_0}{2\pi r} q \vec{v}_{\parallel} I_1$
- Field of magnetic force creates circles around the wire
- Strength of magnetic field from a wire is $B = \frac{\mu_0}{2\pi r} I_1$



Direction of magnetic field

- For positive current direction of magnetic field is determined with rule of right hand

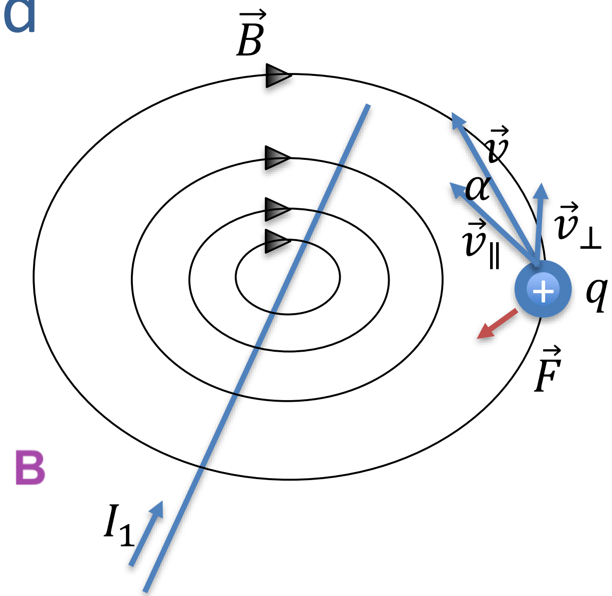


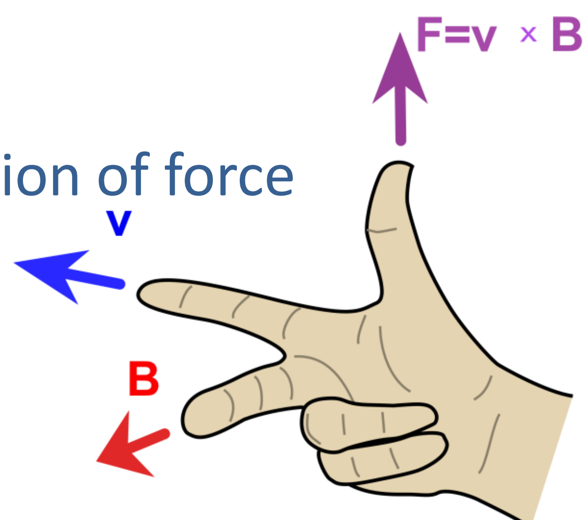
Magnetic Force

- Equation for force from magnetic field

$$\vec{F} = q \cdot \vec{v} \times \vec{B}$$

- Direction of force determined with rule of right hand
  - Index finger: v
  - Middle finger: B
  - Thumb gives direction of force



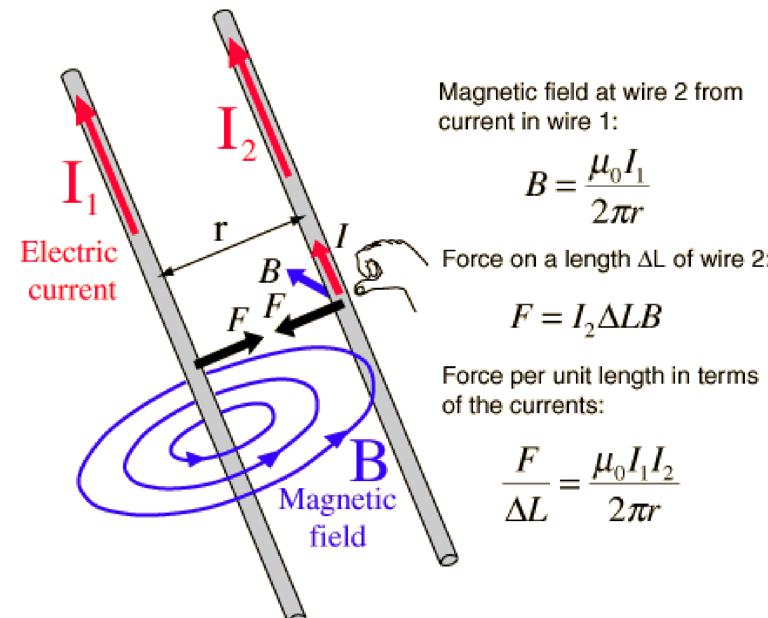


Magnetic Force

- Equation for force from magnetic field

$$\vec{F} = q \cdot \vec{v} \times \vec{B}$$

- Direction of force determined with rule of right hand
  - Index finger: I
  - Middle finger: B
  - Thumb gives direction of force

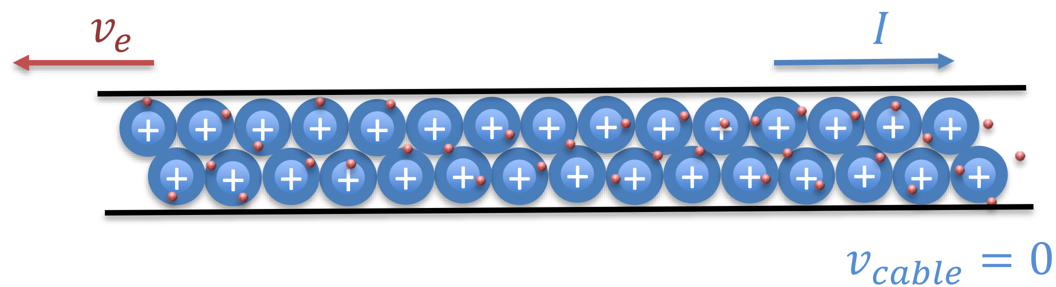


Electromagnetism

- Magnetic force is electrostatic force transformed by relativistic motion of charges

Electromagnetism

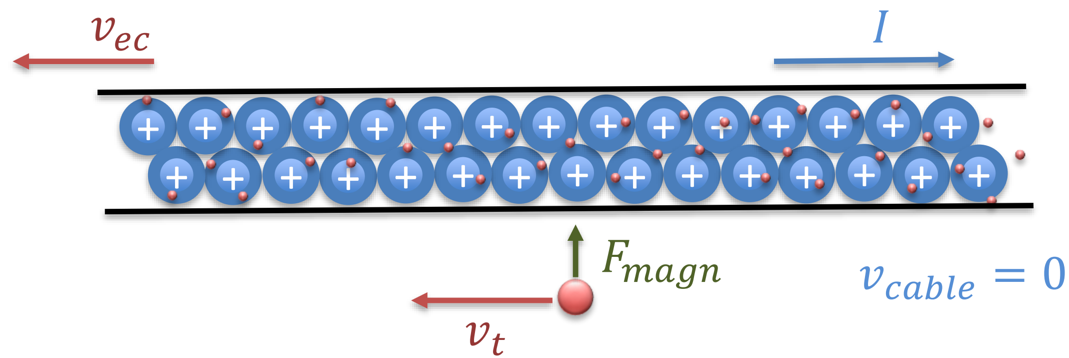
- Magnetic force is electrostatic force transformed by relativistic motion of charges



- Let's take a cable with flowing electric current
  - Positive current to the right
  - Electrons are flowing to the left
- The cable is electrically neutral
- Atoms of the metal are at rest

Electromagnetism

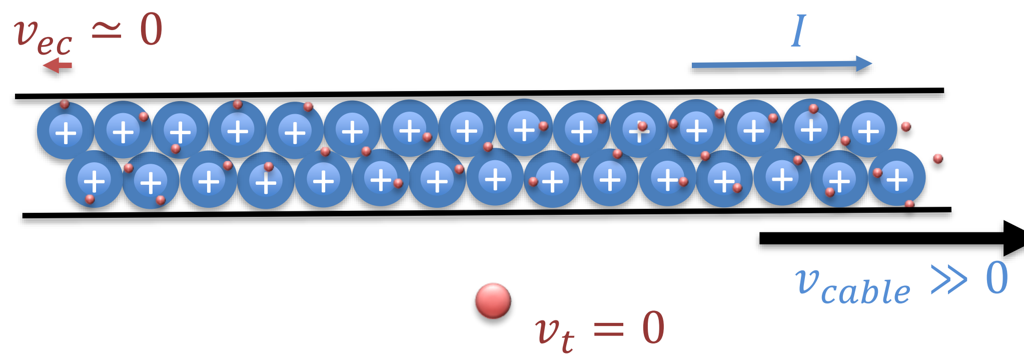
- Magnetic force is electrostatic force transformed by relativistic motion of charges



- A test negative charge moves along the cable to the left
- Magnetic force attracts the test charge to the cable

Electromagnetism

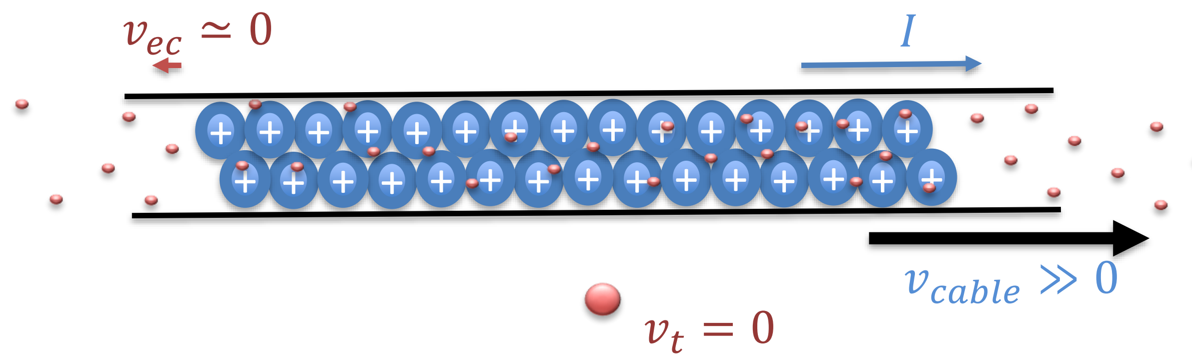
- Magnetic force is electrostatic force transformed by relativistic motion of charges



- Lets move to rest frame of the test charge
- Test charge now is at rest
- Electrons in the cable are almost at rest ($v_t \simeq v_{ec}$)
- Cable atoms move to the right with $-v_t$

Electromagnetism

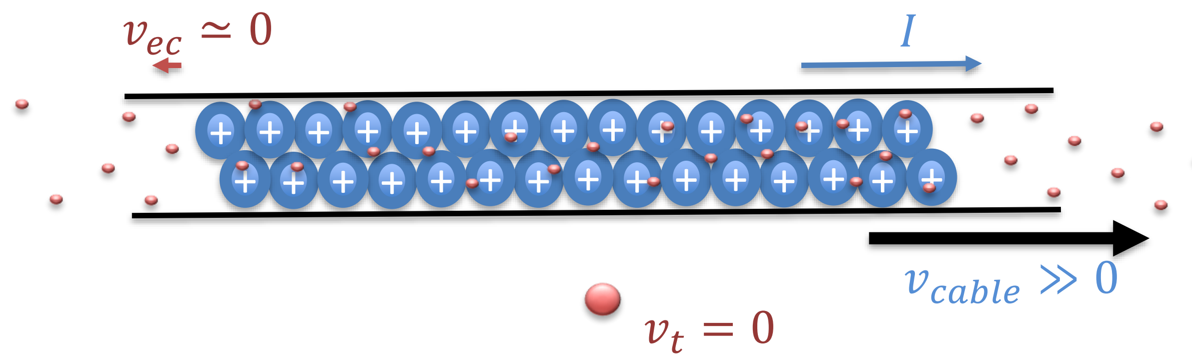
- Magnetic force is electrostatic force transformed by relativistic motion of charges



- Due to relativistic Lorentz contraction the cable gets shorter
- Distance between atoms (positive charges) gets smaller
- Density of positive charges gets larger

Electromagnetism

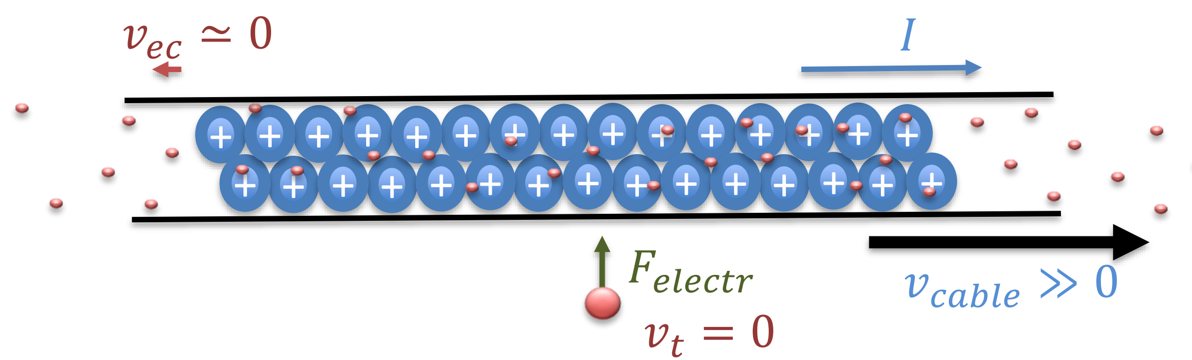
- Magnetic force is electrostatic force transformed by relativistic motion of charges



- Density of positive charges gets larger
- Density of electrons gets smaller
- Therefore, the cable is positively charged

Electromagnetism

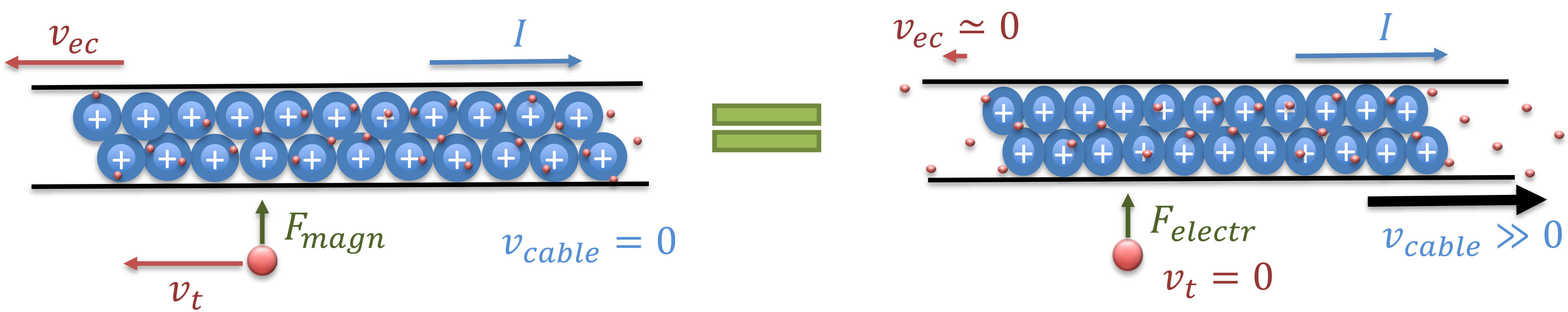
- Magnetic force is electrostatic force transformed by relativistic motion of charges



- Cable is positively charged
- Electrostatic force attracts the test charge to the cable

Electromagnetism

- Magnetic force is electrostatic force transformed by relativistic motion of charges



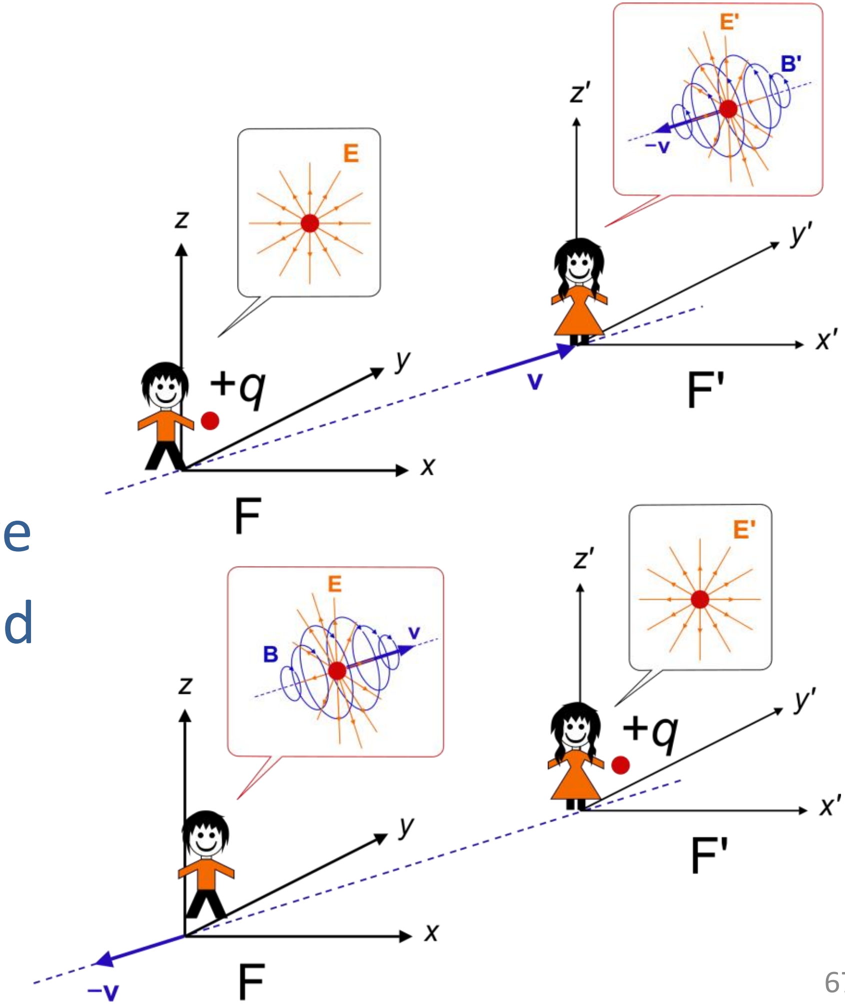
- Magnetic force when cable is at rest is the same as electric force when test charge is at rest
- With reference frame change one field changes to another one

Electromagnetism

- With reference frame change one field changes to another one

$$\mathbf{E}_{\|}{}' = \mathbf{E}_{\|}$$
$$\mathbf{B}_{\|}{}' = \mathbf{B}_{\|}$$
$$\mathbf{E}_{\perp}{}' = \gamma\left(\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B}\right)$$
$$\mathbf{B}_{\perp}{}' = \gamma\left(\mathbf{B}_{\perp} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}\right)$$

- Only transverse components change
- Electric field gets weaker with speed
- Magnetic field gets stronger
- The reason we use magnetic fields at high particle energies



Force strength comparison

- As an example, let's compare "easily achievable" electric and magnetic fields
  - Electric: 1 MV/m (10 MV voltage over 10 cm gap)
  - Magnetic: 1 T

$$\frac{F_{magn}}{F_{elec}} = \frac{qvB}{qE} = \frac{c\beta_{rel}B}{E} = \frac{c\beta_{rel}B}{E} = \beta_{rel}\frac{3\cdot10^{8}\cdot1}{10^{6}} = 300\cdot\beta_{rel}$$

- If β_{rel} is smaller than 1/300 the electric force is stronger
  - At CERN only behind the particle sources and in ELENA

Lorentz force

- The electromagnetic force is called Lorentz force

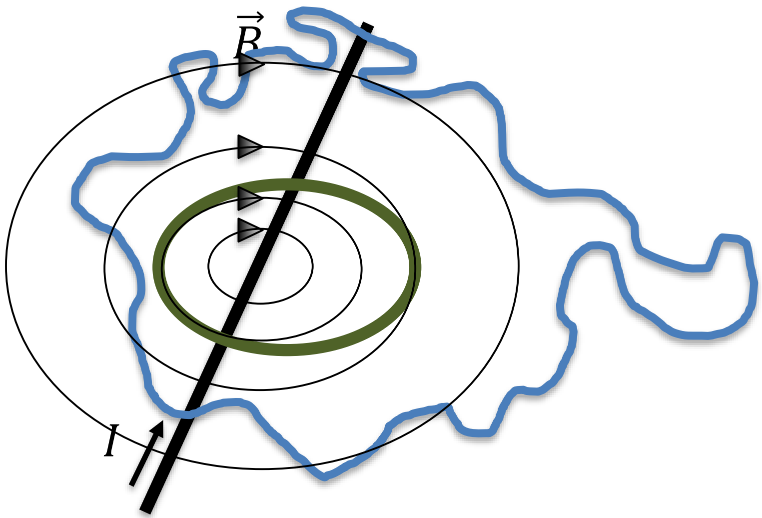
$$\vec{F} = q \cdot \left(\vec{E} + \vec{v} \times \vec{B}\right)$$

- It is the sum of forces due to electric and magnetic fields

Ampere's law

The field integrated around any closed loop is proportional to the current enclosed by the loop irrespective of how that current is distributed

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

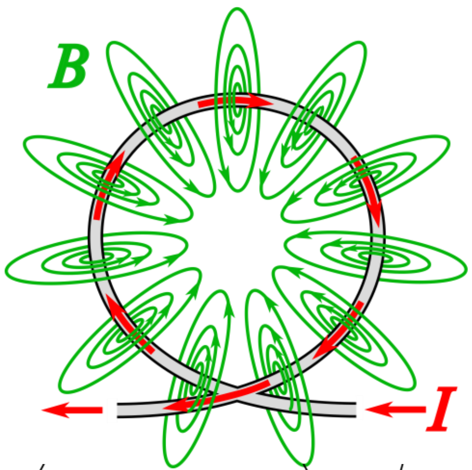


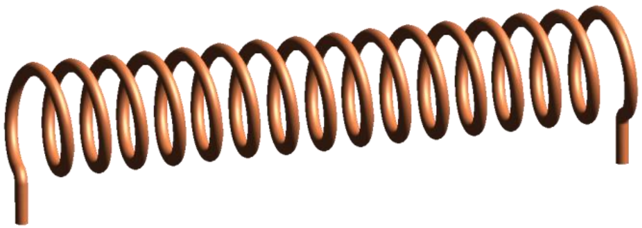
Magnetostatic case!

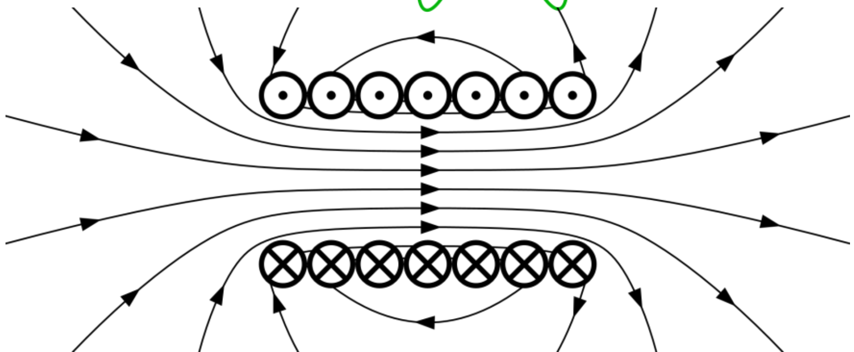
When fields are time-varying additional term needs to be added on R.H.S.

Magnetic coil

- Ampere's law allows to calculate magnetic fields from given distribution of electric currents
  - Or shapes of the wire
- For example, of a
  - Loop
  - Solenoid coil







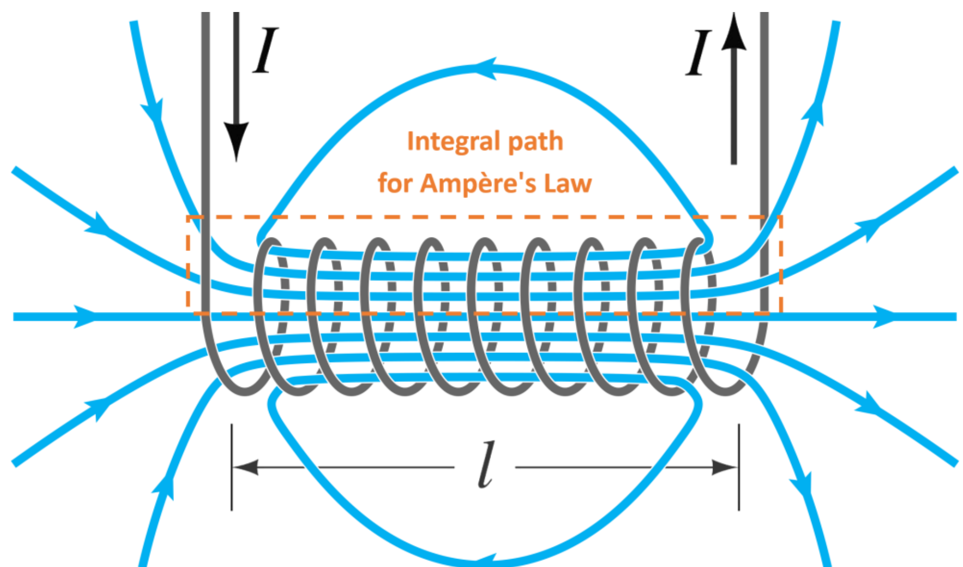
Solenoid coil

- Selecting contour like orange box, only the line inside solenoid counts and it has constant B field

$$Bl = \mu_0 NI,$$

$$B = \mu_0 \frac{NI}{l}.$$

N: number of windings



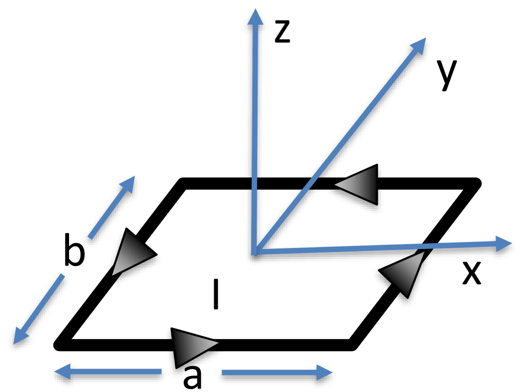
A loop with current

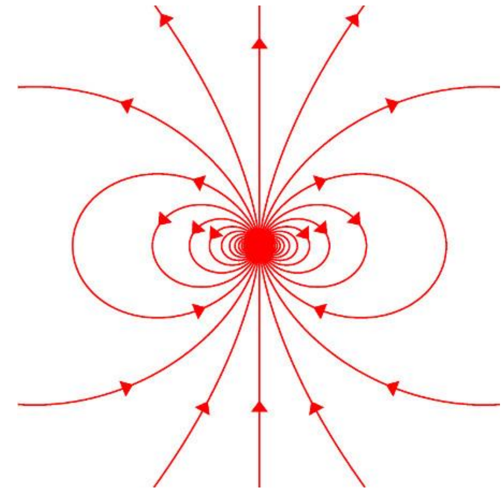
- In many problems it is conceptually useful to split a source of magnetic field into very small loops with current

$$B_x = \frac{Iab}{4\pi\varepsilon_0 c^2}\frac{3xz}{R^5}$$

$$B_y = \frac{Iab}{4\pi\varepsilon_0 c^2}\frac{3yz}{R^5}$$

$$B_x = \frac{Iab}{4\pi\varepsilon_0 c^2}\left(\frac{1}{R^3} - \frac{3z^2}{R^5}\right)$$

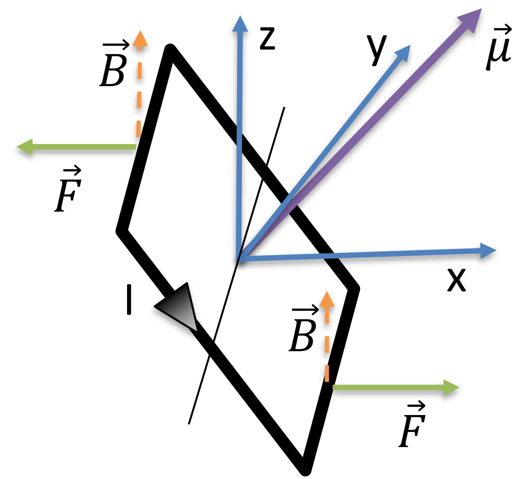




- Iab or IA is called magnetic dipole moment vector or simply magnetic moment

Forces acting on loop with current

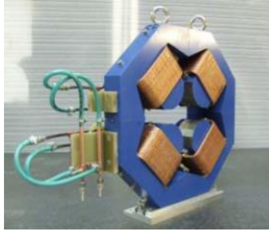
- If put a loop with current in uniform magnetic field the forces will rotate it such that μ is in direction of the field
- Torque $\vec{\tau} = \vec{\mu} \times \vec{B}$
- Energy $\mathrm{U} = -\vec{\mu} \cdot \vec{B}$



Magnetic moments of particles and atoms

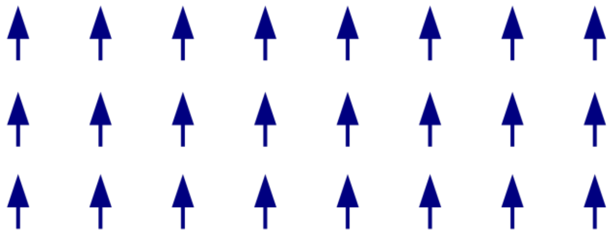
- Charged elementary particles have magnetic moments
- They act like very small loops with current
- One can think of it as charge rotating due to spin
- Usually magnetic moments are distributed randomly
- But some materials can have moments of their atoms aligned, they are magnets

Magnets

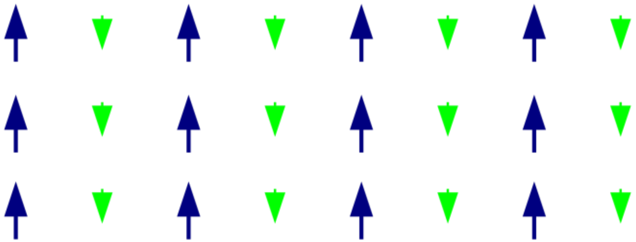


- Magnetic materials can have their moments aligned, they can be magnetized by external magnetic field
- Magnetization can stay forever: permanent magnets
- Or only when external field is present: electromagnets

Ferromagnetic

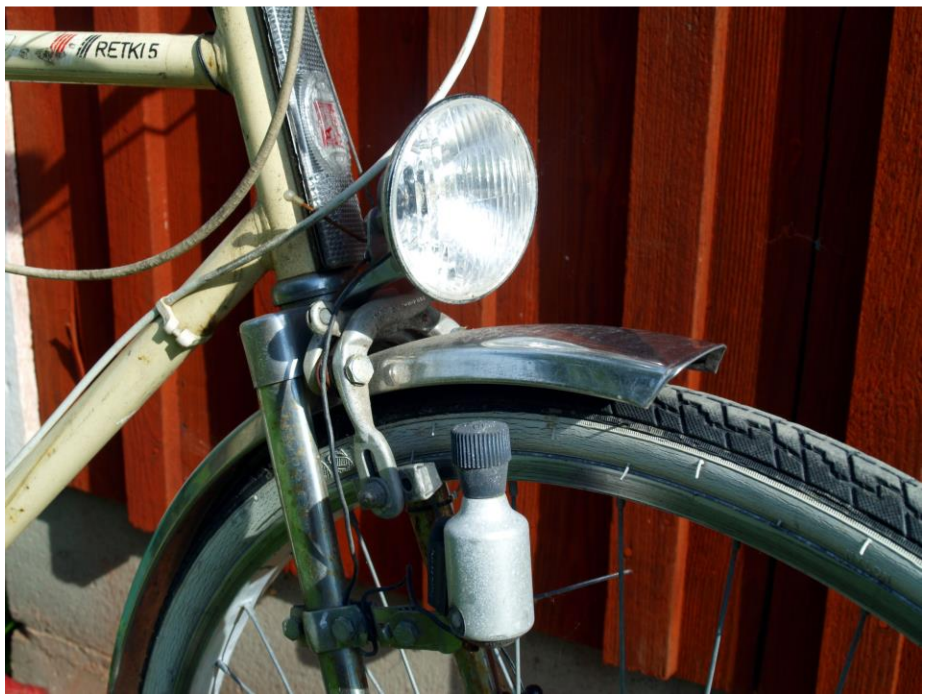


Ferrimagnetic



Magnetic Induction

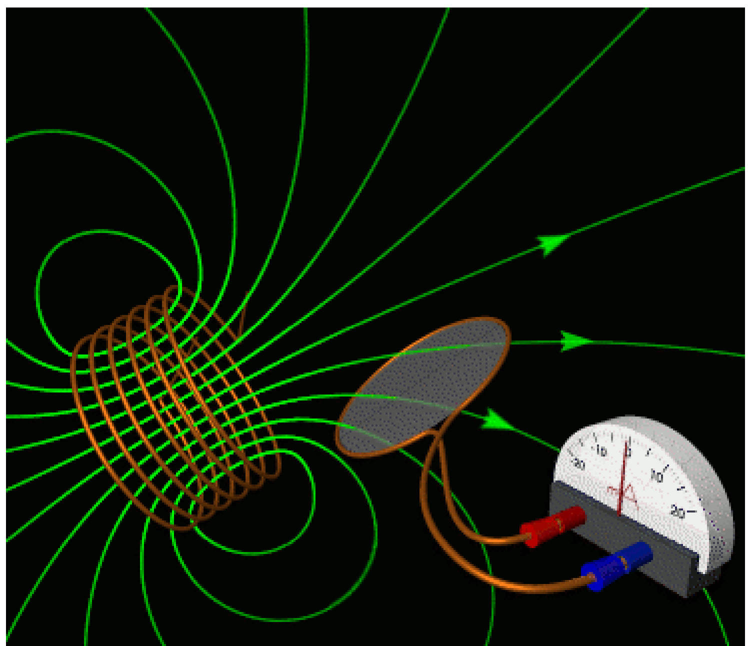
- Real life examples:
  - Electric generator
    - Voltage out of rotating magnet



Induction

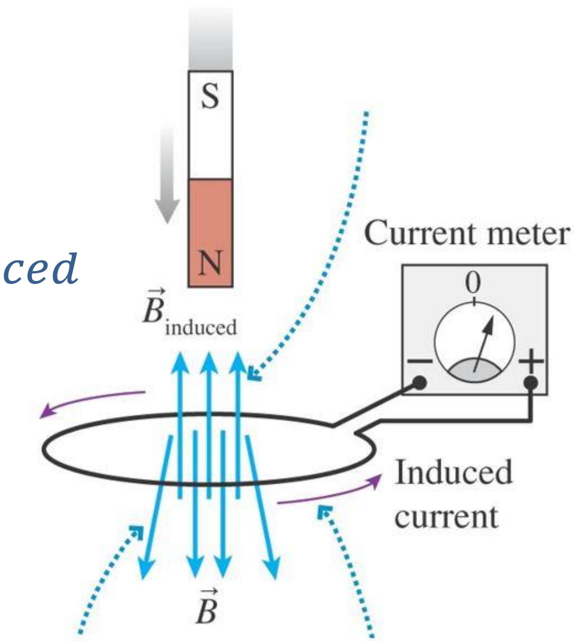
- **Change of magnetic field induces electric field**

- **Change of electric field induces magnetic field**

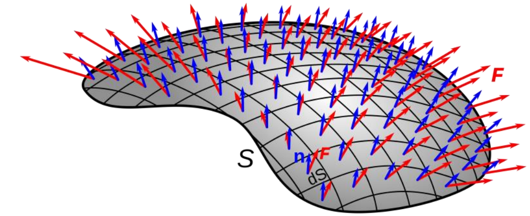


Faraday law

- Change of magnetic field induces electric field
  - When magnet is inserted into the loop then $\vec{B}$ is increasing and electric field is induced along the wire loop
  - Electric field pushes electrons in the wire and generates induced current
  - Induced current creates magnetic field $\vec{B}_{induced}$ such that it is against the external field $\vec{B}$



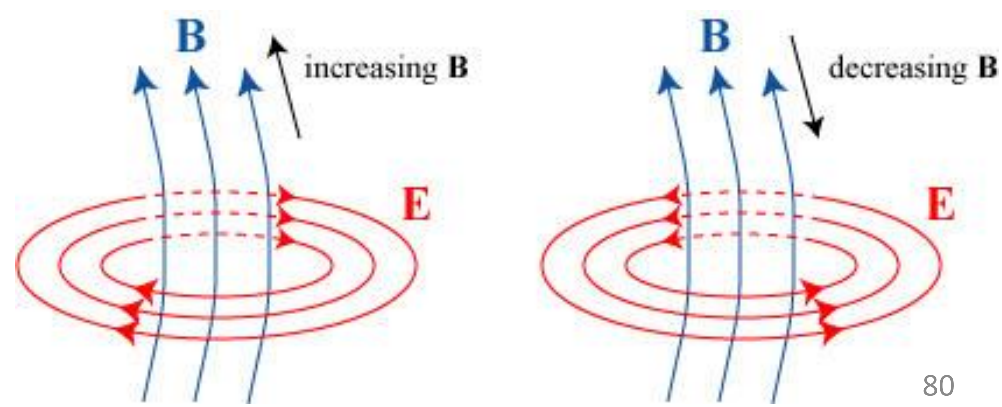
Faraday law



$$\varepsilon_{elecromotive} = \oint_C \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \oiint_S \vec{B} \cdot d\vec{S}$$

- Electromotive force around a closed loop C is opposite to change of magnetic field flux in time
- Unit is Volt [V]
- The sign of B field is not important
- But what is important it is if B field is increasing or if it is decreasing in time

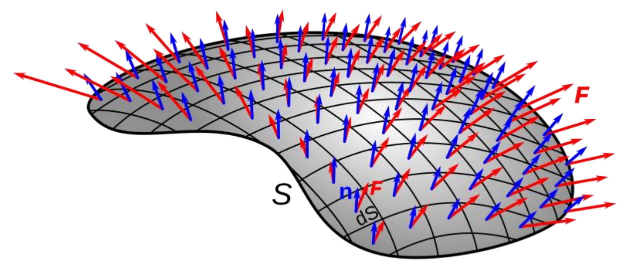


Maxwell's addition to Ampere's law

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Magnetic Field integrated around any closed loop is proportional to the current enclosed by the loop plus change of electric field flux in time

$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{S}$$



Maxwell equations

1. Gauss law for electric field

   - Source of electric field is electric charge

2. Gauss law for magnetic field

   - Magnetic field has no source, there is no “magnetic charges”

3. Faraday law

   - Electric field around a loop (electromotive force) is opposite to change of magnetic field flux through the loop

4. Ampere law

   - Magnetic field around a loop is equal to electric current plus change of electric field flux through the loop

Maxwell equations

- Integral form

$$\oiint_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

$$\oiint_S \vec{B} \cdot d\vec{S} = 0$$

$$\oint_C \vec{E} \cdot d\vec{l} = \oiint_S \overrightarrow{\frac{\partial B}{\partial t}} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \oiint_S \overrightarrow{\frac{\partial E}{\partial t}} \cdot d\vec{S}$$

Credits

- Certain figure were copied from
  - Wikipedia.org
  - xaktly.com by Dr. Jeff Cruzan
    -

Maxwell equations

- Differential form

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \times \vec{E} = \frac{\overrightarrow{\partial B}}{\partial t}$$

$$\nabla \times \vec{B} \; = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\overrightarrow{\partial E}}{\partial t}$$

Curl

- Curl is by definition of integral around infinitesimally small closed loop

$$(\nabla \times \mathbf{F})(p) \cdot \hat{\mathbf{n}} \stackrel{\text{def}}{=} \lim_{A \to 0} \frac{1}{|A|} \oint_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$$

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\boldsymbol{\imath}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\boldsymbol{\jmath}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\boldsymbol{k}} = \begin{bmatrix} \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \\ \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \\ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \end{bmatrix}$$

- It measures how much the field is curling, or circulating