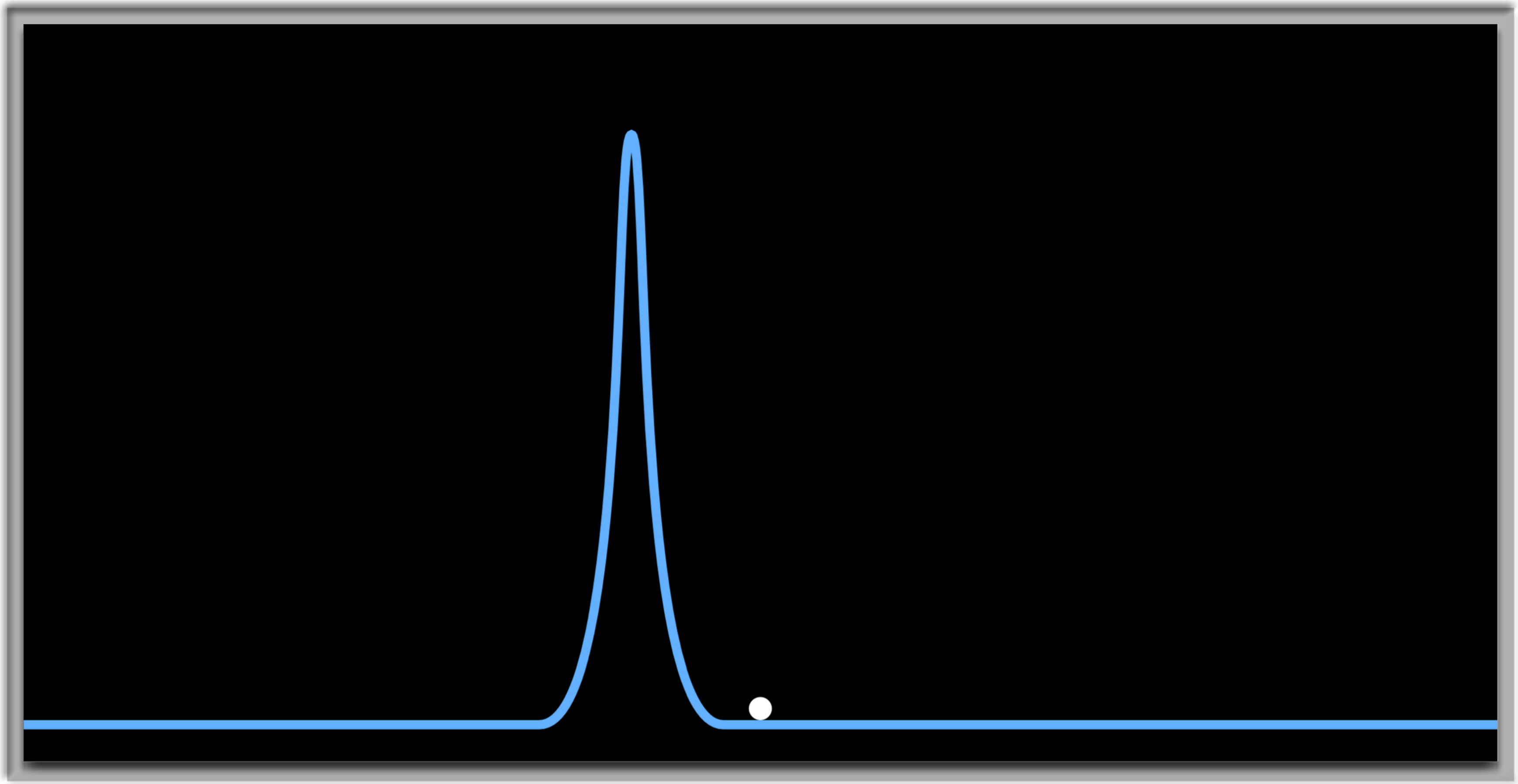
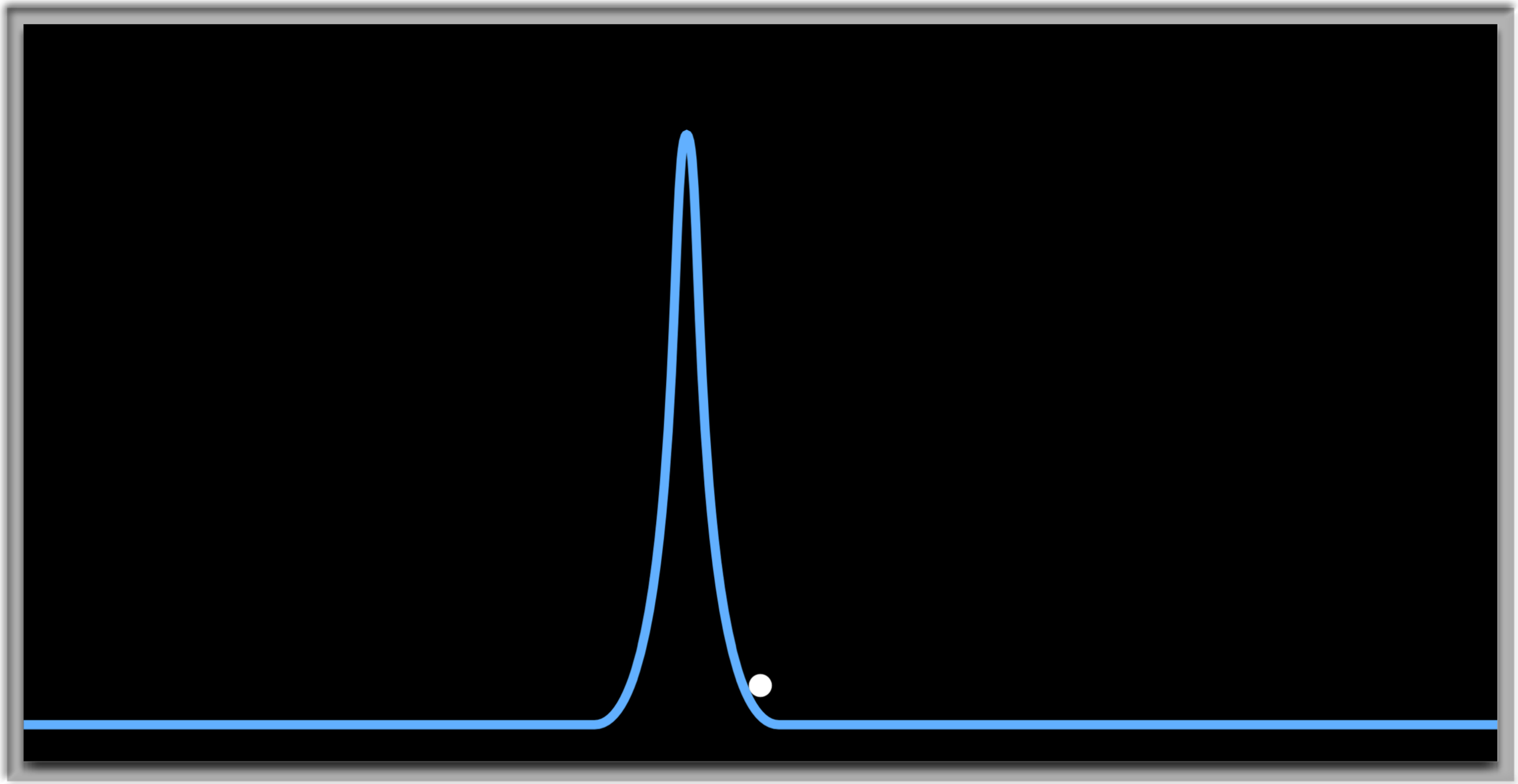
PART II: INJECTION



- Electron at rest is simply overtaken by the plasma wave



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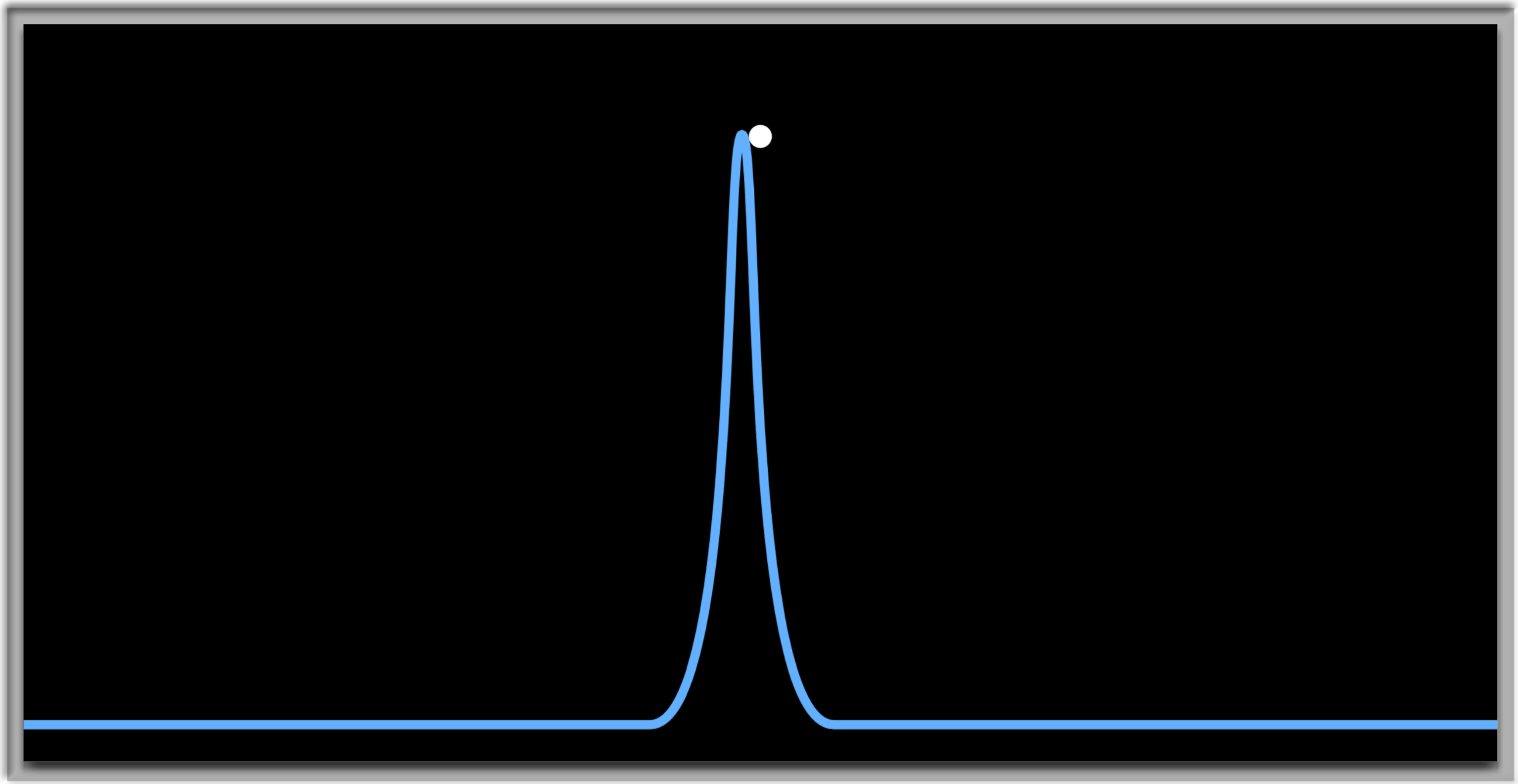

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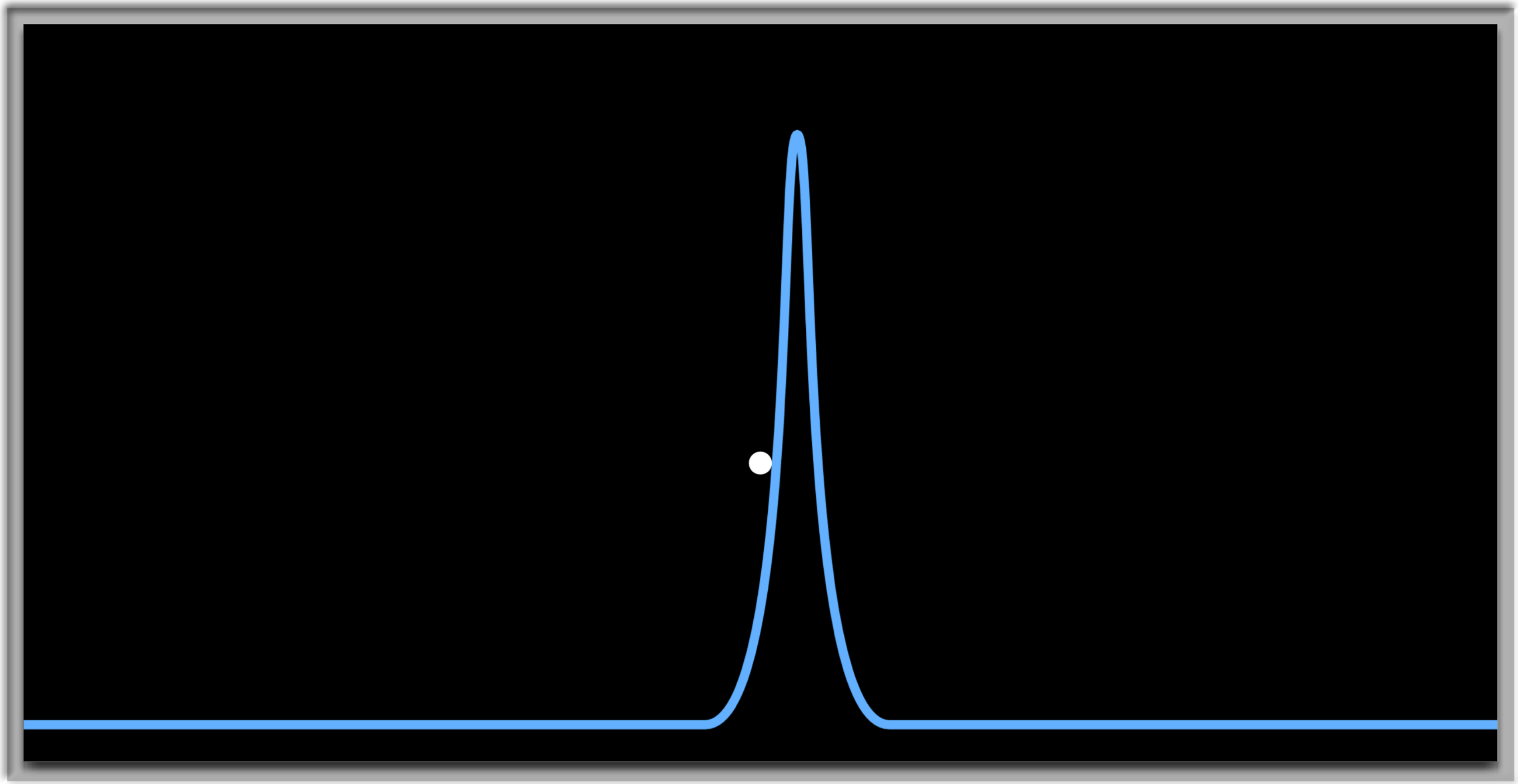

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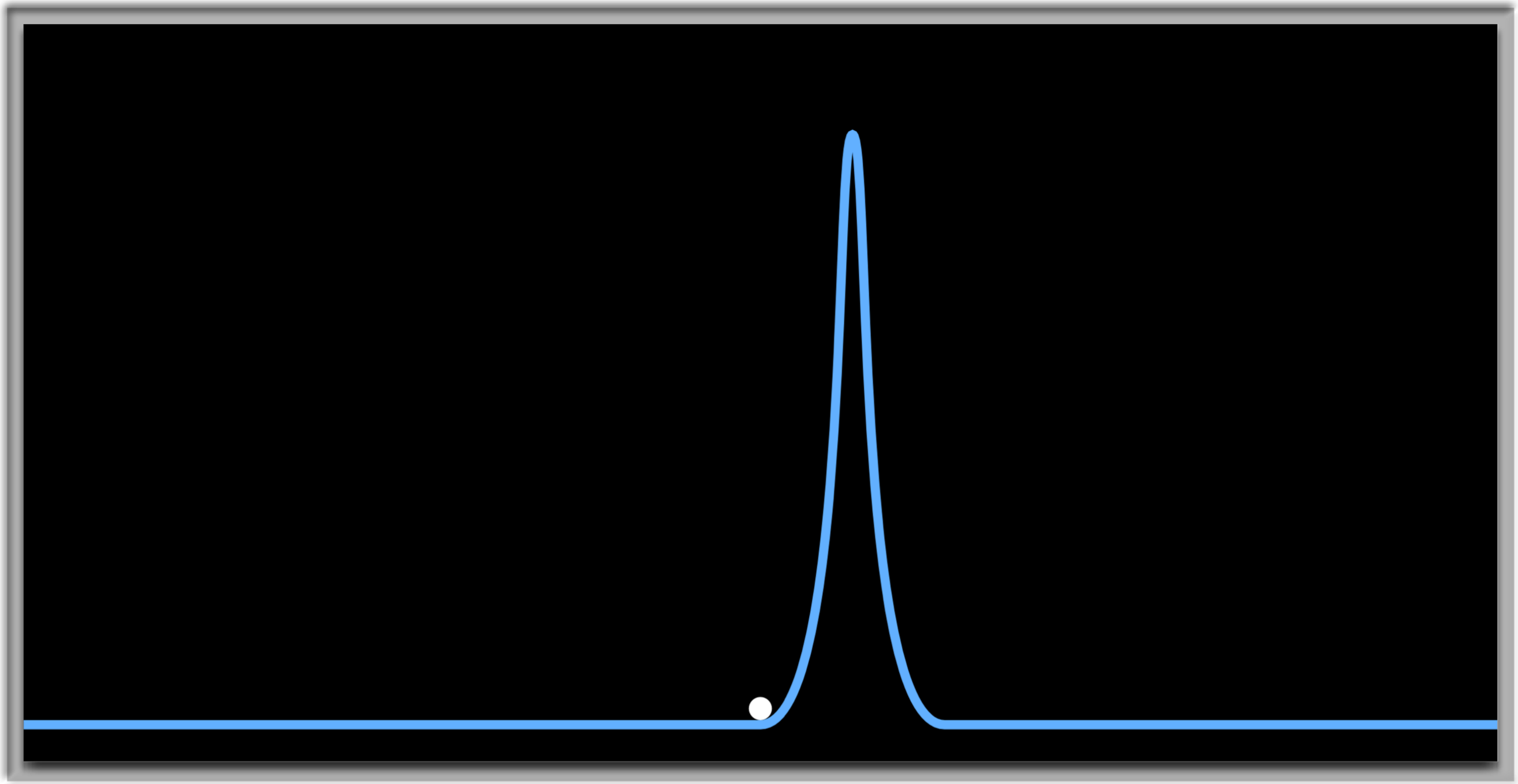

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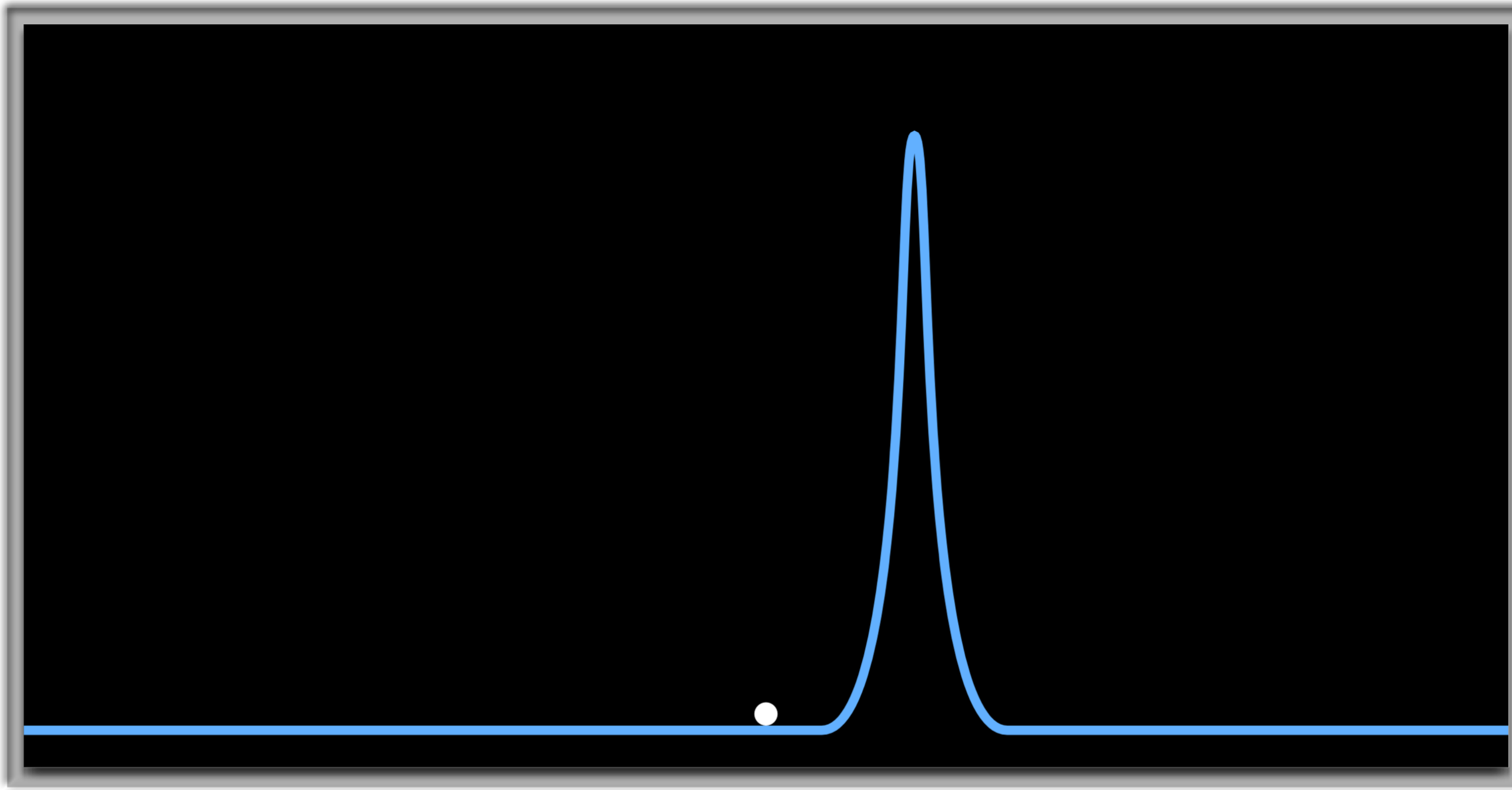
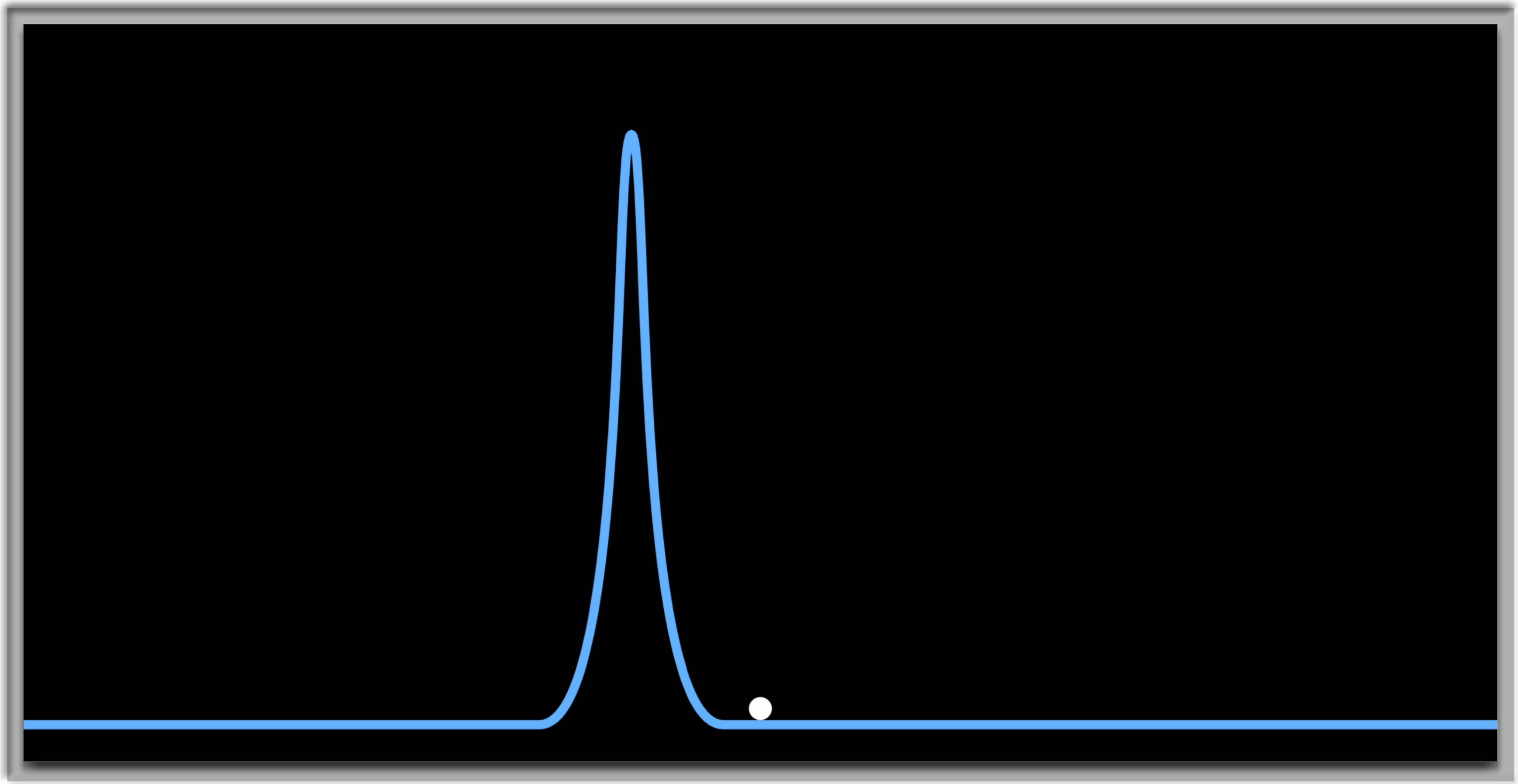
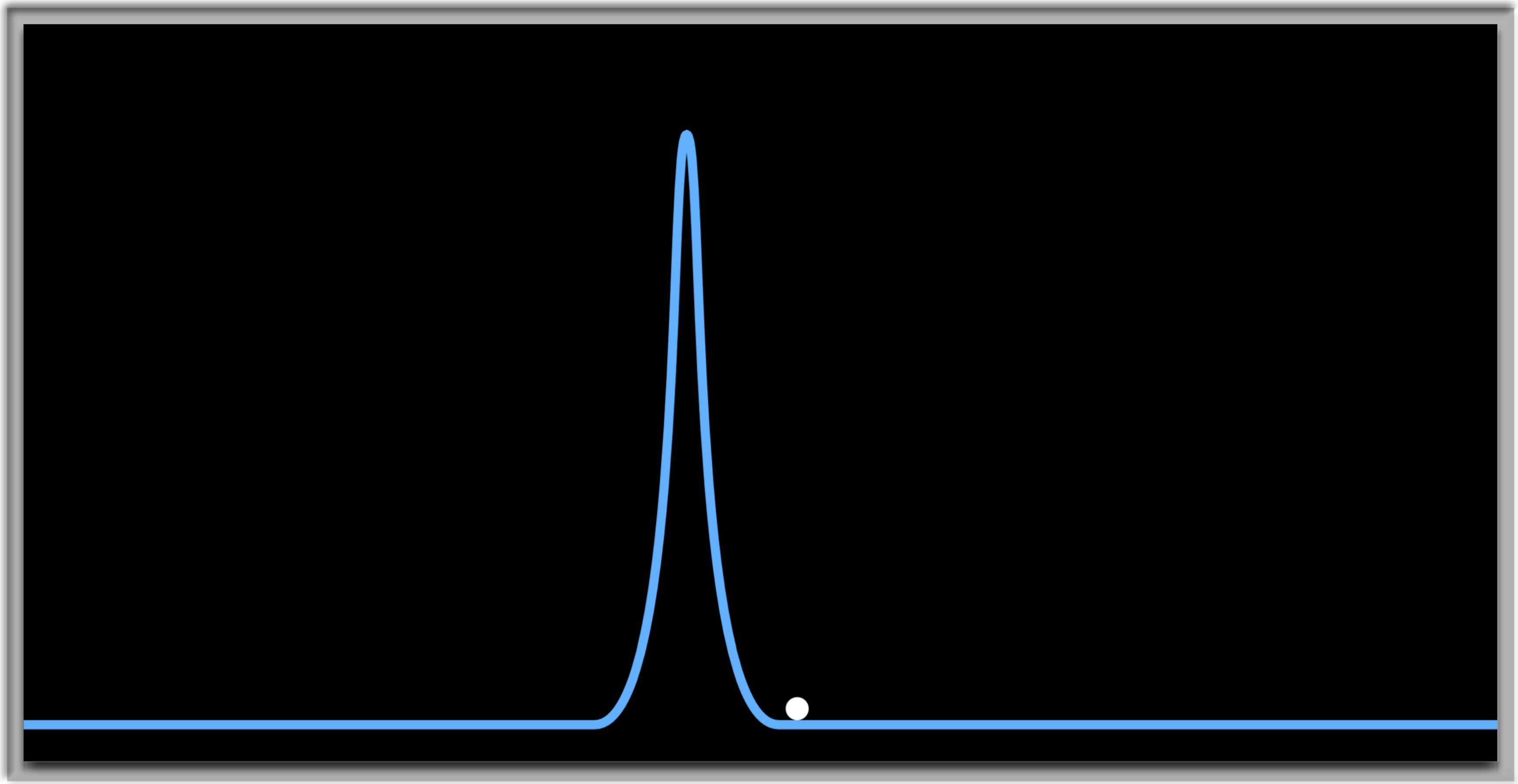


- Electron at rest is simply overtaken by the plasma wave



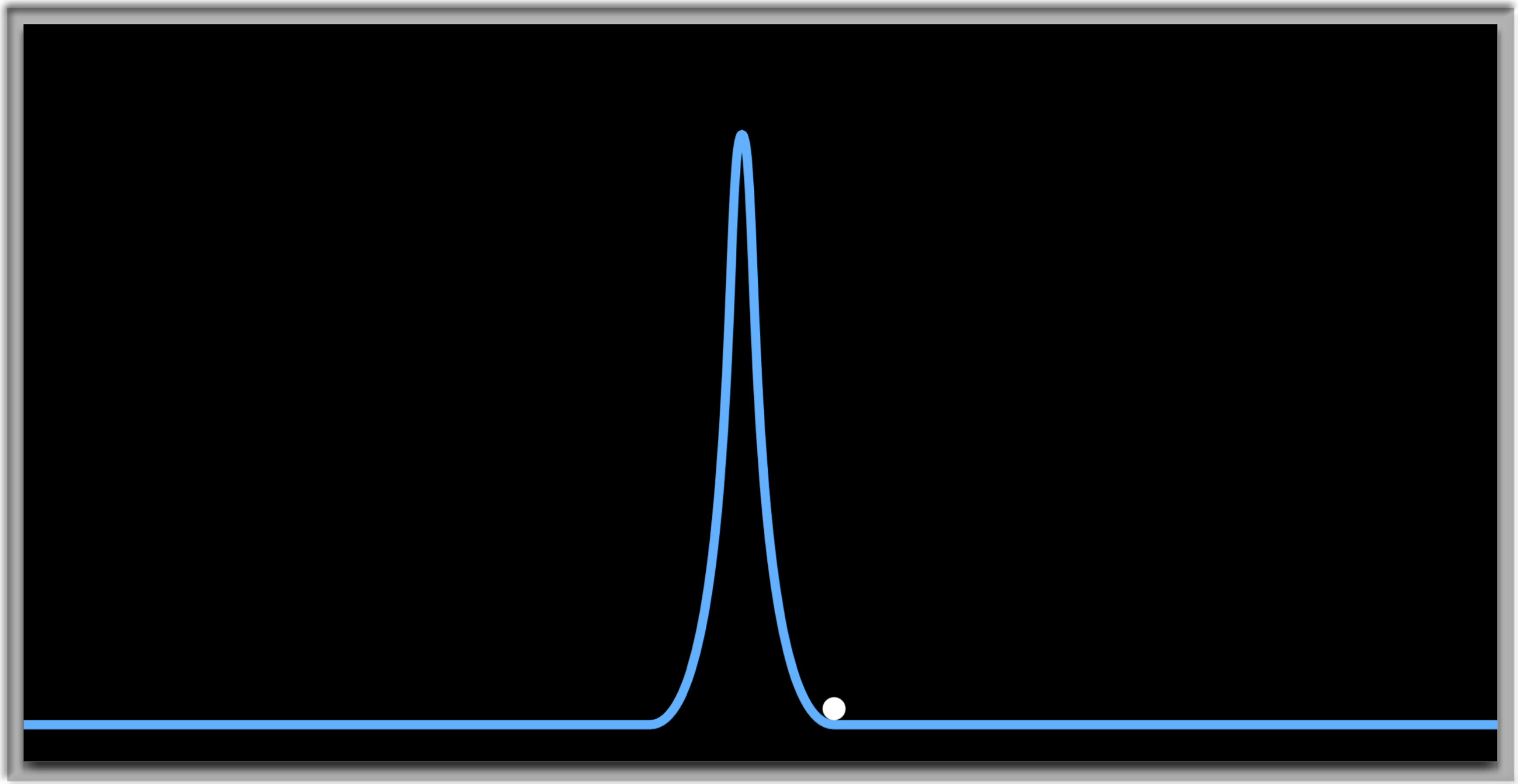
- Electron at rest is simply overtaken by the plasma wave



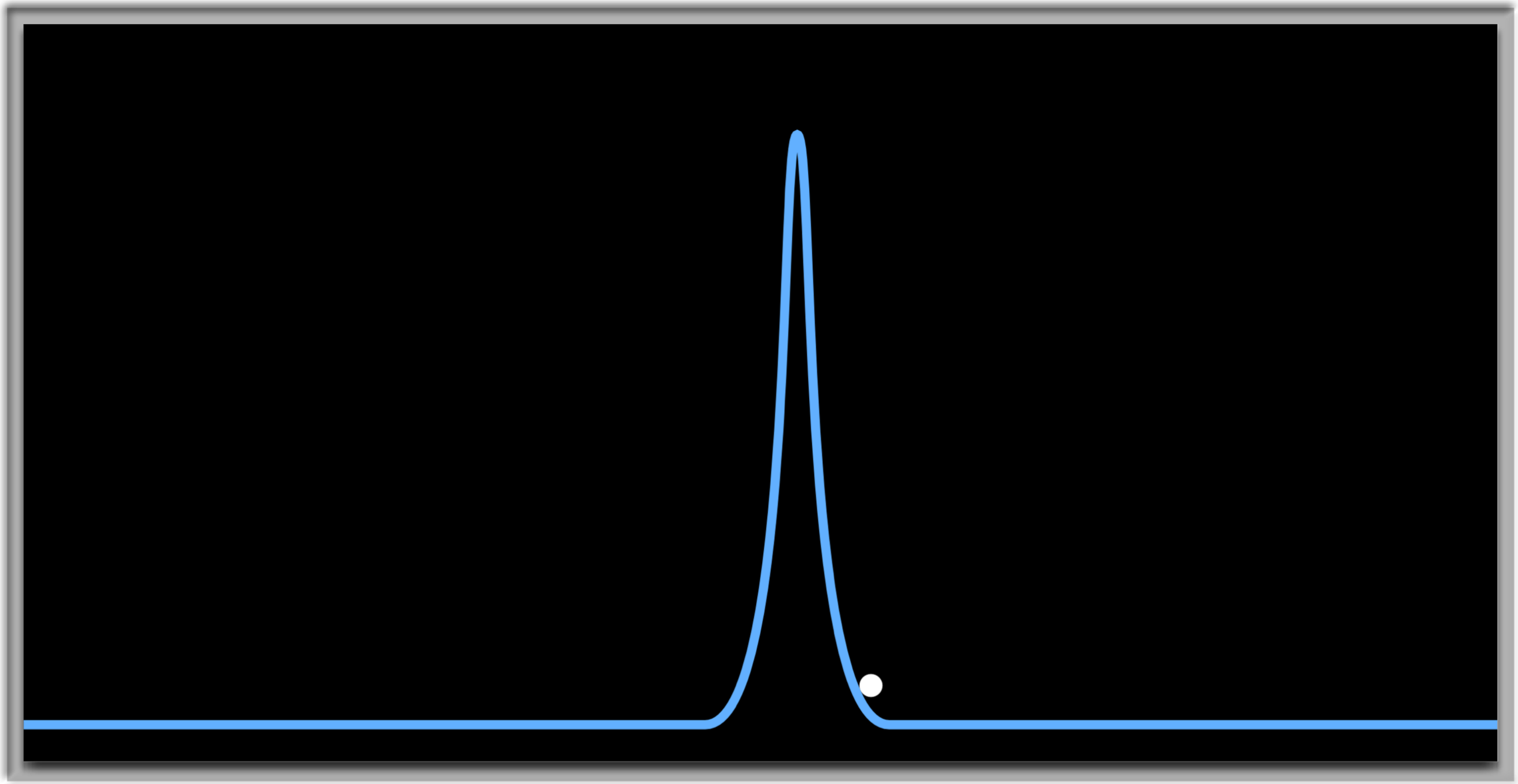
- Electron at rest is simply overtaken by the plasma wave

This is not surprising: after all, the plasma wave is comprised of electrons oscillating around their equilibrium position.

- The wave does not transport mass.
- The phase propagation is imprinted onto the individual particles' phases by the propagating driver.
- Any accelerated electrons have to surf the wave!



- Electron with sufficient initial velocity surfs



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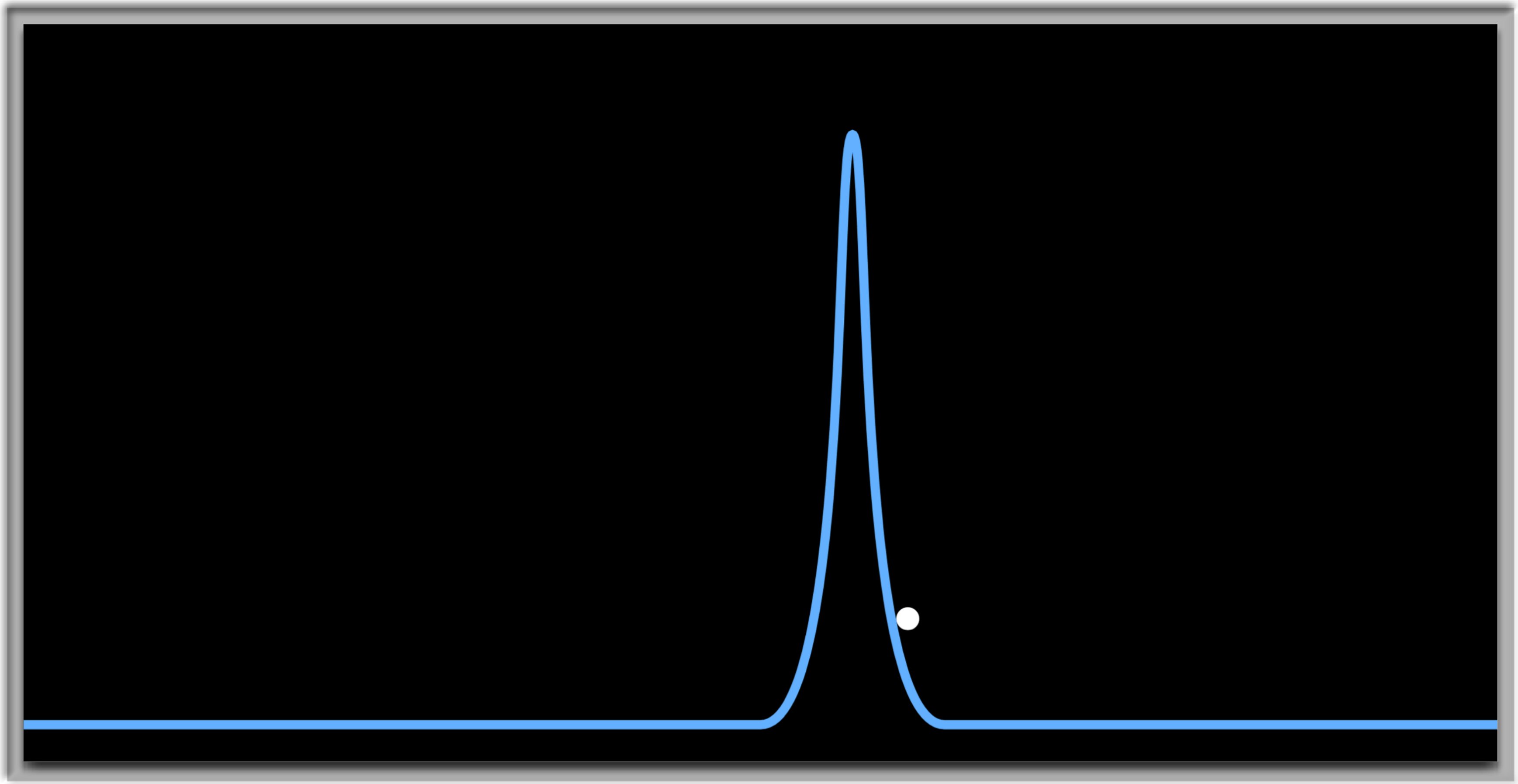

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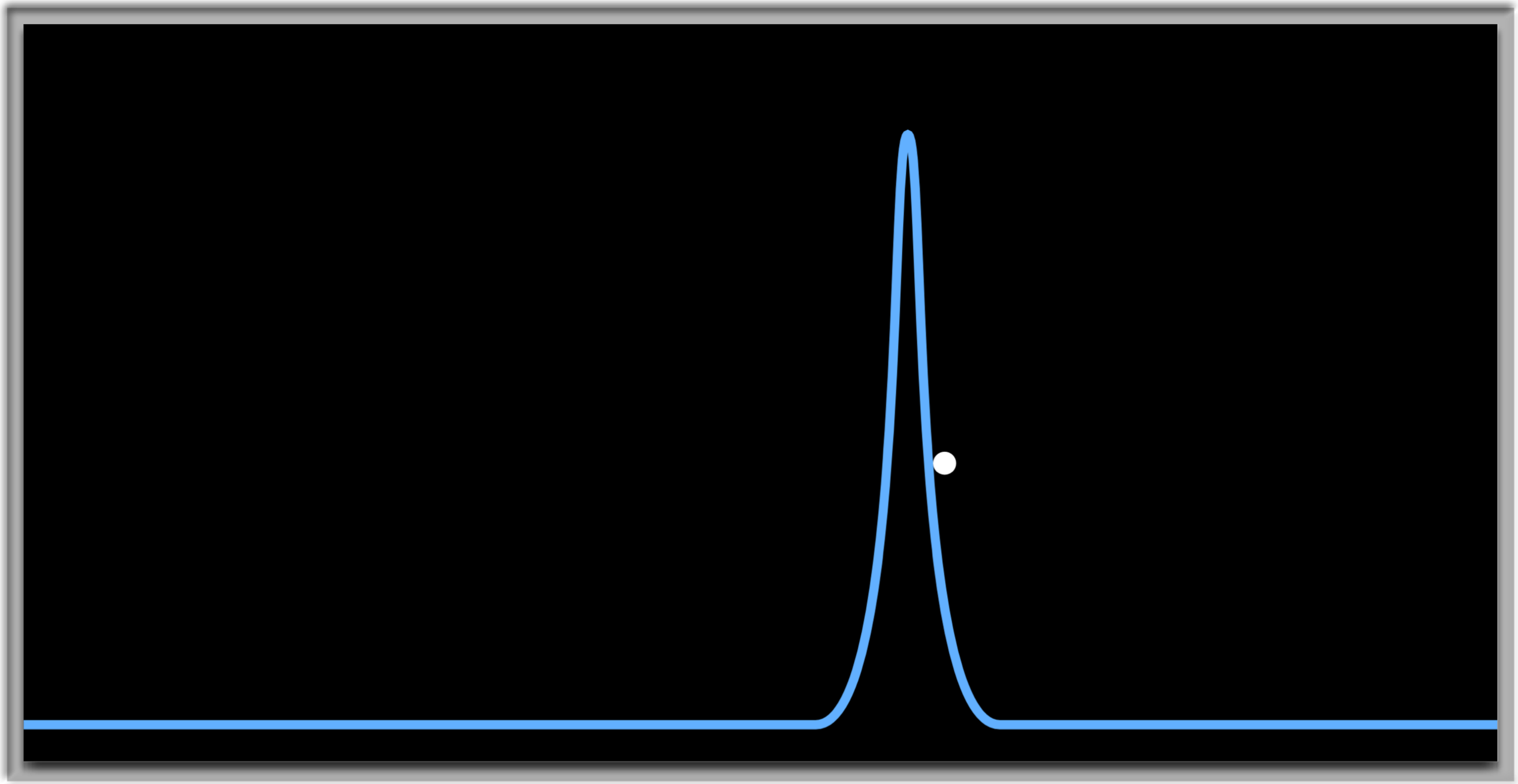

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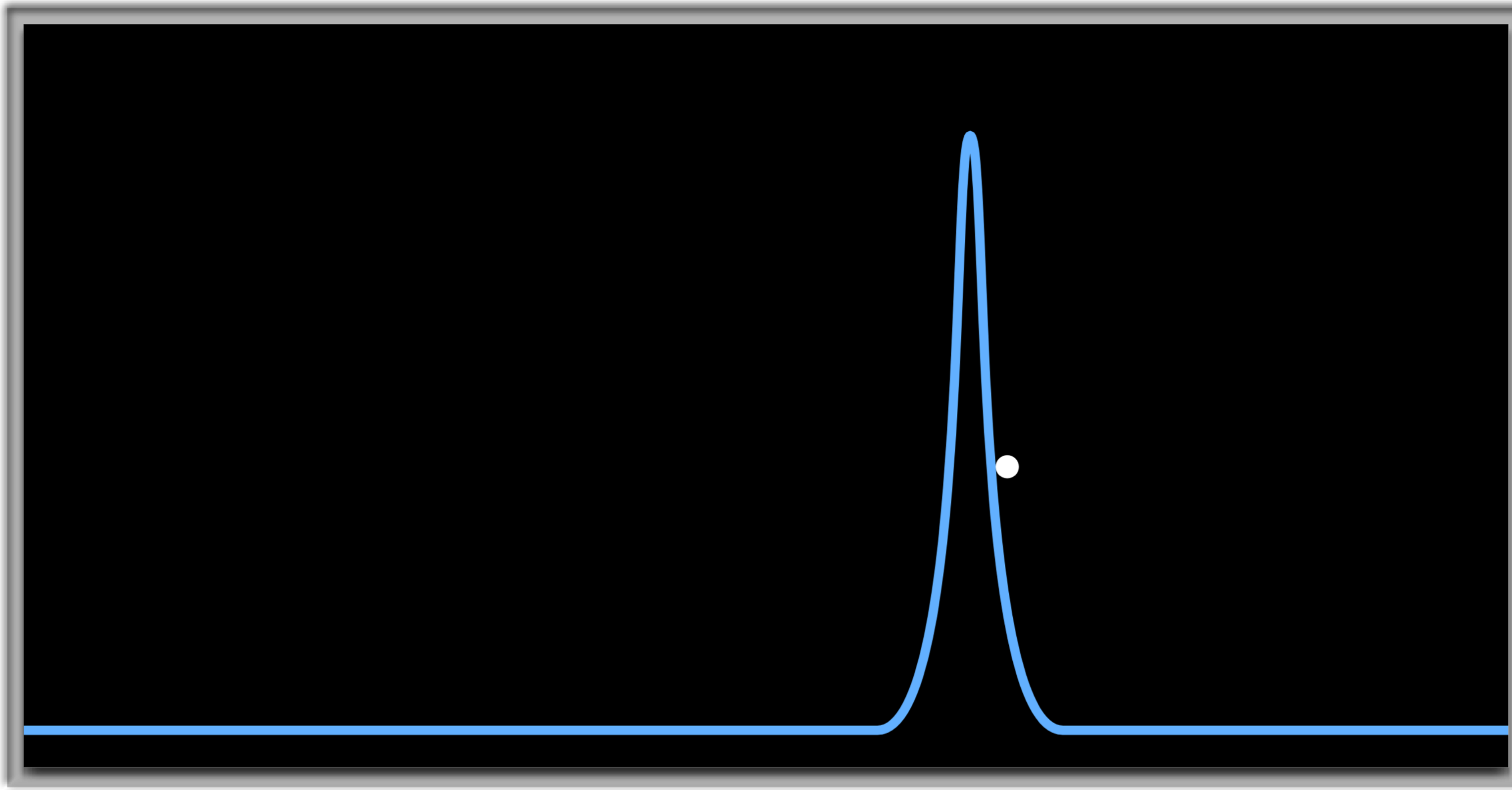
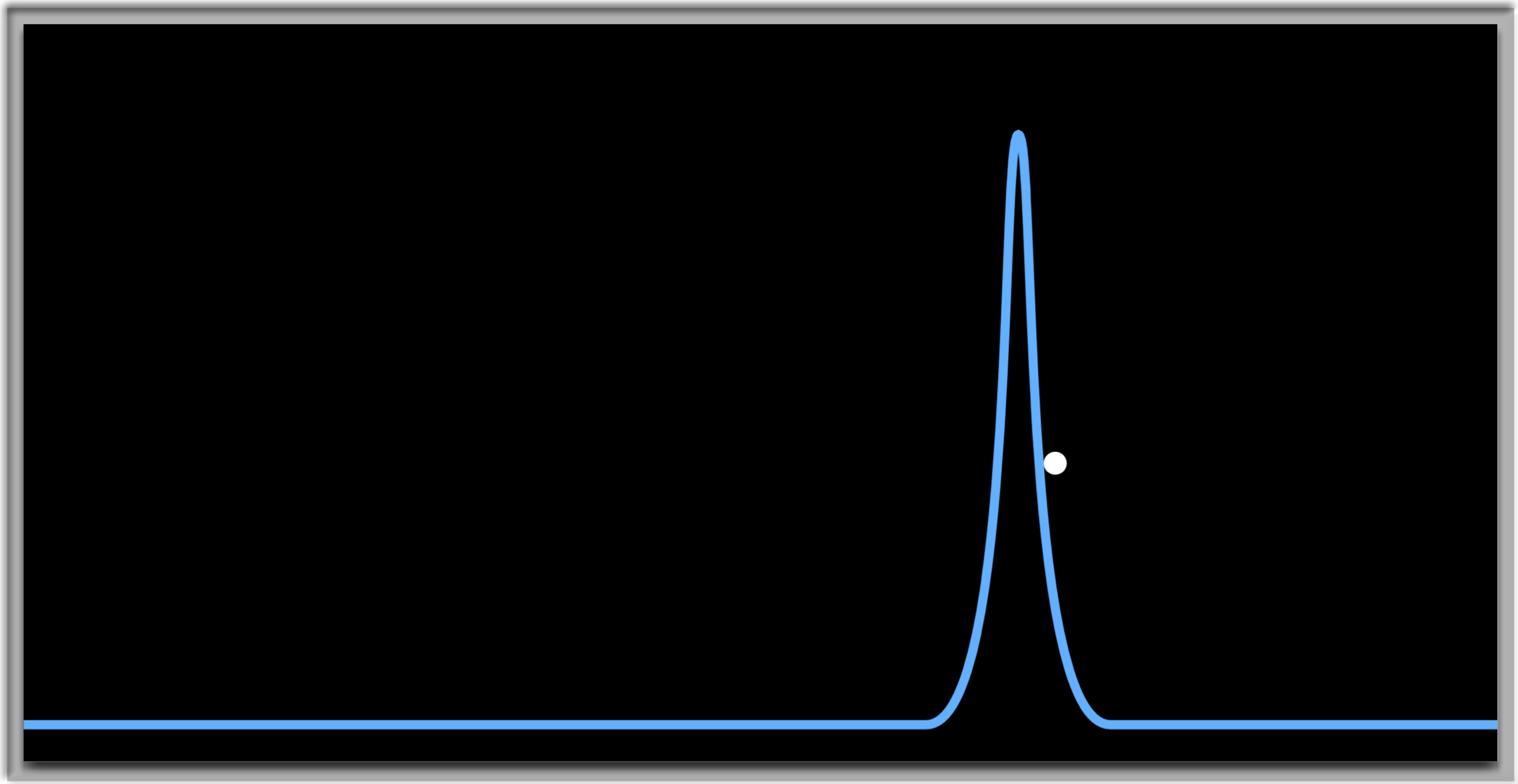


- Electron with sufficient initial velocity surfs



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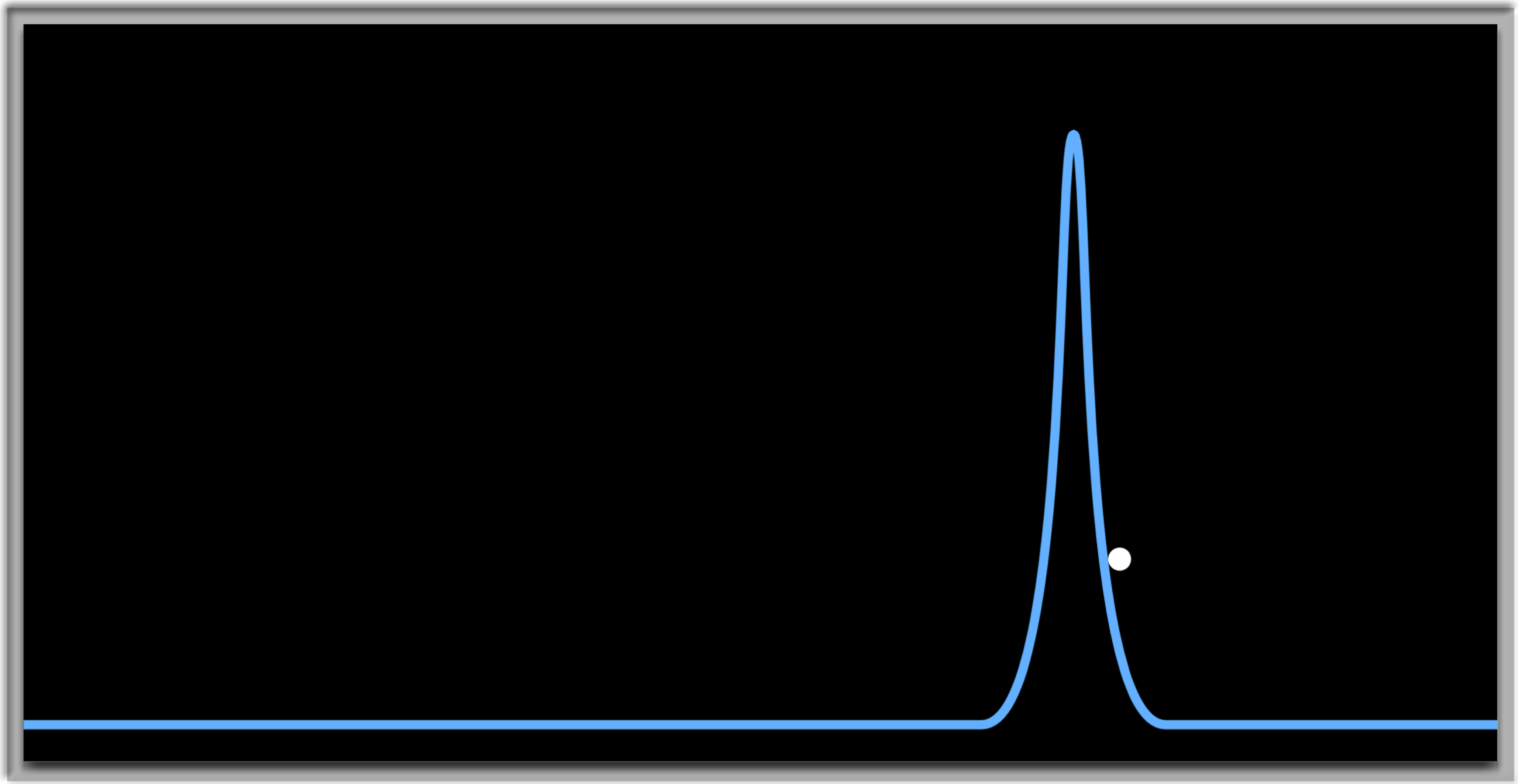

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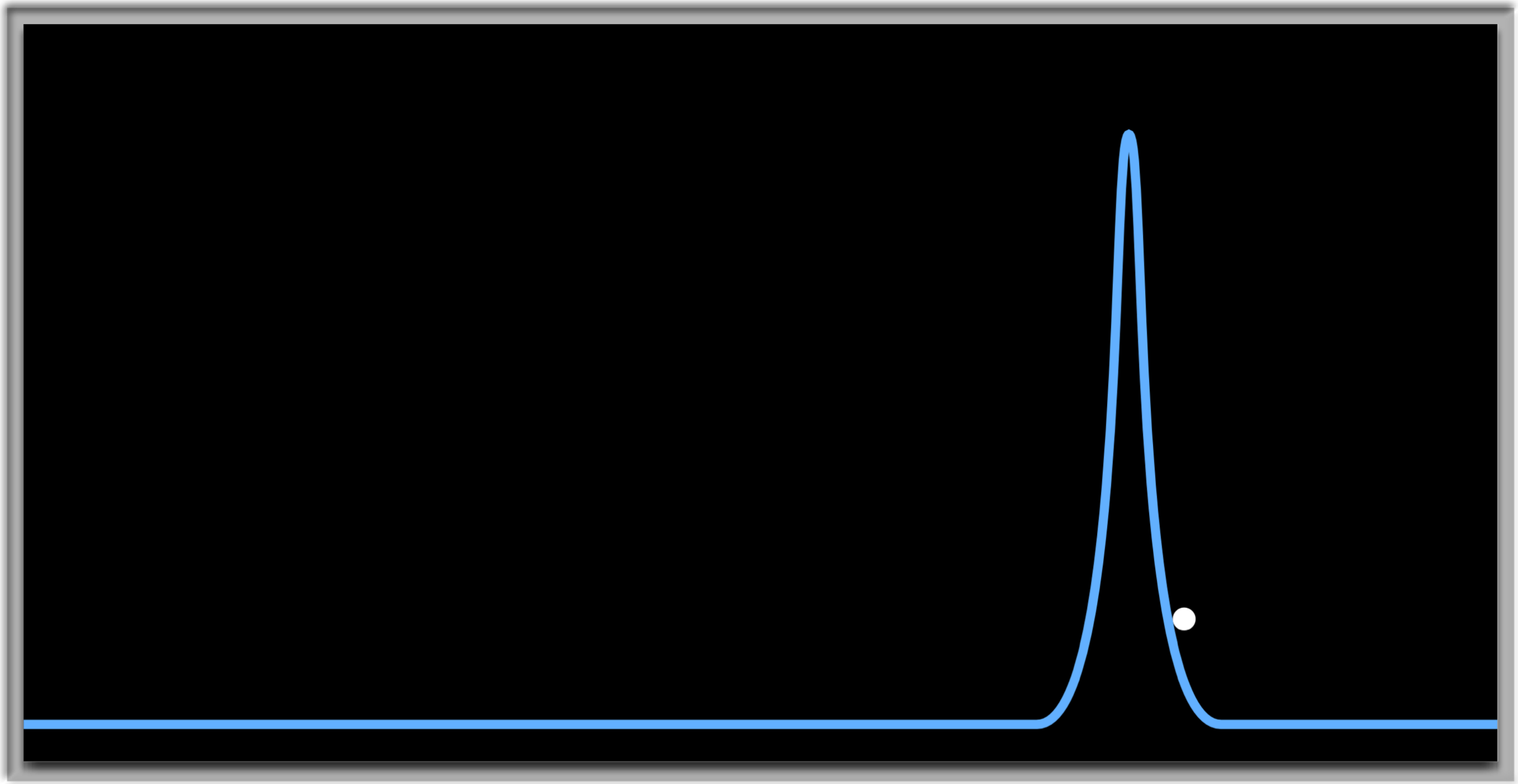

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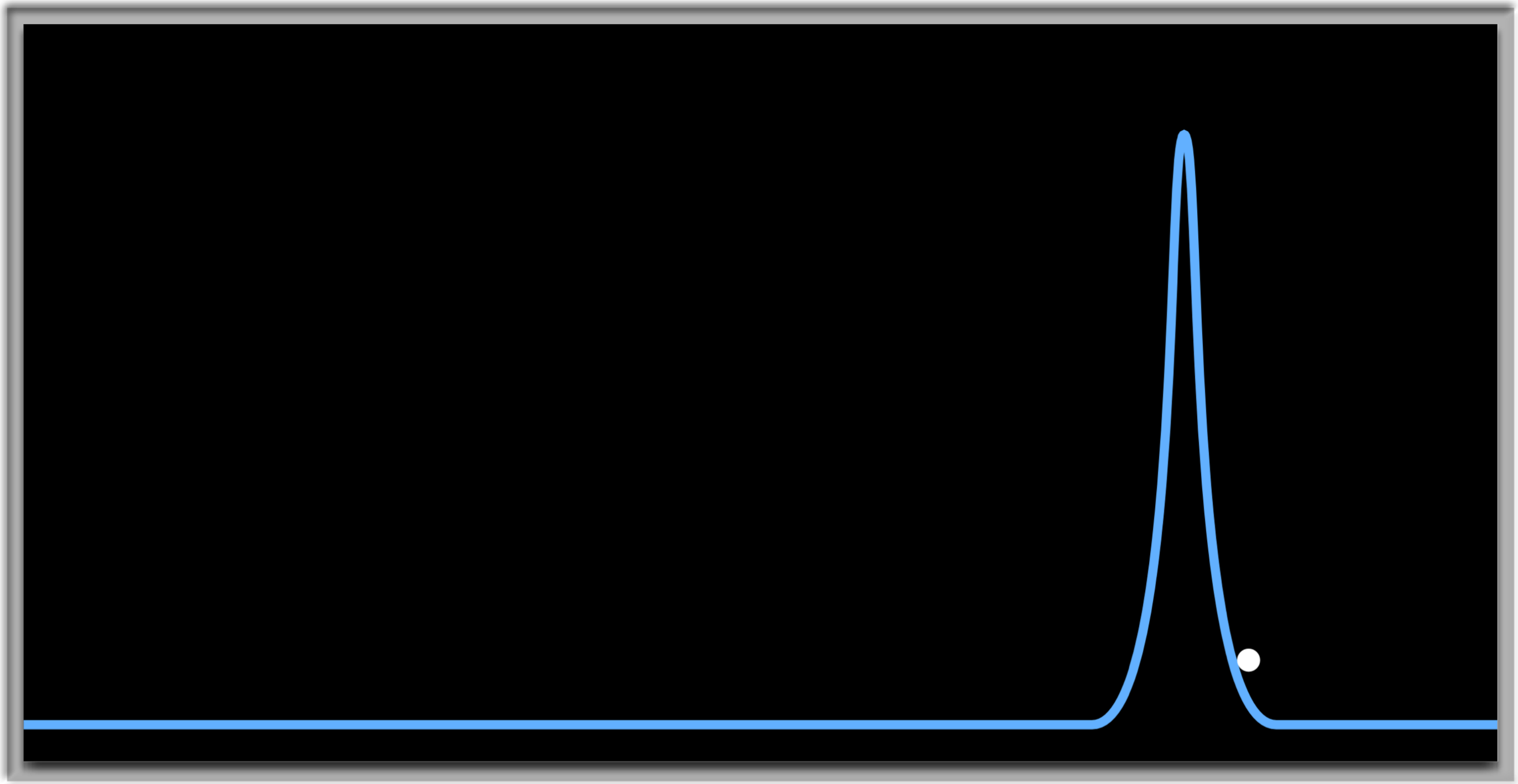

- Electron with sufficient initial velocity surfs

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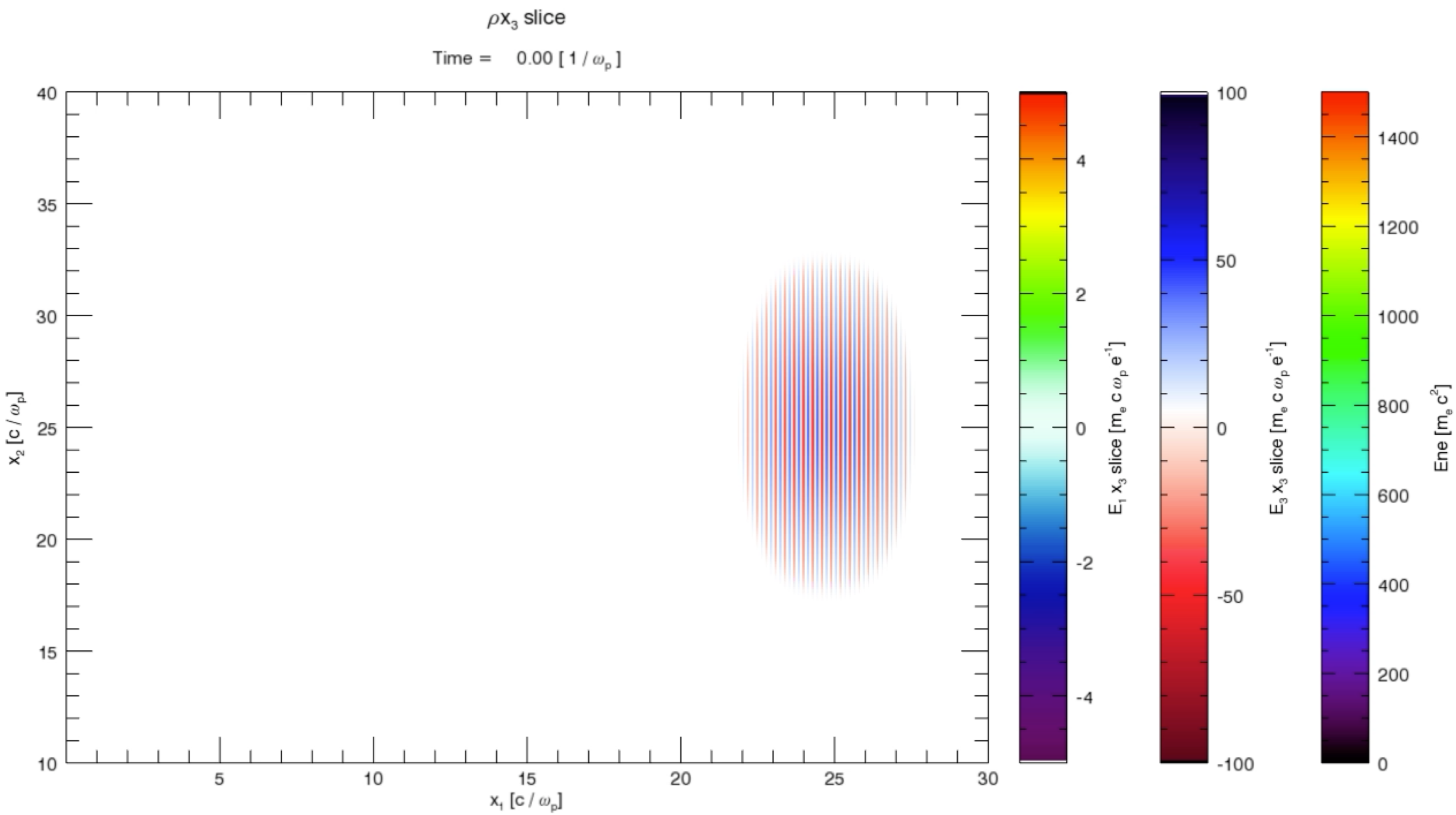
Injection: How do we get particles into the wake?



Trapping condition

When can an electron be trapped in the plasma wave?
Consider Hamiltonian of an electron interacting with the laser field in the presence of a plasma wave (normalized quantities):

$$H\left(z,u_z\right)=\underbrace{\sqrt{1+u_\perp^2+u_z^2}}_{=\gamma}-\phi\left(z-v_g t\right)$$

For an initially resting electron, due to conservation of canonical momentum, $u_\perp = a$. The second term represents the wake's potential. The time dependence can be eliminated by a canonical transformation $(z,u_z) \rightarrow (\xi, u_z)$[1]. The time-independent Hamiltonian then reads:

$$H\left(z,u_z\right)=\sqrt{1+a\left(\xi\right)^2+u_z\left(\xi\right)^2}+\phi\left(\xi\right)-\beta_g u_z\left(\xi\right)$$

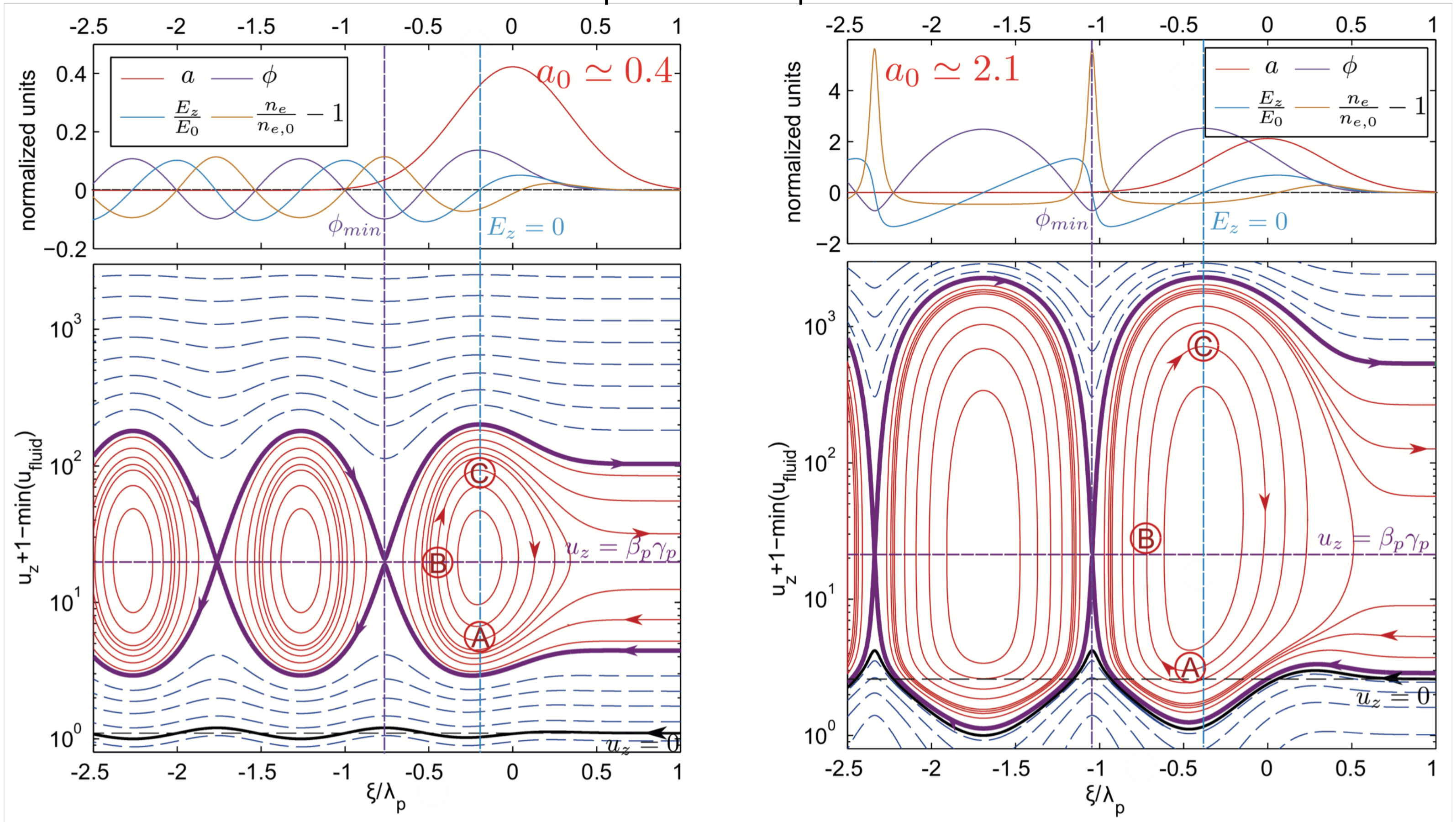
$H(\xi,u_z) = H_0$ = const. describes the motion of an electron with an initial energy $E= H_0$ on a distinct orbit in the plasma wave. Solving the the expression for the Hamiltonian for $u_z(\xi)$ gives the trajectory of the electron in the longitudinal phase space (ξ,u_z):

$$u_z=\beta_g\gamma_g^2\left(H_0+\phi\right)\pm\gamma_g\sqrt{\gamma_g^2\left(H_0+\phi\right)^2-\gamma_\perp^2}$$

$u_z(\xi)$ represents an electron orbit of constant total energy for a given set of $a(\xi)$, $\phi(\xi)$ and H_0

[1]With a generating function $F(z,u_z)=u_z\times(z-vgt)$ the new Hamiltonian reads $H = H' - 1/c\ \partial F/\partial t$

Separatrix plots

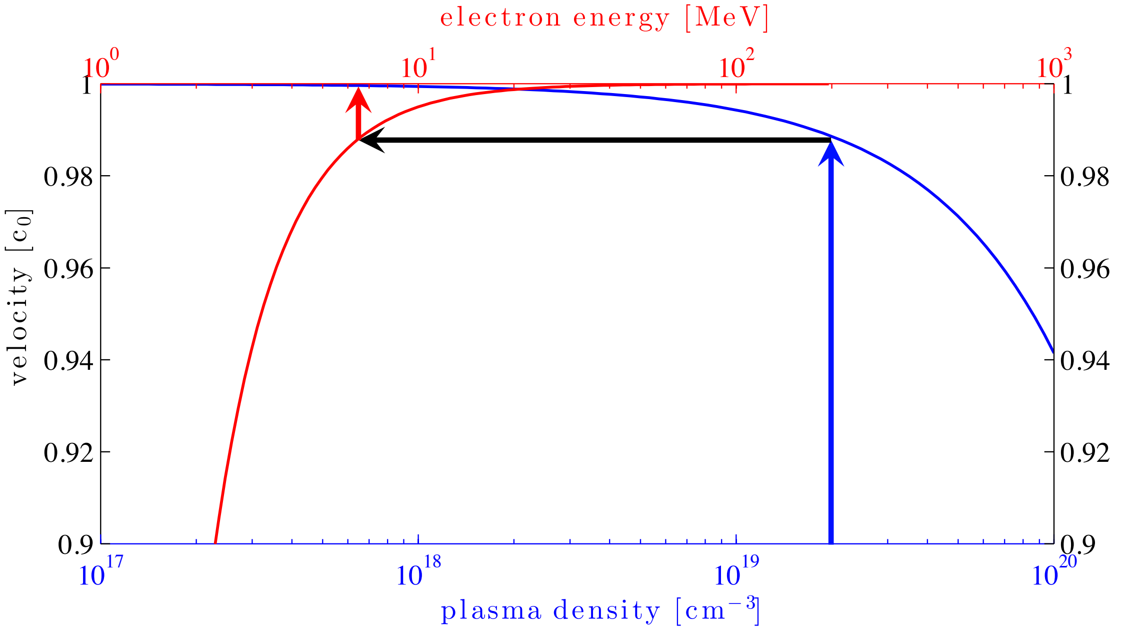


(red): trapped electrons on closed orbit. (blue): untrapped electrons on open orbit. (purple) Separatrix separating open and closed orbits with a radicand equal to zero. It crosses itself at ϕ = min (purple vertical line). The Hamiltonian of the separatrix is given by $H_{sep} = \gamma_\perp(\xi_{min})/\gamma_g - \phi_{min}$. Electrons initially at rest (H_{fluid} = 1, $u_\perp(\xi = +\infty) = u_z(\xi = +\infty) = 0$, black) do not gain momentum from the plasma wave. Electrons with a too low/high initial momentum (dashed blue lines) $|H_0| > |H_{sep}|$ are moving on open orbits

Photo by Mike Baird, published under Creative Commons Licence on Flickr

Surfing is only possible if initial velocity is close to speed of the wave, otherwise the surfer just goes up and down.

The surfer needs to catch the wave in the same way as the electrons ➔ Injection problem



The phase velocity of a cold plasma wave is

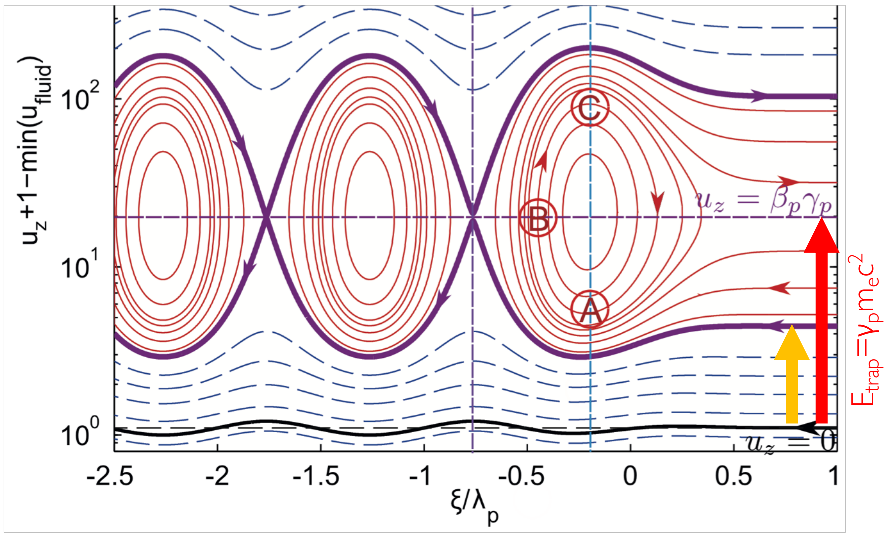
$$\beta_{ph} \simeq 1 - \frac{1}{2}\frac{n}{n_c}$$

While the electron velocity is

$$\beta_e = \sqrt{1 - \gamma^{-2}}$$

Is this the end of the story?

Trapping condition for e⁻ overtaken by wakefield (external injection)



In 1-D, the trapping condition reads:

$$E_{trap} = m_e c^2 \left(\sqrt{1 + u_{z,sep}^2(+\infty)} - 1 \right)$$

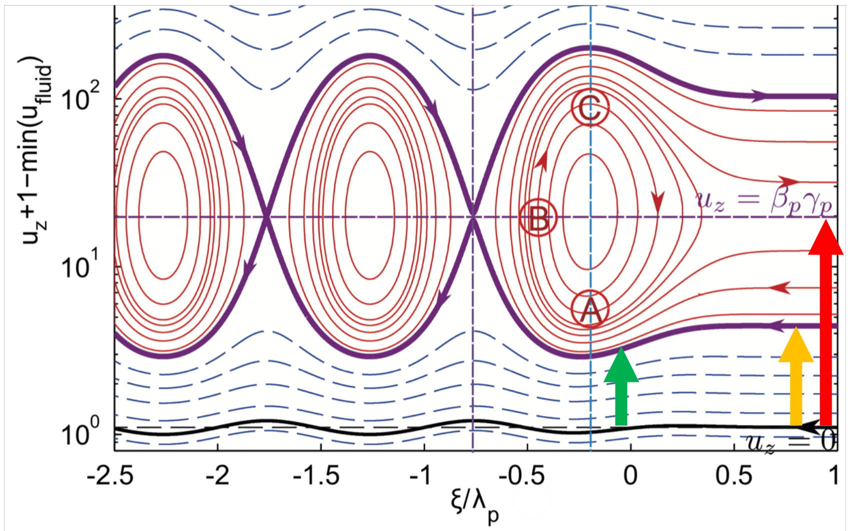
with:

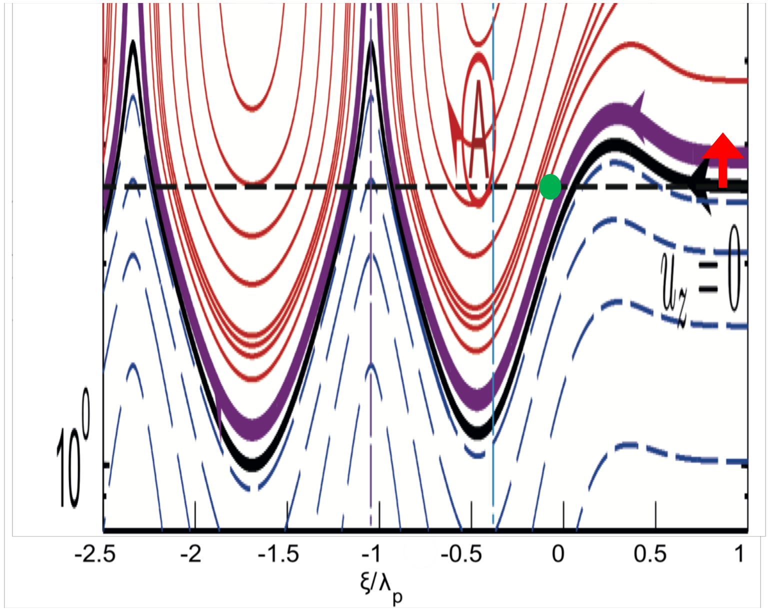
$$u_{z,sep}(+\infty) = \beta_p \gamma_p^2 H_{sep} - \gamma_p \sqrt{\gamma_p^2 H_{sep}^2 - 1}$$

being the separatrix distance in front of the laser ($a_0=\phi=u_\perp=0$)

- Electrons with a forward momentum substantially lower (how much depends on wake amplitude) can be caught and gain maximum energy at point C if acceleration would terminate there.
- Unfortunately, in an unperturbed plasma no such electrons exist in front of the laser pulse (E_{trap}>>$E_{thermal}$).
- Since everything co-moves with velocities close to c, the time to overtake an injected electron at this energy might easily exceed the time the laser stays focused.

How about even lower thresholds?





Electrons gain threshold energy inside wake bucket

Electrons are born inside wake bucket

Colliding pulse injection

Ionization injection

Colliding pulse (beat wave) injection

Consider two counter-propagating, c.p. laser pulses:

$$a_{1/2} = \frac{a_{1/2}(t)}{\sqrt{2}}\left(\cos\left(k_L z \pm \omega_L t\right)\vec{e}_x + \sin\left(k_L z \pm \omega_L t\right)\vec{e}_y\right)$$

where $a_{0,1/2}(t)$ are the temporal pulse shapes for both pulses

With the beat-wave Hamiltonian

$$H_{beat} = \sqrt{1 + u_\perp^2 + u_z^2} = \sqrt{1 + \left(a_1 + a_2\right)^2 + u_z^2}$$

we get a beat-wave separatrix:

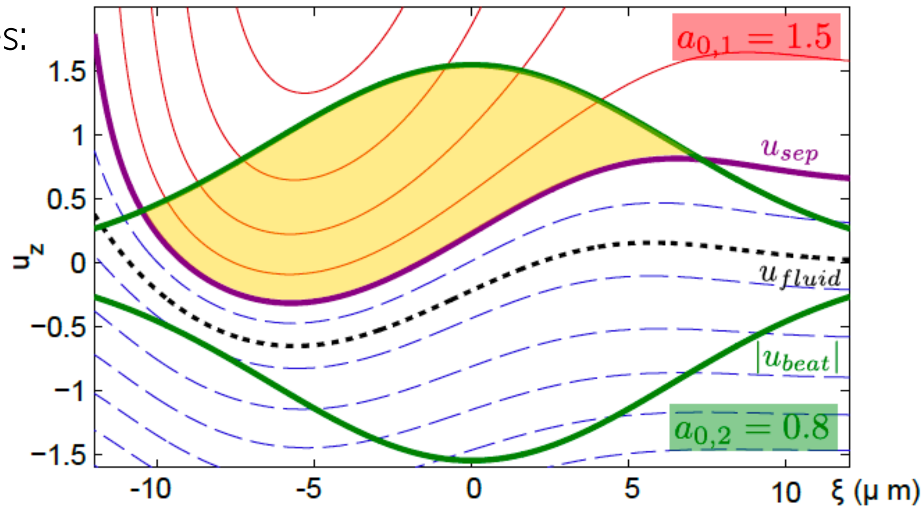
$$u_{beat}(t) = \pm\sqrt{a_{0,1}(t)\, a_{0,2}(t)\left(1 - \cos\left(2\omega_L t\right)\right)}$$

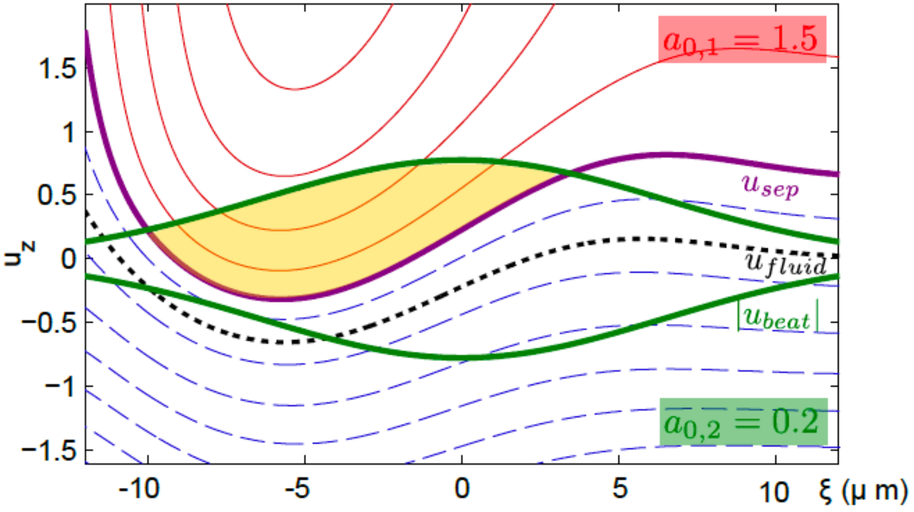
$$u_{beat,\max/\min}(t) = \pm\sqrt{2 a_{0,1}(t)\, a_{0,2}(t)}$$

$$W_{beat}(t) = m_e c^2 \sqrt{1 + u_{beat}(t)^2} - 1$$

Injection if (in co-moving frame):

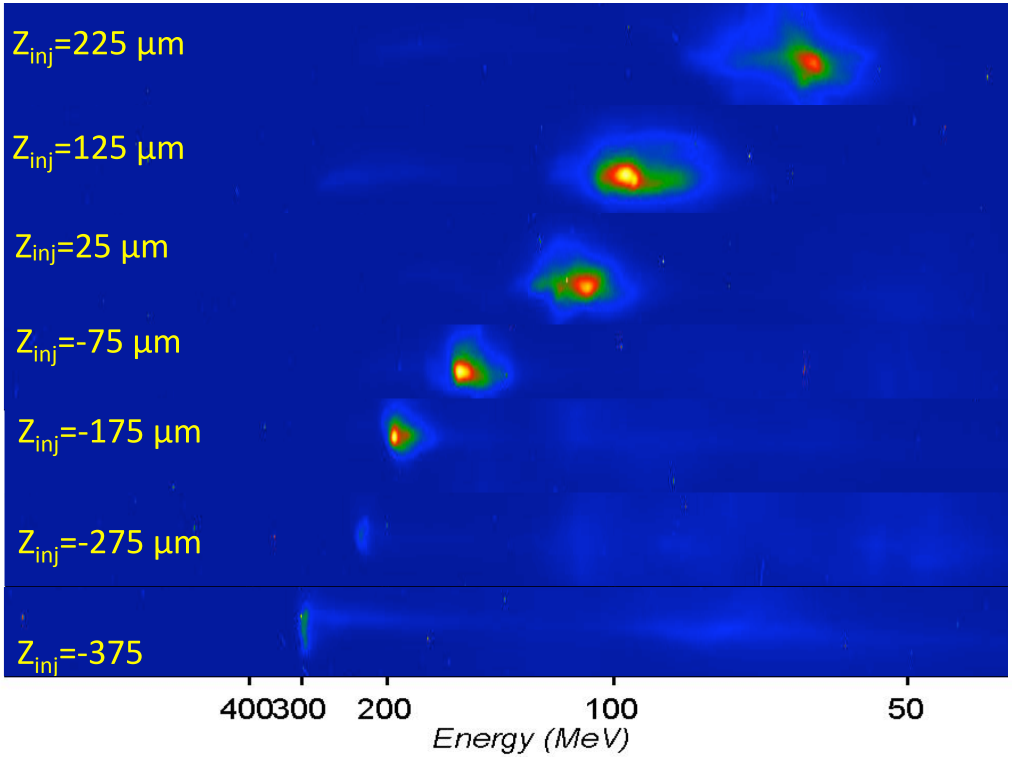
$$u_{beat,\max}(\xi) > u_{sep}(\xi)$$

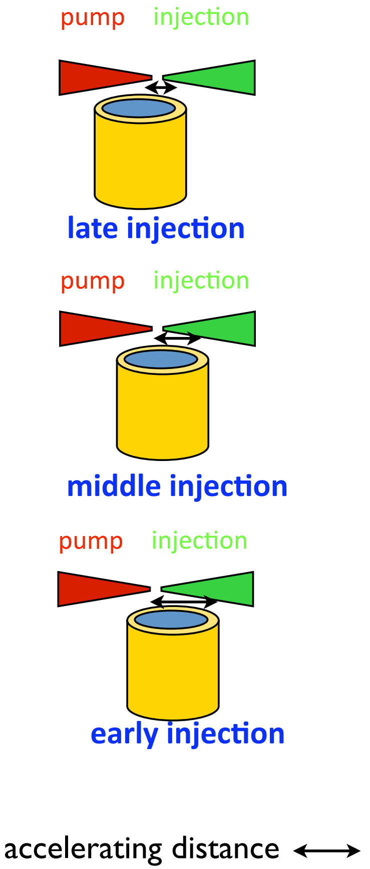




Colliding pulse (beat wave) injection exp.

- Localized injection leading to quasi-monochromatic beams
- Adjustable energy via tuning of collision (injection) position





J Faure et al., Nature 444, 737 (2006)

Ionization injection

Gas target contains traces of high-Z gas, which is ionized by the peak of the laser and born at ξion~0 at rest ($u_z(\xi_{ion})\sim 0$):

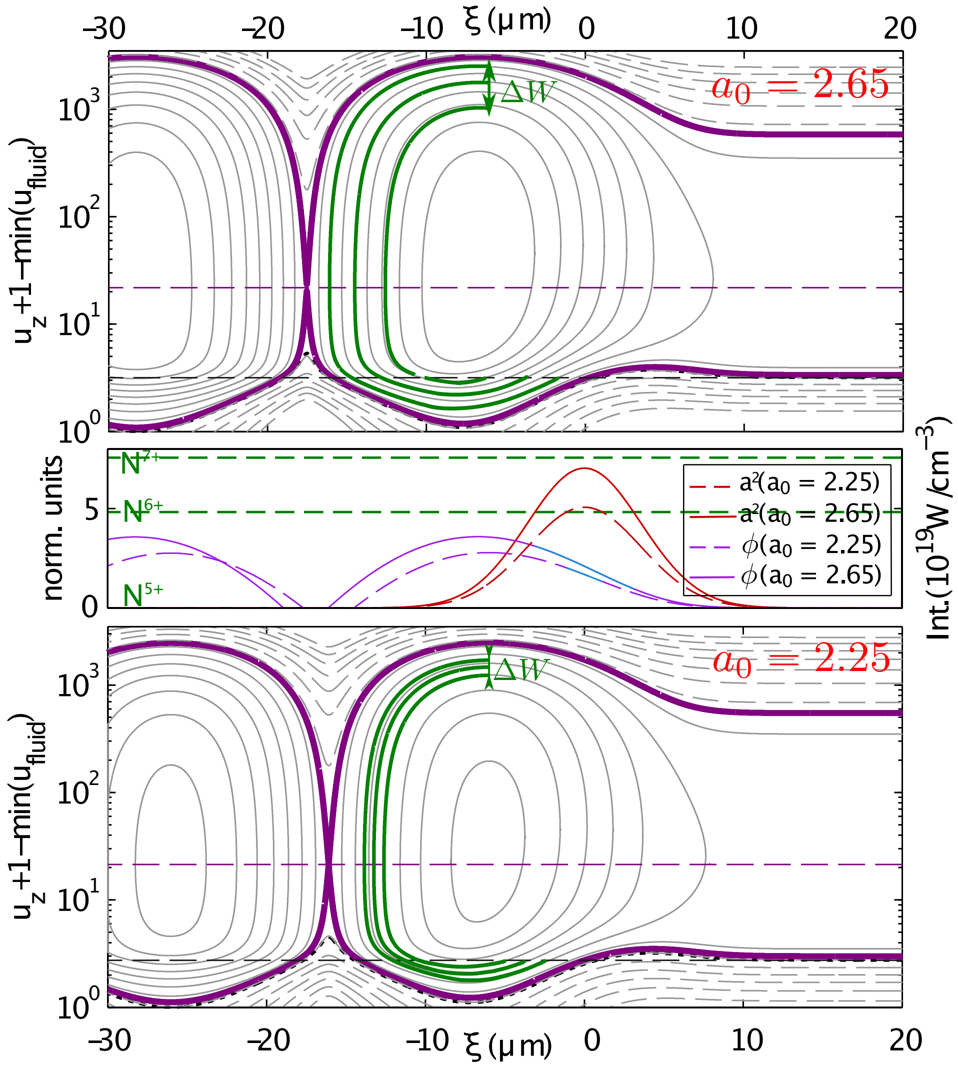
$$H_{ion} = 1 - \phi(\xi_{ion})$$

Trapping condition[1] for sin-envelope pulses:

$$1-\gamma_p^{-1} \le \phi(\xi_{ion}) - \phi_{\min} \le \phi_{\max} - \phi_{\min} \sim \underbrace{\left(\frac{\pi}{8}+\frac{1}{4}\right)}_{\sim 0.64} a_0^2$$

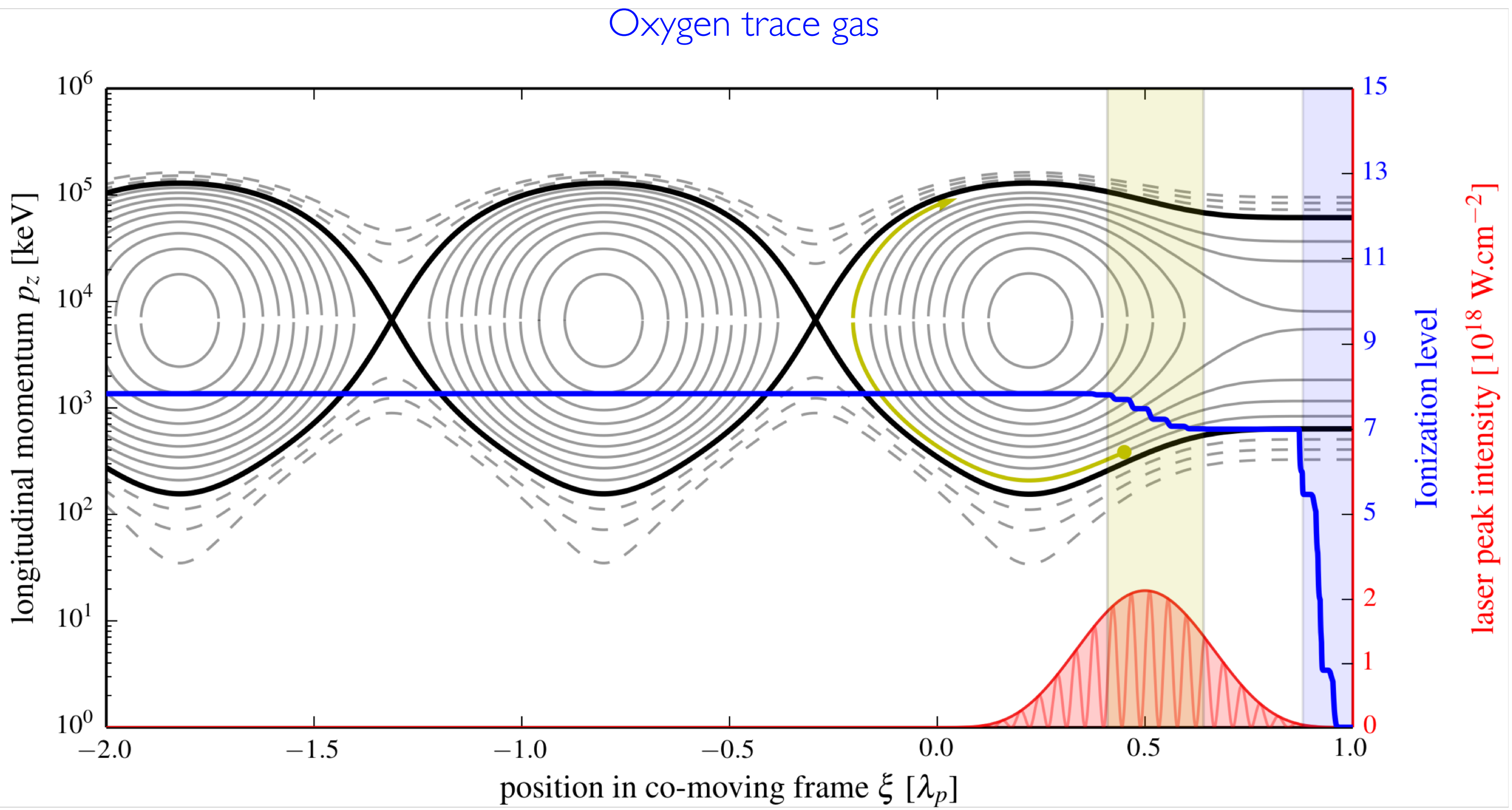
Ionization injection only works for relativistic intensity ($a_0^2 > 1.6$) pulses!

(even if ionization threshold would be lower)



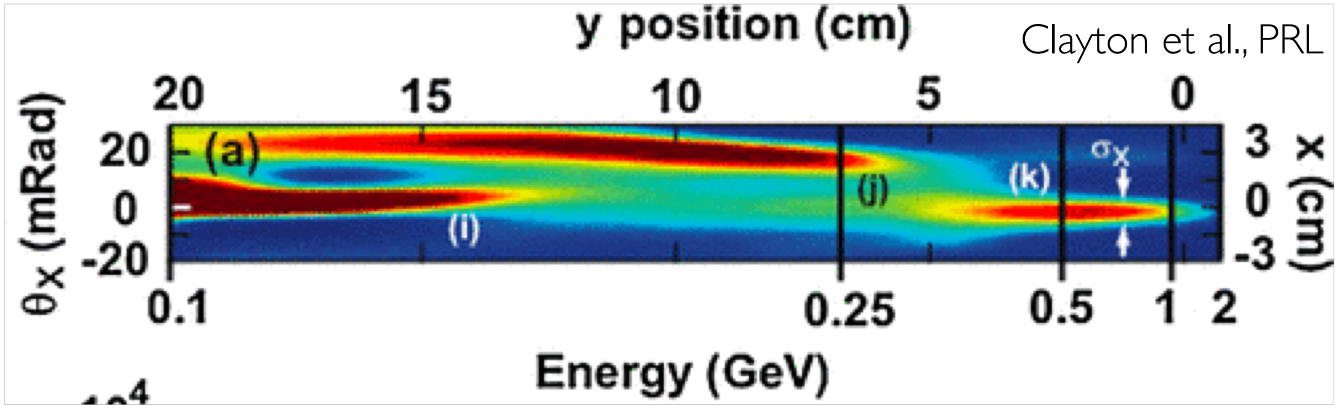
[1]Chen et al, Phys. Plasmas 19, 033101 (2012)

Ionization injection II



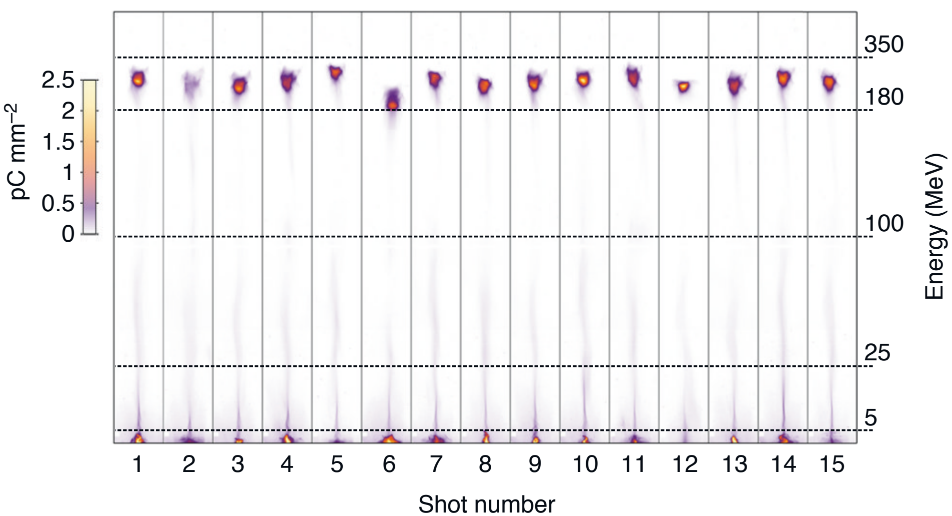
Ionization injection exp.

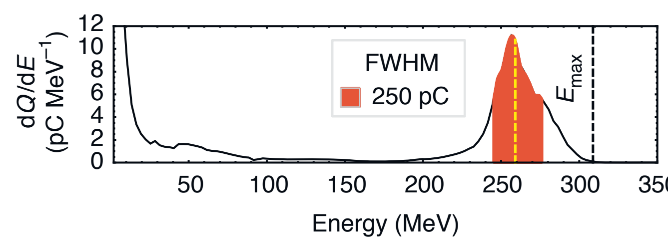
- Constant injection commonly leads to broadband spectra, but high charge...



Clayton et al., PRL 105, 105003 (2010)

- ... which can be used to fully beamload and truncate the injection





Couperus et al, Nat. Comm. 8, 487, 2017

„Longitudinal injection"

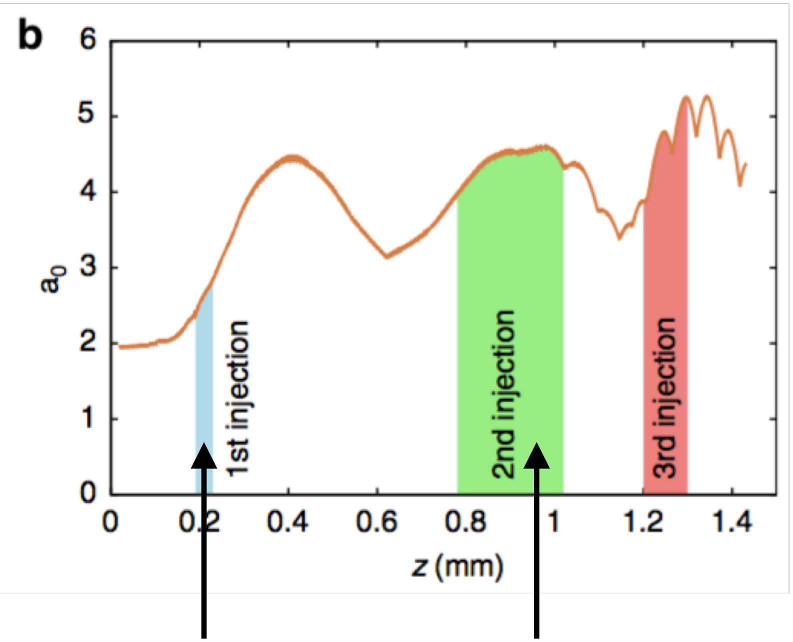
Instead of giving an electron the correct energy at the correct phase, it is possible to shift the wake phase to gobble up electrons from other phase positions.

Any sudden shift in plasma wavelength our driving phase will shift the wake phase.

Shift by laser intensity variation	Shift by density step / slope	Shift by driver swap
↓	↓	↓
longitudinal/transverse self-injection	density down ramp/shock front injection	Hybrid injection

all these schemes will cause the wave to break momentarily or continuously

Longitudinal self-injection vs. transverse self-injection



Wakefield expansion

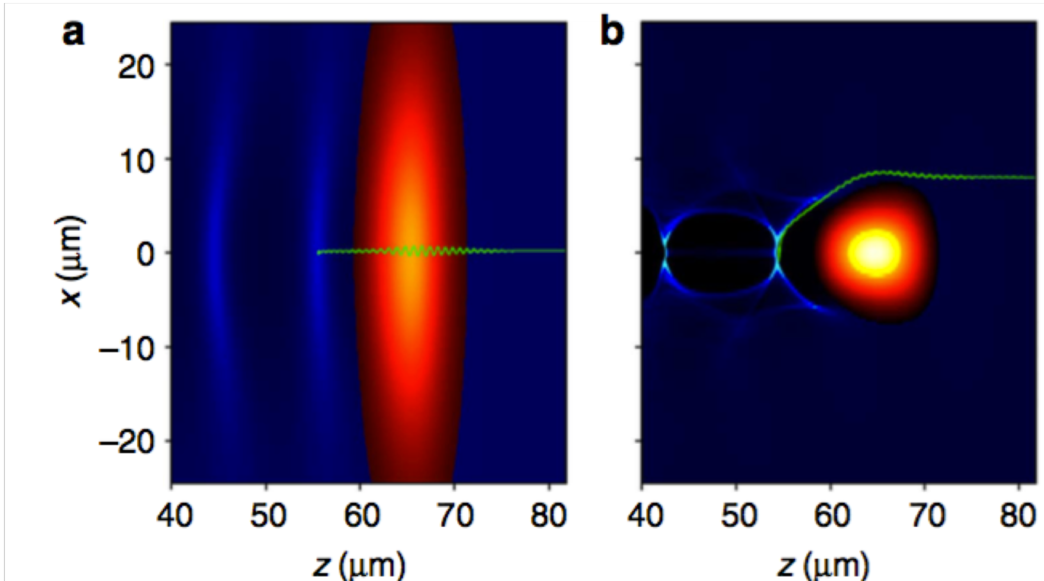
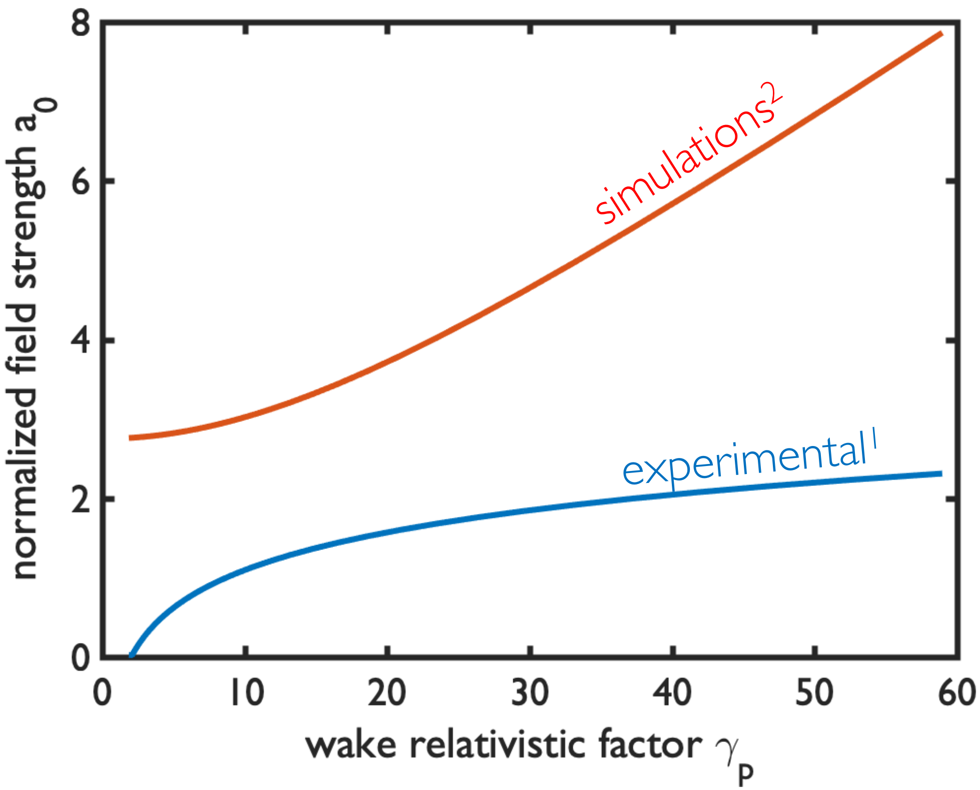


Figure 1 | Schematic for longitudinal and transverse self-injections. (**a**) Typical trajectory of an injected electron in the longitudinal self-injection mechanism. (**b**) Typical trajectory of an injected electron in the transverse self-injection mechanism. The blue colour scale represents the electron density. The red to yelow colour scale indicates the laser intensity. The trajectories are given by the green lines.

Several injection events cause broadband energy distribution

S. Corde et al., Nature Comm. 4, 1501 (2013)

Self-injection threshold



Why such a large difference?

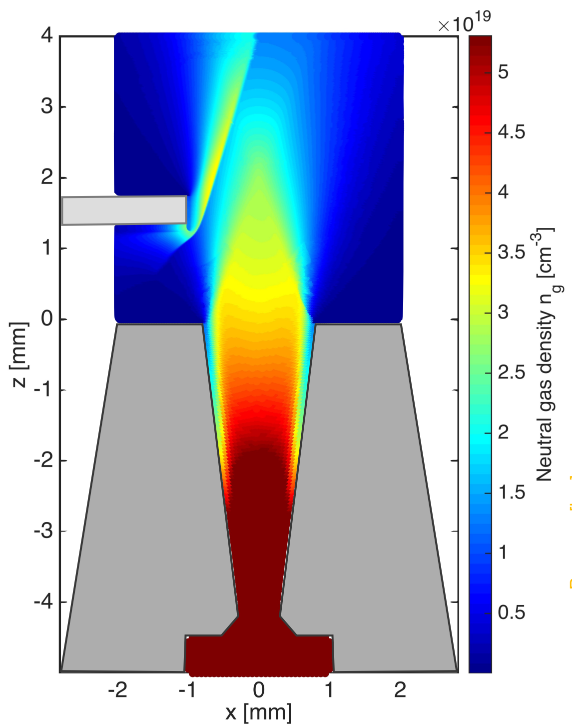
The simulations deliberately omit any laser pulse evolution and therefore give thresholds in terms of a_0 at the injection point.

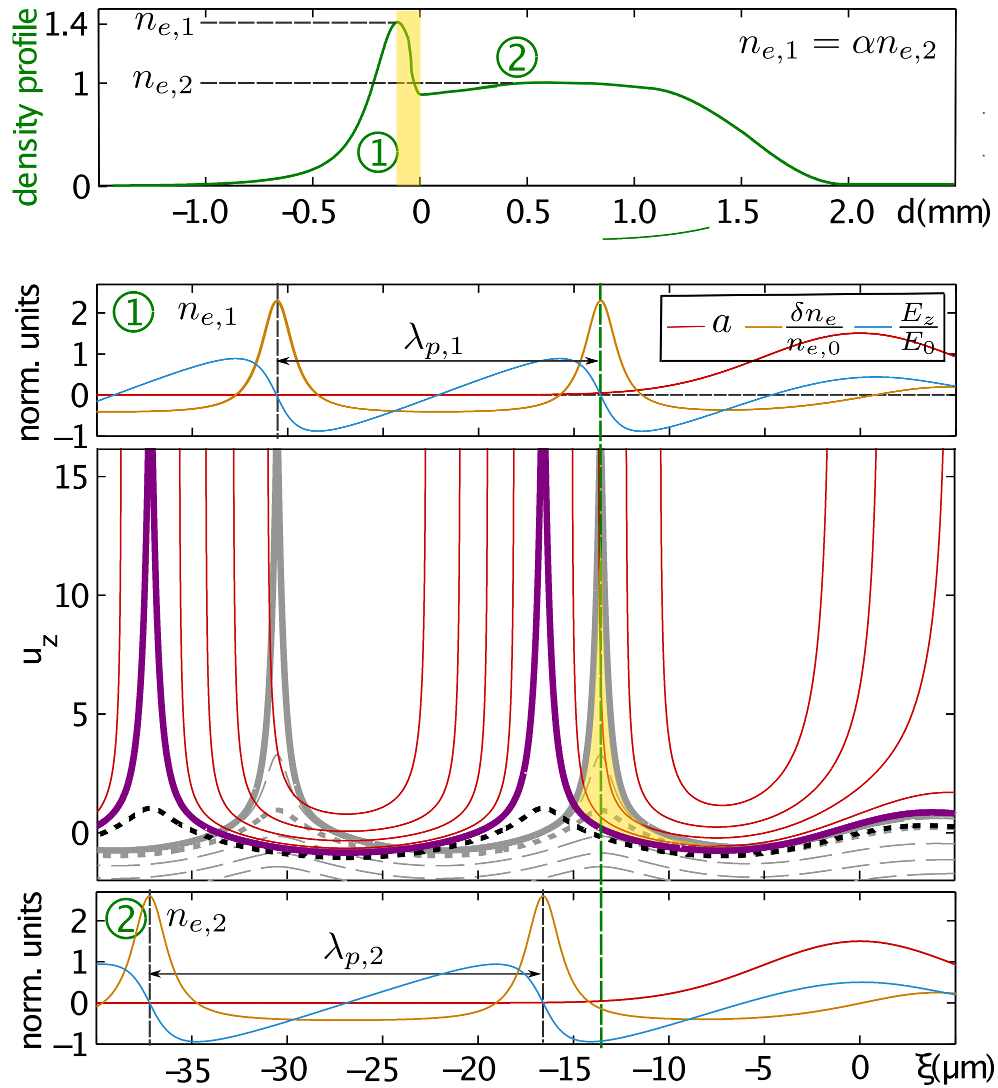
The experimental data includes all these effects and gives thresholds in terms of vacuum a_0, yet only for matched conditions.

[1] Mangles et al., PRSTAB 15, 011302 (2012)
[2] Benedetti et al, Phys. Plasmas 20, 103108 (2013)

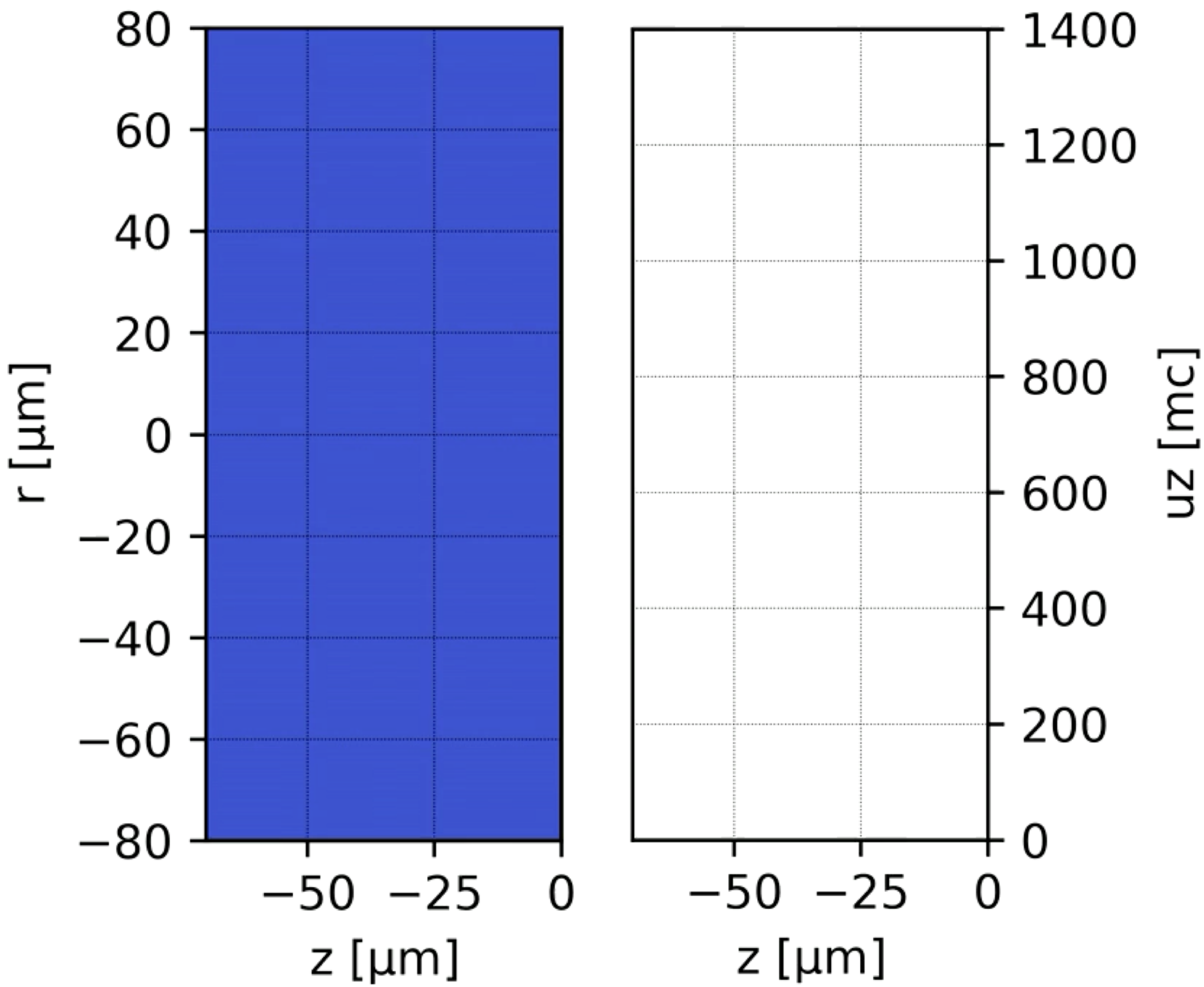
Shock-front/down-ramp injection

Shock front in a gas jet provides sudden density drop. Fluid electrons from first density peak in region 1 rephase and are trapped in region 2.



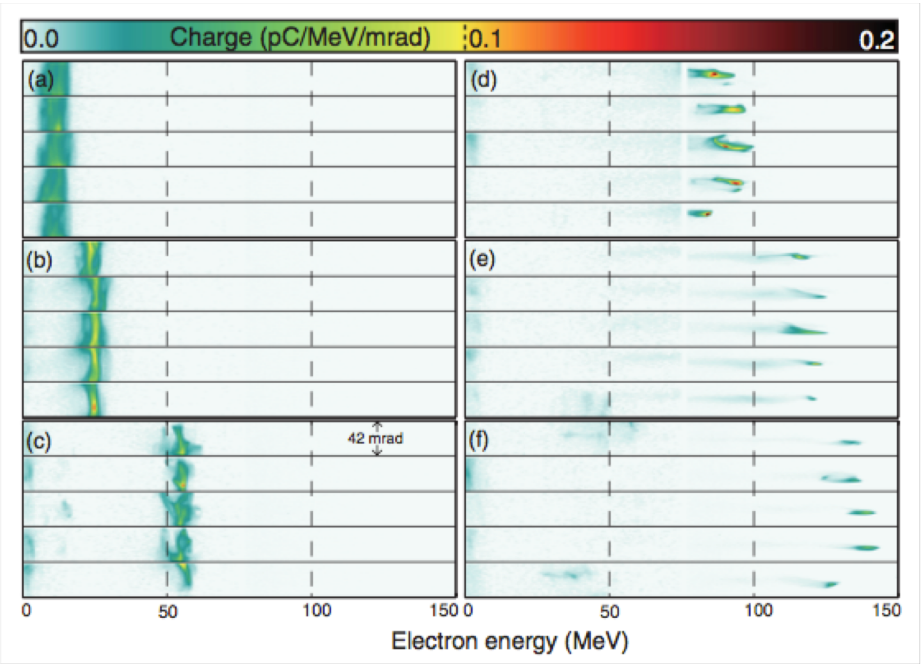


Injection: How do we get particles into the wake?

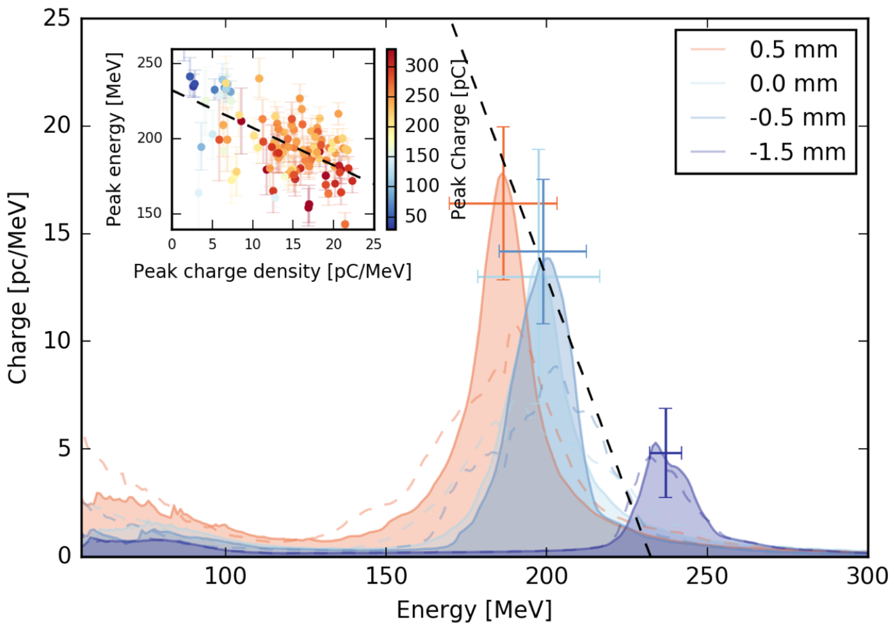


Shock front: Stable / adjustable energy and charge

Moving the blade

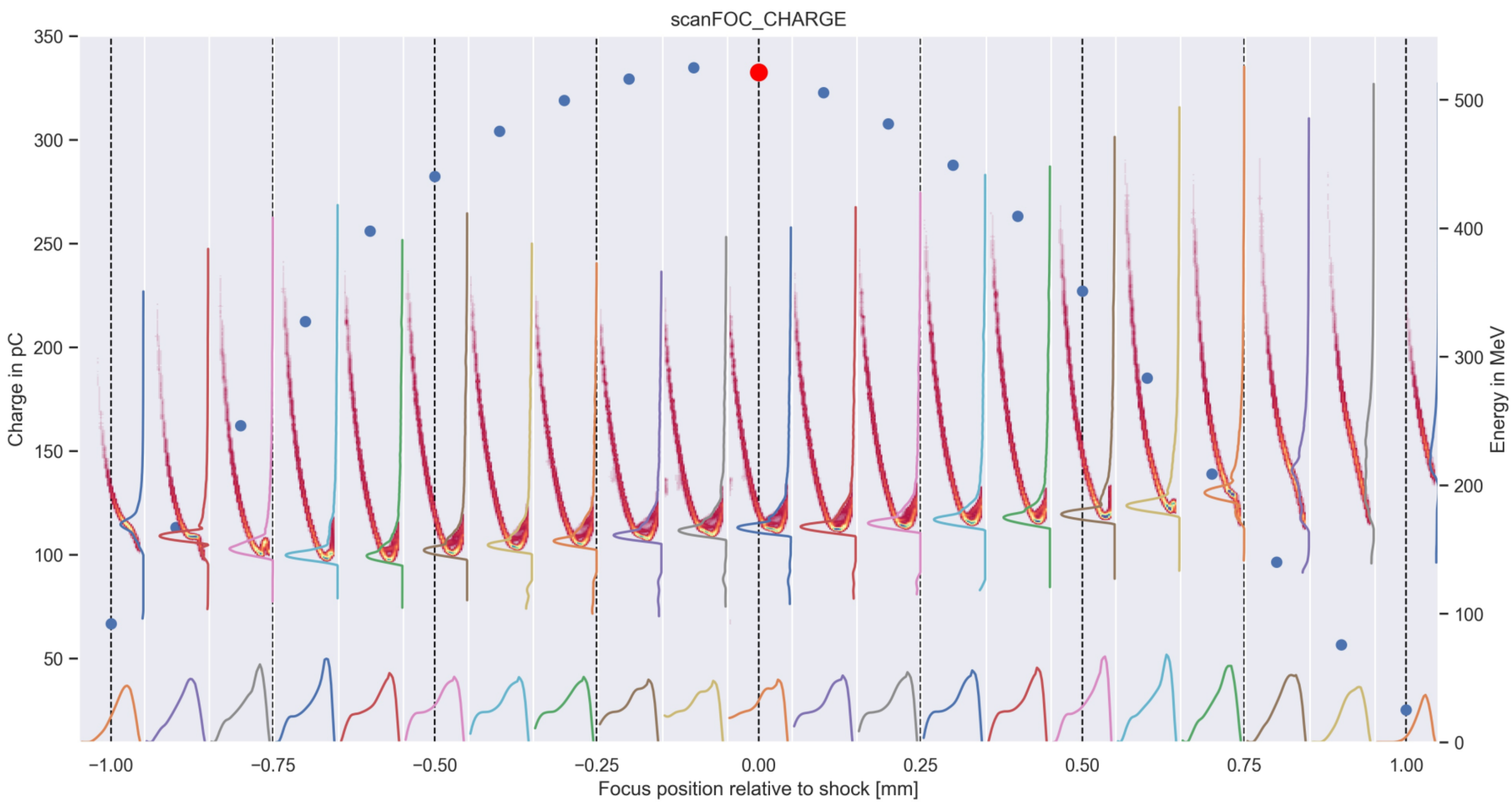


Moving the focus position / tuning a_0 at shock

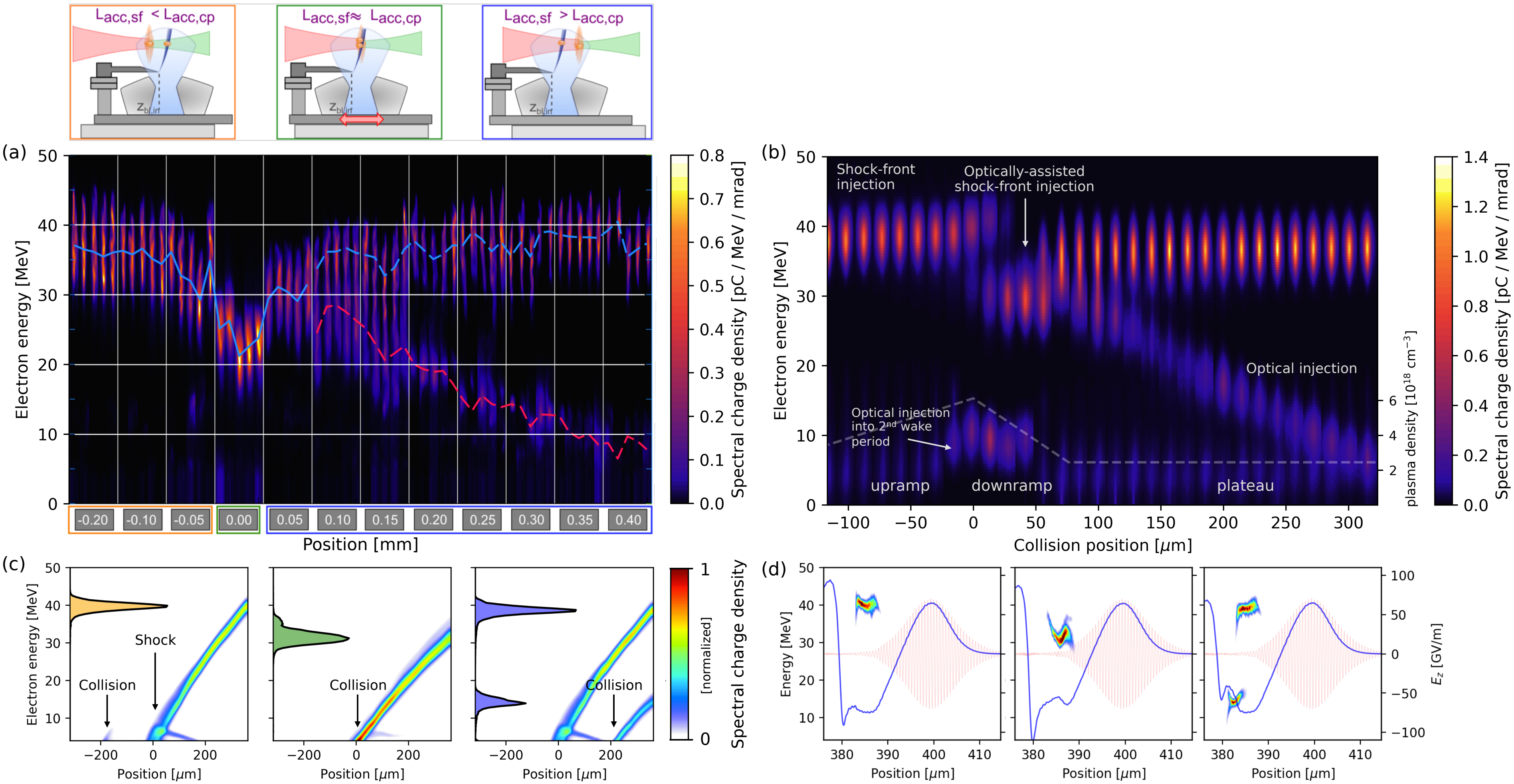


A. Buck et al., PRL 110, 185006 (2013)

Shock front: positive electron chirp



Dual beams via combined injection: colliding pulse + shock



Wenz et al., Nature Photonics, DOI: 10.1038/s41566-019-0356-z (2019)

	a_0	w_0	L_d	L_{pd}	λ_p	$E_z/E_{p,0}$	$\Delta W/m_e c^2$
Linear	< 1	$\frac{2\pi}{k_p}$	$\frac{\pi}{k_p}\frac{\omega_L^2}{\omega_p^2}$	$\frac{c\tau_L}{a_0^2}\frac{\omega_L^2}{\omega_p^2}$	$\frac{2\pi}{k_p}$	a_0^2	$\pi a_0^2 \frac{\omega_L^2}{\omega_p^2}$
1D NL	> 1	$\frac{2\pi}{k_p}$	$\frac{2a_0}{k_p}\frac{\omega_L^2}{\omega_p^2}$	$a_0 \frac{2}{k_p}\frac{\omega_L^2}{\omega_p^2}$	$\frac{4a_0}{k_p}$	$a_0/2$	$a_0^2 \frac{\omega_L^2}{\omega_p^2}$
NL Lu	> 2	$\frac{2\sqrt{a_0}}{k_p}$	$\frac{4}{3}\frac{\sqrt{a_0}}{k_p}\frac{\omega_L^2}{\omega_p^2}$	$c\tau_L \frac{\omega_L^2}{\omega_p^2}$	$\sqrt{a_0}\frac{2\pi}{k_p}$	$\sqrt{a_0}/2$	$\frac{2}{3} a_0 \frac{\omega_L^2}{\omega_p^2}$
NL GP	$> 2\sqrt{\frac{n_c}{n_p}}$	$\frac{\sqrt{a_0}}{k_p}$		$a_0 c\tau_L \frac{\omega_L^2}{\omega_p^2}$		$\sqrt{a_0}$	$a_0^{\frac{3}{2}} \omega_p \tau_L \frac{\omega_L^2}{\omega_p^2}$

Scaling rules for LWFA in the linear and nonlinear 1D and 3D regime as given by (Esarey et al., 2009; Lu et al., 2007; Pukhov et al., 2004)

Quantity	Definition	Engineering Formula
Gaussian Laser Beam Parameters (a_0)		
Focal Spot	$2w_0 = \frac{4\lambda_L}{\pi}\frac{f}{D} = \sqrt{\frac{2}{\ln 2}} d_{FWHM}$	$w_{\frac{1}{e^2}-\varnothing}[\mu\text{m}] = f/\#$ $\quad @\lambda_L = 0.8\,\mu\text{m}$
Confocal Parameter	$2z_R = 2\pi w_0^2/\lambda_L$	$\Delta z[\mu\text{m}] = 2(f/\#)^2$ $\quad @\lambda_L = 0.8\,\mu\text{m}$
Peak Power	$P_0 = 2\sqrt{\frac{\ln 2}{\pi}}\frac{W_L}{t_{FWHM}}$ $P_0 = \frac{\pi}{4\ln 2} d_{FWHM}^2 I_0$	$P_0[\text{TW}] = 940\frac{W_L[\text{J}]}{t_{FWHM}[\text{fs}]}$ $P_0[\text{TW}] = 0.011 d_{FWHM}^2[\mu\text{m}] I_0[10^{18}\,\frac{\text{W}}{\text{cm}^2}]$
Peak Intensity	$I_0 = \left(\frac{4\ln 2}{\pi}\right)^{\frac{3}{2}} \frac{W_L}{t_{FWHM} d_{FWHM}^2[\mu\text{m}]}$ $I_0 = \frac{2\pi^2\epsilon_0 m_e^2 c^5}{e^2}\frac{a_0^2}{\lambda_L^2}$	$I_0[10^{18}\,\frac{\text{W}}{\text{cm}^2}] = 83\times 10^3 \frac{W_L[\text{J}]}{t_{FWHM}[\text{fs}] d_{FWHM}^2[\mu\text{m}]}$ $I_0[10^{18}\,\frac{\text{W}}{\text{cm}^2}] = 1.37\frac{a_0^2}{\lambda_L^2[\mu\text{m}]}$
Vector Potential	$a_0 = \frac{e}{\pi m_e c^2}\sqrt{\frac{I_0}{2\epsilon_0 c}}\lambda_L$	$a_0 = 0.85\sqrt{I_0[10^{18}\,\text{W cm}^{-2}]}\lambda_L[\mu\text{m}]$
Peak Electric Field	$E_0 = \frac{e a_0}{c m_e \omega_L}$	$E_0[10^{12}\,\text{V/m}] = 3.2\frac{a_0}{\lambda_L[\mu\text{m}]}$
Plasma Parameters ($n_e \propto k_p$)		
Plasma Wavelength	$\omega_p = \sqrt{\frac{n_{e,0}e^2}{m_e\epsilon_0}}$	$\lambda_p[\mu\text{m}] = \frac{33.4}{\sqrt{n_{e,0}[10^{18}\,\text{cm}^{-3}]}}$
Wavebreaking Field	$E_{p,0} = \frac{m_e c\omega_p}{e}$	$E_{p,0}[\text{GV m}^{-1}] = 96\sqrt{n_{e,0}[10^{18}\,\text{cm}^{-3}]}$
Plasma Gamma Factor	$\gamma_p = \frac{\omega_L}{\omega_p}$	$\gamma_p = 33.4\frac{1}{n_{e,0}[10^{18}\,\text{cm}^{-3}]\lambda_L[\mu\text{m}]}$
Critical Density	$n_{e,c} = \frac{\epsilon_0 m_e}{e^2}\omega_L^2$	$n_{e,c}[10^{18}\,\text{cm}^{-3}] = \frac{1.1\times 10^3}{\lambda_L^2[\mu\text{m}]}$
LWFA Parameters in the Bubble Regime ($r_b = 2\sqrt{a_0}/k_p$)		
Dephasing Length	$L_d = \frac{2}{3\pi}\sqrt{a_0}\lambda_L\left(\frac{n_c}{n_{e,0}}\right)^{3/2}$	$L_d[\text{mm}] = 7.9\sqrt{a_0}\left(\frac{\lambda_L^{-4/3}[\mu\text{m}]}{n_{e,0}[10^{18}\,\text{cm}^{-3}]}\right)^{3/2}$
Electric Field	$E_p = \frac{m_e c\omega_p}{e}\sqrt{a_0}$	$E_p[\text{GV m}^{-1}] = 96\sqrt{n_{e,0}[10^{18}\,\text{cm}^{-3}]}\sqrt{a_0}$
Electron Energy	$W_{el} = \frac{2a_0}{3}\left(\frac{n_c}{n_{e,0}}\right)m_e c^2$	$W_{el}[\text{MeV}] \approx 380\frac{a_0}{n_{e,0}[10^{18}\,\text{cm}^{-3}]\lambda_L^2[\mu\text{m}]}$
Optimum Charge	$Q_{opt} = \frac{\pi c^3}{e^2}\sqrt{\frac{m_e^3\epsilon_0^3}{n_{e,0}}} a_0^{\frac{3}{2}}$	$Q_{opt}[\text{pC}] = 75\sqrt{\frac{a_0^3}{n_{e,0}[10^{18}\,\text{cm}^{-3}]}}$

E-M wave propagation in plasma I

Maxwell's eqns *in vacuo*:

$$\vec{\nabla}\times\vec{E}_1 = -\frac{\partial \vec{B}_1}{\partial t} \quad (1)$$

$$c^2\vec{\nabla}\times\vec{B}_1 = \frac{\partial \vec{E}_1}{\partial t} \quad (2)$$

Solve by perturbation ansatz:

$$\vec{E},\vec{B} = \underbrace{\vec{E}_0,\vec{B}_0}_{\text{static}} + \underbrace{\vec{E}_1,\vec{B}_1}_{\text{variable}}$$

$$\frac{\partial}{\partial t}(1),\ \vec{\nabla}\times(2) \quad \Rightarrow\ c^2\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}_1\right) = \vec{\nabla}\frac{\partial \vec{E}_1}{\partial t} = -\frac{\partial^2 \vec{B}_1}{\partial t^2}$$

Plane wave ansatz: $\vec{E},\vec{B} \sim \exp\left[i\left(kx-\omega t\right)\right] \ \Rightarrow\ \vec{\nabla}\times \rightarrow i\vec{k},\ \ \frac{\partial}{\partial t}\rightarrow -i\omega$

$$\Rightarrow \omega^2\vec{B}_1 = -c^2\left[\vec{k}\times\left(\vec{k}\times\vec{B}_1\right)\right] = -c^2\left[\underbrace{\vec{k}\cdot\left(\vec{k}\cdot\vec{B}_1\right)}_{=0,\vec{k}\perp\vec{B}} - \vec{k}^2\vec{B}_1\right]$$

$$\Rightarrow \quad \boxed{\omega^2 = k^2c^2}$$

dispersion relation for plane waves *in vacuo*

E-M wave propagation in plasma II

In plasma, eqn. 2 has to be modified by j-term to account for currents driven by external field:

$$\vec{j} = \vec{j}_0 + \vec{j}_1 = \vec{j}_1 \quad \text{(no stationary current)}$$

$$\Rightarrow \quad c^2 \vec{\nabla} \times \vec{B}_1 = \frac{\vec{j}_1}{\varepsilon_0} + \frac{\partial \vec{E}_1}{\partial t}$$

$$\overbrace{\Rightarrow}^{\partial/\partial t} \quad c^2 \vec{\nabla} \times \frac{\partial \vec{B}_1}{\partial t} = \frac{1}{\varepsilon_0} \frac{\partial \vec{j}_1}{\partial t} + \frac{\partial^2 \vec{E}_1}{\partial t^2} \qquad (3)$$

$$\vec{\nabla} \times (1): \quad \vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}_1 \right) = \vec{\nabla} \left(\vec{\nabla} \vec{E}_1 \right) - \vec{\nabla}^2 \vec{E}_1 = -\vec{\nabla} \times \frac{\partial \vec{B}_1}{dt} \qquad (4)$$

(3) + (4), plane waves for E,B,j

eliminate j

$$\Rightarrow -\vec{k} \underbrace{\left(\vec{k} \cdot \vec{E}_1 \right)}_{=0, \vec{E} \perp \vec{k}} + \vec{k}^2 \vec{E}_1 = \frac{i\omega}{\varepsilon_0 c^2} \vec{j}_1 + \frac{\omega^2}{c^2} \vec{E}_1 \quad \Rightarrow \quad \boxed{\left(\omega^2 - c^2 k^2 \right) \vec{E}_1 = -\frac{i\omega}{\varepsilon_0} \vec{j}_1}$$

E-M wave propagation in plasma III

Eliminating j: due to their low mass, the plasma response is governed by the electron current:

$$\vec{j}_1 = -en_e\vec{v}_{e1}$$

Electron current is driven by electric field, write down equation of motion:

$$m_e\frac{\partial\vec{v}_{e1}}{\partial t} = -e\vec{E}_1 \Rightarrow \vec{v}_{e1} = \frac{e\vec{E}_1}{im_e\omega}$$

$$\Rightarrow \vec{j}_1 = \frac{n_e e^2}{im_e\omega}\vec{E}_1 = \frac{\omega_p}{i}\vec{E}_1$$

$$\Rightarrow \left(\omega^2 - c^2\vec{k}^2\right)\vec{E}_1 = \omega_p^2\vec{E}_1 \quad \Leftrightarrow \quad \boxed{\omega^2 = \omega_p^2 + c^2\vec{k}^2}$$

dispersion relation for plane e-m waves in plasma

Phase velocity:

$$v_{ph} := \frac{\omega}{k} = c\frac{\omega}{\sqrt{k^2c^2}} = c\frac{\omega}{\sqrt{\omega^2 - \omega_p^2}} = \frac{c}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} =: \frac{c}{\eta} > c$$

E-M wave propagation in plasma IV

The quantity $\eta := \sqrt{1-\frac{\omega_p^2}{\omega^2}}$ denotes the plasma refractive index. It can assume values between 0 and 1.

Group velocity: $$v_{gr} = \frac{\partial \omega}{\partial k} = \frac{kc^2}{\sqrt{\omega_p^2 + k^2c^2}} = c\frac{kc}{\omega} = c\eta < c$$

$\omega > \omega_p \quad \Rightarrow \quad \eta < 1 \quad \Rightarrow$ wave can propagate

$\omega > \omega_p \quad \Rightarrow \quad \eta = i\eta \quad \Rightarrow$ wave cannot propagate, exp. damping

Only e-m- waves with frequency

$$\omega > \omega_p = \sqrt{\frac{n_e e^2}{m_e \varepsilon_0}}$$

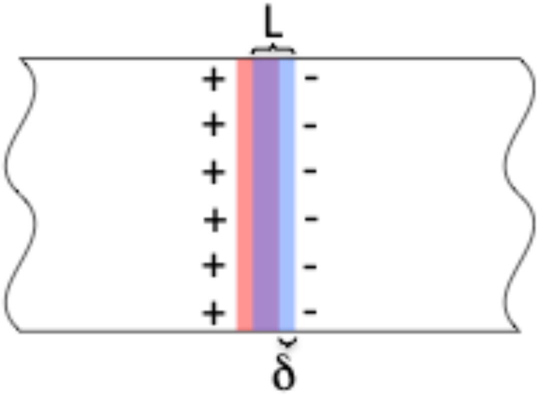
can propagate through a plasma. Lower frequencies are absorbed because the plasma electrons can follow the external field and effectively dissipate its energy..

for a given frequency light can only propagate in a plasma with a density smaller than:

$$n_e < \frac{\omega^2 \varepsilon_0 m_e}{e^2} =: n_{crit}$$

The plasma frequency

Imagine a neutral plasma region with homogeneous density and area A. Now consider the displacement of a slab of electrons with a thickness L by a small distance δ ≪ L, such that one side of the slab charges negatively, the other positively.:



The electrons will experience a restoring force $F = -q_{dis} E = m_{dis} \ddot{\delta}$

where the displaced charge and mass are given by: $q_{dis} = -e n_e A \delta$ $\quad m_{dis} = m_e n_e A \delta$

The electric field and restoring force per unit area are $E = \dfrac{n_e e}{\varepsilon_0} \delta L, \quad F = \dfrac{n_e^2 e^2}{\varepsilon_0} \delta L$

...leading to the equation of motion:

$$\boxed{-\frac{n_e^2 e^2}{\varepsilon_0} \delta L = n_e m L \frac{\partial^2 \delta}{\partial t^2}}$$

(harmonic oscillator)

<u>Plasma frequency:</u>
Frequency that displaced plasma electrons will oscillate at.

$$\boxed{\omega_p = \sqrt{\frac{n_e e^2}{m \varepsilon_0}}}$$

depends on electron density n_e and (relativistic) mass m

Ponderomotive force (non-rel.)

Consider a plane light wave with $E(t) = E_0 \cos(\omega t)$

$\Rightarrow$ equation of motion for electrons: $F(t) = m\ddot{x}(t) = -eE = -eE_0 \cos(\omega t)$

integration yields:

$$\ddot{x}(t) = -\frac{e}{m} E_0 \cos(\omega t) \quad \Leftrightarrow \quad \dot{x}(t) = -\frac{e}{m\omega} E_0 \sin(\omega t) \quad \Leftrightarrow \quad x(t) = \frac{e}{m\omega^2} E_0 \cos(\omega t)$$

spatially varying E-field in first order expansion:

$$E(x,t) = \left(E_0(x) + x(t) \frac{\partial}{\partial x} E_0(x) \right) \cos(\omega t)$$

equation of motion: $F(x,t) = eE(x,t)$

$$= -eE_0(x)\cos(\omega t) - ex(t) \frac{\partial}{\partial x} E_0(x) \cos(\omega t)$$

$$= -eE_0(x)\cos(\omega t) - \frac{e^2}{m\omega^2} E_0(x) \cos^2(\omega t) \frac{\partial}{\partial x} E_0(x)$$

carry-over from previous slide:

$$F = -eE_0(x)\cos(\omega t) - \frac{e^2}{m\omega^2} E_0(x)\cos^2(\omega t)\frac{\partial}{\partial x} E_0(x)$$

averaging over one period yields:

$$F = -\frac{1}{2}\frac{e^2}{m\omega^2} E_0(x)\frac{\partial}{\partial x} E_0(x) = -\frac{1}{4}\frac{e^2}{m\omega^2}\frac{\partial}{\partial x} E_0^2(x)$$

generalization to 3D:

$$\boxed{F = -\frac{1}{4}\frac{e^2}{m\omega^2}\nabla E_0^2(x,y,z)}$$

<u>ponderomotive force:</u>

with this force, we can define a ponderomotive potential:

$$F_{pond} = -\nabla\Phi_{pond} \quad \rightarrow \quad \Phi_{pond} = \frac{e^2}{4m\omega^2} E_0^2$$

with can be identified as the mean kinetic energy of the electrons:

$$\bar{E}_{kin} = \frac{1}{2} m_e \langle v_e^2 \rangle_T = \frac{1}{2} m_e \left(\frac{eE_0}{\omega_L m_e}\right)^2 \underbrace{\frac{1}{T}\int_0^T \sin^2(k_L x - \omega_L t)\,dt}_{=1/2} = \frac{1}{4} m_e \left(\frac{eE_0}{\omega_L m_e}\right)^2 = \Phi_{pond} \propto I$$