

H. Yıldız, Ankara University, Ankara, TURKEY  
A. Kenan Ciftci\*, Ankara University, Ankara, TURKEY  
K. Zengin\*\*, Ankara University, Ankara, TURKEY

## Abstract

The Turkish Accelerator Center (TAC) is a project for accelerator based fundamental and applied researches supported by Turkish State Planning Organization (TSPO). In this presentation, the dynamic aperture calculations for the TAC Synchrotron Storage Ring is made.

## MACHINE APERTURES AND MAGNETS

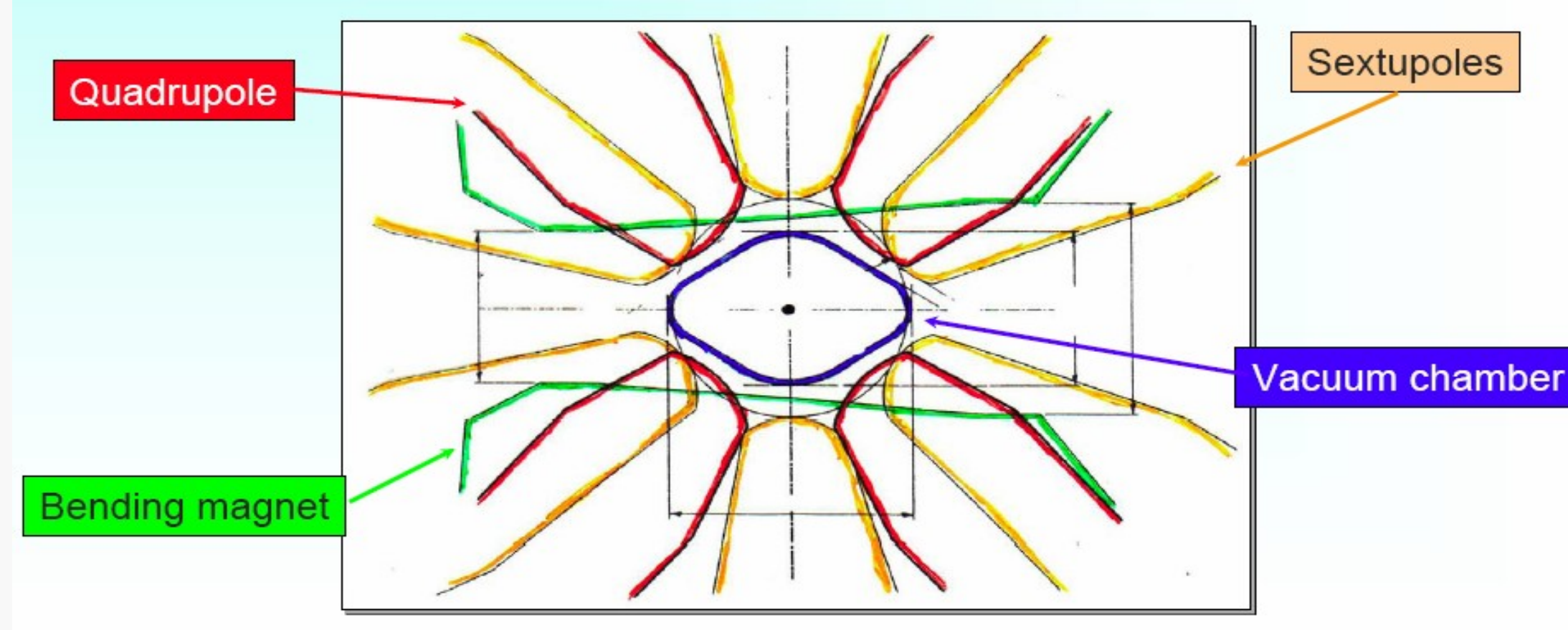


Figure 1. The Schematic view of vacuum chamber and magnets

Sextupole magnets are very important at the design of a machine. Figure 2 displays speciality of sextupole magnets

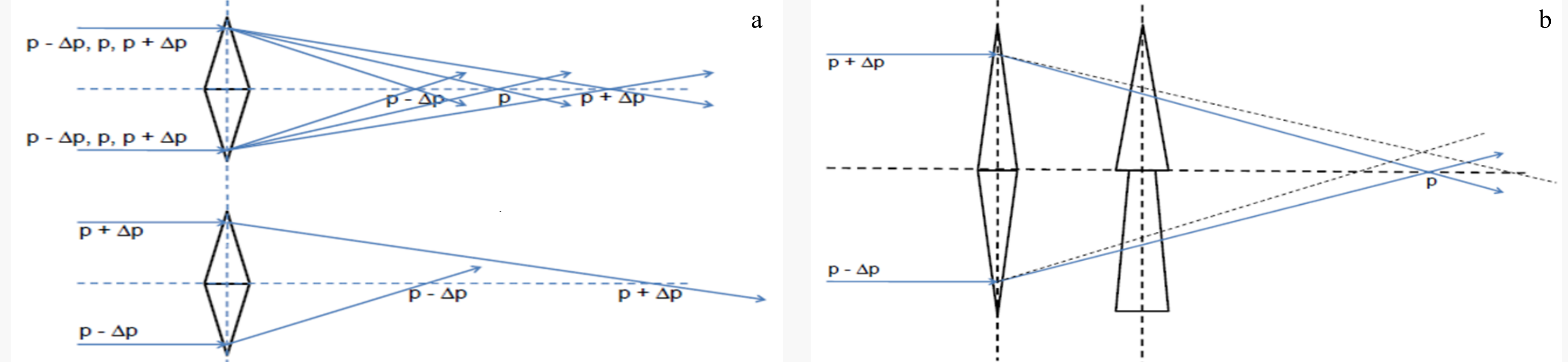


Figure 2. a-) Resultant case, when sextupole magnet isn't used b-) when it is used

## PHYSICAL APERTURE

In a storage ring, a vacuum chamber is composed of various vacuum components, such as vacuum tube, valves, bellows, flanges, kickers, electrostatic separators, RF cavities, diagnostic devices, etc. Charged particles move along a closed orbit in such a chamber. The transverse apertures of these vacuum components, called physical aperture, limit the beam motion along its path. It is very clear that the larger the physical aperture, the less particles loss. But on the other hand, a circular machine with a large physical aperture means a large amount of budget, as well as a big scale of the machine itself. As a result, large gaps and high fields for magnets and other elements are also required. So a suitable physical aperture is crucial to a circular machine. Physical aperture limits swing of a matter, as shown at Figure 3.



Figure 3. Example to physical aperture

## DYNAMIC APERTURE

### Definition of Dynamic Aperture

Dynamic aperture is a kind of amplitude threshold. When the amplitude of the motion of a charged particle is smaller than this threshold, the particle will not get lost as a consequence of single particle dynamics effect. When the amplitude exceeds this threshold, the betatron oscillation of the particle will not have any bounds, and the motion will become unstable. Then, the particle can not circulate in the accelerator. Unlike the physical aperture, the dynamic aperture separating stable and unstable trajectories is not a hard boundary. Otherwise the ideal machine design always requires a dynamic aperture larger than its physical aperture.

### Dynamic Aperture in Phase Space

Working point is very important for dynamic aperture in phase space. For example: CERN-PS with nonlinearities (sextupoles & octupoles) with three different working points: (a) 0.252 (b) 0.249 (c) 0.245 With increasing initial amplitudes, the motion progressively becomes: linear → nonlinear (with or without islands) → chaotic → unstable

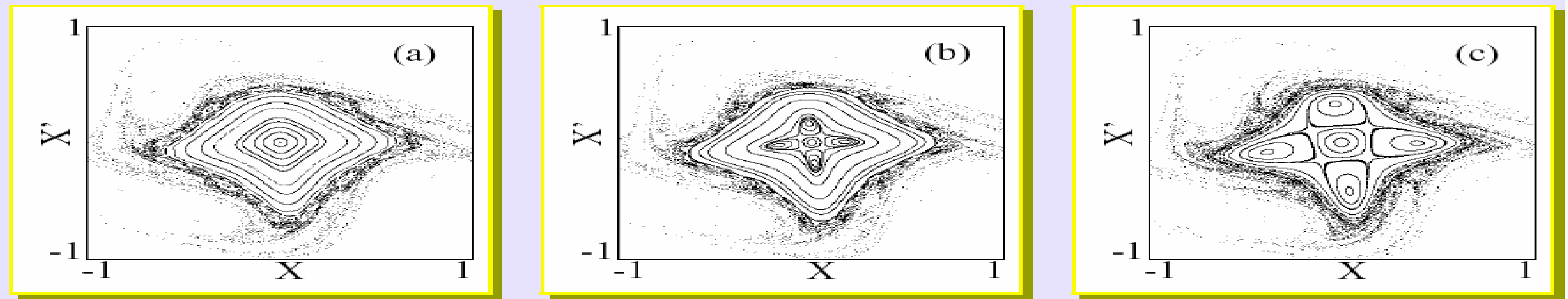


Figure 4. An example of dynamic aperture in Phase space

## DETERMINATION OF DYNAMIC APERTURE

Hamiltonian formalism is often adopted to describe the stability of particle motion and to determine the limit of the stability. The general Hamiltonian for a particle of rest mass  $m_0$ , and charge  $e$ , in a magnetic vector potential  $A$ , and vector potential  $\Phi$ , is expressed as

$$H(q,p,t) = e\Phi + c[(p - eA)^2 + m_0c^2]^{1/2} \quad (3)$$

### Numerical Method for Dynamic Aperture

The numerical method used to simulate the particles motion in accelerator with such a modeling is called tracking. One can get the single particle motion directly from tracking and determine the stable regions of phase space. And numerical simulation based on the particle tracking aims at determining the dynamic aperture with powerful computer.

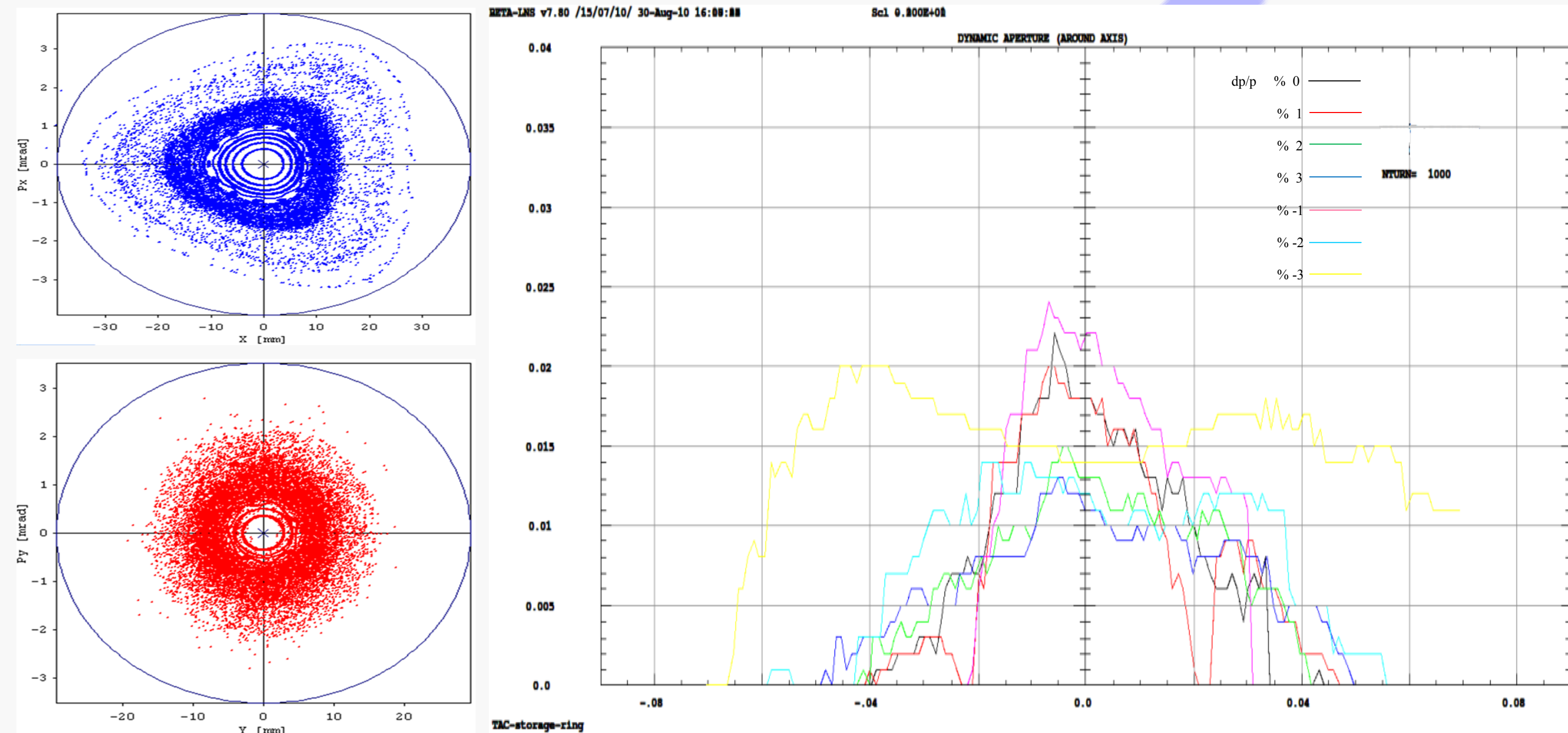


Figure 5. Phase space tracking

Figure 6. Dynamic aperture

Figure 7 shows the tune diagram of the four order of TAC storage ring, where the cross indicates the working point of the machine. TuneX  $\approx 31.64$  and tuneY  $\approx 13.26$ , as shown at figure 8. Otherwise figure 8 shows optical functions of TAC storage ring.  $\beta_x(0) = 9.988$  and  $\beta_y(0) = 8.340$ , Emittance = 3.654 nmrd

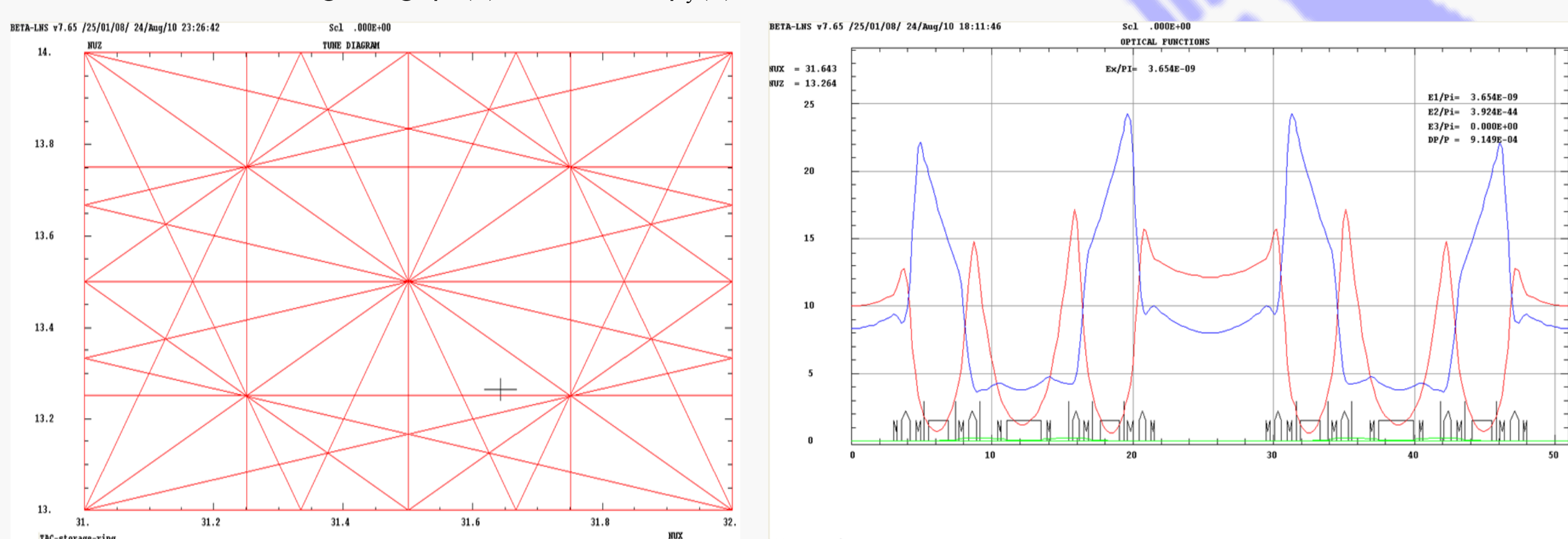


Figure 7. Tune diagram of TAC

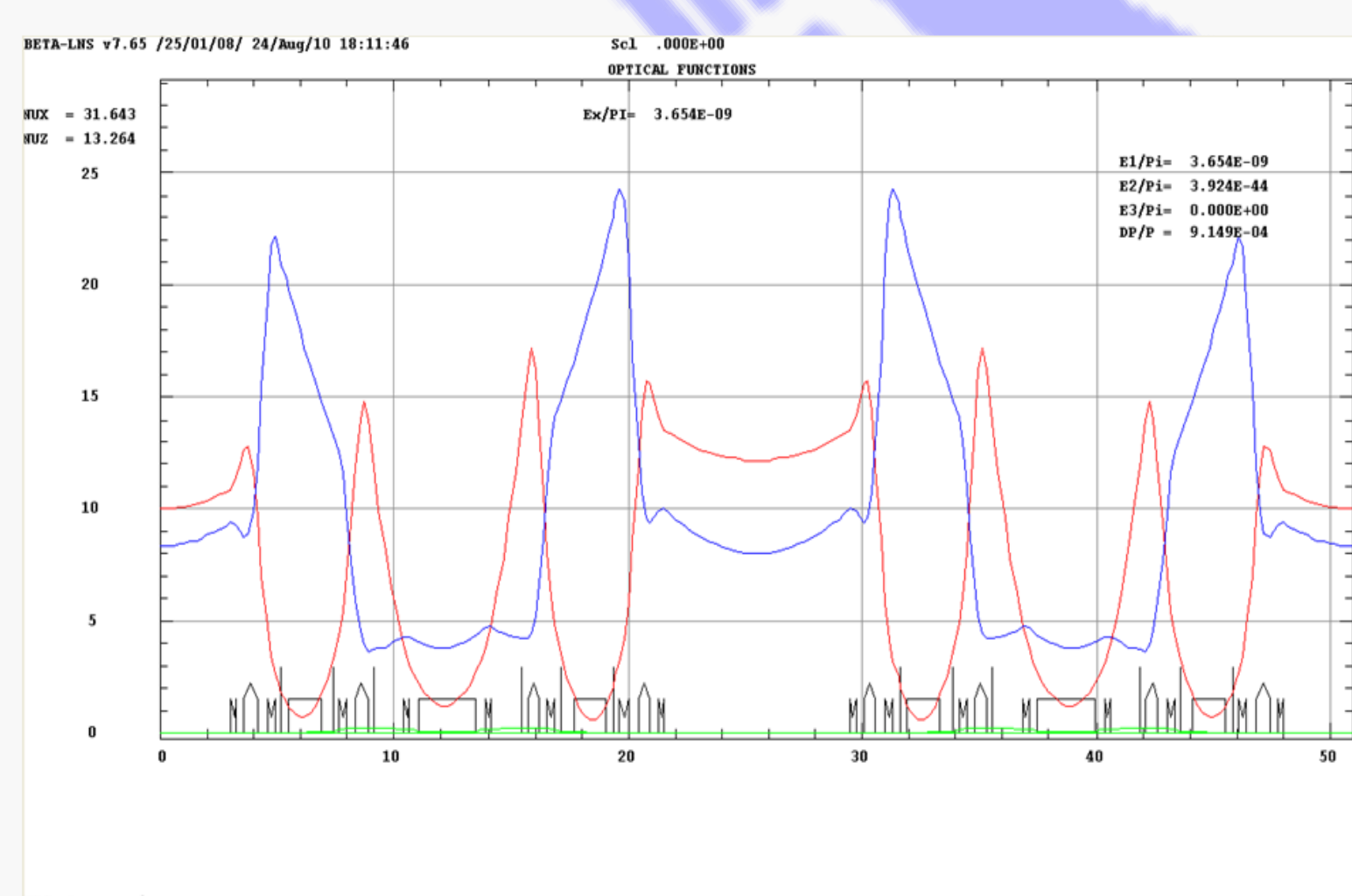


Figure 8. Optical Functions of TAC

### Analytical Method for Dynamic Aperture

The general expression of dynamic aperture in the horizontal plane ( $z=0$ ) of multipole component:

$$A_{dyna,2m} = \sqrt{2\beta_x(s)} \left( \frac{1}{m\beta_x^m(s(2m))} \right)^{1/2(m-2)} \left( \frac{\rho}{|b_{m-1}|L} \right)^{1/(m-2)} \quad (4)$$

$m=3$  for sextupole;

$$A_{dyna,sext} = \sqrt{2J_{max,sext}\beta_x(s)} = \frac{\sqrt{2\beta_x(s)}}{\sqrt{3}\beta_x(s_1)^{3/2}} \left( \frac{\rho}{|b_2|L} \right) \quad (5)$$

At the same time the total dynamic aperture expressed as follows:

$$A_{dyna,total} = \frac{1}{\sqrt{\sum_i 1/A_{dyna,sext,i}^2 + \sum_j 1/A_{dyna,oct,j}^2 + \sum_k 1/A_{dyna,dec,k}^2 + \dots}} \quad (6)$$

Finally, six sextupole of  $S=1$  are located at  $s=s_{1,2,3,4,5,6}$  with  $\beta_x(s_1) = 3.286$  m,  $\beta_x(s_2) = 1.767$  m,  $\beta_x(s_3) = 11.854$  m,  $\beta_x(s_4) = 13.855$  m,  $\beta_x(s_5) = 2.021$  m,  $\beta_x(s_6) = 3.558$  m, respectively, and  $\beta_x(0) = 9.988$  m, in the same time we get  $A_{dyna,sext,1} = 0.43$  m,  $A_{dyna,sext,2} = 1.093$  m,  $A_{dyna,sext,3} = 0.063$  m,  $A_{dyna,sext,4} = 0.05$  m,  $A_{dyna,sext,5} = 0.893$  m,  $A_{dyna,sext,6} = 0.385$  m, so the total dynamic aperture value of TAC expressed as follows:

$$A_{dyna,total} \approx \frac{1}{\sqrt{666}} = \frac{1}{25} = 0,0387 \text{ m} \approx 38,7 \text{ mm} \quad (7)$$

## CONCLUSION

In this study, dynamic aperture for storage ring of the TAC is calculated. We found  $A_{dyna,total} = 0.0387$  m with analytical methods and it compared with the numerical value of 0.037 m. They are good values, but dynamic aperture can be improved by using more sextupoles (harmonic sextupoles) or octupoles to cancel the effect of the chromaticity sextupoles. Otherwise dynamic aperture should be researched and studied for top-up injection and Touschek life time.

## ACKNOWLEDGEMENT

The authors want to thank to Turkish State Planning Organization (DPT) for support. Grant No: DPT2006K-120470

## REFERENCES

- [1] Bengtsson J. "Dynamic Aperture". Proc. of ASAC(Accelerator Systems Advisory Committee), p. 35, USA, 2006.
- [2] Bocchetta C. J. "Beam Lifetime". Proc of CAS, p. 64, Baden, Austria. 2004.
- [3] Chao A.W., Moser H.O., Zhao Z. "Accelerator Physics, Technology and Applications", world scientific, p. 639, Singapore.2004.
- [4] Chao A.W., Tigner M. "Handbook of Accelerator Physics and Engineering", world scientific, p. 650, Singapore. 1998.
- [5] Gao J. "Analytical Estimation of The Dynamic Apertures of Circular Accelerators", Particle Accelerator Conference proceedings, p.1799, Chicago,2001.
- [6] Greiner W. "Classical Mechanics", Springer, p. 579, Germany, 2009.
- [7] Rumolo G. "Machine Apertures", Proc of CAS, p. 49, Zakopane-Poland, 2006.
- [8] Wiedemann H. "Particle Accelerator Physics II", Springer, p. 464, Berlin,1995.
- [9] Wilson M. N. "Magnets", Proc of CAS, p. 58, Frascati-Italy, 2008.
- [10] <http://thm.ankara.edu.tr>, <http://web.cern.ch>, <http://cas.web.cern.ch/cas>.

\* hyildiz@science.ankara.edu.tr



