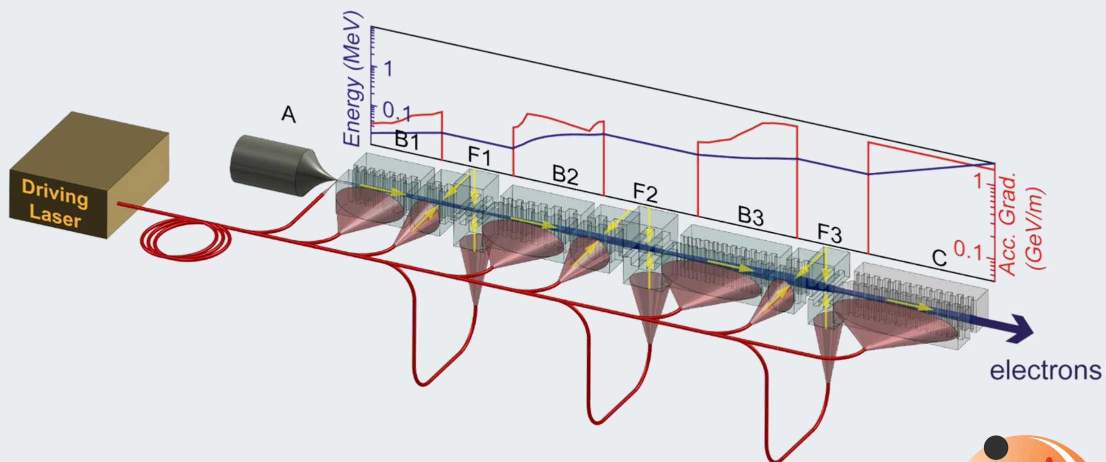
FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG
NATURWISSENSCHAFTLICHE FAKULTÄT

GORDON AND BETTY MOORE FOUNDATION

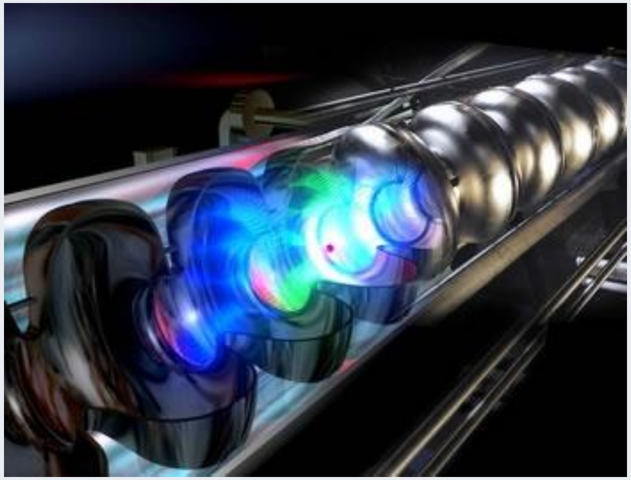
Theory of Dielectric Laser Acceleration

Norbert Schönenberger
Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU), Erlangen

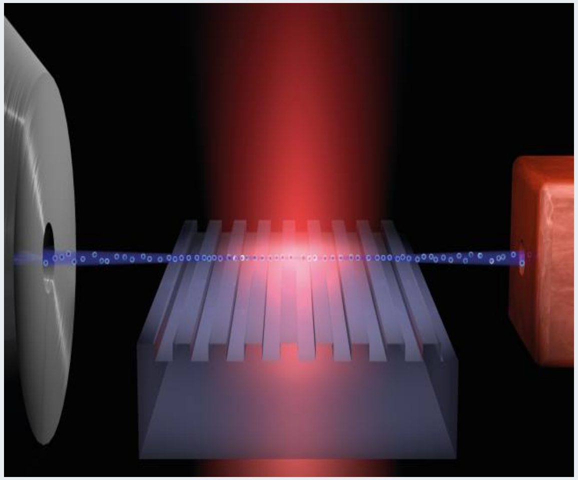
ACHIP
Accelerator on a Chip International Program



Particle accelerators: from RF to optical/photonic drive?

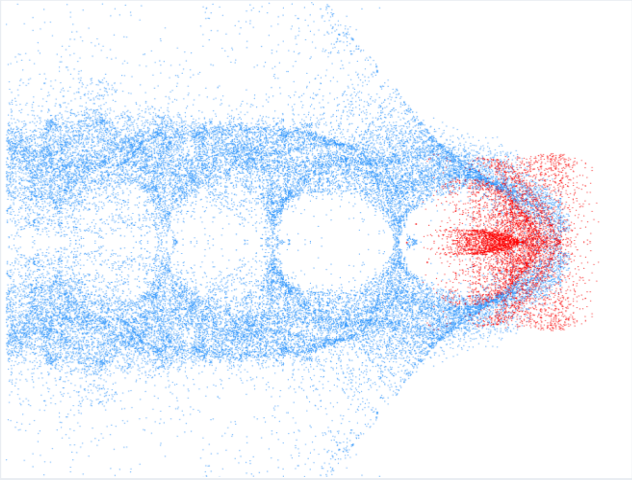


RF cavity (TESLA, DESY)

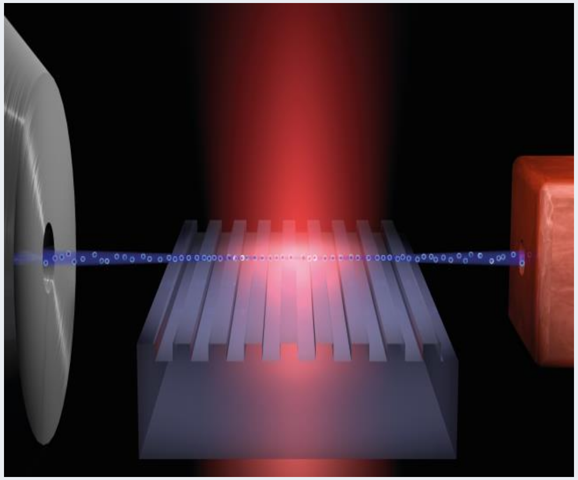


	Conventional linear accelerator (RF)	Laser-based dielectric accelerator (optical)
Based on	(Supercond.) RF cavities	Dielectric nano structures
Peak field limited by	**Surface breakdown: 200 MV/m**	**Damage threshold: 30 GV/m**
Max. achievable gradients	**100 MeV/m**	**10 GeV/m**

Particle accelerators: from RF to optical/photonic drive?

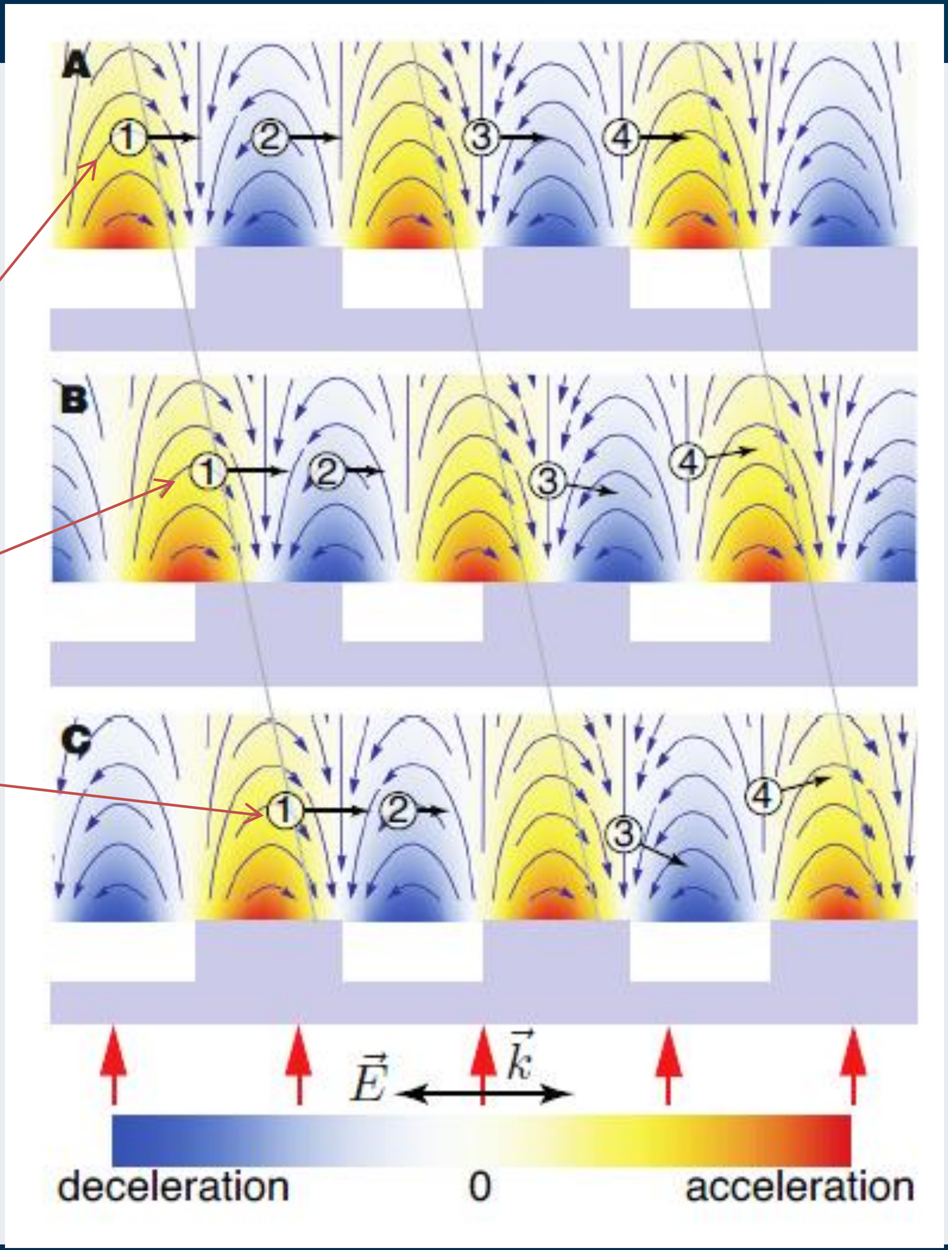


Rasmus Ischebeck



	Plasma wakefield & Laser plasma accelerators	Laser-based dielectric accelerator (optical)
Based on	Plasma	Dielectric nano structures
Driving laser	**4 PW/m**	**100 GW/m (no laser recycle)**
Max. achievable gradients	**10s – 100s GeV/m**	**10 GeV/m**

Preview: where do we want to end up after the lecture



t = 0

t = π/2

t = π

1 acceleration

2 deceleration

3 deflection

4 deflection

An old idea … I

Proposal for an Electron Accelerator Using an Optical Maser

Koichi Shimoda

January 1962 / Vol. 1, No. 1 / APPLIED OPTICS 33

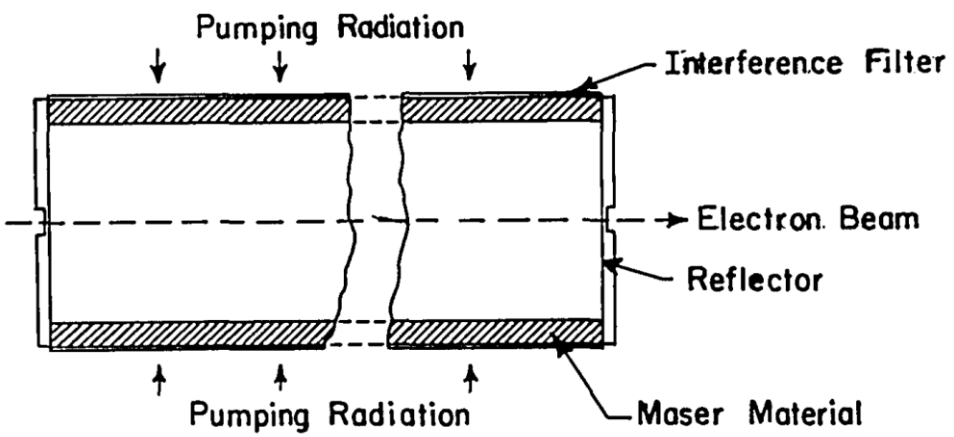


Fig. 1. Schematic diagram of an electron linear accelerator by optical maser.

An old idea … II

IBM TN-5

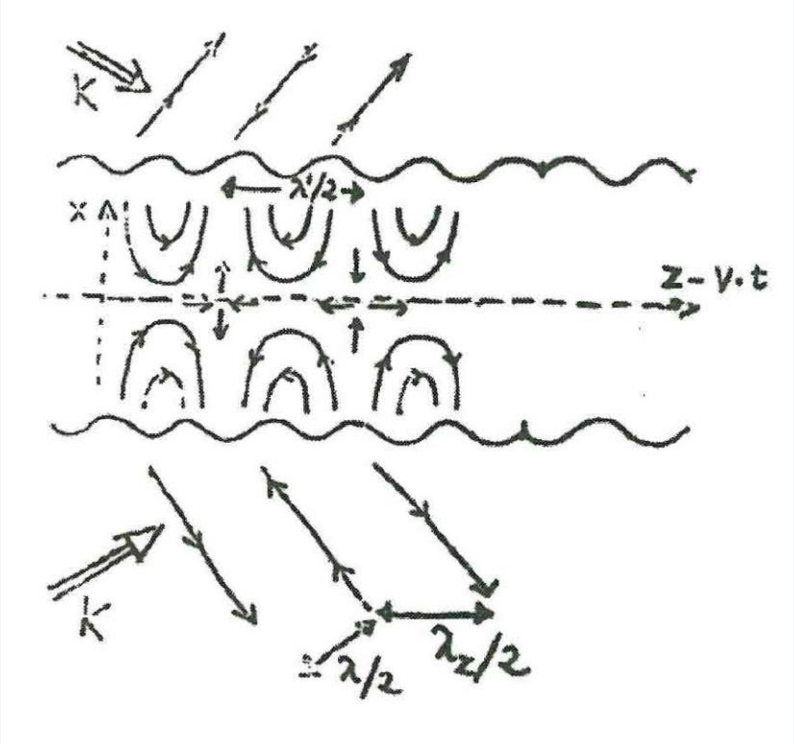
Electron Acceleration by Light Waves

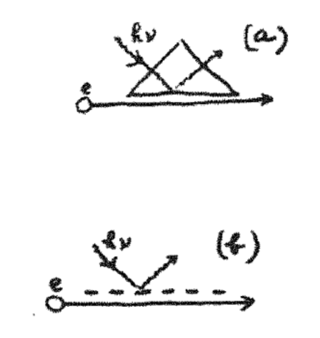
October 3, 1962

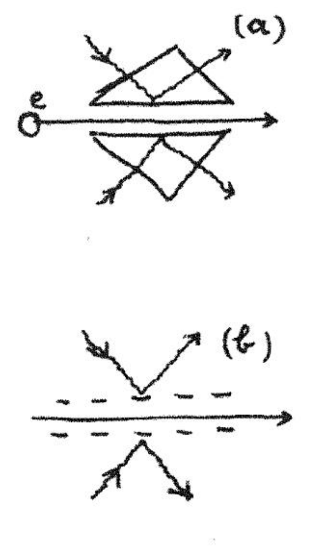
A. Lohmann*

Department 522
Photo-Optics
Technology

GPD Development
Laboratory
San Jose







Aug. 16, 1966 A. W. LOHMANN 3,267,383

PARTICLE ACCELERATOR UTILIZING COHERENT LIGHT

Filed May 27, 1963 2 Sheets-Sheet 2

Electromagnetic waves – Maxwell Equations

Gauss‘ law for electricity

$$\nabla * \vec{E} = \frac{\rho}{\varepsilon_0}$$

- Electric field $\vec{E}$
- Total charge ρ
- Vacuum permittivity ϵ_0
- Divergence $\nabla * \vec{x}$

The electric flux out of a closed surface is proporional to the enclosed charge

Electromagnetic waves – Maxwell Equations

Gauss‘ law for magnetism

- Magnetic field $\vec{B}$

$$\nabla * \vec{B} = 0$$

The magnetic flux out of a closed surface is zero

Electromagnetic waves – Maxwell Equations

Faraday's law of induction

- Curl $\nabla \times \vec{x}$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

The curl of the electric field is equal to the negative rate of change of the magnetic field

Electromagnetic waves – Maxwell Equations

Ampere‘s law

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

- Current $\vec{J}$
- Vacuum permeability μ_0
- Divergence $\nabla \ * \vec{x}$

The curl of the magnetic field is proportional to the electric current flowing through a loop and the rate of change of the electric field

Electromagnetic waves – Maxwell Equations

Gauss‘ law for electricity

$$\nabla * \vec{E} = \frac{\rho}{\varepsilon_0}$$

Gauss‘ law for magnetism

$$\nabla * \vec{B} = 0$$

Faraday‘s law of induction

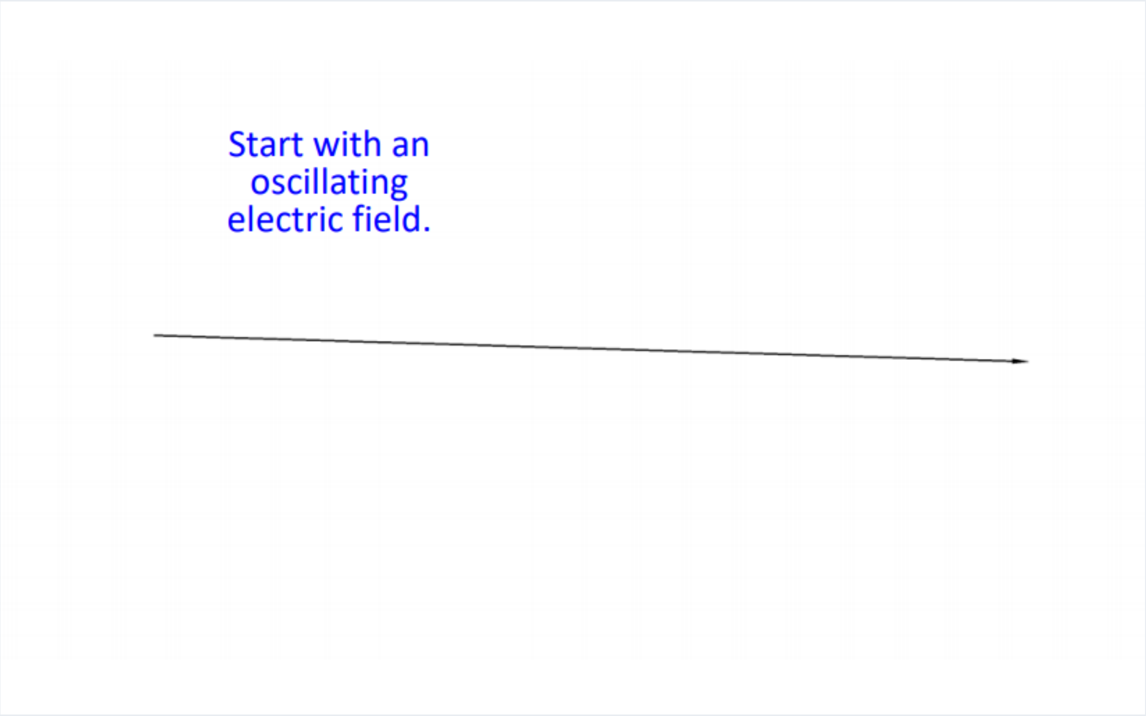
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Ampere‘s law

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

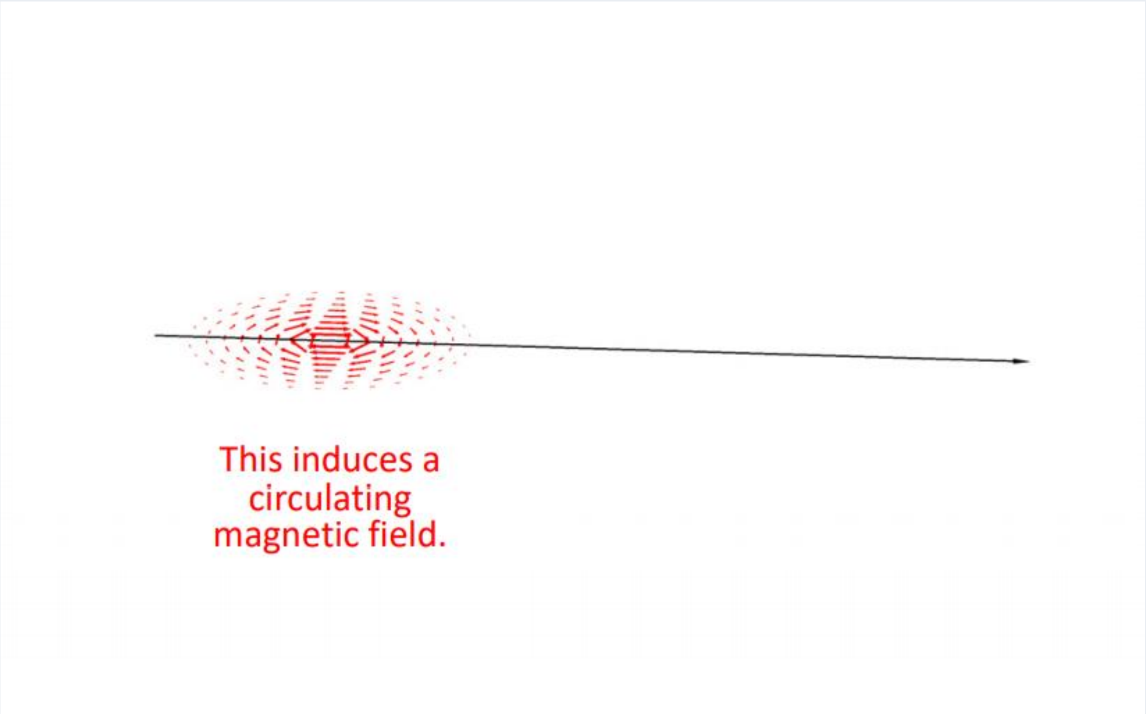
Evaluate in vacuum -> no charges and currents

Maxwell Equations predict waves



Source: The University of Texas at El Paso: EE 4347 Applied Electromagnetics

Maxwell Equations predict waves



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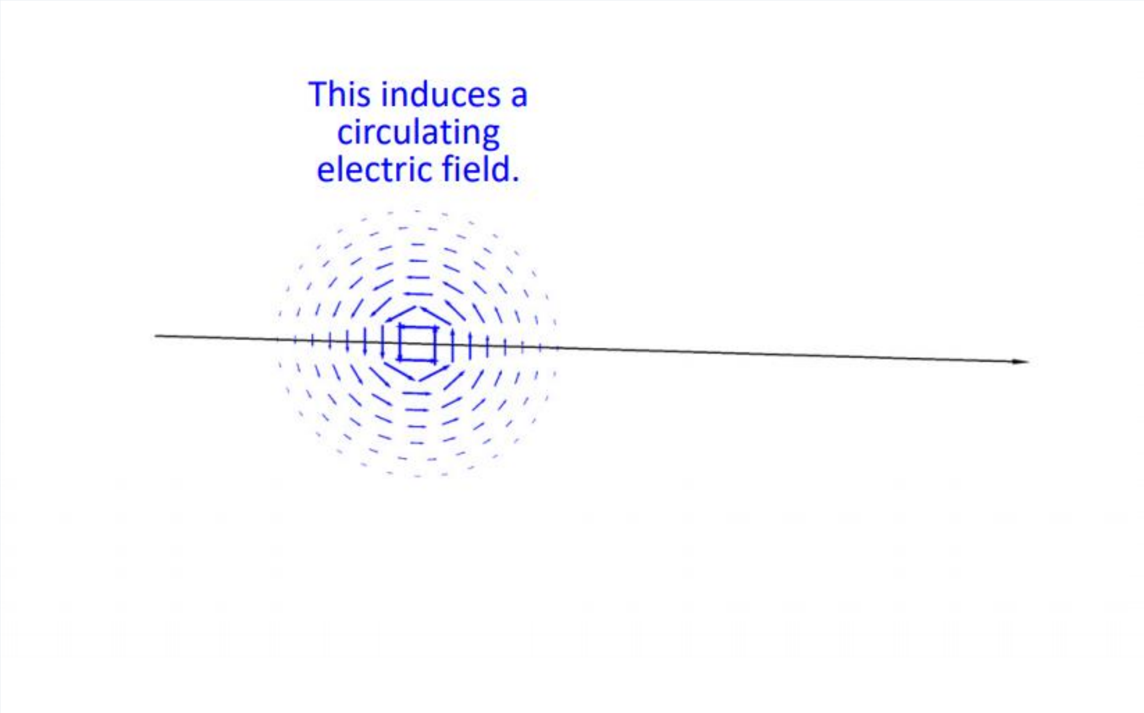
Maxwell Equations predict waves



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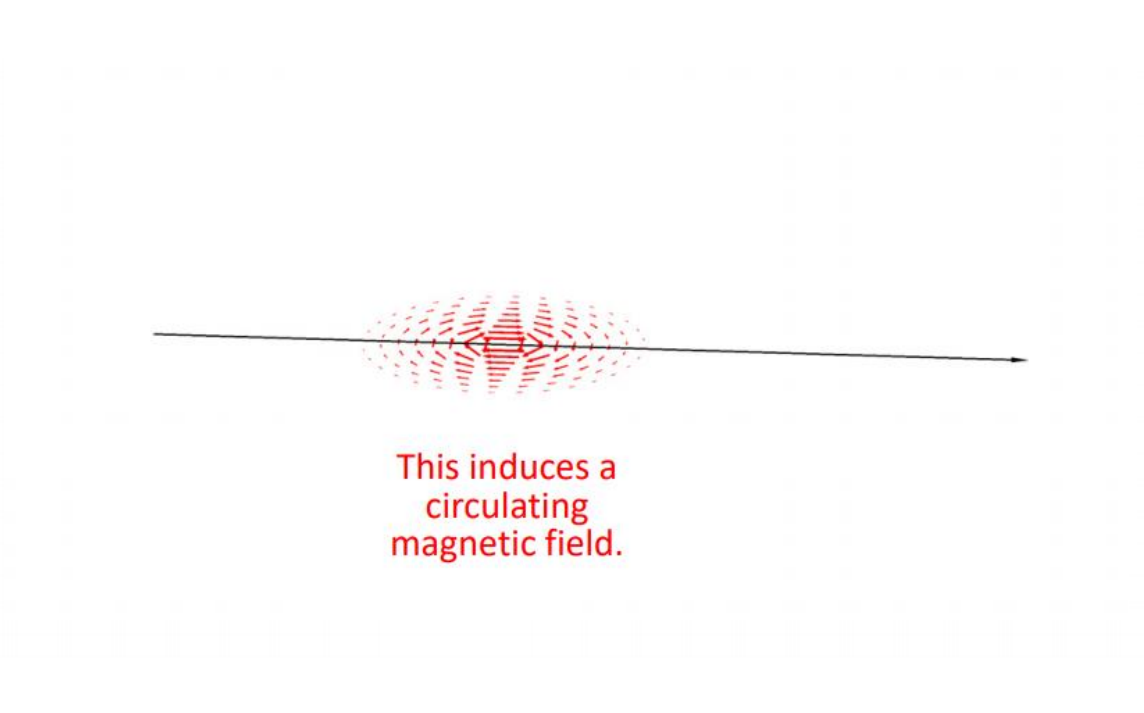
Maxwell Equations predict waves



Source: The University of Texas at El Paso: EE 4347 Applied Electromagnetics

Electromagnetic waves in vacuum

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday‘s law of induction

$$\nabla \times (\nabla \times \vec{E}) = \nabla \times -\frac{\partial \vec{B}}{\partial t}$$

Take the curl

$$\nabla \times \left(\nabla \times \vec{E}\right) = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

Change RHS order of differentiation

But we know already Ampere‘s law $\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Electromagnetic waves in vacuum

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}\left(\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

Substitute Ampere‘s law

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Assume $\mu_0 \epsilon_0$ are not time dependent

With identity $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla * \vec{A}) - \nabla^2 \vec{A}$

$$\nabla(\nabla * \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

But: $\nabla * \vec{E} = 0$

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Electromagnetic waves in vacuum

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

Generaleralized form of the wave equation

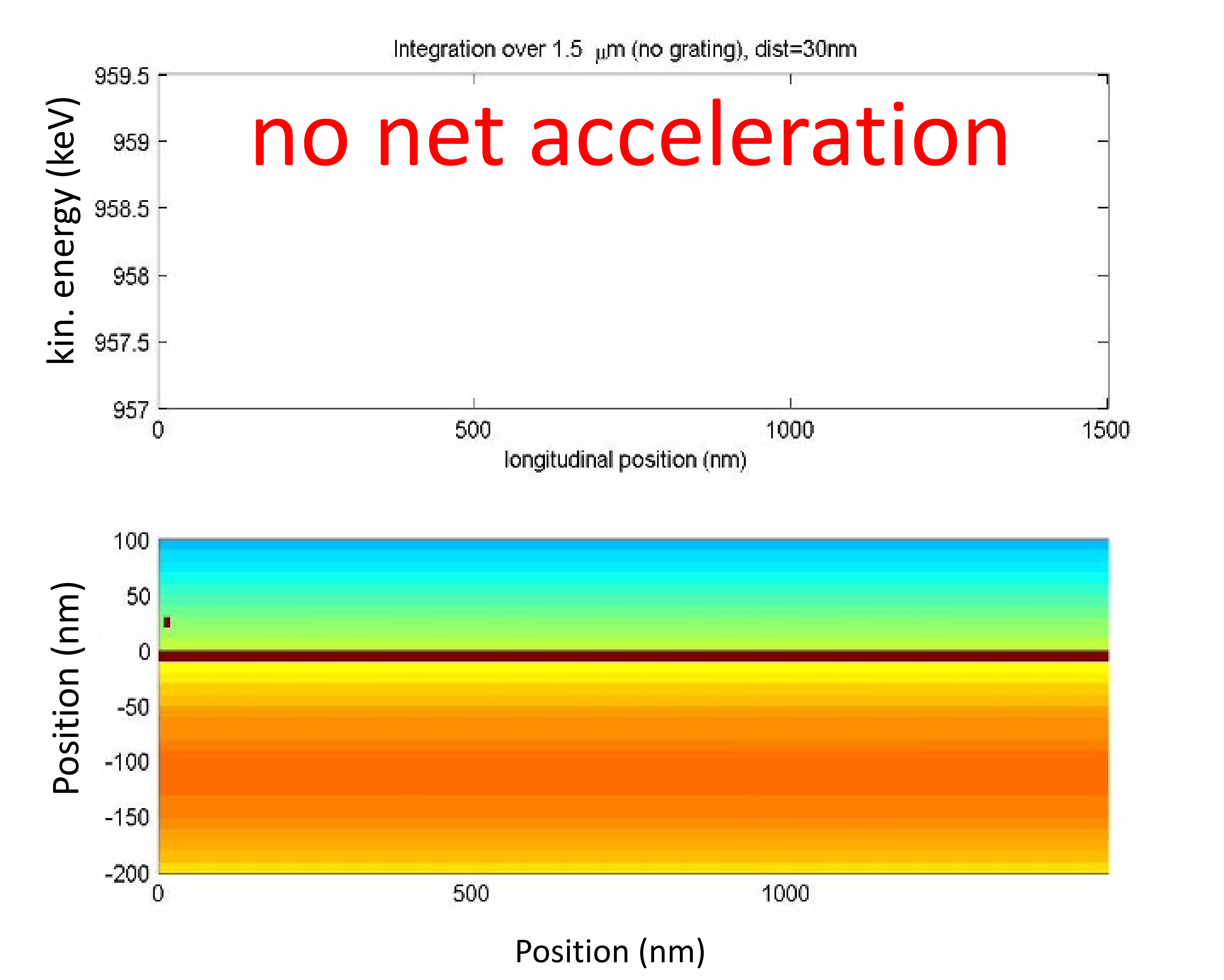
$$\nabla^2 \vec{A} = \frac{1}{v^2} \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$\mathrm{c} = \frac{1}{\mu_0 \epsilon_0}$$

Solution: Plane waves

$$\vec{E}(t,\vec{r}) = E_0 * e^{i\vec{k}\vec{r} - i\omega t} \qquad \vec{B}(t,\vec{r}) = B_0 * e^{i\vec{k}\vec{r} - i\omega t}$$

Electron light interaction in free space



Lawson-Woodward theorem

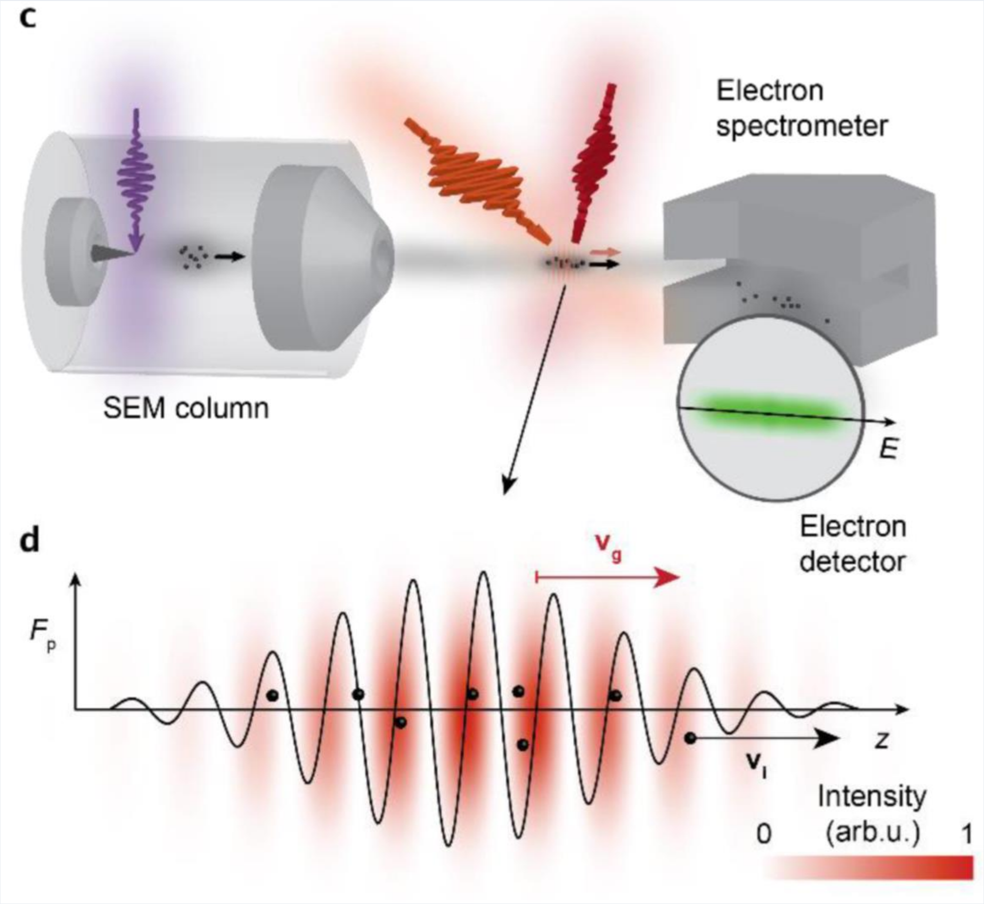
No net acceleration if all the following are true:

- The interaction takes place in vacuum (unity refractive index)
- No boundaries or surfaces are present, i.e., the distance from any source of field is large compared to the wavelength (far-field)
- The particle is moving in a region without other free charges
- (The particle is highly relativistic) Palmer, R. An introduction to acceleration mechanisms. Frontiers of Particle Beams 296, 607{635 (1988).
- No static electric or magnetic fields are present
- The interaction region is infinitely large
- Non-linear forces (e.g., the ponderomotive force) are neglected.

Ponderomotive Generation and Detection of Attosecond Free-Electron Pulse Trains, M. Kozák, et. al., Phys. Rev. Lett. **120**, 103203

Inelastic ponderomotive scattering of electrons at a high-intensity optical travelling wave in vacuum, M. Kozák et. al., *Nature Physics* **volume14**, pages121–125 (2018)

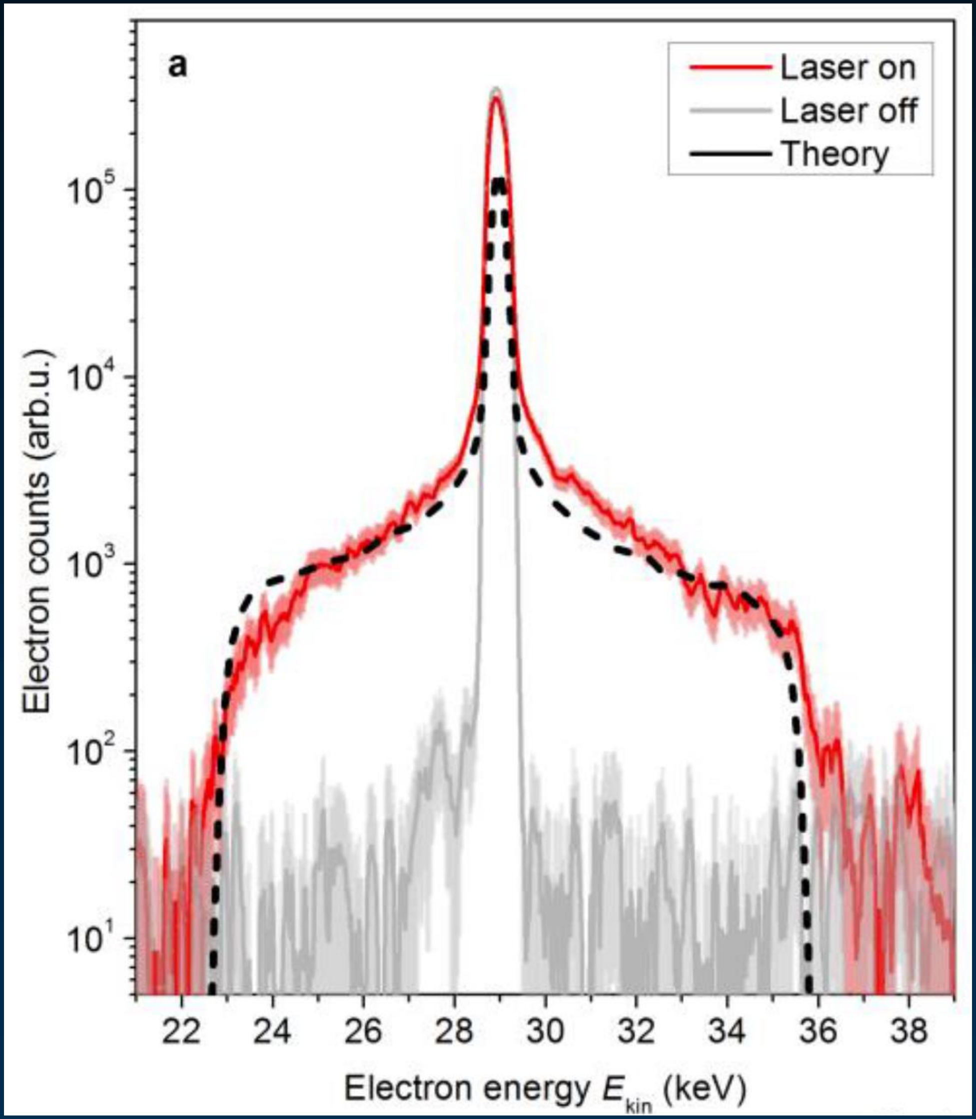
Ponderomotive acceleration



λ_1 = 1356 nm
(0.91 eV)
λ_2 = 1958 nm
(0.63 eV)
α = 41°
β = 107°

$$\lambda_g = 2\pi c/(\omega_1 \cos\alpha - \omega_2 \cos\beta) = 1.41\ \mu m$$

Ponderomotive acceleration



In both pulsed beams:
E_p = 85 μJ
$I_p = 3\ 10^{15}\ W/cm^2$
(rep. rate: 1 kHz)

Gradient: 2.2 GeV/m

- +/- 7 keV broad shoulders
- corresponding to absorption/emission of ~10,000 photons

Electromagnetic waves at interfaces

Boundary conditions:

- $n_{12} \times \left(\overrightarrow{E_2} - \overrightarrow{E_1}\right) = 0$
- $\left(\overrightarrow{D_2} - \overrightarrow{D_1}\right) * n_{12} = \sigma_s$

- $\left(\overrightarrow{B_2} - \overrightarrow{B_1}\right) * n_{12} = 0$
- $n_{12} \times \left(\overrightarrow{H_2} - \overrightarrow{H_1}\right) = \overrightarrow{j_s}$

With:

- n_{12} the normal vector from medium 1 to 2
- $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ the electric displacement field
- σ_s the surface charge
- $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ the magnetic field strength in matter
- $\overrightarrow{j_s}$ the surface current

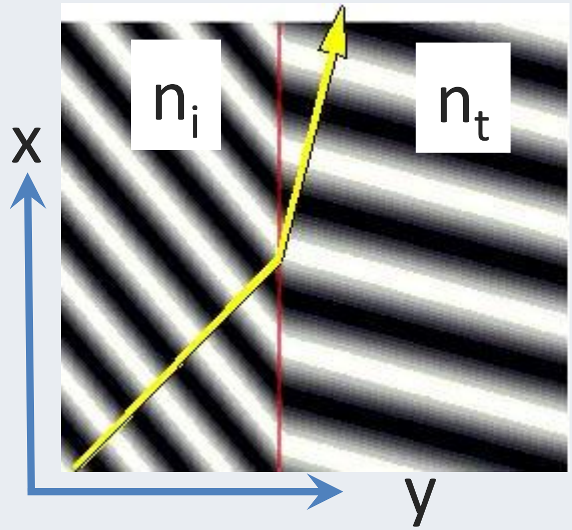
Dielectrics only –> $\sigma_s = 0 = \overrightarrow{j_s}$

Dielectric – Dielectric interface

From the boundary conditions + plane waves:

$$\left(\overrightarrow{k_i} - \overrightarrow{k_r}\right) * \vec{r} = 0$$

$$\left(\overrightarrow{k_i} - \overrightarrow{k_t}\right) * \vec{r} = 0$$



Evaluating the scalar product yields:

$$k_{i,x} = k_{r,x} = k_{t,x}$$

$$k_{i,x} = \left|\overrightarrow{k_i}\right| \sin\phi = \frac{n_i \omega}{c} \sin\phi$$

Angle of incidence ϕ

Dispersion relation

$$k = \frac{n\omega}{c}$$

Similar for transmitted wave:

$$\left|\overrightarrow{k_t}\right| = \frac{n_t \omega}{c} = \sqrt{k_{t,x}^2 + k_{t,y}^2}$$

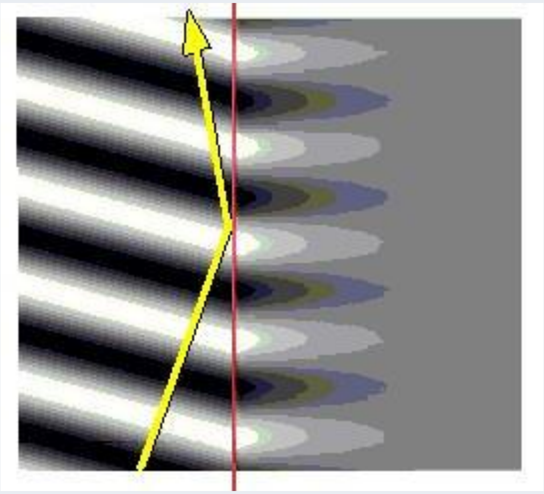
Dielectric – Dielectric interface

Finally: solve for $k_{t,y}$ with $k_{t,x}^2 = k_{i,x}^2$

$$k_{t,y}^2 = \left(\frac{n_t \omega}{c}\right)^2 - \left(\frac{n_i \omega}{c}\right)^2 \sin^2 \phi$$

For $\phi = \sin^{-1} \frac{n_t}{n_i}$

$$k_{t,y} = \pm i k_t \sqrt{\frac{n_i^2}{n_t^2} \sin^2 \phi - 1} = \pm i \beta k_t$$



Transmitted plane wave:

$$\overrightarrow{E_t} = E_0 \, e^{-\beta k_t y} e^{i k_{t,x} x - i \omega t}$$

Acceleration with evanescent fields in vacuum

Phase matching:

$$v_{ph} = \frac{c}{n \ \sin\phi} \qquad v_e = c\beta$$

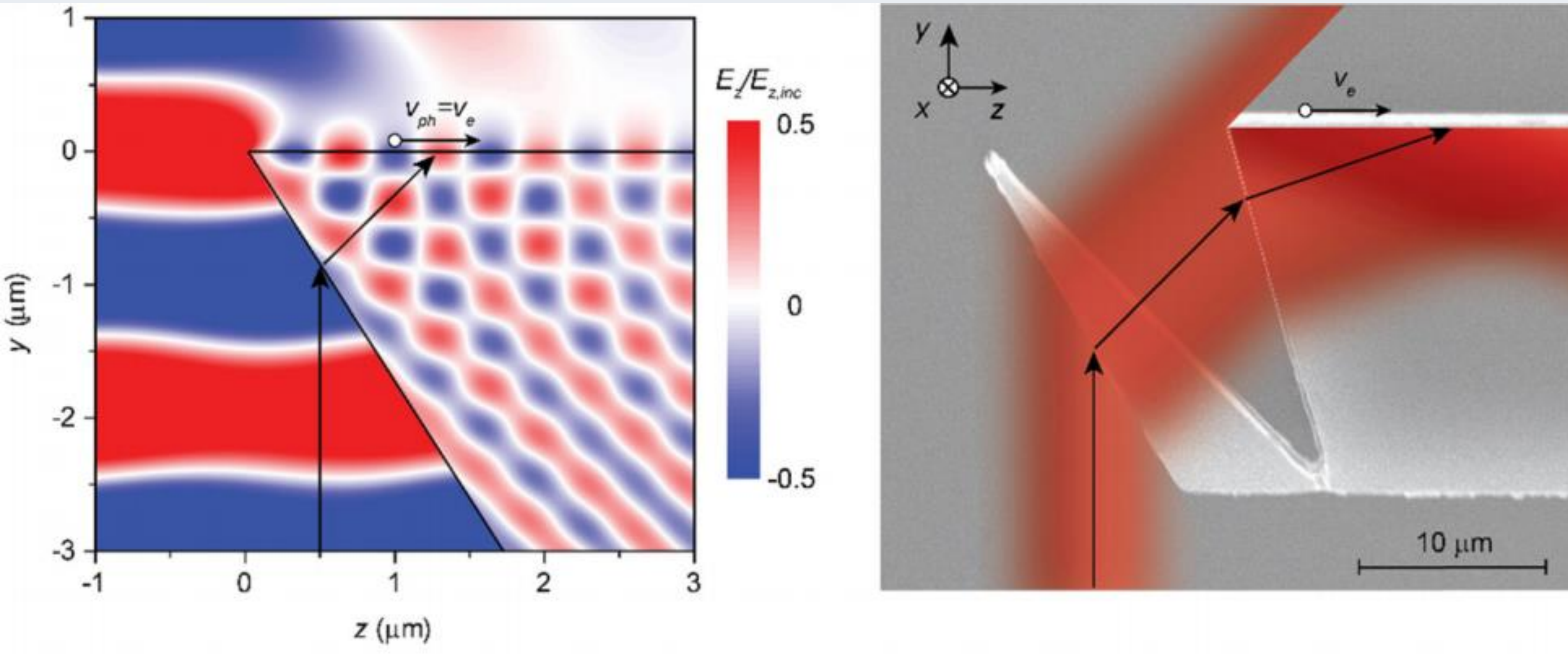
Decay length

$$\Gamma = \frac{c}{\omega\sqrt{n^2 \sin^2\phi - 1}}$$

$$\Gamma = \frac{1}{2\pi}\gamma\beta\lambda$$

Acceleration of sub-relativistic electrons with an evanescent optical wave at a planar interface, M. Kozák et. al., **Optics Express** 25 (2017), S. 19195-19204

Acceleration with evanescent fields in vacuum



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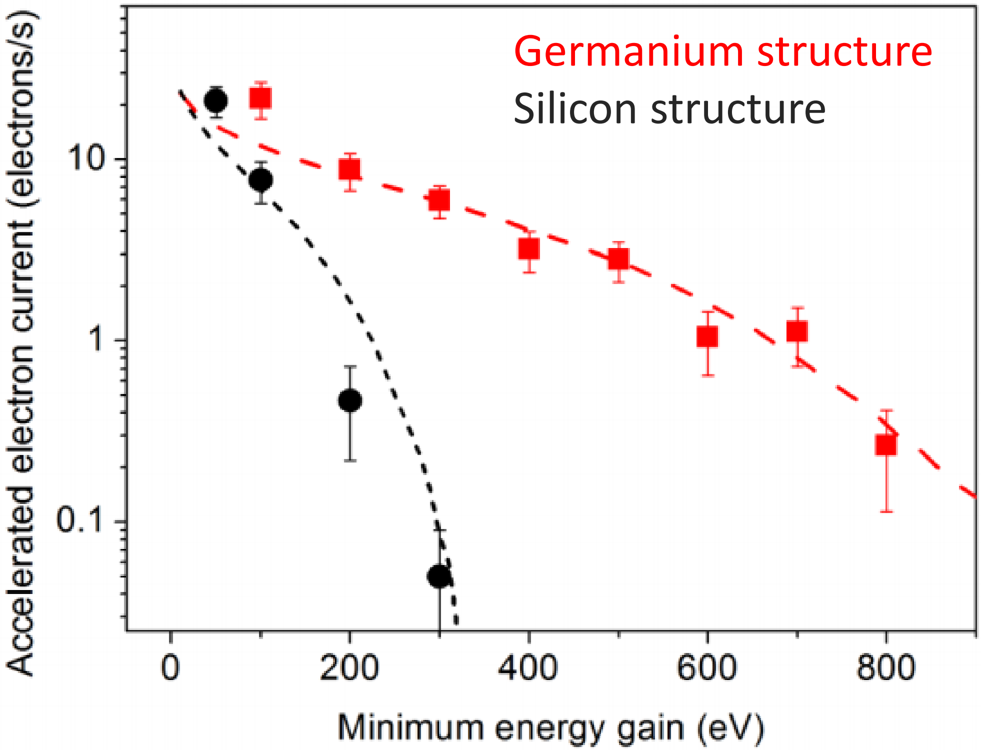
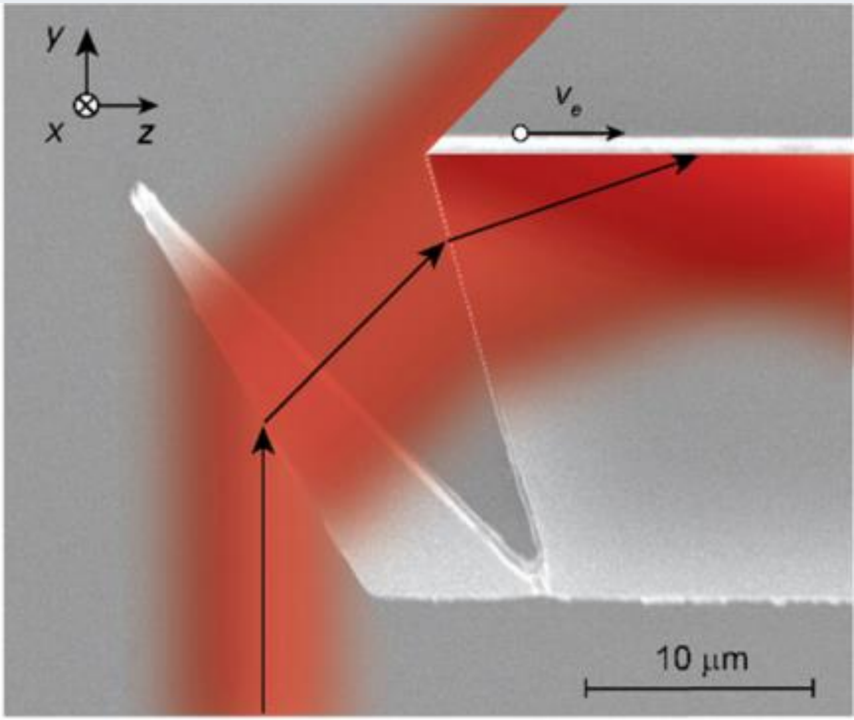
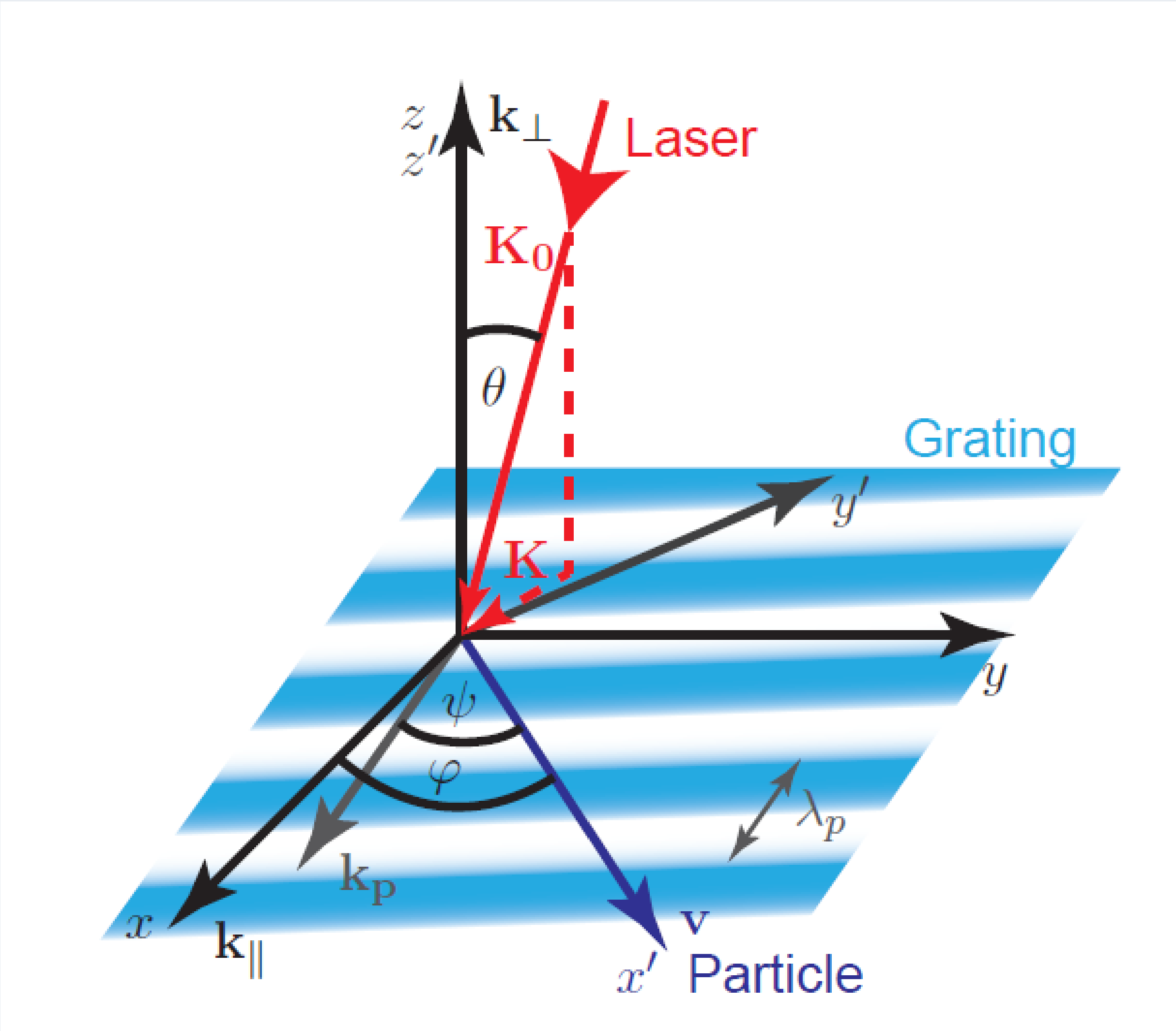




Control only via refractive index n and incidence angle ϕ

Acceleration of sub-relativistic electrons with an evanescent optical wave at a planar interface, M. Kozák et. al., **Optics Express** 25 (2017), S. 19195-19204

Fields at dielectric gratings



Assume infinite plane grating of periodicity λ_P

Diffracted light creates spatial harmonics $\overrightarrow{k_{||}^{n}} = \vec{K} + n\overrightarrow{k_P}$

With:

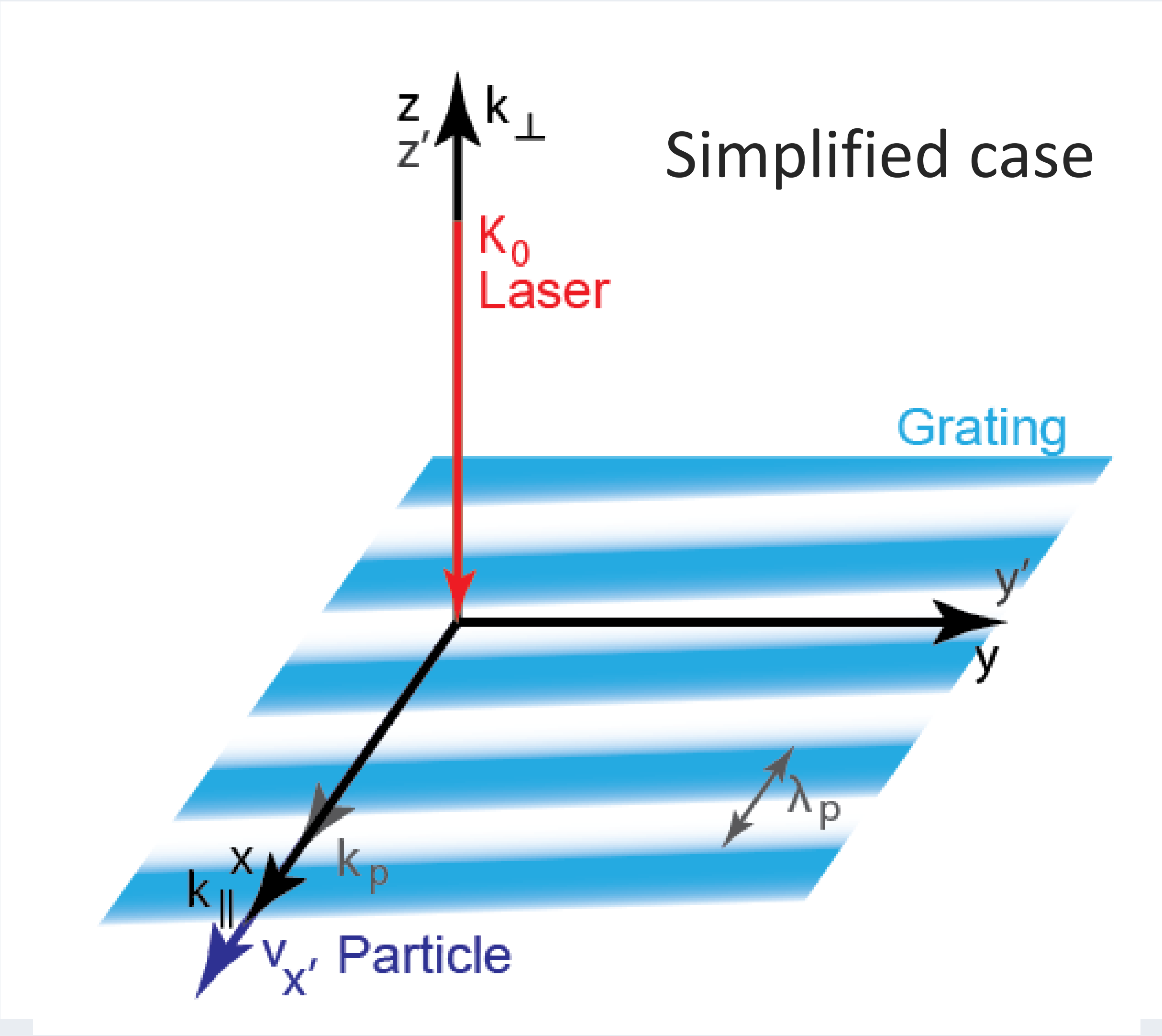
$\overrightarrow{K_0}$ incident wave vector

$\vec{K}$ component parallel to surface

$\overrightarrow{k_{||}}$ parallel diffracted component

$\overrightarrow{k_{\perp}}$ perpendicular diffrected component

Fields at dielectric gratings



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Fields at dielectric gratings

Grating fields can be described as:

$$\vec{A}(\vec{r},t) = \sum_{n=-\infty}^{\infty} \overrightarrow{A_n}\, e^{i(k_{\perp}^{n} z + k_{||}^{n} r - \omega t + \theta)}$$

➢ total field is comprised of a series of spatial harmonics

For phase matching, electrons ($v = \beta c$) and the grating mode ($v_{ph} = \omega / k_{||} \cos\phi$) have to have the same speed:

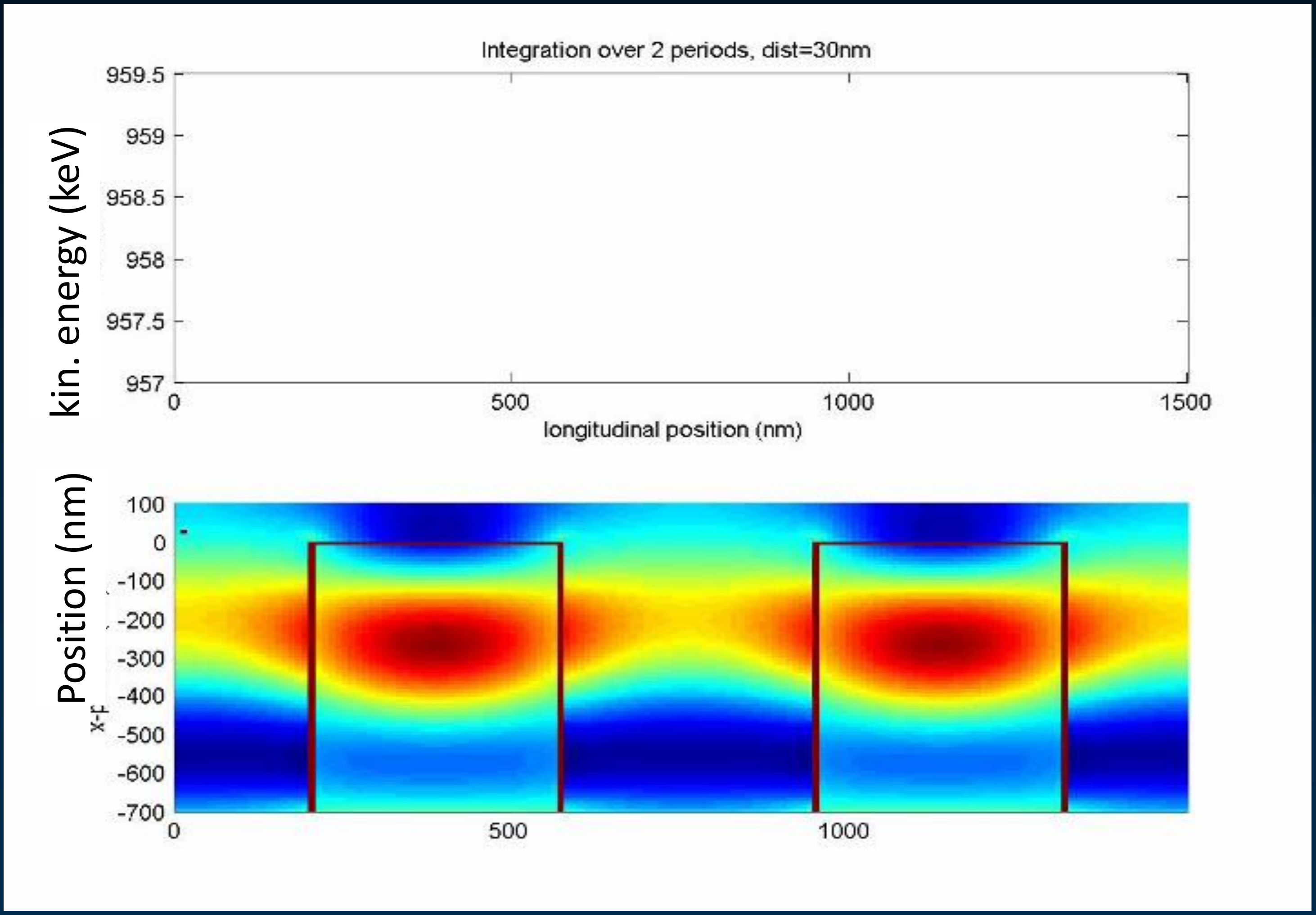
$$k_{||} = \frac{\omega}{\beta c \cos\phi} = \frac{k_0}{\beta \cos\phi}$$

with the dispersion relation $k_0 = \omega / c$.

Assuming particle trajectory is parallel to grating vector k_P, and laser is incident perpendicular on grating surface -> $\vec{K} = 0$

Synchronicity condition: $$\lambda_p = n\beta\lambda$$

Electron light interaction



net acceleration of 1.1 GeV/m

Fields and forces at dielectric gratings

Using $k_{||}$ and $k_{\perp}$ in Ampere's and Faraday's laws, we obtain:

$$\vec{E} = \begin{pmatrix} icB_y/(\tilde{\beta}\tilde{\gamma}) \\ E_y \\ -cB_y/\tilde{\beta} \end{pmatrix} \qquad \vec{B} = \begin{pmatrix} icE_y/(\tilde{\beta}\tilde{\gamma}) \\ B_y \\ E_y/(\tilde{\beta}\tilde{\gamma}) \end{pmatrix}$$

From these fields we can calculate the Lorentz force:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = q\begin{pmatrix} icB_y/(\tilde{\beta}\tilde{\gamma}) + \tan\phi E_y \\ 0 \\ -cB_y(1-\tilde{\beta}^2)/\tilde{\beta} + i\tan\phi E_y/\tilde{\gamma} \end{pmatrix}$$

$$\vec{F} = q\begin{pmatrix} icB_y/(\beta\gamma) \\ 0 \\ -cB_y/\beta\gamma^2 \end{pmatrix}$$

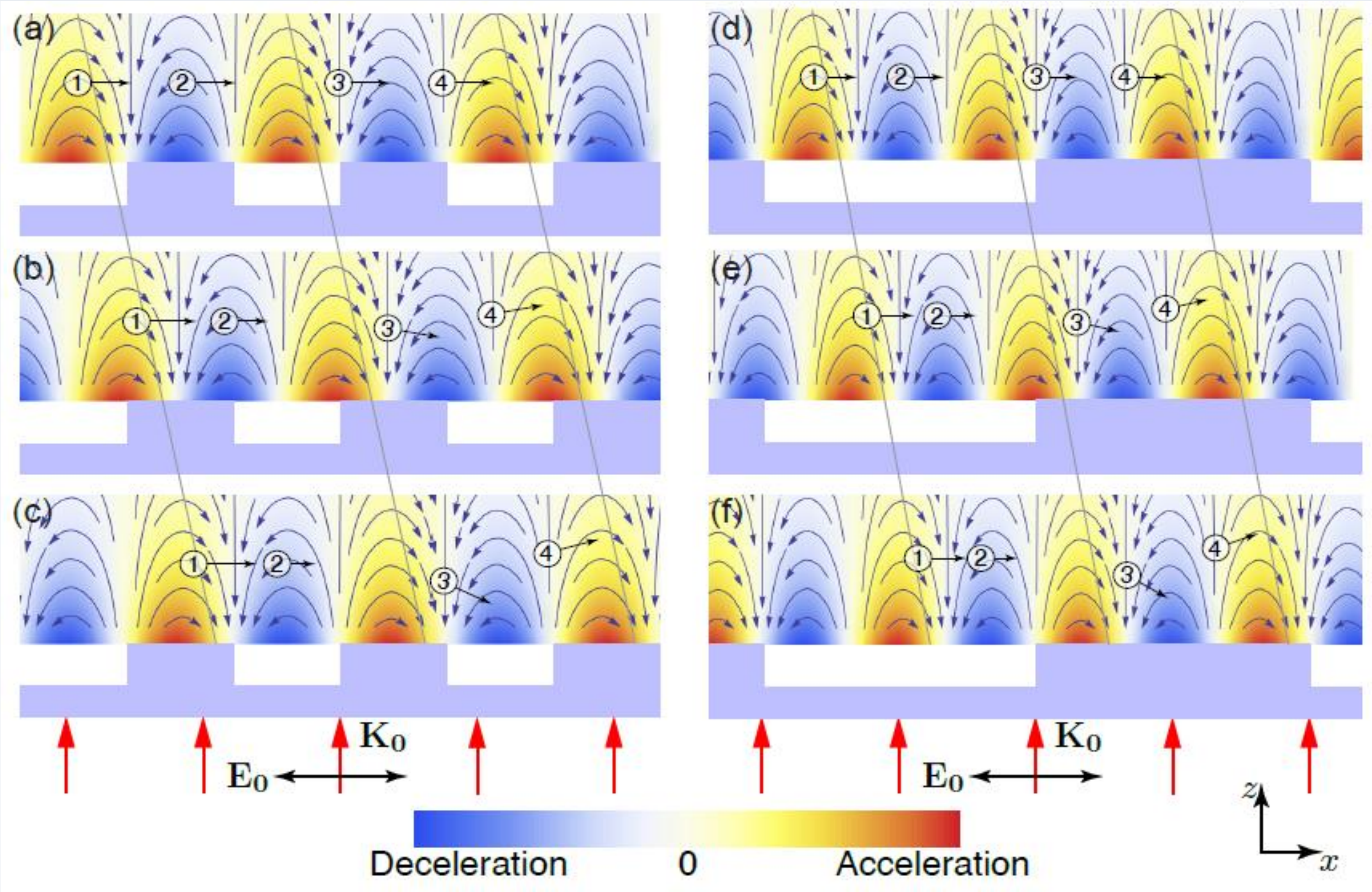
Acceleration at dielectric gratings

Fields of a dielectric laser accelerator based on a one sided grating structure. Depicted are 3 moments in time, t = 0 (a, d), t = π/2 (b, e), t = π (c, f)

Electrons injected in different phases experience different fields:
1: Acceleration
2: Deceleration
3: Deflection to structure
4: Deflection to vacuum

left: first spatial harmonic
right: third spatial harmonic
-> 1/3 decay length

$$\lambda_p = n\beta\lambda$$



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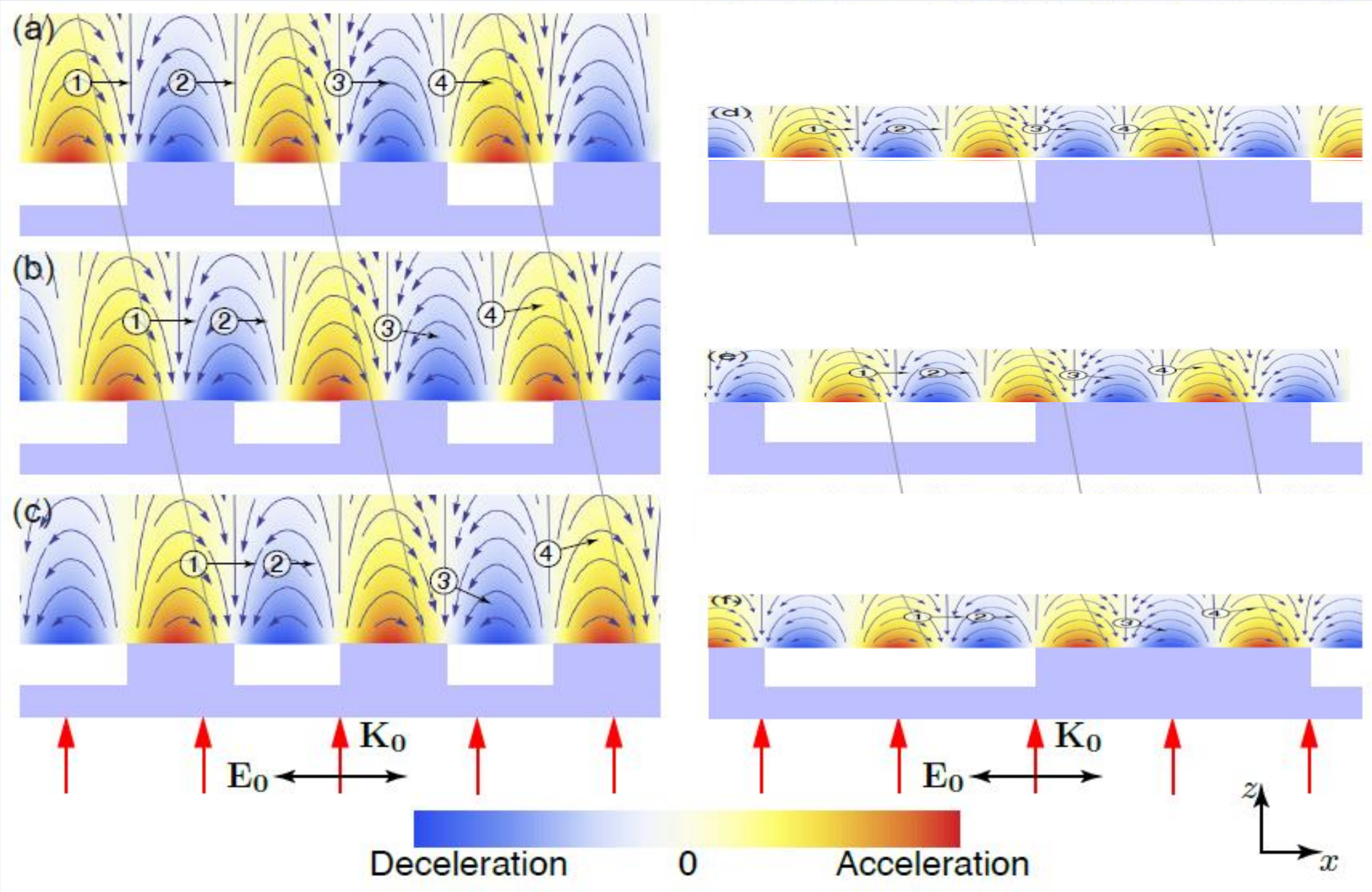
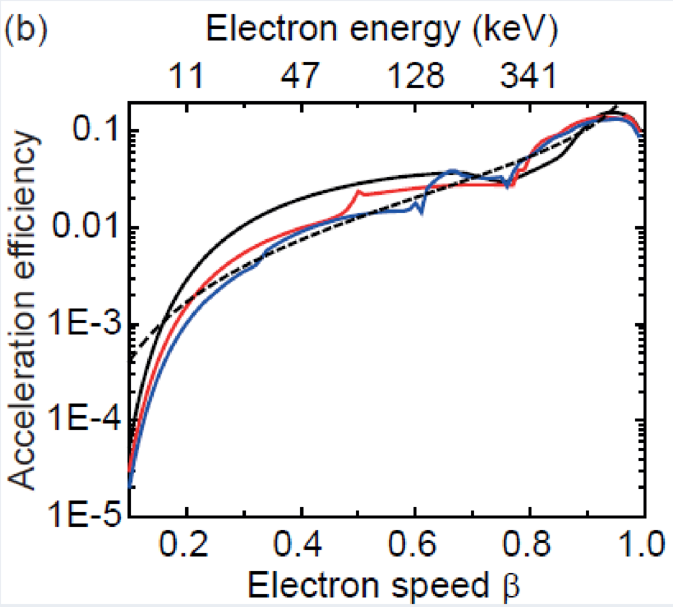


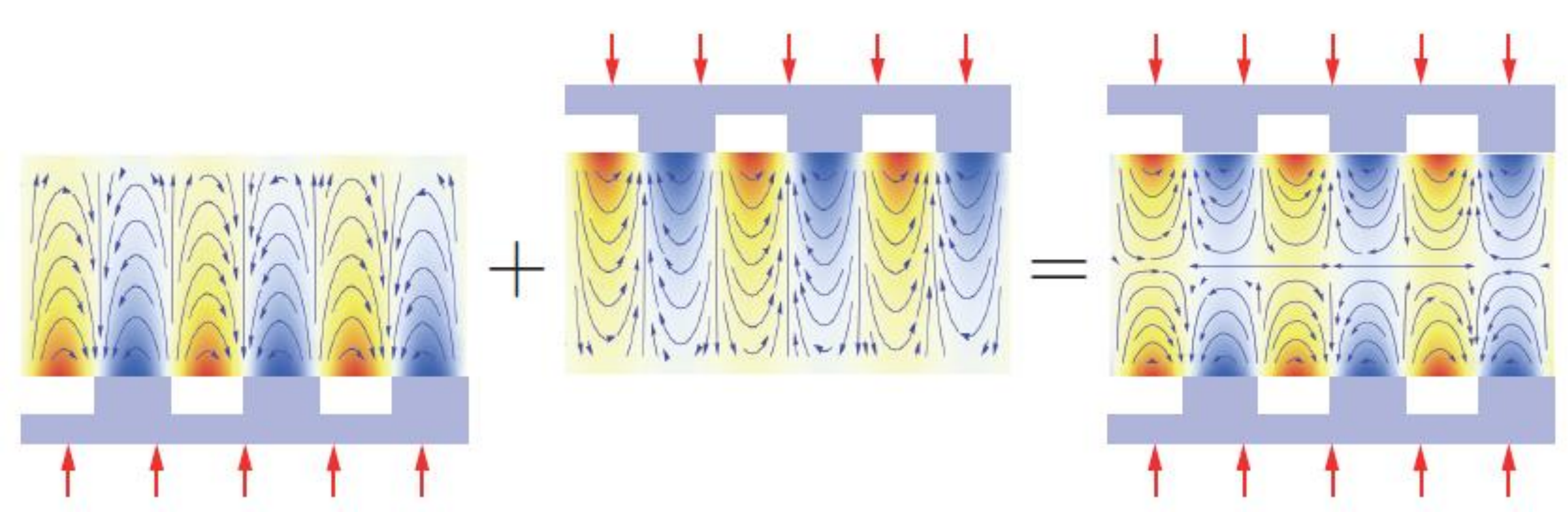
Implications of these forces and fields

- There is a transversal force component
  - At this geometry the transversal position of the electrons is non recoverable, due to the evanescent nature of the fields
- There is no light speed mode, a mode capable of accelerating $\beta = 1$ electrons, in the case of a single sided grating, since the solution would require a linearly increasing electric field extending to infinity



Solution: double sided grating

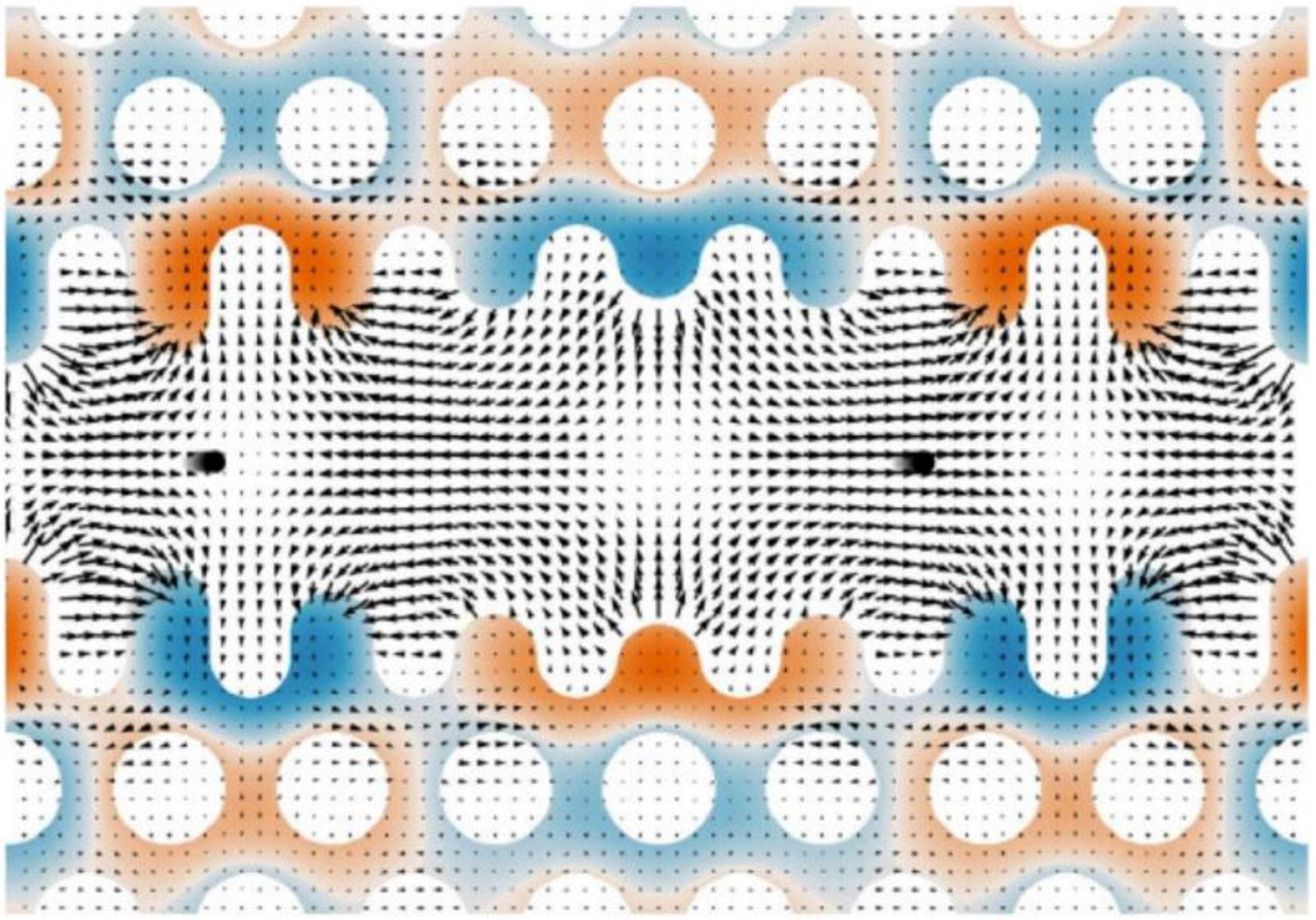
Adding a second grating, inverted, on the other side, creates a symmetric field with either a cosh or sinh mode.
While deflecting forces are not mittigated, the symmetric field profile can be used to confine the electron beam.
More later with Alternating Phase Focusing (APF)



Bonus: double sided structures support speed of light mode

Galaxie: Using multiple modes

A synchronous mode is used for acceleration while an asynchronous mode confines the bunches



Stable Charged-Particle Acceleration and Focusing in a Laser Accelerator Using Spatial Harmonics, B. Naranjo et. al., PRL 109, 164803 (2012)
Animations: http://rodan.physics.ucla.edu/PhysRevLett.109.164803/

Simulations

We use different simulation tools to compute the characteristics of out accelerating devices:

- Finite Difference Time Domain (FDTD) code to calculate exact fields
- The resulting fields can be broken down into kicks per period to approximate the accelerator
- For special cases we use a PIC implementation to look at wakefields in dielectric accelerators

Electron tracking:

- Integrate computed kicks
- Runge-Kutta motion solver (with spacecharge)
- PIC code for self consistent solution