Wakefield Computations for the FERMI FEL Undulator

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Abstract

As the electron bunches proceed through the undulator, resistive, roughness and geometric wakefields act back onto its core and tail. Concerning the longitudinal wakefields the induced energy spread would enhance the range of the charge density modulation resulting thus in broader frequency bands around the harmonics. Calculations of the resistive-wall and surface-roughness wake functions are presented, together with energy spread estimations for the FERMI FEL bunches (Fig.1).

Resistive-wall wakefields

Longitudinal wake functions are found for three possible cross-sections performing an inverse Fourier transform of the longitudinal impedance. Aluminium and Copper are chosen as wall material for exemplification in Fig.2. The induced energy spread is found convolving the bunch profiles in Fig.1 with these wake functions as shown in Fig.3 only for the medium bunch and Aluminium (AC conductivity).

Circular cross-section

For a circular vacuum chamber of inner radius b with $Z_0 = 120\,\pi\ [\Omega]$ and k the wave number, the resistive-wall longitudinal impedance is written as [1]:

$$Z_{\parallel}(k)[\Omega/m] = \frac{Z_0 c}{4\pi}\times\frac{1}{cb}\frac{2}{\lambda(k)/k - ikb/2} \qquad \lambda(k,\sigma)[m^{-1}] = \sqrt{\frac{Z_0\sigma k}{2}}(i+\mathrm{sgn}(k))$$

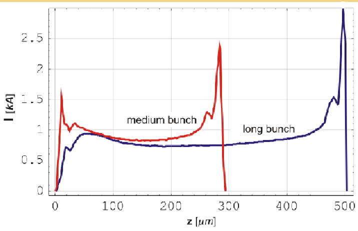

Fig.1: Longitudinal profile of the FERMI FEL bunches.

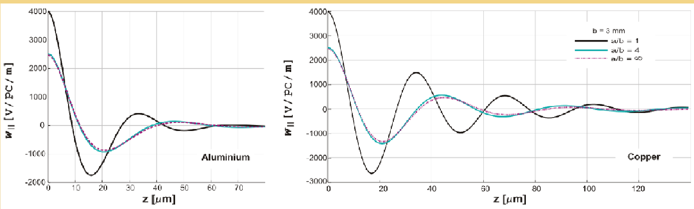

Fig.2: Longitudinal resistive wake functions.

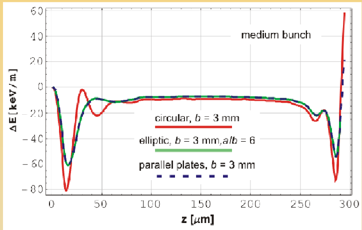

Fig.3: Wall resistivity induced energy spread.

Elliptic cross-section

For this case the wake function is obtained by developing a system of solutions to the Maxwell's equations both in the vacuum and in the resistive wall and then imposing the boundary conditions on the wall surface. The mathematical formulation can be found in [4].

Rectangular cross-section

Taking a vertical half-gap b and large enough width, the longitudinal impedance is defined as [2]:

$$Z_{\parallel}(k)[\Omega/m] = \frac{Z_0 c}{4\pi}\times\frac{1}{c}\int_{-\infty}^{\infty}\Big[\frac{\lambda(k)}{k}\cosh^2(bx) - i\frac{k}{x}\cosh(bx)\sinh(bx)\Big]^{-1}dx$$

Surface roughness wakefields

Considering a circular cross-section vacuum chamber of inner radius b, the Stupakov's resonator and statistical models [5] have been applied to a surface modeled as a sinusoidal corrugation, following in-house surface profile measurements [6]. The longitudinal surface profile was filtered into a low-frequency component with the rms height $h_{\mathrm{rms}} = 250\ nm$ and the periodicity $\lambda = 30\ \mu m$ (to be treated within the resonator model) and a high-frequency component with the rms height $h_{\mathrm{rms}} = 40\ nm$ and the periodicity $\lambda = 10\ \mu m$ (to be treated within the statistical model).

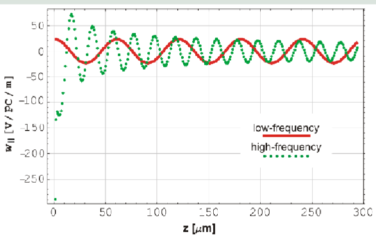

Fig.4: Longitudinal roughness wake functions.

For the low-frequency component the wake function represents a synchronous (with the bunch) mode:

$$w_{\parallel}^{low}(z) = \frac{Z_0 c}{4\pi}\frac{h_{rms}^4(2\pi/\lambda)^6}{64}\cos\Big(\frac{\pi}{\lambda}z\Big)$$

while for the high-frequency component the wake function describes the impedance of tiny sine-like bumps:

$$w_{\parallel}^{high}(z) = \frac{Z_0 c}{4\pi}\frac{h_{rms}^2(2\pi/\lambda)^2}{b}\frac{1}{2\sqrt{\pi}}\frac{\partial}{\partial z}\frac{\cos(\pi z/\lambda)+\sin(\pi z/\lambda)}{\sqrt{2\pi z/\lambda}}$$

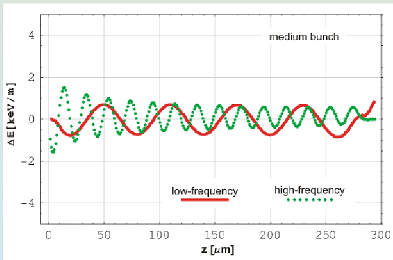

Fig.5: Surface roughness induced energy spread.

Conclusions

It has been shown that in what concerns the wall resistivity, it is preferable to have a rectangular or elliptic cross-section vacuum chamber with Aluminium made walls knowing the fact that its a shorter relaxation time (in AC electrical conductivity) makes the wakes ring less. As for the surface induced wakes, calculations indicate that the induced energy spread would account for about 12 % and 15 % of the resistivity induced energy spread, respectively for the medium and long FERMI FEL bunch.

Contribution	medium bunch	long bunch
Low-frequency corrugation	0.74 keV/m	0.20 keV/m
High-frequency corrugation	0.99 keV/m	0.97 keV/m
Total surface roughness	1.73 keV/m	1.17 keV/m
Wall resistivity	11.71 keV/m	9.35 keV/m
Grand total (resistivity+roughness)	13.44 keV/m	10.52 keV/m

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