

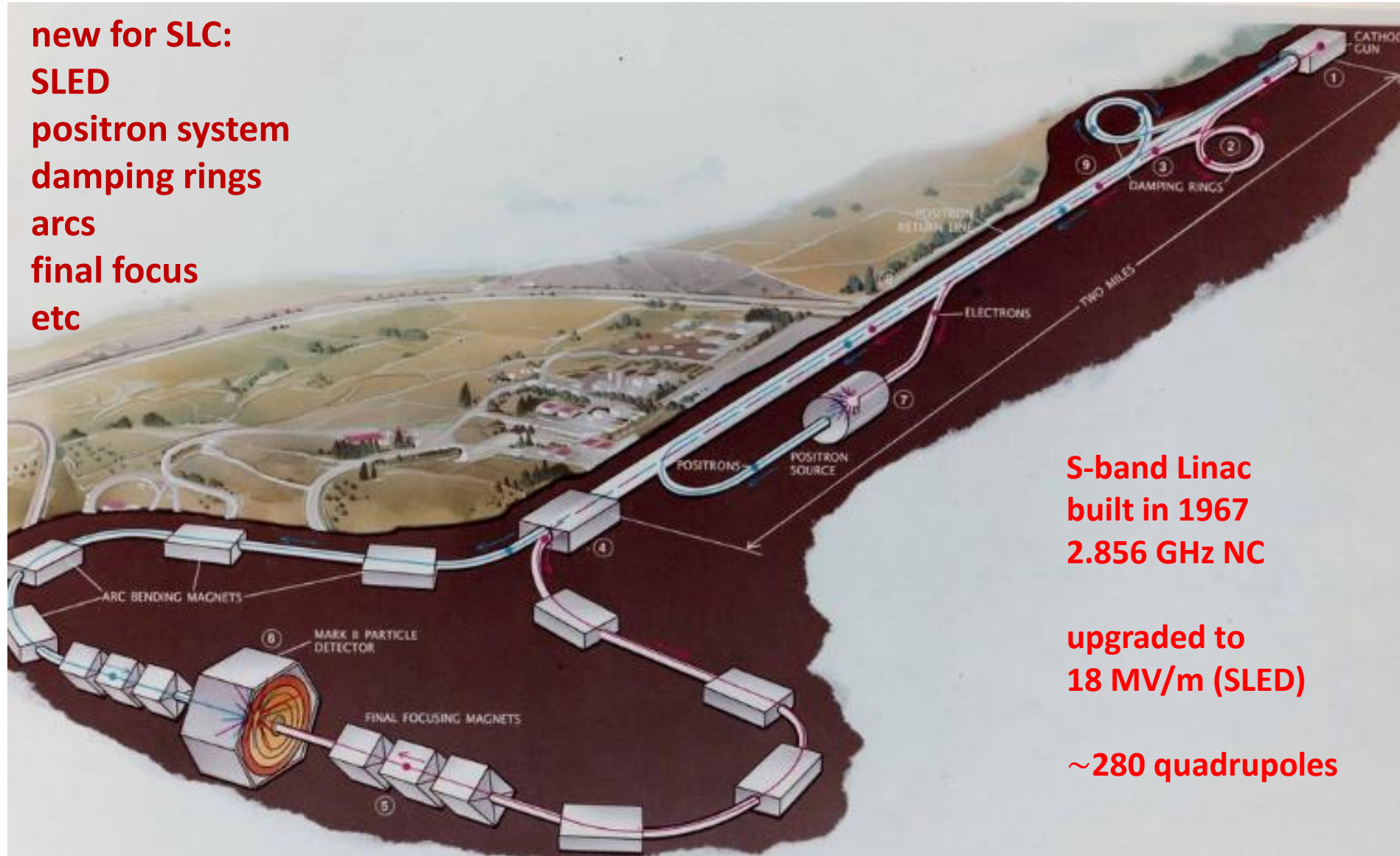
# lessons from the SLC

Frank Zimmermann

CERN Accelerator School on Future Colliders

Friday 2 March 2018

new for SLC:  
SLED  
positron system  
damping rings  
arcs  
final focus  
etc



S-band Linac  
built in 1967  
2.856 GHz NC

upgraded to  
18 MV/m (SLED)

~280 quadrupoles

SLAC Linear Collider - The first and only linear collider (1987/89-1998);  
~two years to produce 1<sup>st</sup> Z's; competitor circular LEP started in 1989

SLC was proposed in 1979  
by Burt Richter



(who a few years earlier  
had proposed LEP....)

*What SLC is trying to do is "like shooting two guns at one another from 100 miles apart and getting the bullets to hit".*

New Scientist, 19 May 1988



# Physics Dream Machine Is Imperiled

Technical problems plague Stanford's Linear Collider, threatening its ability to produce breakthroughs in particle physics. Expectations were running high. For months, the Stanford Linear Collider (SLC), an innovative particle accelerator nearing completion at the Stanford Linear Accelerator Facility (SLAC) in Palo Alto, Calif. had been preparing for its debut. This was the machine that would mint a million  $Z^0$  particles a year. Close study of the  $Z^0$ —its mass, for example—

By Robert Crease | September 5, 1988

The Scientist Magazine

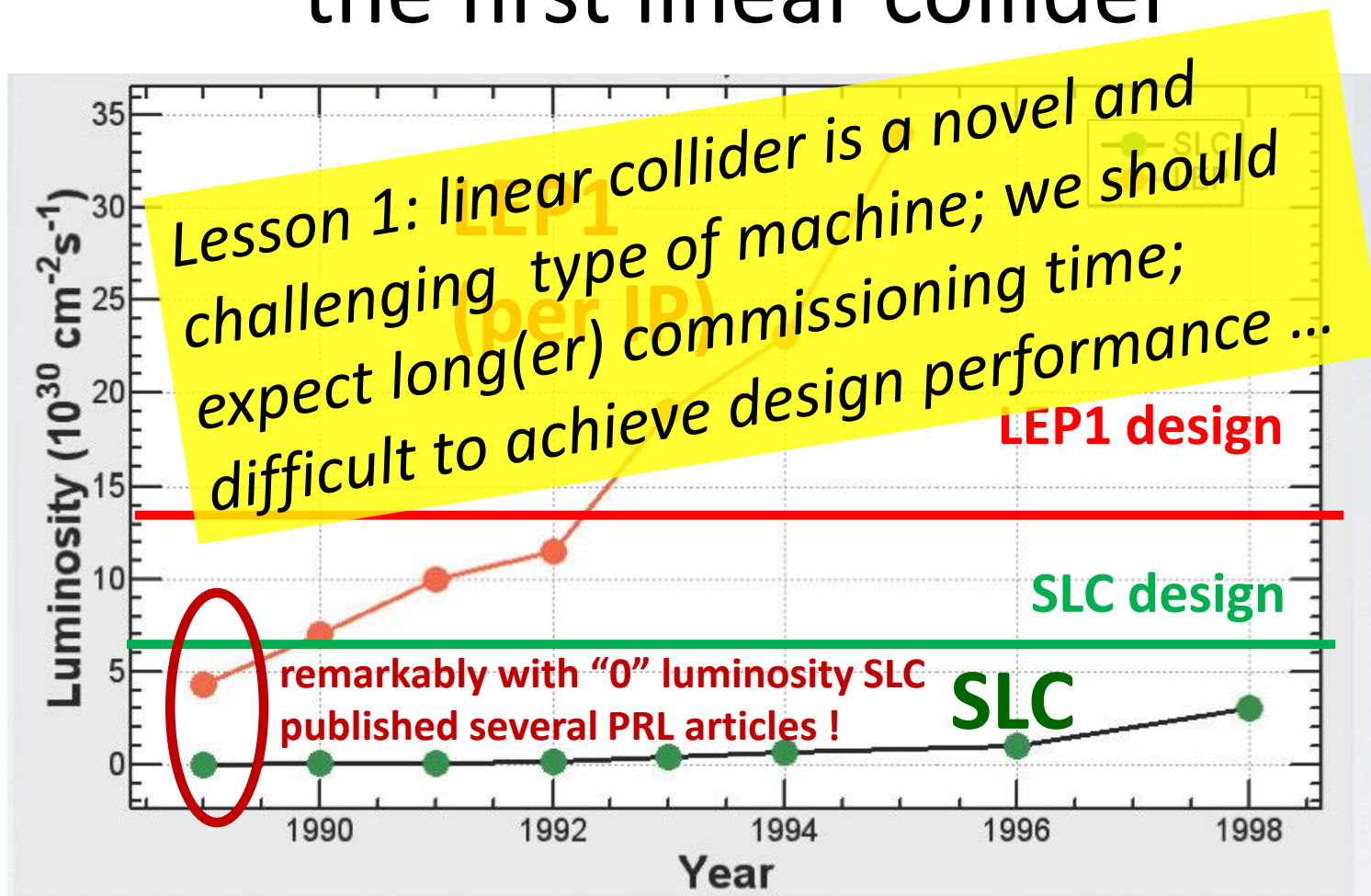
Laboratory management supercharged the already exuberant atmosphere by issuing news releases advertising the imminent birth of  $Z^0$  particles. The media was duly impressed. **"For the first time in five years," gushed the New York Times on July 19, "high-energy physicists in the United States are poised to seize a commanding lead from colleagues in Europe."**

No such luck. On June 24, the machine was switched on for what was called "a physics research run," but the experiment never got rolling.

"Nobody believed that, with a new kind of accelerator, we'd flip a switch and be up to specs," says lab director Burton Richter. "But nobody believed we'd have this much trouble getting started either."

"We'll make it go," says Richter, "but don't ask me when. We have a lot of work to do." Meanwhile, the delay is eating away at the SLC's lead over its European rival, the Large Electron Positron (LEP) accelerator at CERN in Geneva, Switzerland. And even though the SLC had already proved the value of its innovative design, its troubles are threatening not only to harm the reputations of the Stanford scientists, but to wipe out their daring approach to accelerator...

# commissioning time & performance of the first linear collider



CERN-SL-2002- 009 (OP), SLAC-PUB-8042

SLC

- peak  $\sim 1/2$  design after 11 years, average  $\sim 1/4$  design

# PRL articles from the early years

VOLUME 62, NUMBER 20

PHYSICAL REVIEW LETTERS

15 MAY 1989

## First Observation of Beamstrahlung

G. Bonvicini, E. Gero, R. Frey, and W. Koska

*University of Michigan, Ann Arbor, Michigan 48109*

VOLUME 62, NUMBER 25

PHYSICAL REVIEW LETTERS

19 JUNE 1989

*Stanford Linear Accelerator*

## Observation of Beam-Beam Deflections at the Interaction Point of the SLAC Linear Collider

CE P. Bambade,<sup>(1),(a)</sup> R. Erickson,<sup>(1)</sup> W. A. Koska,<sup>(2)</sup> W. Kozanecki,<sup>(1)</sup> N. Phinney,<sup>(1)</sup> and S. R. Wagner<sup>(3)</sup>

<sup>(1)</sup>Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

<sup>(2)</sup>University of Michigan, Ann Arbor, Michigan 48109

<sup>(3)</sup>University of Colorado, Boulder, Colorado 80309

(Received 21 February 1989)

Collisions of electron and positron  
led to the first detected emission

VOLUME 63, NUMBER 7

PHYSICAL REVIEW LETTERS

14 AUGUST 1989

## Initial Measurements of Z-Boson Resonance Parameters in $e^+e^-$ Annihilation

G. S. Abrams,<sup>(1)</sup> C. E. Adolphsen,<sup>(2)</sup> R. Aleksan,<sup>(3)</sup> J. P. Alexander,<sup>(3)</sup> M. A. Allen,<sup>(3)</sup> W. B. Atwood,<sup>(3)</sup>  
D. Averill,<sup>(4)</sup> J. Ballam,<sup>(3)</sup> P. Bambade,<sup>(3)</sup> B. C. Barish,<sup>(5)</sup> T. Barklow,<sup>(3)</sup> B. A. Barnett,<sup>(6)</sup> J. Bartelt,<sup>(3)</sup>  
S. Bethke,<sup>(1)</sup> D. Blockus,<sup>(4)</sup> W. de Boer,<sup>(3)</sup> G. Bonvicini,<sup>(7)</sup> A. Boyarski,<sup>(3)</sup> B. Brabson,<sup>(4)</sup>  
A. Breakstone,<sup>(8)</sup> M. Breuer,<sup>(3)</sup> J. M. Brown,<sup>(3)</sup> K. L. Brown,<sup>(3)</sup> F. Bulos,<sup>(3)</sup> P. R. Burchat,<sup>(2)</sup>  
D. L. Burke,<sup>(3)</sup> R. J. Cence,<sup>(8)</sup> J. Chapman,<sup>(7)</sup> M. Chmeissani,<sup>(7)</sup> J. Clendenin,<sup>(3)</sup> D. Cords,<sup>(3)</sup>  
D. P. Coupal,<sup>(3)</sup> P. Dauncey,<sup>(6)</sup> N. R. Dean,<sup>(3)</sup> H. C. DeStaebler,<sup>(3)</sup> D. E. Dorfman,<sup>(2)</sup> J. M. Dorfman,<sup>(3)</sup> P. S. Drell,<sup>(1)</sup>  
D. C. Drewier,<sup>(6)</sup> F. Dydak,<sup>(3)</sup> S. Ecklund,<sup>(3)</sup> R. Elia,<sup>(3)</sup> R. A. Erickson,<sup>(3)</sup> J. Fay,<sup>(1)</sup> G. J. Feldman,<sup>(3)</sup>  
D. Fernandes,<sup>(3)</sup> R. C. Field,<sup>(3)</sup> T. H. Fieguth,<sup>(3)</sup> G. E. Fischer,<sup>(3)</sup> W. T. Ford<sup>(9)</sup>  
C. Fordham,<sup>(3)</sup> R. Frey,<sup>(7)</sup> D. Fujino,<sup>(1)</sup> VOLUME 70, NUMBER 17

PHYSICAL REVIEW LETTERS

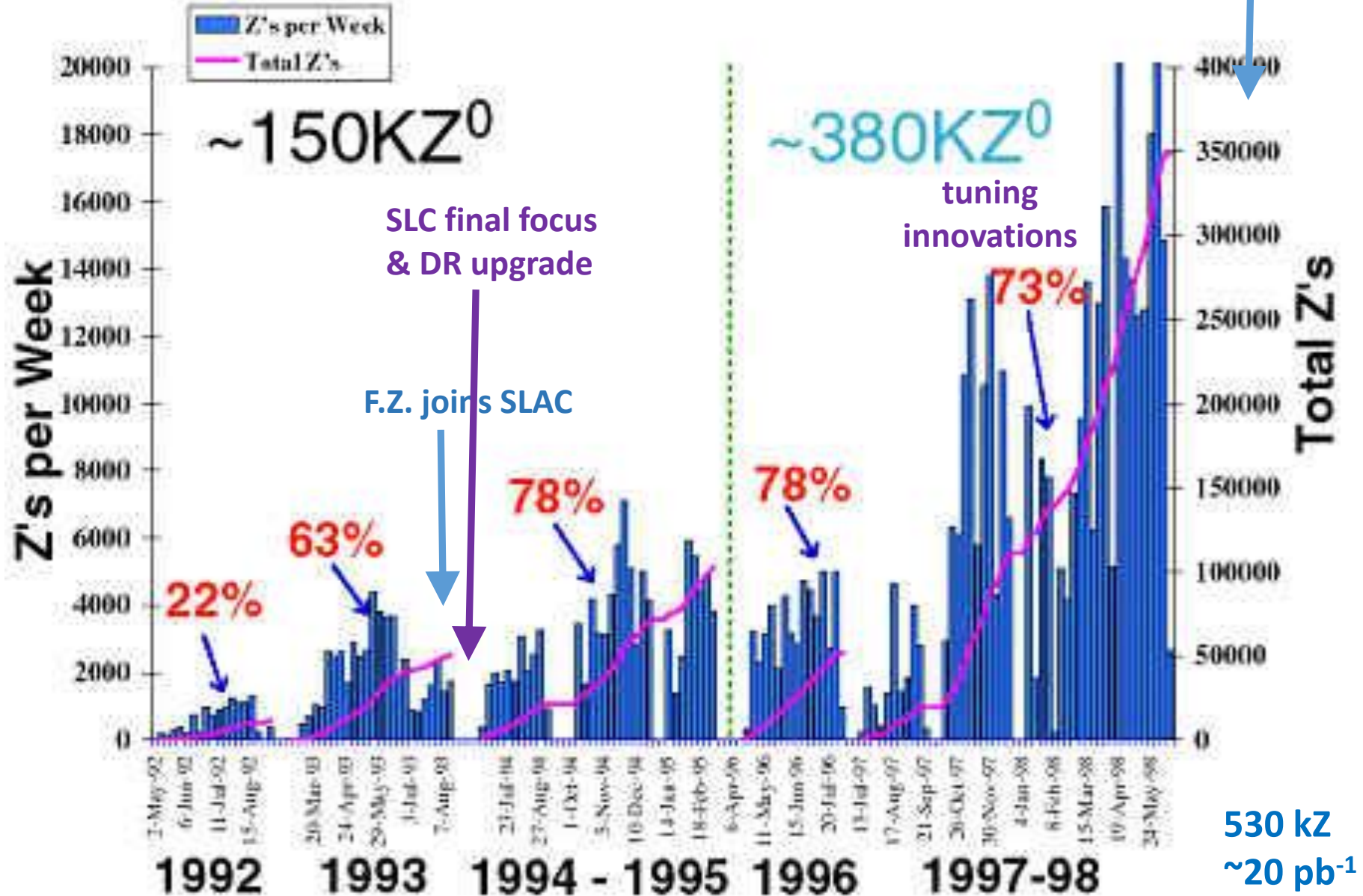
26 APRIL 1993

## First Measurement of the Left-Right Cross Section Asymmetry in Z Boson Production by $e^+e^-$ Collisions

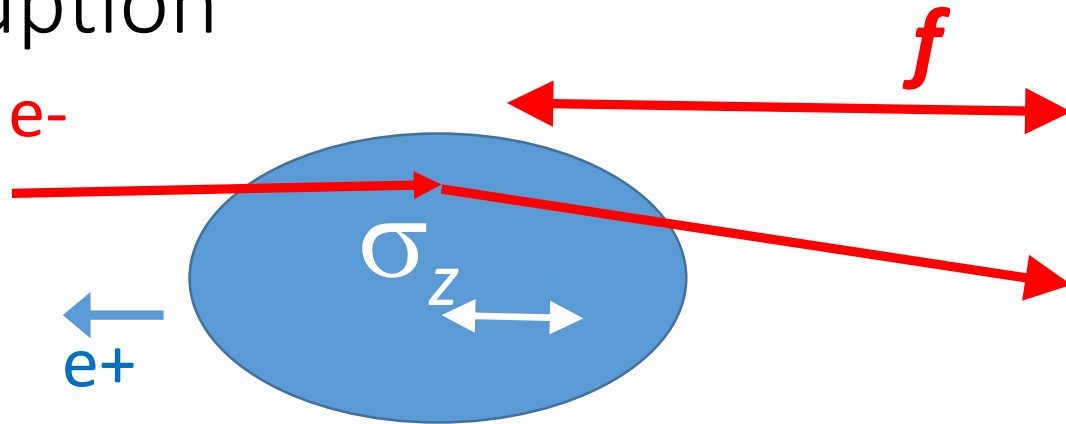
K. Abe,<sup>(29)</sup> I. Abt,<sup>(15)</sup> P. D. Acton,<sup>(5)</sup> C. E. Adolphsen,<sup>(27)</sup> G. Agnew,<sup>(5)</sup> C. Alber,<sup>(10)</sup> D. F. Alzofon,<sup>(27)</sup> P. Antilogus,<sup>(27)</sup> C. Arroyo,<sup>(11)</sup> W. W. Ash,<sup>(27)</sup> V. Ashford,<sup>(27)</sup> A. Astbury,<sup>(32)</sup> D. Aston,<sup>(27)</sup> Y. Au,<sup>(11)</sup> D. A. Axen,<sup>(4)</sup> N. Bacchetta,<sup>(22)</sup> K. G. Baird,<sup>(25)</sup> W. Baker,<sup>(27)</sup> C. Baltay,<sup>(35)</sup> H. R. Band,<sup>(34)</sup> G. Baranko,<sup>(10)</sup> O. Bardon,<sup>(18)</sup> F. Barrera,<sup>(27)</sup> R. Battiston,<sup>(23)</sup> A. O. Bazarko,<sup>(11)</sup> A. Bean,<sup>(7)</sup> G. Beer,<sup>(32)</sup> R. J. Belcinski,<sup>(19)</sup> R. A. Bell,<sup>(27)</sup> R. Ben-David,<sup>(35)</sup> A. C. Benvenuti,<sup>(2)</sup> R. Berger,<sup>(27)</sup> S. C. Berridge,<sup>(28)</sup> S. Bethke,<sup>(17)</sup> M. Biasini,<sup>(23)</sup> T. Bienz,<sup>(27)</sup> G. M. Bilei,<sup>(23)</sup> F. Bird,<sup>(27)</sup> D. Bisello,<sup>(22)</sup> G. Blaylock,<sup>(8)</sup> R. Blumberg,<sup>(27)</sup> J. R. Bogart,<sup>(27)</sup> T. Bolton,<sup>(11)</sup> S. Bougerolle,<sup>(4)</sup> G. R. Bower,<sup>(27)</sup> R. F. Boyce,<sup>(27)</sup> J. E. Brau,<sup>(21)</sup> M. Breidenbach,<sup>(27)</sup> T. E. Browder,<sup>(27)</sup> W. M. Bugge,<sup>(28)</sup> B. Burgess,<sup>(27)</sup> D. Burke,<sup>(27)</sup> T. H. Burnett,<sup>(33)</sup> P. N. Burrows,<sup>(18)</sup> W. Busza,<sup>(18)</sup> B. L. Byers,<sup>(27)</sup> A. Calcaterra,<sup>(13)</sup> D. O. Caldwell,<sup>(7)</sup> D. Calloway,<sup>(27)</sup> B.



# 1992 - 1998 SLD Polarized Beam Running



# luminosity enhancement from beam-beam “disruption”



“disruption parameter” = bunch length / focal length

$$D_{x,y} = \frac{\sigma_z}{f_{x,y}} = \frac{2r_p N_b \sigma_z}{\gamma \sigma_{x,y} (\sigma_x + \sigma_y)}$$

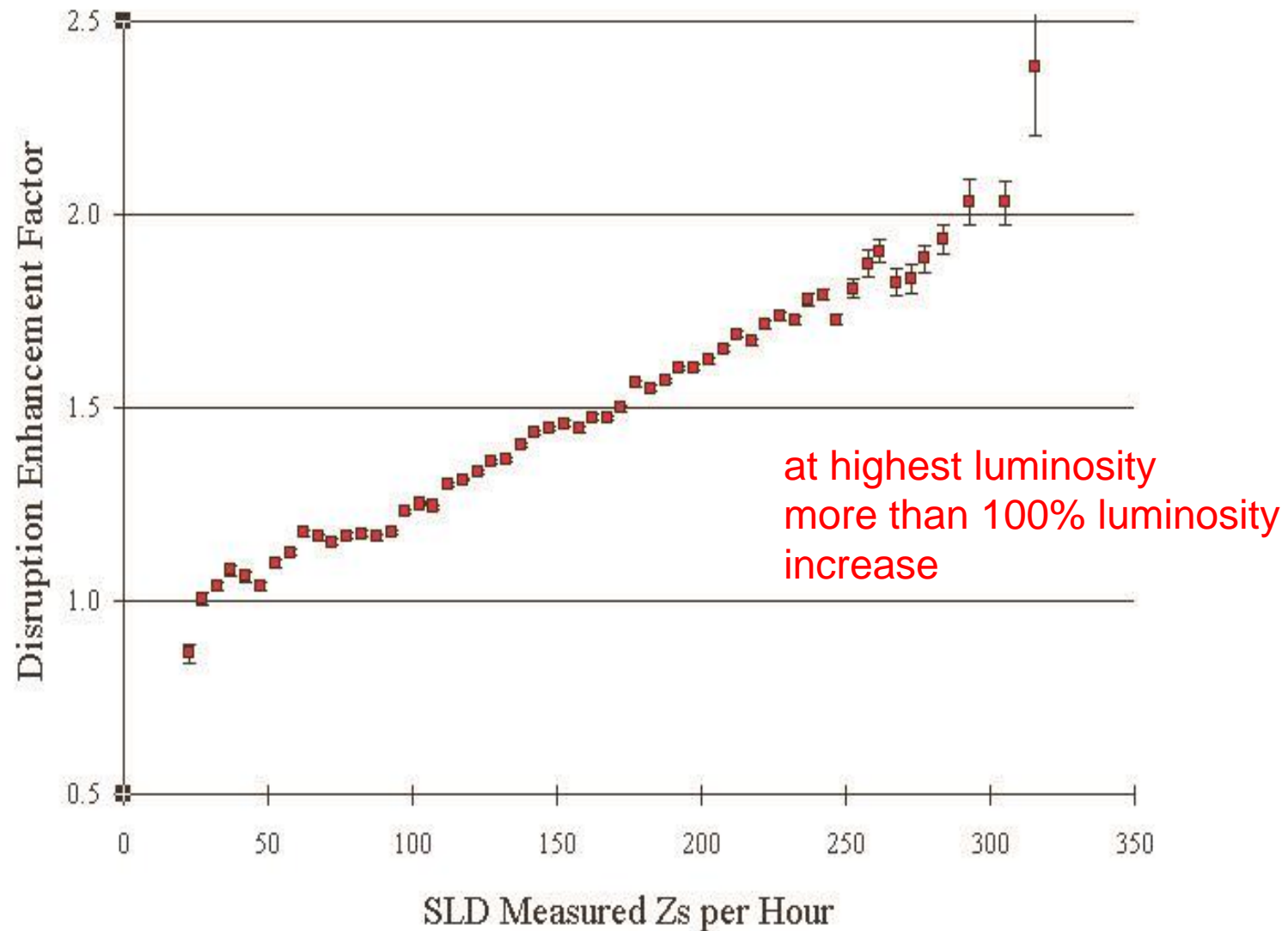
$$L = L_0 H_D \left( D_x, D_y, \sigma_z / \sigma_y, \sigma_z / \beta_y^* \right)$$

$L_0$ : luminosity for  
constant beam size

$H_D$ : disruption enhancement factor



# beam-beam disruption at the SLC

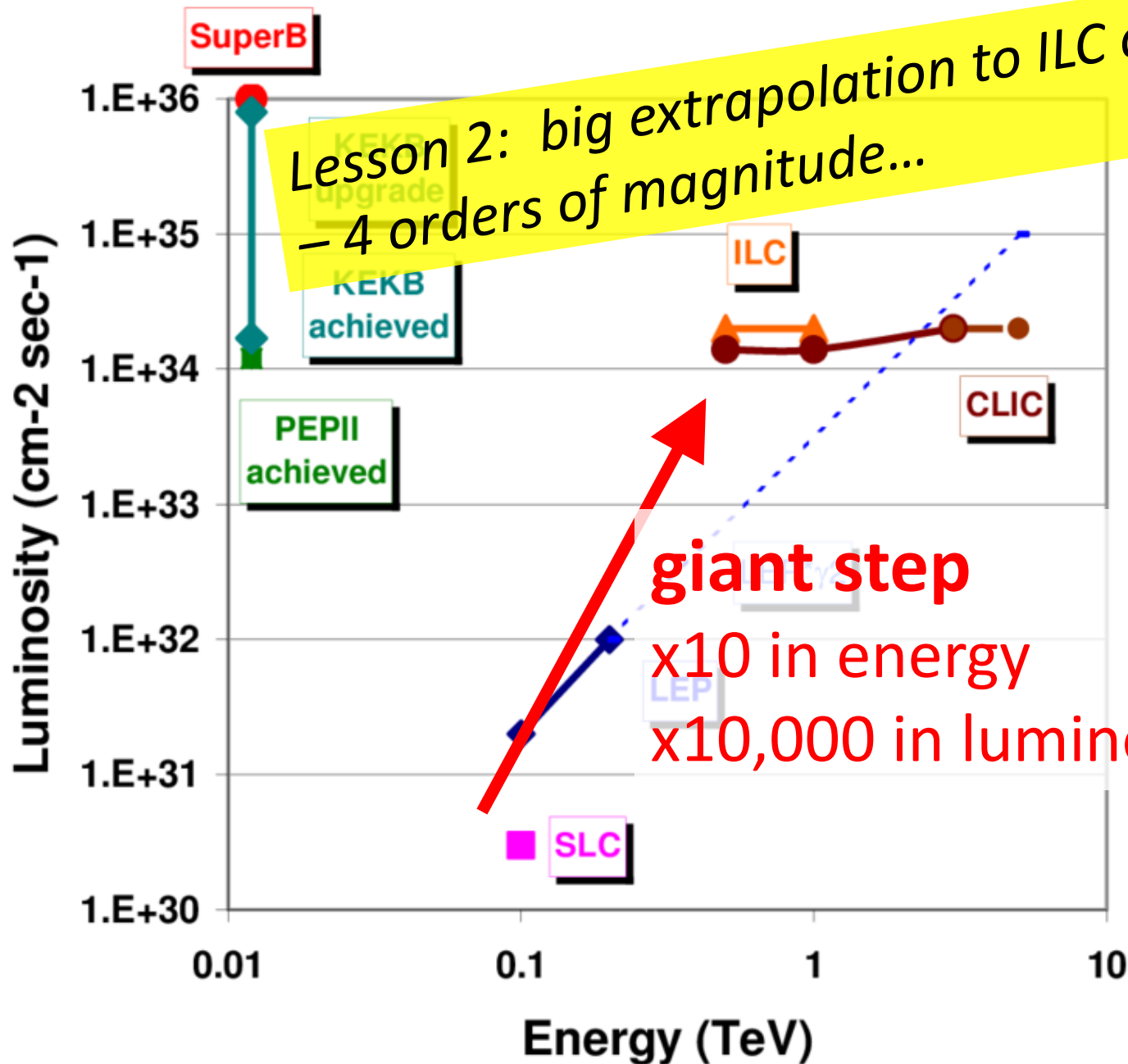


# design versus reality

parameter	design*	achieved 1998
repetition rate [Hz]	180	<b>120</b>
bunch population [ $10^{10}$ ]	7.2	<b>3.7 (e<sup>-</sup>), 3.4 (e<sup>+</sup>)</b>
norm.emittance in final focus $\gamma\epsilon_{x,y}$ [ $\mu\text{m}$ ]	42, 42	<b>54, 10</b>
IP bunch length [mm]	1.0	1.0
IP beta function $\beta_{x,y}^*$ [mm]	5, 5	<b>2.8, 1.5</b>
IP beam size $\sigma_{x,y}^*$ [ $\mu\text{m}$ ]	1.65, 1.65 (round)	1.84, 0.98 measured** [1.34, 0.39 expected**]
e <sup>-</sup> polarization at IP	0	<b>73% (1998)</b>
disruption enhancement	2.2	<b>2.0</b>
luminosity [ $10^{30} \text{ cm}^{-2}\text{s}^{-1}$ ]	6.0	<b>1.4</b> (average Jan-May'98)

\*SLC Design Handbook, December 1984

\*\*SLAC-CN-418 (1998)





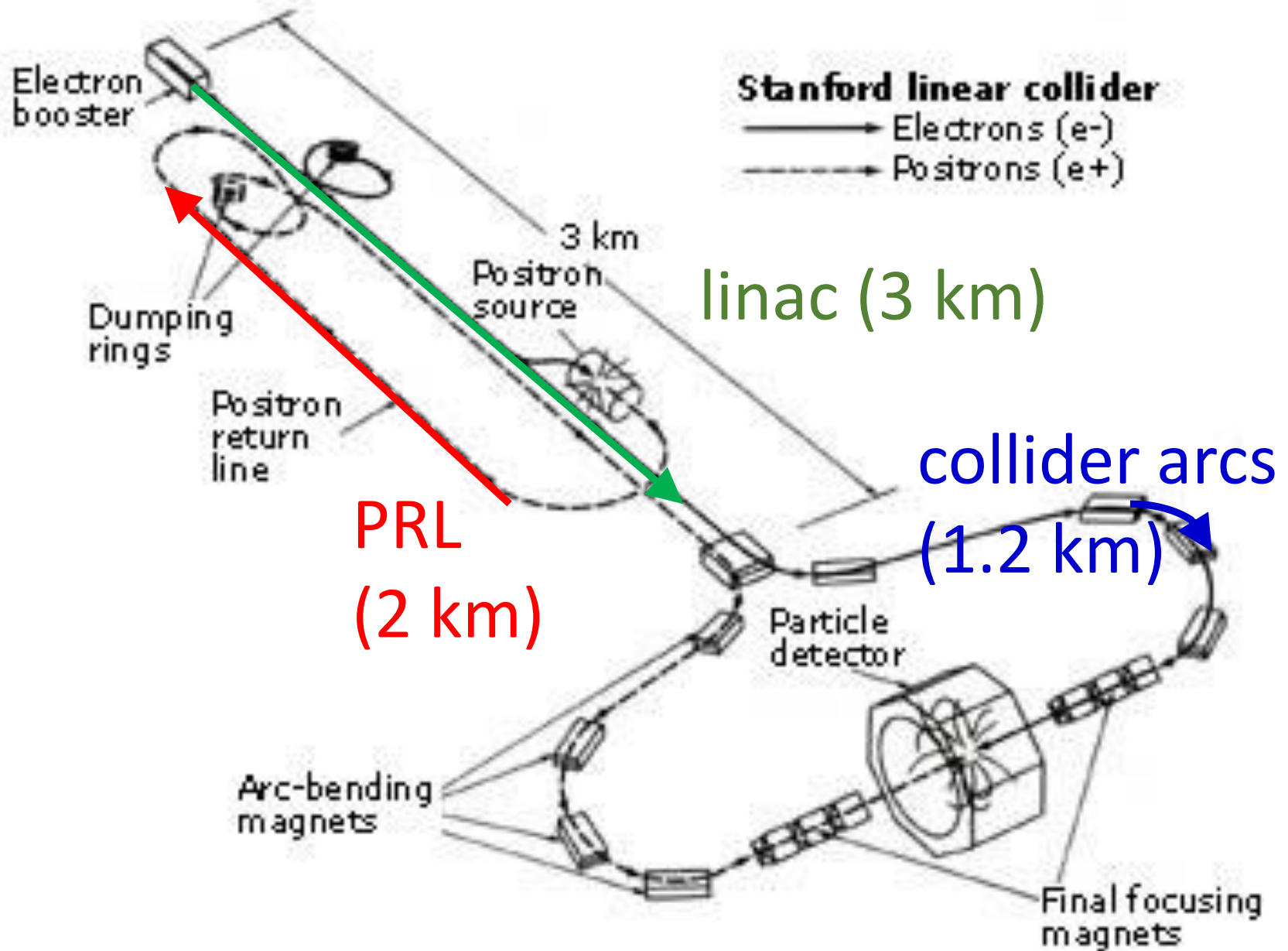
# a few selected SLC challenges & highlights

- **fighting resonances**  
e+ return, two-beam linac wake, arc spin rotator
- **keeping the beam stable**  
jitter, e+, vibration, damping ring instability
- **making a small beam spot**  
design, errors/tuning, the unknowns
- **beam halo & detector background**  
modelling halo, suppressing muon background

# resonances in a linear collider ... ?

Resonance of Trump's tweets  
among the Twitter publics

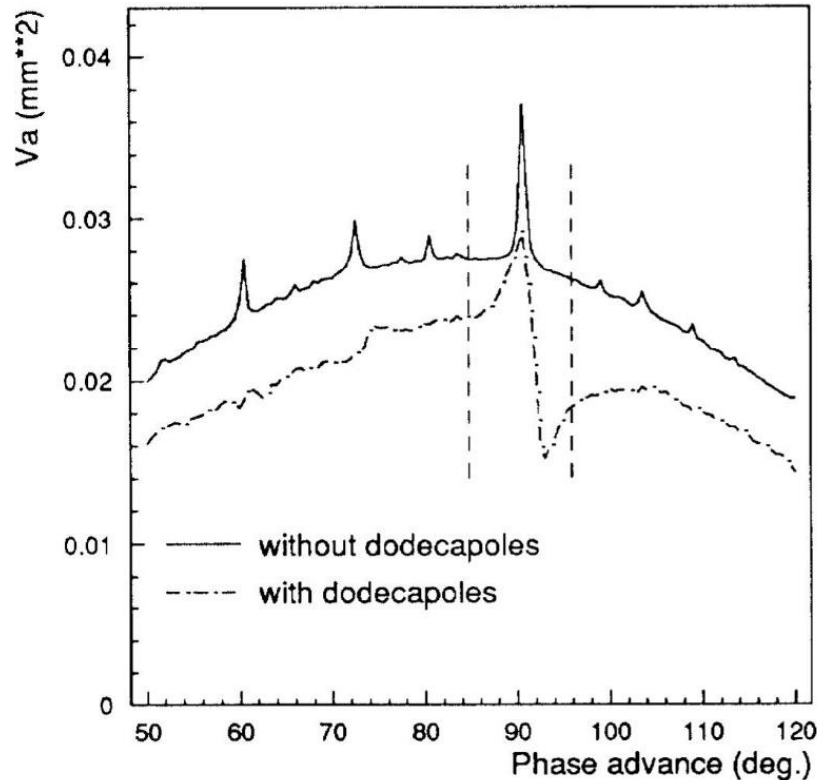




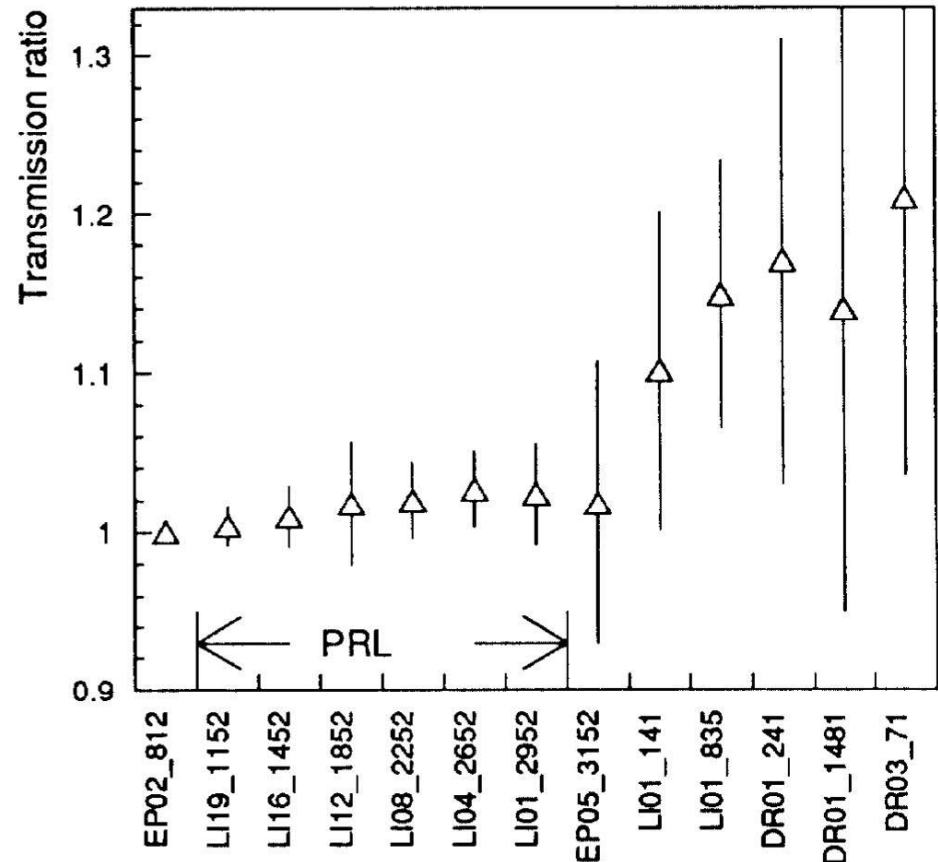


# positron return line (PRL)

2 km long transport line), consisting of **75 FODO cells**, nonlinear field errors (12-pole in quadrupoles), chromatic errors, etc.



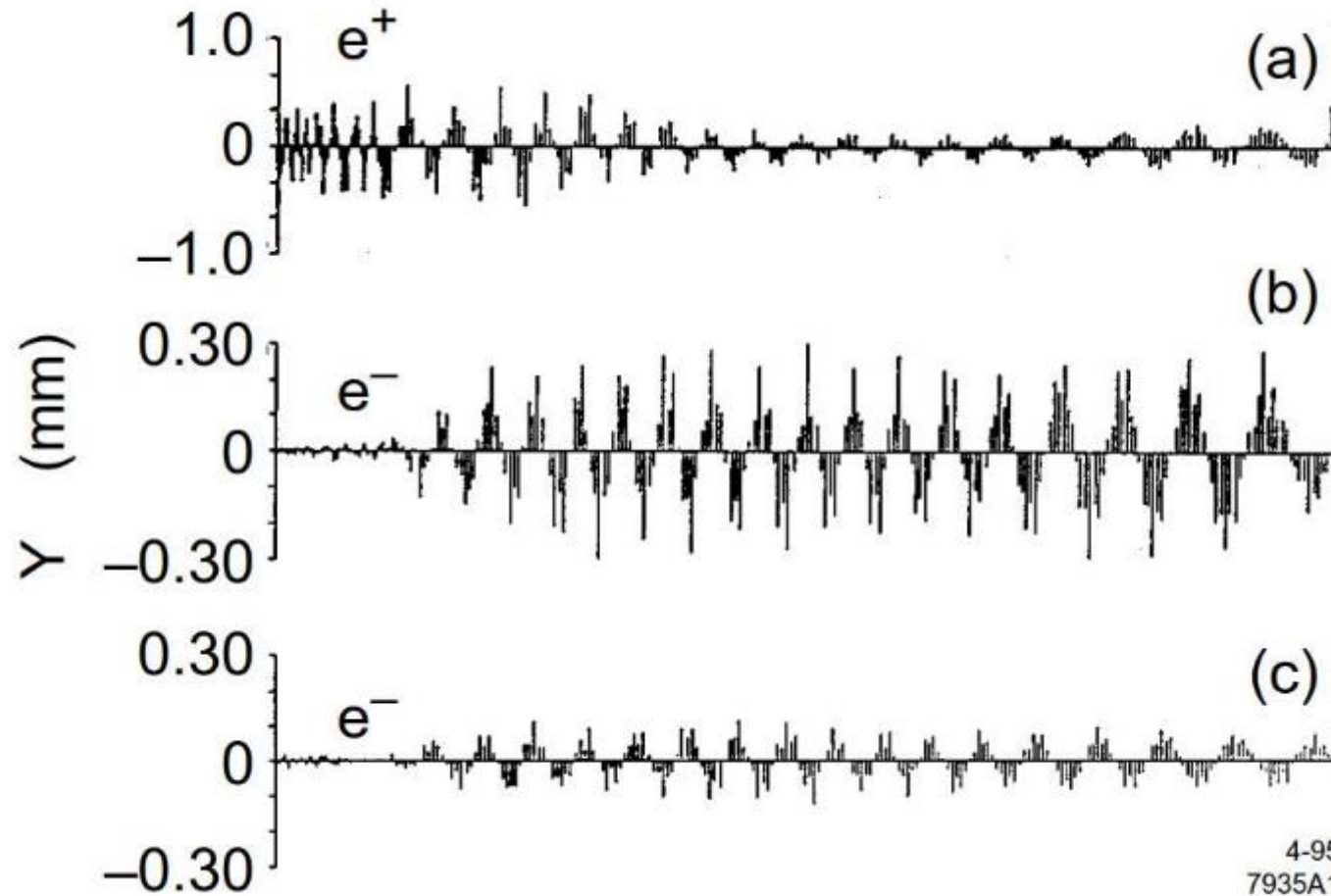
PRL acceptance as a function of phase advance per FODO cell



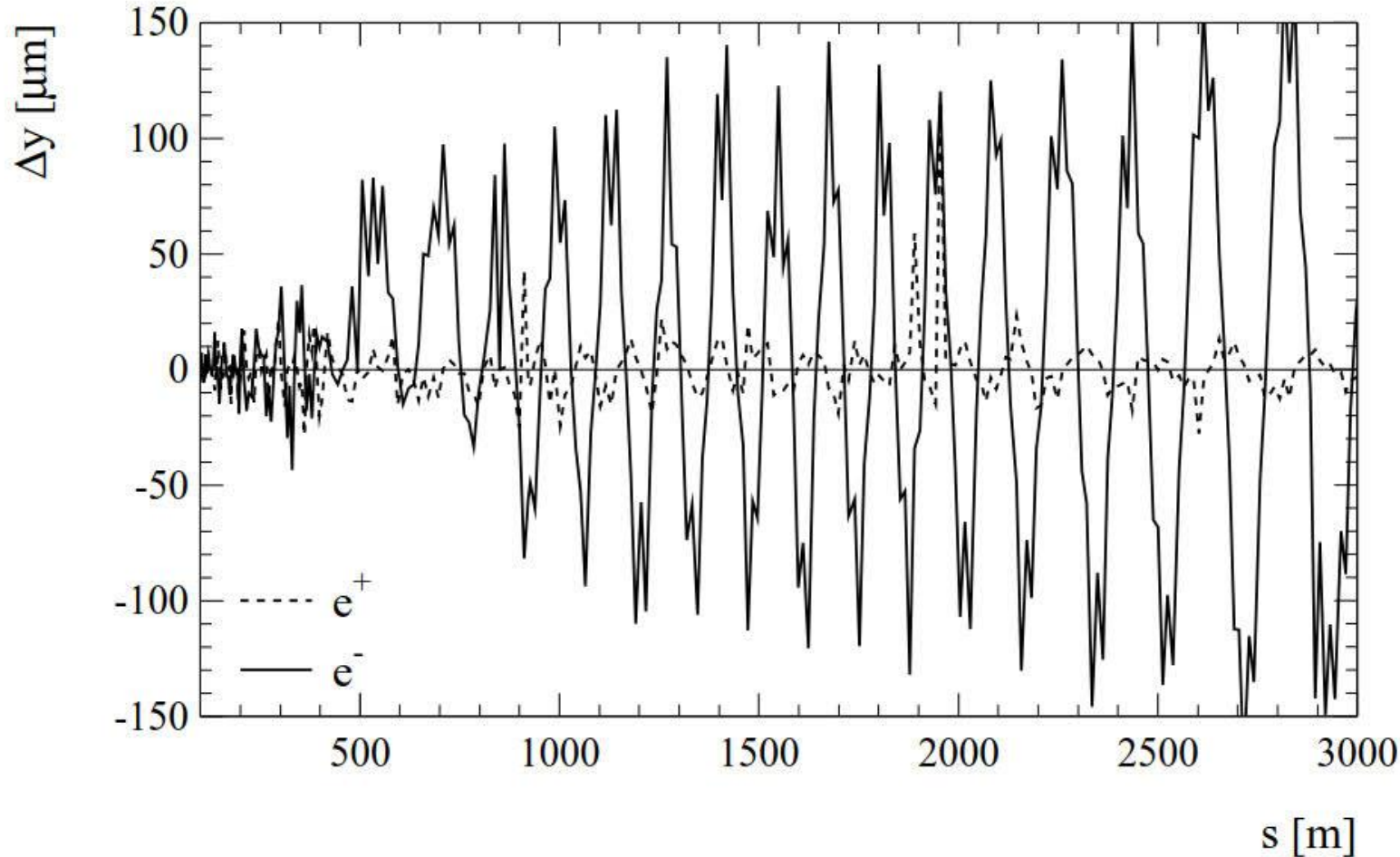
Transmission of the 75' lattice normalized to the transmission of the 90" lattice

phase advance of 90 degree / cell drives resonance

# e<sup>+</sup> and e<sup>-</sup> bunches in main linac

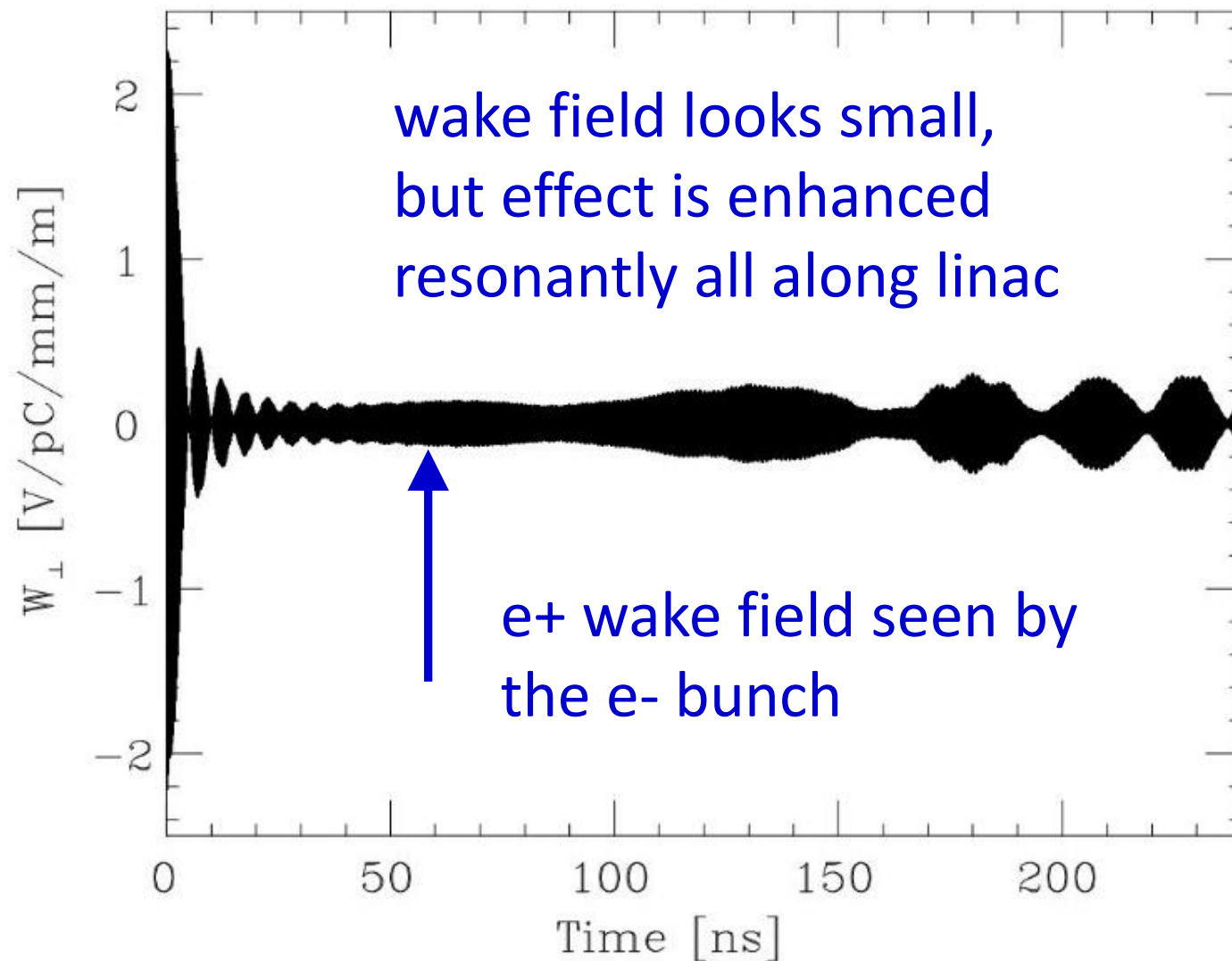


vertical  $e^+$  oscillation introduced before the linac (a) and the long range wakefield induced  $e^-$  oscillation (b) before and (c) after implementation of the split tune lattice



difference orbit in the electron beam by moving the leading positron bunch by one bucket ( $\sim 0.3$  ns) from 59 ns

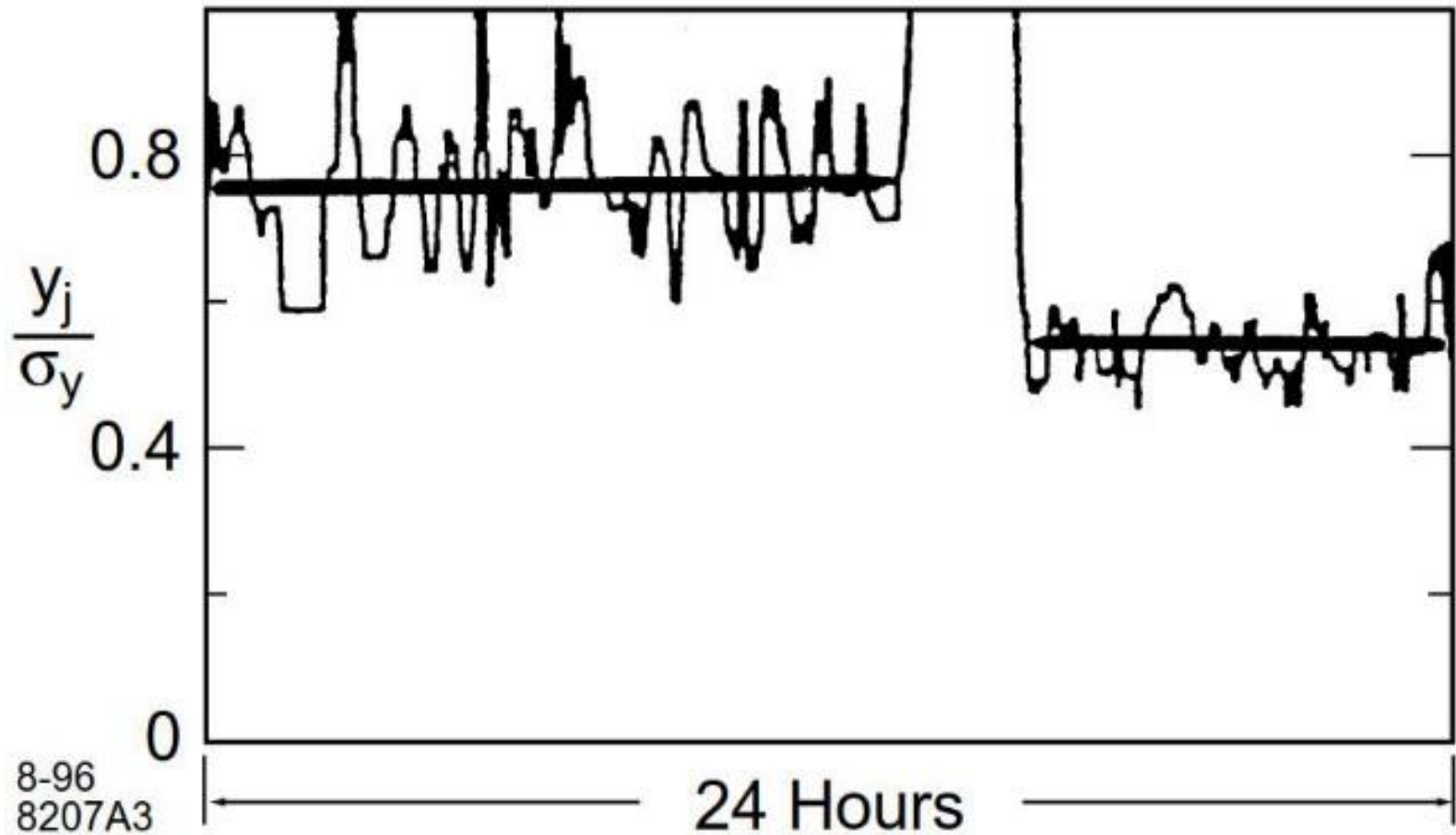




theoretical calculation of the transverse wakefield vs  
time for the lowest dipole modes of the SLAC structure

solution: split phase advances in x and y

$$\Delta\varphi_x^{e-} = \Delta\varphi_y^{e+}, \Delta\varphi_y^{e-} = \Delta\varphi_x^{e+} \text{ but } \Delta\varphi_{x(y)}^{e-} \neq \Delta\varphi_{x(y)}^{e+}$$



jitter reduction after the introduction of the split-tune lattice

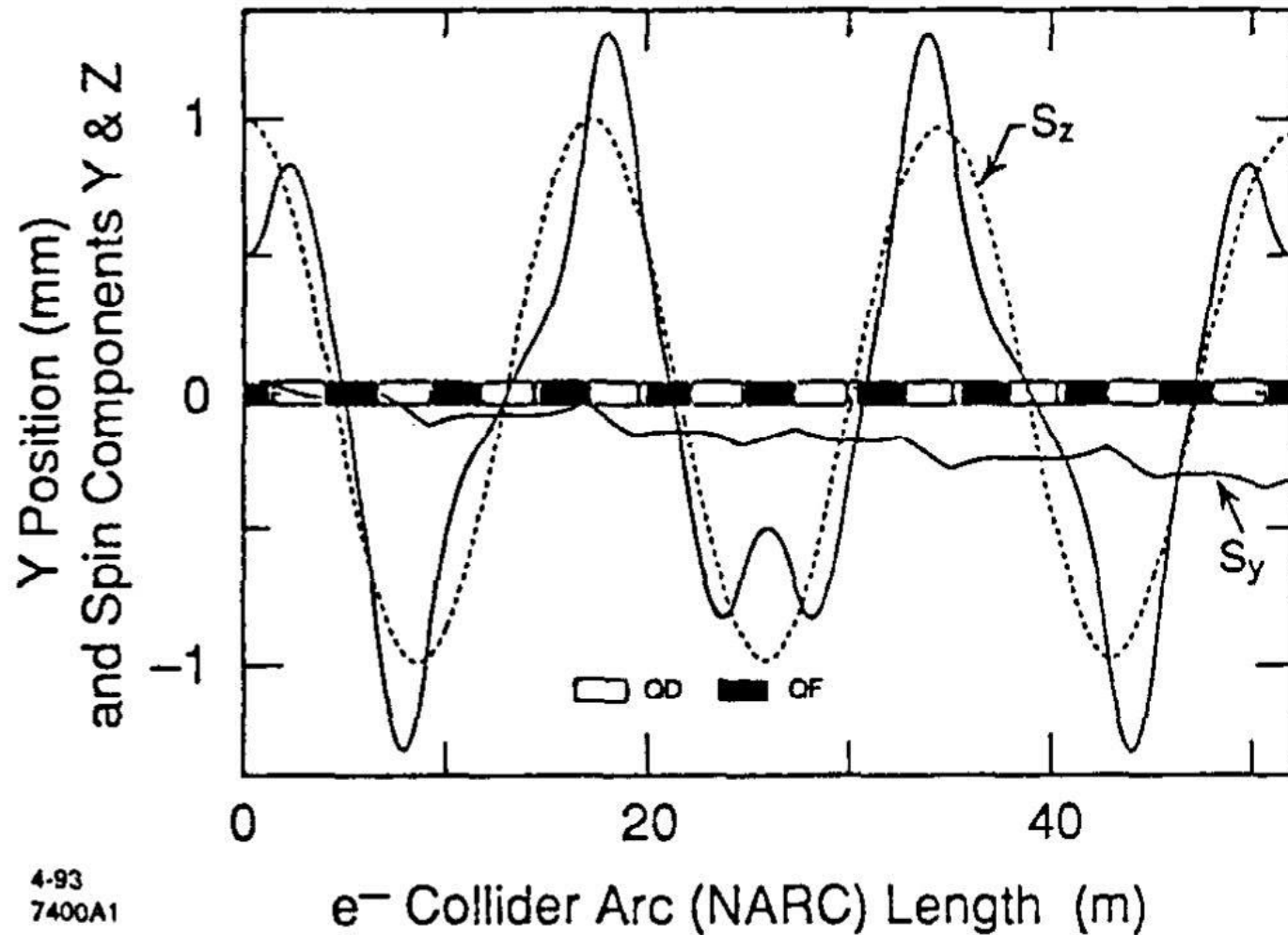
## spin-betatron resonance in the SLC North arc

if an electron is deflected in a transverse magnetic field by an angle  $\varphi$ , the spin is rotated around the field axis by

$$\phi = a\gamma\varphi$$

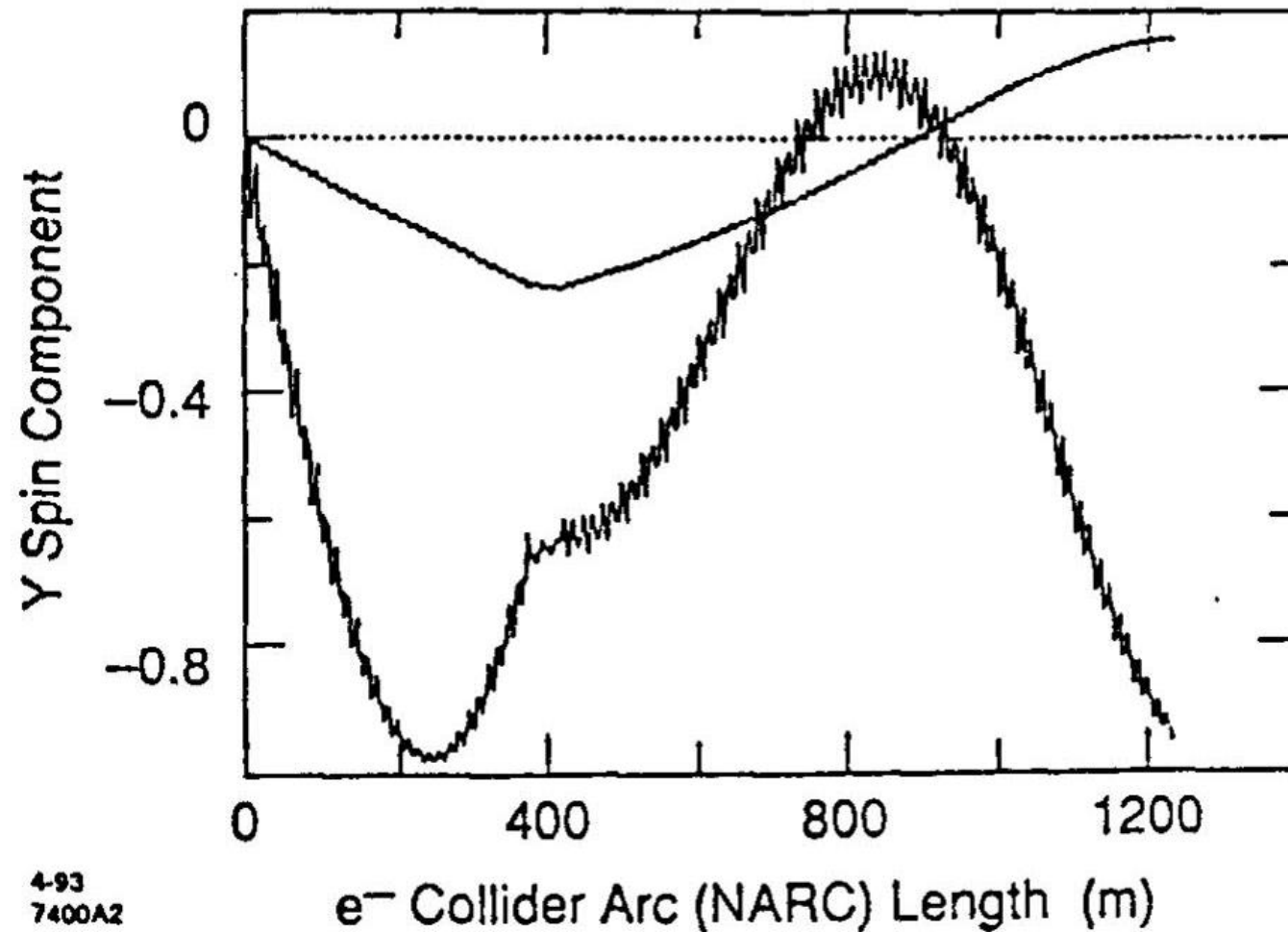
where  $a = \frac{\alpha}{2\pi} \approx 0.0011614$  is the anomalous magnetic moment of the electron and  $\gamma$  the Lorentz factor

at beam energy corresponding to peak Z boson production (45.60 GeV) spin phase advance  $\Delta\phi =$  vertical betatron phase advance (108 deg/cell)



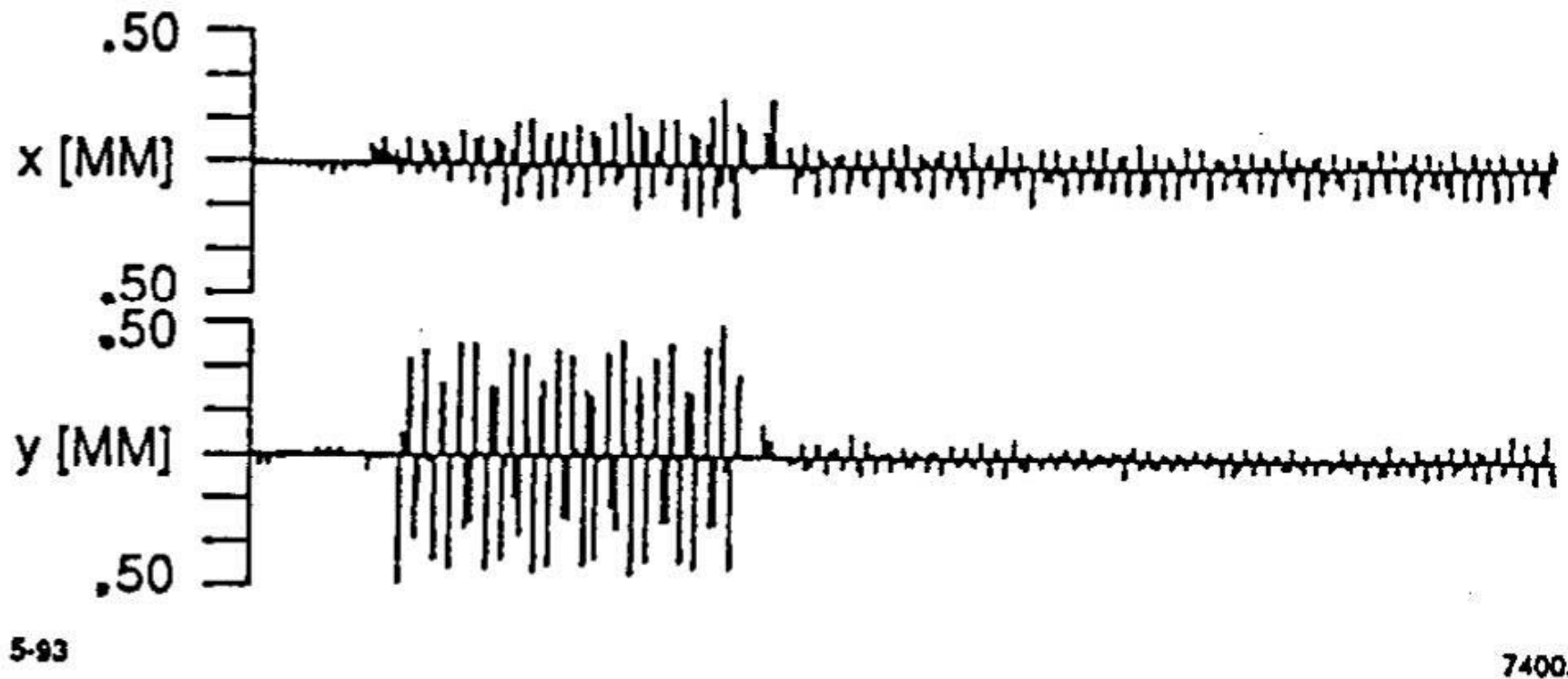
vertical orbit and vertical & longitudinal spin components over the first of twenty-three achromatic sections of the arc; the particle is launched with a vertical offset of 0.5 mm, the spin with longitudinal orientation

vertical spin component over entire arc for  
particles of 0.5 & 0.05 mm vertical launch offset

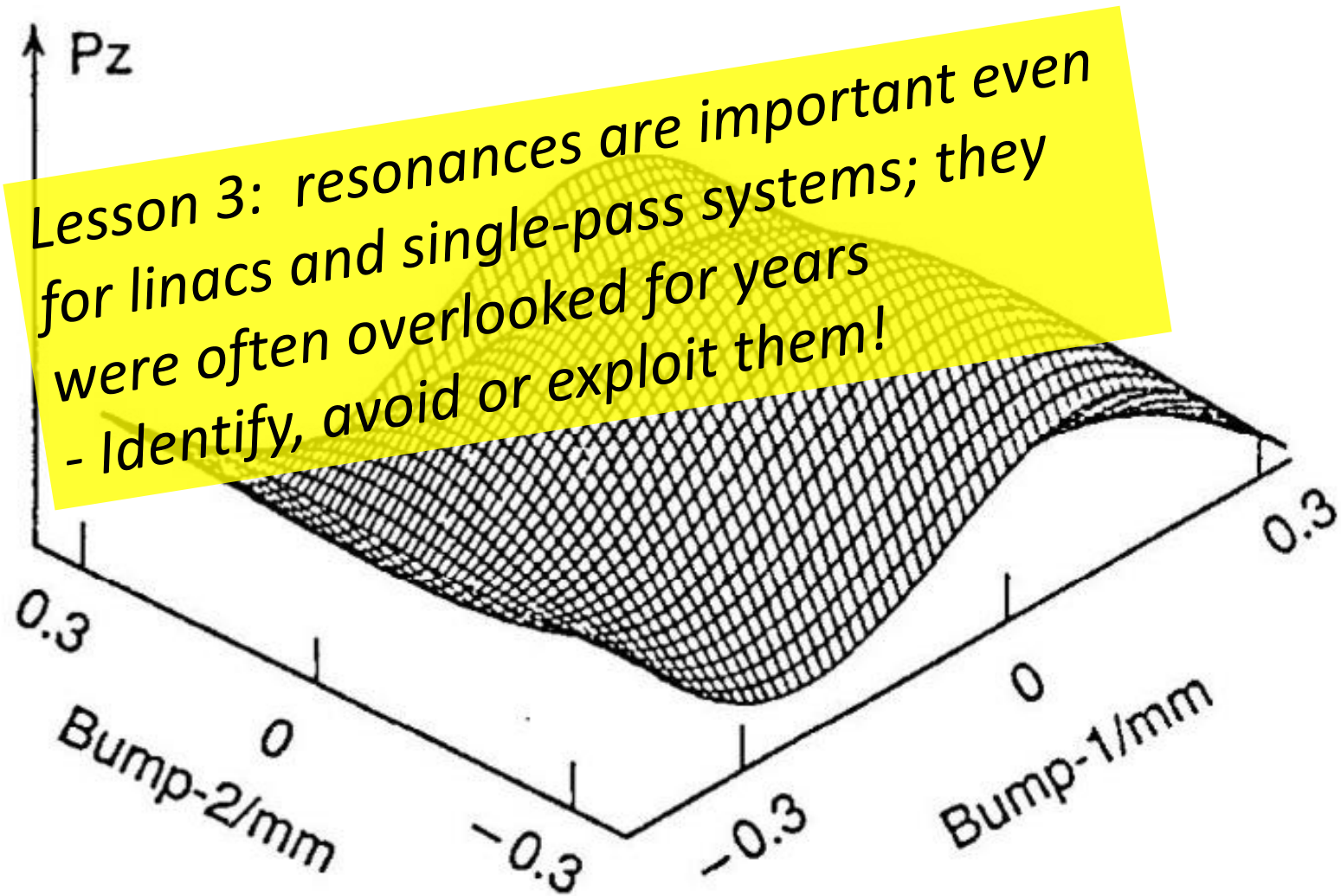




# IP polarization controlled with arc spin bumps

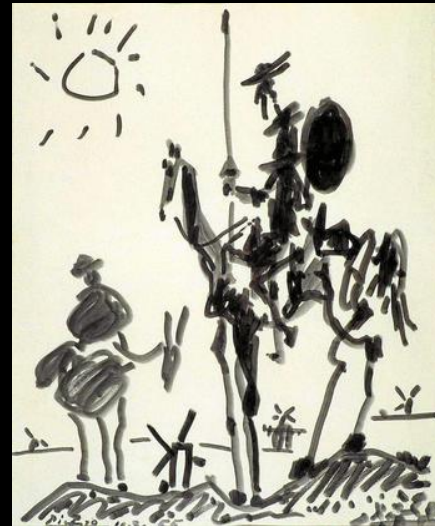


difference orbit in the North Arc showing typical spin bump; this bump rotates the spin by 60 degrees. Also note the x-y coupling due to arc rolls

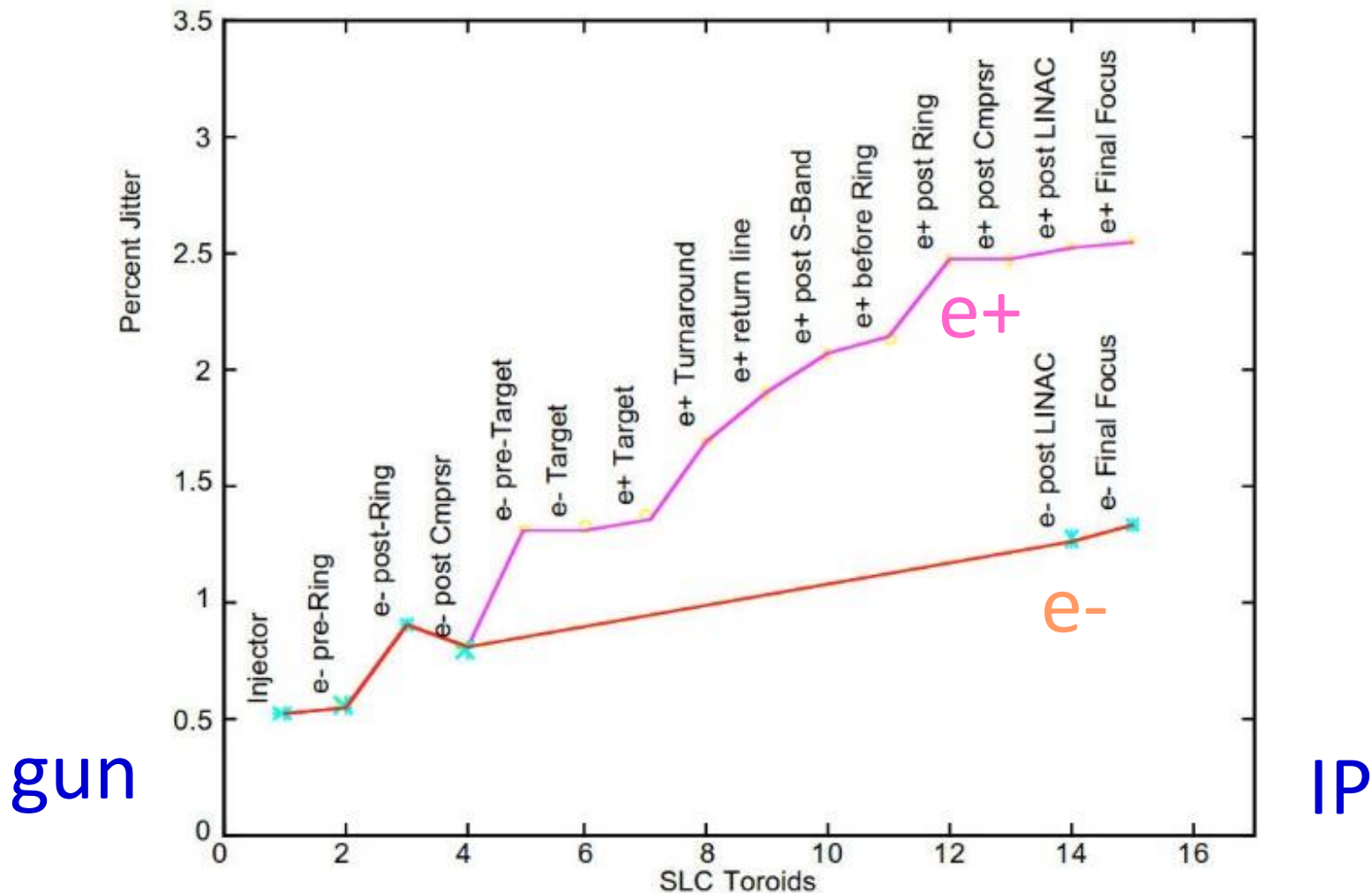


fit to 9-point grid scan with arc polarization bumps

... the SLC jitter wars ...



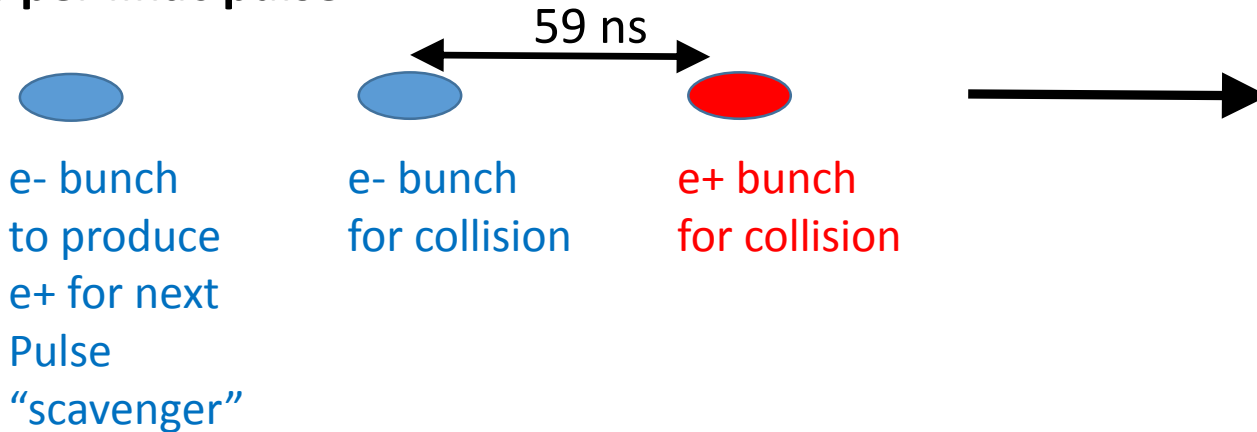
# rms intensity jitter (1994)



the value plotted at each toroid is the mean of the rms jitter recorded over the entire 1994 run; **in addition, beam orbit jitter  $\Delta y \geq \sigma_y/2$ , and also beam size variation of similar size**

# e<sup>+</sup> production instability

3 bunches per linac pulse



beam loading in the linac: each bunch extracts energy from the RF structures

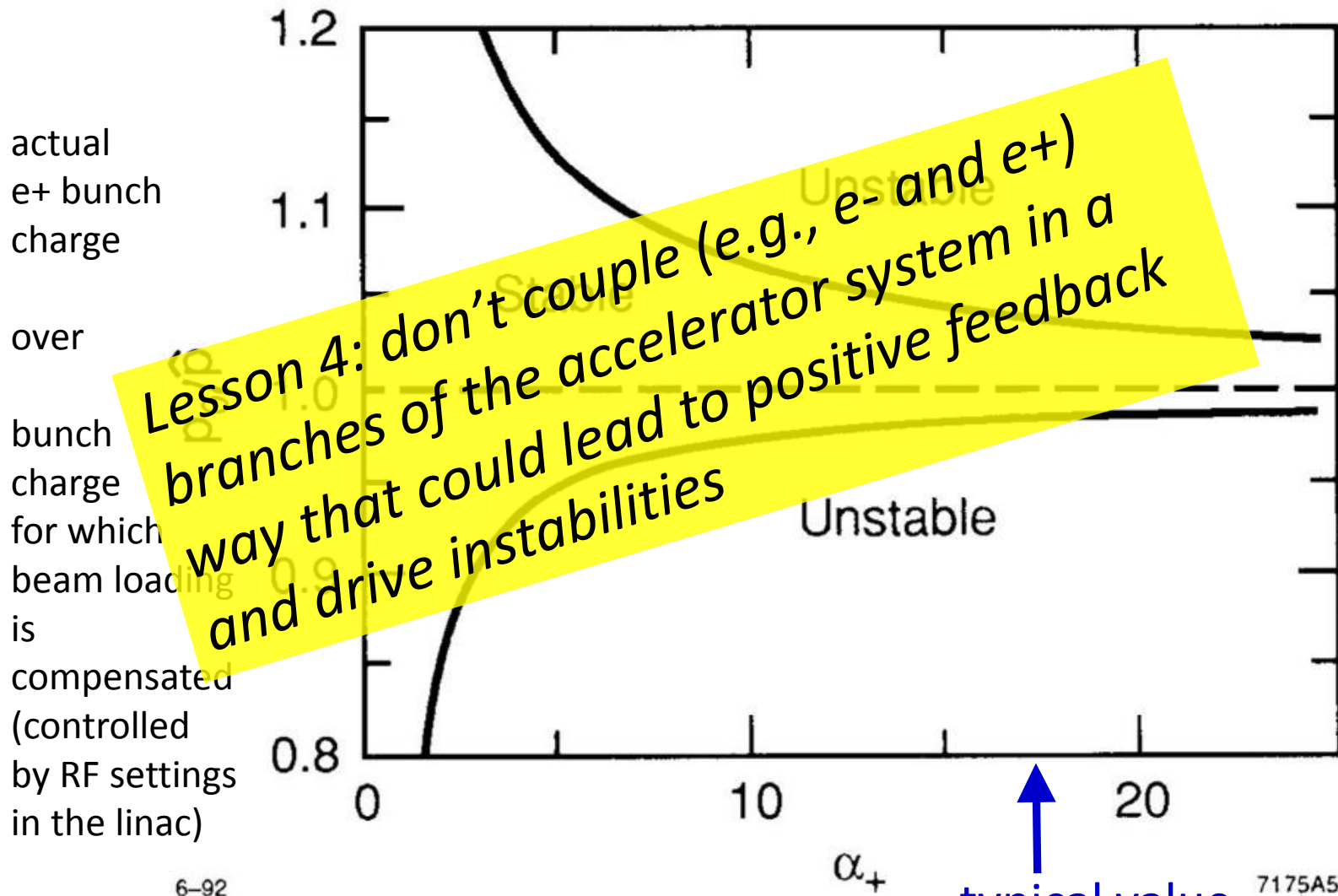
if  $N_{e^+}$  too high or too low, the e- bunch(es) will see too low or too high a field and will have the wrong energy at extraction, which will make them lose more particles in the energy-aperture limited line upstream of the e<sup>+</sup> production target  
→ next e<sup>+</sup> bunch will have low(er) intensity

$$\Delta N_{e^+} = \pm 1\% \rightarrow \Delta E_{e^-}/E_{e^-} = 3.3 \times 10^{-5} \rightarrow \Delta N_{e^+} = -0.2\% \text{ on next pulse, quadratic dependence}$$

$$\text{logistic equation: } x_n = 1 - Cx_{n-1}^2$$

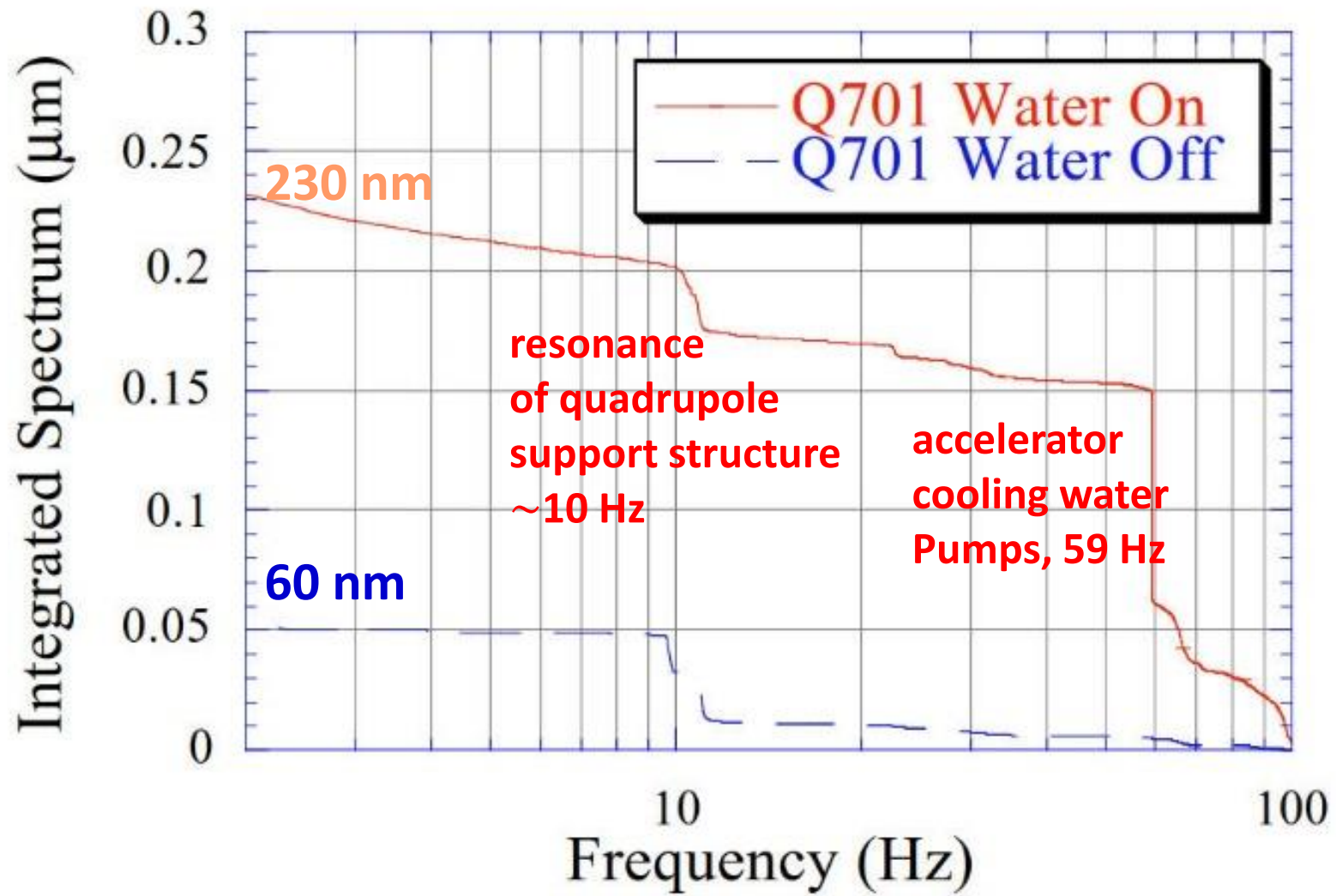


# e<sup>+</sup> production – local stability analysis

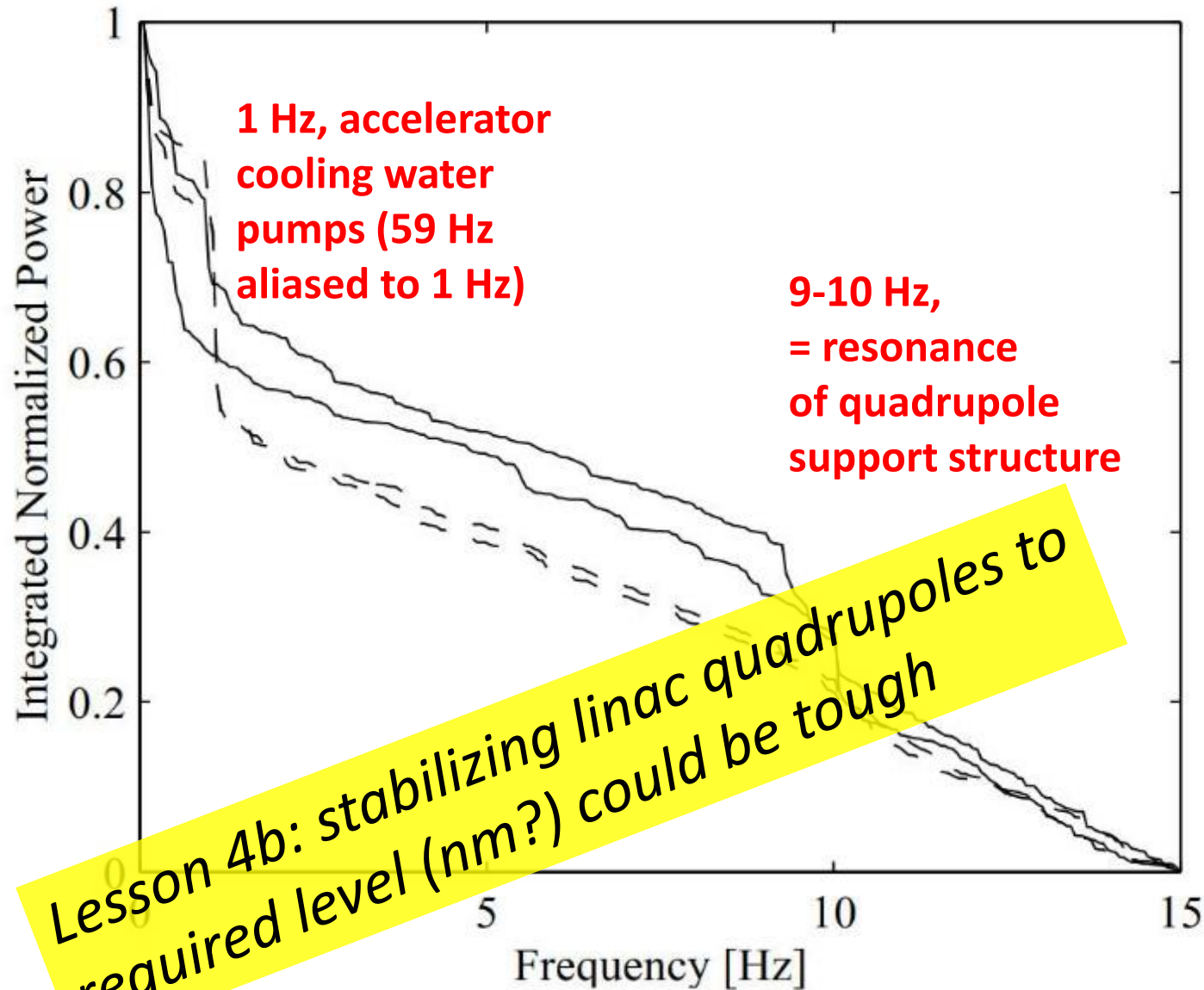


typical value  
for SLC

# linac quadrupole vibration



# power spectrum of linac beam orbit jitter

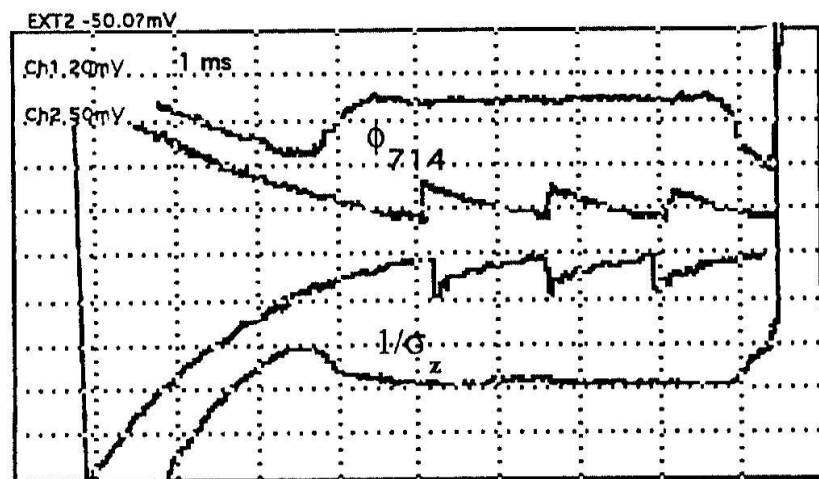


Lesson 4b: stabilizing linac quadrupoles to required level (nm?) could be tough

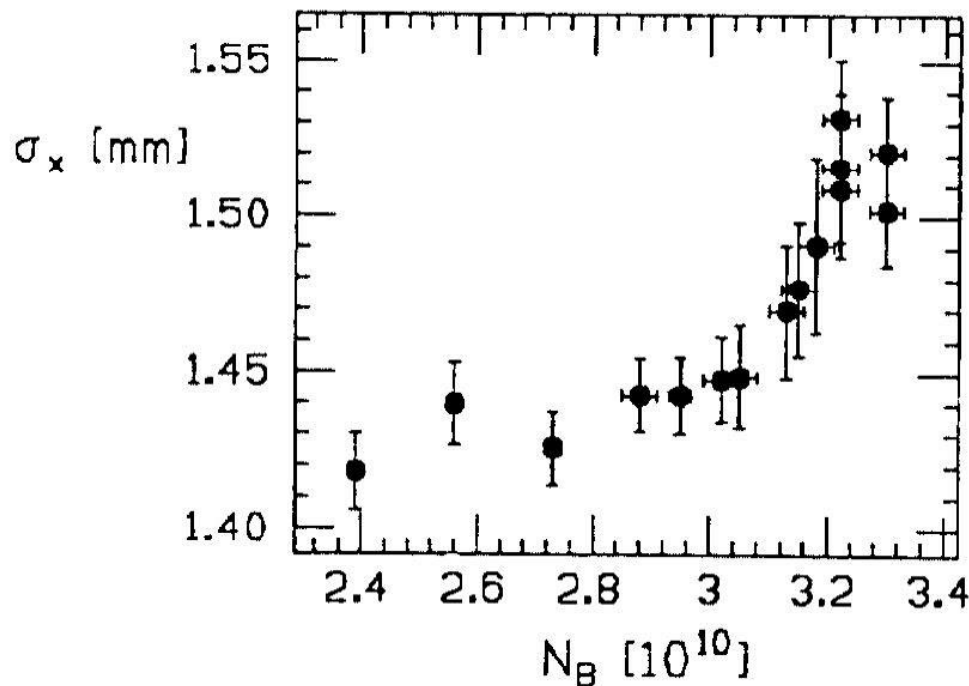
one *amplifier of jitter* was the linac wake field (already seen); one *random source* was the longitudinal microwave instability “sawtooth instability”) in the damping ring

P. Krejcik et al., PAC95

### 1993 data



inverse bunch length and beam phase signals exhibit sawtooth behavior during the instability



energy spread ( $\sigma_x$  at  $D \neq 0$ ) shows instability onset at  $N_{th} \sim 3 \times 10^{10}$

# SLC DR vacuum chamber upgrade 1994

Table 1. Vacuum Chamber Inductance (nH)

Element	Old Chamber <sup>†</sup>	New Chamber*
Synch. Radiation Masks	9.5	----
Bellows	----	1.1
Quadrupole to Dipole Chamber Transitions	9.3	2.4
Ion Pump Slots	9.2	1.05
Kicker Magnet Bellows	4.1	----
Flex Joints	3.6	----
Beam Position Monitors	3.5	0.2
Other	2.4	2.4
<b>TOTAL</b>	33	6

traditional wisdom

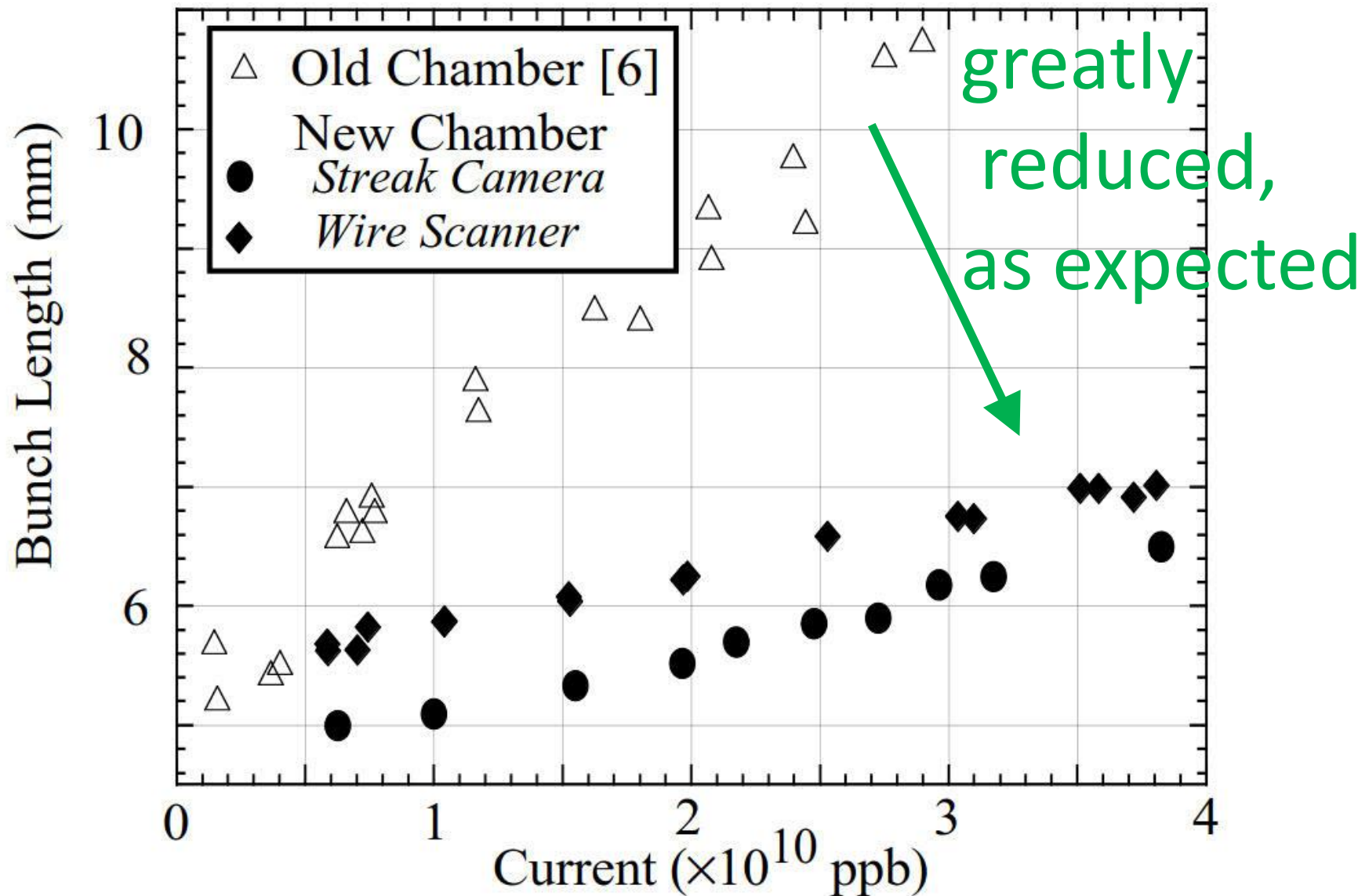
$$N_{th} \propto 1/Z$$

<sup>†</sup> From ref 1. Bellows included in Table 1 of [1] were shielded in a previous upgrade. Changes to that table from recent calculations are included here.

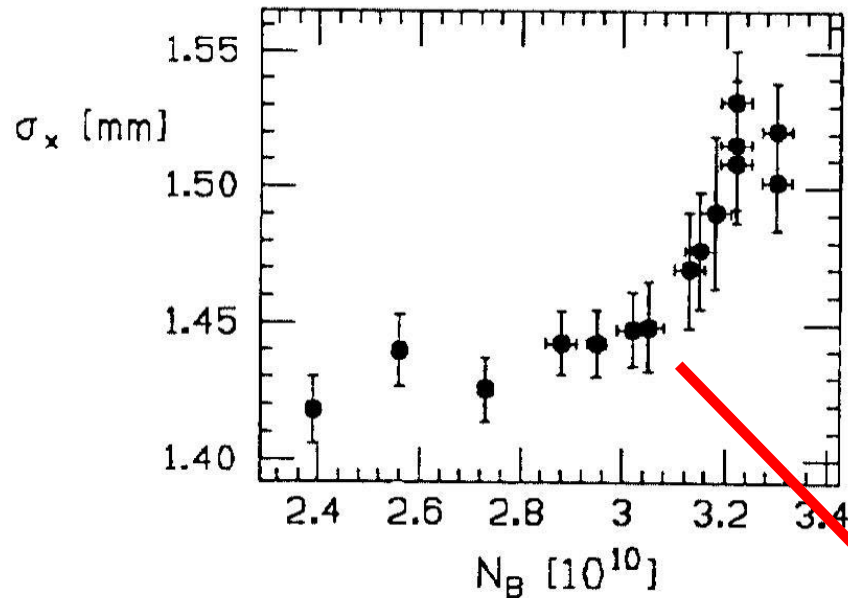
\* Many of the impedance calculations are in ref 2.



# bunch length dependence on current



# threshold of sawtooth instability



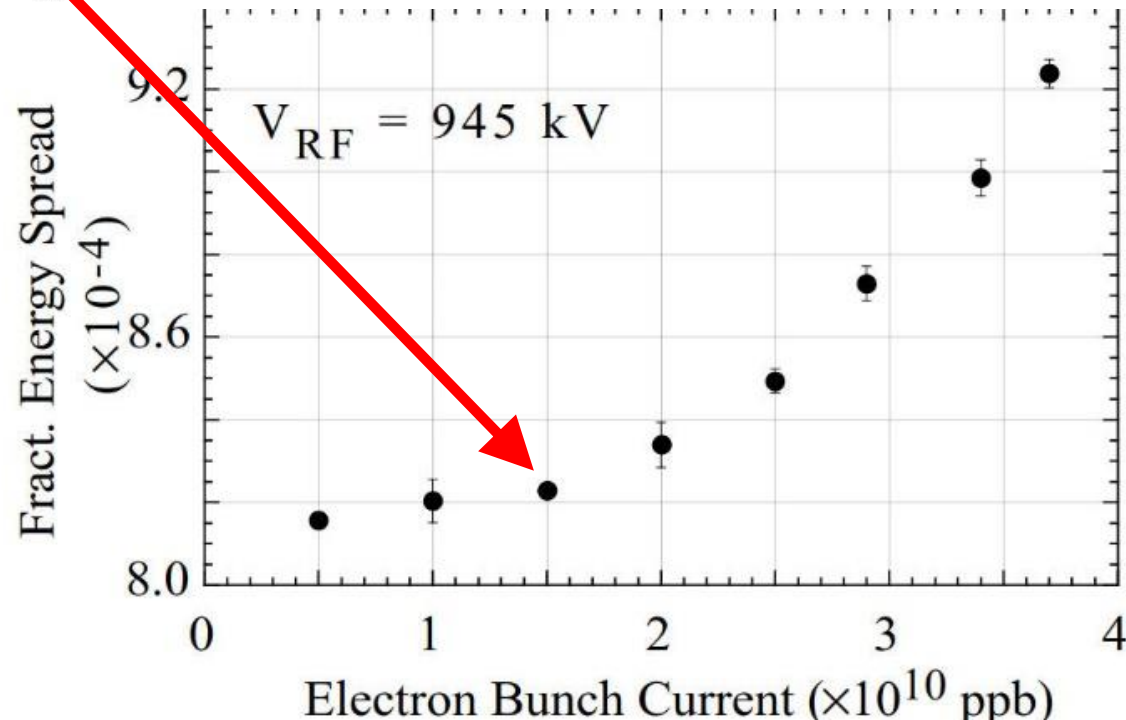
**1993 old chamber**

$$N_{th} \sim 3 \times 10^{10}$$

**threshold reduced!  
by  $\sim 1/2$ , not expected!?**

**1994 new chamber**

$$N_{th} \sim 1.5 \times 10^{10}$$



# K. Oide's theory had actually predicted that instability threshold decreases if ratio $\text{Re}Z/\text{Im}Z$ increases!



KEK Preprint 90- 10  
April 1990  
A



KEK Preprint 94-138  
November 1994  
A

Longitudinal Single-Bunch Instability in Electron Storage Rings

A Mechanism of Longitudinal  
Single-Bunch Instability in Storage Rings

formalism

KATSUNOBU OIDE

and

KAORU YOKOYA

KEK, National Laboratory for High Energy Physics  
Oho, Tsukuba, Ibaraki 305, Japan

ABSTRACT

A new method was investigated to obtain the threshold intensity of longitudinal single-bunch instability of an electron beam in a storage ring. The deformation of the equilibrium distribution in a bunch due to the wakefield induced by itself is included in this analysis. The results become significant for the comparison with those without deformation, and are compared with a single-particle tracking simulation.

PACS number(s): 41.80.+t, 41.80.Ee

\* Submitted to Phys. Rev. Lett.

prediction

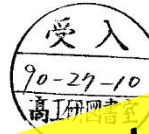
KATSUNOBU OIDE

KEK, National Laboratory for High Energy Physics  
Oho, Tsukuba, Ibaraki 305, Japan

ABSTRACT

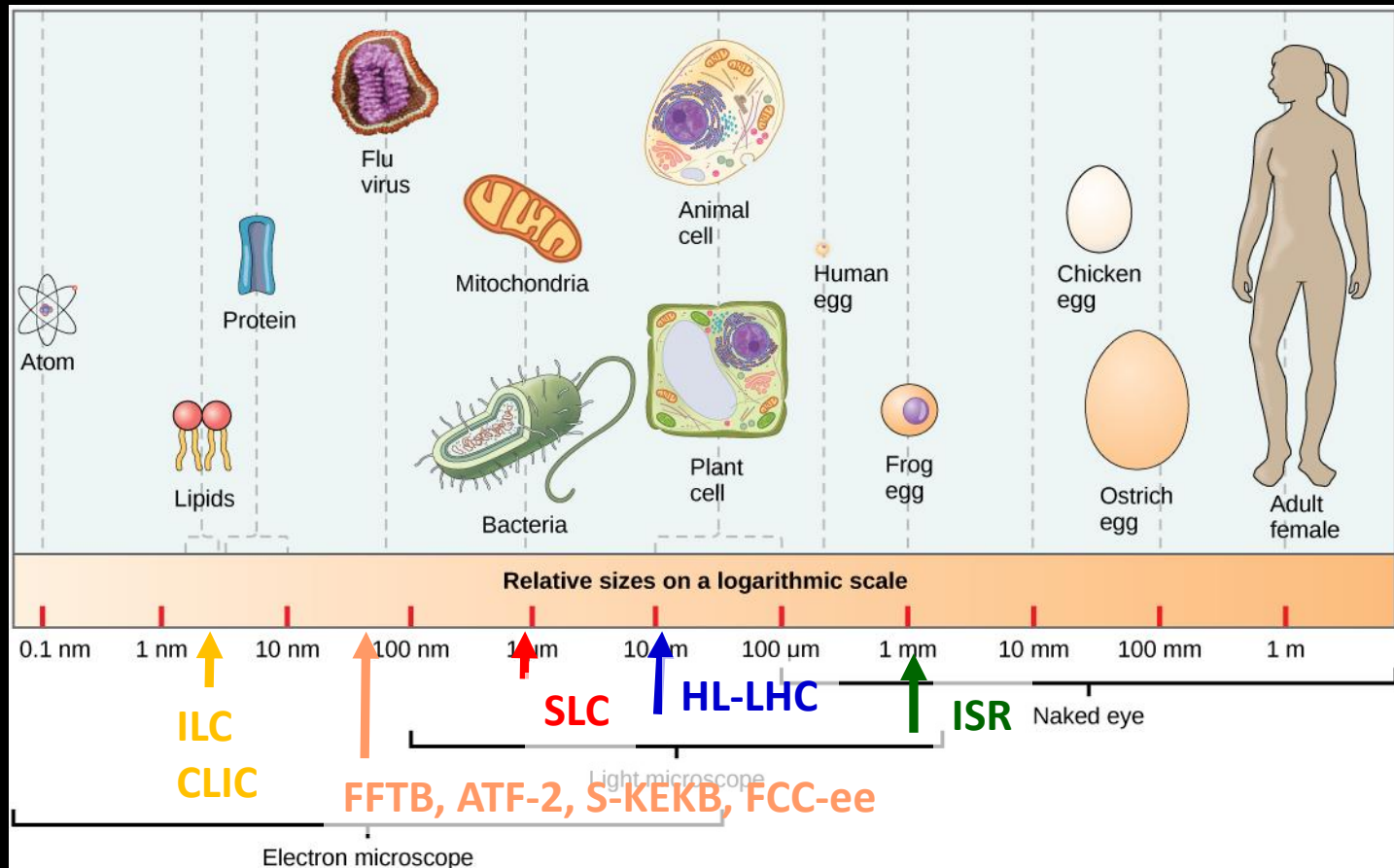
A new type of longitudinal single-bunch instability in storage rings is found. This instability is resulted from an interaction of two coherent synchrotron motions with different amplitudes. The frequency spread of incoherent synchrotron motion in a bunch generated by the potential-well distortion plays an essential role in this instability. The system becomes unstable when synchrotron frequencies of two different amplitudes degenerate. In an extreme case with the pure-resistive ( $\delta$ -function) wake potential, it is shown that the system is always unstable, i.e., the threshold intensity is zero.

Submitted to Particle Accelerators.

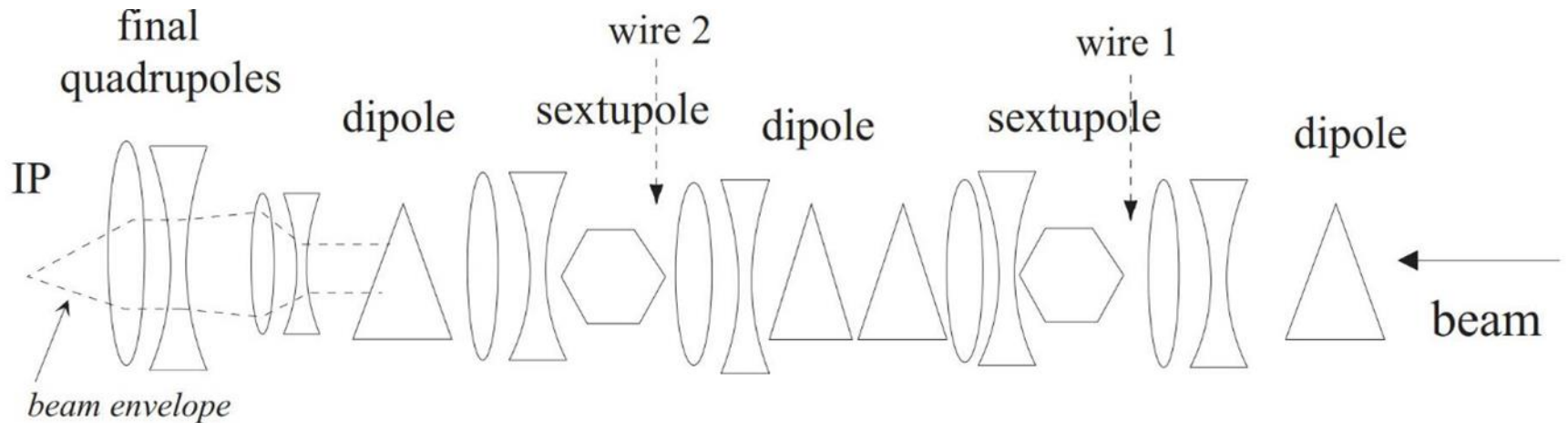


Lesson 5: cross-check, simulate, and prepare for the unexpected

# ... and the nanobeam challenge



# schematic of final focus system

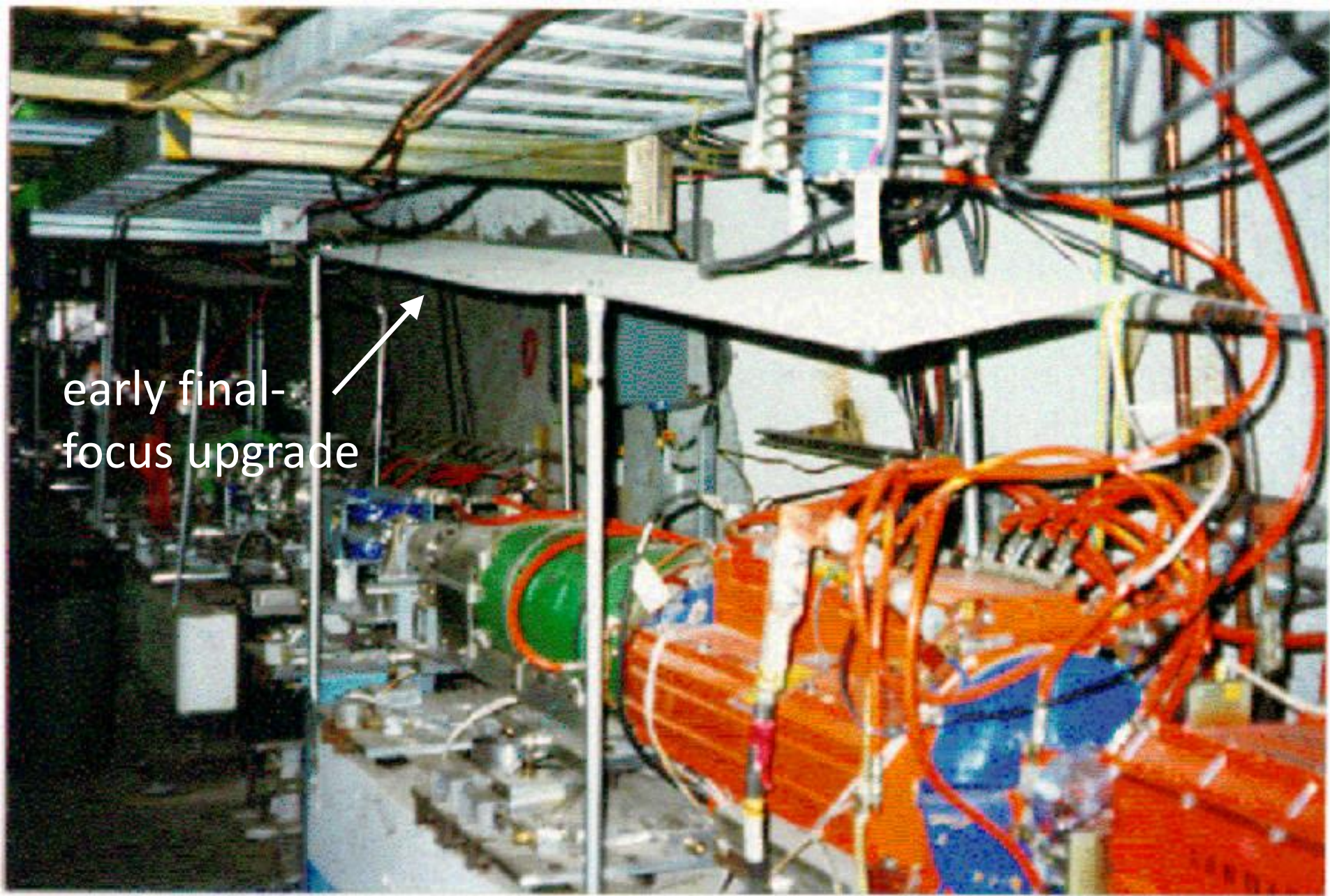


IP

final triplet

dipoles,  
quadrupoles,  
sextupoles, ...





early final-  
focus upgrade

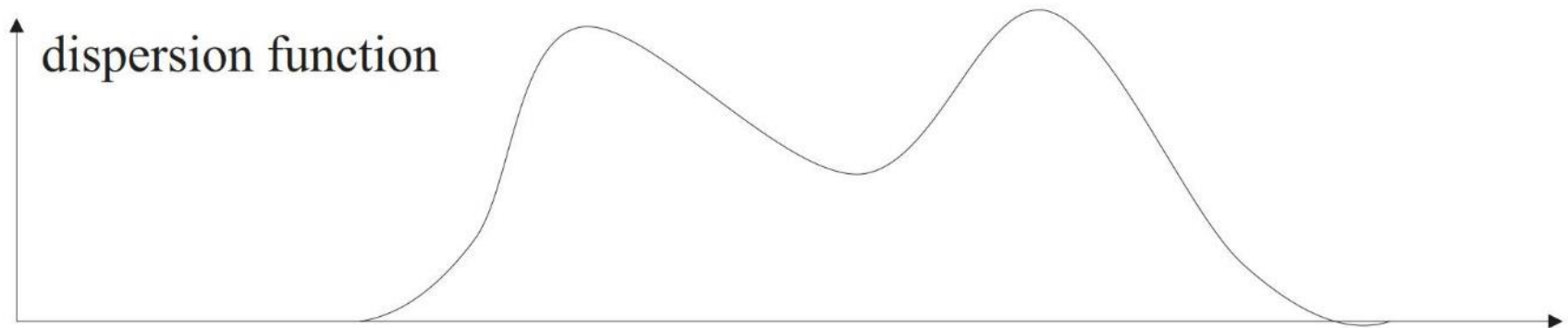
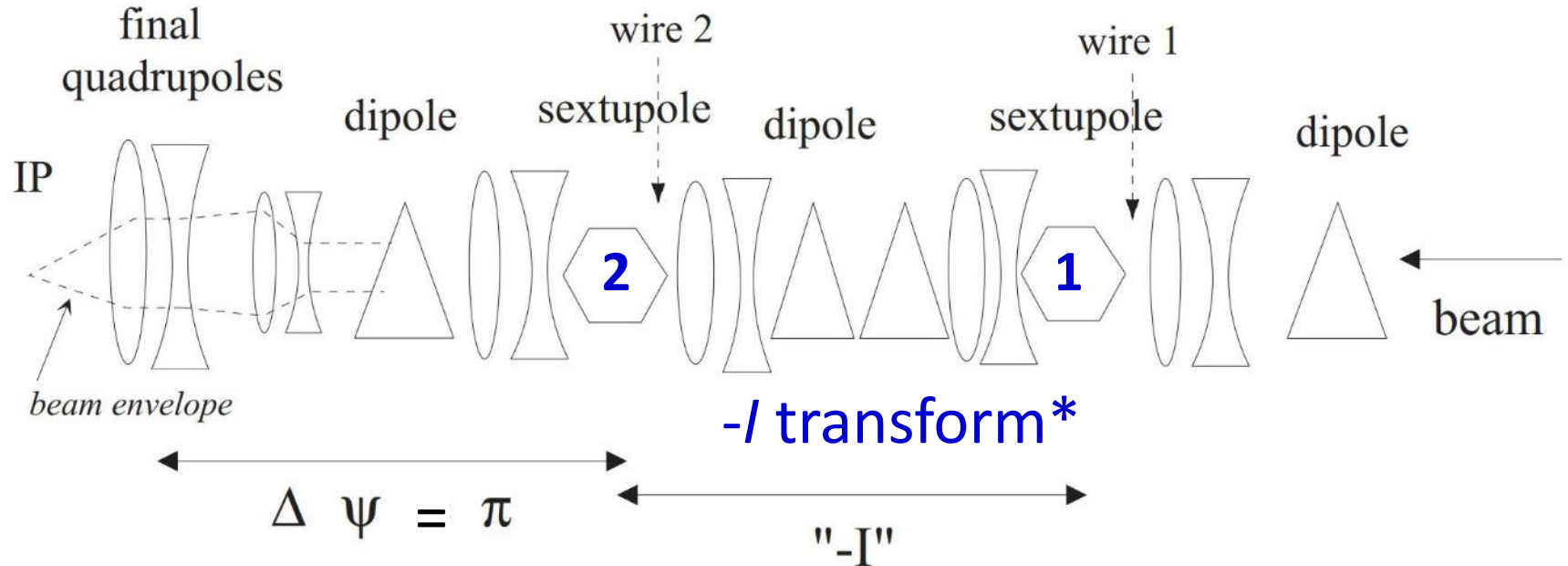
photograph of the SLC final focus, ~1996





Paul Emma and  
Olivier Napoly  
inspecting the  
spare  
superconducting  
final triplet of the  
SLC, in the SLD  
collider hall,  
1996

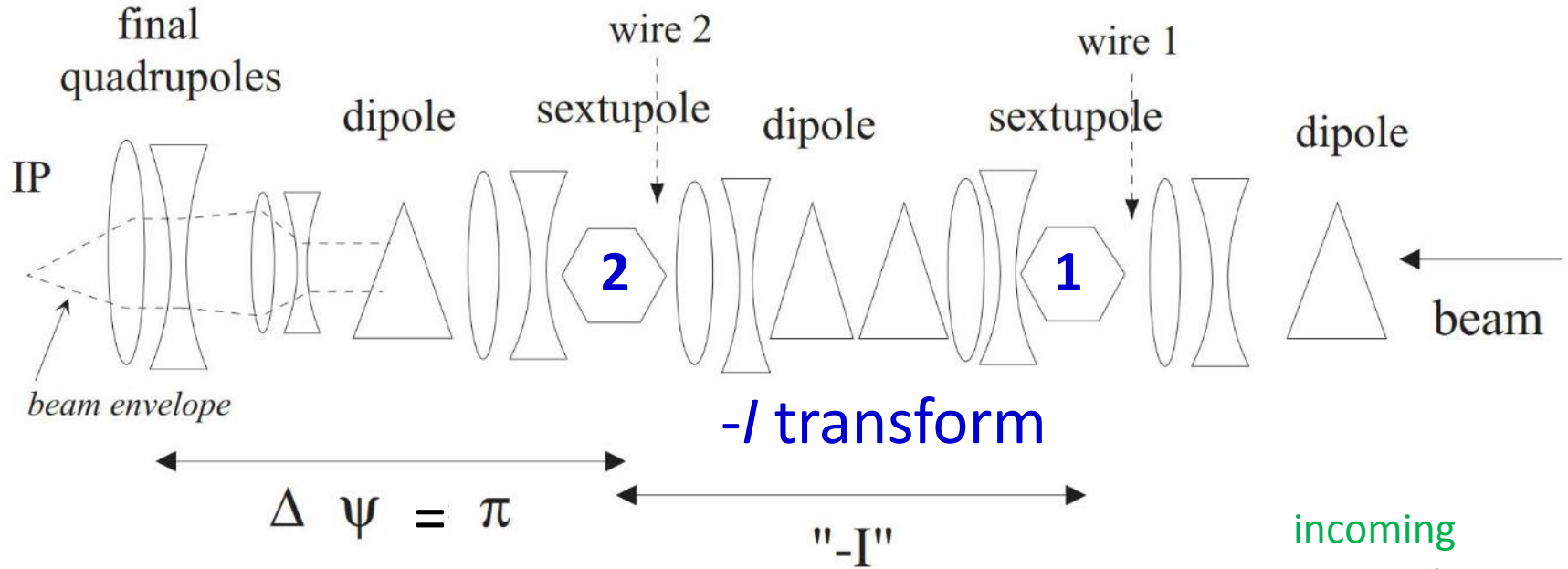
# schematic final focus system



$$x'_2 = -x'_\beta - \frac{1}{2}K_{s,1}(x_\beta^2 + 2x_\beta D_i \delta + D_i^2 \delta^2) + \frac{1}{2}K_{s,2}(x_\beta^2 - 2x_\beta D_i \delta + D_i^2 \delta^2)$$

\*patented by K. Brown

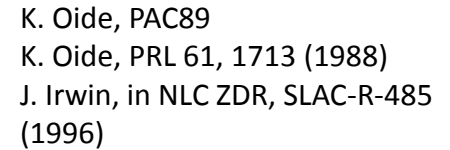
# chromatic correction



$$x'_2 = -x'_\beta - \frac{1}{2}K_{s,1}(x_\beta^2 + 2x_\beta D_i \delta + D_i^2 \delta^2) + \frac{1}{2}K_{s,2}(x_\beta^2 - 2x_\beta D_i \delta + D_i^2 \delta^2)$$

$$K_{s,1}=K_{s,2}: \quad x'_2 = -x'_\beta - 2K_s x_\beta D_i \delta.$$

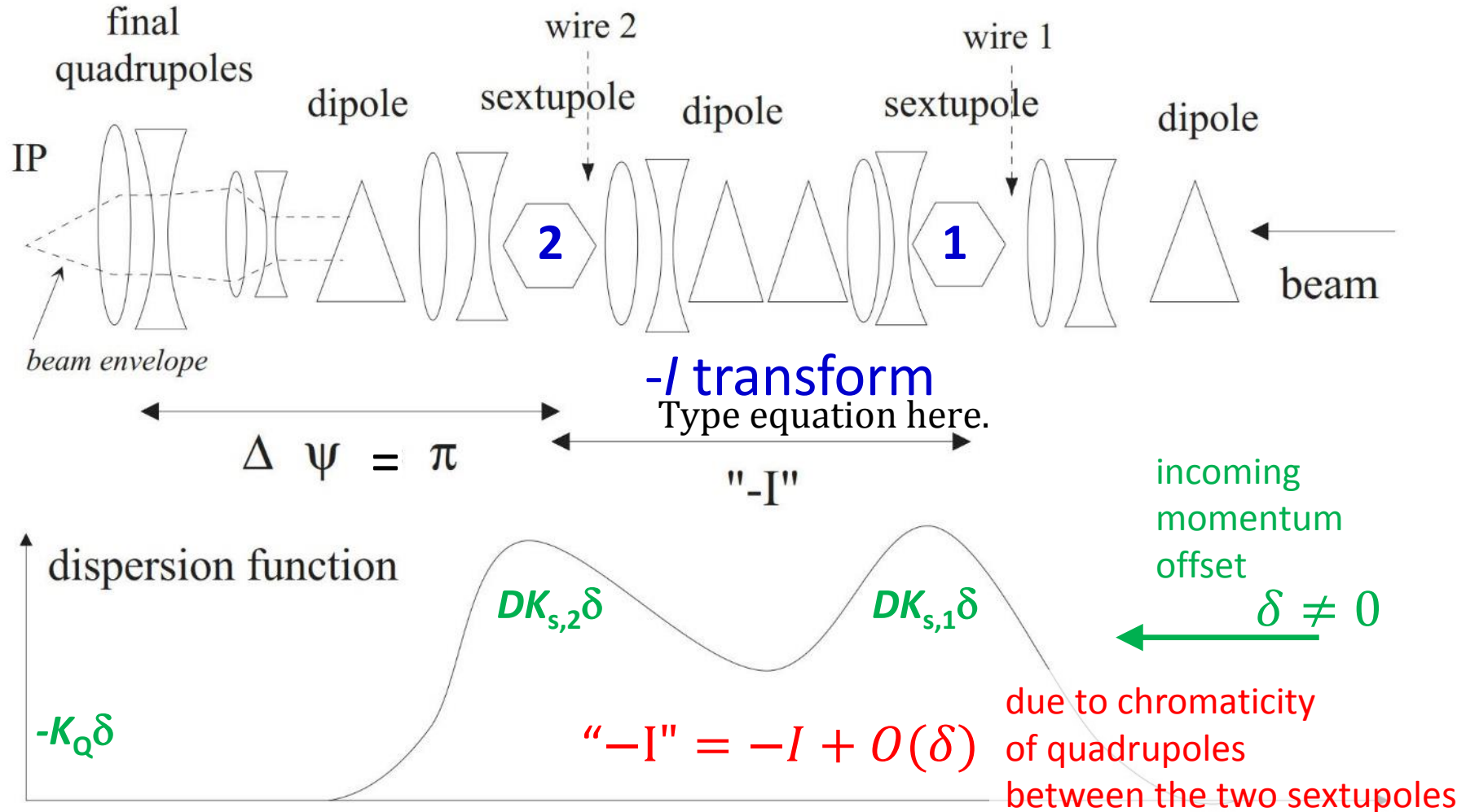
## “Oide effect”



energy loss due to SR in final quadrupole  
and final dipole(s) not chromatically corrected



# another problem: chromatic breakdown of -/



$$x'_2 = -x'_\beta - \frac{1}{2}K_{s,1}(x_\beta^2 + 2x_\beta D_i \delta + D_i^2 \delta^2) + \frac{1}{2}K_{s,2}(x_\beta^2 - 2x_\beta D_i \delta + D_i^2 \delta^2)$$

$$K_{s,1}=K_{s,2}: \quad x'_2 = -x'_\beta - 2K_s x_\beta D_i \delta + c_1 x x' \delta + c_2 x^3 \delta + \dots$$



# from “TRANSPORT” Description (SLC design)

## A First- and Second-Order Matrix Theory for the Design of Beam Transport Systems and Charged Particle Spectrometers



Karl L. Brown

SLAC Report-75

June 1982

Under contract with the  
Department of Energy  
Contract DE-AC03-76SF00515

```
*BEAM*      1.      299.79248 MEV
0.000 CM
0.000 2.000 N
0.000 3.000 R 0.000
0.000 4.000 N 0.000 0.000
0.000 5.000 R 0.000 0.000 0.000
0.000 6.000 N 0.000 0.000 0.000 0.000
0.000 7.000 N 0.000 0.000 0.000 0.000 0.000

*2ND ORDER* 17.      GAUSSIAN DISTRIBUTION 2. 2.
*EPS*      16.  "BETA"  1.  0.25000E+00
*BEND*      4.      74.05000 CM  10.00000 KG  0.50000 ( 100.000 CM , 42.428 DEG )
74.050 CM
*TRANSFORM 1*
0.86602 0.70712 0.00000 0.00000 0.00000 0.26796
-0.35356 0.86602 0.00000 0.00000 0.00000 0.70712
0.00000 0.00000 0.86602 0.70712 0.00000 0.00000
0.00000 0.00000 -0.35356 0.86602 0.00000 0.00000
-0.70712 -0.26796 0.00000 0.00000 1.00000 -0.06675
0.00000 0.00000 0.00000 0.00000 0.00000 1.00000

O*2ND ORDER TRANSFORM*
1 11 -6.101E-02
1 12 6.440E-01 1 22 1.220E-01
1 13 0.000E+00 1 23 0.000E+00 1 33 -2.992
1 14 0.000E+00 1 24 0.000E+00 1 34 3.158
1 15 0.000E+00 1 25 0.000E+00 1 35 0.000
1 16 3.749E-01 1 26 1.592E-01 1 36 0.000E

3 11 0.000E+00
3 12 0.000E+00 3 22 0.000E+00
3 13 -1.220E-01 3 23 -6.316E-02 3 33 0.000E+00
3 14 6.440E-01 3 24 2.440E-01 3 34 0.000E+00 3 44 0.000E+00
3 15 0.000E+00 3 25 0.000E+00 3 35 0.000E+00 3 45 0.000E+00 3 55 0.000E+00
3 16 0.000E+00 3 26 0.000E+00 3 36 1.131E-01 3 46 9.340E-02 3 56 0.000E+00 3 66 0.000E+00

2 11 1.610E-01
2 12 1.220E-01 2 22 -3.220E-01
2 13 0.000E+00 2 23 0.000E+00 2 33 -1.579
2 14 0.000E+00 2 24 0.000E+00 2 34 1.220
2 15 0.000E+00 2 25 0.000E+00 2 35 0.000
2 16 4.003E-01 2 26 -1.131E-01 2 36 0.000E

4 11 0.000E+00
4 12 0.000E+00 4 22 0.000E+00
4 13 1.579E-02 4 23 5.984E-03 4 33 0.000E+00
4 14 -1.280E-01 4 24 -3.158E-02 4 34 0.000E+00 4 44 0.000E+00
4 15 0.000E+00 4 25 0.000E+00 4 35 0.000E+00 4 45 0.000E+00 4 55 0.000E+00
4 16 0.000E+00 4 26 0.000E+00 4 36 3.384E-01 4 46 1.251E-01 4 56 0.000E+00 4 66 0.000E+00

5 11 -2.245E-04
5 12 -1.310E-01 5 22 -3.698E-01
5 13 0.000E+00 5 23 0.000E+00 5 33 -1.557E-02
5 14 0.000E+00 5 24 0.000E+00 5 34 1.190E-01 5 44 -3.057E-01
5 15 0.000E+00 5 25 0.000E+00 5 35 0.000E+00 5 45 0.000E+00 5 55 0.000E+00
5 16 -3.202E-02 5 26 -2.801E-01 5 36 0.000E+00 5 46 0.000E+00 5 56 0.000E+00 5 66 -1.814E-03

-14.050 18.969 N
-46.687 53.231 R 0.918
0.000 10.080 N 0.000 0.000
0.000 11.472 R 0.000 0.000 0.386
0.000 14.802 N 0.000 0.000 0.000 0.000
0.000 7.000 N 0.099 0.093 0.000 0.000 -0.032
```

“R-matrix”  
(linear)

“T-matrix”  
(second order)

+ “U-matrix” (3<sup>rd</sup> order, not shown)

# to “Lie algebra” description and Optimization (SLC final focus upgrade, N. Walker, J. Irwin, 1993)



John Irwin, The  
Application of Lie algebra  
techniques to beam  
transport design, Nucl.  
Instrum. Meth. A298  
(1990) 460-472

## The application of Lie algebra techniques to beam transport design \*

John Irwin

*Stanford Linear Accelerator Center, P O. Box 4349, Stanford University, Stanford, California 94309, USA*

Using a final focus system for high-energy linear colliders as an example of a beam transport system, we illustrate for each element, and for the interplay of elements, the connection of Lie algebra techniques with usual optical analysis methods. Our analysis describes, through fourth order, the calculation and compensation of all important aberrations.

### 1. Introduction

The techniques described here can be used for beamline design in quite general circumstances. We are introducing them for a final focus system design in linear colliders because this is where we have applied them and to provide a specific context for our discussion. Other optical systems may require modifications or extensions to the methods we introduce here. We have kept the formalism and mathematics to a bare minimum, hoping to simplify the presentation and clarify its connection with other methods. The territory we sketch is the tip of the iceberg of Lie algebraic methods. In the last section we describe briefly a broader context, though the interested reader will need to consult the extensive literature on this subject [1]. As far as we know the particulars we present here are original, however the essence of the method comes from Alex Dragt and collaborators.

### 2. Hamiltonians, kicks, and Poisson brackets

#### 2.1. Hamiltonian reminders

The elegant and powerful formulation of classical mechanics given by Hamilton is summed up in pairs of first order differential equations:

$$\frac{dx}{dt} = \frac{\partial H}{\partial p_x}, \quad \frac{dp_x}{dt} = -\frac{\partial H}{\partial x}. \quad (1)$$

The state of motion of a particle is described by giving a position and a momentum, which can be identified with a point in a  $2n$ -dimensional space of these variables, for  $n$  degrees of freedom. The velocity of this point in this space is prescribed by one function defined in this

space, the Hamiltonian, as indicated above. Thus one function determines completely the ensuing motion once initial conditions are specified.

The Hamiltonian function for the motion of particles in magnetic optical elements can be derived, after introduction of the appropriate coordinates and approximations, from the Hamiltonian for motion in a general electromagnetic field. This procedure has been described in many places [2]. The principal elements in our beamline consist of static transverse magnetic fields. If one chooses the distance  $s$  measured along the design orbit as the time-like variable, then within uniform elements with no dipole field the Hamiltonian can be transformed to

$$H = \left[ p^2 - p_x^2 - p_y^2 \right]^{1/2} - \frac{e}{c} A_s(x, y), \quad (2)$$

where  $p_x$  and  $p_y$  are the transverse components of the momentum,  $p$  is the total momentum of the particle,  $e$  is the electronic charge,  $c$  is the speed of light and  $A_s$  is the magnetic vector potential in the  $s$  direction.

For  $p$  much greater than  $p_x$  or  $p_y$  the square root can be expanded in powers of  $(p_x^2 + p_y^2)/p^2$  as follows:

$$H = p \left( 1 + \frac{1}{2} \left( \frac{p_x^2 + p_y^2}{p^2} \right) - \frac{1}{8} \left( \frac{p_x^2 + p_y^2}{p^2} \right)^2 + \dots \right) - \frac{e}{c} A_s. \quad (3)$$

For particles of constant  $p$  the first term can be dropped and the transverse momentum variables changed from  $p_x$  and  $p_y$  to  $p_x/p \equiv x' (\approx dx/ds)$  and  $p_y/p \equiv y'$ . The latter is a scale transformation for which Hamilton's equations remain intact if  $H$  is scaled by  $p$ . The new  $H$  is

$$H = \frac{1}{2} (x'^2 + y'^2) - \frac{1}{8} (x'^2 + y'^2)^2 + \dots - \frac{e}{pc} A_s. \quad (4)$$

The scale transformation we performed here is not appropriate if the total energy is needed as a dynamical variable for the third degree of freedom. However, since

\* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

# reminder of classical mechanics

Hamiltonian  $H$

equations of motion  $\frac{dx}{ds} = \frac{\partial H}{\partial p_x} \quad \frac{dp_x}{ds} = -\frac{\partial H}{\partial x}$

consider a function  $U(x, p_x)$ ;

change of  $U$  as particle move around in phase space

$$\frac{dU}{ds} = \frac{dU}{dx} \frac{dx}{ds} + \frac{dU}{dp_x} \frac{dp_x}{ds} = \frac{dU}{dx} \frac{\partial H}{\partial p_x} - \frac{dU}{dp_x} \frac{\partial H}{\partial x} \equiv [U, H] = -[H, U]$$

in particular

$$\frac{dx}{ds} = -[H, x] \quad \frac{dp_x}{ds} = -[H, p_x] \quad \frac{dy}{ds} = -[H, y] \quad \frac{dp_y}{ds} = -[H, p_y]$$

integration over a magnet / element from  $s_1$  to  $s_2 = s_1 + L$

$$x(s_1 + L) = \sum_{n=0}^{\infty} \frac{L^n}{n!} \frac{d^n x}{ds^n} = \exp\left(L \frac{d}{ds}\right) x \equiv \exp(L : -H : ) x \Big|_{x=x_1, p_x=p_{x1}, \dots}$$

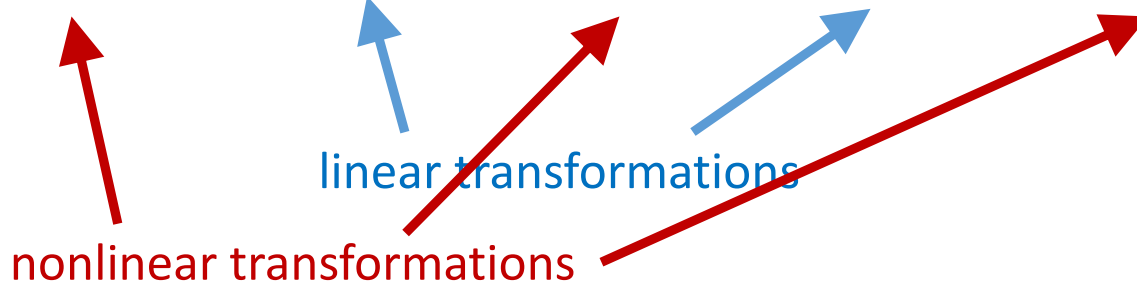
where  $: -H :^n x = [-H, [-H, \dots, [-H, x]] \dots]$

in the following I will drop the colons

# Lie algebra illustration ....

beam line with Hamiltonians in local coordinates

$$M = \dots \exp\left(-H_i(x_i, p_{xi}, y_i, p_{yi}, \delta)\right) R_{ji} \exp\left(-H_j(x_j, p_{xj}, y_j, p_{yj}, \delta)\right) R_{kj} \exp\left(-H_k(x_k, p_{xk}, y_k, p_{yk}, \delta)\right) \dots$$



example: sextupole Hamiltonian

$$H_s(x_s, p_{xs}, y_s, p_{ys}, \delta) = \frac{1}{6} K_s (x_s^3 - 3x_s y_s^2) \left(1 - \underbrace{\frac{\delta}{1 + \delta}}_{\equiv \bar{\delta}}\right)$$

“kicks” from sextupole

$$\Delta p_{xs} = [-H_s, p_{xs}] = -\frac{\partial H_s}{\partial x_s} = -\frac{1}{2} K_s (x_s^2 - y_s^2) (1 - \bar{\delta})$$

$$\Delta p_{ys} = [-H_s, p_{ys}] = -\frac{\partial H_s}{\partial y_s} = K_s x_s y_s (1 - \bar{\delta})$$



beam line with Hamiltonians in IP coordinates

“Poisson bracket”:

$$[a, b] \equiv \frac{\partial a}{\partial x} \frac{\partial b}{\partial p_x} - \frac{\partial a}{\partial p_x} \frac{\partial b}{\partial x}$$

$$M \approx \exp\left(-H_{\text{NL,tot},*}(x^*, p_x^*, y^*, p_y^*, \delta)\right) R_{0,\text{init}}$$

the regrouping is achieved with the help of two transformations:

similarity transformation

$$\exp A \exp B \exp(-A) = \exp C \text{ where } C = (\exp A)B$$

Campbell-Baker-Hausdorff formula\*

$$\exp A \exp B = \exp C$$

where  $C = A + B + \frac{1}{2}[A, B] + \text{higher order terms}$

*“heart of the usefulness of Lie algebra approach”*

\*Henry Baker: Proc Lond Math Soc (1) 34 (1902) 347–360; ibid (1) 35 (1903) 333–374; ibid (Ser 2) 3 (1905) 24–47.  
John. Campbell: Proc Lond Math Soc 28 (1897) 381–390; ibid 29 (1898) 14–32.  
Felix Hausdorff: Berl Verh Saechs Akad Wiss Leipzig 58 (1906) 19–48.

chromatic breakdown of  $-/$  can be described by a similarity transformation

$-/$  transform between sextupoles



$$\exp\left(-\frac{1}{2}K_s\beta_{x,s}^{1/2}\beta_{y,s}\bar{x}'^*\bar{y}'^{*2}\right)\exp\left(\frac{1}{2}\bar{\delta}K_q\beta_{y,q}\bar{y}^{*2}\right)\exp\left(\frac{1}{2}K_s\beta_{x,s}^{1/2}\beta_{y,s}\bar{x}'^*\bar{y}'^{*2}\right)$$

intermediate chromaticity in IP phase

$$\text{where } \bar{x}^* \equiv x^*/\sqrt{\beta_x^*}, \bar{y}^{*2} \equiv \sqrt{\beta_x^*}x'^*$$

→ leading new term in total

$$\left[-\frac{1}{2}K_s\beta_{x,s}^{1/2}\beta_{y,s}\bar{x}'^*\bar{y}'^{*2}\right]$$

Lesson 6: Lie algebra methods are a great approach for understanding final-focus aberrations and their interaction

chromogeometric aberration affecting  $\sigma_y$  and depending on  $\beta_x^*$ !



effect of aberrations on IP beam size

$$\Delta y^* \approx \frac{\partial H_{\text{NL,tot},*}}{\partial p_y^*}$$

and

$$\Delta \sigma_y^{*2} \approx \langle (\Delta y)^2 \rangle - \langle \Delta y \rangle^2$$

$\langle \dots \rangle$ : average over (initial/linearly transformed) bunch distribution, often assumed as Gaussian

$$\langle y^2 \rangle = \sigma_y^2 \quad \langle y^4 \rangle = 3\sigma_y^4 \quad \langle y^6 \rangle = 15\sigma_y^6$$

# design contributions to IP spot size

for all types of final focus system there are aberrations due to the chromatic breakdown of the “-I” transforms between sextupoles

famous LC expert told me  
“ $\sigma_y^*$  does not depend on  $\beta_x^*$ ”  
??!?

$$\sigma_y^{*2} =$$

$$\beta_y^* \varepsilon_y$$

linear spot size

$$+ \Delta \sigma_{y,SR}^{*2}$$

contribution from SR  
in bends and quadrupoles

“long sextupole effect”  
(K. Oide)

$$+ a \frac{\varepsilon_x}{\beta_x^*} \sqrt{\frac{\varepsilon_y}{\beta_y^*}} + \dots + b \sigma_\delta \sqrt{\frac{\varepsilon_x}{\beta_x^*}} \sqrt{\frac{\varepsilon_y}{\beta_y^*}} + \sum_{ikl} c_{ikl} \left( \frac{\varepsilon_x}{\beta_x^*} \right)^{i/2} \left( \frac{\varepsilon_y}{\beta_y^*} \right)^{k/2} \sigma_\delta^l$$

“chromatic breakdown of linear optics”

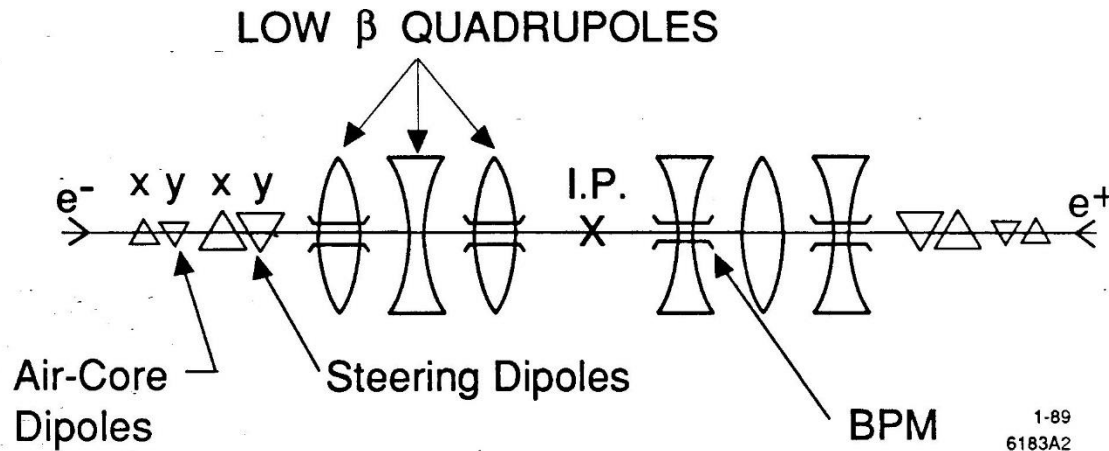
higher-order aberrations scale with higher powers  
x and y divergence and momentum spread

aside from design imperfections additional dilutions arises from errors, spurious dispersion, waist shift, etc. ;

we need to constantly cancel additional aberrations by scanning, correcting or “tuning” ; this requires:

1. (quasi-orthogonal) tuning knobs; at the SLC we used nonlinear “Irwin knobs”
2. a tuning signal; at the SLC we used: beam-beam deflection (two beams), laser wires (single beam), beamstrahlung, luminosity

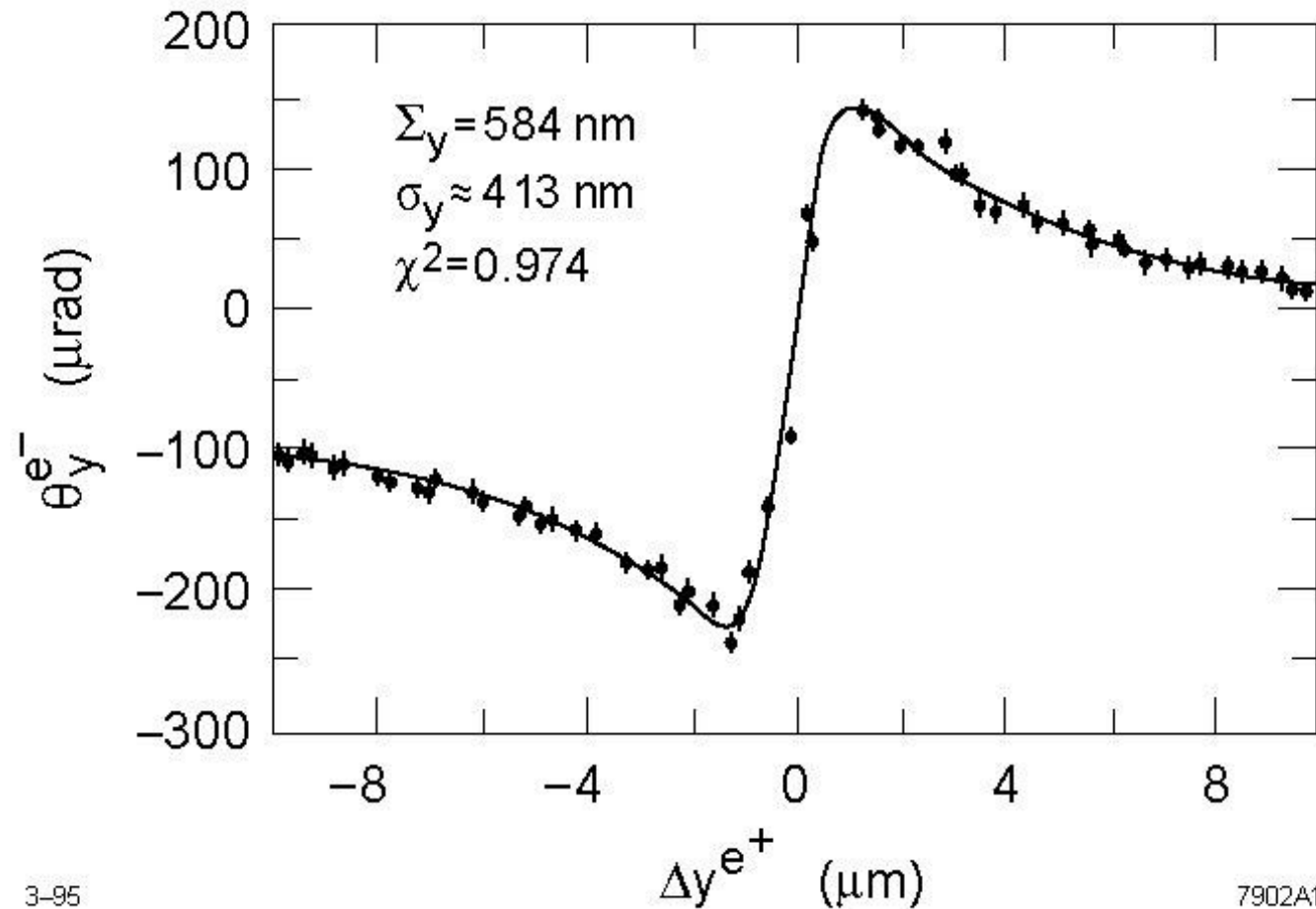
# beam-beam deflection scan at SLC



$$\begin{aligned} & \Delta y' + i\Delta x' \\ &= -\frac{\sqrt{2\pi}N_b r_p}{\gamma} \frac{1}{\sqrt{\sigma_x^2 - \sigma_y^2}} \left\{ w \left( \frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right. \\ & \quad \left. - \exp \left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right) w \left( \frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right\} \end{aligned}$$

Note: centroid deflection is obtained by replacing  $\sigma \rightarrow \Sigma \equiv \sqrt{\sigma_{e-}^2 + \sigma_{e+}^2}$

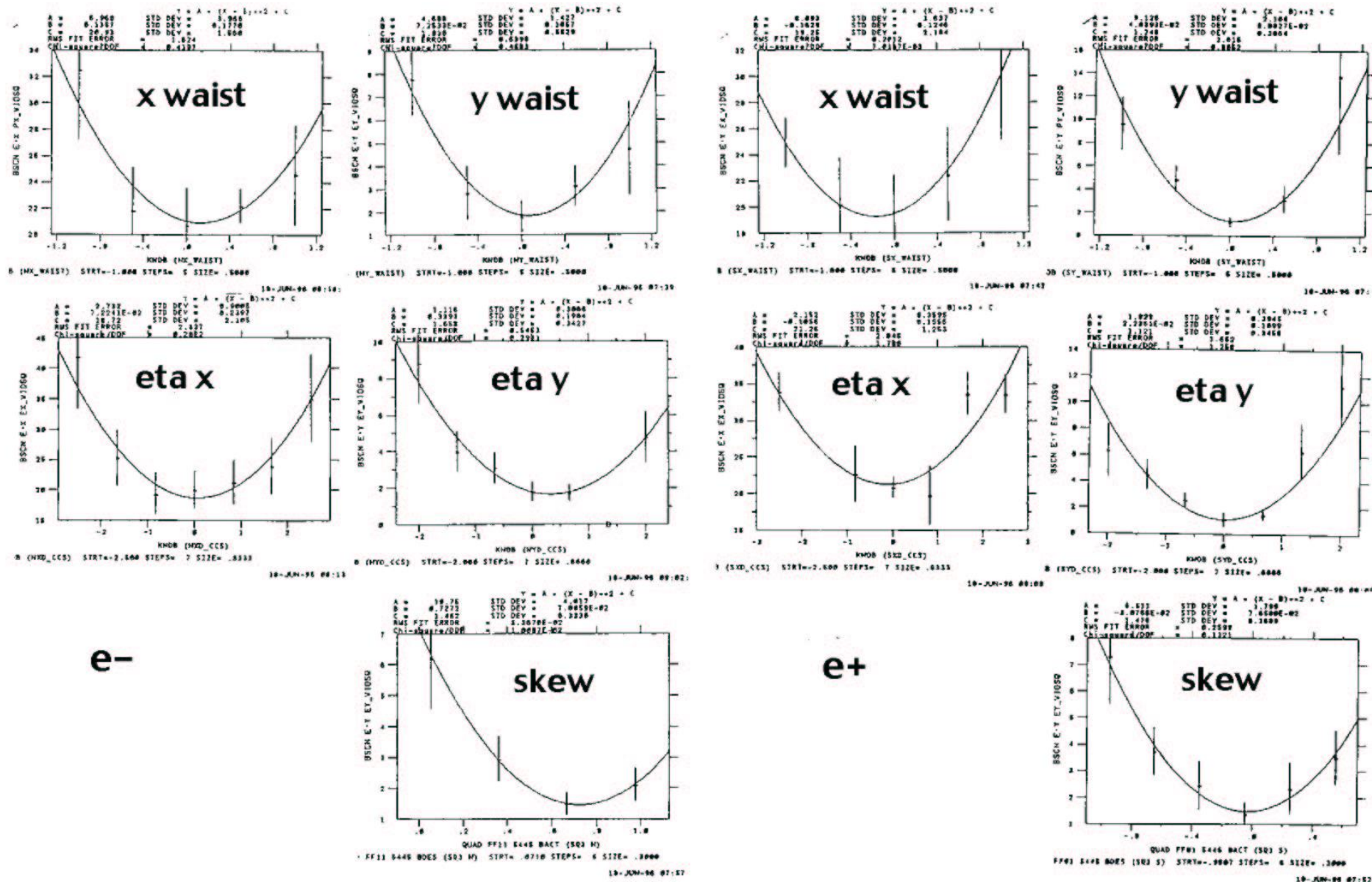
# beam-beam deflection with flat beams



3-95

7902A1

SLC vertical beam-beam deflection scan at low bunch charge, demonstrating a single-beam size of about 410 nm (1994/95) as expected

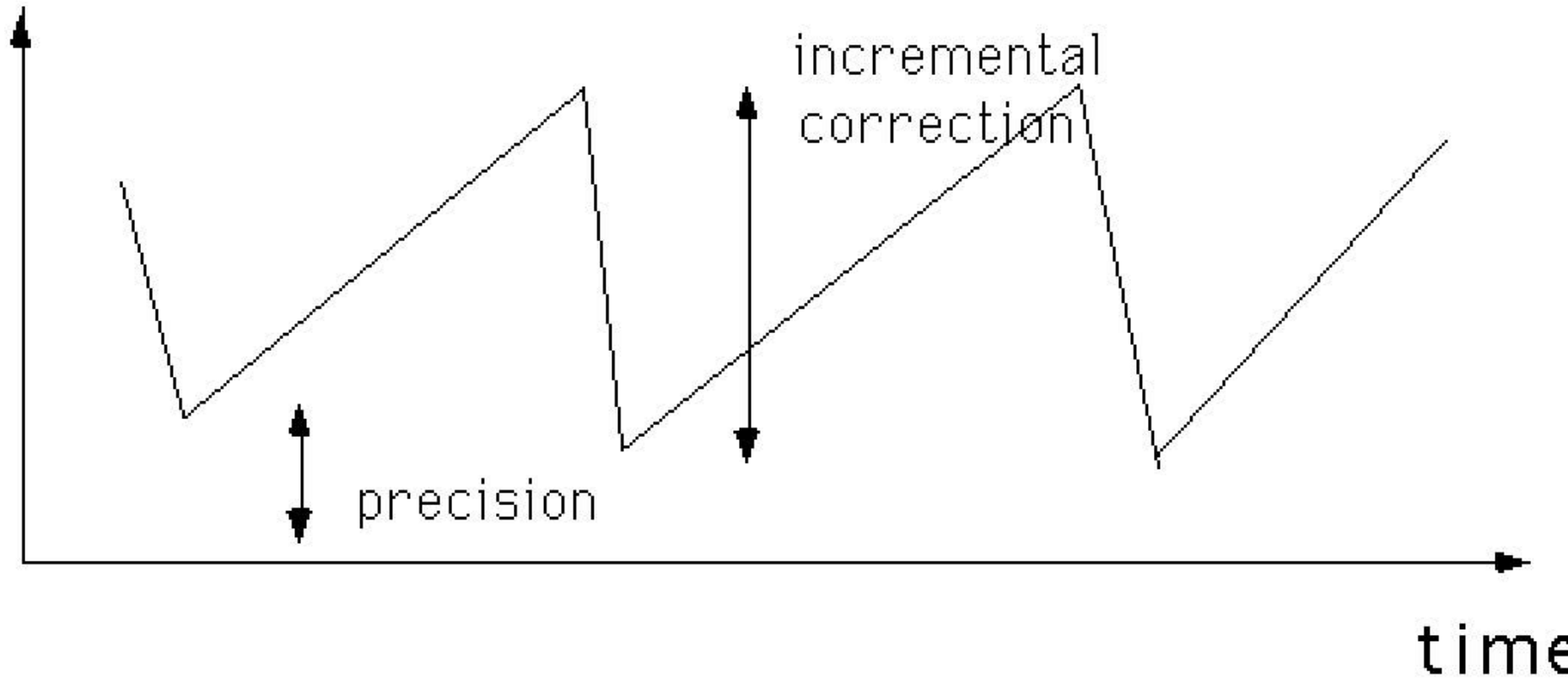


## aberration scans at SLC collision point

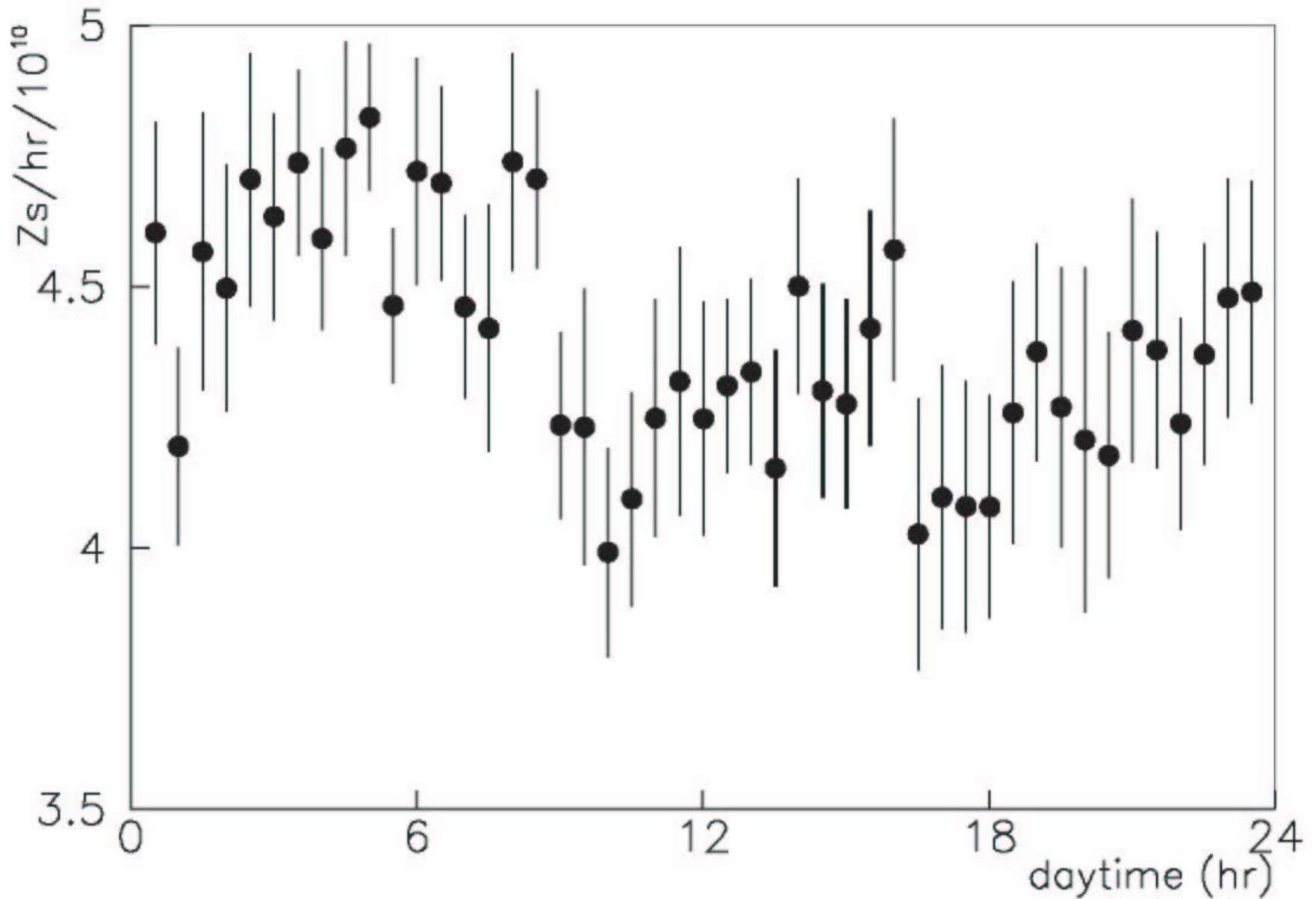
tuning knobs control **waists**, **dispersion**, **coupling**, **sextupolar aberrations**,  $\xi$ , **higher-order terms**,... tuning optimization was repeated **in regular intervals** (hours)



$\Delta\sigma/\sigma$

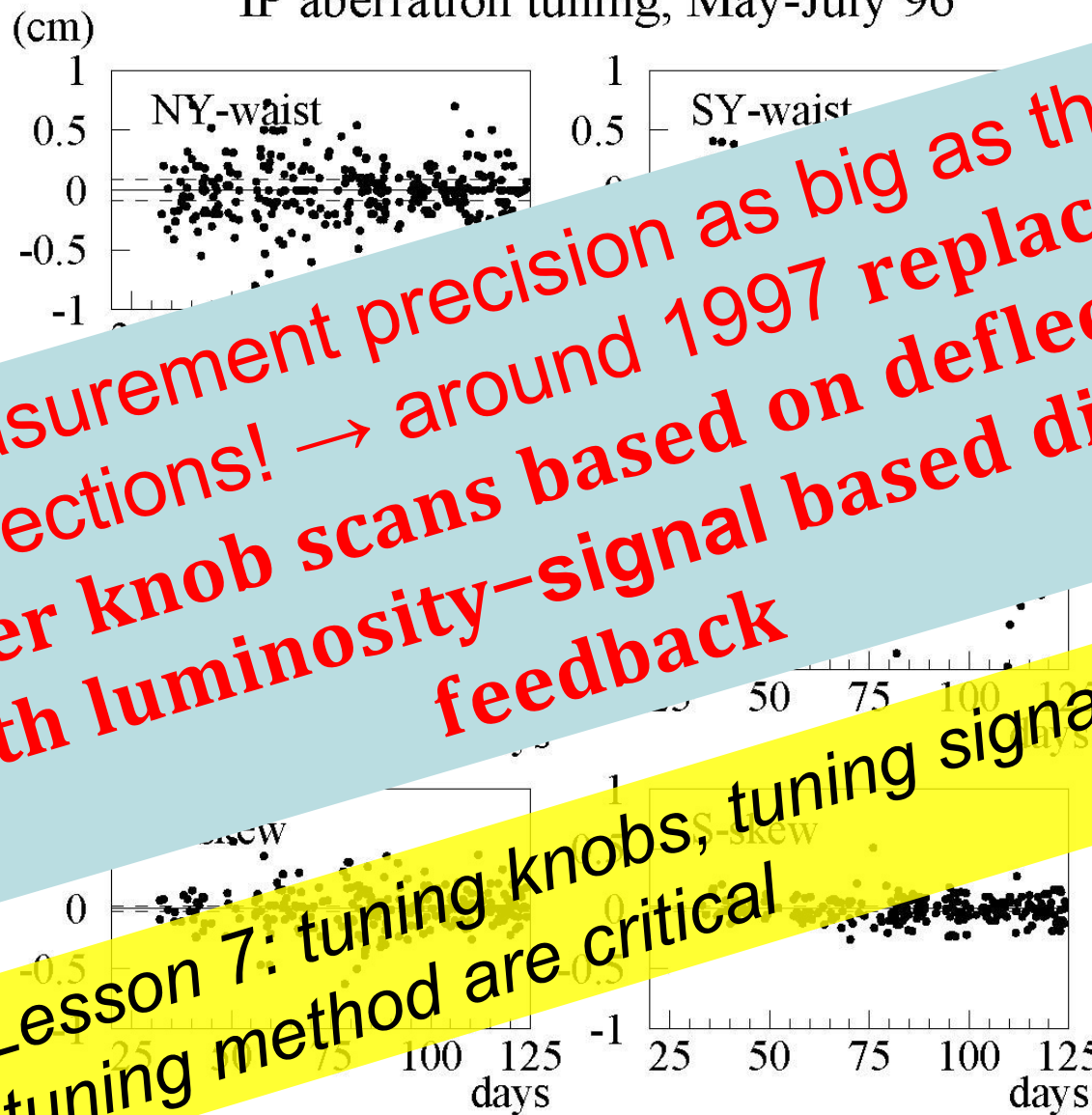


schematic of tuning effect and spot-size  
increase between tunings



diurnal normalized luminosity during the 1996 SLC run;  
steady increases in day & swing shifts, drops at 8&16 h

IP aberration tuning, May-July 96



measurement precision as big as the  
corrections! → around 1997 replaced  
wider knob scans based on deflection  
with luminosity-signal based dither  
feedback

Lesson 7: tuning knobs, tuning signal +  
tuning method are critical

incremental IP corrections of waist, dispersion  
and coupling during the 1996 SLC run

# validating the final focus optics

SLC examples:

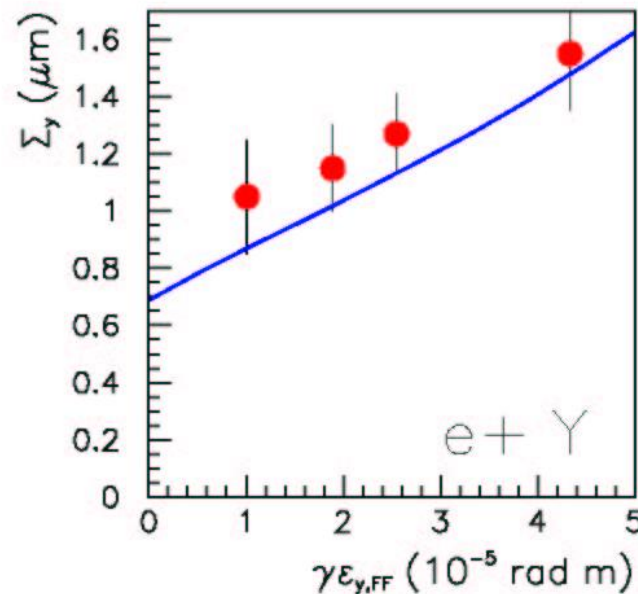
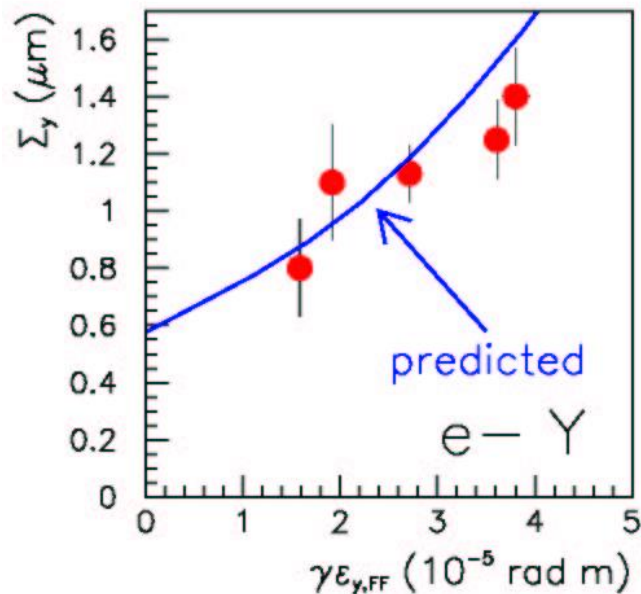
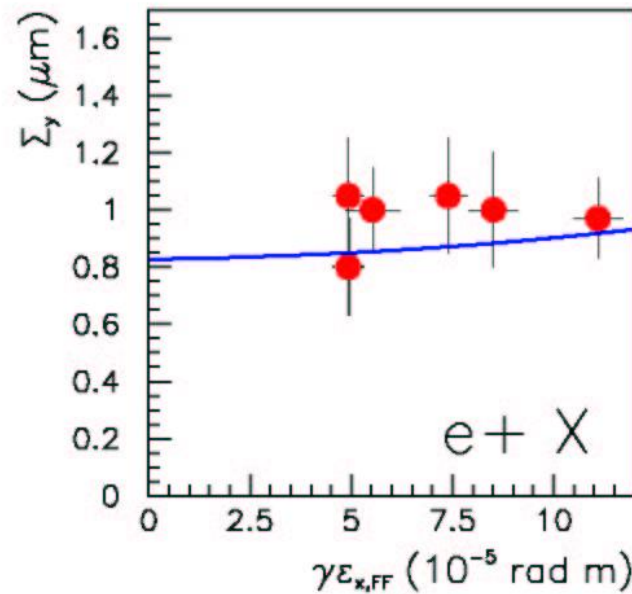
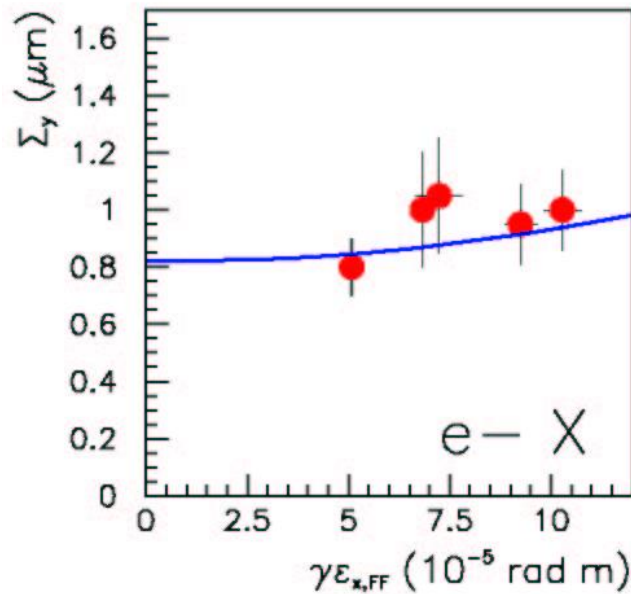
**spot size vs. beam emittance**

(varied in the damping ring)

**spot size vs. beam energy**, launch orbit, etc.

**spot size vs  $\beta^*$**  and intensity

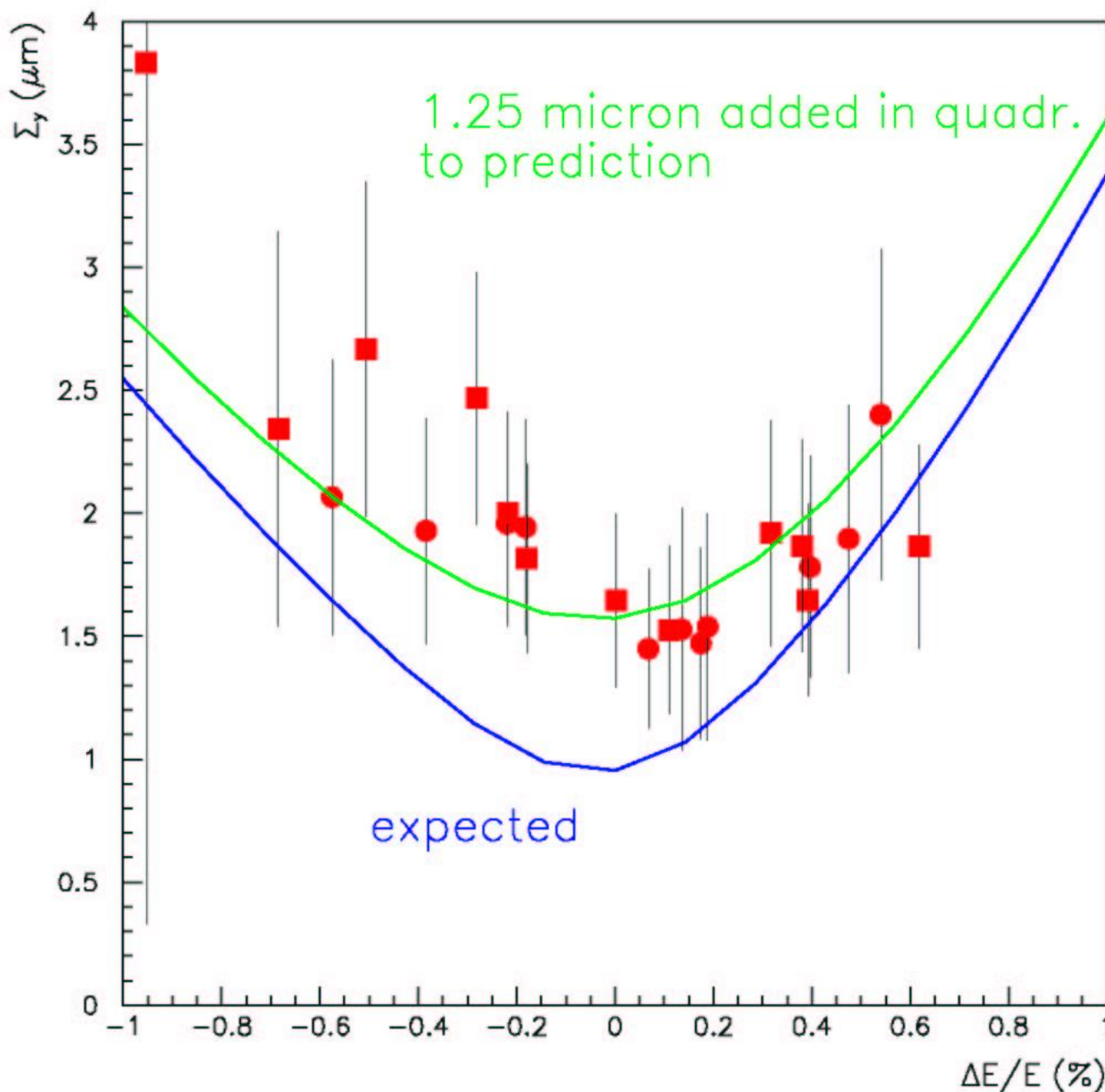
## Y Spot Size vs. Final-Focus Emittances



vertical  
convoluted  
IP spot size  
measured  
**at low charge**  
as function  
of the four  
beam  
emittances;  
solid line  
is SLC final-  
focus flight-  
simulator  
prediction

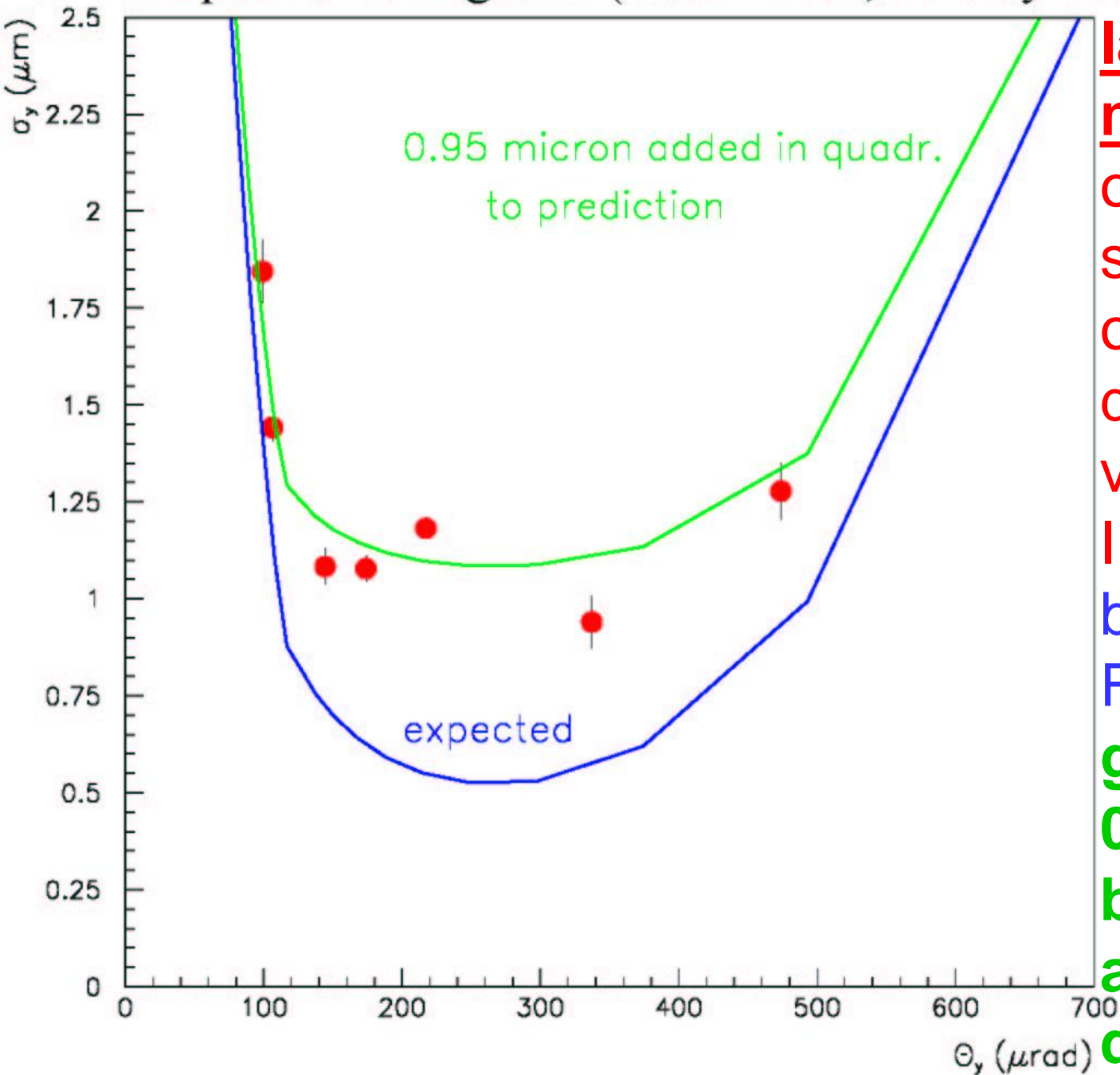
# FF energy bandpass (18 June 1996)

SLAC-CN-410 (1996)



vertical  
convoluted  
IP spot size  
measured  
at nominal  
SLC current  
as function  
of centroid  
energy; rms  
energy spread  
 $\sim 0.1\%$ ;  
blue line is  
FFFS prediction  
green line has  
1.25  $\mu\text{m}$  added





**laser wire**  
**measurement**  
of e+ IP beam  
size as function  
of vertical IP  
divergence,  
varied with  
Irwin knobs;  
blue line is  
FFFS prediction,  
green line has  
0.95  $\mu\text{m}$  single-  
beam dilution  
added in  
quadrature

# 1998 - last year of LHC operation

Lesson 8: average IP spot size always above expectation (impact on luminosity)

20 Jan – 20 May 1998, SLAC-CN-418

parameter	symbol	expected		measured	
		mean	rms	mean	rms
horizontal convoluted spot size	$\Sigma_x^* [\mu\text{m}]$	1.80	0.22	2.60	1.21
vertical convoluted spot size	$\Sigma_y^* [\mu\text{m}]$	0.55	0.13	1.38	0.91

$$\sigma_{y(x)} = \sqrt{\sigma_{y(x),0}^2 + \Delta\sigma_{y(x)}^2}$$

$$\Delta\sigma_x = 1.89 \mu\text{m}$$

$$\Delta\sigma_y = 1.27 \mu\text{m}$$

added in quadrature

remark: discrepancy similar for two-beam measurements (deflection scans) and single-beam tuning (laser wire)

# how do we get a small spot size?

## small emittances

- from damping ring
- emittance preservation in linac (wake fields, spurious dispersion)

## final focus minimizing aberrations and synchrotron radiation effects

- **SLC type** (interleaved sextupoles)
- **FFTB type** (modular, non-interleaved sextupoles)
- **compact ILC/CLIC/ATF-2** type w extremely local chromatic correction

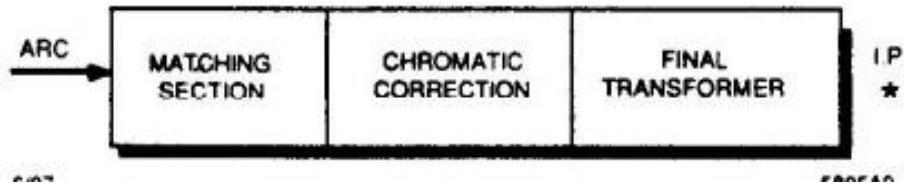
## tuning recipe

- scanning orthogonal tuning knobs
- beam-beam deflection scans or luminosity dither feedback

## stability

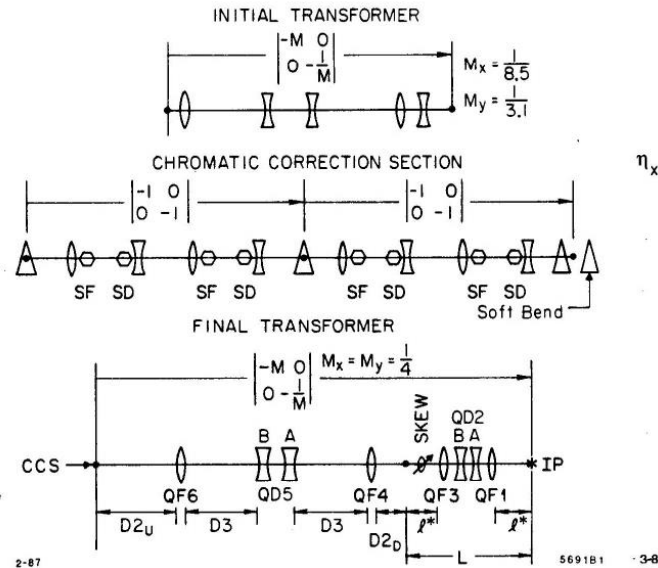
- pulse-to-pulse orbit stability (damping ring instability, wakes, kicker)
- pulse-to-pulse beam size / emittance stability

# final-focus evolution

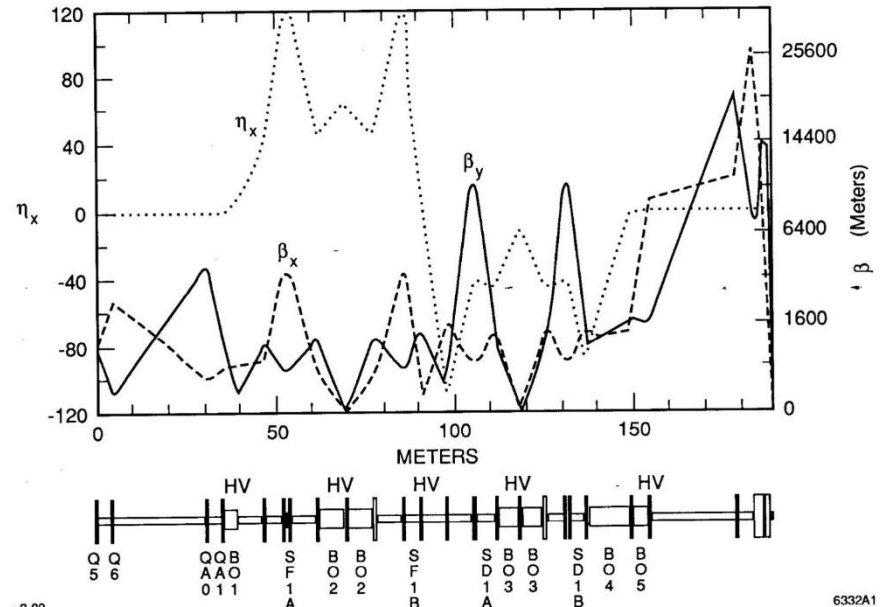


## SLC final focus,

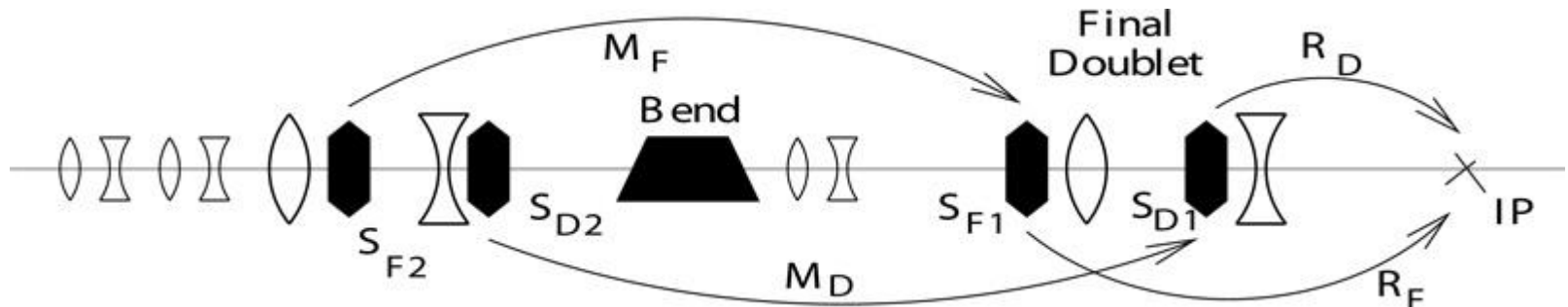
J. Murray,  
K. Brown  
T. Fieguth,  
PAC1987

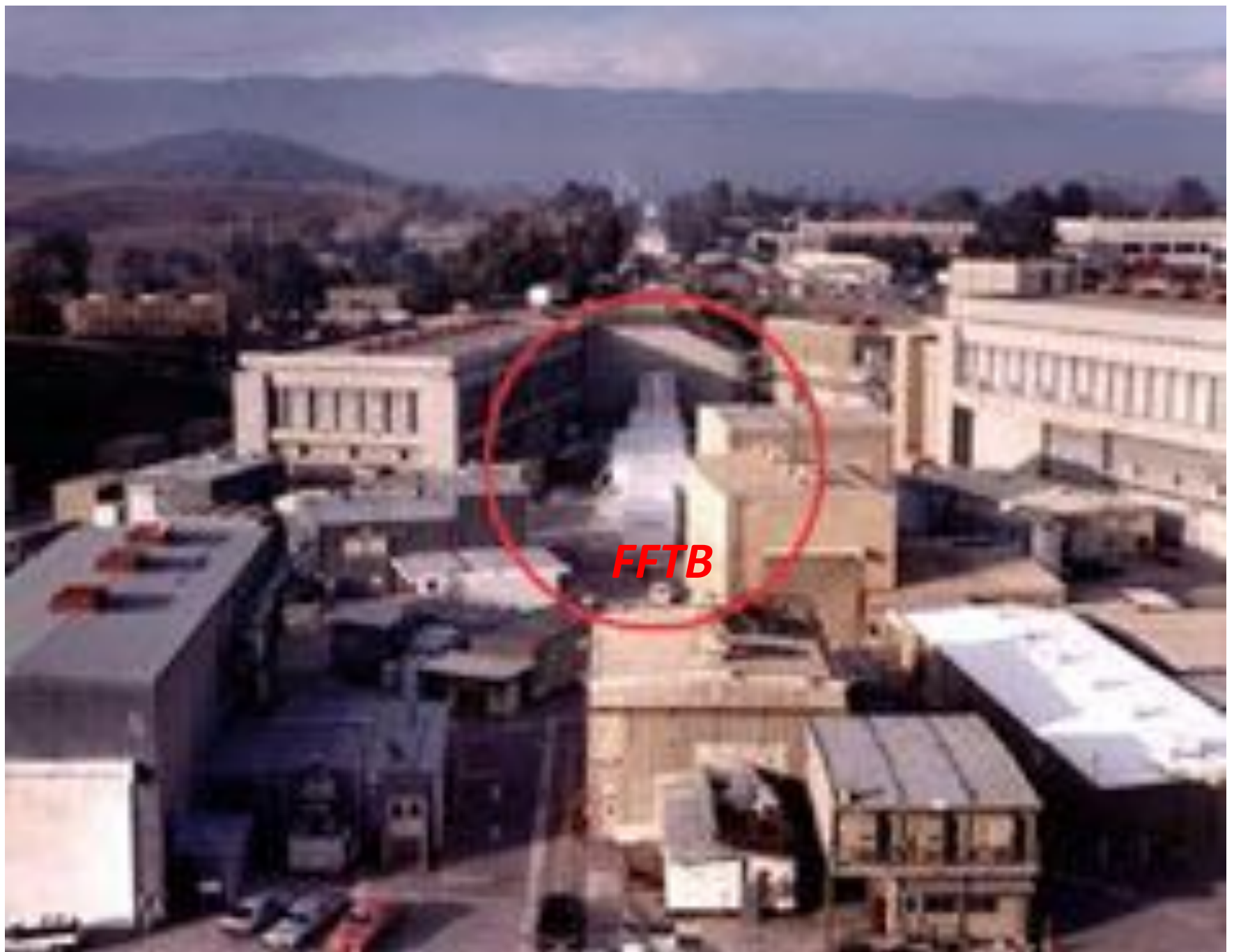


Non-interleaved modular final focus for the **FTB**, K. Oide, IEEE PAC 1989

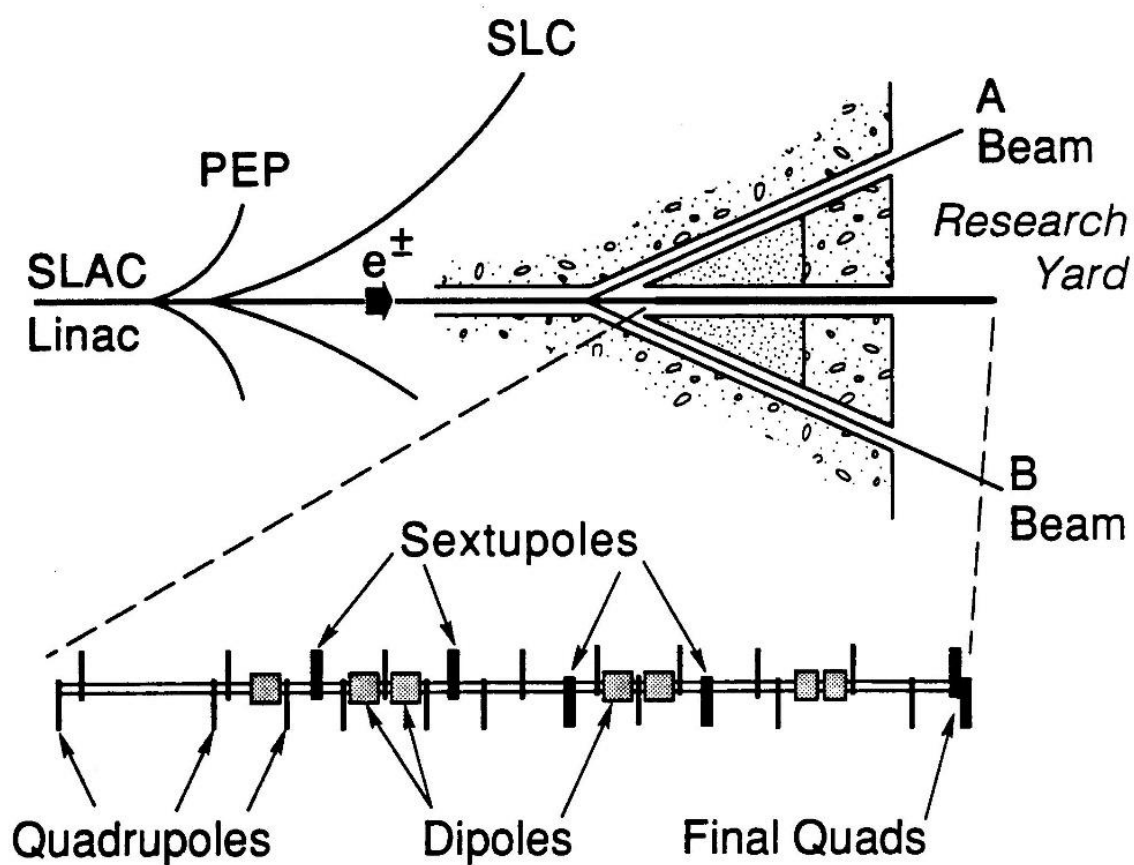


Compact final focus (**ATF-2**/ILC/CLIC), P. Raimondi, A. Seryi, Phys. Rev. Lett. 86, 3779 (2001)











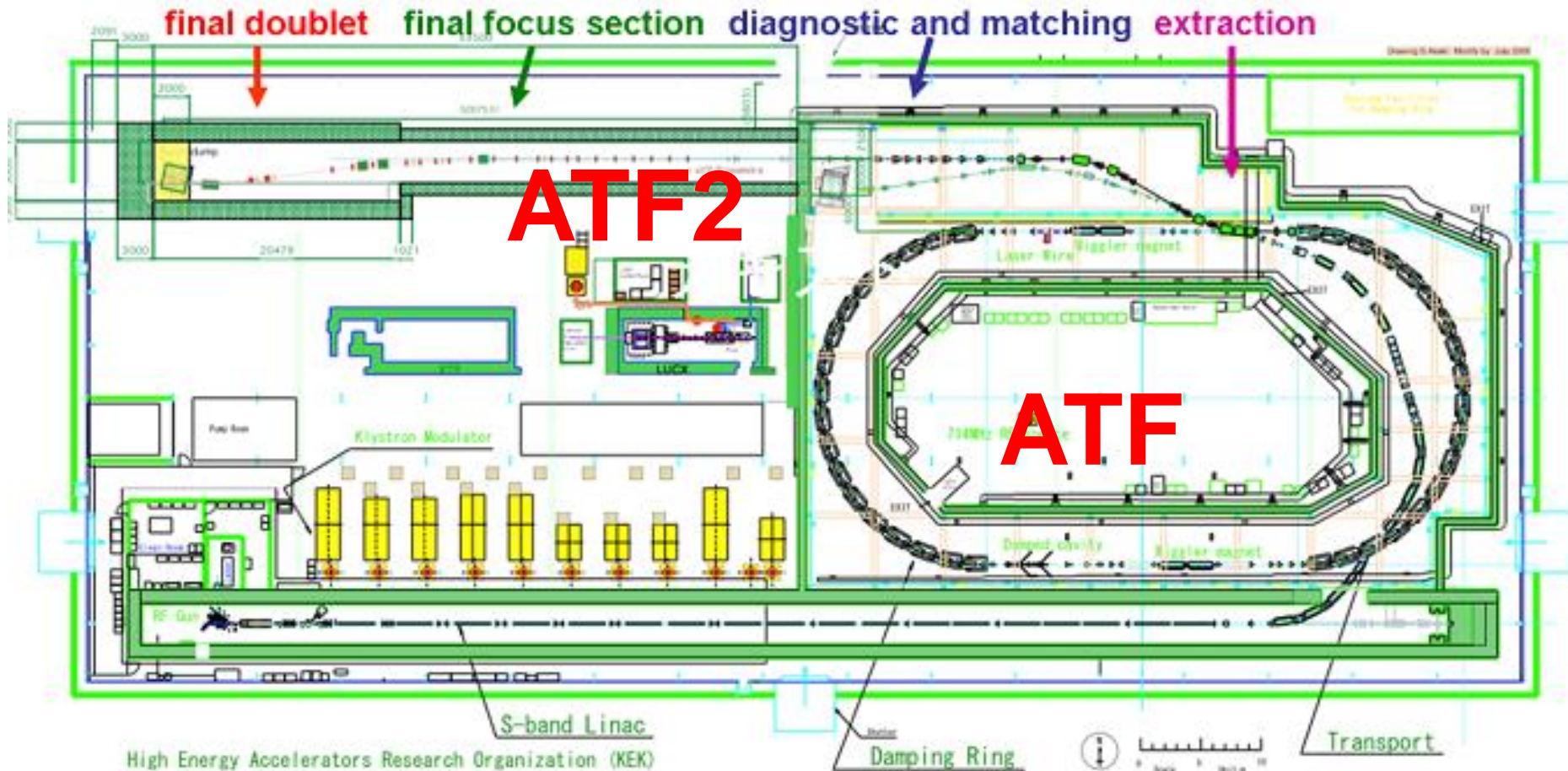
# FFTB spot size results

date	spring 1994	fall 1994	fall 1995	spring 1997
time	3 weeks	2 weeks	1.5 weeks	1.5 weeks
<b>expected</b>	<b>50 nm</b>	<b>50 nm</b>	<b>50 nm</b>	<b>50 nm</b>
<b>minimum spot size measured</b>	<b>70 nm</b>	<b>70 nm</b>	<b>100-120 nm</b>	<b>120 nm</b>
<b>residual &amp; possible origin</b>	<b>40 nm jitter/vibration? (RFBPM/laser-monitor housing)</b>	<b>40 nm jitter/vibration?</b>	<b>100 nm collimator wakes?</b>	<b>100-110 nm jitter + sextupole aberration?</b>

*“The [FFTB] difference from 40 nm was attributed to significant jitter of the focused beam and was also partly due to limited accuracy in tuning the linear optics and the aberrations.” [ATF2 proposal]*

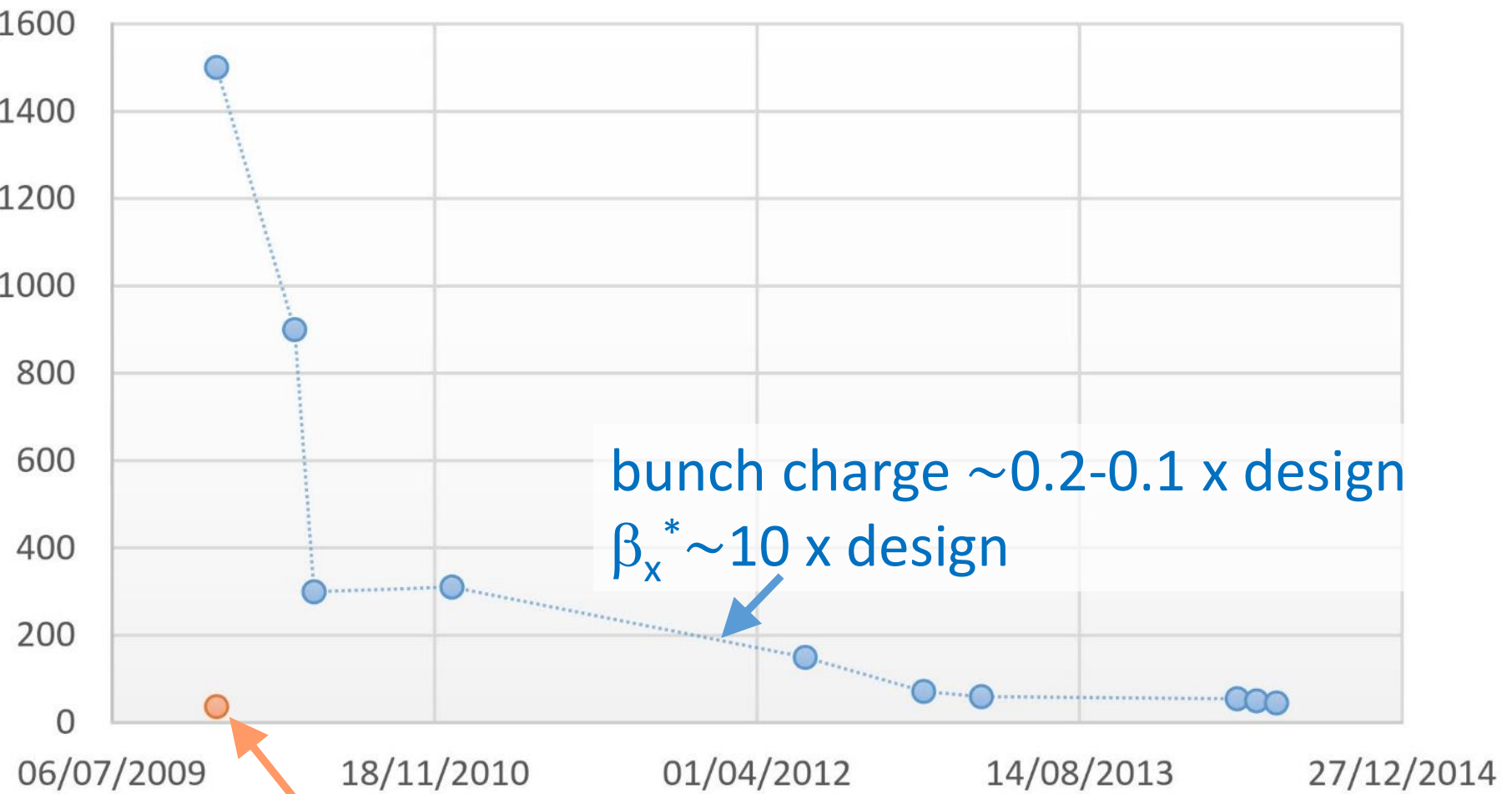
D. Burke, LC97

KEK / ATF (linac & damping ring) in operation since ~1995,  
ATF-2 (final focus) since ~2009 [proposed by A. Seryi et al.]



ATF-2: test facility for small spot size & viability of compact final focus

## minimum rms vertical beam size at ATF-2



design value at original  
target date

# ATF2 parameters & spot size

	2010	2015
bunch population		<b>~0.05</b>
$\epsilon_x$ [nm]		1.7
$\epsilon_y$ [pm]		<10
$\beta_x^*$ [mm]		<b>40.0</b>
$\beta_y^*$ [mm]	0.1	<b>1.0</b>
$\sigma_y^*$ [nm]	37	<b>300</b>
$\sigma_y^*$ [nm] expected	-	100
$\Delta\sigma_y^*$ [nm] residual	-	<b>280</b>

Lesson 9: test facilities FFTB and ATF-2 had similar experience as SLC: getting expected spot size is much easier at low intensity and with relaxed parameters; residual spot size increases not fully explained or understood

mission  
accomplished?



world record  
spot size  
achieved!

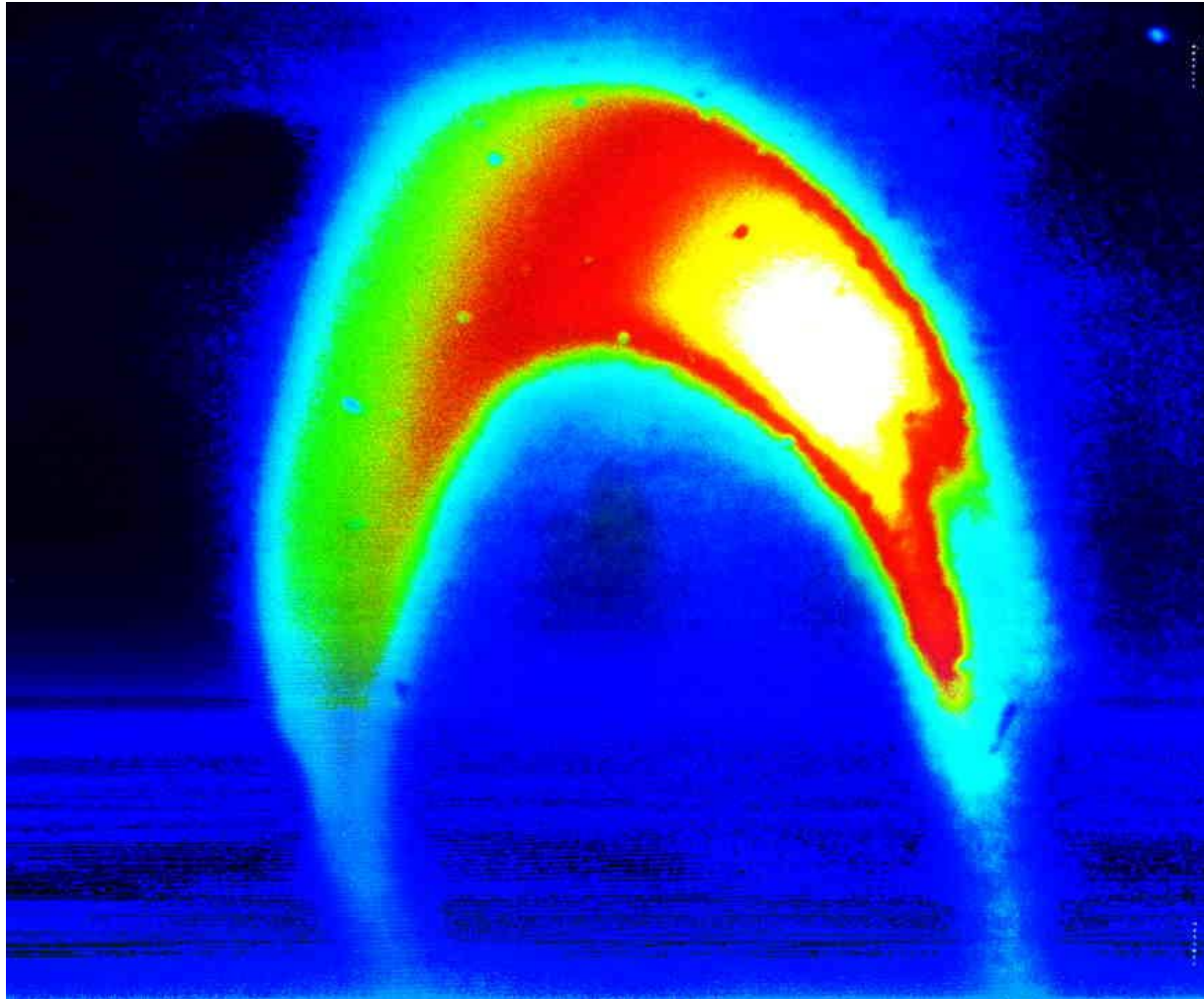
but  $\leq 1\%$  of design  
luminosity if this  
were a linear collider

halo and background ...





## *Is there halo in linear colliders?*



Yes, measured beam distribution at the end of the SLAC linac (projection on the x-y plane)!



let us look at the SLC prediction...

# S L C

## DESIGN HANDBOOK

STANFORD LINEAR ACCELERATOR CENTER  
STANFORD UNIVERSITY

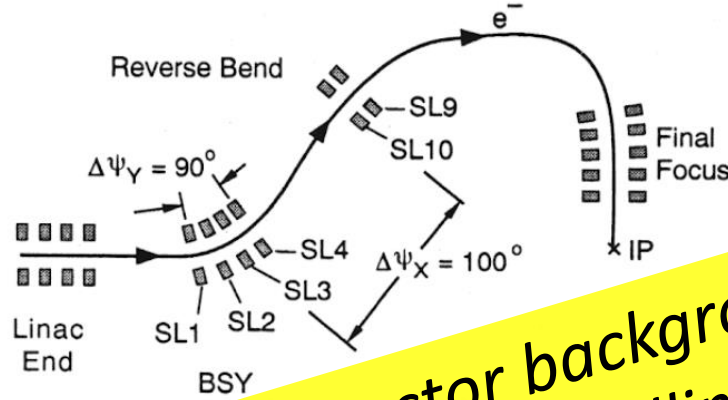
Stanford, California 94305

December 1984

Prepared for DOE under Contract  
No. DE AC03-76SF 00515

**9.3 BACKGROUNDS** This section is not yet ready for distribution.

# collimation and beam-loss induced muon background



SLC solutions:

many collimators +  
large toroidal magnets  
installed  
around the beam line  
to deflect particles  
traveling  
outside the beam pipe

Lesson 10: detector background may  
created by upstream collimation may  
seriously impact detector operation  
and constrain luminosity; large, expensive  
spoiler systems ("tunnel fillers") required for  
next generation of linear colliders

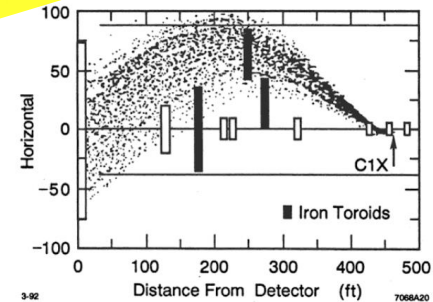
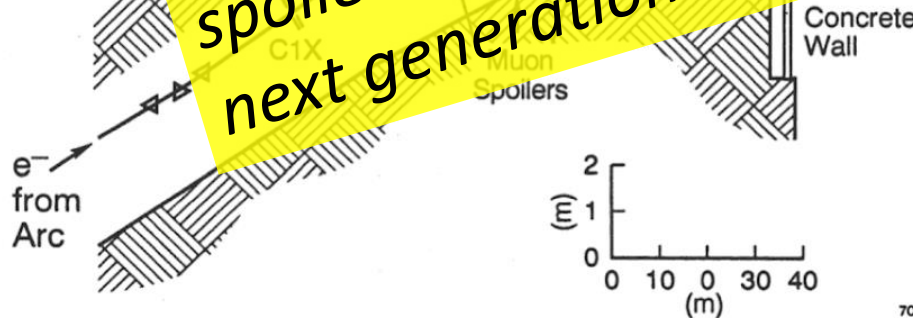


Figure 23. Trajectories of muons created in the collimator C1X that would reach the detector in the absence of toroidal fields.

Figure 22. Layout of toroidal muon spoilers in the Final Focus tunnel at the SLC. The collimators C1X and PC12 are slits used to shadow the apertures of the final quadrupole lenses shown near the detector.

a few conclusions

SLC's prime challenges and opportunities:

Halo ! Resonances ! Jitter ! Spot size ! Polarization !

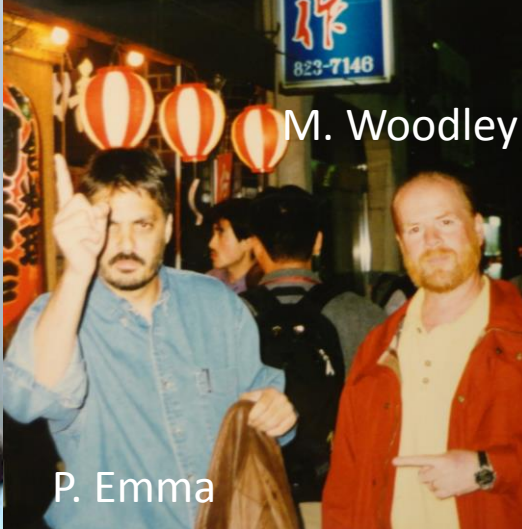
SLC shows linear colliders = **paradise for accelerator folks**  
→ new diagnostics, new mathematics, new tools, new methods, new phenomena, ...

SLC spot-size puzzles continued at FFTB and ATF-2





N. Phinney



M. Woodley

P. Emma



P. Tenenbaum



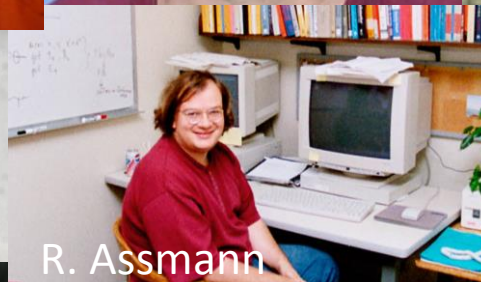
D. Burke



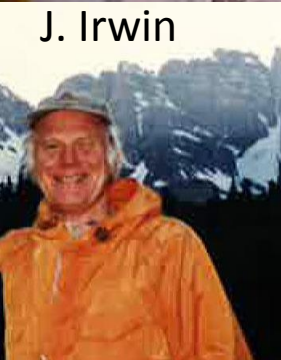
P. Raimondi



T. Raubenheimer



R. Assmann



J. Irwin



M. Ross



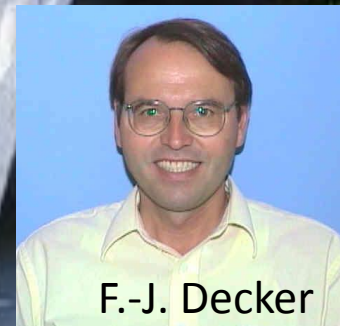
K. Bane



K. Oide

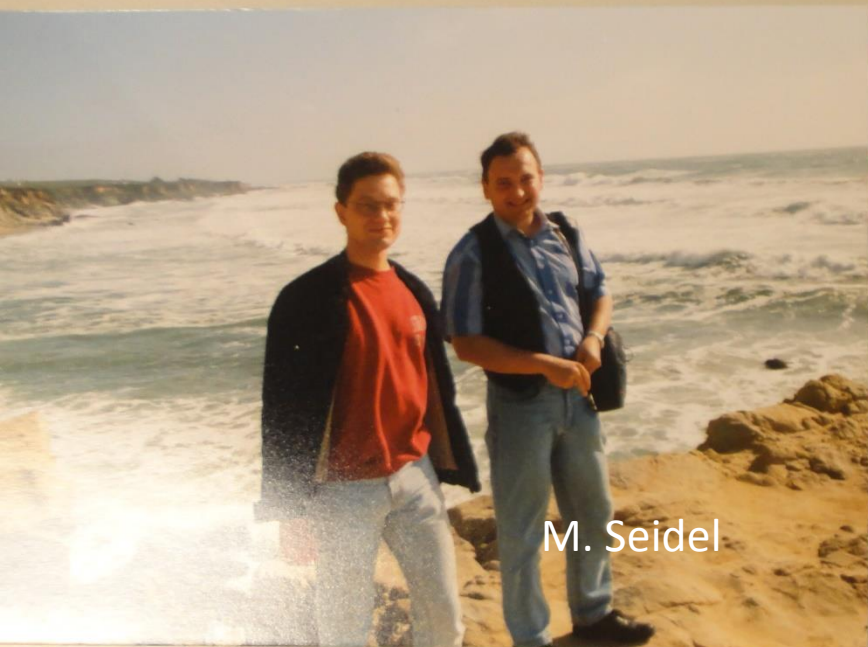


J. Seeman

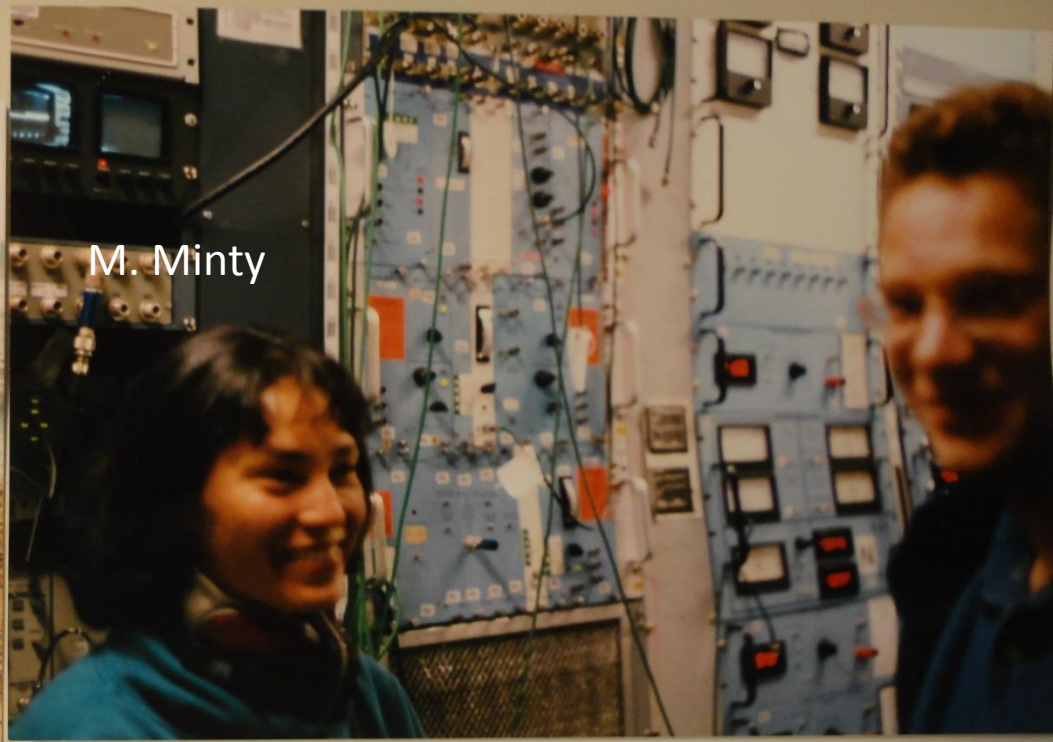


F.-J. Decker





M. Seidel



M. Minty



K. Bane

*warm thanks to all the  
great SLC colleagues & friends and many others!*



A. Chao

N. Toge

R. Brinkmann