

# Kickers and Septa

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SLS



PROSCAN

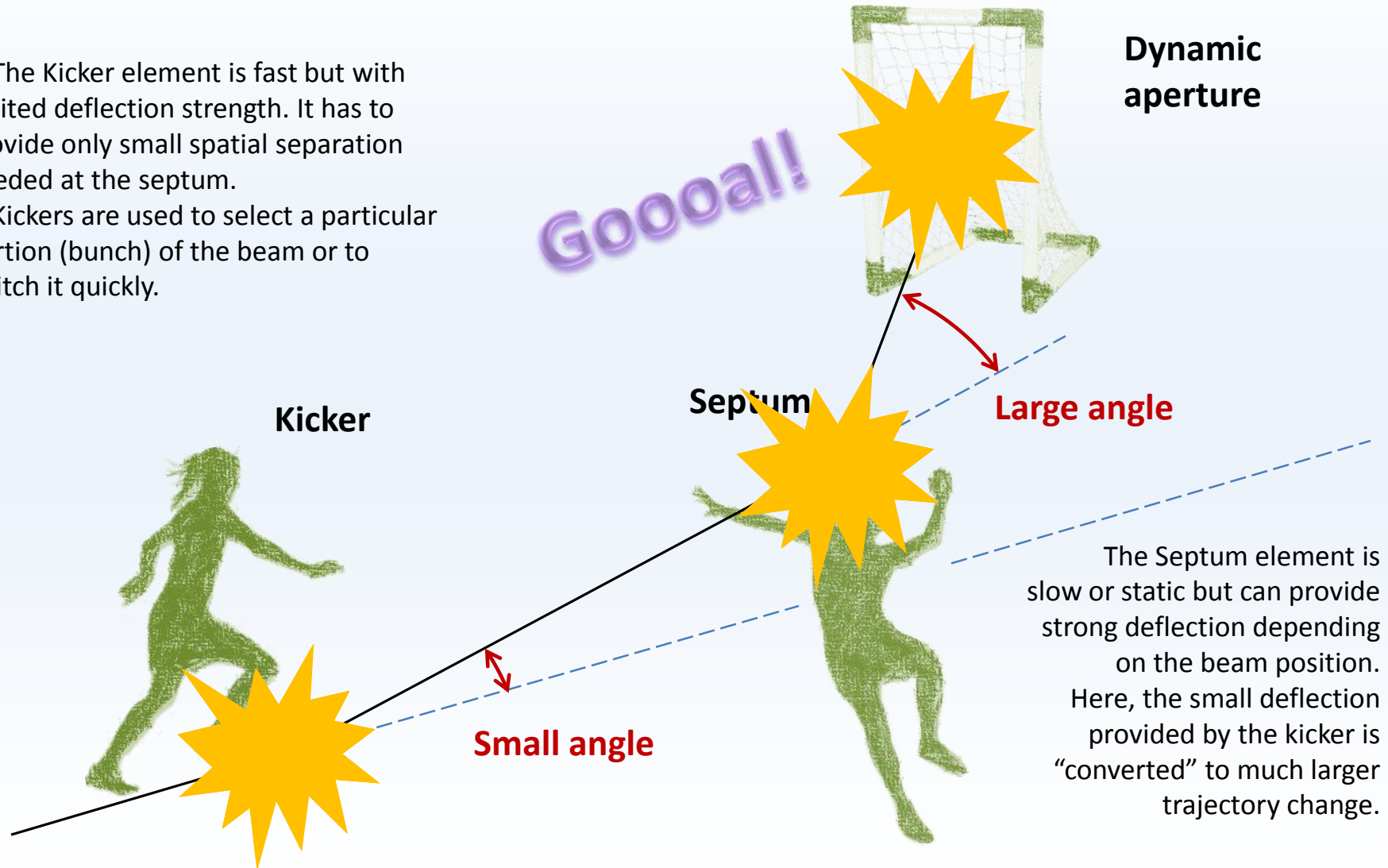
HIPA

Accelerators at  
Paul Scherrer Institute

## General picture – Single bunch extraction

The Kicker element is fast but with limited deflection strength. It has to provide only small spatial separation needed at the septum.

Kickers are used to select a particular portion (bunch) of the beam or to switch it quickly.



# Deflecting charged particles

How do we deflect charged particles beams?

Electric, magnetic and “exotic” (e.g. bent crystals - plane channeling\*)

Lorentz force\*\* – the force exerted on a point charge by electromagnetic fields.

$$\vec{F} = \underbrace{q\vec{E}}_{\text{Electric}} + \underbrace{q\vec{v} \times \vec{B}}_{\text{Magnetic}}$$

$\vec{F}$  – Force exerted on the point charge [N]

$q$  – Electric charge [C]

$\vec{E}$  – Electric field [V/m]

$\vec{v}$  – Velocity of the point charge [m/s]

$\vec{B}$  – Magnetic flux density [T]



Hendrick Lorentz  
1853 - 1928

\*Plane channeling of protons (> 10 GeV) in Si mono-crystals<sup>[26, 32]</sup>

\*\*First derivation is often attributed to Oliver Heaviside or James Maxwell

- ❑ Often charged particles in accelerators move with relativistic speeds – *relativistic dynamics* should be applied.

## Electrostatic deflection

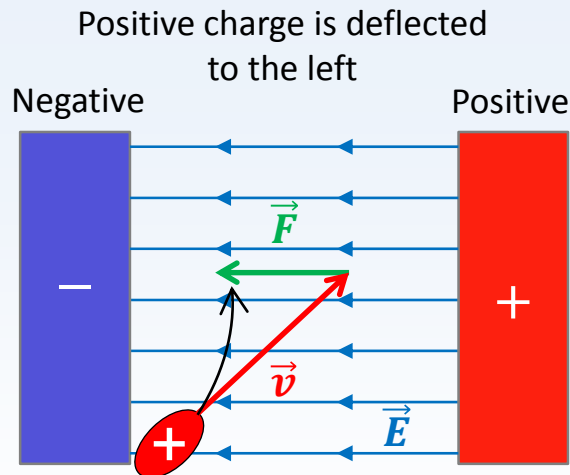
Deflecting force is collinear with the electric field – positive charges are deflected in the direction of the electric field lines, negative charges are deflected on the opposite direction.

### Conventions:

- Force on a *positive* point charge.
- Electric field lines go from **positive** electrode to the **negative** one.
- *Opposite* electric charges attract each other and *like* electric charges repel.

Electrostatic deflection angle  $\theta_E$  of particles with positive elementary charge <sup>[19, 33]</sup>

$$\theta_E \approx \frac{E \cdot l_{eff}}{10^9 \cdot \beta \cdot p}$$



Where:

$\theta_E$  – electrostatic bending angle\* [rad]

$E$  – deflecting electric field [V/m]

$l_{eff}$  – effective length of the septum [m], usually different from the mechanical length due to fringe fields

$\beta$  – relativistic coefficient that gives the fraction of the speed of light at which the particles travel [-]

$p$  – particles' momentum [GeV/c]

\*Small angle approximation:  $\tan(\theta) \approx \theta$  up to  $\sim 0.17$  rad ( $\sim 10^\circ$ ) error is  $< 1\%$

Equation derivation – in “Additional material” at the end of the presentation.

# Deflecting charged particles

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$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Electric
Magnetic

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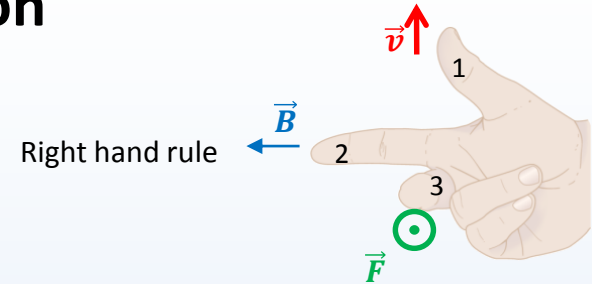
- ❑ Often charged particles in accelerators move with relativistic speeds – *relativistic dynamics* should be applied.

# Magnetic deflection

Deflecting force  $F$  is cross product of  $v$  and  $B$

## Conventions:

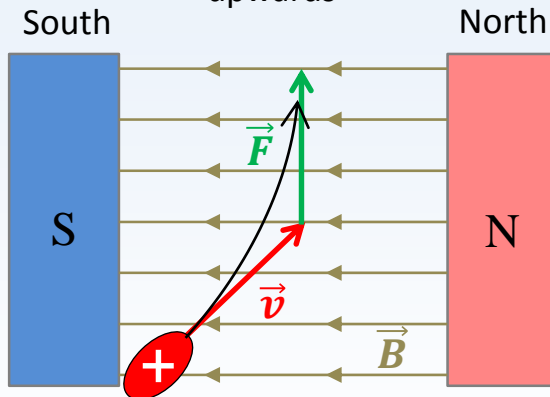
- Force on a positive point charge.
- Magnetic field lines go from **North pole** to the **South pole** of the magnet.
- Right hand rule: If the thumb points in the direction of motion and the index finger is in the direction of the magnetic field, the force goes in the direction of the middle finger.



Magnetic deflection angle  $\theta_M$  of particles with positive elementary charge [15, 19, 33, 30]

$$\theta_M \approx \frac{0.3 \cdot B \cdot l_{eff}}{p}$$

Positive charge is deflected upwards



Positive charge moving into plane of page

Where:

$\theta_M$  – magnetic bending angle\* [rad]

$B$  – deflecting magnetic flux density [T]

$l_{eff}$  – effective length of the septum [m], usually different from the mechanical length due to fringe fields

$p$  – particles' momentum [GeV/c]

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Electric      Magnetic

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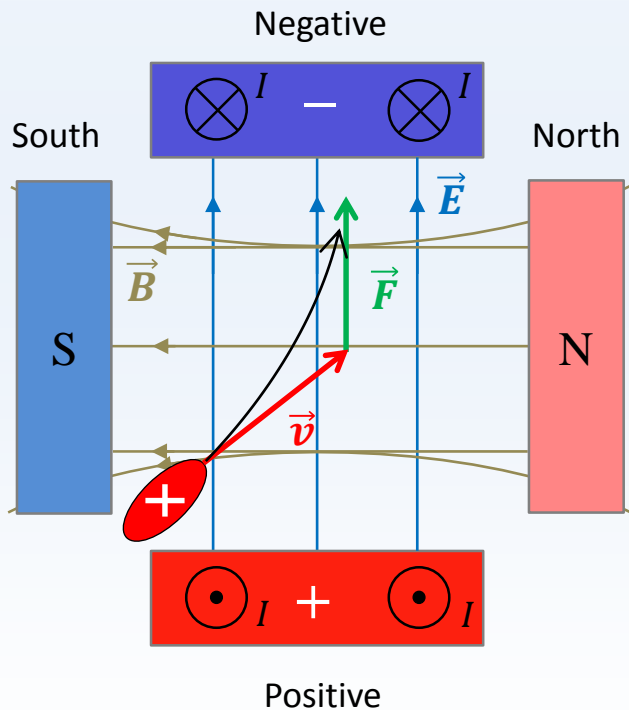
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# Electromagnetic deflection

If the current in upper conductor is flowing into the plane of page and the current in lower conductor is flowing out of the plane of page magnetic and electric force add-up.

Positive charge is deflected upwards



Positive charge moving into plane of page

Electromagnetic bending angle  $\theta_{EM}$  of particles with positive elementary charge <sup>[10, 18]</sup>

$$\theta_{EM} \approx \frac{l_{eff}}{p} \left( \frac{E}{10^9 \cdot \beta} \pm 0.3 \cdot B \right)$$

Where:

$\theta_M$  – magnetic bending angle\* [rad]

$l_{eff}$  – effective length of the septum [m], usually different from the mechanical length due to fringe fields

$p$  – particles momentum [GeV/c]

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\*Small angle approximation:  $\tan(\theta) \approx \theta$  up to  $\sim 0.17$  rad ( $\sim 10^\circ$ ) error is  $< 1\%$



# Electrostatic vs Magnetic deflection

Which one to use? How to compare?<sup>[22]</sup>

*Duality of electromagnetism*

$$B [\text{T}] = \frac{3.3}{\beta} E [\text{GV/m}]$$

- ❑ It is more practical to use **magnetic field!**
- ❑ Too high electric field in vacuum could provoke electric breakdown. It is widely accepted that 10 MV/m is a practical limit<sup>[24]</sup>.
- ❑ Electric deflection could be beneficial for **non-relativistic** particles (e.g. low energy beams, heavy ions etc.)

Comparison using volumetric energy density

Deflecting field	Stored energy per unit volume (free space)	Scalar form of Lorentz force
Electric	$W_E = \frac{\epsilon_0 E^2}{2}$	$F_E = qE$
Magnetic	$W_M = \frac{B^2}{2\mu_0}$	$F_M = qvB$

For  $W_E = W_M$  ( $E = cB$ ) and **relativistic** particles ( $v = c$ )

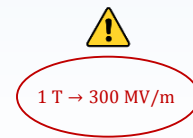
$F_E = qcB = F_M$

Comparison between electric and magnetic deflection for beams with different momentum  $p$

$\beta, -$	$\gamma, -$	$p_{electrons}, \text{ MeV}/c$	$p_{protons}, \text{ GeV}/c$	Electric field, MV/m	Equivalent magnetic field, T
0.001	1.000	0.0005	0.0009	10.00	33.356
0.01	1.000	0.0051	0.0094	10.00	3.336
0.1	1.005	0.0514	0.0944	10.00	0.334
0.3	1.048	0.1607	0.2955	10.00	0.111
0.5	1.155	0.2950	0.5425	10.00	0.067
0.9	2.294	1.0552	1.9401	10.00	0.037
0.99	7.089	3.5864	6.5944	10.00	0.034
0.999	22.366	11.4185	20.9955	10.00	0.033
0.9999	70.712	36.1328	66.4386	10.00	0.033
0.99999	223.607	114.2698	210.1114	10.00	0.033
0.999999	707.107	361.3552	664.4349	10.00	0.033

Non-relativistic

Relativistic



Equation derivation – in “Additional material” at the end of the presentation.

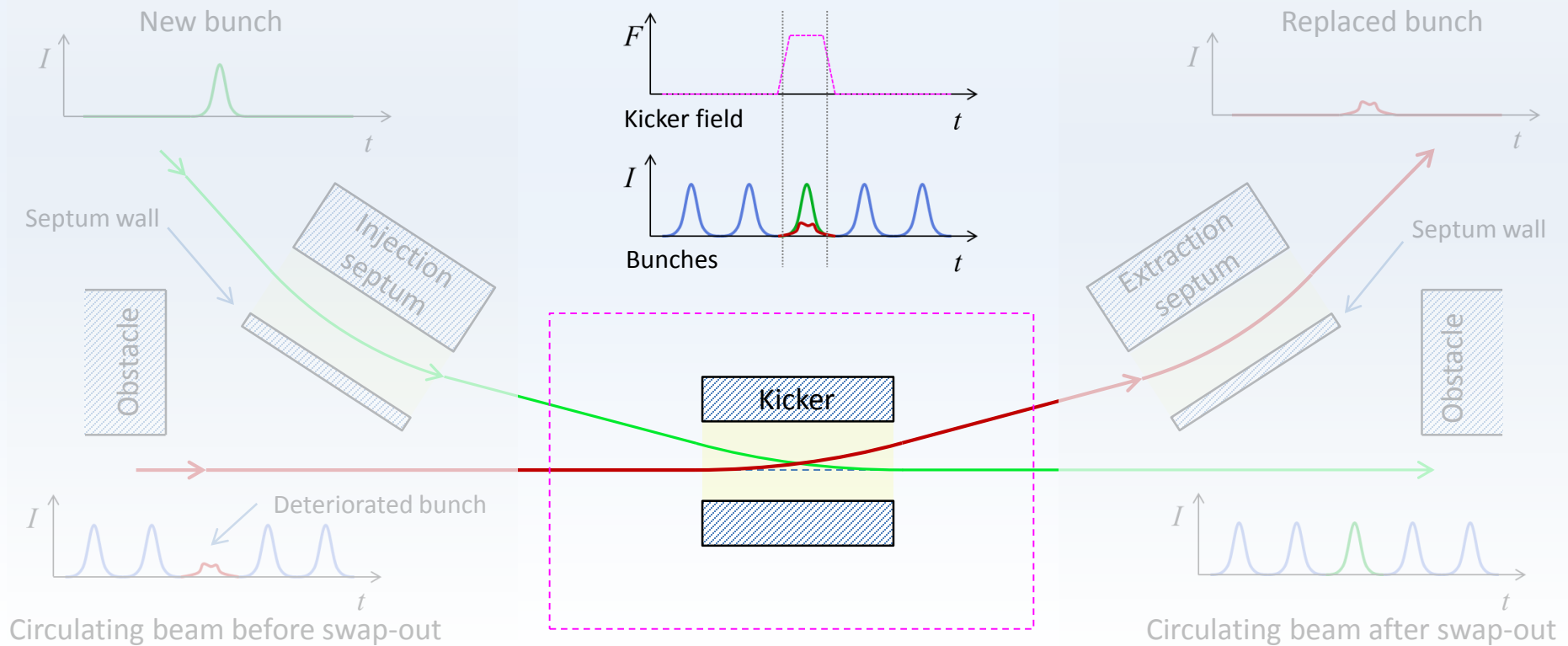
# Kickers

## Kicker - function

A kicker is a fast acting (1 ns ... 1  $\mu$ s ) beam deflection element, usually providing relatively small deflection. It is used for beam injection, extraction, feedback, dilution and dump.

Example: “Swap-out” injection scheme

The kicker deflects only one bunch. Its field should rise and fall between bunches and should remain zero for the rest of the time, not to disturb the circulating beam.

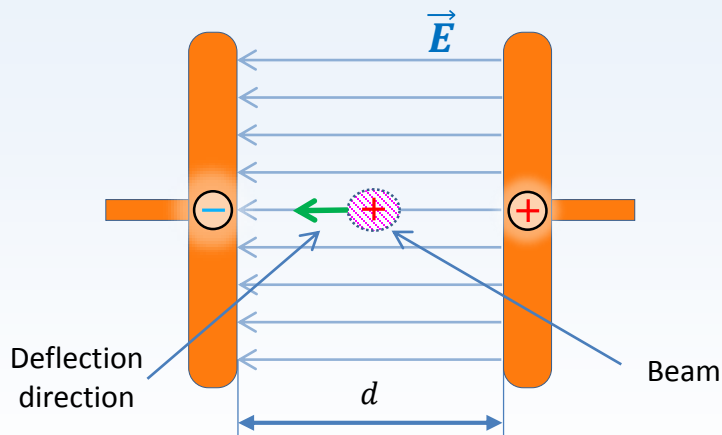


## Lumped element electric kicker

Electrostatic deflection angle  $\theta_M$  of particles with positive elementary charge :

$$\theta_E \approx \frac{U \cdot l_{eff}}{10^9 \cdot \beta \cdot p \cdot d}$$

where  $U$  is deflecting voltage,  $l_{eff}$  is effective length of the kicker,  $\beta$  is relativistic coefficient (fraction of the speed of light),  $p$  is particles momentum [GeV/c] and  $d$  is distance between electrodes.



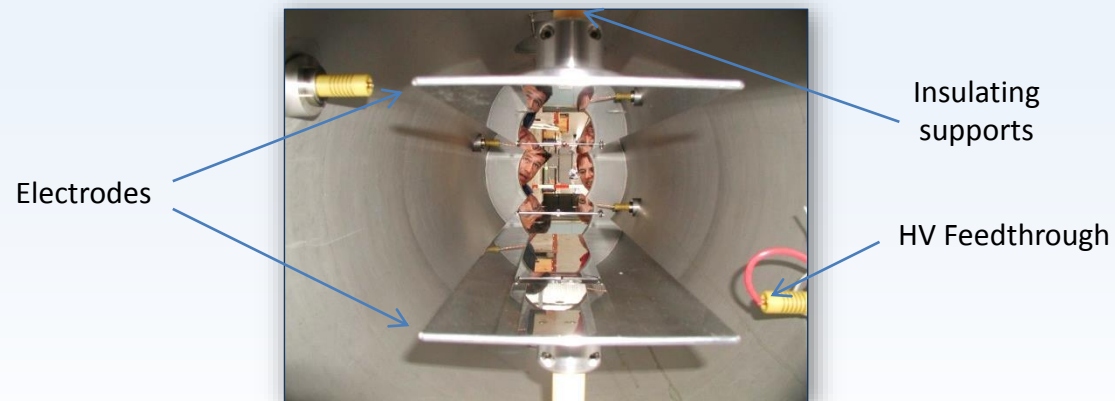
Lumped element electric kicker

Electric kickers are used to deflect heavy particles (muons, ions, etc.) – even at high energy they travel significantly slower than speed of light ( $\beta$  is in denominator) or where magnetic field should not be used (e.g. to prevent spin rotation)

Exponential risetime of the deflecting voltage  $U$ :

$$U(t) = U_0(1 - e^{-t/\tau}), \quad \tau = R_0 C_K$$

where  $U_0$  is the charging voltage,  $R_0$  is the charging impedance and  $C_K$  is kicker capacitance.



Courtesy of R. Carey

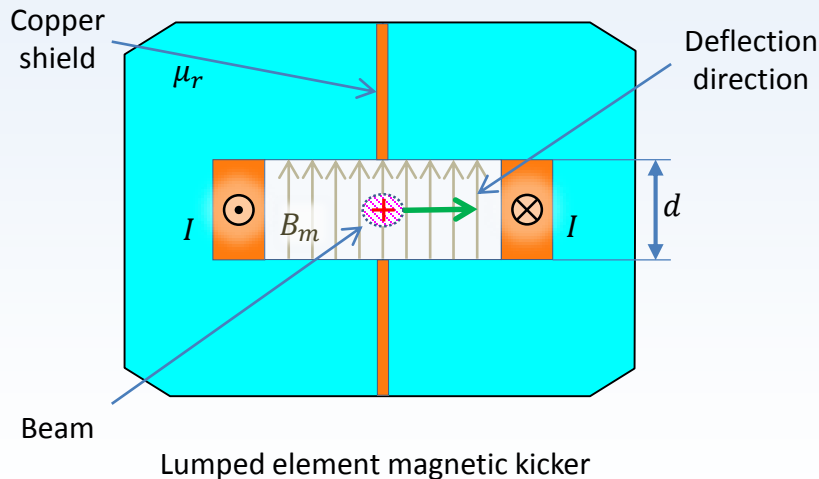
Muon electric kicker for MuLan experiment (PSI) – voltage  $\pm 12$  kV, field 0.16 MV/m, capacitance 74 pF, risetime 40 ns, minimum pulse length 160 ns<sup>[25, 17]</sup>

## Lumped element magnetic kicker

Magnetic deflection angle  $\theta_M$  of particles with positive elementary charge moving into the plane of page, for  $\mu_r \gg \mu_0$ :

$$\theta_M \approx \frac{0.3 \cdot \mu_0 \cdot l_{eff} \cdot n \cdot I}{p \cdot d}$$

where  $\mu_0$  is free space permeability,  $l_{eff}$  is effective length of the kicker,  $n$  is number of turns,  $I$  is deflecting current,  $p$  is particles' momentum [GeV/c] and  $d$  is magnetic gap.



Magnetic kickers are more efficient for relativistic beams and are widely used. Exponential risetime of the deflecting current  $I$ :

$$I(t) = I_0(1 - e^{-t/\tau})$$

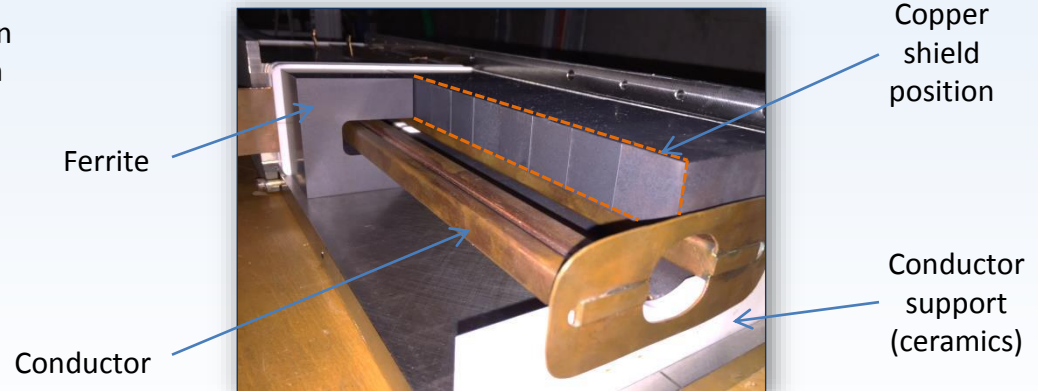
Terminated case:

$$I_0 = \frac{U_0}{2Z_0} \text{ and } \tau = \frac{L_K}{2Z_0}$$

Low impedance (“shorted”) case:

$$I_0 = \frac{U_0}{Z_0} \text{ and } \tau = \frac{L_K}{Z_0}$$

where  $U_0$  is charging voltage,  $Z_0$  is supply line impedance,  $\tau$  is corresponding time constant and  $L_K$  is kicker inductance.



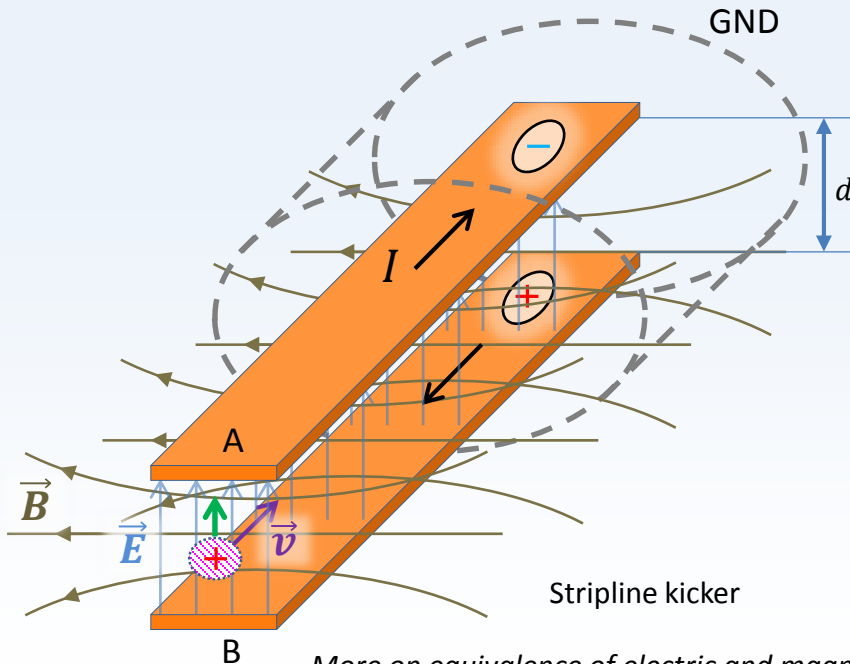
SLS Extraction kicker (PSI) - Kicker inductance 1  $\mu\text{H}$ , driving impedance 12.5  $\Omega$ , risetime 200 ns, pulse duration 1  $\mu\text{s}$ , 22 kV, 1.7 kA<sup>[29]</sup>

# Transmission line kicker - topology

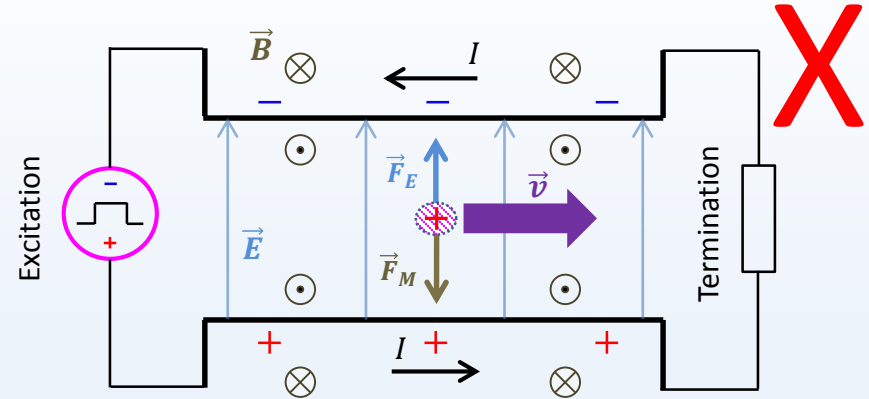
Electromagnetic deflection angle  $\theta_{EM}$  of particles with positive elementary charge:

$$\theta_{EM} = \theta_E \pm \theta_M \approx \frac{l_{eff}}{p} \left( \frac{E}{10^9 \cdot \beta \cdot d} \pm 0.3 \cdot B \right)$$

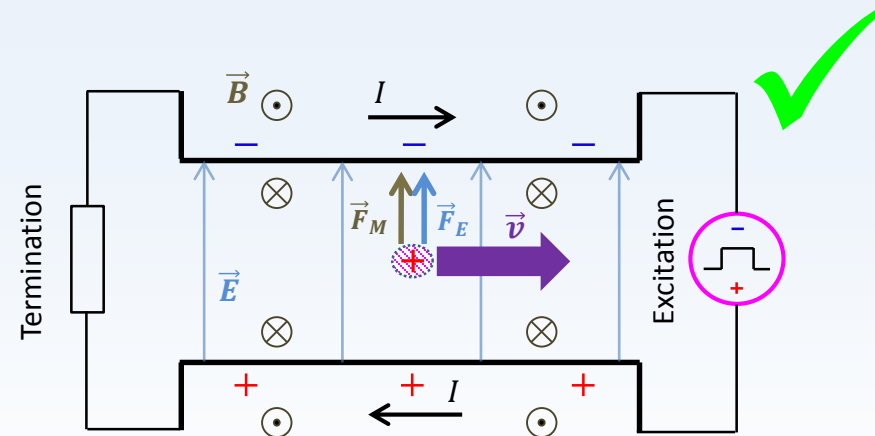
For relativistic particles in free space (no dielectric or ferrite) the electric and magnetic force components are roughly the same.



More on equivalence of electric and magnetic deflection – in “Additional material” at the end of the presentation.



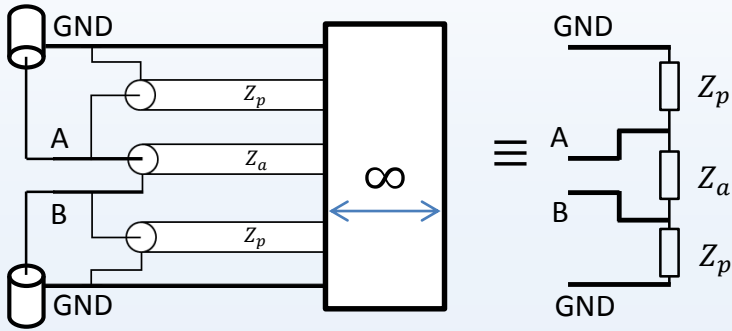
**Up-stream excitation** – electric and magnetic force cancel out.



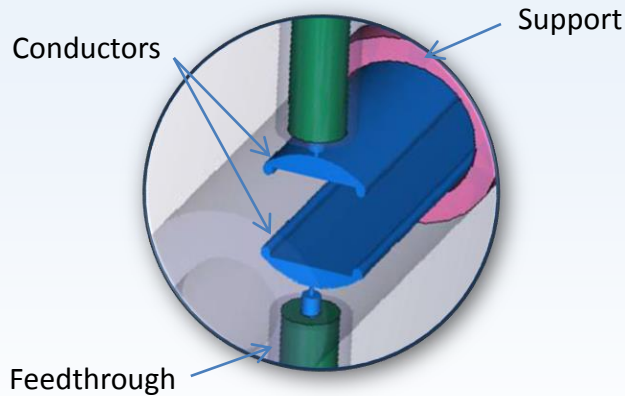
**Down-stream excitation** – electric and magnetic force add up<sup>[18, 3, 14, 21]</sup>

# Transmission line kicker – even and odd mode

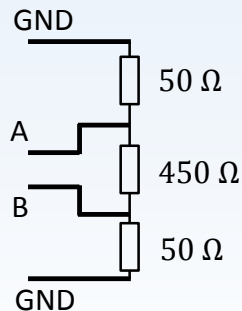
If transmission lines are matched there is no reflection, respectively no beam disturbance after the pulse finishes.



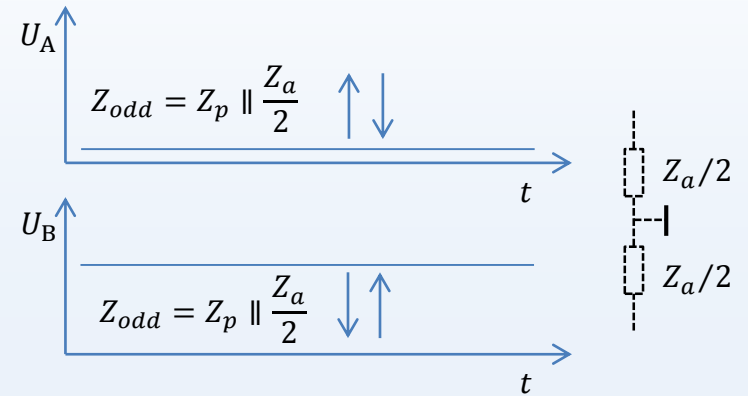
Placing the transmission line ( $Z_a$ ) inside conducting pipe (GND) creates two more parasitic transmission lines ( $Z_p$ )



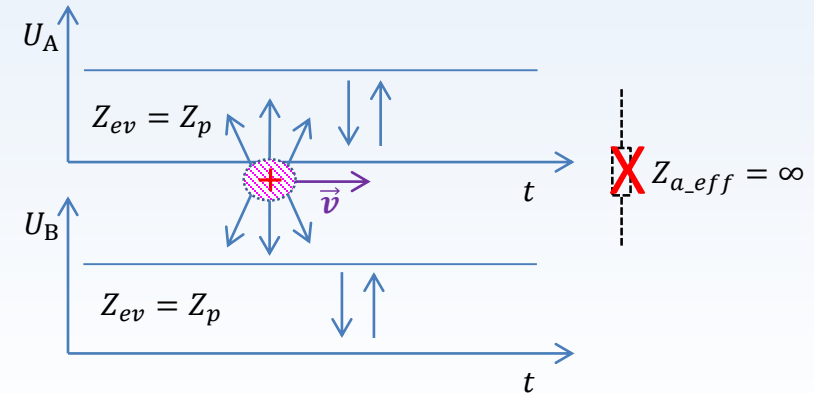
Courtesy of C. Aguilar



Termination and impedance components  $Z_a = 450 \Omega$ ,  $Z_p = 50 \Omega$ ,  $Z_{ev} = 50 \Omega$  and  $Z_{odd} = 40.9 \Omega$  [14]



Odd (or differential) mode – used to kick the beam



Even (or common) mode – induced by the beam back into the kicker conductors.

## Stripline kicker

Due to the lack of lumped inductance stripline kickers preserve the rise- and falltime of the excitation pulse thus can provide very fast deflection of the beam.

Since the total flux of terminated transmission line kicker is:

$$\Phi = \int (V_{In} - V_{Out}) dt$$

Kicker effective risetime :

$$t_r = t_{r\_pulse} + t_f$$

$$t_f = l/c$$

Where:

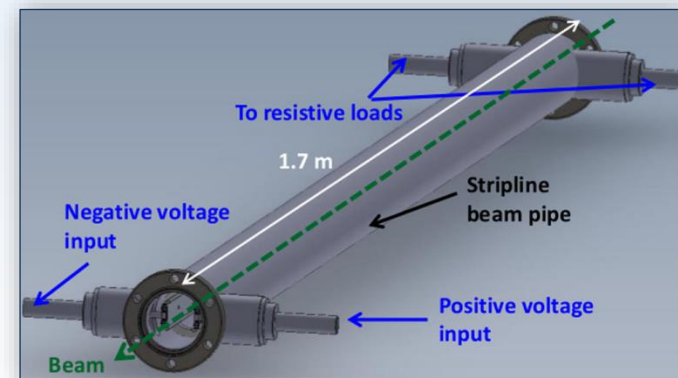
- $\Phi$  – total magnetic flux [Wb]
- $V_{In}, V_{Out}$  – input and output voltage [V]
- $t_{r\_pulse}$  – excitation pulse risetime [s]
- $t_f$  – transmission line kicker filling time [s]
- $l$  – length of the transmission line kicker [m]
- $c$  – speed of light [m/s]

Stripline configuration of a transmission line kicker provides compact and simple design.

Shape of stripline structure and the feedthroughs should be carefully optimized for field homogeneity, impedance and discontinuity.

Special care should be taken to avoid electrical discontinuity (change of characteristic impedance) especially in the feedthroughs to avoid reflections.

For the same length, voltage and current the deflection of transmission line kicker could be up to 15 times weaker compared to lumped element kicker (but fast)<sup>[18]</sup>



Courtesy of C. Aguilar

Model of extraction kicker for CLIC damping rings<sup>[14]</sup>



## Ferrite loaded transmission line kicker

In order to benefit from stronger magnetic field ferrite material is added. This increases the inductance of the transmission line (TL) and respectively its impedance. To keep the impedance low additional capacitance is added using conducting plates or lumped capacitors. This makes a periodic structure of  $n L_c C_c$  cells that has properties similar to these of a distributed TL at the expense of reduced propagation speed and limited bandwidth.

Impedance  $Z_0$ :

$$Z_0 = \sqrt{L_c / C_c}$$

Filling time:

$$t_f = n \sqrt{L_c C_c}$$

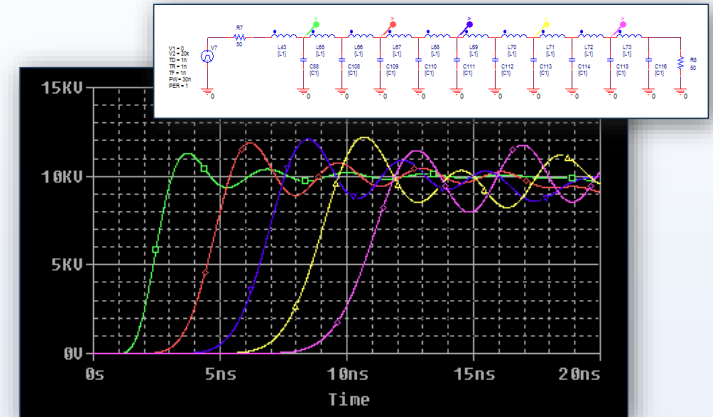
Pulse risetime is limited by Bragg cutoff frequency:

$$f_b = 1 / \pi \sqrt{L_c C_c}$$

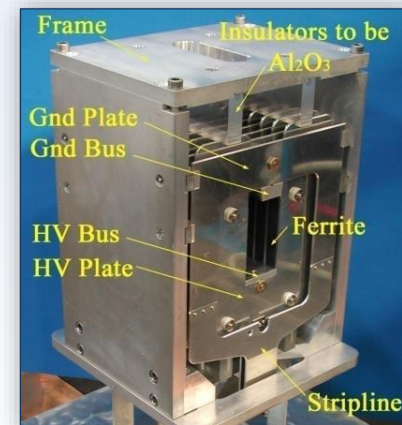
Kicker effective risetime:

$$t_r = t_{r\_pulse} + t_f$$

Ferrite loaded TL kicker is complex and requires care to cope with HV challenges and beam coupling.



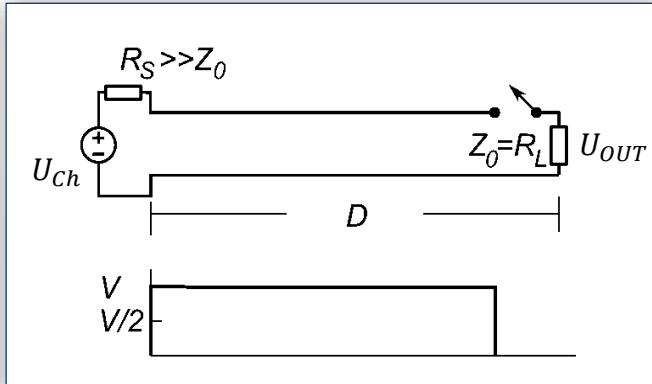
Propagation delay, ripple and risetime degradation after 1<sup>st</sup>, 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup> and 9<sup>th</sup> cell (pSpice). Pulse risetime 1 ns,  $L_c = 50$  nH,  $C_c = 20$  pF,  $Z_0 = 50 \Omega$ ,  $f_b = 318$  MHz.



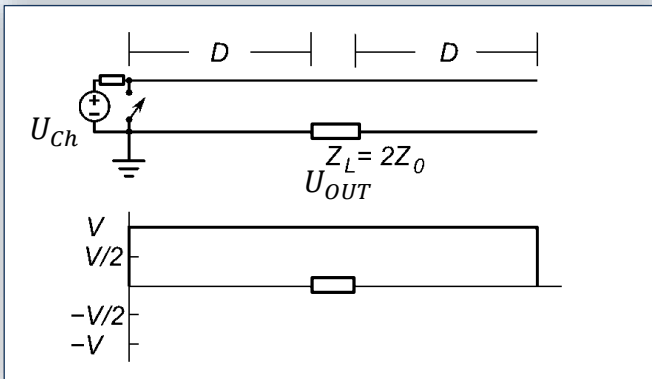
Courtesy of M. Barnes

Prototype of TRIUMF Kicker Magnet <sup>[20]</sup>

## Pulse forming lines (PFL)



Simple PFL configuration with matched load. Pulse **amplitude is half** of charging voltage and pulse **duration is twice** the electrical length of PFL.



Blumlein PFL configuration with matched load. Pulse **amplitude is equal** to charging voltage and pulse **duration is equal** to the electrical length of PFL

Pulse length:

$$t_p = 2t_{line}$$

Output voltage  $U_{out}$  and current  $I_{out}$   
Matched ( $R_L = Z_0$ , no reflection):

$$U_{out} = U_{Ch}, I_{out} = \frac{U_{Ch}}{2Z_0}$$

“Shorted” ( $R_L = 0$ , full inverted reflection) :

$$U_{out} = 0, I_{out} = \frac{U_{Ch}}{Z_0}$$

where  $t_p$  is electrical length of the PFL and  $U_{Ch}$  is charging voltage

“Shorted” systems require termination on the other end of PFL.



SLS Extraction kicker PFL (4 + 1x 50  $\Omega$ , 100 m, PSI) in “shorted” configuration and termination unit:  $t_p = 1 \mu\text{s}$ ,  $Z_0 = 12.5 \Omega$ ,  $U_{Ch} = 22 \text{ kV}$ ,  $I_{out} = 1.7 \text{ kA}$ <sup>[29]</sup>

## Pulse forming networks (PFN)

Pulsed Forming Networks (PFN) provide a more compact pulse forming solution. They use an electrical approximation of distributed transmission line (like a coaxial cable) built with a series of elementary LC cells.

Since the approximation is not perfect PFNs have limited bandwidth and are prone to ripples. Adjustment of each individual cell may be needed for best performance.

Impedance  $Z_0$ :

$$Z_0 = \sqrt{L_c/C_c}$$

Electrical length:

$$l_e = n\sqrt{L_c C_c}$$

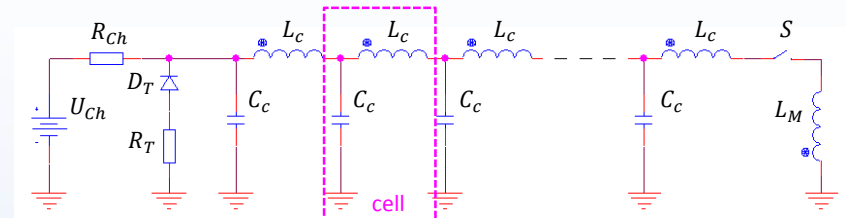
Pulse length:

$$t_p = 2l_e$$

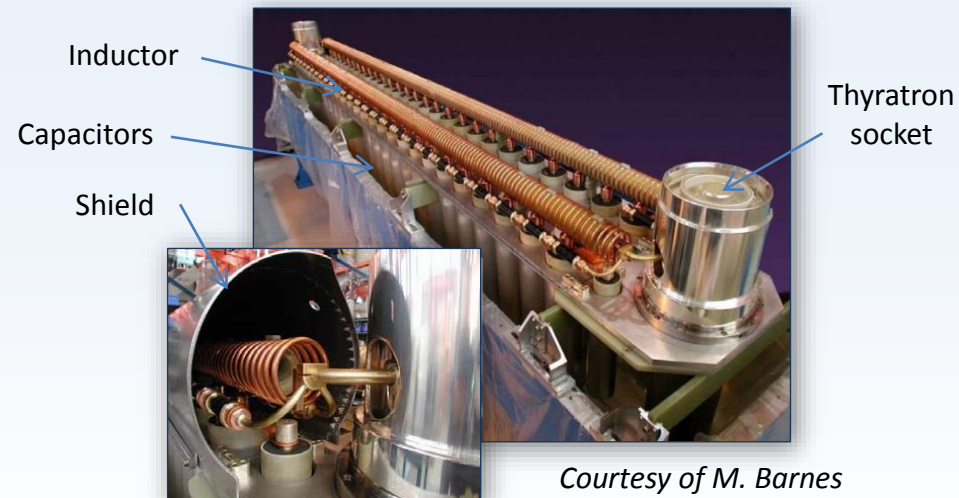
Pulse risetime is limited by Bragg cutoff frequency:

$$f_b = 1/\pi\sqrt{L_c C_c}$$

Where  $L_c$  is cell inductance,  $C_c$  is cell capacitance and  $n$  is number of cells.



PFN consists of many cells connected in series. It is charged by voltage source  $U_{Ch}$  and charging resistor  $R_{Ch}$ . When switch  $S$  operates the PFN is discharged through the magnet  $L_M$ . In order to absorb inverted reflection there is a reversed diode  $D_T$  and termination resistor  $R_T$  at the end of the PFN.



Courtesy of M. Barnes

LHC Injection PFN (CERN)  $t_p \leq 7 \mu\text{s}$ ,  $L_c = 1.7 \mu\text{H}$ ,  $C_c = 19.6 \text{ nF}$ ,  $n = 26$ ,  $t_r = 900 \text{ ns}$ ,  $Z_0 = 5 \Omega$  (two  $10 \Omega$  PFN in parallel),  $U_{Ch} = 54 \text{ kV}$ ,  $I_{out} = 5.4 \text{ kA}$ <sup>[20, 31]</sup>

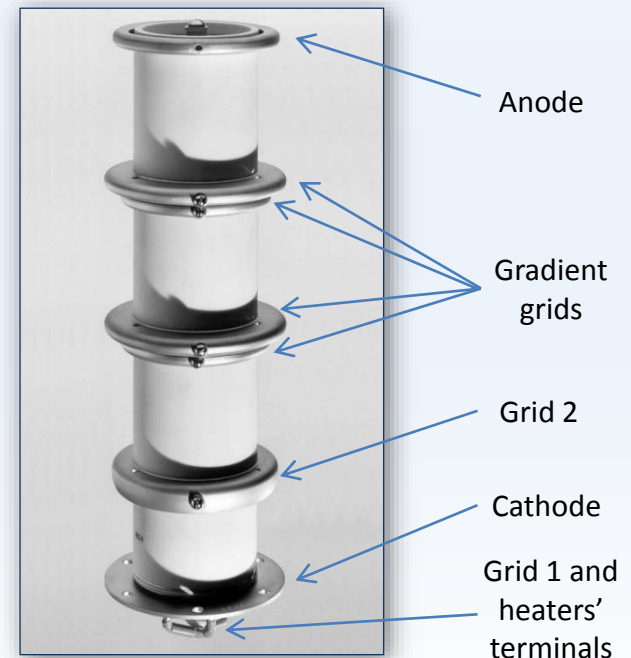
## PFL/PFN switch

A PFL/PFN requires (at least) one closing switch able to operate reliably at the charging voltage and discharge current. Since voltages and currents are generally high the choice of the switches is limited to deuterium thyratrons and thyristor (in systems where slower turn-on is possible).

With the recent advancement of the semiconductors technology (especially the emergence of HV SiC MOSFETs) there is a continuous effort to develop reliable solid state replacement of the thyratrons.

Thyratrons switches:

- Capable of switching HV and high currents (considering not only normal operation but breakdown or fault scenarios)
- Fast turn-on (tens of nanoseconds) but **no commanded turn-off** possible
- Adequate housing (cooling, low inductance, **X-ray emission**)
- Measures against **spontaneous turn-on** (fast charging of PFL/PFN) or **misfiring**
- Supporting circuits (cathode heater, reservoir heater and triggering, especially if the switch is in “**floating configuration**”)
- Limited lifetime
- Sensitive to **reverse conduction** (some models are capable of tolerating some reverse conduction)



E2V Three-Gap Deuterium-Filled Ceramic Thyratron CX1171, Blocking voltage 80 kV (peak 105 kV), Average current 2 A, Peak current 3 kA (single shot @0.1 Hz 10 kA), 30 ns risetime, time jitter 1..5 ns

## PFL vs PFN



Pulse Forming Line (PFL)

- Simple design
- Short risetimes (switch or cable dispersion limited)
- Short pulses (maximum pulse length defined by the electrical length of the cable)
- Ripple-free (flat) pulses
- Cable attenuation becomes problematic (pulses  $> 3 \mu\text{s}$  and droop  $< 1\%$ )
- Bulky:  $1 \mu\text{s}$  pulse  $\sim 100$  m of cable
- Require high voltage and low-loss coaxial cable (often custom made) [8]



Pulse Forming Network (PFN)

- Compact design
- Low droop (long pulses  $> 3 \mu\text{s}$ )
- Complex construction
- More cells, better transmission line representation
- Risetime limited (Bragg cut-off frequency)
- Pulses are prone to ripples (degraded flat-top) - may require adjustments of each individual cell : difficult and time consuming
- Require high voltage capacitors

## Kickers recap

- **Fast acting** beam deflecting element (electric, magnetic or combined)
- Usually providing relatively **small deflection**
- Essential part of beam **injection** and **extraction** systems, beam **dump** and **dilution** systems, **feedback** systems
- **Complex mechanical** and **electrical design**, often a limiting factor for overall system performance
- **Parasitic coupling** to the beam – unwanted energy exchange with the beam – goes in both directions: degrades beam quality / stability and could damage / disable the kicker element or its systems
- **Electric** – fast but not efficient for highly relativistic beams
- **Magnetic** – efficient but slow due to its lumped inductance
- **Transmission line** (travelling wave ) kicker – fast, combines electric and magnetic deflection, limited by electrical impedance and electrical vacuum breakdown
- Often crucial **field time profile** – fast rise- and fall- times, precise pulse duration, stringent tail-effect requirements etc.
- **Demanding power supply systems** – high speed, high voltage and current switches, complex intermediate energy storage elements (PFL/PFN, HV capacitors)
- **Stringent reliability requirements** – especially when part of safety and machine protection systems (emergency beam dump, beam dilution etc.)



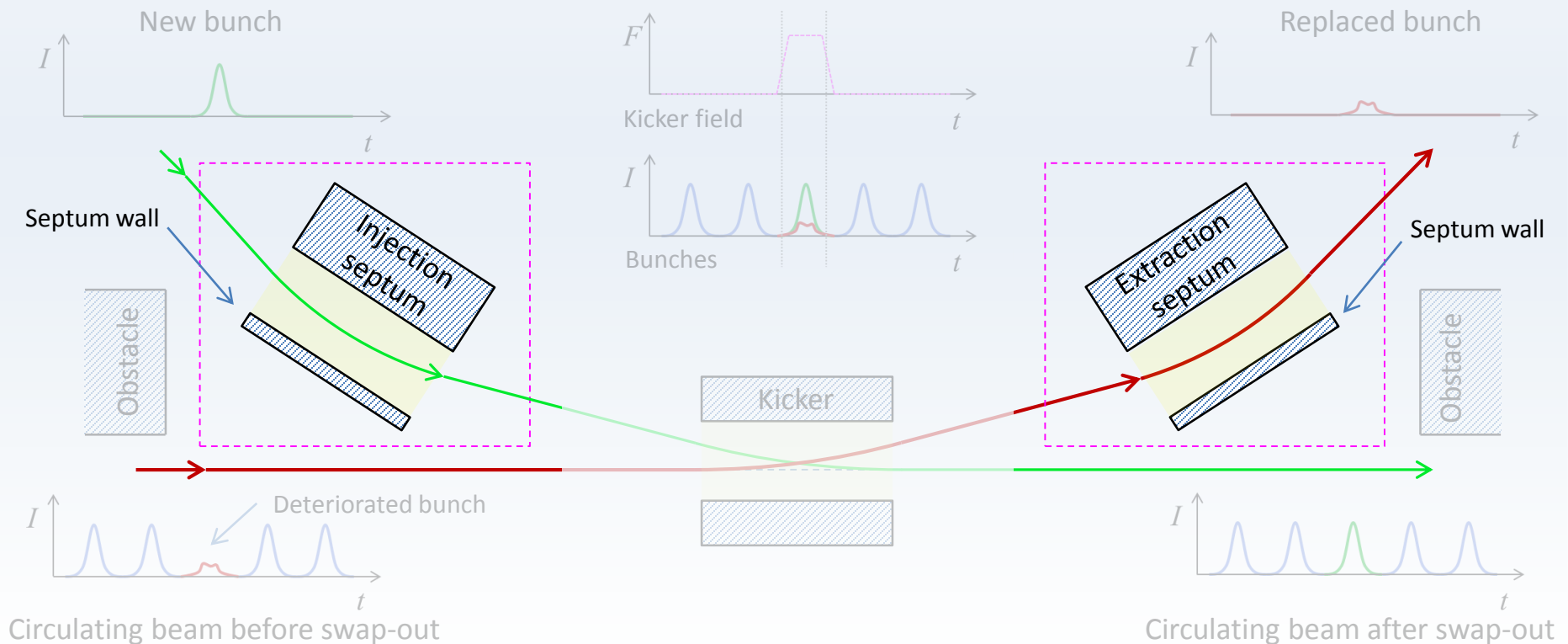
# Septa

## Septum - function

A septum is a selective beam deflection element, usually DC or slow pulse. It is used for beam injection, extraction and beam dump.

Example: “Swap-out on axis” injection scheme

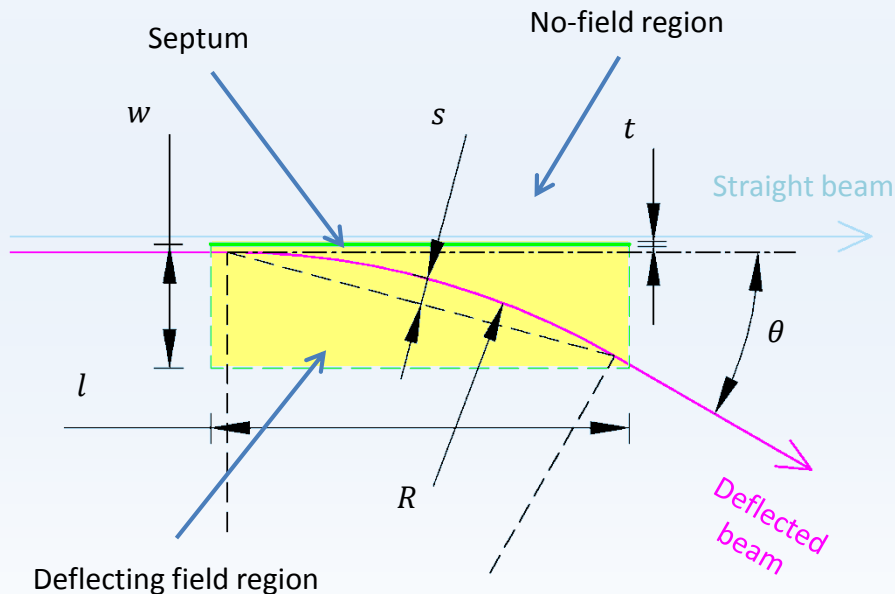
Septa have abruptly changing deflection field depending on spatial position. Ideally, on one side of the septum wall there is zero deflecting field and on the other, there is strong homogeneous deflecting field. Usually septa provide the necessary additional deflection to the “kicked” beam to clear other machine components.





## Basic concept and terminology

- ❑ A septum shares a lot in common with dipole (bending) magnets
- ❑ It has an abrupt field change between field and no-field region



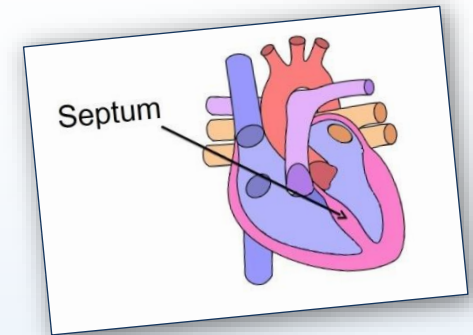
Schematic representation of a septum

$\theta$  – Bending angle  
 $R$  – Bending radius  
 $s$  – Sagitta  
 $w$  – Deflecting gap width  
 $t$  – Septum thickness  
 $l$  – Septum length

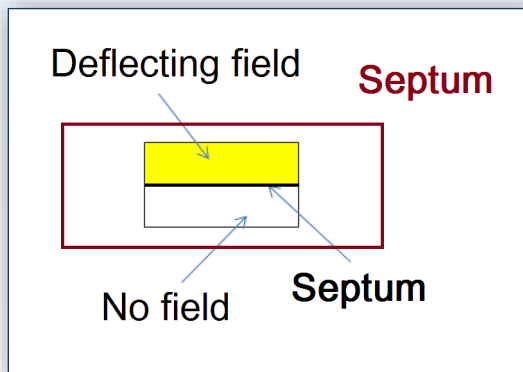
If deflecting gap does not follow the trajectory of the deflected beam (as shown here) the deflecting gap should be wide enough to accommodate the deflected beam trajectory.

## Septum

- ❑ A **septum** (plural **septa**) is a partition, a wall, a barrier that separates two cavities or two chambers (biology, mechanics, part. physics, etc.).
- ❑ Latin origin - *saepio* (*sēpiō*) - surround, enclose, fence in.
- ❑ In particle accelerators, a **septum** separates two distinctive field regions in order to selectively deflect particle beams.
- ❑ Used for **injection** and **extraction** of the beam



The septum separates the left and the right side of the heart (biology)



Schematic representation of a septum device

- ❑ Often the device that embodies the **septum** is called **septum** as well (*electrostatic septum*, *septum magnet*, etc.)

~~Septums~~

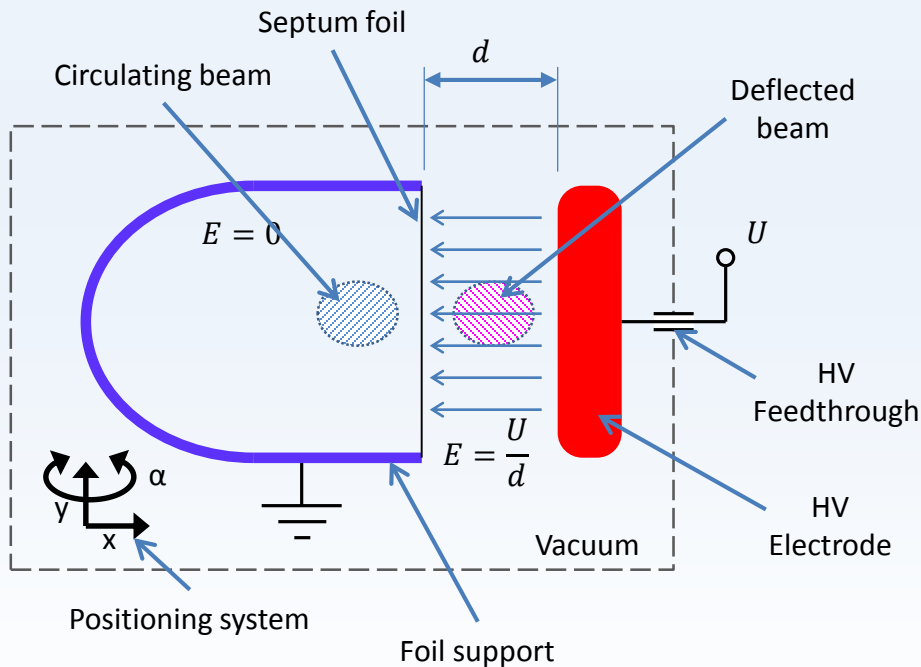
✓ Septa

## Electrostatic septum

Electric field is established between a HV electrode and a septum foil. The extracted beam passes through the electric field region and it is deflected.

Using Faraday cage effect the foil and the foil support create a zero-field region for the circulating beam that goes straight.

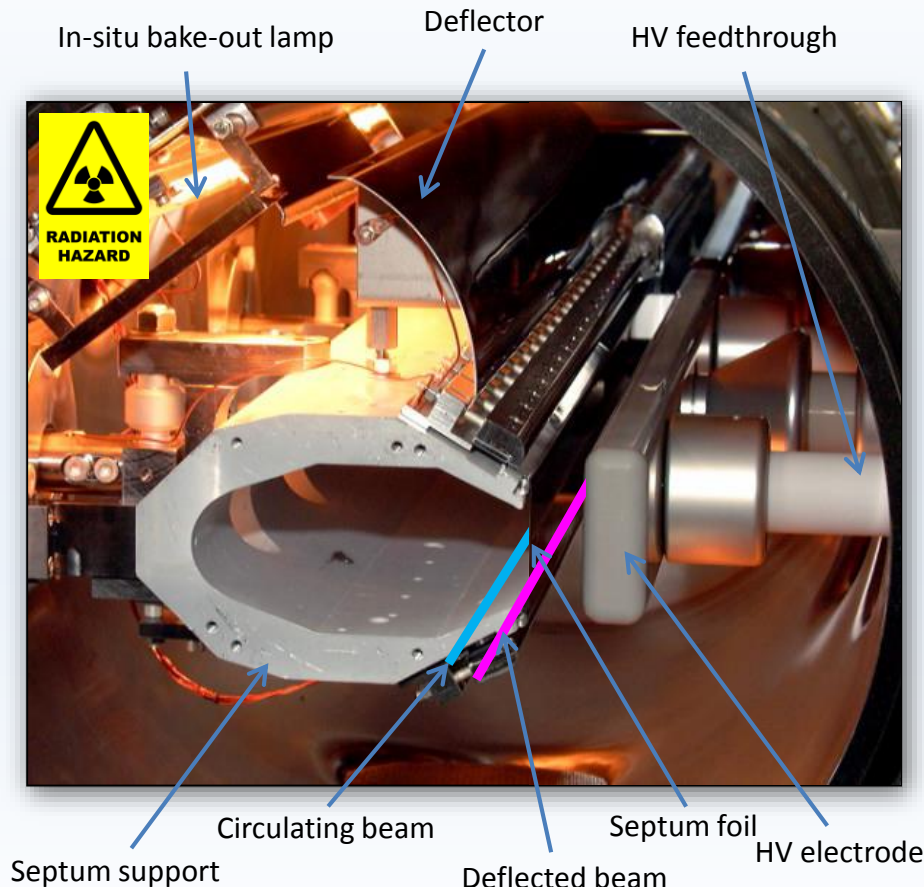
Electrostatic septa provide the smallest septum thickness.



Faraday cage used to protect a person while demonstrating discharges from Tesla transformer

- Thin foil is used to minimize the interaction with beam (reduce beam losses and radiation levels)
- Surface treatment of the HV exposed components for higher electric field
- To utilize precise alignment with respect to the circulating beam often the septum is mounted on precision mover system
- Care should be taken to ensure good vacuum conduction in order to maintain low background pressure
- Often septa are the most irradiated devices in an accelerator. **They get radioactively activated.** Safety precautions should be taken when maintaining or discarding the devices

## Foil electrostatic septum



*Courtesy of CERN Septa Section*

PS "Septum 23" (CERN) – gap 17 mm, Septum thickness 100  $\mu\text{m}$ , voltage up to 260 kV, electric field up to 15 MV/m <sup>[27, 28]</sup>

In-vacuum electrostatic foil (wire) septum gives the thinnest possible septum barrier.

Deflectors are used to improve the field homogeneity in the gap and to protect the HV elements from titanium sublimation pumps <sup>[27]</sup>

Holes in the septum support should ensure good vacuum conductivity, respectively low background pressure

HV conditioning to reach maximum electric field

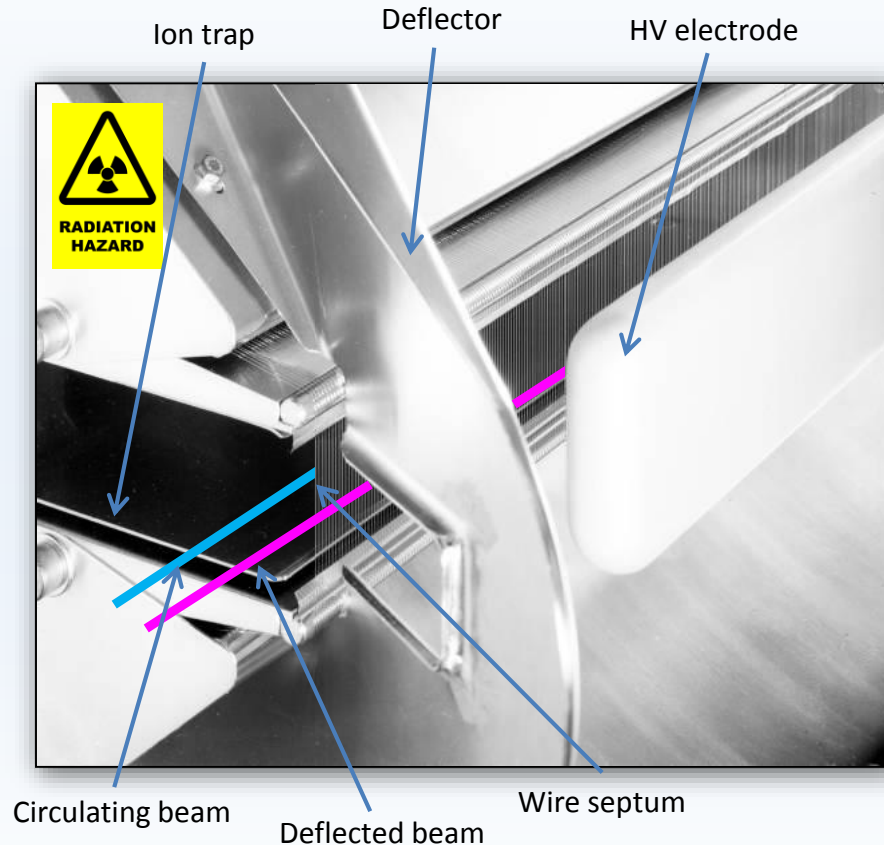
Depending on required vacuum level provisions for in-situ bake-out should be implemented

Depending on the irradiation dose the insulating oil of the HV feedthroughs and the close HV cables have to be exchanged (radiation deterioration)

Maintenance - radioactive activation

## Wire electrostatic septum

The thin foil septum is substituted by array of thin wires for even lower interaction with beam



*Courtesy of CERN Septa Section*

SPS wire septum ZS (CERN) – gap 20 mm, septum wire diameter 50 – 100  $\mu\text{m}$ , wire spacing 1.5 mm, voltage up to 220 kV, electric field up to 11 MV/m, Ion traps voltage: 3 to 6.5 kV <sup>[23]</sup>

An array of wires reduces the effective density of the septum, decreasing beam loss and radiation

High electric field possible

Field leakage in no-field region

Wires array increase the vacuum conductivity to the screened volume

Individual tensioner on each wire

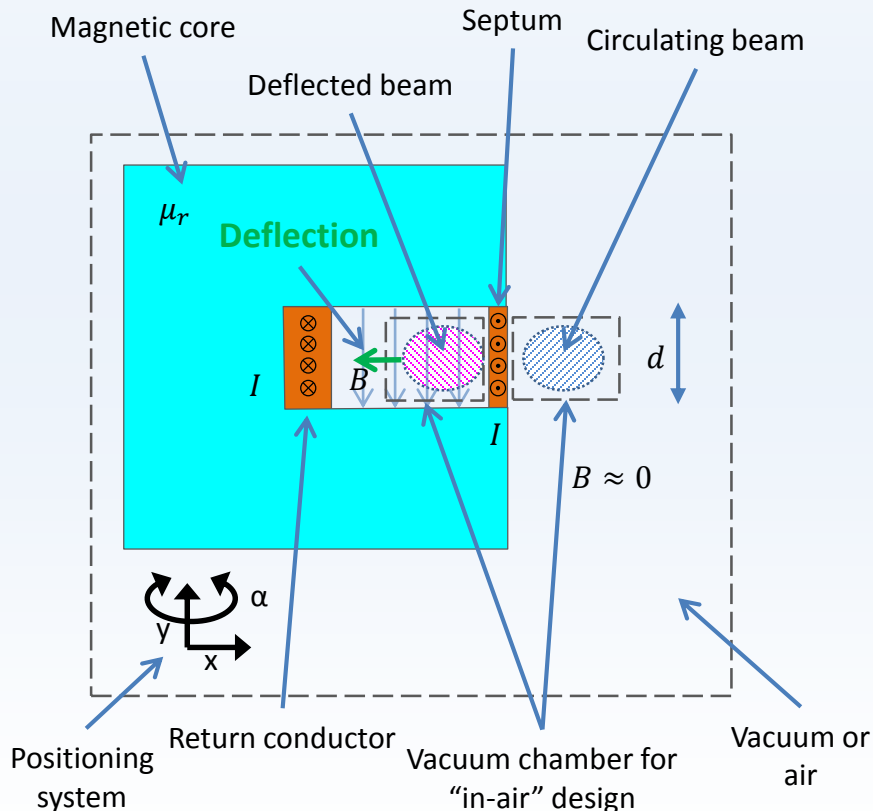
Ionization of residual gas in the field-free region can provoke a HV breakdown. Care should be taken to remove ions from the volume

White array interaction with beam could cause excessive heating and consecutive destruction of the septum array. Proper protection mechanisms should be implemented to avoid damage.

Maintenance - radioactive activation

## Magnetic septum

The deflected beam goes through homogeneous magnetic field that is established between two magnetic poles. The circulating (straight) beam passes next to main magnetic circuit “seeing” as little as possible magnetic field. Often magnetic screening techniques are used to shield the straight beam.



$$B \approx \frac{\mu_0 \cdot N \cdot I}{d}$$

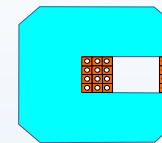
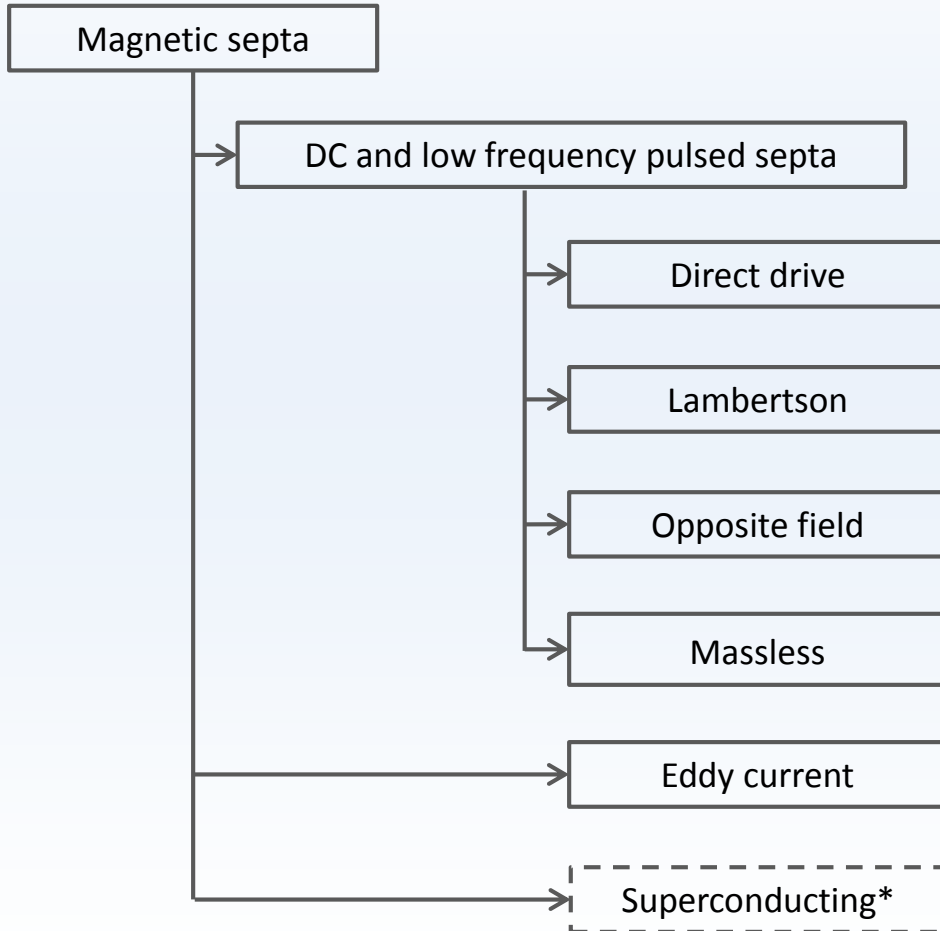
Follows from Hopkinson's law (analogous to Ohm's law), for  $\mu_r \gg 1$

Where:

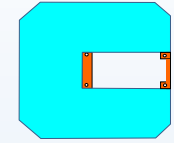
- $B$  – magnetic flux density [T]
- $\mu_0$  – vacuum permeability [H/m]
- $\mu_r$  – core relative permeability [-]
- $N$  – number of turns [-]
- $I$  – current [A]
- $d$  – gap [m]

# Types of magnetic septa

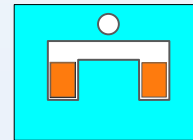
Classification according to magnetic field variation in time. Basically each type can be “in-vacuum” or “in-air design”



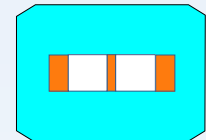
Direct drive DC



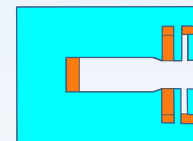
Direct drive LF pulsed



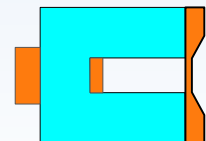
Lambertson



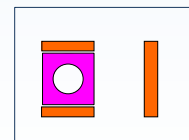
Opposite field



Massless



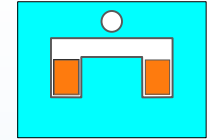
Eddy current



Superconducting

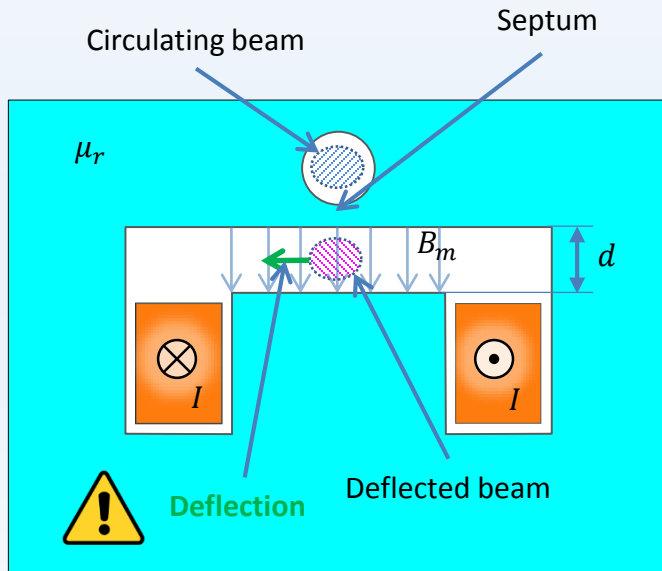
\* Proposed design

# Lambertson septum



Lambertson septum

Due to magnetic circuit symmetry the circulating beam area has very low leakage field.



Lambertson septum

Thin septum possible

Low leakage field. To further reduce the leakage field a mu-metal screen could be added

More complex geometry

Poor vacuum conduction at circulating beam volume. NEG coating might be necessary.

DC or LF pulsed design

**Deflection perpendicular to beam displacement**

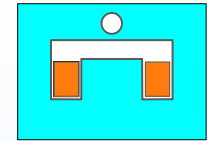


(In shown example:)

Kicker magnet is used to deflect the beam **vertically** (Down) and then the Lambertson septum deflects the beam **horizontally** (To the left)

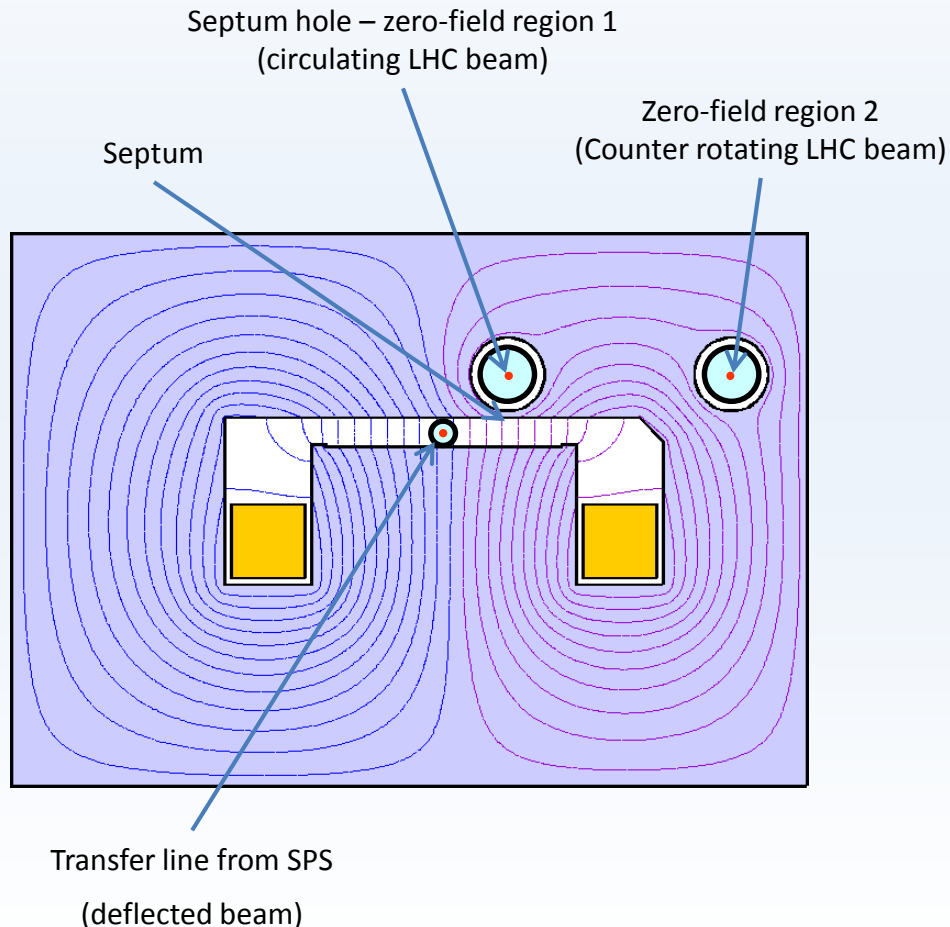


## Lambertson septum - example



Lambertson septum

Design specifics of MSIA (LHC, CERN)<sup>[27, 16]</sup>



“In-air” design

Two zero-field regions for circulating beam  
and for counter rotating beam

Mu metal chambers (thickness: 0.9 mm, 0.8  
T saturation) for additional screening

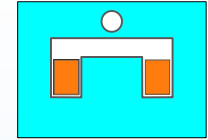
Chamber is copper coated (0.4 mm) to  
improve beam impedance

Vacuum chamber is NEG coated

In-situ bake-out 200°C

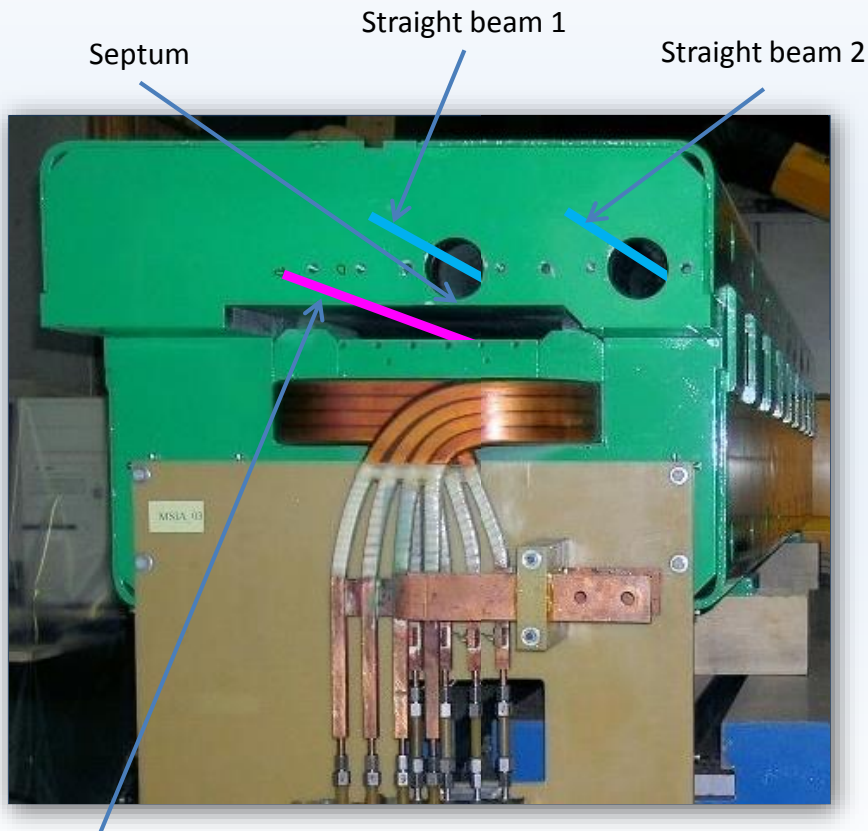
Top yoke side (with zero-field regions)  
extends 175 mm on each side to screen the  
fringe fields

## Lambertson septum - example



Lambertson septum

Construction and technical data of “in-air” Lambertson septum MSIA (LHC, CERN)



Deflected beam

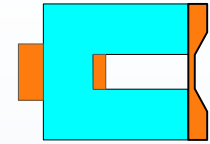
*Courtesy of CERN Septa Section*

Lambertson septum MSIA (LHC, CERN)<sup>[27, 16]</sup>

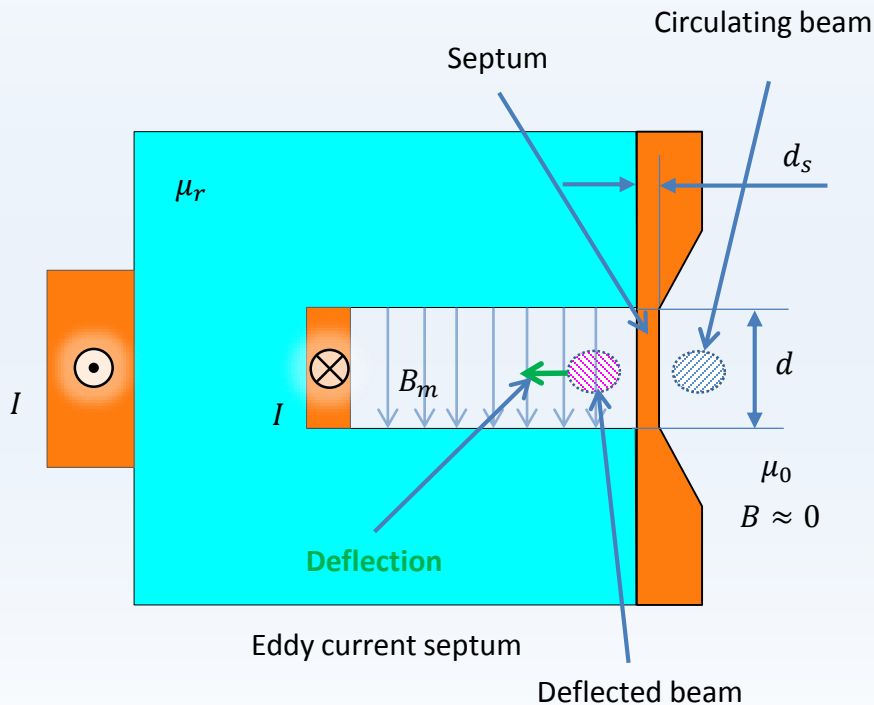
- Vacuum:  $10^{-7}$  mbar
- Field length : 3650 mm
- Gap height: 25 mm
- Gap width: 230 mm
- Beam momentum: 450 GeV/c
- Deflection angle: 1.846 mrad
- Septum thickness: 6 mm
- Current: 950 A x 16 turns
- Magnetic flux density: 0.76 T
- Cooling water flow rate: 7.9 L/min
- Dissipated power: 10.6 kW

## Eddy current septum

Eddy currents in the septum conductor cancel the changing magnetic field (eddy currents screening)



Eddy current septum



Thin septum possible

Eddy currents dissipate power as well (edge cooling might be necessary)

Doesn't work for DC magnets

Low leakage fields

Maximum of the leakage field appears after certain delay

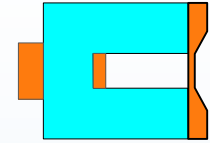
More complex pulsed power supplies (short pulses)

Low inductance magnets (single turn)

Combined with thin mu-metal screening brings the ratio main field to leakage field to  $>1000:1$

# AC magnetic field penetration

Eddy currents always flow in such direction that their magnetic field opposes the change of the magnetic field that produces them (Lenz's law)



Eddy current septum

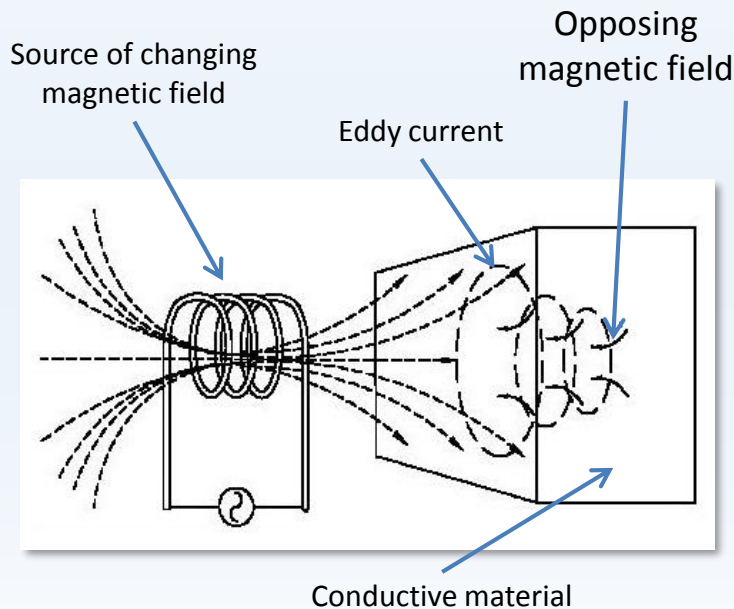


Illustration of magnetic field cancelation due to eddy currents in bulk conductor

Skin depth (field penetration)  $\delta$  [m] – distance after the AC current is reduced 63%.

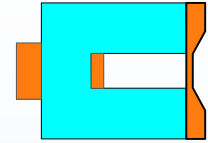
$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \mu_r \sigma}}$$

Where:

- $f$  – magnetic field frequency [Hz]
- $\mu_0$  – vacuum permeability [H/m]
- $\mu_r$  – relative permeability [-]
- $\sigma$  – material conductivity [S/m]

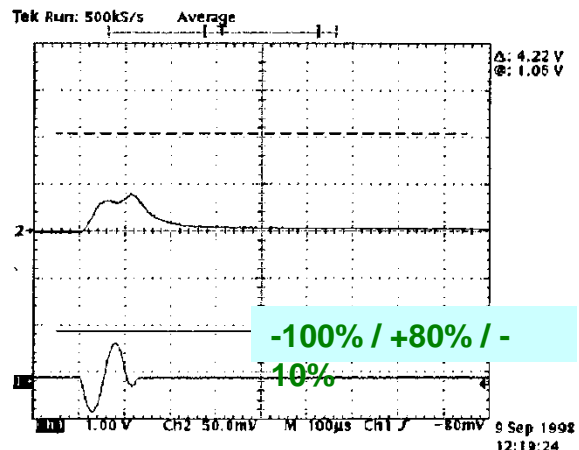
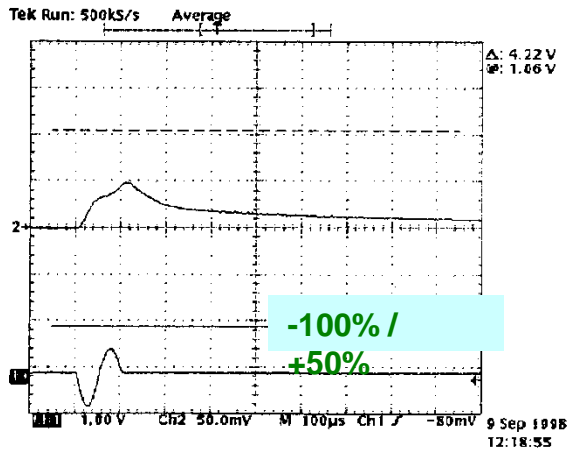
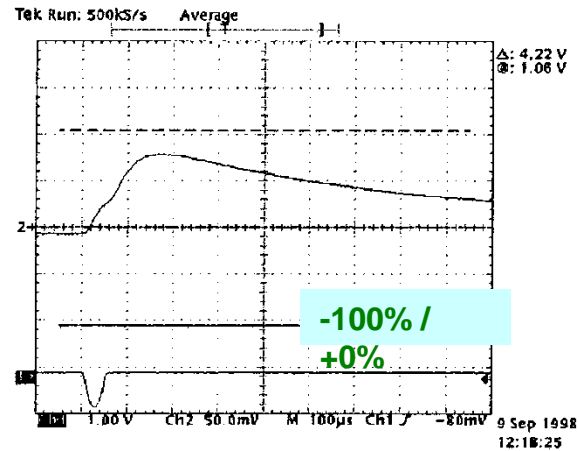
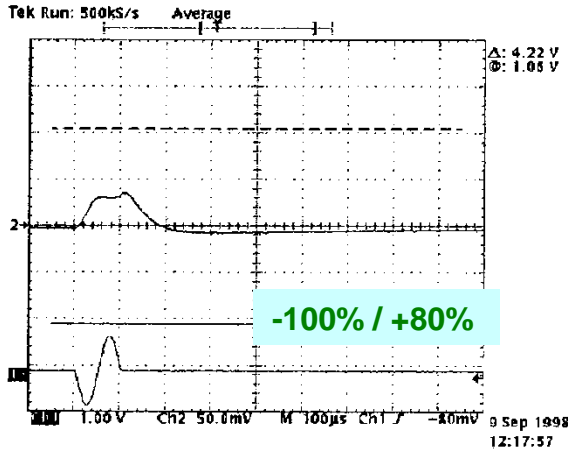
- High frequencies penetrate less
- High conductivity materials screen better
- Penetration in magnetic materials is smaller

# Magnetic pulse waveform form



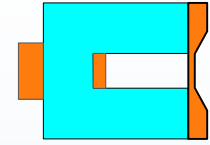
What is the best magnetic pulse waveform?

Eddy current septum



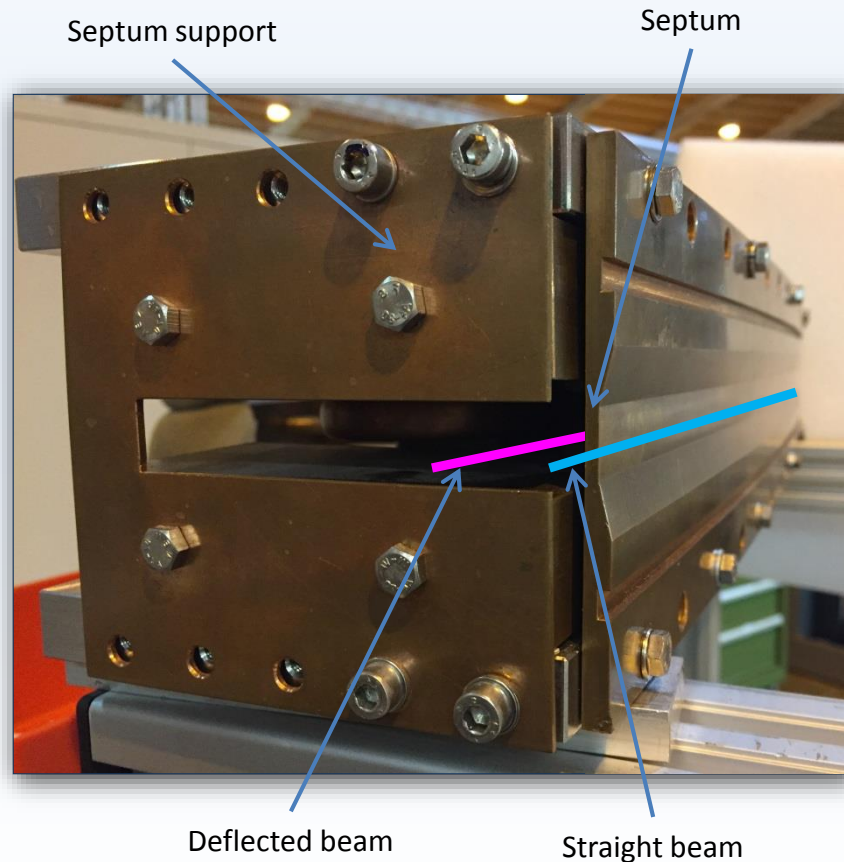
Courtesy of C. Gough

## Eddy current septum - example



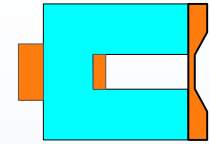
Eddy current septum

Construction and technical data of “in-vacuum” eddy current septum for SLS (PSI)<sup>[24]</sup>



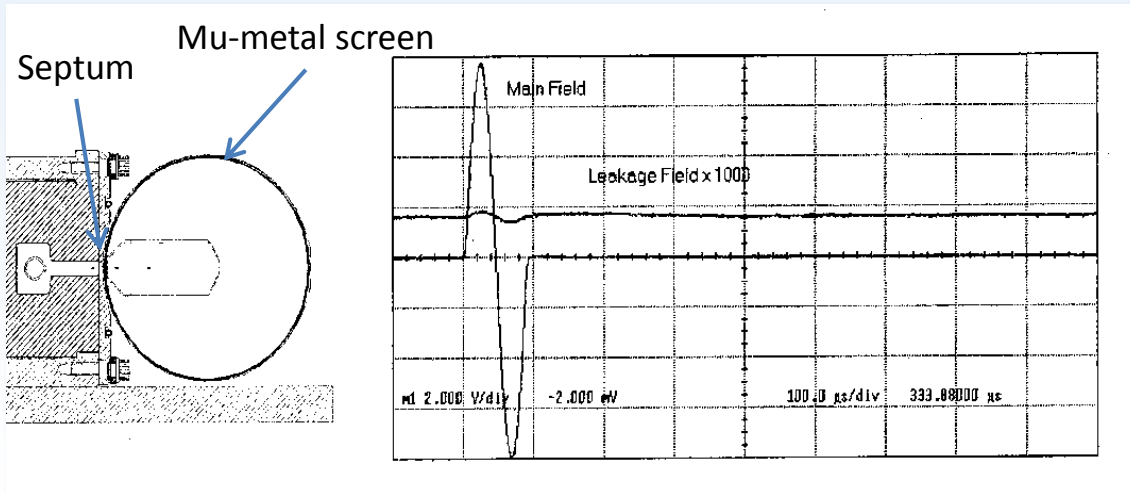
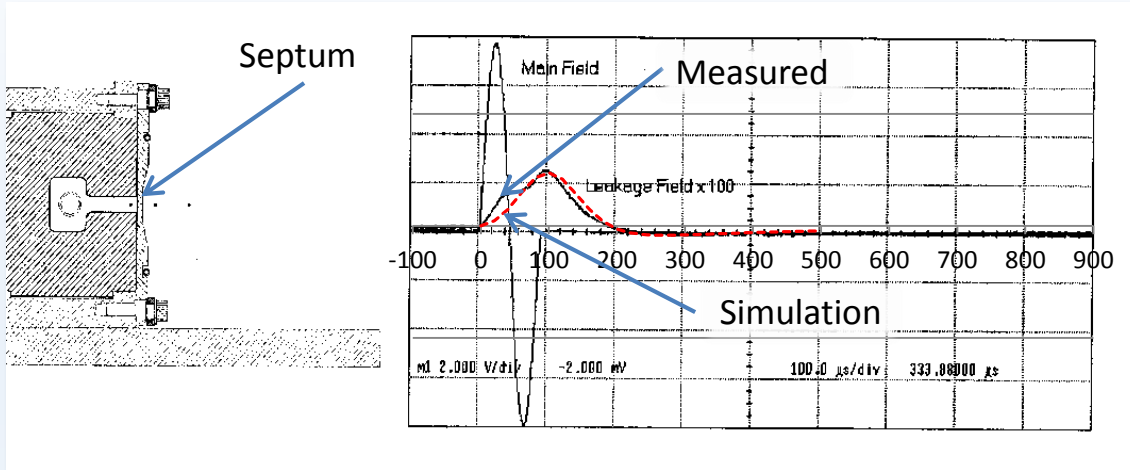
- Vacuum:  $10^{-7}$  mbar
- Field length : 600 mm
- Gap height: 6 mm
- Gap width: 20 mm
- Beam momentum: 2.4 GeV/ $c$
- Deflection angle: 70 mrad
- Septum thickness: 2.5 mm
- Current: 4.3 kA (full sine 0.16 ms)
- Magnetic flux density: 0.9 T
- Eddy currents septum extends 50 mm on each side of the magnet to screen the fringe fields

# Eddy current septum - example



Eddy current septum

Leakage field measurements with and without mu-metal screen<sup>[24]</sup>



- Leakage field is reduced below 0.01%
- Leakage field only varies by 2:1 over entire horizontal volume
- With mu-metal screen, integral leakage dominated by screen end effect (integral leakage  $<1e-6$  is possible)

Courtesy of C. Gough

## Septa recap

- **Slow acting or DC** beam deflecting element (electric or magnetic)
- Usually providing **large deflection**
- Essential part of beam **injection, extraction** and **beam dump** systems
- **Complex mechanical, thermal and vacuum design**, often working close to materials' damage threshold (mechanical, electrical, thermal).
- **Field homogeneity and beam impedance** (injection/extraction septa)
- Usually **highest radiation point** in the machine – gets radioactively activated. Materials degradation due to radiation (liquid and solid insulators degradation)
- **Electric** – thinnest septum possible but not efficient for highly relativistic beams
- **Magnetic** – efficient but at the cost of thicker septum
- Has to **accommodate the deflected beam trajectory** (sagitta)
- **Low leakage field** for the straight beam (in some cases leakage field could appear long after the main septum pulse due to slow field penetration through septum)
- **Interaction with adjacent magnetic elements** (beam optics components, dipoles) due to large magnetic field leakage
- **Stringent reliability requirements** – especially when part of safety and machine protection systems (emergency beam dump, beam dilution etc.)
- **Economics** – dissipated power, supporting systems



A photograph of a long, curved tunnel, likely a particle accelerator, with a bright light source at the end. The tunnel is illuminated with a greenish-yellow light, and the walls are made of concrete or metal. The perspective is from the entrance, looking down the length of the tunnel.

# Future colliders

# Future colliders

Future colliders pose huge resources and technological challenges but they are the frontiers to the tomorrow's science and physics in particular.

With the discovery of a Higgs boson, after > 50 years of superb theoretical and experimental work the SM is now complete. However major questions remain.



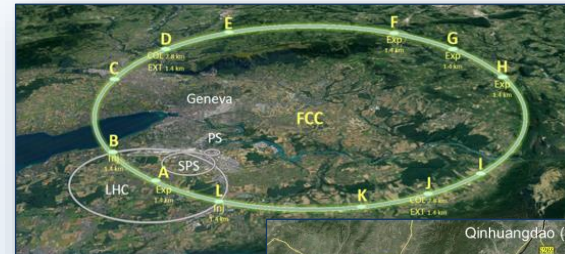
Fabiola Gianotti  
CERN director general

The full exploitation of the LHC, and more powerful future accelerators, will be needed to address them and to advance our knowledge of fundamental physics.

No doubt that future high-E colliders are extremely challenging projects Didn't the LHC also look close-to-impossible in the '80s ??

However: the correct approach, as scientists, is not to abandon our exploratory spirit, nor give up to financial and technical challenges. The correct approach is to use our creativity to develop the technologies needed to make future projects financially and technically affordable

High Energy LHC (HE-LHC)  
Future Circular Collider (FCC)



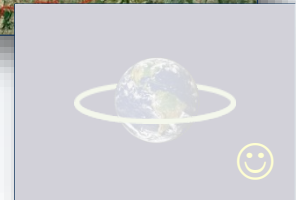
Circular Electron Positron Collider (CepC)  
Super Proton Proton Collider (SppC)



Compact Linear Collider (CLIC)



Earth Orbit Collider (EOC) 😊



Fabiola Gianotti [13]

## Planned future colliders<sup>[1, 2, 6, 12]</sup>

Parameter	LHC	HE-LHC	FCC	SppC	CLIC	CepC
Type	pp	pp	pp	pp	e-e+	e-e+
Length, km	<b>27</b>	27	100	54	48	54
Max beam energy, TeV	<b>7</b>	14	50	35	1.5	0.12
Magnetic rigidity, T.km	<b>23</b>	47	167	117	5	0.4
Stored energy / beam, MJ	<b>360</b>	1300	8400	6400	0.280	0.350

Magnetic rigidity of charged particles beam:

$$BR \approx 3.3 \cdot p$$

Where:

- $BR$  – magnetic rigidity [T.m]
- $B$  – magnetic flux density [T]
- $R$  – bending radius [m]
- $p$  – beam momentum [GeV/c]

High energy beams are difficult to deflect (due to high magnetic beam rigidity) and require high magnetic fields.

If HLC had to be built with 1 T magnets it would have had at least 23 km radius or would have required 144 km tunnel.

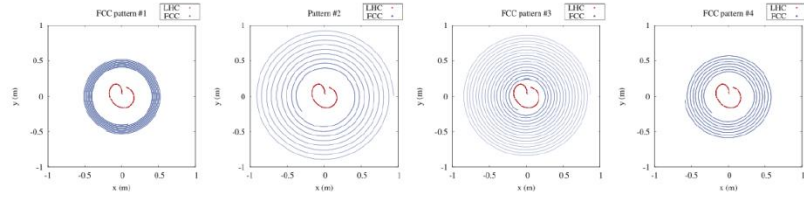
With 8.4 T (present design) bending radius is 2.74 km or minimum circumference (no straight sections ~17 km)

# Challenges of future colliders

- High beam energy** - high magnetic rigidity, high magnetic fields are required
- High beam intensity** – (together with high energy) large amount of beam stored energy
- High machine reliability** – a failure in beam steering could be pretty destructive
- High stability** – to maintain a stable particle beam

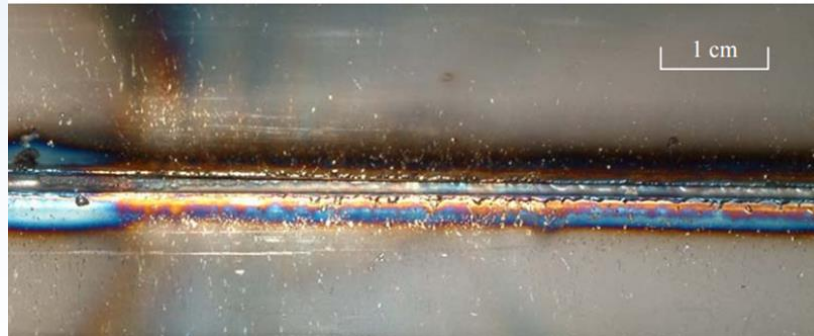
	MKB frequency modulation	Frequency	B·dl <sup>a)</sup>	Distance between neighbouring bunches	Distance between neighbouring branches
#1 <sup>b)</sup>	No	32.8 kHz	34 Tm	2.00–2.64 mm	1.6 cm
#2	No	32.8 kHz	56 Tm	1.87–4.70 mm	6.5 cm
#3 <sup>c)</sup>	No	50.9 kHz	53 Tm	1.83–6.95 mm	4.0 cm
#4 <sup>c)</sup>	Yes	20–43 kHz	39 Tm	1.90 mm	3.7 cm

<sup>a)</sup> For a dump line length of 2.5 km. <sup>b)</sup> See F. Burkart, FCC Dump Meeting, 02/07/2015. <sup>c)</sup> See F. Burkart, FCC Dump Meeting 02/12/2015.



Courtesy of W. Bartmann

Over 1 m length of SPS vacuum chamber destroyed after impact of 2 MJ proton beam (2004)<sup>[9]</sup>



Courtesy of B. Goddard

Different FCC dilution scenarios compared to the existing LHC one<sup>[7]</sup>

FCC stored beam energy 8.4 GJ  
 $\cong$   
 4.2x A320-200  
 @840 km/h



For comparison: energy of A320-200 (73000 kg) @ 840 km/h – 2 GJ



# Prepare for the future

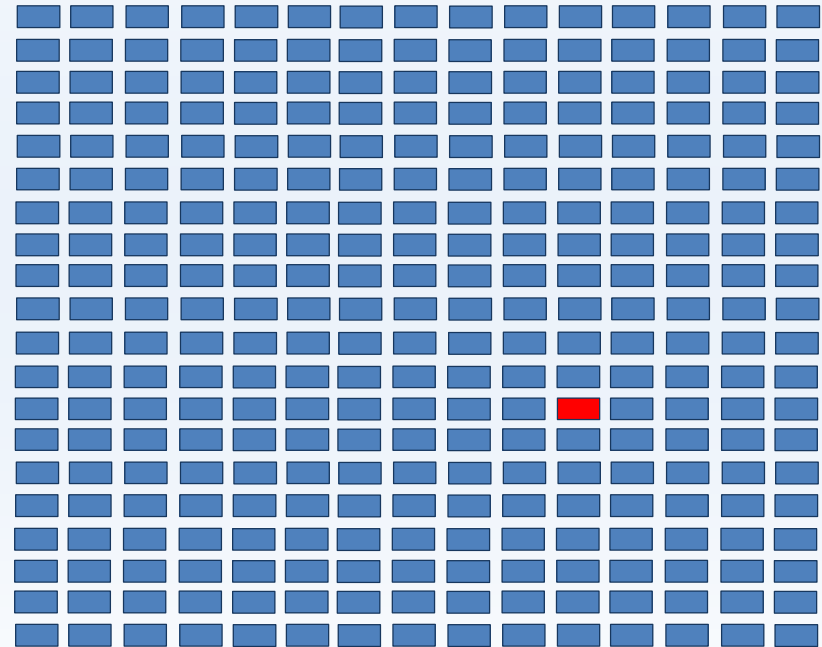
## Segmented beam dump kicker

FCC beam dump kicker <sup>[11, 3, 7, 5]</sup>:

- Extremely high energy stored in the beam 8.4 GJ. Even fraction of a bunch can cause damage.
- High magnetic rigidity 167 T.km (More than 7 times larger than the one of LHC)
- Follow the beam energy
- Fast reaction time
- High reliability (solid state switches)

Possible solution: Segmented dump kicker:

Number of devices:	300
Current:	7.5 kA
Voltage:	10 kV
Magnet length:	300 mm
Magnetic field:	0.25 T
Rise time:	1 $\mu$ s



Array of 300 elements (for illustrative purposes)

Single spontaneous fire will cause very small deflection ( $\sim 0.3\%$ ) - less than one sigma beam oscillation. Single failure will cause reduced kick by the same small amount – the beam still has to clear the septum.

## Solid state kicker drivers

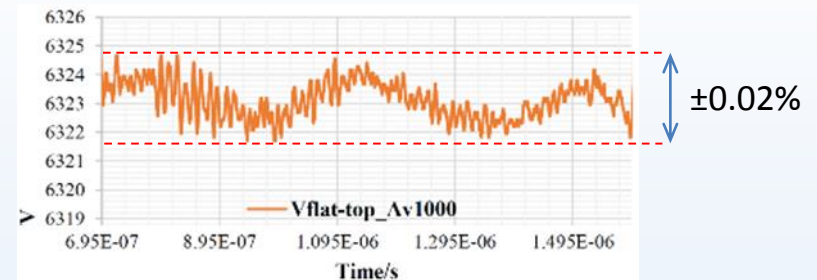
CLIC Damping Ring Kickers drivers<sup>[34]</sup>:

- Pulse-to-pulse repeatability better than  $1 \cdot 10^{-4}$
- Pulse flatness (droop and ripple) less than  $2 \cdot 10^{-4}$
- High speed (rise/fall time) 100 ns
- High reliability

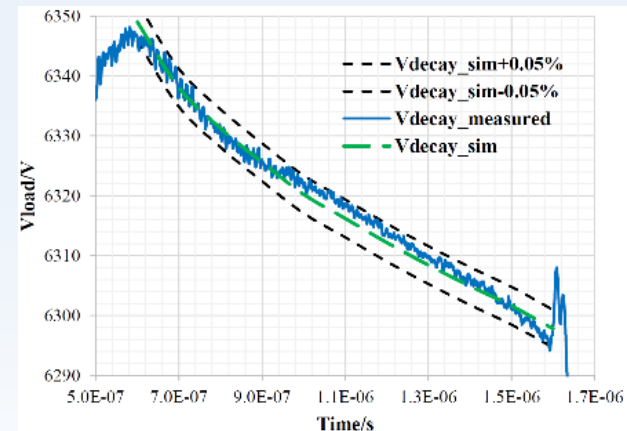


*Courtesy of J. Holma*

First full-scale, 20-layer, 12.5 kV CLIC DR extraction  
kicker inductive adder



Pulse flat-top measurement at 6.3 kV with active  
compensation of droop and ripple<sup>[34]</sup>



Controlled decay pulse waveform with active  
amplitude modulation<sup>[34]</sup>

- Flat-top repeatability at low voltage (475 V)  $\pm 0.0004\%$   
( $\pm 4 \cdot 10^{-6}$ ) measured with differential measurement  
method developed at PSI<sup>[34, 35]</sup>

## Superconducting septum

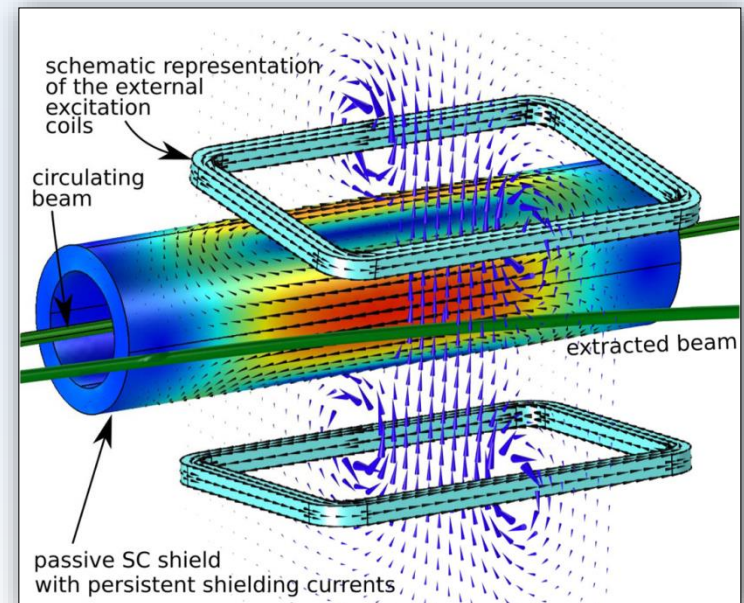
FCC beam extraction/dump septum<sup>[4]</sup>:

- High magnetic field
- Follow the beam energy (quasi-DC mode) - avoid excessive power consumption. Scaling in length LHC Lambertson technology (1 T) will require  $\sim 6$  MW
- Low leakage field in wide operating range
- Homogeneous deflecting field

Possible solution: Superconducting septum:

Septum thickness:	25 mm
Field homogeneity:	0.5...1%
Relative leakage field:	$1.25 \cdot 10^{-4}$
Magnetic field:	0 ... 3 T

If the magnet's shield transits to superconducting state in zero-field conditions it will act as a "perfect" magnetic screen. Any change of the external field will induce currents in the superconducting material that will prevent magnetic field penetration.



*Courtesy of D. Barna*

Schematic illustration of superconducting septum magnet.



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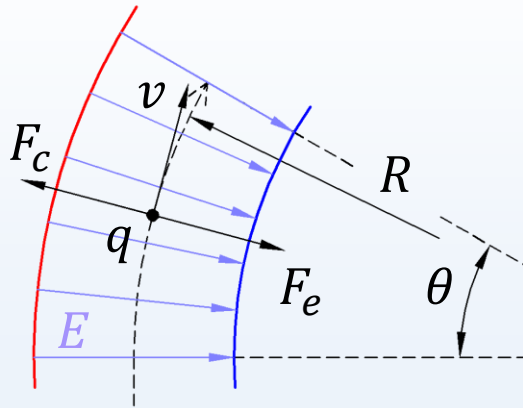
Special thanks to  
**Dr. Mike Barnes (CERN)**  
for providing very useful material

Thank you for your  
attention

## Literature

- [1] M. Mangano, "Physics at Future Circular Colliders", Future Colliders for Fresh Physicists, CERN, Switzerland, 2017
- [2] M. Dam, "Physics at Future Colliders 1", Summer Student Lecture Programme Course, CERN, Switzerland, 2017
- [3] M. Barnes, et al., "Kicker Systems -Part 2 -Hardware: Existing and Future", Beam Injection, Extraction and Transfer, CAS 2018, Erice, Italy, 2017
- [4] D. Barna, "High field septum magnet using a superconducting shield for the Future Circular Collider", Phys. Rev. Accel. Beams 20, 2017
- [5] T. Kramer, et al., "Considerations for the injection and extraction kicker systems of a 100 TeV centre-of-mass FCC-hh collider", IPAC 2016, Busan, Korea, 2016
- [6] F. Su, et al., "SPPC Parameter Choice and Lattice Design", IPAC 2016, Busan, Korea, 2016
- [7] W. Bartmann, et al., "FCC-hh Dump concepts", The second Annual Meeting of the Future Circular Collider study, Rome, Italy, 2016
- [8] J. Borburgh, "An introduction to HV cables", Group meeting TE-ABT, CERN, Switzerland, 2016
- [9] B. Goddard, "Surviving an Asynchronous Beam Dump?", The second Annual Meeting of the Future Circular Collider study, Rome, Italy, 2016
- [10] C. Aguilar, "Development of Stripline Kickers for Low Emittance Rings: Application to the Beam Extraction Kicker for CLIC Damping Rings", PhD Thesis, Spain, 2015
- [11] T. Kramer, et al., "Considerations for the Beam Dump System of a 100 TeV Centre-of-mass FCC hh Collider", IPAC 2015, Richmond, VA, USA, 2015
- [12] R. Schmidt, "Concept & architecture of the machine protection systems for FCC", First Annual Meeting of the Future Circular Collider study, USA, 2015
- [13] F. Gianotti, "Physics opportunities at future high-energy colliders", Invisibles15 Workshop, Madrid, Spain, 2015
- [14] C. Aguilar, et al., "Stripline design for the extraction kicker of Compact Linear Collider damping rings", Phys. Rev. ST Accel. Beams 17, 071003, 2014
- [15] B. Holzer, "Introduction to Transverse Beam Dynamics", CAS-CERN Accelerator School, Slovakia, 2012
- [16] A. Gabard, et al., "Radiation Hard Magnets at the Paul Scherrer Institute", IPAC 2012, pp 3518-3520, USA, 2012
- [17] R. Carey, "The MuLan Experiment", The 19th Particles and Nuclei International Conference, Cambridge, USA, 2011
- [18] M. Plum, "Injection and extraction single, turn injection", USPAS Class on Injection and Extraction, USA, 2009
- [19] M. Barnes, "Beam Transfer Devices Septa", CAS-CERN Accelerator School, Belgium, 2009
- [20] M. Barnes, et al., "Injection and extraction magnets: kicker magnets", CAS 2009: Specialised Course on Magnets, Bruges, Belgium, 2009
- [21] A. Mikhailichenko, "Fast kicker", Colliding Beam Notes 2009, Cornell University, USA, 2009
- [22] M. Reiser, "Theory and Design of Charged Particle Beams", Wiley, p. 18, 2008
- [23] J. Wenninger, "Introduction to Slow Extraction to the North Targets", Training for operation, CERN, Switzerland, 2007
- [24] C. Gough, "Minimizing Leakage Field from Eddy Current Septum Magnets", Presentation at SINAP 2005, 2005
- [25] M. Barnes, "A 25-kV 75-kHz Kicker for Measurement of Muon Lifetime", IEEE Transactions on Plasma Science, Vol. 32, Issue: 5, 2004
- [26] A. Afonin, et al., "Proton Extraction from IHEP Accelerator Using Bent Crystals", Int. WS "Relativistic. Channeling and Related Coherent Phenomena", Italy, 2004
- [27] J. Borburgh, et al., "Final results on the CERN PS electrostatic septa consolidation program", PAC 2003, USA, 2003
- [28] J. Borburgh, et al., "Consolidation project of the electrostatic septa in the CERN PS ring", PAC 2001, USA, 2001
- [29] C. Gough, et al., "Septum and Kicker Systems for the SLS" PAC 2001, USA, 2001
- [30] K. Wille, "The Physics of Particle Accelerators: An Introduction", Oxford University Press, p. 148, 2000
- [31] M. Barnes, "Design of the injection kicker magnet system for CERN's 14 TeV proton collider LHC", IPPC 95, Albuquerque, USA, 1995
- [32] S. Baker, et al., "First Operation with a Crystal Septum to Replace a Magnet in a Charged Particle Beam", NIM A234 (1985), pp 602-605, 1985
- [33] C. Bovet, et al., "A selection of formulae and data useful for the design of A.G. synchrotrons", CERN, Switzerland, 1970
- [34] J. Holma, "Inductive adders for the CLIC damping ring kickers", First Topical Workshop on Injection and Injection systems, Berlin, Germany, 2017
- [35] M. Paraliev, C. Gough, "Towards Sub-ppm Shot-to-shot Amplitude Stability of SwissFEL Resonant Kicker", 2016 IEEE PMHVC, San Francisco, CA, USA, 2016

## Additional material – Electrostatic deflection derivation



Derivation of electrostatic deflection of moving charged particle with velocity  $v$ , rest mass  $m_0$  and elementary charge  $q_e$ , in electric field  $E$  perpendicular to particle's direction of travel. The electric force  $F_e$  balances the centrifugal force  $F_c$  and the particle travels on an arc trajectory with radius  $R$ .

Bending angle  $\theta$  is found using bending radius  $R$  and the length of the field  $l$  in the limits of small angle approximation  $\tan(\theta) \approx \theta$

Where:

$F_c$  - centrifugal force [N]

$F_e$  - electrostatic force [N]

$E$  - electric field [V/m]

$R$  - bending radius [m]

$q_e$  - particle charge (elementary charge) [C]

$m_0$  - particle's rest mass [kg]

$p$  - beam momentum [kg.m/s] or [GeV/c]

$\gamma$  - relativistic gamma [-]

$\beta$  - relativistic beta [-]

$c$  - speed of light [m/s]

$l$  - length of the field [m]

$$F_e = q_e E, \quad F_c = \frac{\gamma m_0 v^2}{R}, \quad p = \gamma m_0 \beta c, \quad v = \beta c$$

and

$$p \left[ \frac{\text{kg} \cdot \text{m}}{\text{s}} \right] = \frac{10^9 q_e}{c} p \left[ \frac{\text{GeV}}{c} \right]$$

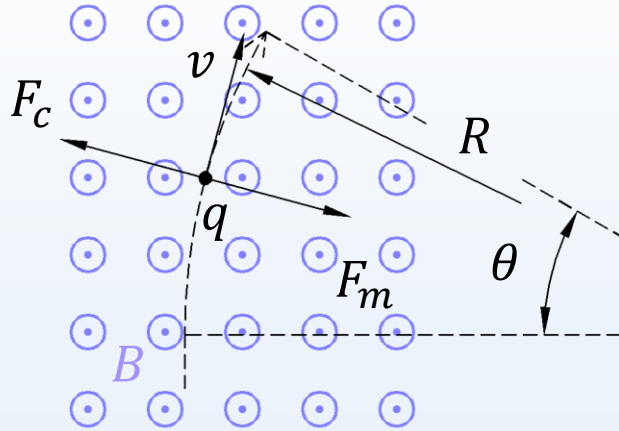
$\therefore$

$$R = \frac{\gamma m_0 v^2}{q_e E} = \frac{p \left[ \frac{\text{kg} \cdot \text{m}}{\text{s}} \right] \beta c}{q_e E} = \frac{10^9 p \left[ \frac{\text{GeV}}{c} \right] \beta}{E}$$

$\therefore$

$$\theta \approx \frac{l}{R} = \frac{El}{10^9 \cdot \beta \cdot p \left[ \frac{\text{GeV}}{c} \right]}$$

## Additional material – Magnetic deflection derivation



Where:

$F_c$  - centrifugal force [N]

$F_m$  - magnetic force [N]

$B$  - magnetic flux density [T]

$R$  - bending radius [m]

$q_e$  - particle charge (elementary charge) [C]

$m_0$  - particle's rest mass [kg]

$p$  - beam momentum [kg.m/s] or [GeV/c]

$\gamma$  - relativistic gamma [-]

$\beta$  - relativistic beta [-]

$c$  - speed of light [m/s]

$l$  - length of the field [m]

Derivation of magnetic deflection of moving charged particle with velocity  $v$ , rest mass  $m_0$  and elementary charge  $q_e$ , in magnetic field  $B$  going out of plane of paper. The magnetic force  $F_m$  balances the centrifugal force  $F_c$  and the particle travels on an arc trajectory with radius  $R$ .

Bending angle  $\theta$  is found using bending radius  $R$  and the length of the field  $l$  in the limits of small angle approximation  $\tan(\theta) \approx \theta$

$$F_m = q_e v B, \quad F_c = \frac{\gamma m_0 v^2}{R}, \quad p = \gamma m_0 \beta c, \quad v = \beta c$$

and

$$p \left[ \frac{\text{kg} \cdot \text{m}}{\text{s}} \right] = \frac{10^9 q_e}{c} p \left[ \frac{\text{GeV}}{c} \right]$$

$\therefore$

$$R = \frac{\gamma m_0 v}{q_e B} = \frac{p \left[ \frac{\text{kg} \cdot \text{m}}{\text{s}} \right]}{q_e B} = 3.3 \frac{p \left[ \frac{\text{GeV}}{c} \right]}{B}$$

$\therefore$

$$\theta \approx \frac{l}{R} = \frac{0.3 B l}{p \left[ \frac{\text{GeV}}{c} \right]}$$

## Additional material – Equivalence of electrostatic and magnetic deflection

Comparing capabilities of electric and magnetic field with same volumetric energy density (in vacuum) to deflect relativistic charged particles

Where:

$W_E$  - electric field energy density [J/m<sup>3</sup>]

$W_M$  - magnetic field energy density [J/m<sup>3</sup>]

$\epsilon_0$  - vacuum permittivity [F/m]

$\mu_0$  - vacuum permeability [H/m]

$q_e$  - particle charge (elementary charge) [C]

$E$  - electric field [V/m]

$B$  - magnetic flux density [T]

$c$  - speed of light [m/s]

$F_e$  - electric force [N]

$F_m$  - magnetic force [N]

$$\text{Electric field energy density } W_E = \frac{\epsilon_0 E^2}{2}$$

$$\text{Magnetic field energy density } W_M = \frac{B^2}{2\mu_0}$$

$$\text{For } W_E = W_M \text{ and } c^2 = \frac{1}{\epsilon_0 \mu_0} \therefore$$

$$\frac{\epsilon_0 E^2}{2} = \frac{B^2}{2\mu_0} \quad \text{or} \quad E = cB$$

$$\text{Magnetic (Lorentz) deflection force } F_m = q_e v B$$

$$\text{Relativistic regime } v \cong c \therefore$$

$$F_m = q_e v B \cong q_e c B = q_e E$$

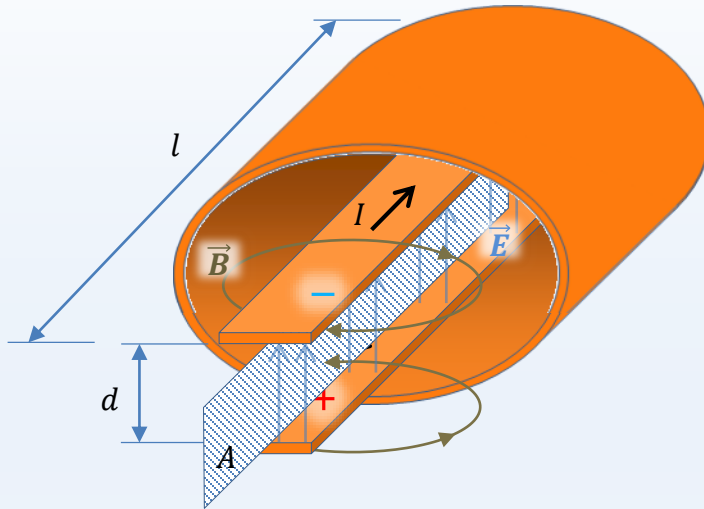
$$\text{Electrostatic (Lorentz) deflection force } F_e = q_e E$$

or

$$F_e \cong F_m$$

Electric and magnetic field, with same energy density, have same effectiveness in deflecting relativistic charged particles.

## Additional material – Equivalence of electric and magnetic deflection for relativistic particles in constrained vacuum transmission line



Where:

$E$  - electric field [V/m]

$l$  - transmission line length [m]

$\beta$  - relativistic factor ( $\beta = 1$ )

$p$  - beam momentum [GeV/c]

$U$  - voltage between electrodes [V]

$d$  - distance between electrodes [m]

$B$  - magnetic flux density [T]

$\Phi$  - total magnetic flux [Wb]

$c$  - speed of light ( $c \approx 3 \cdot 10^8$  m/s)

Electrostatic deflection

$$\theta_E \approx \frac{E \cdot l}{10^9 \cdot \beta \cdot p} \approx \frac{U \cdot l}{10^9 \cdot \beta \cdot p \cdot d}$$

Magnetic deflection

$$\theta_M \approx \frac{0.3 \cdot B \cdot l}{p}$$

In the middle of constrained transmission line:  $\vec{B} \approx const$  and crosses perpendicularly symmetry plane  $A$ , or total flux  $\Phi$  is:

$$\Phi \approx B \cdot d \cdot l$$

Assuming voltage step  $U$ , started at  $t = 0$  has just filled the line for time  $t = l/c$ ,  $\beta = 1$  and  $\Phi(0) = 0$ :

$$U = \frac{\Delta\Phi}{\Delta t} = \frac{\Phi}{t} = B \cdot d \cdot c$$

$$\theta_E \approx \frac{B \cdot d \cdot c \cdot l}{10^9 \cdot \beta \cdot p \cdot d} = \frac{B \cdot 3 \cdot 10^8 \cdot l}{10^9 \cdot 1 \cdot p} = \frac{0.3 \cdot B \cdot l}{p}$$

$$\theta_E \approx \theta_M$$