# Cyclotrons

**CERN Accelerator School – Introductory Course**
**High Tatras, September 19, 2019**

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Paul Scherrer Institut

# Cyclotrons/FFA - Outline

- the classical cyclotron

  **history of the cyclotron, basic concepts and scalings, focusing, stepwidth, classification of cyclotron-like accelerators**

- synchro-cyclotrons

  **concept, synchronous phase, example**

- isochronous cyclotrons **( → sector cyclotrons )**

  **isochronous condition, focusing in Thomas-cyclotrons, spiral angle, classical extraction: pattern/stepwidth, space charge**

- applications and examples of existing cyclotrons

  **TRIUMF, RIKEN SRC, PSI Ring, PSI medical cyclotron**

**Part II**

- cyclotron subsystems

  **Injection/extraction schemes, RF systems/resonators, magnets, vacuum issues, instrumentation, FFA specific magnets, FFA resonators**

- FFA = Fixed Focus Alternating Gradient Accelerators

  **motivation & applications, scaling FFA's, non-scaling and linear FFA, FFA subsystems**

- discussion

  **classification of circular accelerators, Pro's and Con's of cyclotrons / FFA for different applications**

# The Classical Cyclotron

two capacitive electrodes „Dees", two gaps per turn

internal ion source

homogenous B field

**constant revolution time**

(for low energy, $\gamma \approx 1$)

$$\omega_c = \frac{eB_z}{m}$$
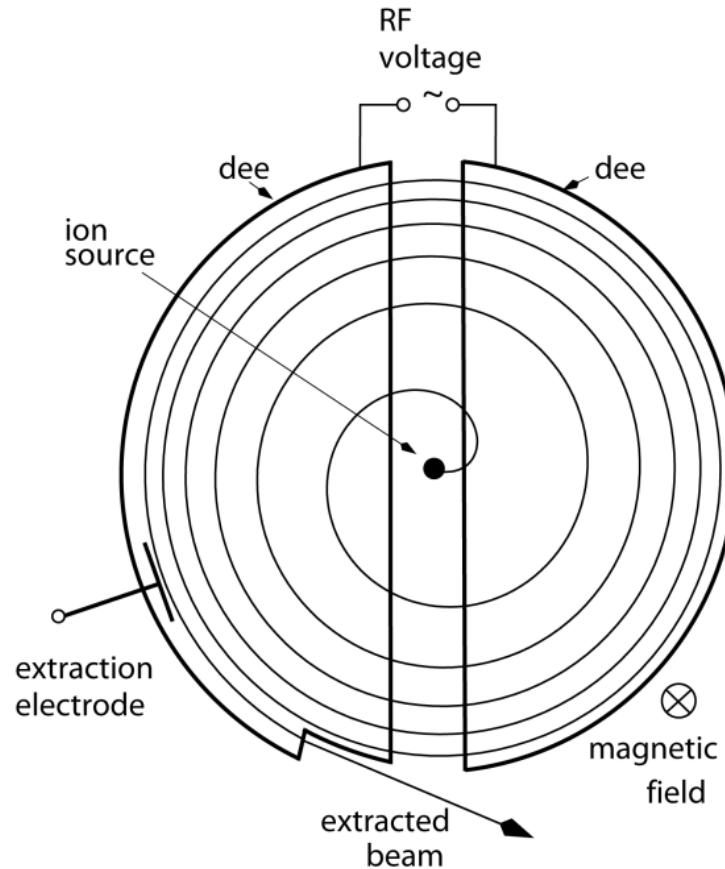
**powerful concept:**

➔ **simplicity, compactness**

➔ **continuous injection/extraction**
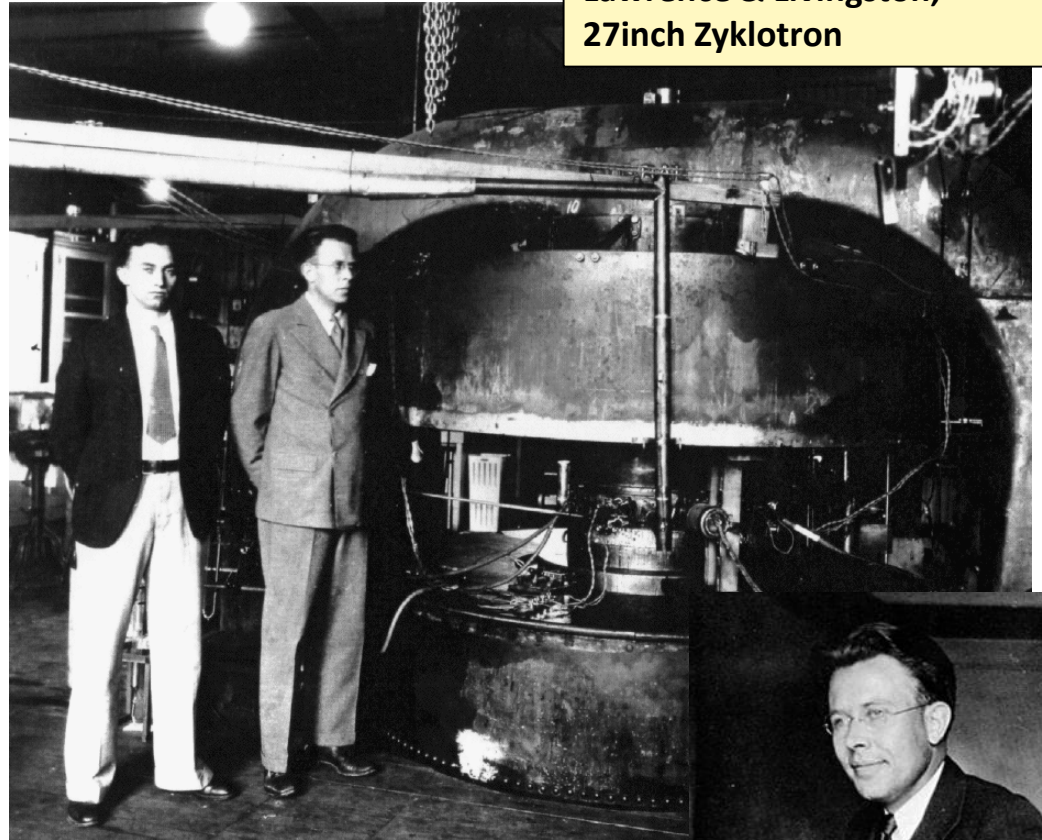
➔ **multiple usage of accelerating voltage**

# some History …

first cyclotron: 1931, Berkeley
1kV gap-voltage 80keV Protons

Lawrence & Livingston,
27inch Zyklotron

Ernest Lawrence, Nobel Prize 1939
*"for the invention and development of the cyclotron and for results obtained with it, especially with regard to artificial radioactive elements"*

John Lawrence (center), 1940'ies
*first medical applications: treating patients with neutrons generated in the 60inch cyclotron*

PSI Ring Cyclotron & Crew

# cyclotron frequency and *K* value

- **cyclotron frequency** (homogeneous) B-field:

$$\omega_c \;=\; \frac{eB}{\gamma m_0}$$

- **cyclotron *K*-value**:

→ *K* is the **kinetic energy reach** for protons **from bending strength** in non-relativistic approximation:

$$K \;=\; \frac{e^2}{2m_0}(B\rho)^2$$

→ *K* can be used to rescale the energy reach of protons to other charge-to-mass ratios:

$$\frac{E_k}{A} \;=\; K\left(\frac{Q}{A}\right)^2$$

→ *K* in [MeV] is often used for naming cyclotrons

examples:     K-130 cyclotron / Jyväskylä

cyclone C230 / IBA

# cyclotron - isochronicity and scalings

**continuous acceleration → revolution time should stay constant, though $E_k$, $R$ vary**

magnetic rigidity:

$$BR = \frac{p}{e} = \beta\gamma\frac{m_0 c}{e}$$

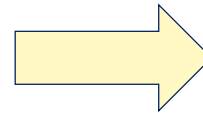orbit radius from isochronicity:

$$R = \frac{c}{\omega_c}\beta = R_\infty\beta$$

deduced scaling of $B$:

$$\longrightarrow B(R) \propto \gamma(R)$$

**to be isochronous, $B$ must be raised $\propto \gamma(R)$**
**→ this contradicts the focusing requirements!**

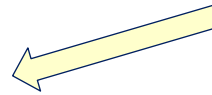main difficulty to be overcome by cyclotron & FFA variants.

# field index

the field index describes the (normalized)
radial slope of the bending field:

$$k \;=\; \frac{R}{B}\frac{dB}{dR}$$

$$=\; \frac{\beta}{\gamma}\frac{d\gamma}{d\beta}$$

$$=\; \gamma^2 - 1$$

from isochronous condition:
$B \propto \gamma, \; R \propto \beta$

→ **thus k > 0 (positive slope of field) to keep beam isochronous!**

# focusing in a classical cyclotron

centrifugal force mv²/r

Lorentz force qv×B

$$m\ddot{r} = mr\dot{\theta}^2 - qr\dot{\theta}B_z$$

focusing: consider small deviations *x* from beam orbit *R*  (*r = R+x*):

$$\ddot{x} + \frac{q}{m}vB_z(R+x) - \frac{v^2}{R+x} = 0,$$

$$\ddot{x} + \frac{q}{m}v\left(B_z(R) + \frac{\mathrm{d}B_z}{\mathrm{d}R}x\right) - \frac{v^2}{R}\left(1 - \frac{x}{R}\right) = 0,$$

$$\ddot{x} + \omega_{\mathrm{c}}^2(1+k)x = 0.$$

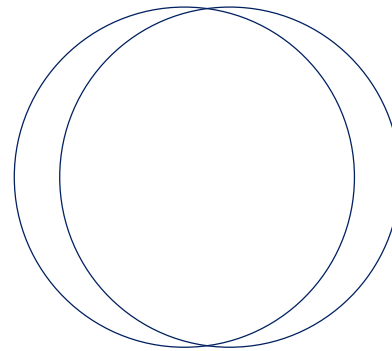using: $\omega_{\mathrm{c}} = qB_z/m = v/R,\ r\dot{\theta} \approx v,\ k = \frac{R}{B}\frac{dB}{dR}$

# betatron tunes in cyclotrons

thus in radial plane:

$$\omega_r = \omega_c\sqrt{1+k} = \omega_c\nu_r$$
$$\nu_r = \sqrt{1+k}$$

using isochronicity condition

$$\approx \gamma$$

note: simple case for $k = 0$:  $\nu_r = 1$
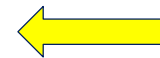(one circular orbit oscillates w.r.t the other)

using Maxwell to relate $B_z$ and $B_R$:

$$\mathrm{rot}\,\vec{B} = \frac{dB_R}{dz} - \frac{dB_z}{dR} = 0$$

in vertical plane:

$$\nu_z = \sqrt{-k}$$

$k<0$ to obtain vertical focus.

**thus: in classical cyclotron $k < 0$ required for vert. focus; however this violates isochronous condition $k = \gamma^2\text{-}1 > 0$**

# cyclotron stepwidth classical (nonrelativistic, B const)

centrifugal f.

Lorentz f.

equation of motion for ideal centroid orbit *R*,
→ relation betw. **energy and radius**

$$m\ddot{R} = m\frac{v^2}{R} - qvB_z = 0$$

$$qRB_z = \sqrt{2mE_k}$$

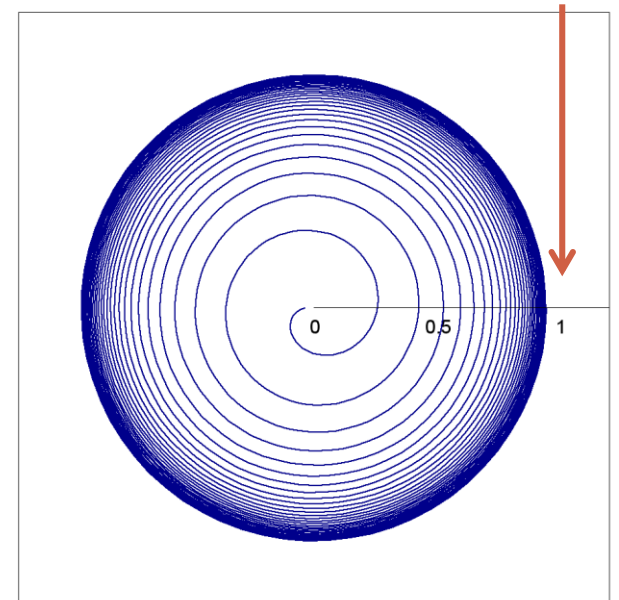$$\frac{dR}{R} = \frac{1}{2}\frac{dE_k}{E_k}$$

"cyclotron language"

$$R_\infty = R/\beta$$

use: $\Delta E_k = \text{const}; B_z = \text{const}; E_k \propto R^2$

thus:

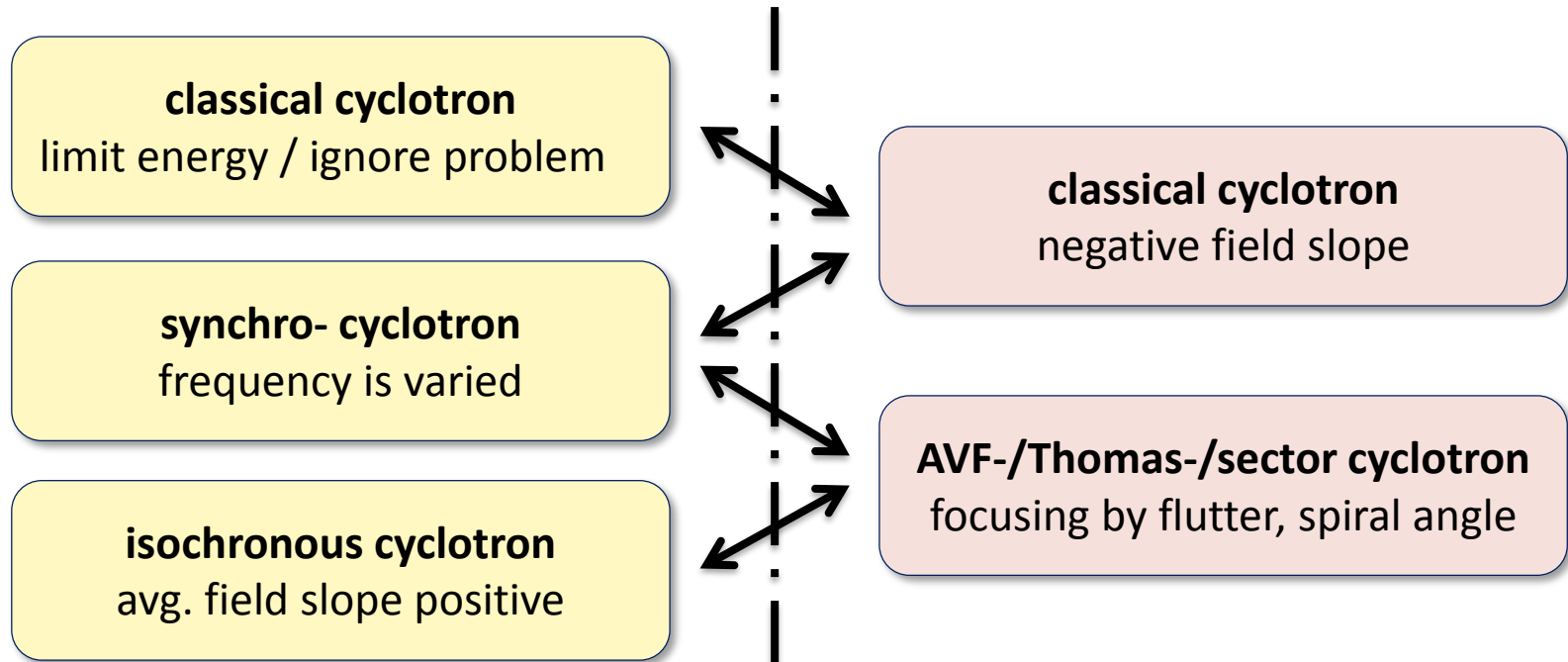$$\Delta R \propto \frac{R}{E_k} \propto \frac{1}{R}$$

radius increment per turn decreases with increasing radius
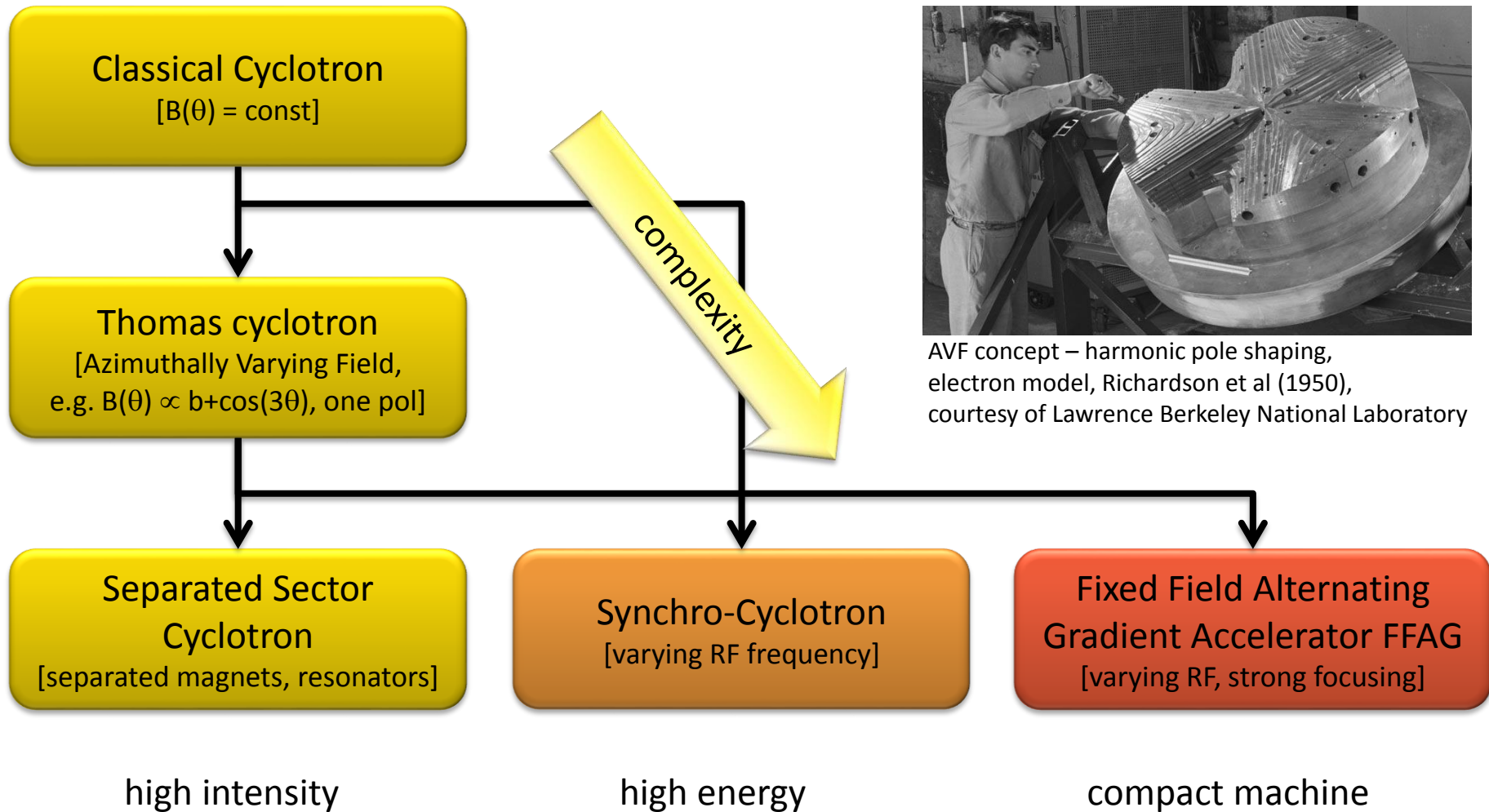→ **extraction becomes more and more difficult at higher energies**

# naming conventions of cyclotrons …

## 1.) resonant acceleration

**classical cyclotron**
limit energy / ignore problem

**synchro- cyclotron**
frequency is varied

**isochronous cyclotron**
avg. field slope positive

## 2.) transverse focusing

**classical cyclotron**
negative field slope

**AVF-/Thomas-/sector cyclotron**
focusing by flutter, spiral angle

# classification of cyclotron like accelerators

**Classical Cyclotron**
[B(θ) = const]

**Thomas cyclotron**
[Azimuthally Varying Field,
e.g. B(θ) ∝ b+cos(3θ), one pol]

complexity



AVF concept – harmonic pole shaping,
electron model, Richardson et al (1950),
courtesy of Lawrence Berkeley National Laboratory

**Separated Sector Cyclotron**
[separated magnets, resonators]

**Synchro-Cyclotron**
[varying RF frequency]

**Fixed Field Alternating Gradient Accelerator FFAG**
[varying RF, strong focusing]

high intensity

high energy

compact machine
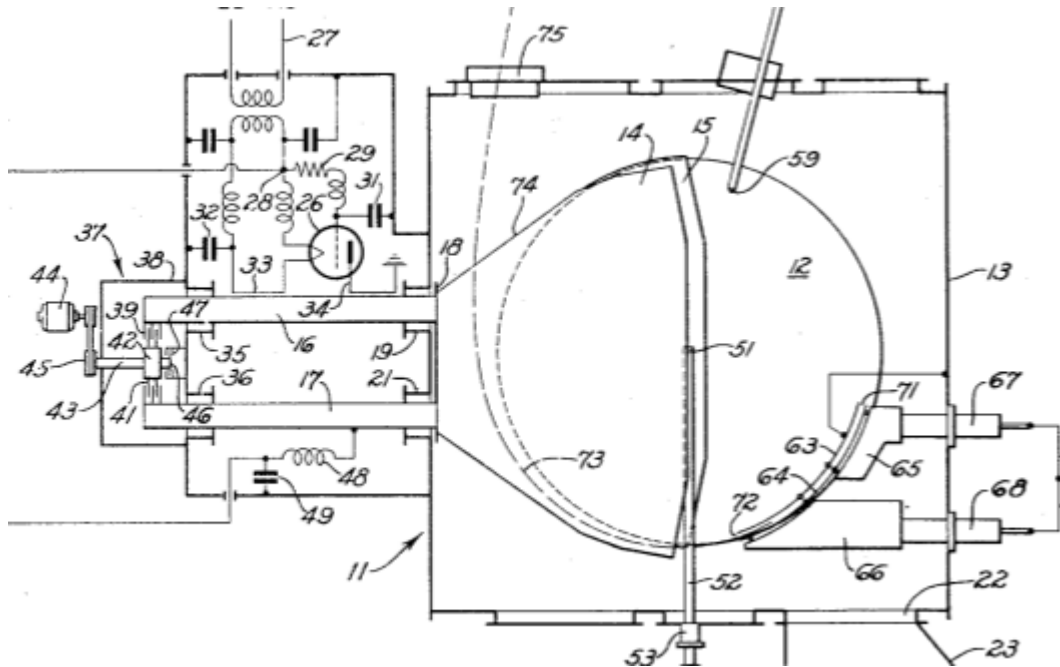
next: **synchro-cyclotrons**
- **concept and properties**
- **frequency variation and synchronous phase**
- **an example for a modern synchrocyclotron**
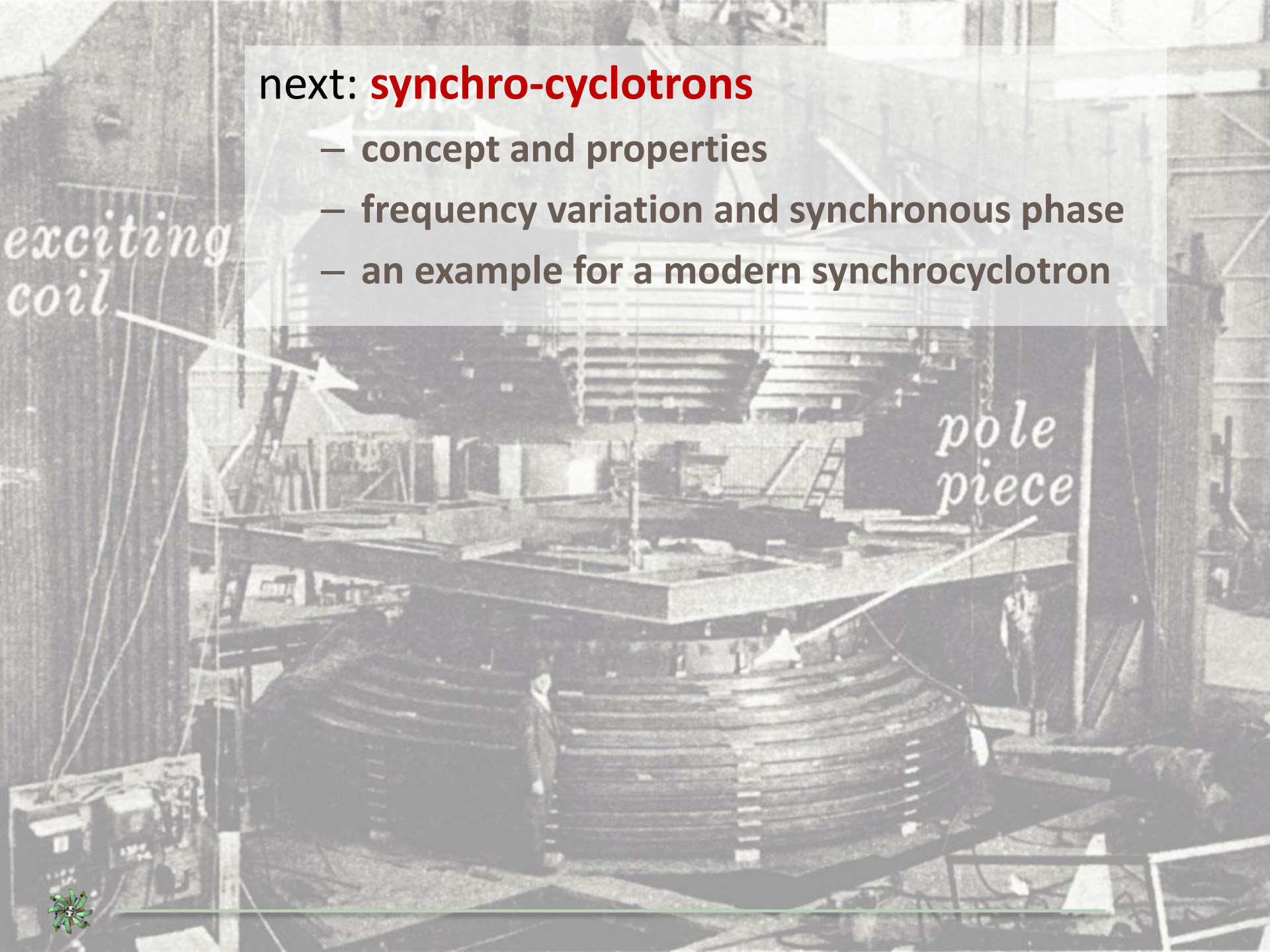
# Synchrocyclotron -concept



**first proposal by Mc.Millan, Berkeley**

- accelerating frequency is variable, is reduced during acceleration
- negative field index (= negative slope) ensures sufficient focusing
- operation is pulsed, thus avg. intensity is low
- bending field constant in time, thus rep. rate high, e.g. 1kHz

# Synchrocyclotron continued

| advantages | disadvantages |
|---|---|
| - high energies possible (≥1Gev)<br>- focusing by field gradient, no complicated flutter required → thus compact magnet<br>- only RF is cycled, fast repetition as compared to synchrotron | - low intensity, at least factor 100 less than CW cyclotron<br>- complicated RF control required<br>- weak focusing, large beam |

**numerical example**
**field and frequency vs. radius**:

- 230MeV p, strong field
- RF curve must be programmed in some way
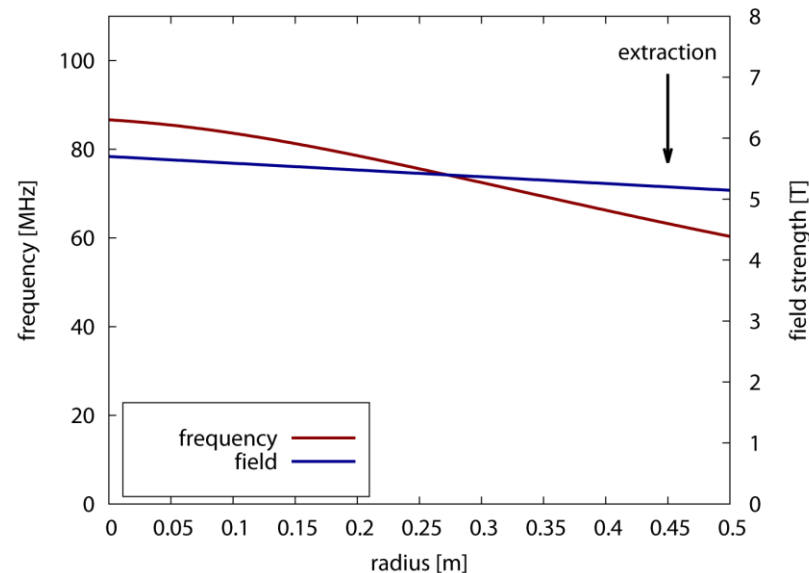
# Synchrocyclotron and synchronous phase

- internal source generates continuous beam; only a fraction is captured by RF wave in a phase range around a synchronous particle
- in comparison to a synchrotron the "storage time" is short, thus in practice no synchrotron oscillations
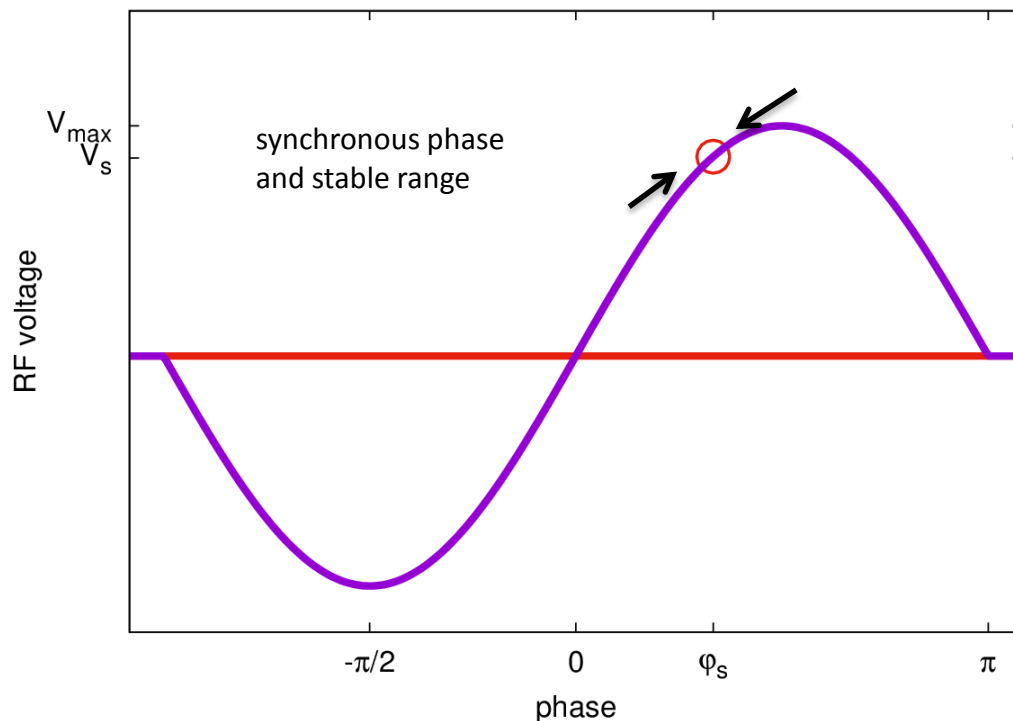


relation of
    energy gain per turn and
        rate of frequency change

$$\frac{qU_0 N \cos \varphi_s}{E_k + E_0} = -\frac{2\pi}{\omega^2}\frac{d\omega}{dt}$$

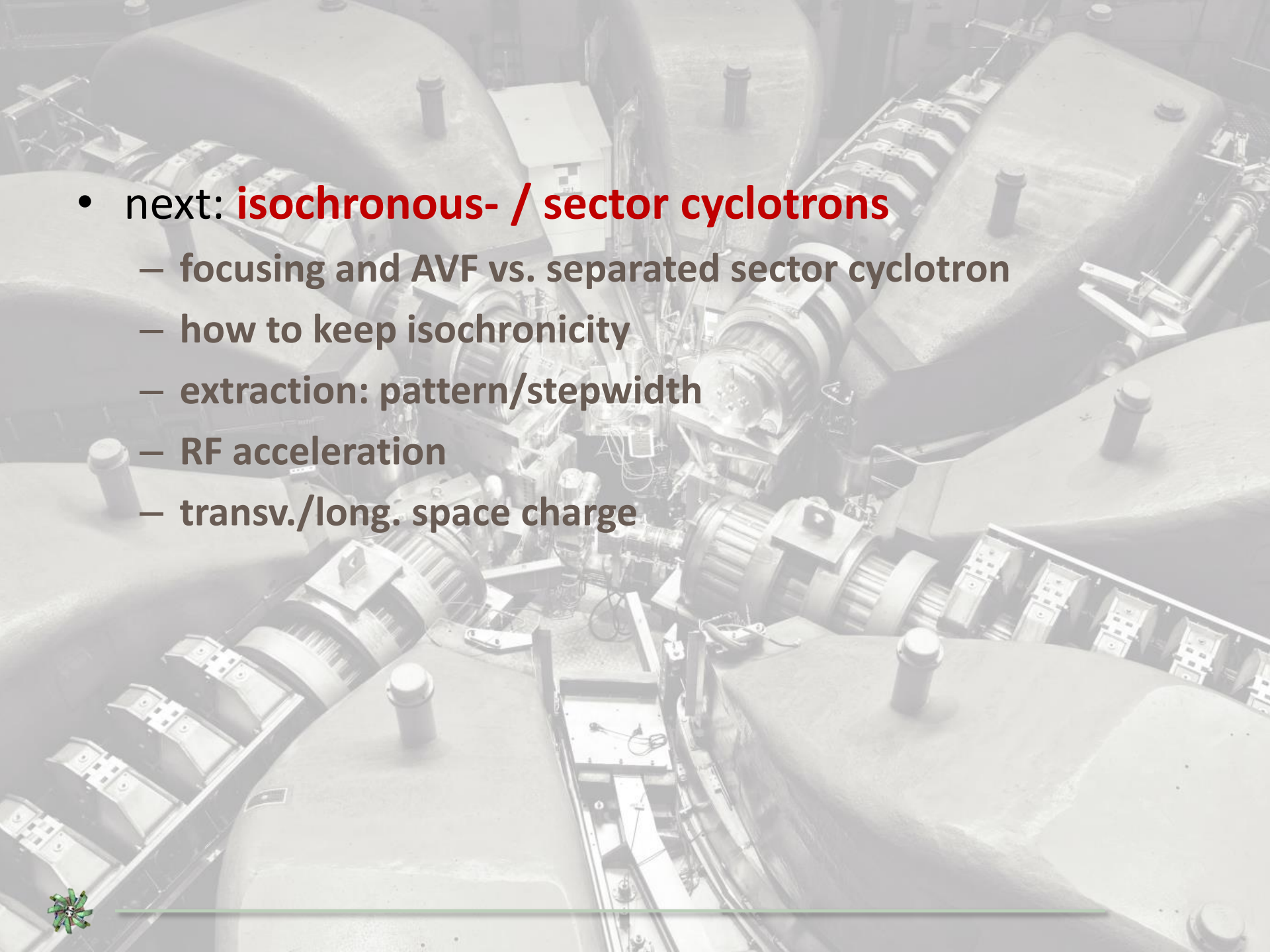# A modern synchrocyclotron for medical application – IBA S2C2

$\rightarrow$ at the same energy synchrocyclotrons can be build more compact and with lower cost than sector cyclotrons; however, the achievable current is significantly lower

| energy | 230 MeV |
|---|---|
| current | 20 nA |
| dimensions | $\varnothing$2.5 m  x  2 m |
| weight | < 50 t |
| extraction radius | 0.45 m |
| s.c. coil strength | 5.6 Tesla |
| RF frequency | 90...60 MHz |
| repetition rate | 1 kHz |



Feb/2013, courtesy: P.Verbruggen,IBA

# compact treatment facility using the high field synchro-cyclotron
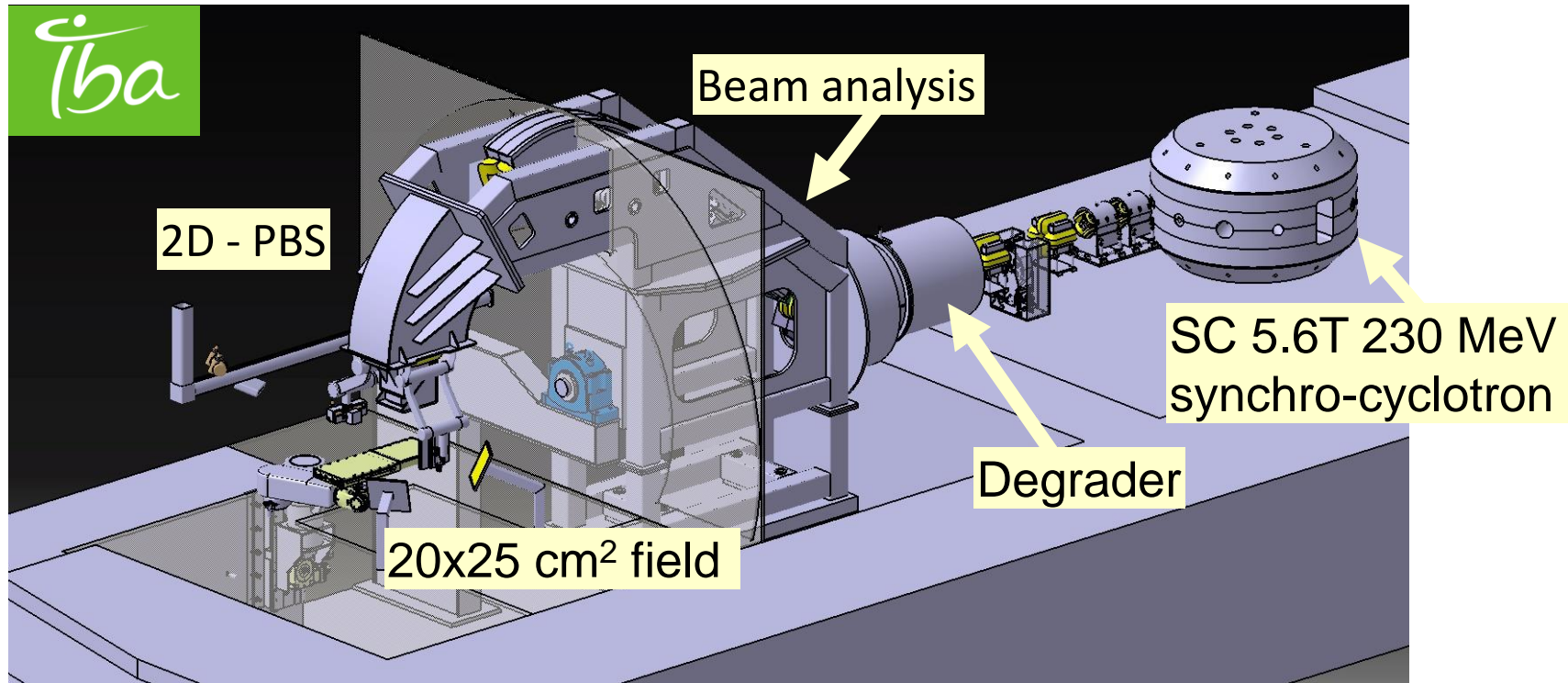


- required area: 24x13.5m$^2$ (is small)
- 2-dim pencil beam scanning

- next: **isochronous- / sector cyclotrons**
  - focusing and AVF vs. separated sector cyclotron
  - how to keep isochronicity
  - extraction: pattern/stepwidth
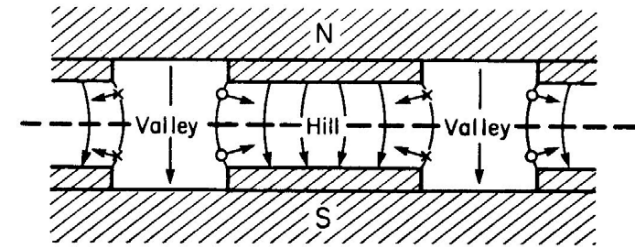  - RF acceleration
  - transv./long. space charge

# focusing in sector cyclotrons

**hill / valley variation of magnetic field (Thomas focusing) makes it possible to design cyclotrons for higher energies**

Flutter factor:

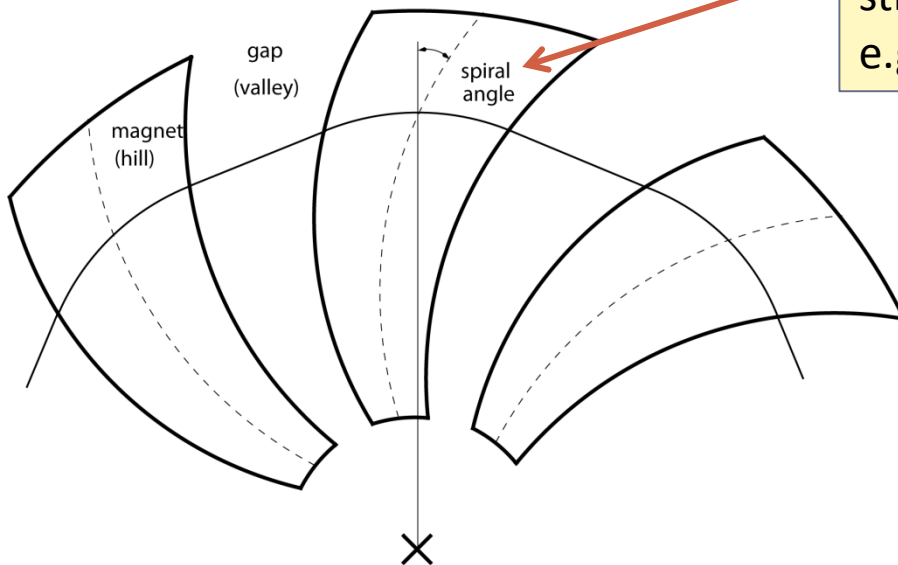$$F^2 = \frac{\overline{B_z^2} - \overline{B_z}^2}{\overline{B_z}^2}$$



[illustration of focusing at edges]

with flutter and additional spiral angle
of bending field:

$$\nu_z^2 = -\frac{R}{B_z}\frac{dB_z}{dR} + F^2(1 + 2\tan^2\delta)$$

strong term
e.g.: $\delta = 27°$:  $2\tan^2\delta = 1.0$



gap
(valley)

spiral
angle

magnet
(hill)

# Azimuthally Varying Field vs. Separated Sector Cyclotrons



**PSI/Varian comet: 250MeV sc. medical cyclotron**



**PSI Ring cyclotron**

- **AVF = single pole with shaping**
- often **spiral poles** used
- **internal source** possible
- **D-type RF electrodes**, rel. low energy gain
- **compact**, cost effective
- depicted Varian cyclotron: 80% extraction efficiency; **not suited for high power**
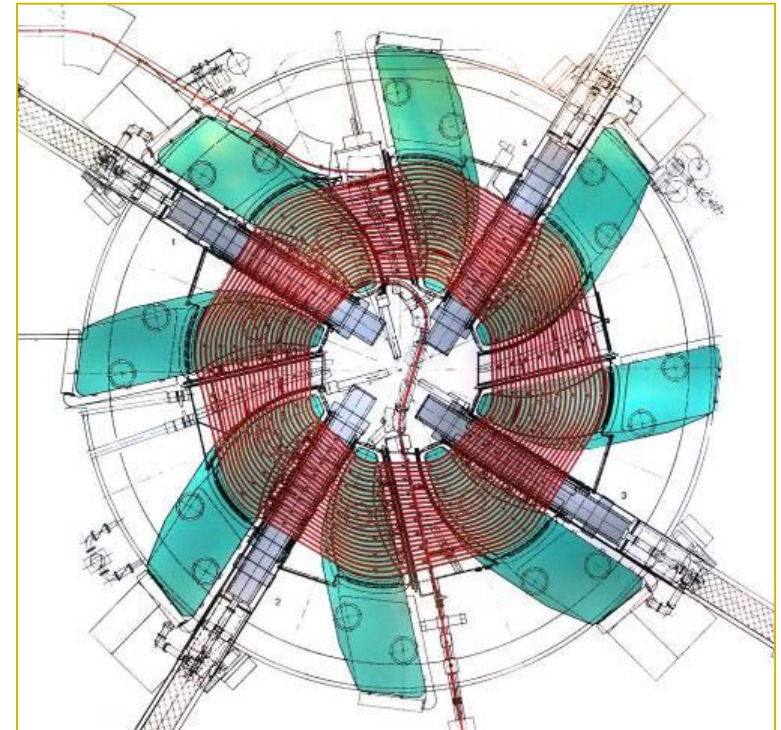
- **modular layout,** larger cyclotrons possible, sector magnets, box resonators, stronger focusing, injection/extraction in straight sections
- **external injection** required, i.e. pre-accelerator
- **box-resonators** (high voltage gain)
- high **extraction efficiency** possible:
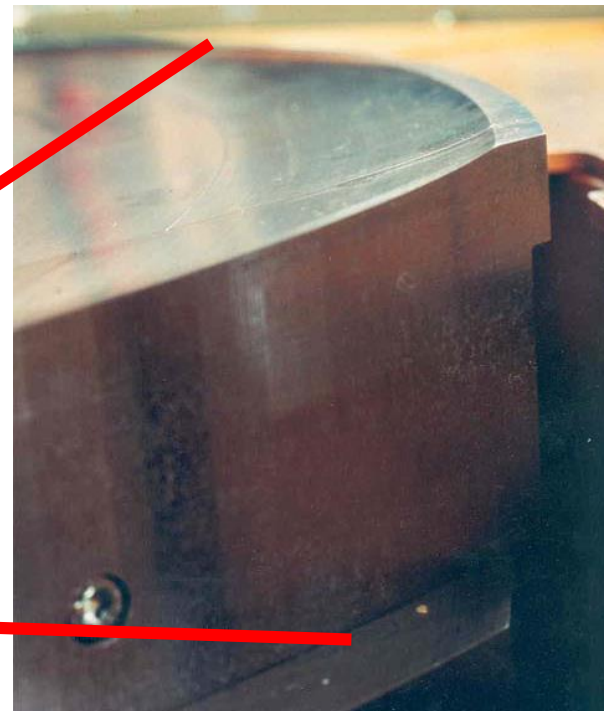  e.g. PSI: $99.98\% = (1 - 2{\times}10^{-4})$

# three methods to raise the average magnetic field with γ

**remember:**

$$
\begin{array}{rrcl}
\text{rev.time}: & R & \propto & \beta \\
\text{momentum}: & BR & \propto & \beta\gamma \\
\text{thus}: & B & \propto & \gamma
\end{array}
$$

1.) broader hills (poles) with radius

2.) **decrease pole gap with radius**

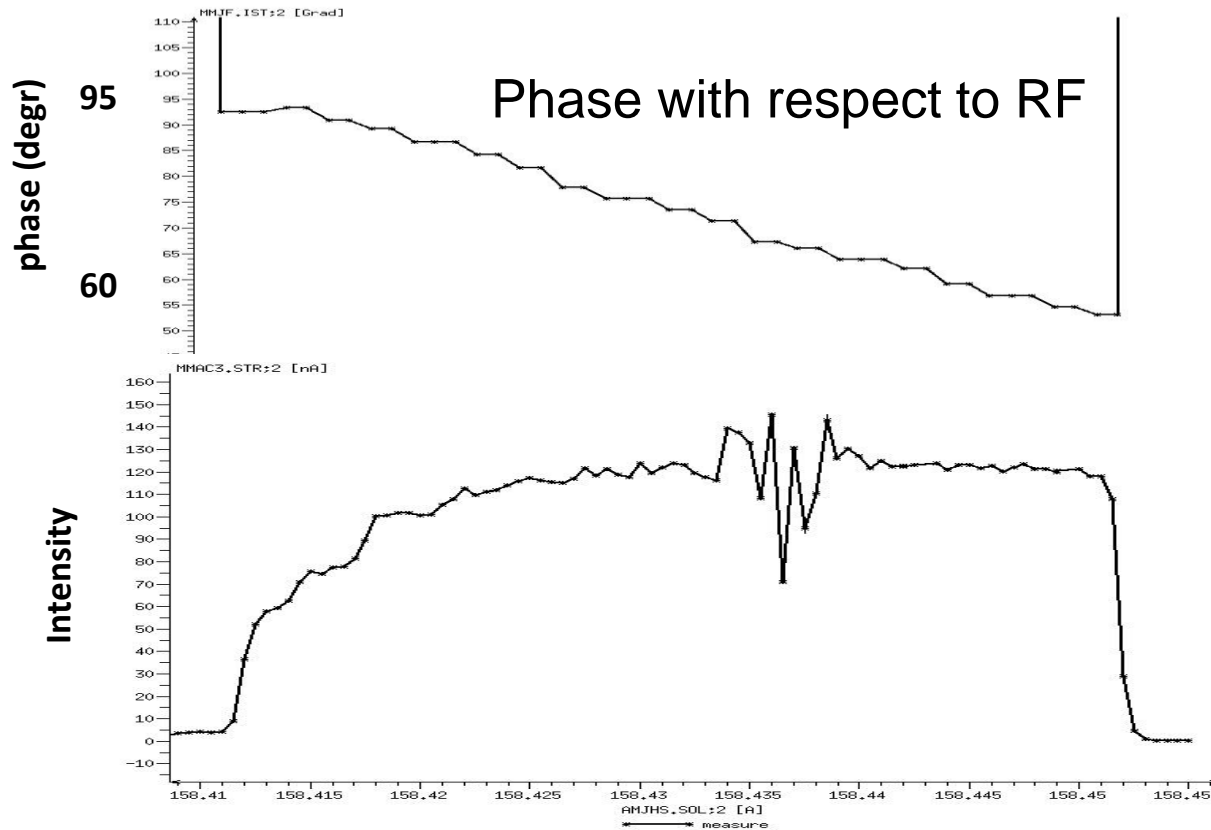3.) s.c. coil arrangement to enhance field at large radius (in addition to iron dominated field)



(photo: S. Zaremba, IBA)

# field stability is critical for isochronicity

example: medical Comet cyclotron (PSI)

Phase with respect to RF

$$\Delta\phi_{RF} \quad \propto \quad n_{\text{turn}} \frac{\Delta B}{B}$$

$$\text{e.g.} : \quad n_{\text{turn}} \quad = \quad 600$$

**phase (degr)**

**95**

**60**

**Intensity**

| 158.41 | 158.43 | 158.45 |

**Current in main coil (A)**

# derivation of (relativistic) turn separation in a cyclotron

starting point: bending strength
$\rightarrow$ compute total log.differential
$\rightarrow$ use field index $k = R/B \cdot dB/dR$

$$BR = \sqrt{\gamma^2 - 1}\, \frac{m_0 c}{e}$$

$$\frac{dB}{B} + \frac{dR}{R} = \frac{\gamma\, d\gamma}{\gamma^2 - 1}$$

$$\frac{dR}{d\gamma} = \frac{\gamma R}{\gamma^2 - 1}\frac{1}{1 + k}$$

radius change per turn

$$\frac{dR}{dn_t} = \frac{dR}{d\gamma}\frac{d\gamma}{dn_t}$$

$[U_t$ = energy gain per turn$]$

$$= \frac{U_t}{m_0 c^2} \frac{\gamma R}{(\gamma^2 - 1)(1 + k)}$$

$\left.\right\}$ isochronicity not conserved (last turns)

$$= \frac{U_t}{m_0 c^2} \frac{R}{(\gamma^2 - 1)\gamma}$$

$\left.\right\}$ isochronicity conserved (general scaling)

# turn separation - discussion

for clean extraction a large stepwidth (turn separation) is of utmost importance; in the PSI Ring most efforts were directed towards maximizing the turn separation

**general scaling at extraction:**

$$\Delta R(R_{\mathrm{extr}}) = \frac{U_t}{m_0 c^2} \frac{R_{\mathrm{extr}}}{(\gamma^2 - 1)\gamma}$$
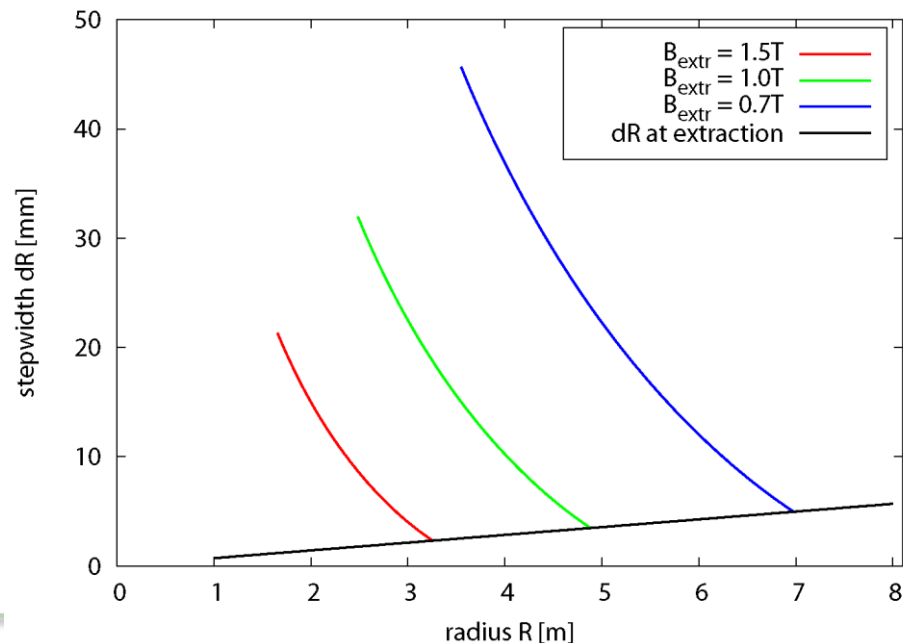
desirable:
- limited energy (< 1GeV)
- large radius $R_{\mathrm{extr}}$
- high energy gain $U_t$

**scaling during acceleration:**

$$\frac{dR}{dn_t} \approx \frac{U_t}{m_0 c^2} \frac{R}{\beta^2} \rightarrow \Delta R(R) \propto \frac{1}{R}$$
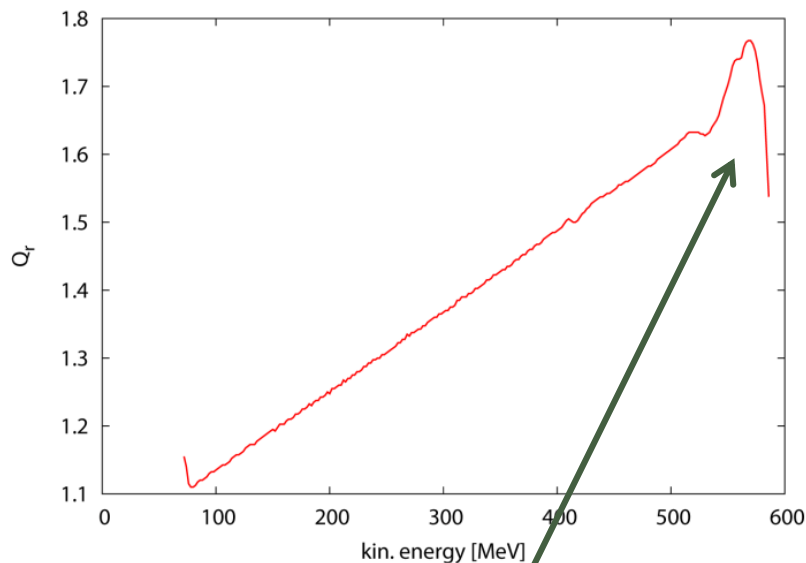
illustration:
**stepwidth vs. radius** in cyclotrons of different sizes but same energy;
100MeV inj $\rightarrow$ 800MeV extr



26

# extraction with off-center orbits

betatron oscillations around the "closed orbit" can be used to increase the radial stepwidth by a factor 3 !

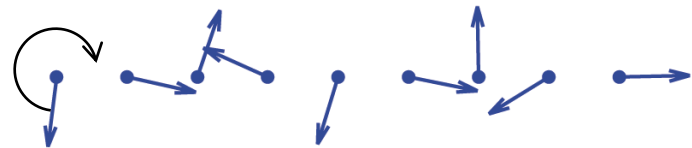**without orbit oscillations**: stepwidth from $E_k$-gain (PSI: 6mm)

**with orbit oscillations**: extraction gap; up to 3 x stepwidth possible for $\nu_r=1.5\pi$ (phase advance)
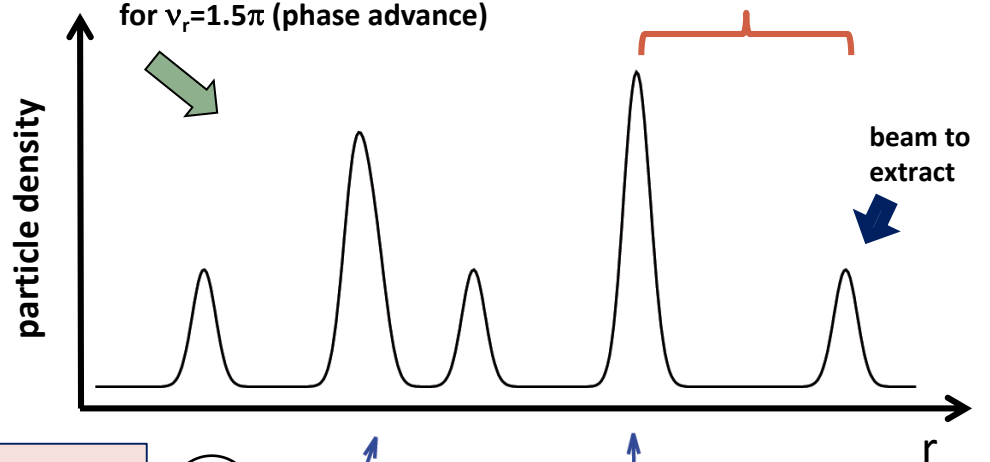
**beam to extract**

**particle density**

r

**radial tune vs. energy (PSI Ring)**
typically $\nu_r \approx \gamma$ during acceleration; but decrease in outer fringe field

phase vector of orbit oscillations (r,r')

$Q_r$

kin. energy [MeV]

# extraction profile measured at PSI Ring Cyclotron

**red: tracking simulation [OPAL]**
**black: measurement**
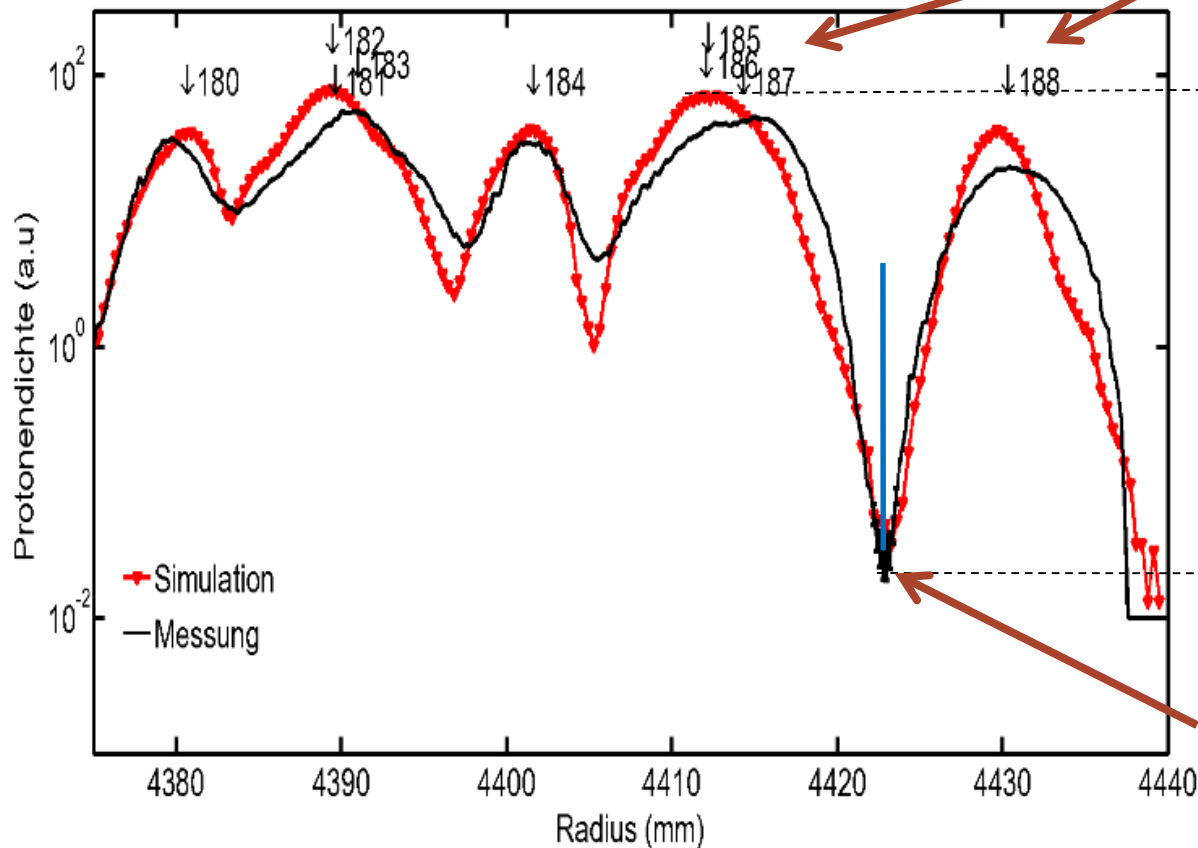
turn numbers

from simulation



**dynamic range:**
factor 2.000 in
particle density

position of extraction septum

d=50μm

[Y.Bi et al]

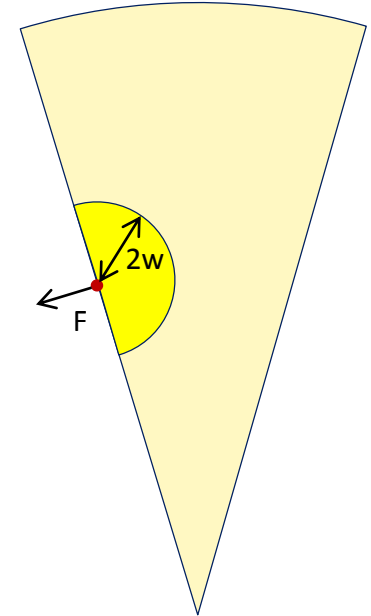# longitudinal space charge

**sector model (W.Joho, 1981)**:

→ accumulated energy spread transforms into transverse tails
- consider rotating uniform sectors of charge (overlapping turns)
- test particle "sees" only fraction of sector due to shielding of vacuum chamber with gap height 2$w$

two factors are proportional to the number of turns:
1) the charge density in the sector
2) the time span the force acts

$$\Delta U_{sc} \;=\; \frac{8}{3} e I_p Z_0 \ln\left(4\frac{w}{a}\right) \cdot \frac{n_{\max}^2}{\beta_{\max}} \approx 2.800\,\Omega \cdot e I_p \cdot \frac{n_{\max}^2}{\beta_{\max}}$$

**derivation see: High Intensity Aspects of Cyclotrons, ECPM-2012, PSI**

in addition:
3) the inverse of turn separation at extraction: $\dfrac{1}{\Delta R_{\mathrm{extr}}} \quad \propto \quad n_{\max}$

▶ thus the attainable current at constant losses scales as $n_{\max}^{-3}$

# longitudinal space charge; evidence for third power law

- at PSI the maximum attainable current indeed scales with the third power of the turn number

- maximum energy gain per turn is of utmost importance in this type of high intensity cyclotron

→ **with constant losses at the extraction electrode the maximum attainable current indeed scales as:**
$$I_{\max} \propto n_t^{-3}$$
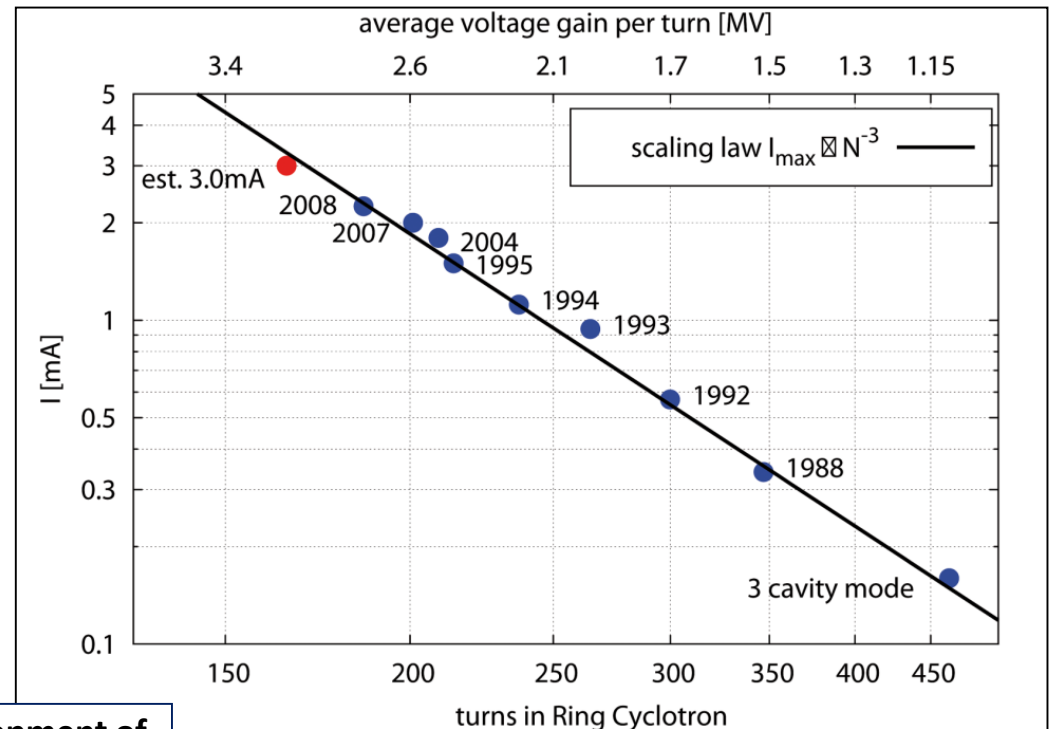


historical development of current and turn numbers in PSI Ring Cyclotron

next: **cyclotron examples**

- **compact cyclotrons**
- **TRIUMF, RIKEN SRC, PSI-Comet, PSI-HIPA**

# compact cyclotrons for Isotope production





CYCLONE 30 (IBA) : H- 15 à 30 MeV

## Vertical setup

# some cyclotrons

| | TRIUMF | RIKEN SRC (supercond.) | PSI Ring | PSI medical (supercond.) |
|---|---|---|---|---|
| particles | H- → p | ions | p | p |
| K [MeV] | 520 | 2600 | 592 | 250 |
| magnets (poles) | (6) | 6 | 8 | (4) |
| peak field strength [T] | 0.6 | 3.8 | 2.1 | 3.8 |
| $R_{inj}/R_{extr}$ [m] | 0.25/3.8…7.9 | 3.6/5.4 | 2.4/4.5 | -/0.8 |
| $P_{max}$ [kW] | 110 | 1 (86Kr) | 1300 | 0.25 |
| extraction efficiency (tot. transmission) | 0.9995 (0.70) | (0.63) | 0.9998 | 0.80 |
| extraction method | stripping foil | electrostatic deflector | electrostatic deflector | electrostatic deflector |
| comment | variable energy | ions, flexible | high intensity | compact |

# cyclotron examples: TRIUMF / Vancouver

photo: iron poles with spiral shape ($\delta_{max}$=70deg)

- p, 520MeV, up to 110kW beam power
- diameter: 18m (largest n.c. cyclotron worldwide)
- extraction by stripping H$^-$ → variable energy; multiple extraction points possible
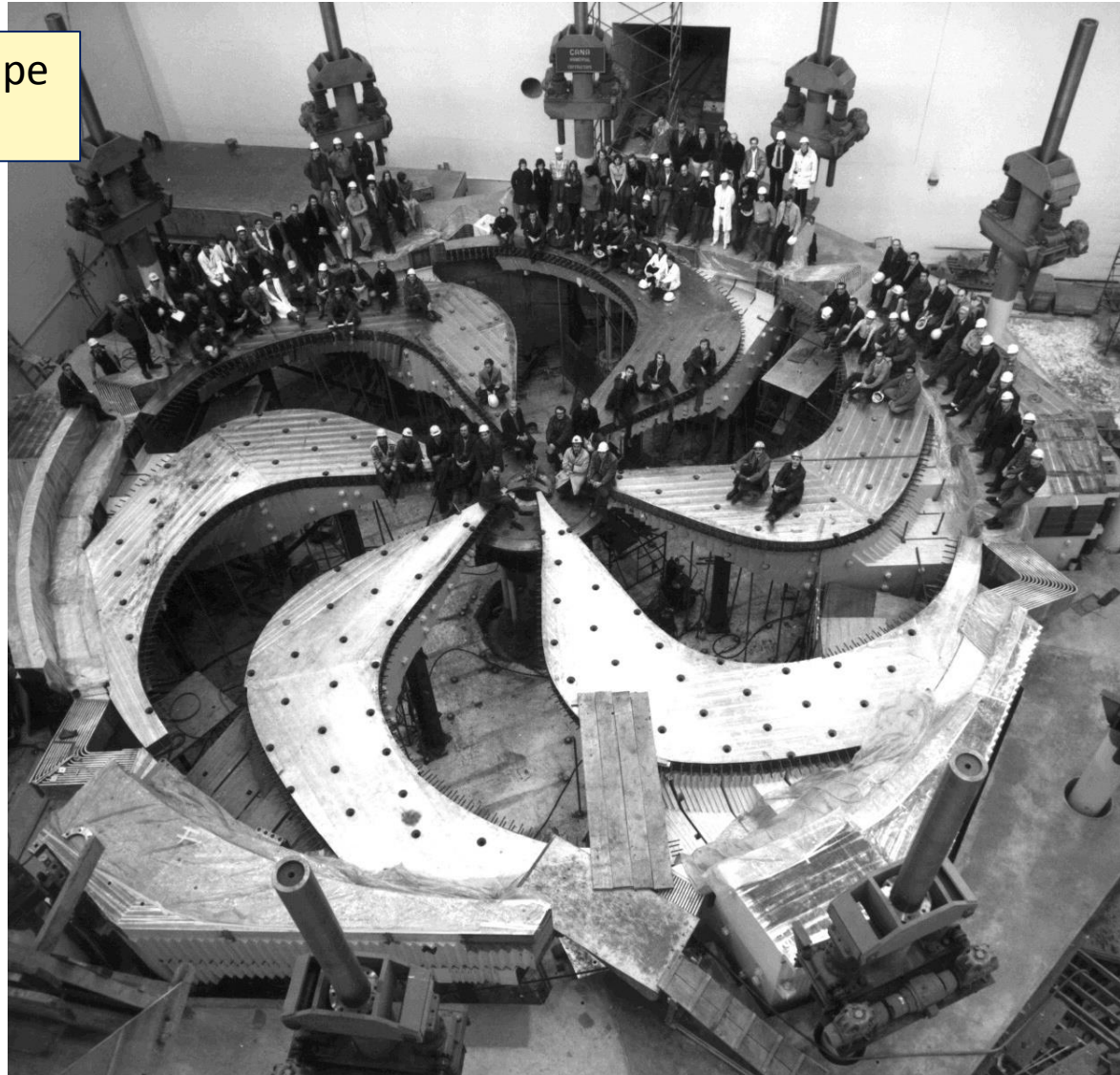
# example: RIKEN (Jp) superconducting cyclotron

**K = 2,600 MeV**
Max. Field: 3.8T (235 MJ)
RF frequency: 18-38 MHz
Weight: 8,300 tons
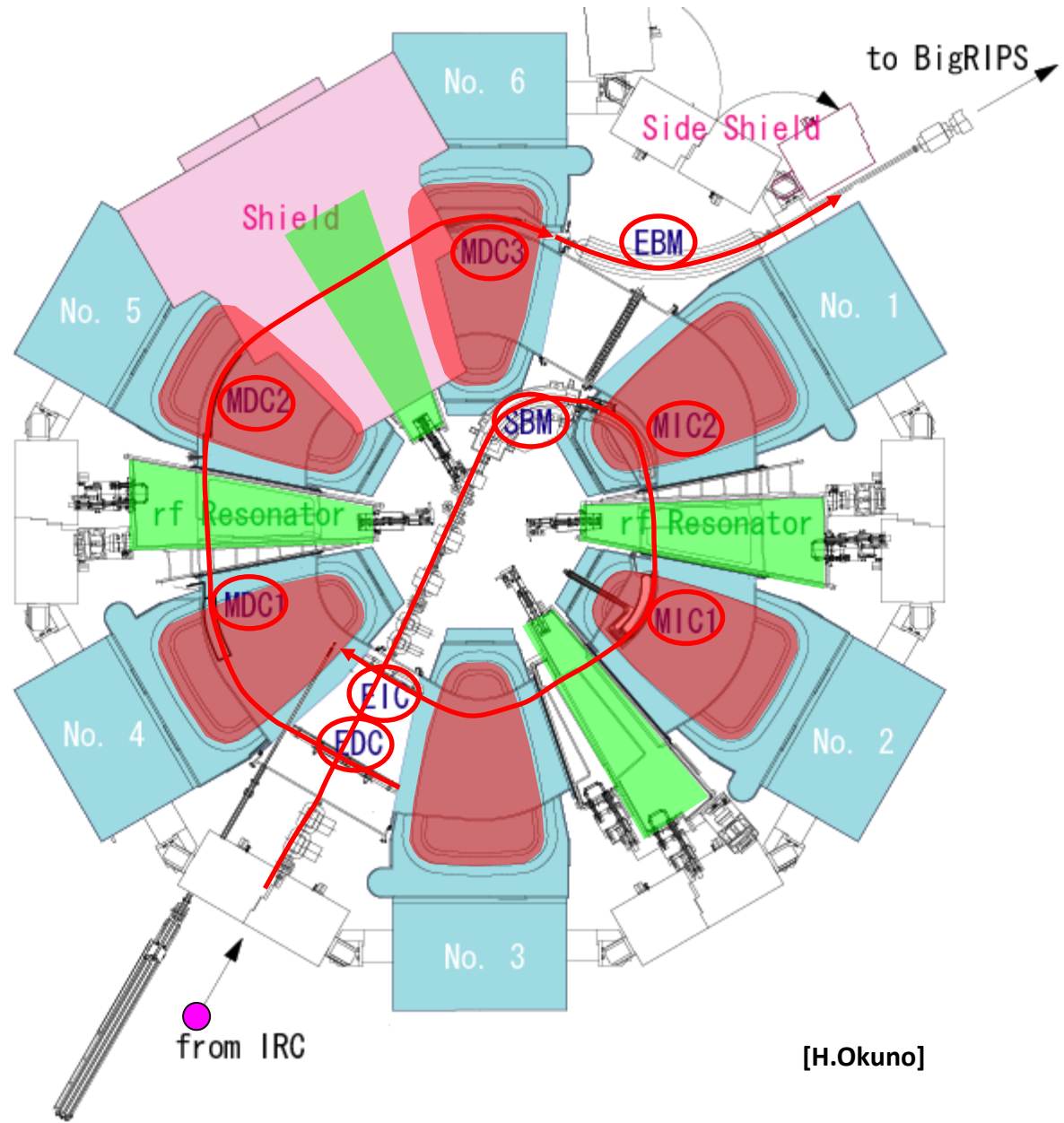Diameter: 19m
Height: 8m

superconducting
Sector Magnets :6
RF Resonator :4
Injection elements.
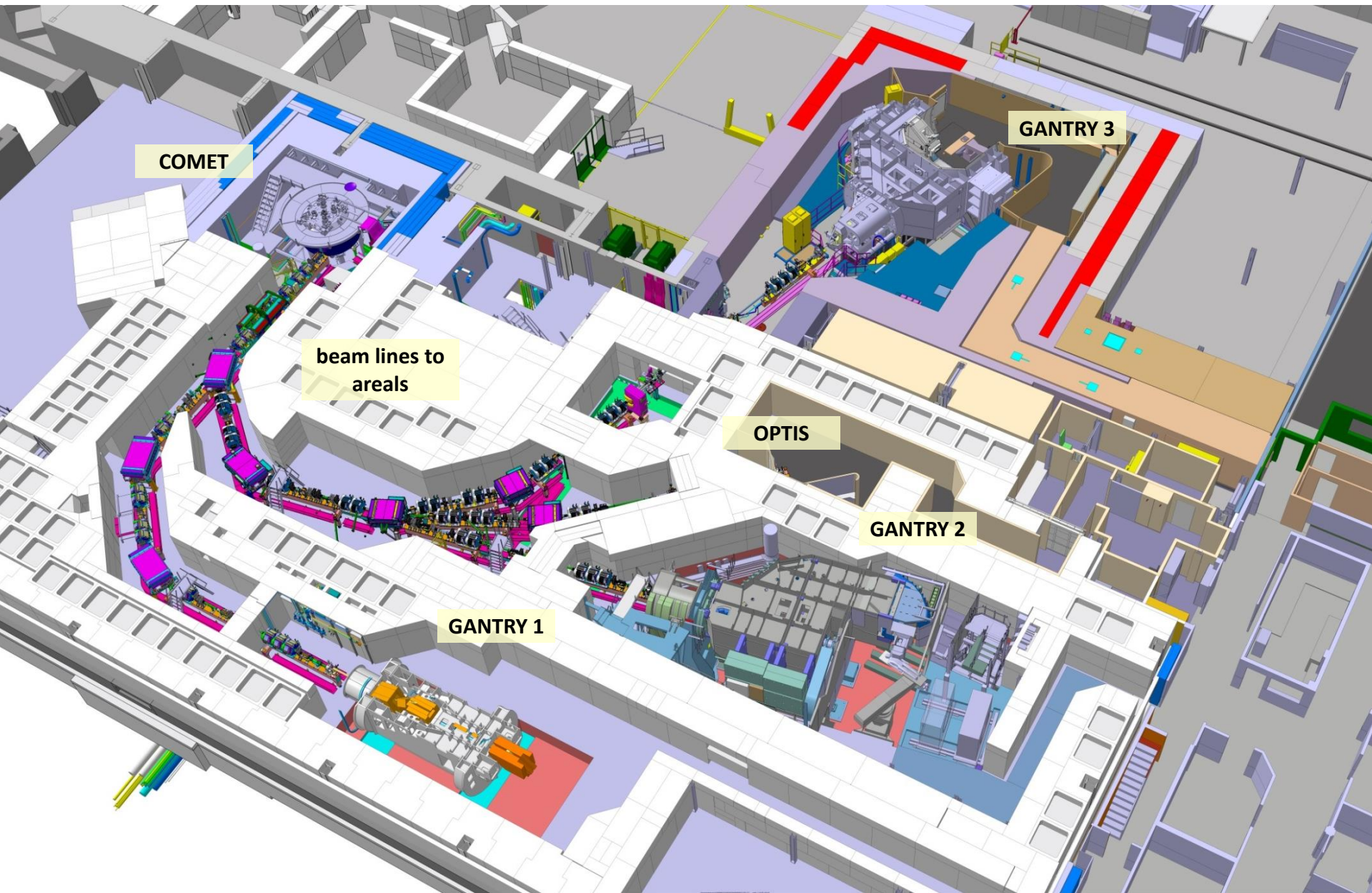Extraction elements.
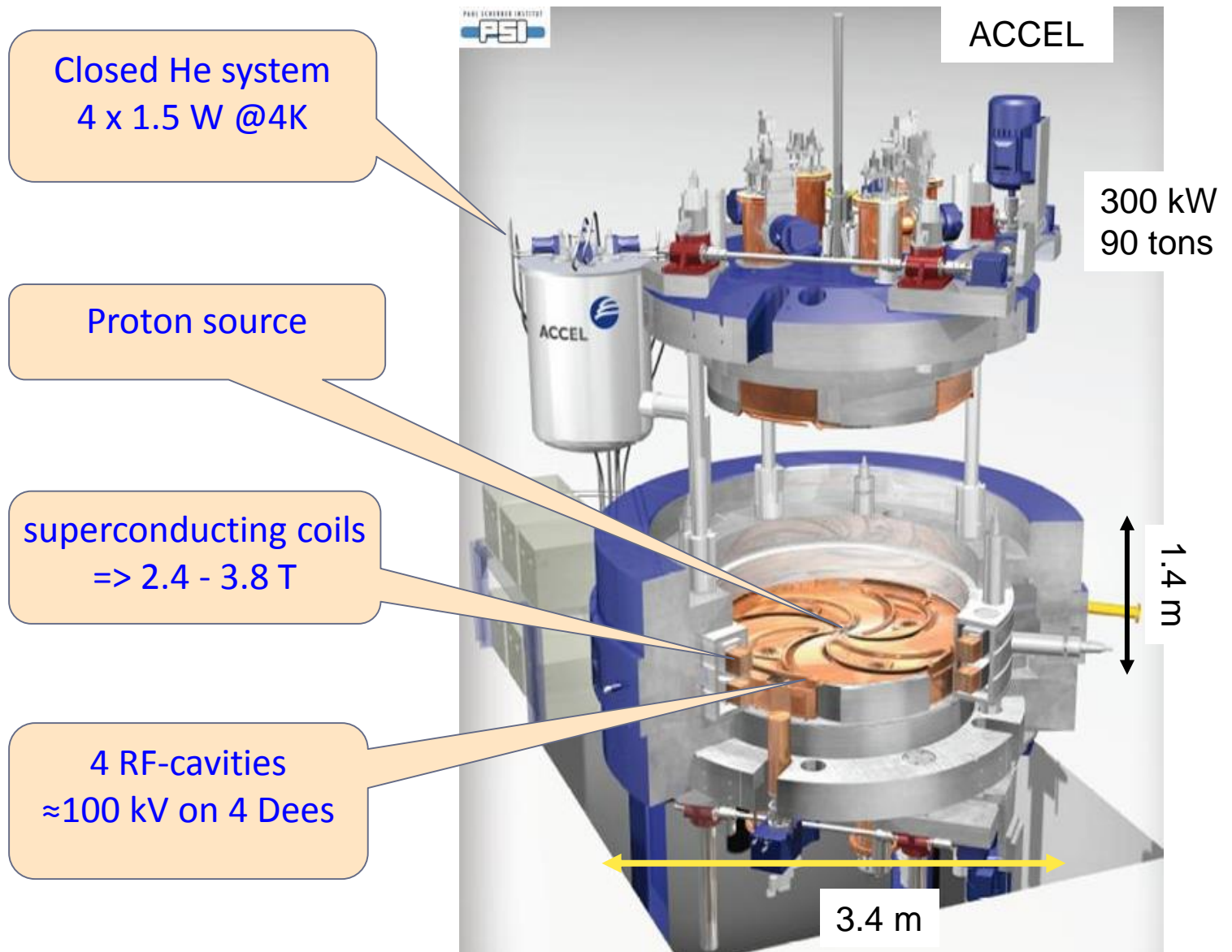
*utilization:*
*broad spectrum of*
*ions up to Uranium*



[H.Okuno]

# RIKEN SRC in the vault

# PSI Proton Therapy Facility

# 250 MeV proton cyclotron (ACCEL/Varian)

Closed He system
4 x 1.5 W @4K

Proton source

superconducting coils
=> 2.4 - 3.8 T

4 RF-cavities
≈100 kV on 4 Dees
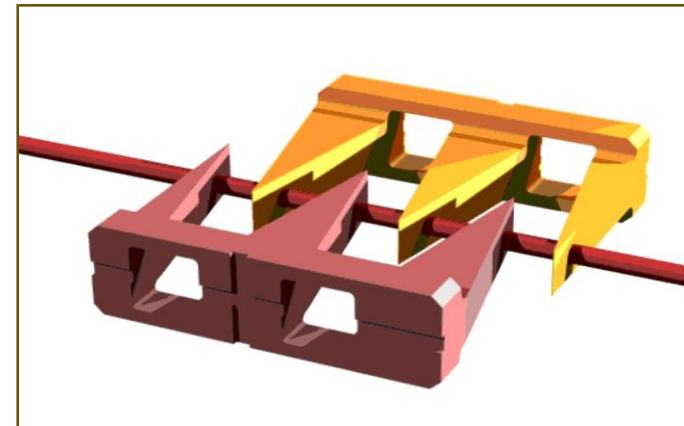
ACCEL

300 kW
90 tons

1.4 m

3.4 m

# Cyclotron needs degrader :

- cyclotron has fixed energy; need **degrader** for energies down to 70MeV

- collimation after degrader to keep emittance → lose intensity with degrader



measured transmission with "OPTIS" collimators

Behind E-slits
$\Delta p/p = +/-1\%$
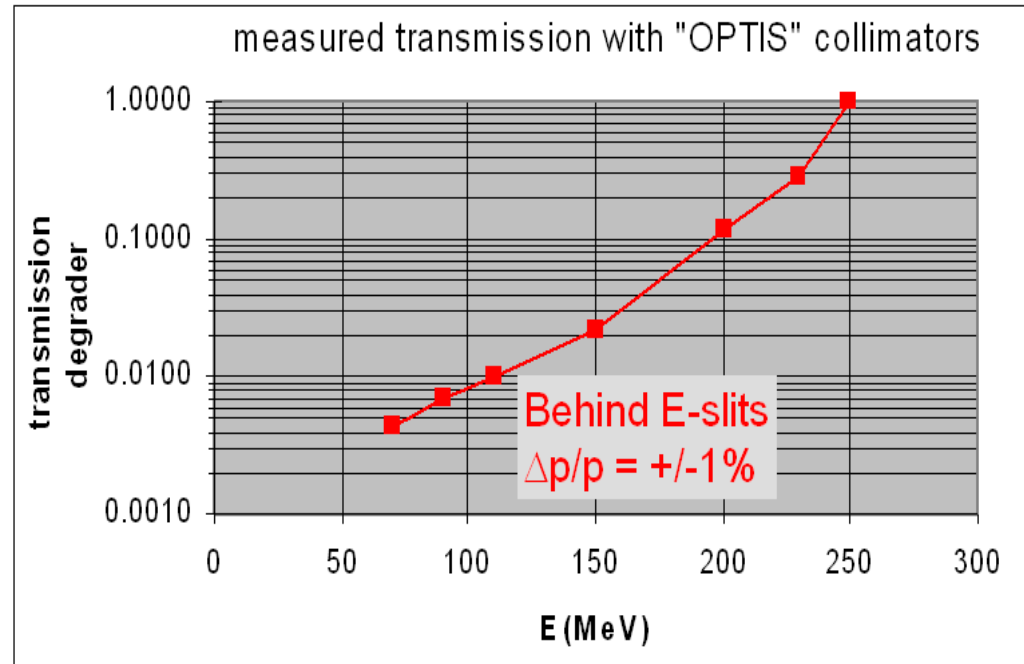
degrader: (carbon wedges in vacuum) and laminated beam line magnets for fast energy changes < 80 ms / step

# examples: PSI High Intensity Proton Accelerator



Ring Cyclotron 590 MeV
2.4mA / 1.4MW
diameter: 15m

SINQ
spallation source

meson production
targets

proton therapie center
[250MeV sc. cyclotron]

dimensions:

120 x 220m$^2$

# Outlook: Cyclotrons II & FFA

- cyclotron subsystems

  **extraction schemes, RF systems/resonators, magnets, vacuum issues, instrumentation**

- FFA = Fixed Focus Alternating Gradient Accelerators

  **motivation & applications, scaling FFA's, non-scaling and linear FFA, FFA subsystems**

- discussion

  **classification of circular accelerators, cyclotron vs. FFAG, Pro's and Con's of cyclotrons for different applications**