TRANSITION RADIATION BY ELECTRON BEAMS AND BEAM TRANSVERSE SIZE EFFECTS: RESULTS AND BEAM DIAGNOSTICS

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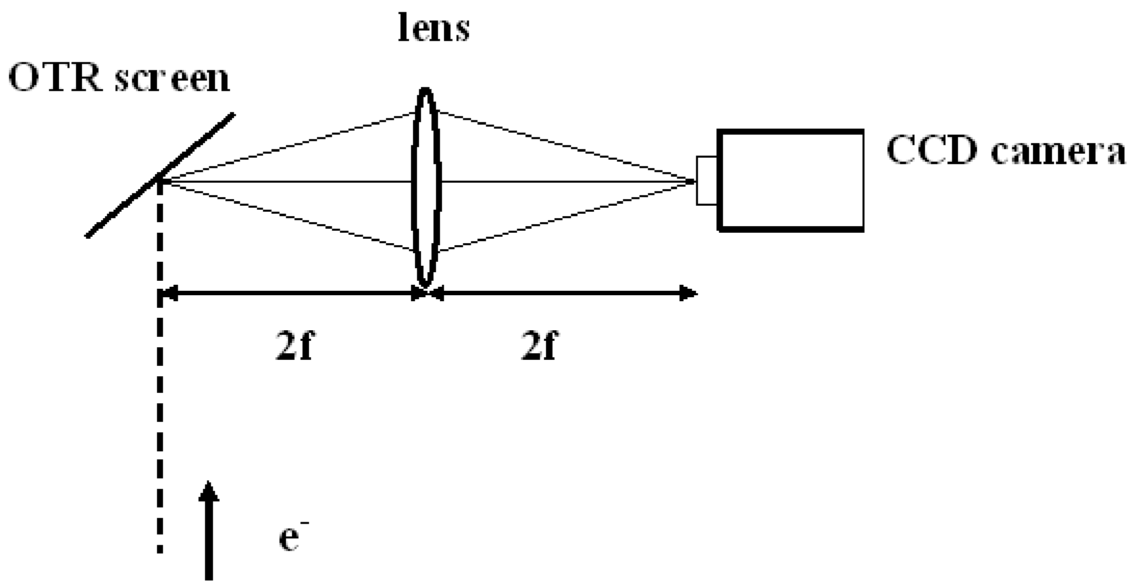
Abstract: Effects of the beam finite transverse size on electromagnetic radiation emitted by relativistic charged beam are commonly observed in condition of temporal incoherence in several electromagnetic radiative phenomena such as synchrotron radiation in an electron storage ring or photon bremsstrahlung in a positron-electron collider. Similarly, beam transverse size effects can be also predicted to affect the transition radiation emission by an electron beam in a particle accelerator. The main theoretical arguments in favour of such a sort of "source size" effect in the transition radiation energy spectrum and the main consequences in the field of the beam diagnostics in a particle linear accelerator will be described.

TRANSITION RADIATION (**TR**) ORIGINATED BY THE COUPLING OF THE VIRTUAL QUANTA FIELD BY A RELATIVISTIC CHARGE WITH THE CHARGE INDUCED CONDUCTION ELECTRONS ON A VACUUM-METAL INTERFACE.

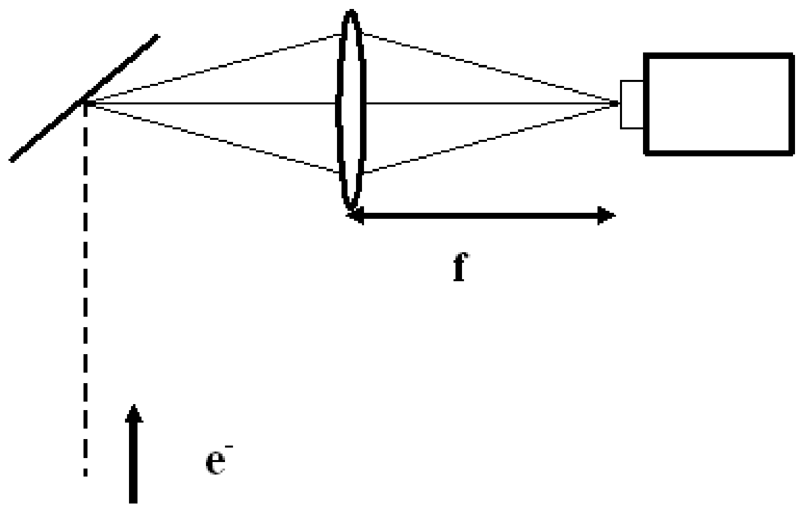
OTR (**O**ptical **T**ransition **R**adiation) **e-BEAM DIAGNOSTICS** IN A LINAC:

- **BEAM TRANSVERSE SIZE MONITOR** BY IMAGING THE TRACE LIGHT SPOT AT THE RADIATOR SURFACE;

- **BEAM ENERGY MONITOR** BY MAPPING THE RADIATION ANGULAR DISTRIBUTION.



BEAM -TRANSVERSE-SIZE IMAGING SET-UP



BEAM ENERGY MONITOR SET-UP

TR-ENERGY SPECTRUM BY A N-ELECTRON BEAM, CLASSICAL FORMULATION:

$$\frac{d^2I}{d\Omega d\omega} = \frac{d^2I_e}{d\Omega d\omega}[N + N(N-1)F(\vec{k})]$$

$$F(\vec{k}) = \left| \int d\vec{r}\, \rho(\vec{r}) e^{i\vec{k}\cdot\vec{r}} \right|^2$$

$$\frac{d^2I^e}{d\Omega d\omega} = \frac{(e\beta)^2}{\pi^2 c} \frac{sin^2\theta}{(1-\beta^2 cos^2\theta)^2}$$

OTR-ENERGY SPECTRUM: LINEAR ADDITION OF CONTRIBUTIONS FROM N ISOLATED AND INDIVIDUALLY RADIATING ELECTRONS

$$\frac{d^2I^{OTR}}{d\Omega d\omega} = N \frac{d^2I_e}{d\Omega d\omega}$$

OTR ENERGY SPECTRUM INDEPENDENT OF THE BEAM TRANSVERSE SIZE

QUESTION: MAY THE NUMBER AND THE ANGULAR DISTRIBUTION OF THE OTR PHOTONS EMITTED AT A GIVEN WAVELENGTH DEPEND ON THE BEAM TRANSVERSE SIZE?

PHENOMENOLOGICAL ANALOGIES: BEAM-TRANSVERSE-SIZE DEPENDENT VARIATIONS OF THE SYNCHROTRON RADIATION **BRILLIANCE** IN A STORAGE RING AND OF THE PHOTON BREMSSTRAHLUNG **LUMINOSITY** IN AN ELECTRON-POSITRON COLLIDER

TR KINEMATICS: RELATIVISTIC CHARGED BEAM COLLIDING WITH THE CHARGE INDUCED CONDUCTION ELECTRONS $\Longleftrightarrow$ PHOTON BREMSSTRAHLUNG BY COLLISION OF A RELATIVISTIC ELECTRON BEAM WITH AN IDENTICAL CHARGE IMAGE AT REST IN THE LABORATORY FRAME OF REFERENCE

BEAM TRANSVERSE SIZE EFFECTS IN TR-ENERGY SPECTRUM

HOW COME?: VIRTUAL QUANTA FIELD AS A DIRECT FUNCTION OF THE BEAM PARTICLE DENSITY INSTEAD OF THE INDIVIDUAL SINGLE ELECTRON

$$\frac{d^2I}{d\Omega d\omega} = \frac{d^2I^\sigma}{d\Omega d\omega}\,[N + N(N-1)F(\omega)] \qquad (1)$$

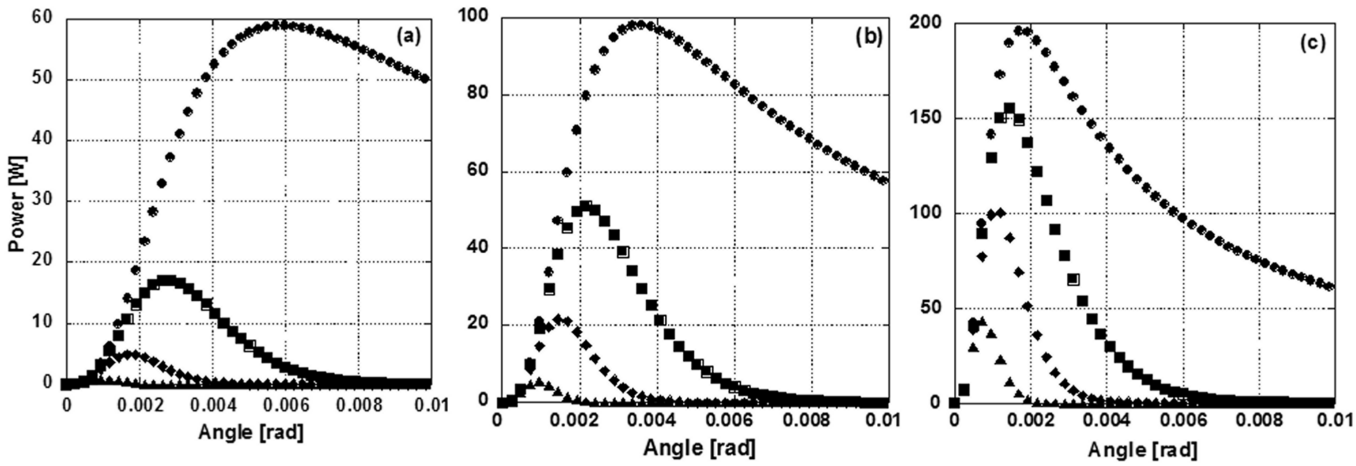
$$F(\omega) = \left| \int dz\, \rho_z(z) e^{-i(\omega/w)z} \right|^2$$

and, in case of an infinite radiator surface,

$$\frac{d^2I^\sigma}{d\Omega d\omega} = \frac{(e\beta)^2}{\pi^2 c} \frac{sin^2\theta\, |\rho_x(k_x)|^2\, |\rho_y(k_y)|^2}{(1-\beta^2 cos^2\theta)^2}$$

where, in case of a gaussian beam, the **transverse form factor** reads

$$|\rho_x(k_x)|^2\, |\rho_y(k_y)|^2 = e^{-k^2 sin^2\theta(\sigma_x^2 cos^2\phi + \sigma_y^2 sin^2\phi)}$$



Polar angle distribution of the transition radiation power integrated in the wavelength band 0.4-0.7 μm by a 10 ps long 1nC gaussian electron bunch having a cylindrical symmetry ($\sigma_x = \sigma_y = \sigma$) for different values of the beam energy (E=150 (a), 250 (b) and 500 (c) MeV). In each frame, the radiation power expected at each energy is calculated for different values of the beam transverse size ($\sigma = 100$ (▲), 50 (⧫), 25 (■) and 0 (•) μm).

EQ.(1) OBTAINED FROM

$$\frac{d^2 I}{d\Omega d\omega} = \frac{cR^2}{4\pi^2} \sum_{\mu=x,y} \Big(\sum_{j=1}^{N} \left| H_{\mu,j} \right|^2 + \\ + \sum_{j,l(j\neq l)=1}^{N} e^{i(\omega/w)(z_{0l}-z_{0j})} H_{\mu,j} H^*_{\mu,l} \Big)$$

UNDER CONTINUOUS LIMIT FORMULATION

$$F(\omega) \propto < e^{-i(\omega/w)z_{0j}} >_{avg} < e^{i(\omega/w)z_{0l}} >_{avg}$$

where

$$< e^{i(\omega/w)z_{0j}} >_{avg} = \int dz_0 \, \rho_z(z_0) e^{i(\omega/w)z_0}$$

$$\frac{d^2 I^\sigma}{d\Omega d\omega} \propto \\ \propto \sum_{\mu=x,y} \left| \int_S d\vec{\rho} \int d\vec{\tau} \, \frac{\tau_\mu \, \langle e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}} \rangle_{avg}}{\tau^2 + \alpha^2} \, e^{i(\vec{\tau}-\vec{\kappa})\cdot\vec{\rho}} \right|^2$$

$$\langle e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}} \rangle_{avg} = \\ = \Big(\int dx_0 \, \rho_x(x_0) e^{-i\tau_x x_0}\Big)\Big(\int dy_0 \, \rho_y(y_0) e^{-i\tau_y y_0}\Big) \\ = \rho_x(\tau_x)\rho_y(\tau_y)$$

VIRTUAL QUANTA FIELD DIRECTLY EXPRESSED IN TERMS OF THE PARTICLE DENSITY⇒ TR FIELD READS:

$$E^{tr}_{x,y}(\vec{\kappa},\omega) = \sum_{j=1}^{N} H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j})\, e^{-i(\omega/w)z_{0j}}$$

where $\vec{\kappa} = (k_x, k_y)$, $[\vec{\rho}_{0j} = (x_{0j}, y_{0j}), z_{0j}]$ $j = 1, .., N$ are the electron coordinates.

SINGLE ELECTRON FIELD AMPLITUDE:

$$H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j}) = \frac{i\,e\,k}{2\pi^2 R w} \int_S d\vec{\rho} \int d\vec{\tau}\, \frac{\tau_{x,y}\, e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}}}{\tau^2 + \alpha^2}\, e^{i(\vec{\tau}-\vec{\kappa})\cdot\vec{\rho}}$$

IMPLICIT INTEGRAL DEFINITION OF A SPECIAL FUNCTION WHOSE FORMAL EXPRESSION IS, IN PRINCIPLE, UNKNOWN BUT INTRINSICALLY DEPENDENT ON THE BEAM ENERGY (via $\alpha = \frac{\omega}{w\gamma}$), THE ELECTRON TRANSVERSE COORDINATES $\vec{\rho}_{0j}$ AND THE SHAPE AND SIZE OF THE RADIATOR SURFACE S.

SINGLE ELECTRON AMPLITUDE AND RELATIVE PHASE STRUCTURE OF TR-FIELD PHYSICALLY COMPATIBLE WITH:

- **TEMPORAL CAUSALITY PRINCIPLE**: TEMPORAL DURATION AND SHAPE OF THE TR LIGHT PULSE ONLY DEPENDENT ON THE TEMPORAL SEQUENCE OF THE PARTICLE COLLISIONS ON THE RADIATOR $\Leftrightarrow$ RELATIVE PHASE OF THE SINGLE ELECTRON FIELD AMPLITUDES ONLY DEPENDENT ON THE ELECTRON LONGITUDINAL COORDINATES

- **HUYGENS-FRESNEL PRINCIPLE**: LUMINOSITY OR BRILLIANCE OF A RADIATION SOURCE IS A FUNCTION OF BOTH THE BEAM ENERGY AND THE TRANSVERSE SIZE $\Leftrightarrow$ TR-FIELD STRENGTH - I.E., FIELD AMPLITUDE - HAS TO DEPEND NOT ONLY ON THE BEAM ENERGY BUT ALSO ON THE BEAM TRANSVERSE SIZE

e-BEAM TRANSVERSE SIZE EFFECTS ON TR AND BEAM DIAGNOSTICS IN A LINAC:

- BOTH BEAM ENERGY AND TRANSVERSE SIZE CAN BE ESTIMATED BY MEASURING THE OTR ANGULAR DISTRIBUTION

- MEASUREMENTS OF THE BEAM ENERGY AND TRANSVERSE SIZE ARE NOT INDEPENDENT AS LONG AS THE RATIO OF THE BEAM TRANSVERSE SIZE TO THE OBSERVED WAVELENGTH IS NOT NEGLIGIBLE

- THE SHORTER THE BEAM TRANSVERSE SIZE σ, THE HIGHER THE NUMBER OF TR-PHOTONS AT A GIVEN WAVELENGTH AND THE WIDER THE BROADENING OF THEIR ANGULAR DISTRIBUTION $\Rightarrow$ THE HIGHER THE SPATIAL RESOLUTION OF A MEASUREMENT OF σ BY DETECTING THE RADIATION INTENSITY AND ANGULAR DISTRIBUTION

Conclusions: Beam transverse size effects are currently observed in electromagnetic radiative mechanisms by charged beams. The beam-transverse-size induced variations of the synchrotron radiation Luminosity in an electron storage ring or the photon bremsstrahlung Brilliance in an electron-positron collider constitute the phenomenological evidence of such a statement. Based on the virtual quanta method newly formulated in terms of the beam electron density instead of the individual single electron, a recently proposed theoretical model for the transition radiation emission by an electron bunch predicts that the imprinting of the beam transverse size can be observed in the transition radiation energy spectrum even at a short wavelength, in particular, in the visible optical region. The observation of a possible effect of the beam transverse size on the spectral and angular distribution of the transition radiation energy spectrum in the short wavelength region would open new perspectives in the electron beam diagnostics in a linear accelerator. The possibility to estimate both the beam energy and transverse size from a measurement of the angular distribution of the radiation intensity in the short wavelength region can be indeed envisaged as well as the possibility to perform beam transverse size measurements with a high spatial resolution taking advantage of the beam-transverse-size induced variations of the number and the angular distribution of the transition radiation photons emitted in the visible optical spectral region.

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