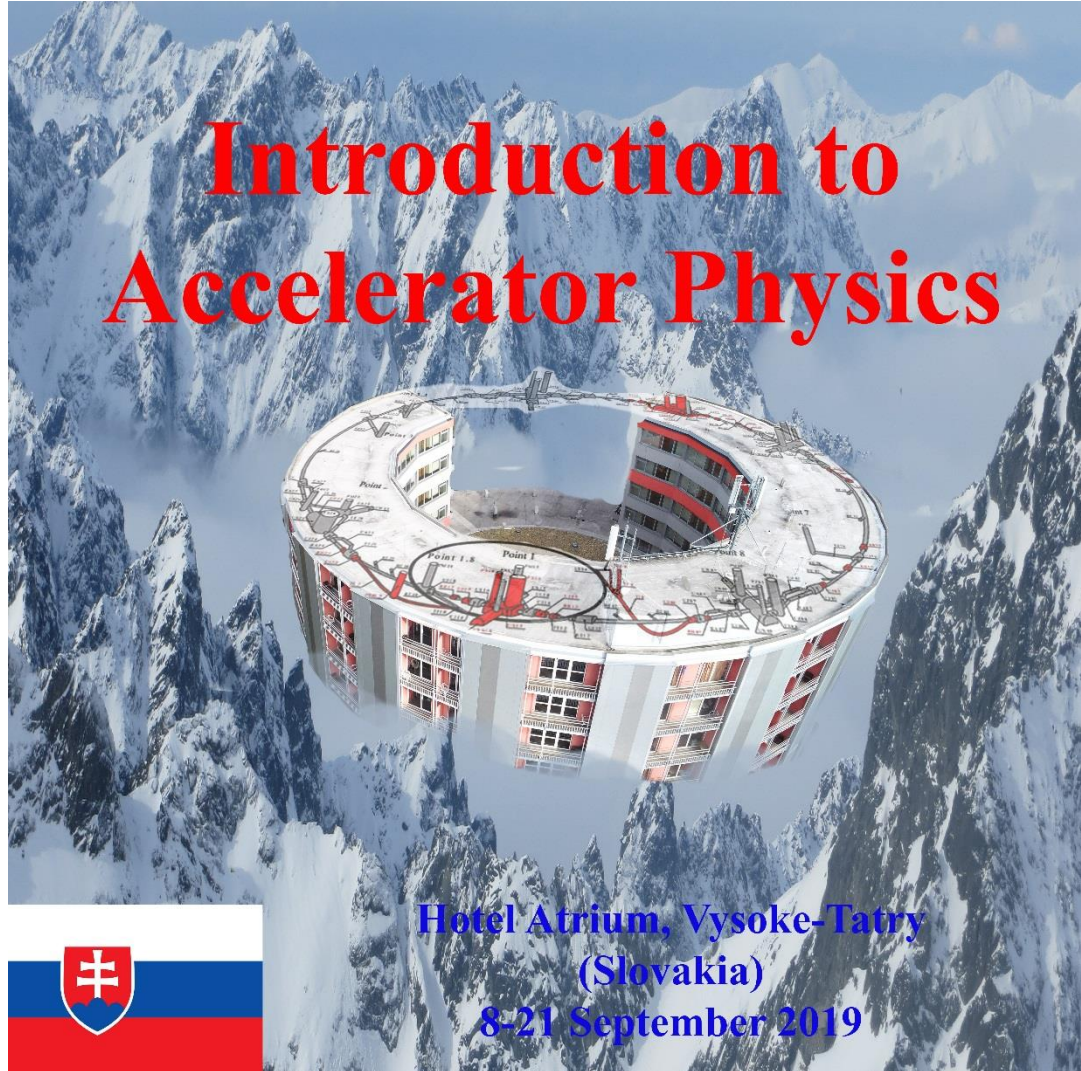




The CERN Accelerator School

Closing remarks

Introduction to Accelerator Physics





The “minimum takeaway”

- **Transverse and longitudinal beam dynamics**
 - trajectory, closed orbit, synchronous particle
 - horizontal and vertical phase/trace-space, preserved action
 - Twiss-parameters: Beta-function, Phaseadvance, tunes (H+V+synchrotron)
 - Dispersion-function, momentum compaction, slip factor
 - transverse and longitudinal focusing
 - chromaticity: origin and correction
 - transport matrix, tracking, dynamic aperture, bucket-area
- **Emittance**
 - emittance = average action of all particles
 - Liouville Theorem
 - RMS emittance, geometrical emittance
 - adiabatic damping, radiation damping
- **Imperfections**
 - dipole displacement: OK, dipole tilt: vertical deflection
 - quadrupole offset: extra deflection; quadrupole tilt: coupling
 - sextupole offset: extra quadrupole, sextupole tilt: coupling
- **Beam instrumentation**
 - Basic BPM functionality
 - How to measure losses, profiles
 - time and frequency domain signals, tune measurement
- **Collective effects:** Head-Tail, Wakefields, Direct Space Charge, Instabilities
- **Types of accelerators:** Linacs, Cyclotrons, Synchrotrons, Colliders, Lightsources

Sorry...many requests for a short
primer of transverse beam dynamics
and emittance

(taken from my summary in the advanced course)

Linear Optics – Hamiltonian (3/3)

Example: Mass-spring system

$$H = T + V = \frac{1}{2} k x^2 + \frac{p^2}{2m} = E$$

Hamiltonian formalism to obtain the equations of motion:

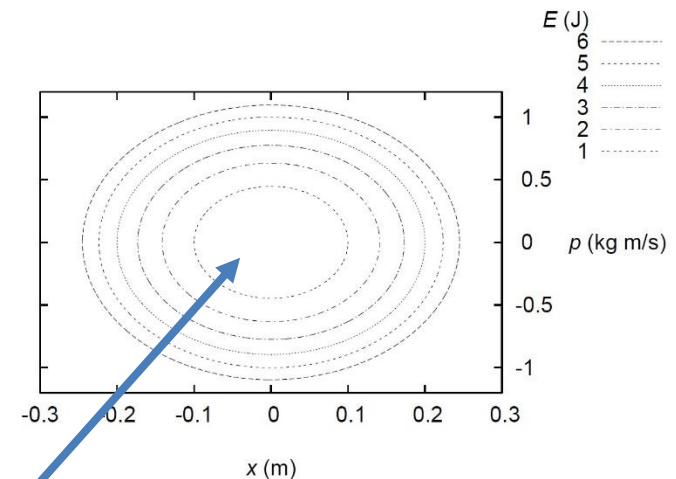
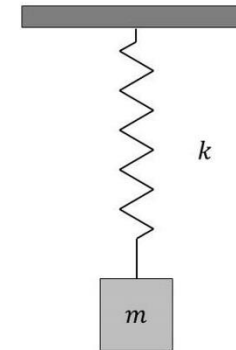
$$\frac{\delta x}{\delta t} = \dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m} \text{ or } p = m\dot{x} = mv$$

$$\frac{\delta p}{\delta t} = \dot{p} = -\frac{\partial H}{\partial x} = -kx$$

We are used to start with the force equation:

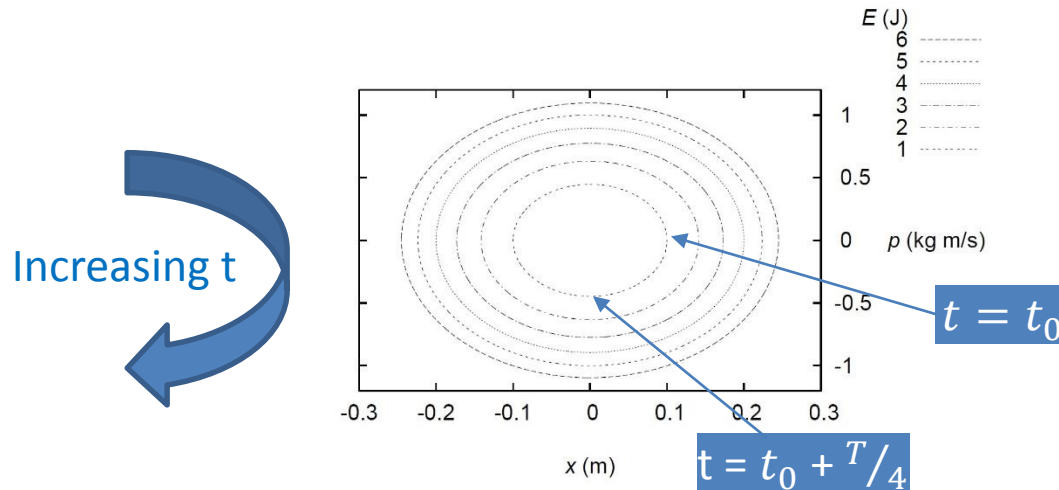
$$F = ma = m\ddot{x} = -kx$$

With the well known sinusoidal solution for $x(t)$.



Instead we look at the trajectory of the system in a phase space.
In this simple case the Hamiltonian itself is the equation of the ellipse.

A further look at phase-space plots



- The particle follows in phase space a trajectory, which has an elliptic shape.
- In the example, the free parameter along the trajectory is time (we are used to express the space-coordinate and momentum as a function of time)
- This is fine for a linear one-dimensional pendulum, but it is not an adequate description for transverse particle motion in a circular accelerator
→ we will choose soon “s”, the path length along the particle trajectory as free parameter
- Any linear motion of the particle between two points in phase space can be written as a matrix transformation: $\begin{pmatrix} x \\ x' \end{pmatrix}(s) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}(s_0)$
- In matrix annotation we define an action “J” as product $J := \frac{1}{2} \begin{pmatrix} x \\ x' \end{pmatrix}(s) \begin{pmatrix} x \\ x' \end{pmatrix}(s_0)$.
- **J is a motion invariant** and describes an ellipse in phase space. The area of the ellipse is $2\pi J$

Why all this? Later we will define the emittance of a beam as the average action variable of all particles... but for the moment we stick to single particles ... and we follow them through magnetic elements.

Why “Hamiltonian” treatment (1/2)?

- Why not just Newton’s law and Lorentz force?
Newton requires rectangular coordinates and time ; for curved trajectories one needs to introduce “reaction forces”.
- Several people use Hill’s equation as starting point, but
- always needs an “Ansatz” for a (periodic) solution:

$$\frac{d^2x}{ds^2} + \left(\frac{1}{\rho(s)^2} - k_1(s) \right) x = 0 \qquad \frac{d^2y}{ds^2} + k_1(s) y = 0$$

No real accelerator has only periodic forces (beam-beam...)

- Hill’s equation follows directly out of a simplified Hamiltonian description (later slide)
- no direct way to extend the treatment to non-linearities
- Hamiltonian equations of motion are two systems of first order <-> Lagrangian treatment yields one equation of second order.
- Hamiltonian equations use the canonical variables p and q , Lagrangian description uses q and $\partial q / \partial t$ and t
 p, q are independent, the others not.

Why “Hamiltonian” treatment (2/2)?

- From each point in an accelerator we can come to the next point by applying a map (or in the linear case a matrix).

$$\begin{pmatrix} x \\ x' \end{pmatrix}(s) = M \begin{pmatrix} x \\ x' \end{pmatrix}(s_0)$$

Linear case: $\begin{pmatrix} x \\ x' \end{pmatrix}(s) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}(s_0)$

- The map M must be symplectic \leftarrow energy conservation
- The maps can be calculated from the Hamiltonian of the corresponding accelerator component.
- We “know” the Hamiltonian for each individual accelerator component (drift, dipole, quadrupole...)
- This way we generate a piecewise description of the accelerator instead of trying to find a general continuous mathematical solution. This is ideal for implementation in a computer code.
- Unfortunately it needs some complex mathematical framework to be able to derive the formalism on how to get symplectic maps from the Hamiltonian. This is dealt with in some detail later in this course.
The next 2 slides show 2 examples.

Particle Motion through accelerator components

Drift space - for the enthusiastic

The exact Hamiltonian in two transverse dimensions and with a relative momentum deviation δ is (full Hamiltonian with $\vec{A}(\vec{x}, t) = 0$):

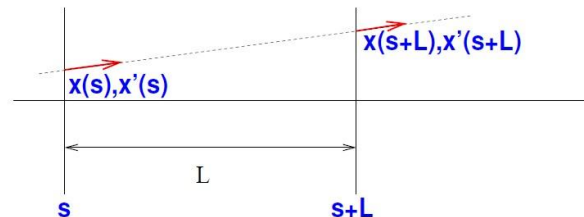
$$H = -\sqrt{(1 + \delta)^2 - p_x^2 - p_y^2}$$

The exact map for a drift space is now (do not use x and x' !):

$$\begin{aligned} x^{new} &= x + L \cdot \frac{p_x}{\sqrt{(1 + \delta)^2 - p_x^2 - p_y^2}} \\ p_x^{new} &= p_x \end{aligned}$$

Most of the time we use the linear approximation, which we get from simple geometry:

A drift space (one dimension only) of length L , starting at position s and ending at $s + L$



The simplest description (1D, using x, x') is (should be in 3D of course):

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s+L} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \circ \begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} x + x' \cdot L \\ x' \end{pmatrix}$$

Starting from:

$$f_{quad} = -\frac{L}{2}(kx^2 + p^2) \quad \text{f is here the generator } L * H$$

we finally have obtained:

$$e^{\dot{f}} : x = \cos(\sqrt{k}L) \cdot x + \frac{1}{\sqrt{k}} \sin(\sqrt{k}L) \cdot p$$

$$e^{\dot{f}} : p = -\sqrt{k} \sin(\sqrt{k}L) \cdot x + \cos(\sqrt{k}L) \cdot p$$

➡ **Thick, focusing quadrupole, 1D !**

Comes directly from the Hamiltonian from first principles, no need to assume a solution of an equation of motion ...

Much more on this: Werner Herr, Non linear Dynamics I- III, advanced general CAS, for example Egham 2017

Hamiltonians of some machine elements (3D)

In general for multipole n :

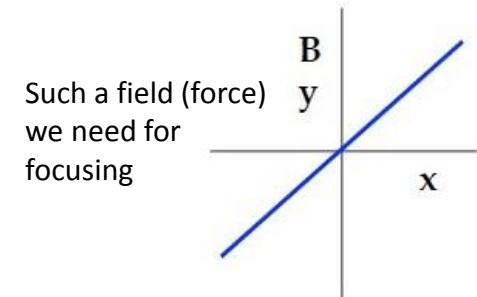
$$H_n = \frac{1}{1+n} \text{Re} [(k_n + ik_n^{(s)})(x + iy)^{n+1}] + \frac{p_x^2 + p_y^2}{2(1+\delta)}$$

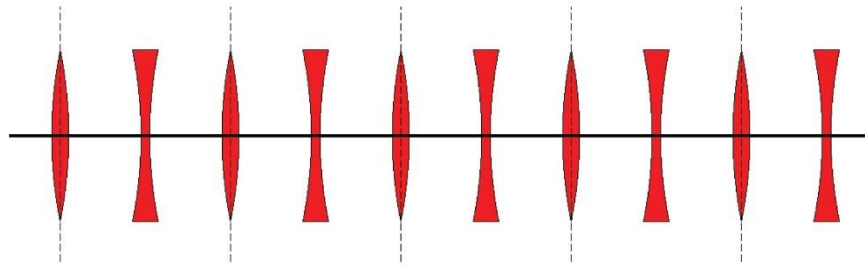
We get for some important types (normal components k_n only):

dipole: $H = -\frac{-x\delta}{\rho} + \frac{x^2}{2\rho^2} + \frac{p_x^2 + p_y^2}{2(1+\delta)}$

quadrupole: $H = \frac{1}{2}k_1(x^2 - y^2) + \frac{p_x^2 + p_y^2}{2(1+\delta)}$

sextupole: $H = \frac{1}{3}k_2(x^3 - 3xy^2) + \frac{p_x^2 + p_y^2}{2(1+\delta)}$

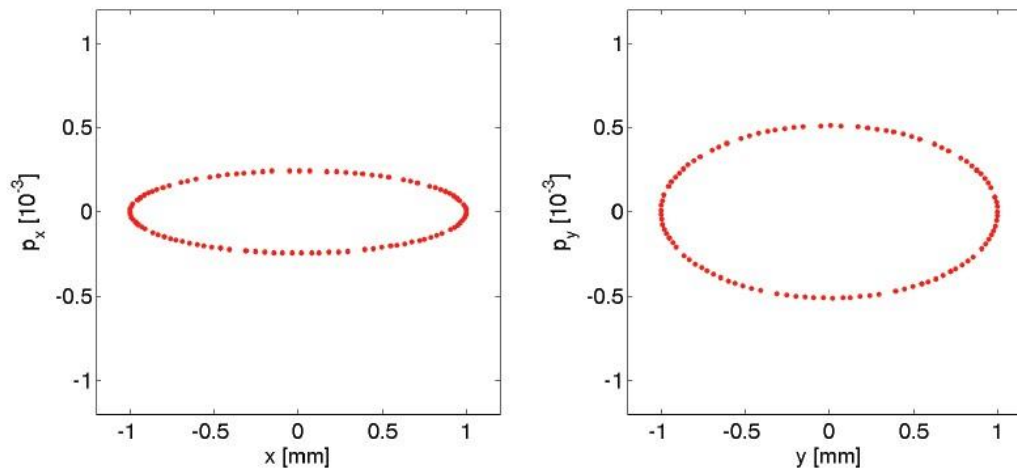




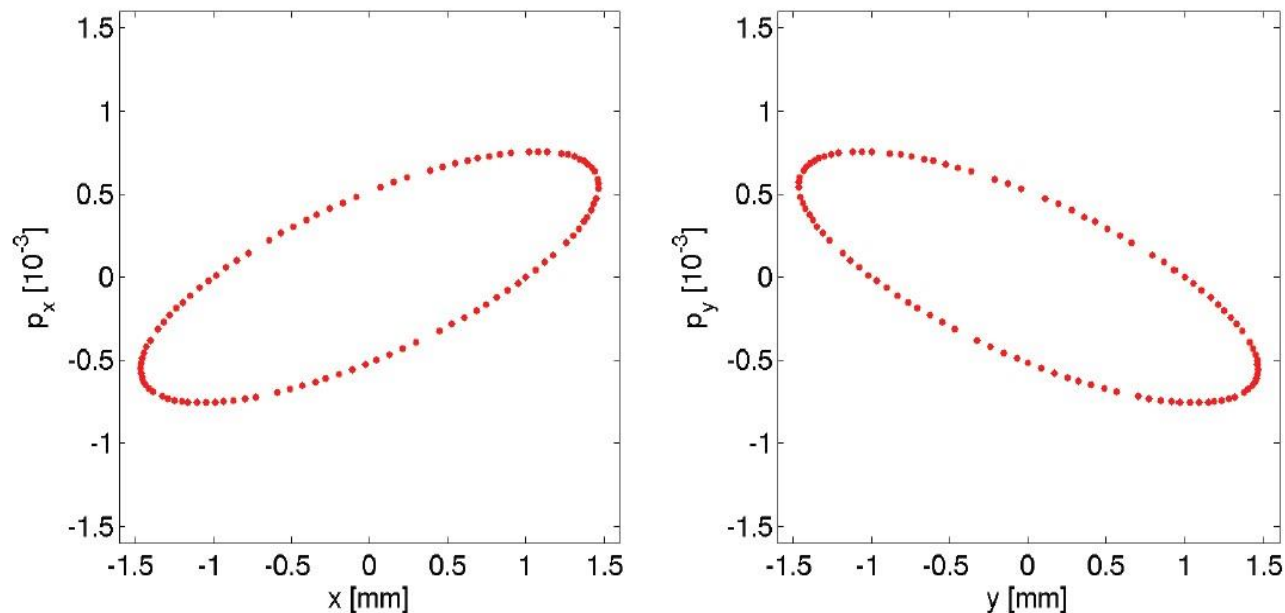
Let us consider the case $L = 1$ m, $f_0 = \sqrt{2}$ m. Take a particle with initial coordinates at the start of a FODO cell:

$$x = 1 \text{ mm}, \quad p_x = 0, \quad y = 1 \text{ mm}, \quad p_y = 0$$

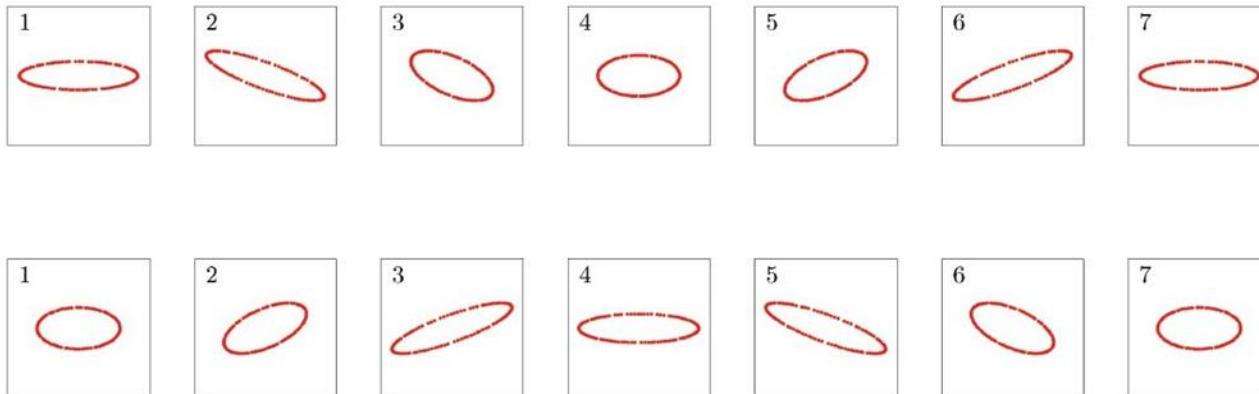
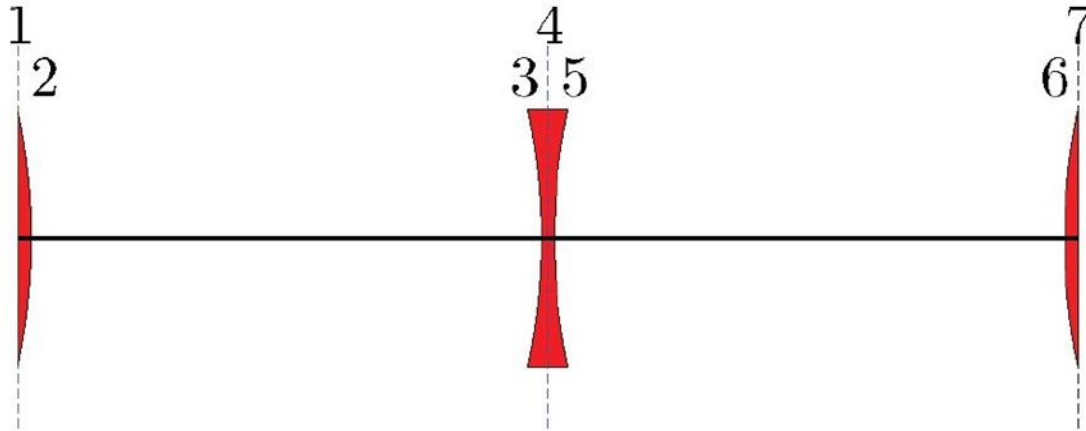
Now track the particle through 100 FODO cells by applying the transfer matrix to the vector constructed from the coordinates, and plot p_x vs x , and p_y vs y :



What happens if we repeat the exercise, but starting the FODO cell at the center of the drift before the (horizontally) defocusing quadrupole? Again, we plot ellipses, but this time, they are tilted:



Evolution of the Phase Space Ellipse in a FODO Cell



Courant – Snyder formalism / Twiss parameters

- Same beam dynamics
- Introduced in the late 50's
- The classical way to parametrize the evolution of the phase space ellipse along the accelerator

Basic concept of this formalism:

1) Write the transfer matrix in this form (2 dimensional case):

$$M = I \cos\mu + S \cdot A \sin\mu$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \quad A = \begin{pmatrix} \gamma & \alpha \\ \alpha & \beta \end{pmatrix}$$

2) M must be symplectic $\rightarrow \beta\gamma - \alpha^2 = 1$

3) Four parameters: $\alpha(s)$; $\beta(s)$; $\gamma(s)$ and $\mu(s)$, with one interrelation (2)
 \rightarrow Three independent variables

4) Again, the preserved action variable J describes an ellipse in phase-space:

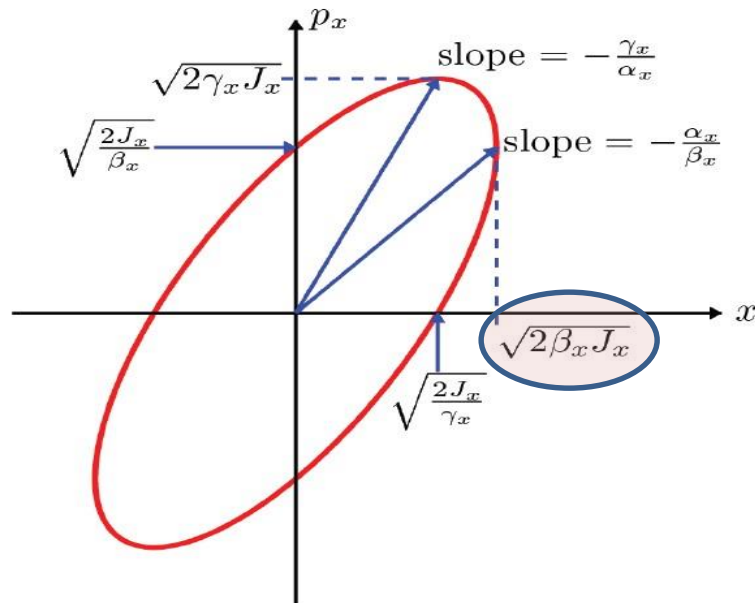
$$J = \frac{1}{2} (\gamma x^2 + 2\alpha xp + \beta p^2)$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = M^* \begin{pmatrix} x \\ x' \end{pmatrix}_{s0} \quad M = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}$$

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_s = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & SC' + CS' & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}$$

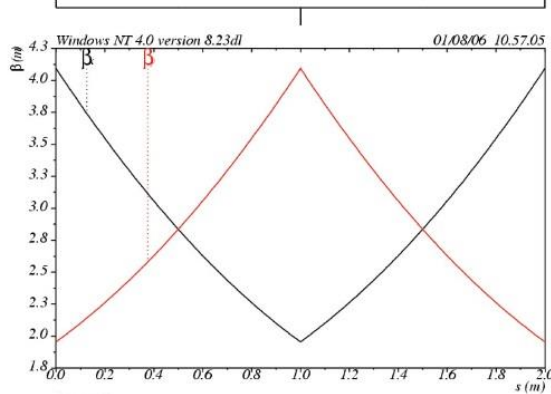
The Phase Space Ellipse

$$J_x = \frac{1}{2} (\gamma_x x^2 + 2\alpha_x x p_x + \beta_x p_x^2) \quad \text{Area} = 2\pi J_x \quad (2.2)$$



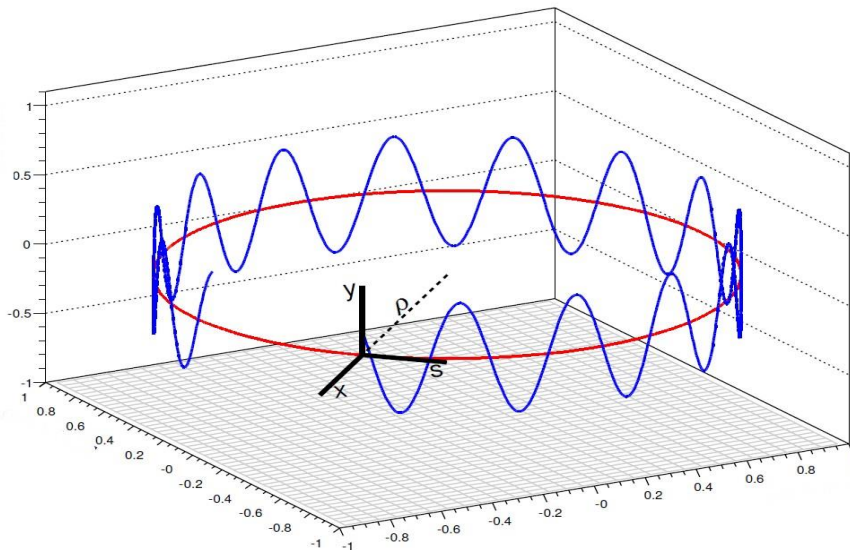
Interpretation of the Twiss parameters (1/2)

1) Horizontal and vertical beta function $\beta_{H,V}(s)$:

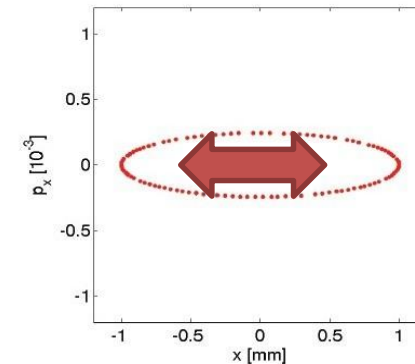


- Proportional to the square of the projection of the phase space ellipse onto the space coordinate
- Focusing quadrupole \rightarrow low beta values

Although the shape of phase space changes along s , the rotation of the particle on the phase space ellipse projected onto the space co-ordinate looks like an harmonic oscillation with variable amplitude: called **BETATRON-Oscillation**



$$x(s) = \text{const} \cdot \sqrt{\beta(s)} \cdot \cos\{\mu(s) + \varphi\}$$



Interpretation of the Twiss parameters (2/2)

2.) $\alpha = -\frac{1}{2} \frac{d\beta}{ds}$

α indicates the rate of change of β along s
 α zero at the extremes of beta (waist)

3.) $\mu = \int_{s1}^{s2} \frac{1}{\beta} ds$

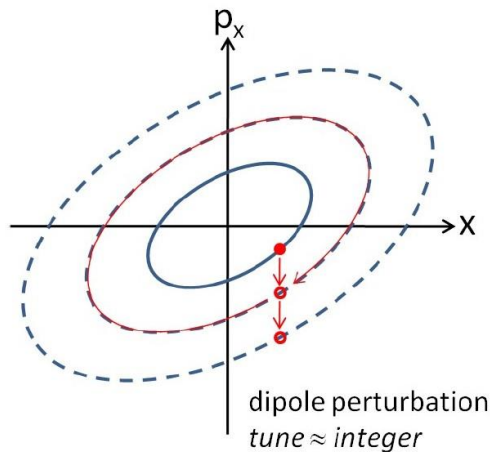
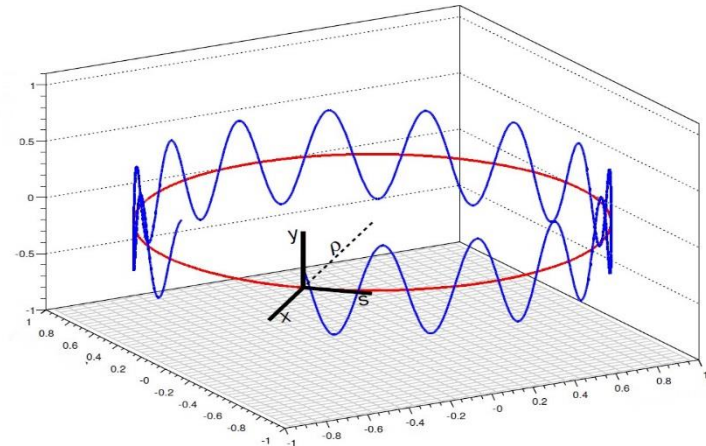
Phase Advance: Indication how much a particle rotates in phase space when advancing in s

Of particular importance: Phase advance around a complete turn of a circular accelerator, called the **betatron tune Q (H,V)** of this accelerator

$$Q_{H,V} = \frac{1}{2\pi} \int_0^C \frac{1}{\beta_{H,V}} ds$$

The betatron tunes $Q_{H,V}$

- Part of the most important parameters of a circular accelerator
- The equivalent in a linac is called “phase advance per cell”
- For a circular accelerator it is the phase advance over one turn in each respective plane.
- In large accelerators the betatron tunes are large numbers (LHC ~ 65), i.e. the phase space ellipse turns about 65 times in one machine turn.
- We measure the tune by exciting transverse oscillations and by spectral analysis of the motion observed with one pickup.
This way we measure the **fractional part of the tune; often called $q_{H,V}$**



- Integer tunes (fractional part= 0) lead to resonant infinite growth of particle motion even in case of only small disturbances.

Importance of betatron tunes

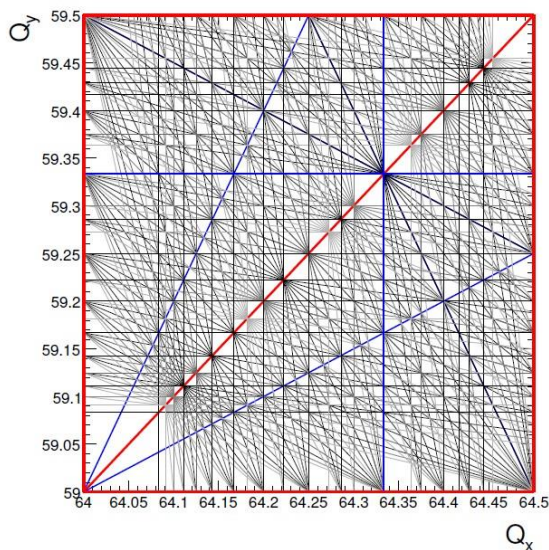
If we include vertical as well as horizontal motion, then we find that resonances occur when the tunes satisfy:

$$m_x \nu_x + m_y \nu_y = \ell,$$

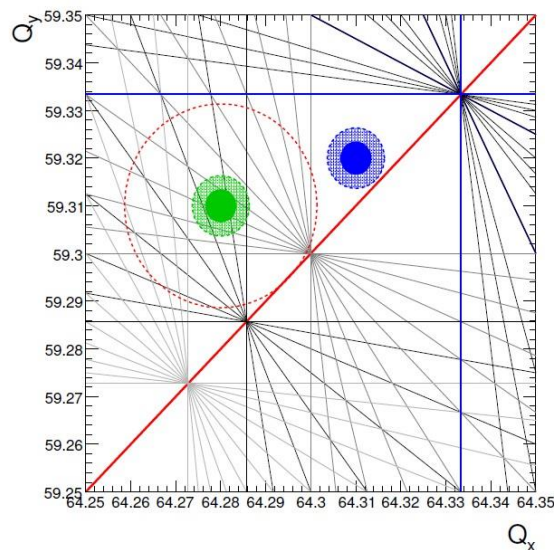
where m_x , m_y and ℓ are integers.

The order of the resonance is $|m_x| + |m_y|$.

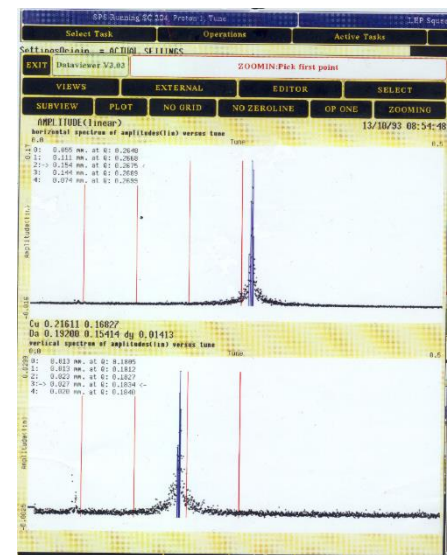
The couple (Q_H, Q_V) is called the working point of the accelerator. Below: tune measurement example from LEP



(a) Full tune diagram



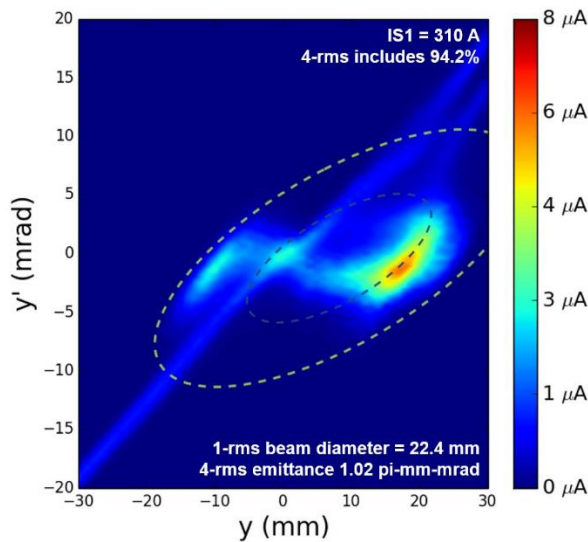
(b) Zoom around LHC Q working points



Finally: a beam

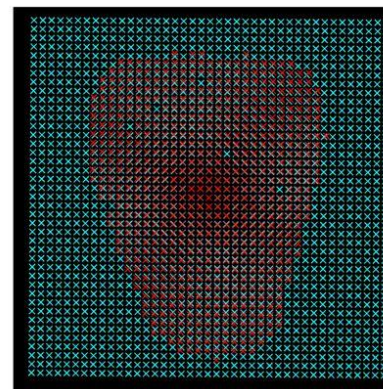
We focus on “bunched” beams, i.e. many (10^{11}) particles bunched together longitudinally (much more on this in the RF classes).

From the generation of the beams the particles have transversally a spread in their original position and momentum.

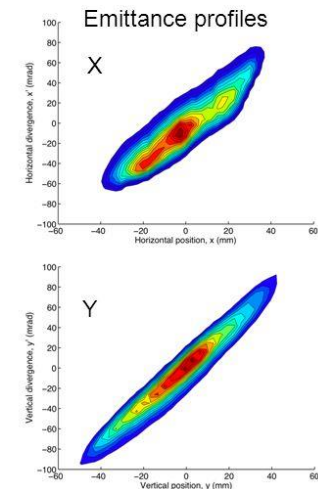


Source: ISODAR (Isotope at rest experiment)

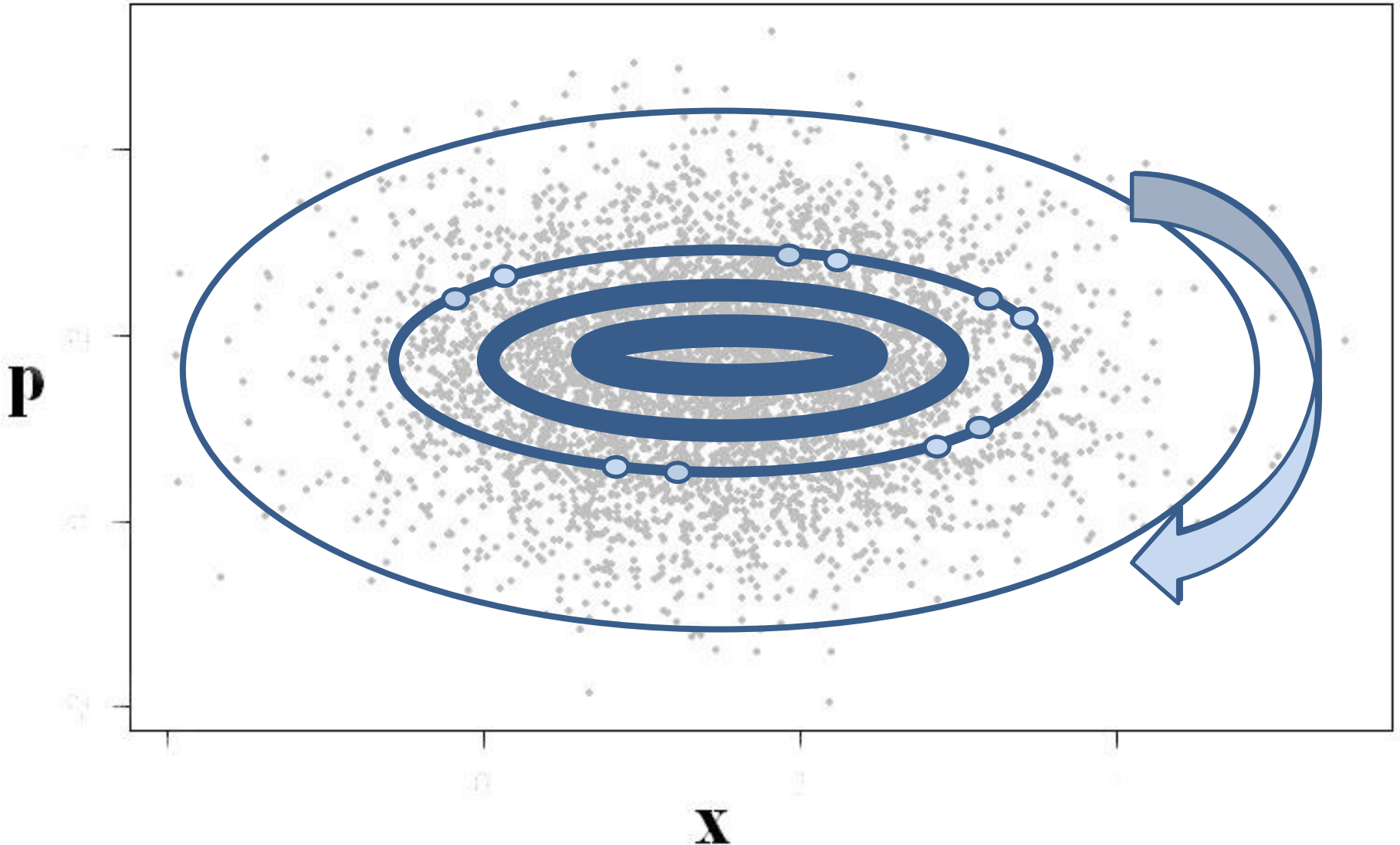
Pepperpot Emittance Extraction



Pepperpot image spots: hole positions (blue) and beam spots (red)

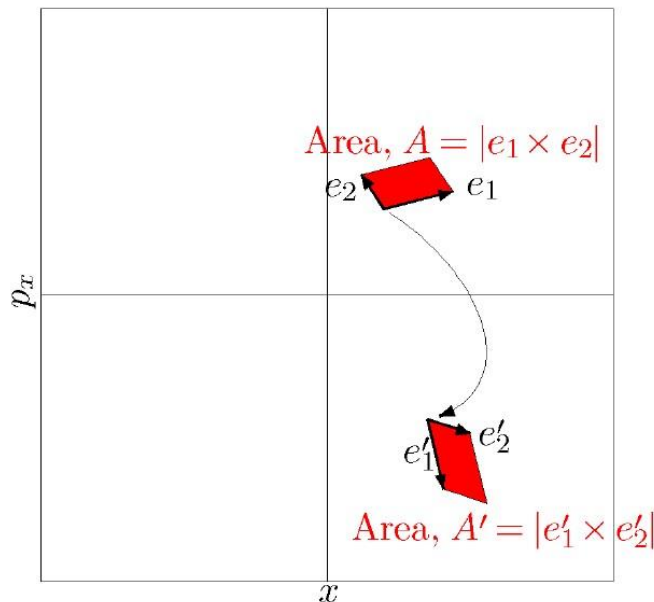


Gaussian beam profile in x and p



Liouville's Theorem (1/2)

1. All particles rotate in phase space with the same angular velocity (in the linear case)
2. All particles advance on **their** ellipse of constant action
3. All constant action ellipses transform the same way by advancing in “s”



Physically, a symplectic transfer map conserves phase space volumes when the map is applied.

This is Liouville's theorem, and is a property of charged particles moving in electromagnetic fields, in the absence of radiation.

→ Since volumes in phase space are preserved, (1)-(3) means That the whole beam phase space density distribution transforms **the same way** as the individual constant action ellipses of individual particles.

We now define the **emittance** of a beam as the **average action** of all particles!

→ Since the action J of a particle is constant and the phase space area A covered by the action ellipse is $A = 2\pi J$, we can represent the whole beam in phase space by an ellipse with a surface $= 2\pi \langle J \rangle$ *

→ all equations for the propagation of the phase space ellipse apply equally for the whole beam

!!! In case we talk about a single particle, the ellipse we draw is “empty” and any particle moves from one point to another; if we consider a beam, the ellipse is full of particles!!!

- *. There are several different definitions of the emittance ε , also different normalization factors. This depends on the accelerator type, but the above definition describes best the physics.

- Another often used definition is called RMS emittance

$$\varepsilon = \text{const} * \langle x^2 \rangle \langle p^2 \rangle - \langle xp \rangle^2 \quad \text{or} \quad \varepsilon = \text{const} * \langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2$$

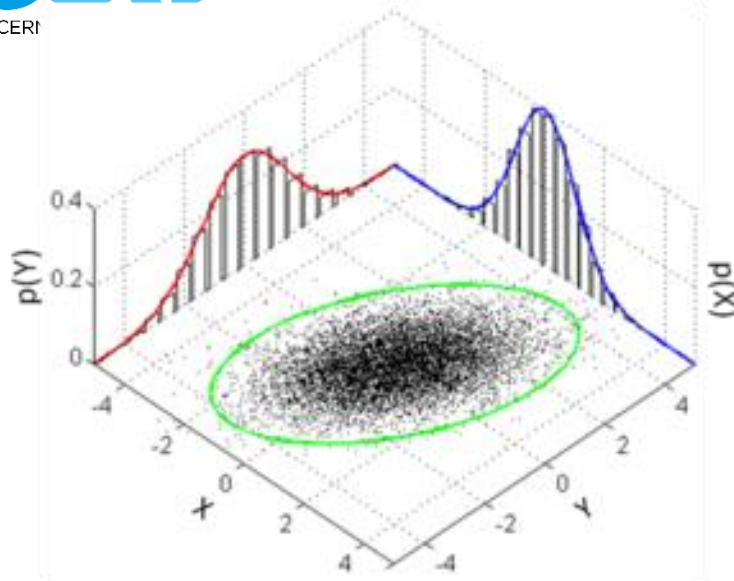
attention: the first definition describes well the physics, the second describes what we eventually can measure

Remarks

1. We have already identified the action as a preserved quantity in a conservative system \leftrightarrow
the emittance of a particle beam is preserved in a conservative beam line.
2. The sentence above is often quoted as Liouville's theorem, but this is incorrect. Liouville's theorem describes the preservation of phase space volumes, the preservation of the phase space of a beam is then just results from the Hamiltonian description.
3. We can identify the constant in the previous equation:

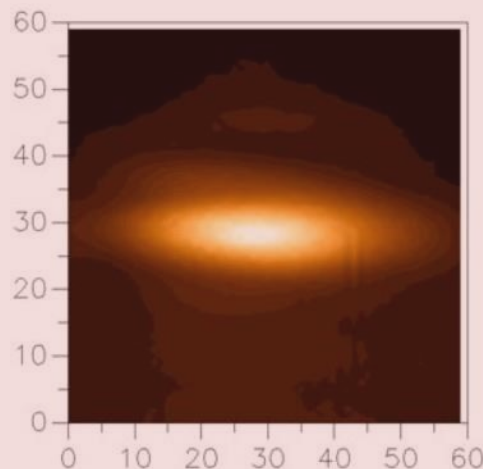
$$x(s) = \sqrt{\varepsilon} \cdot \sqrt{\beta(s)} \cdot \cos\{\mu(s) + \varphi\}$$

What do we normally measure from the phase-space ellipse?

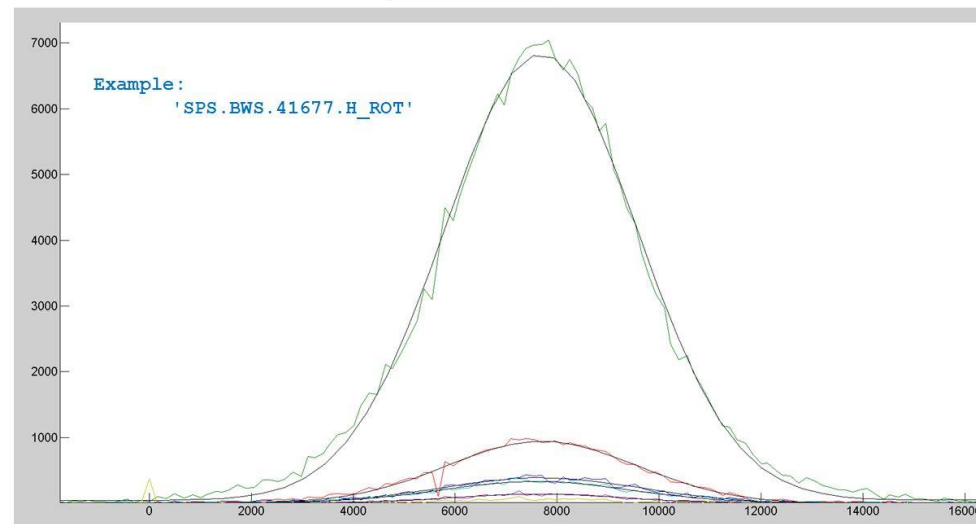


- At a given location in the accelerator we can measure the position of the particles, normally it is difficult to measure the angle...so we measure the projection of the phase space ellipse onto the space dimension:
→ called a profile monitor

Attention! The standard 2 D image of a synchrotron light based beam image is NOT a phase space measurement



FITTING



Some background info

- Last course 1 years ago (Constanta, Romenia)
- Next course in Sept/Oct 2020...somewhere in Serbia.
- Visit and choice of hotel in autumn 2018
- Program committee meeting at CERN
 - decisions: 13 days, focus on beam dynamics, hands-ON courses for transverse and longitudinal beam dynamics
 - list of speakers and subjects
 - NO PROCEEDINGS until the year
- 9 iterations of program

Statistics

- 68 participants
- 25 different nationalities (new record!)
- Age span: 22 ...52
- 55 males/13 females

Feedback Discussion I

- Comments to the program
- Balance of topics
- Balance between accelerator types
- Hands-ON Courses

- Level of the lectures

Project “CAS videos”

Presently two major attempts to produce MOOC's in the field of accelerator physics:

- Nordic Accelerator School
- ARIES

CAS proposes to film its lectures and to put them onto our website including an electronic index baptized “CASopedia”

- first attempt: introductory in Budapest; no index
- next: most likely next introductory course;
(provided we get the necessary resources)

MOOC: Massive Open Online Course


Our website: <http://cas.web.cern.ch/>

Author: Anastasiya

Our major depository of information...large effort to keep the site up to date

Sign inDirectory

Registration is open for "RF for Accelerators", 22 March – 4 April 2020 in Kaunas, Lithuania




The CERN Accelerator School

HomeAboutUpcoming schoolsPrevious schoolsFrequently asked questionsPhoto galleryContact

Welcome to the CERN Accelerator School!

The CERN Accelerator School holds training courses on accelerator physics and associated technologies for physicists, engineers, technicians and students. The courses take place in different member states of CERN and consist of a programme of lectures and tutorials spread over a period of one to two weeks.

We welcome participants from all countries world-wide!



RIGHT NOW

Introduction to Accelerator Physics

ongoing until 21 September 2019 in Vysoke-Tatry, Slovakia

New registration procedure

With the upcoming advanced and then introductory general CAS courses we propose a new procedure for all student registrations external to CERN:

The registration opens 4-5 months before the course and stays open for 3 months. All students registering with payment are informed within a week about the acceptance of their application. For the first half of the payment period a price advantage of 300 CHF for the whole course is offered. Later payment will require the full sum. This also gives ample time for VISA formalities etc.

Grant students will be informed at the end of the registration period, because all grant applications will be considered together and compared to each other. For those typically a 2 months period remains for VISA formalities and travel arrangements.

For students applying as grant student AND as paying student the grant application is automatically removed.

A student, who has applied for a grant, which could not be considered for a course, can NOT apply afterwards as normal paying student.

Hermann Schmickler - CAS Vysoke-Tatry

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Our CAS video on the website

<https://cas.web.cern.ch/>



Feedback

VACUUM FOR PARTICLE ACCELERATORS

6-16 June, 2017

Glumslov, Sweden

YOUR IMPRESSIONS OF THE PROGRAMME

Please mark each lecture with a number 1 to 5 in each of the three columns labelled "Level, Content and Presentation". The meaning of the numbers is as shown below. Please return this sheet to Barbara Strasser or Roger Bailey as soon as possible when completed. Your answers are confidential.

LEVEL	CONTENT	PRESENTATION
1 – Much too low	1 – Completely uninteresting	1 – Very poor
2 – Low	2 – Uninteresting	2 – Poor
3 – Just right	3 – Of some interest	3 – Fair
4 – Too high	4 – Interesting	4 – Good
5 – Much too high	5 – Very interesting	5 – Very good

TITLE	LEVEL	CONTENT	PRESENTATION
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We keep the feedback open until
Wednesday next week...last chance!

- Please help us

- Very important

- For

- For

- About


- The lectures
- The tutorials
- The place
- Anything else

Introduction to Cryogenics			
Cryopumping			
Industrial Vacuum Applications			
Beam Induced Desorption			
Beam-Gas Interaction			
Surface Characterisation			
Interactions between Beams and Vacuum System Walls			
Surface Cleaning & Finishing			
Thin-Film Coating			
Controlling Particles/Dust in Vacuum Systems			
Beam Induced Radioactivity & Radiation Hardness			
Radiation Damage and its Consequence			
Control & Diagnostic			
Vacuum Design Aspects			
Manufacturing & Assembly for Vacuum Technology			
The Real Life of Operation			
Challenges for Vacuum Technology of Future Accelerators			

“Testimonials” on the CAS website



What our students say about us



“ For a beginner like me, it was a very informative and helpful school, I could interact with people from different parts of the world and realize the opportunities ahead of me. ”

— **Aqsa Shaikh**, SAMEER
Student of JAS on RF Technologies, Japan 2017



“ I enjoyed the multinational environment of great people and a great deal of knowledge that I got out of the lectures. ”

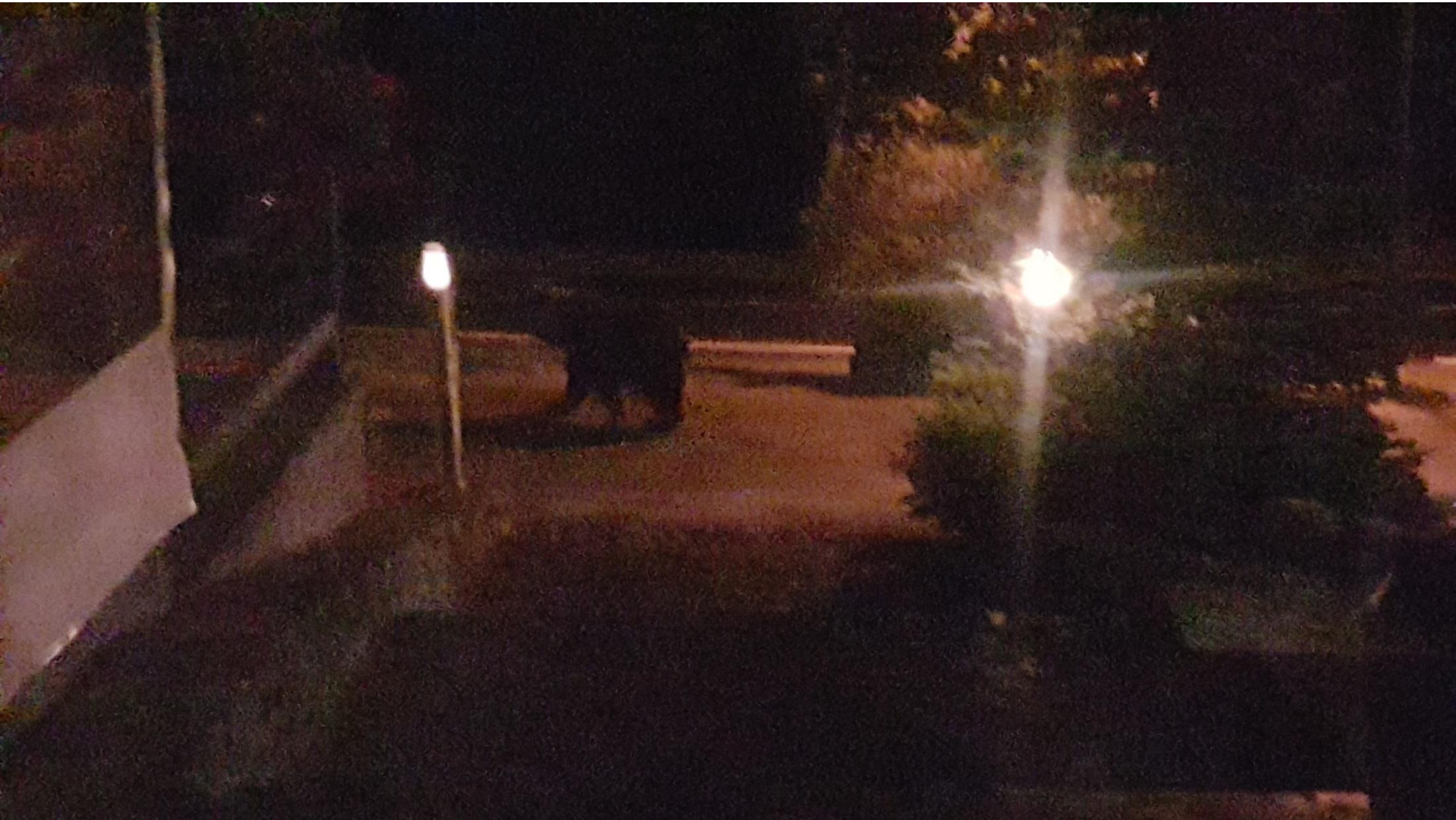
— **Marcin Knafel**, NSRC SOLARIS
Student of JAS on RF Technologies, Japan 2017

- All it needs:
 - a photo
 - name + affiliation + CAS course
 - “a sentence”

Social life during course:

- Next to the course teaching the most important aspect of the school
“electronic training will never replace CAS courses”
- What happened:
 - people socialising (and even working) up to late in the evenings
 - lots of interactions students <-> teachers
 - cinema evening
 - excursion
 - bear viewing

Bear-viewing



Last not least:

This course would not have happened without:

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 - Maria Fillipova
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• YOU