

Superconductive Undulators: Mechanical deviations and their influence on the phase error

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Introduction

- Due to constructive interference of photons emitted by a single electron passing through an undulator magnetic field, the undulator emits a line spectrum:

$$\lambda_L = \frac{\lambda_u}{2k\gamma^2} \left(1 + \frac{K^2}{2}\right). \quad (1)$$

with the period length λ_u , the relative beam energy γ , the harmonic number of the emitted radiation k ($k = 1, 3, 5, \dots$), the deflection parameter $K = 0.0934 \cdot \lambda_u[\text{mm}] \cdot \tilde{B}[\text{T}]$ and the amplitude of the magnetic field \tilde{B} .

- In superconductive undulators the alternating field is produced by two coils

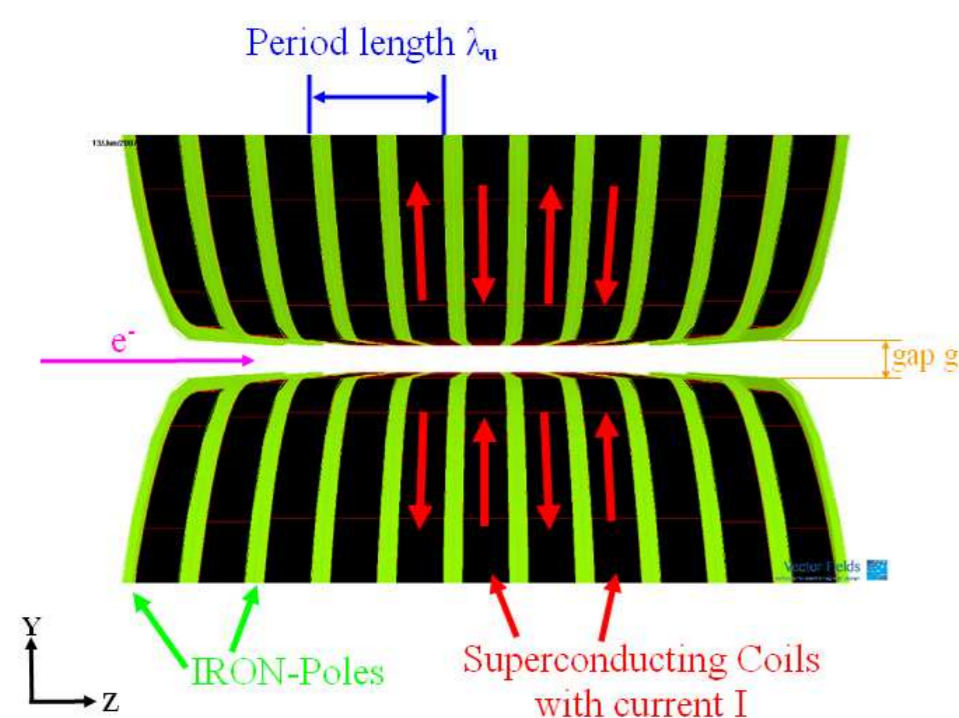


Figure 1: Opera3d [1] modell of a superconductive undulator with iron poles and superconductive wire bundles

- For a given gap and a given period length superconductive undulators have a higher field strength compared to permanent magnet undulators

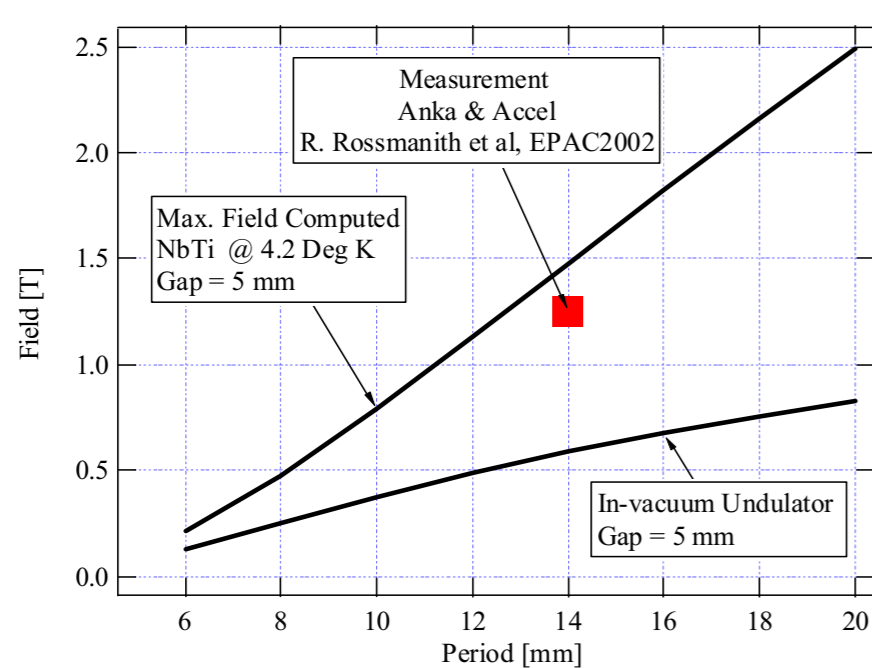


Figure 2: Comparison of the achievable calculated max. field strength for superconductive NbTi and in-vacuum permanent magnet undulators [2].

Monte-Carlo simulations

- Mechanical deviations of the poles and the wire bundles can be assumed as normally distributed along the undulator, due to the given mechanical tolerances in the construction process.
- Therefore the phase errors of different undulators constructed with the same mechanical tolerances vary.
- The required mechanical tolerances to obtain a specific phase error, can be achieved by Monte-Carlo simulations with several thousand undulators.

Phase requirements

- To obtain the maximum brilliance a constant difference between the phase of the emitted photon and the phase of the oscillating electron in each undulator period is required.
- A phase slip between the electron and the photon would disturb the constructive interference and, thus, cause a broadening and intensity reduction of the emission lines
- Therefore field errors of undulators have to be corrected

Phase difference between photon and electron for the period i :

$$\Phi_i = \frac{2\pi}{\lambda_u} \left(\frac{2 \left(\frac{e}{m_e c}\right)^2 J(z_i) - K^2 (z_i - z_0)}{2 + K^2} \right), \quad (2)$$

with

$$J(z_i) = \int_{z_0}^{z_i} \left(\int_{z_0}^z B_y(z') dz' \right)^2 dz.$$

$B_y(z)$, the y-component of the magnetic field along z .

Phase error for an undulator with n periods:

$$\Phi_{error} = \sqrt{\frac{\sum_{i=1}^n (\Phi_i)^2}{n}}. \quad (3)$$

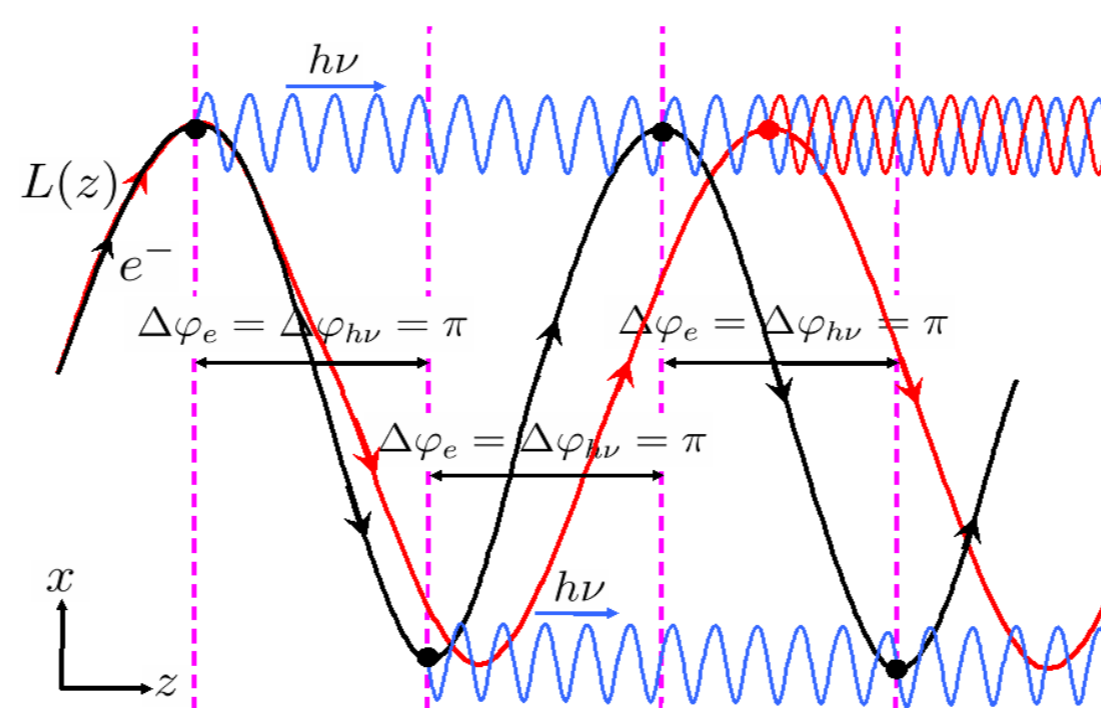


Figure 3: The variation of the period length or the amplitude of the magnetic field causes a slip between the electron phase and photon phase. In the worst case this leads to destructive interference.

Mechanical deviations

- In superconductive undulators phase errors are caused by mechanical deviations of the pole position and the position of the wire bundle relative to the gap center (y-direction).

Pole deviations

- The displacement of a pole in y-direction causes a local field disturbance

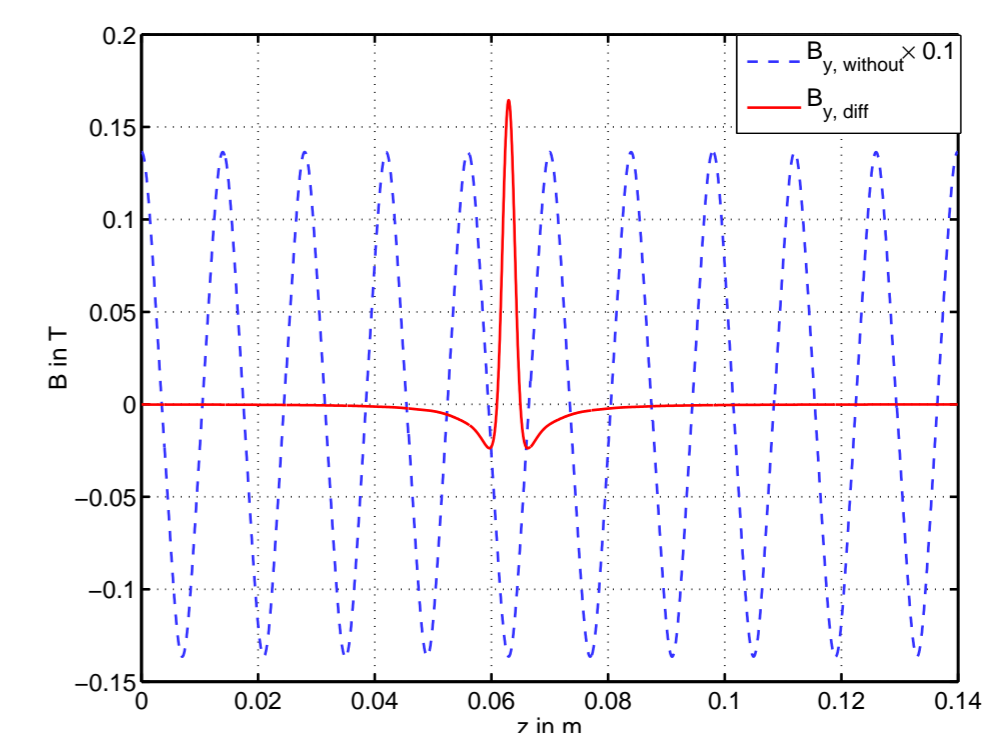


Figure 4: Field deviation caused by the mechanical displacement of one pole ($500\mu\text{m}$ in y-direction).

Wire bundle deviations

- The displacement of a wire bundle in y-direction causes an anti-symmetric local field disturbance

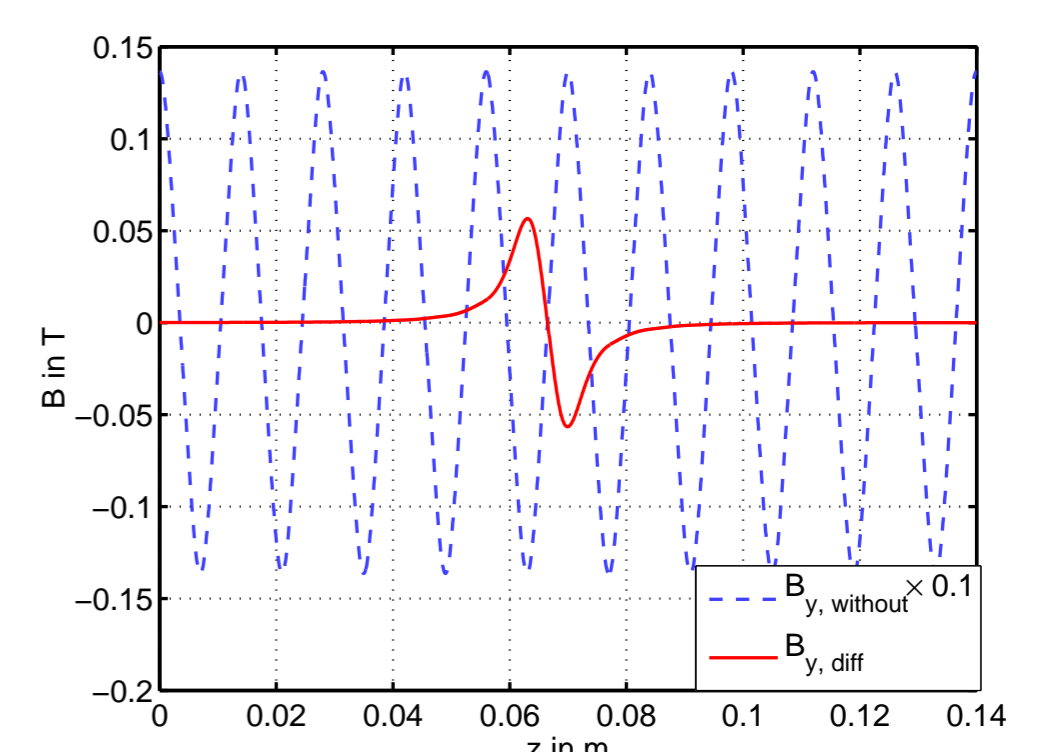


Figure 5: Field deviation caused by the mechanical displacement of one wire bundle ($500\mu\text{m}$ in y-direction).

(50 periods, $\lambda_u = 14\text{mm}$, $\tilde{B} = 1.24\text{T}$, 1000 undulators)

Variation	Φ_{error} (pole)	Φ_{error} (conductor bundle)
$5\mu\text{m}$	$0 - 4^\circ$	$0 - 1.5^\circ$
$14\mu\text{m}$	$1.5 - 10^\circ$	$1.5 - 4.5^\circ$
$50\mu\text{m}$	$10 - 100^\circ$	$5 - 15^\circ$

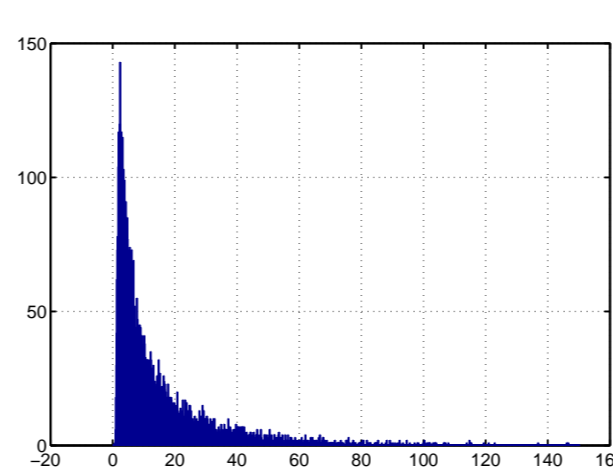


Figure 6: Monte-Carlo simulation of 10000 undulators with normally distributed pole and coil displacement ($5\mu\text{m}$ in y-direction).

Acknowledgments

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