Tracking Code with 3D Space Charge Calculations Taking into account the elliptical Shape of the Beam Pipe*

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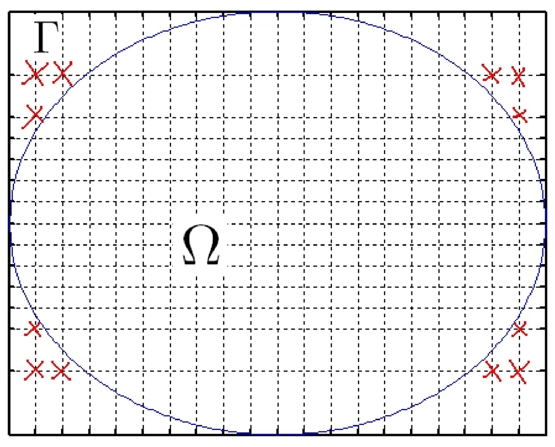
Introduction

In the particle-mesh method the calculation of the space charge fields from spatially distributed charges requires the solution of the Poisson equation. The solution can be strongly influenced by the applied boundary conditions (b.c.). Conducting b.c. are often applied on the rectangular boundaries of the calculation domain. However, the rectangular cross section is not the best approximation to the real geometry of the beam pipe. We present an algorithm applying conducting b.c. on an arbitrary elliptic cross section of the beam pipe. Further we present results from our tracking routine, which employs the new space charge solver.

CONDUCTING B.C. ON BEAM PIPES WITH ELLIPTICAL CROSS SECTION

Considering beam pipes with elliptical cross section, we solve the Poisson equation on the cylindrical domain , $\Gamma = [-a,a] \times [-b,b] \times [-c,c]$

$$-\Delta\varphi = \frac{\varrho}{\varepsilon_0} \quad \text{in } \Omega \subset \mathbb{R}^3,$$
$$\varphi = 0 \quad \text{on } \partial\Omega_1,$$
$$\frac{\partial\varphi}{\partial n} + \frac{1}{r}\varphi = 0 \quad \text{on } \partial\Omega_2,$$



$\partial\Omega_1$ is the surface of the cylinder

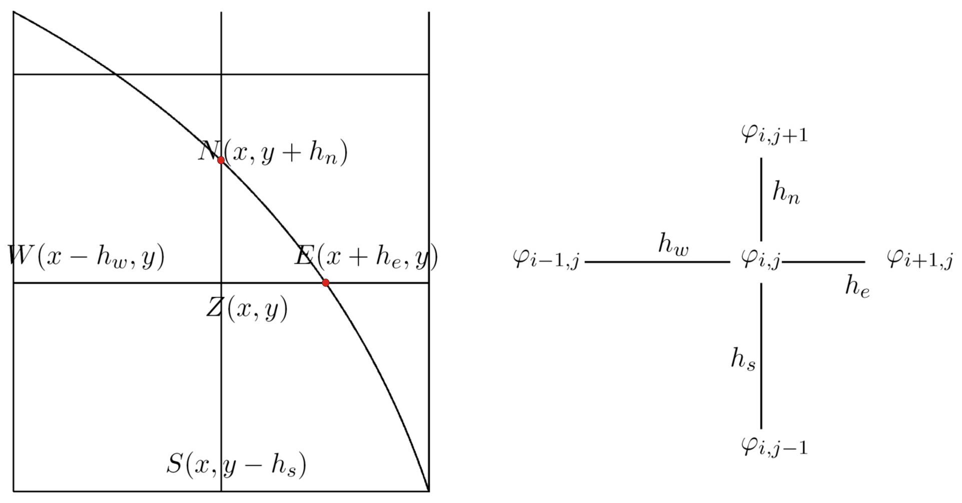
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and } -c < z < c,$$

$\partial\Omega_2$ are the two elliptical bases of the cylinder satisfying

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \text{ and } z = \pm c$$

discretized in N_x N_y and N_z steps

Boundary adapted 7-point-star of grid points inside

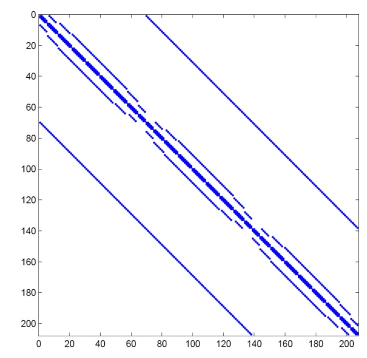


Linear system

$$\boldsymbol{Au} = \boldsymbol{b}$$

System matrix $\boldsymbol{A}$ is:
- block structured
- nonsymmetric
- positve definite [2]

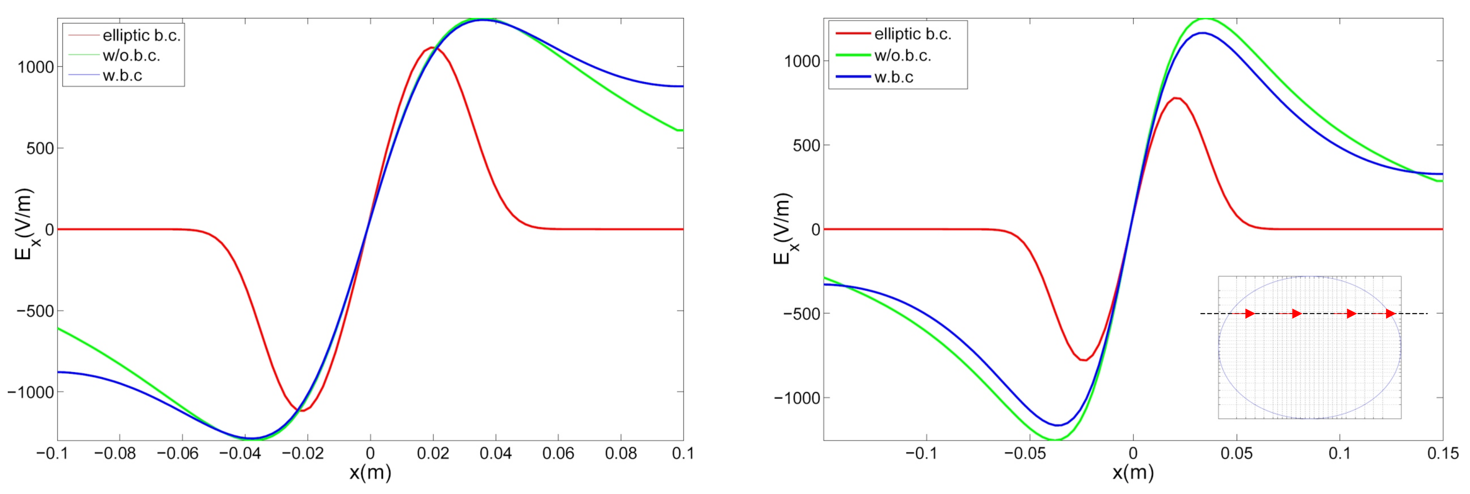
Use of BiCGSTAB to solve the system



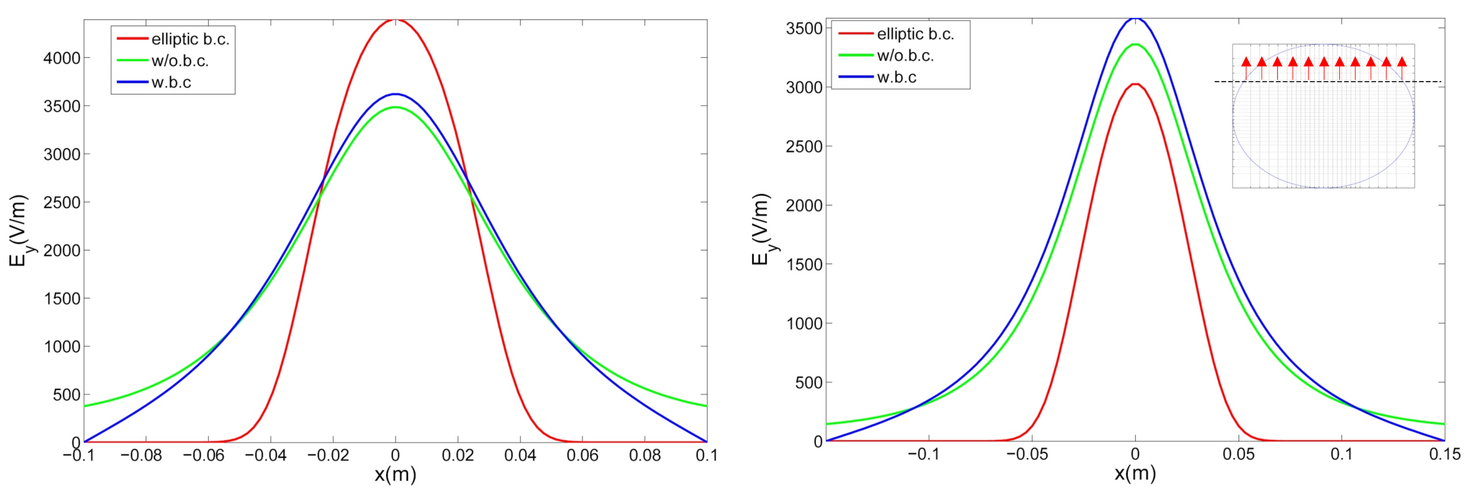
RESULTS

Space charge simulation

Example to compare the calculated *E* fields with open and conducting b.c. on a rectangular and elliptic beam pipe
- spherical bunch (*r*<<*a*,*b*)
- uniformly distributed charge of -1nC



Electric field E_x along $y = b/2$ of a square $a = b$ (left) and a rectangular box $a = 1.5b$ (right), computed with open (w/o.b.c.) and conducting b.c. (w.b.c.) on a rectangular and elliptic (elliptic b.c.) pipe.



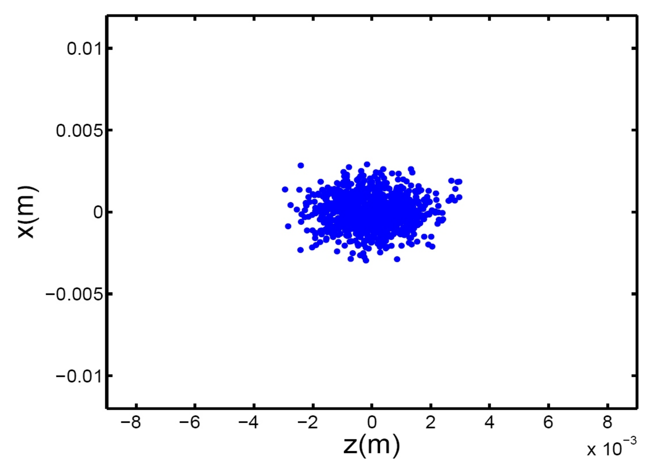
Electric field E_y along $y = b/2$ of a square $a = b$ (left) and a rectangular box $a = 1.5b$ (right), computed with open (w/o.b.c.) and conducting b.c. (w.b.c.) on a rectangular and elliptic (elliptic b.c.) pipe.

First tracking results

Comparison of the bunch after a drift of 3m simulated with open b.c. on a rectangular domain and conuducting circular pipe (r=1cm)

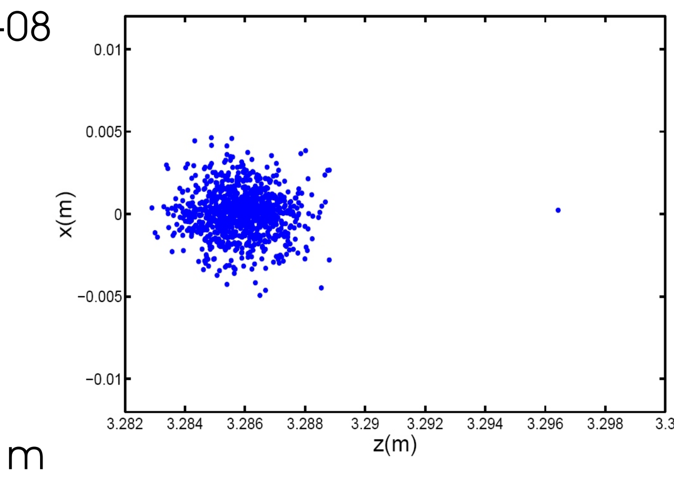
Initial bunch parameter
(from Astra's [1] generator program)
- Gaussian bunch, 1000 macro particles
- total charge of -1nC
- average kinetic energy 5 MeV
- horizontal beam size $_x$= 1 mm
- vertical beam size $_y$= 1mm
- longitudinal beam size $_z$= 1 mm
- beam position:

x=-2.807E-08 m, y=1.364E-07 m, z=3.727E-08



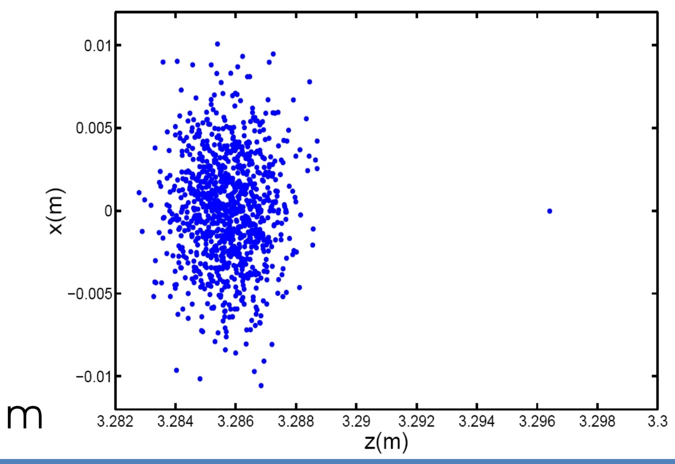
Drift with open boundary conditions
- horizontal beam size $_x$= 1.4 mm
- vertical beam size $_y$= 1.4 mm
- longitudinal beam size $_z$= 1.1 mm
- beam position:

x=-4.250E-05 m, y=5.000E-05 m, z=3.2859 m



Drift in a beam pipe
- horizontal beam size $_x$= 3.3 mm
- vertical beam size $_y$= 3.3 mm
- longitudinal beam size $_z$= 1.1 mm
- beam position:

x=-2.223E-06 m, y=1.266E-06 m, z=3.2857 m



Conclusions

Field differences between simulations with different b.c. and different geometries get significant as we move towards the boundary of the pipe. Consequently a bunch tracked with different b.c. has a different transversal expansion, as shown with our first tracking results.

References

[1] K. Flöttmann, "Astra", DESY, Hamburg, www.desy.de/~mpyflo, 2000.
[2] A. Markovik, "A numerical computation of space-charge fields of electron bunches in a beam pipe of elliptical shape" TESLA-Report 2005-21, DESY Hamburg

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