JÜLICH
FORSCHUNGSZENTRUM

Introduction to Plasma Physics

CERN Accelerator School on High Gradient Wakefield Accelerators

Sesimbra, Portugal, 11-22 March 2019 | Paul Gibbon

Outline

- Lecture 1: Introduction – Definitions and Concepts
- Lecture 2: Laser-plasmas: Electron Dynamics and Wave Propagation

Lecture 1: Introduction

What is a plasma?

Simple definition: a *quasi-neutral* gas of charged particles showing *collective behaviour*.

Quasi-neutrality: number densities of electrons, n_e, and ions, n_i, with charge state Z are *locally balanced*:

$$n_e \simeq Z n_i. \tag{1}$$

Collective behaviour: long range of Coulomb potential ($1/r$) leads to nonlocal influence of disturbances in equilibrium.

Macroscopic fields usually dominate over microscopic fluctuations, e.g.:

$$\rho = e(Z n_i - n_e) \Rightarrow \nabla . \boldsymbol{E} = \rho / \varepsilon_0$$

Where are plasmas found?

1. cosmos (99% of visible universe):
   - interstellar medium (ISM)
   - stars
   - jets
2. ionosphere:
   - $\leq$ 50 km = 10 Earth-radii
   - long-wave radio
3. Earth:
   - fusion devices
   - street lighting
   - plasma torches
   - discharges - lightning
   - *plasma accelerators and radiation sources*!

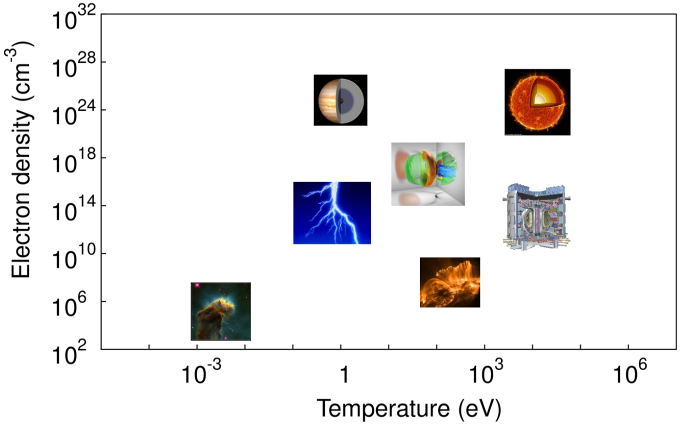
Plasma properties

Type	Electron density n_e (cm^{-3})	Temperature T_e (eV*)
Stars	10^{26}	2×10^3
Laser fusion	10^{25}	3×10^3
Magnetic fusion	10^{15}	10^3
Laser-produced	$10^{18} - 10^{24}$	$10 - 10^3$
Discharges	10^{12}	1-10
Ionosphere	10^6	0.1
ISM	1	10^{-2}

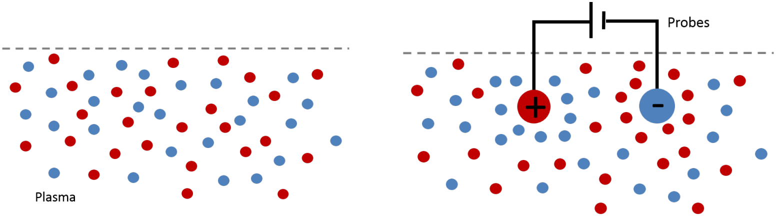
Table 1: Densities and temperatures of various plasma forms

* 1eV $\equiv$ 11600K

Plasma classification



Debye shielding



What is the potential $\phi(r)$ of an ion (or positively charged sphere) immersed in a plasma?

Debye shielding (2): ions vs electrons

For equal ion and electron temperatures ($T_e = T_i$), we have:

$$\frac{1}{2}m_e v_e^2 = \frac{1}{2}m_i v_i^2 = \frac{3}{2}k_B T_e \qquad (2)$$

Therefore,

$$\frac{v_i}{v_e} = \left(\frac{m_e}{m_i}\right)^{1/2} = \left(\frac{m_e}{Am_p}\right)^{1/2} = \frac{1}{43} \qquad \text{(hydrogen, Z=A=1)}$$

Ions are almost stationary on electron timescale!
To a good approximation, we can often write:

$$n_i \simeq n_0,$$

where the material (eg gas) number density, $n_0 = N_A \rho_m / A$;
$N_A =$ Avogadro number, $\rho_m =$ mass density.

Debye shielding (3)

In thermal equilibrium, the electron density follows a Boltzmann distribution): $n_e = n_i \exp(e\phi/k_B T_e)$, where n_i is the ion density and k_B is the Boltzmann constant - see, eg: F. F. Chen, p. 9.

Gauss' law in spherical geometry:

$$\nabla^2\phi = \frac{1}{r^2}\frac{d}{dr}(r^2\frac{d\phi}{dr}) = -\frac{e}{\varepsilon_0}(n_i - n_e) = -\frac{en_0}{\varepsilon_0}\{1 - exp(e\phi/k_B T_e)\}$$

Solving for ϕ, requiring $\phi \to 0$ at $r = \infty$, we obtain a solution:

$$\phi_D = \frac{1}{4\pi\varepsilon_0}\frac{e^{-r/\lambda_D}}{r}. \qquad (3)$$

Potential is shielded on characteristic scale $= \lambda_D$,

Debye length

$$\lambda_D = \left(\frac{\varepsilon_0 k_B T_e}{e^2 n_e}\right)^{1/2} \simeq 743 \left(\frac{T_e}{\mathrm{eV}}\right)^{1/2} \left(\frac{n_e}{\mathrm{cm}^{-3}}\right)^{-1/2} \mathrm{cm} \qquad (4)$$

Debye sphere

An **ideal plasma** has many particles per Debye sphere:

$$N_D \equiv n_e \frac{4\pi}{3} \lambda_D^3 \gg 1. \tag{5}$$

$\Rightarrow$ Prerequisite for collective behaviour.

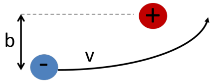
Alternatively, can define **plasma parameter**:

$$g \equiv \frac{1}{n_e \lambda_D^3}$$

Classical plasma theory based on assumption that $g \ll 1$, which also implies dominance of collective effects over collisions between particles.

Collisions in plasmas

At the other extreme, where $N_D \leq 1$, screening effects are reduced and collisions dominate. A quantitative measure of this is the



Electron-ion collision rate

$$\nu_{ei} = \frac{\pi^{\frac{3}{2}} n_e Z e^4 \ln\Lambda}{2^{\frac{1}{2}}(4\pi\varepsilon_0)^2 m_e^2 v_{te}^3} \mathrm{s}^{-1}$$
$$\simeq 2.91 \times 10^{-6} Z n_e T_e^{-3/2} \ln\Lambda \ \mathrm{s}^{-1} \quad (6)$$

Collision frequency: details

$$\nu_{ei} = \frac{\pi^{\frac{3}{2}} n_e Z e^4 \ln\Lambda}{2^{\frac{1}{2}}(4\pi\varepsilon_0)^2 m_e^2 v_{te}^3} \mathrm{s}^{-1}$$

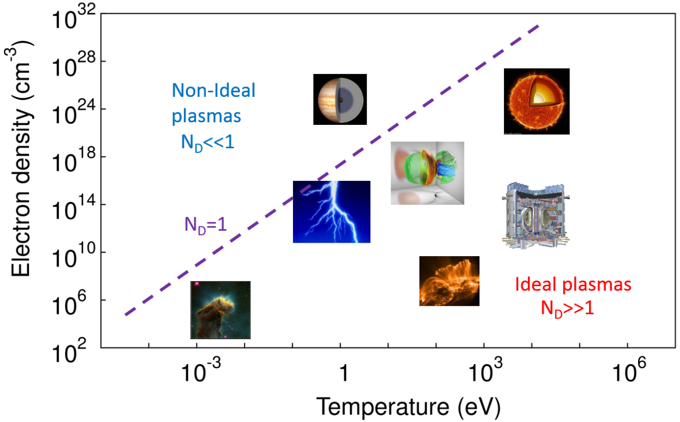
$v_{te} \equiv \sqrt{k_B T_e / m_e}$, electron thermal velocity
$Z =$ number of free electrons per atom (ionization degree)
$n_e =$ electron density in cm^{-3}
$T_e =$ electron temperature in eV
$\ln\Lambda \sim O(2 \rightarrow 10)$ is the **Coulomb logarithm**. Can show that

$$\frac{\nu_{ei}}{\omega_p} \simeq \frac{Z \ln\Lambda}{10 N_D} \quad (7)$$

with

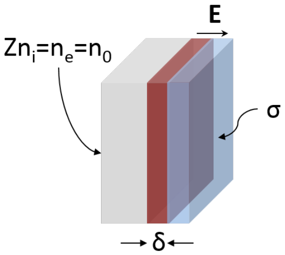
$$\Lambda = \frac{b_{\max}}{b_{\min}} = \lambda_D . \frac{k_B T_e}{Z e^2} \simeq 9 N_D / Z$$

Plasma classification - quantified



N_D characterises plasma 'collectiveness' – see Eq.(5)

Plasma oscillations: capacitor model



Consider electron layer displaced from plasma slab by length δ. This creates two 'capacitor' plates with surface charge $\sigma = \pm e n_e \delta$, resulting in an electric field:

$$\boldsymbol{E} = \frac{\sigma}{\varepsilon_0} = \frac{e n_e \delta}{\varepsilon_0}$$

Capacitor model (2)

The electron layer is accelerated back towards the slab by this restoring force according to:

$$m_e \frac{dv}{dt} = -m_e \frac{d^2\delta}{dt^2} = -eE = \frac{e^2 n_e \delta}{\varepsilon_0}$$

Or:

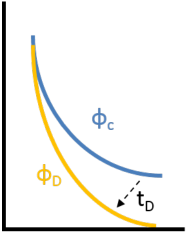
$$\frac{d^2\delta}{dt^2} + \omega_p^2 \delta = 0,$$

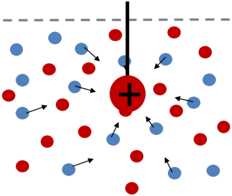
where

Electron plasma frequency

$$\omega_p \equiv \left(\frac{e^2 n_e}{\varepsilon_0 m_e} \right)^{1/2} \simeq 5.6 \times 10^4 \left(\frac{n_e}{\mathrm{cm}^{-3}} \right)^{1/2} \mathrm{s}^{-1}. \quad (8)$$

Response time to create Debye sheath





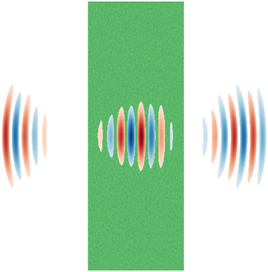
For a plasma with temperature T_e (and thermal velocity $v_{te} \equiv \sqrt{k_B T_e / m_e}$), one can also define a characteristic *reponse time* to recover quasi-neutrality:

$$t_D \simeq \frac{\lambda_D}{v_{te}} = \left(\frac{\varepsilon_0 k_B T_e}{e^2 n_e} \cdot \frac{m}{k_B T_e} \right)^{1/2} = \omega_p^{-1}.$$

Plasma response time ω_p^{-1} dictates type of interaction with time-varying external fields - eg: laser

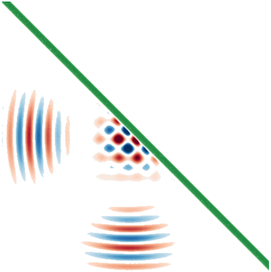
Underdense plasma, $\omega > \omega_p$:

- slow plasma response
- nonlinear refractive medium



Overdense plasma, $\omega < \omega_p$:

- radiation shielded out
- mirror-like optics



The critical density

To make this more quantitative, consider ratio:

$$\frac{\omega_p^2}{\omega^2} = \frac{e^2 n_e}{\varepsilon_0 m_e} \cdot \frac{\lambda^2}{4\pi^2 c^2}.$$

Setting this to unity defines the wavelength for which $n_e = n_c$, or

Critical density

$$n_c \simeq 10^{21} \lambda_\mu^{-2} \text{ cm}^{-3} \tag{9}$$

above which radiation with wavelengths $\lambda > \lambda_\mu$ will be reflected.
cf: radio waves from ionosphere.

Plasma creation: field ionization

At the Bohr radius

$$a_B = \frac{4\pi\varepsilon_0\hbar^2}{me^2} = 5.3 \times 10^{-11} \text{ m},$$

the electric field strength is:

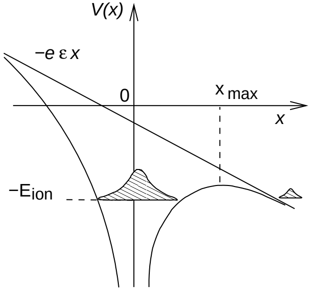
$$\begin{aligned} E_a &= \frac{e}{4\pi\varepsilon_0 a_B^2} \\ &\simeq 5.1 \times 10^9 \text{ Vm}^{-1}. \end{aligned} \tag{10}$$

This leads to the **atomic intensity**:

$$\begin{aligned} I_a &= \frac{\varepsilon_0 c E_a^2}{2} \\ &\simeq 3.51 \times 10^{16} \text{ Wcm}^{-2}. \end{aligned} \tag{11}$$

A laser intensity of $I_L > I_a$ will *guarantee ionization* for any target material, though in fact this can occur well below this threshold value (eg: $\sim 10^{14}$ Wcm^{-2} for hydrogen) via *multiphoton* effects .

Tunnelling ionization: barrier suppression model



Potential barrier tipped below ionization energy $E_{\rm ion}$ by external electric field ε

- Hydrogen: $Z = 1$

$$E_{\rm ion} = E_h = \frac{e^2}{2a_B} = 13.61 \text{ eV}$$

- Critical field for hydrogen:

$$\varepsilon_c = \frac{E_h^2}{4e^3} = \frac{e}{16a_B^2} = \frac{E_a}{16}$$

Appearance intensity of hydrogen ions

$$I_{\rm app} = \frac{I_a}{256} \simeq 1.4 \times 10^{14} \text{ Wcm}^{-2} \tag{12}$$

Relativistic field strengths

Classical equation of motion for an electron exposed to a linearly polarized laser field $\boldsymbol{E} = \hat{\boldsymbol{y}} E_0 \sin \omega t$:

$$\frac{dv}{dt} \simeq \frac{-eE_0}{m_e} \sin \omega t$$

$$\rightarrow v = \frac{eE_0}{m_e \omega} \cos \omega t = v_{\rm os} \cos \omega t \tag{13}$$

Dimensionless oscillation amplitude, or 'quiver' velocity:

$$a_0 \equiv \frac{v_{\rm os}}{c} \equiv \frac{p_{\rm os}}{m_e c} \equiv \frac{eE_0}{m_e \omega c} \tag{14}$$

Relativistic intensity

The laser intensity I_L and wavelength λ_L are related to E_0 and ω by:

$$I_L = \frac{1}{2}\varepsilon_0 c E_0^2; \quad \lambda_L = \frac{2\pi c}{\omega}$$

Substituting these into (14) we find :

$$\begin{aligned} I_L &= \frac{2\pi^2 \varepsilon_0 m^2 c^5}{e^2} \frac{a_0^2}{\lambda_L^2} \\ &\simeq 1.37 \times 10^{18} a_0^2 \lambda_\mu^2 \ \mathrm{Wcm}^{-2} \quad (15) \end{aligned}$$

Exercise

where $\lambda_\mu = \frac{\lambda_L}{\mu m}$.

Implies that we will have **relativistic electrons**, $v_{\rm os} \sim c$, for $I_L \geq 10^{18}$ Wcm^{-2}, $\lambda_L \simeq 1$ μm.

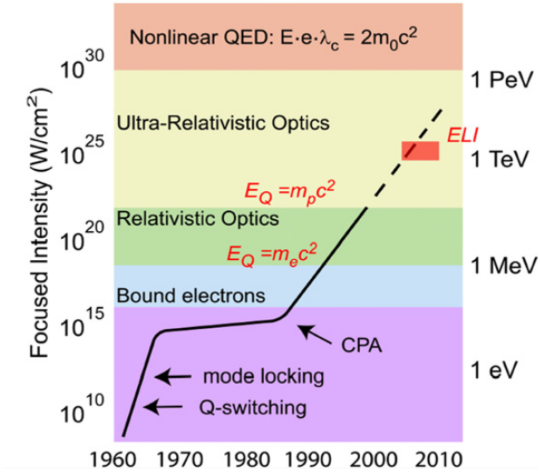
Compare *thermal* velocities $v_{te}/c = \sqrt{k_B T_e / m_e c^2} = 0.01$ for $T_e = 50$eV.

Summary

- Ideal, thermal plasmas possess intrinsic length scale: λ_D
- Characteristic timescale: ω_p^{-1}
- Frequency ratio ω_p/ω_0 determines nature of interaction:
  - $\omega_p/\omega_0 < 1 \rightarrow$ propagation
  - $\omega_p/\omega_0 > 1 \rightarrow$ reflection
- Plasma can be created by laser intensities $I_L > 10^{14}$ Wcm^{-2}
- Relativistic effects kick in when $I_L\lambda^2 > 10^{-18}$ Wcm$^{-2}\mu$m^2

Laser-plasma interactions

G. Mourou et al.,Plasma Physics & Contr. Fus. 49, (2007)



Further reading

1. F. F. Chen, *Plasma Physics and Controlled Fusion*, 2nd Ed. (Springer, 2006)
2. R.O. Dendy (ed.), *Plasma Physics, An Introductory Course*, (Cambridge University Press, 1993)
3. J. D. Huba, *NRL Plasma Formulary*, (NRL, Washington DC, 2007) `http://www.nrl.navy.mil/ppd/content/nrl-plasma-formulary`

Constants

Name	Symbol	Value (SI)	Value (cgs)
Boltzmann constant	k_B	1.38×10^{-23} JK^{-1}	1.38×10^{-16} erg K^{-1}
Electron charge	e	1.6×10^{-19} C	4.8×10^{-10} statcoul
Electron mass	m_e	9.1×10^{-31} kg	9.1×10^{-28} g
Proton mass	m_p	1.67×10^{-27} kg	1.67×10^{-24} g
Planck constant	h	6.63×10^{-34} Js	6.63×10^{-27} erg-s
Speed of light	c	3×10^{8} ms^{-1}	3×10^{10} cms^{-1}
Dielectric constant	ε_0	8.85×10^{-12} Fm^{-1}	—
Permeability constant	μ_0	$4\pi \times 10^{-7}$	—
Proton/electron mass ratio	m_p/m_e	1836	1836
Temperature = 1eV	e/k_B	11604 K	11604 K
Avogadro number	N_A	6.02×10^{23} mol^{-1}	6.02×10^{23} mol^{-1}
Atmospheric pressure	1 atm	1.013×10^{5} Pa	1.013×10^{6} dyne cm^{-2}

Standard formulae

Name	Symbol	Formula (SI)	Formula (cgs)
Debye length	λ_D	$\left(\frac{\varepsilon_0 k_B T_e}{e^2 n_e}\right)^{\frac{1}{2}}$ m	$\left(\frac{k_B T_e}{4\pi e^2 n_e}\right)^{\frac{1}{2}}$ cm
Particles in Debye sphere	N_D	$\frac{4\pi}{3}\lambda_D^3$	$\frac{4\pi}{3}\lambda_D^3$
Plasma frequency (electrons)	ω_{pe}	$\left(\frac{e^2 n_e}{\varepsilon_0 m_e}\right)^{\frac{1}{2}}$ s^{-1}	$\left(\frac{4\pi e^2 n_e}{m_e}\right)^{\frac{1}{2}}$ s^{-1}
Plasma frequency (ions)	ω_{pi}	$\left(\frac{Z^2 e^2 n_i}{\varepsilon_0 m_i}\right)^{\frac{1}{2}}$ s^{-1}	$\left(\frac{4\pi Z^2 e^2 n_i}{m_i}\right)^{\frac{1}{2}}$ s^{-1}
Thermal velocity	$v_{te} = \omega_{pe}\lambda_D$	$\left(\frac{k_B T_e}{m_e}\right)^{\frac{1}{2}}$ ms^{-1}	$\left(\frac{k_B T_e}{m_e}\right)^{\frac{1}{2}}$ cms^{-1}
Electron gyrofrequency	ω_c	eB/m_e s^{-1}	eB/m_e s^{-1}
Electron-ion collision frequency	ν_{ei}	$\frac{\pi^{\frac{3}{2}} n_e Z e^4 \ln\Lambda}{2^{\frac{1}{2}}(4\pi\varepsilon_0)^2 m_e^2 v_{te}^3}$ s^{-1}	$\frac{4(2\pi)^{\frac{1}{2}} n_e Z e^4 \ln\Lambda}{3 m_e^2 v_{te}^3}$ s^{-1}
Coulomb-logarithm	$\ln\Lambda$	$\ln\frac{9N_D}{Z}$	$\ln\frac{9N_D}{Z}$

Useful formulae

Plasmafrequency	$\omega_{pe} = 5.64 \times 10^4 n_e^{\frac{1}{2}}\ \mathrm{s}^{-1}$
Critical density	$n_c = 10^{21} \lambda_{\mathrm{L}}^{-2}\ \mathrm{cm}^{-3}$
Debye length	$\lambda_D = 743\ T_e^{\frac{1}{2}} n_e^{-\frac{1}{2}}\ \mathrm{cm}$
Skin depth	$\delta = c/\omega_p = 5.31 \times 10^5 n_e^{-\frac{1}{2}}\ \mathrm{cm}$
Elektron-ion collision frequency	$\nu_{ei} = 2.9 \times 10^{-6} n_e T_e^{-\frac{3}{2}} \ln \Lambda\ \mathrm{s}^{-1}$
Ion-ion collision frequency	$\nu_{ii} = 4.8 \times 10^{-8} Z^4 \left(\frac{m_p}{m_i}\right)^{\frac{1}{2}} n_i T_i^{-\frac{3}{2}} \ln \Lambda\ \mathrm{s}^{-1}$
Quiver amplitude	$a_0 \equiv \frac{p_{osc}}{m_e c} = \left(\frac{I\lambda_L^2}{1.37 \times 10^{18} \mathrm{Wcm}^{-2} \mu\mathrm{m}^2}\right)^{\frac{1}{2}}$
Relativistic focussing threshold	$P_c = 17.5 \left(\frac{n_c}{n_e}\right)\ \mathrm{GW}$

T_e in eV; n_e, n_i in cm^{-3}, wavelength λ_L in μm

Maxwell's Equations

Name	(SI)	(cgs)
Gauss' law	$\nabla . \boldsymbol{E} = \rho/\varepsilon_0$	$\nabla . \boldsymbol{E} = 4\pi\rho$
Gauss' magnetism law	$\nabla . \boldsymbol{B} = 0$	$\nabla . \boldsymbol{B} = 0$
Ampère	$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$	$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}$
Faraday	$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$	$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}$
Lorentz force per unit charge	$\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}$	$\boldsymbol{E} + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}$